"Give me your Tired, your Poor," so I can Prosper: Immigration in Search Equilibrium

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Abstract

We analyze the impact of immigration on the host country within a search and matching model that allows for skill heterogeneity, endogenous skill acquisition, differential search cost between immigrants and natives, capital-skill complementarity and different degree of substitutability between unskilled natives and immigrants. Within such a framework, we find that although immigration raises the overall welfare, it may have distributional effects. Specifically, skilled workers gain in terms of both employment and wages. Unskilled workers, on the other hand, gain in terms of employment but may lose in terms of wages. Nevertheless, in one version of the model, where unskilled workers and immigrants are imperfect substitutes, we find that even the unskilled wage may rise. These results accommodate conflicting empirical findings.

Keywords: Search; Unemployment; Immigration; Skill-heterogeneity

JEL Classification: F22; J61; J64

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"Give me your tired, your poor,
Your huddled masses yearning to breathe free,
The wretched refuse of your teeming shore.
Send these, the homeless, tempest-tost to me,
I lift my lamp beside the golden door!"

From The New Colossus
By Emma Lazarus

(The poem appears on an engraved plaque by the Statue of Liberty).

1 Introduction

The impact of immigration on the labor market outcomes in the host country has long been a subject of debate among economists. The standard model of a competitive labor market offers a clear and intuitive prediction: immigration should lower the marginal product and thus the wage of natives that compete with immigrants; it should also raise the marginal product and the wage of natives whose labor is complementary to that of immigrant workers. Nevertheless, the results provided by a large number of careful empirical studies on this subject are often contradictory. For example, Borjas (2003) and Borjas et al. (2010) find a large negative wage effect on natives, whereas Card (2009) and Ottaviano and Peri (2010) find this effect to be relatively small, suggesting that other features, not captured by the standard competitive model, may be responsible for "counter-balancing" effects.[1]

Given that the literature has not yet reached a consensus, in this paper we revisit the question of the impact of immigration, but adopt a different approach. We conduct our analysis within a model that belongs to the general family of search and matching models of the labor market (e.g., Diamond, 1982 and Mortensen and Pissarides, 1994). In this class of models unemployment exists due to search frictions and job entry responds endogenously to the incentives provided by the market. Thus, contrary to the competitive paradigm, our approach allows for the analysis of the unemployment and wage effects that come from the impact of changes in the availability of jobs on the bargaining position of workers. An additional advantage of our model is that it incorporates features that can provide theoretical rationale for both positive and negative effects of immigration. Thus,

[1] See also Ortega and Verdugo (2010) for positive impact of immigration on the wages of competing natives and Kugler and Yuksel (2008) for recent evidence that immigrants have no wage or employment effect on natives.

it encompasses both views found in the literature and allows us to study the impact of immigration in a more systematic way.

Aside from explicitly taking into account the effects that come from the impact of immigration on job creation, our model has the following key features. First, the presence of differential search costs between natives and immigrants, besides adding further realism to the model, is a key factor in explaining the equilibrium wage gap between otherwise identical native and immigrant workers. This feature allows for the possibility that immigration improves the employment and wage prospects of competing natives. This is so because an immigration influx lowers the wages firms need to pay on average, leading to more job entry. Second, we allow for heterogeneity in terms of skills among native workers as well as between natives and immigrants. This allows us to analyze distributional effects of immigration on different skill groups. Third, we consider endogenous skill acquisition on behalf of native workers, which gives them the opportunity to react to the negative pressure of immigration on wages and unemployment. Fourth, the presence of capital as an independent factor of production serves as an additional channel of adjustment to immigration-induced changes in labor supply. Finally, our model adopts a generalized production technology that allows for the analysis of the impact of immigration under different assumptions regarding the degrees of capital-skill and across-skill complementarity.

We calibrate the model to the US economy and find that the impact of an increase in the number of unskilled immigrants on the overall welfare of natives is positive. As expected, it lowers the unemployment and raises the wage rate of skilled native workers, mainly because it encourages skilled job entry by raising the marginal product of skilled labor. However, we also find that it encourages unskilled job entry, leading to a smaller unemployment rate for unskilled workers as well. The increase in unskilled job entry is due to firms anticipating that, with a higher number of immigrants searching for jobs, they will have to pay lower wages on average. As regards the wage of unskilled native workers, on the one hand, the higher availability of unskilled jobs strengthens their bargaining position and pushes their wage up. On the other hand, the fall in their marginal product due to higher competition from immigrants causes their wage to fall. In our baseline calibration we let unskilled immigrants and natives be perfect substitutes in production and find the overall impact on the wage of unskilled natives to be negative. However, once we allow for a lower degree of substitutability between natives and immigrants we find the impact on unskilled natives to be positive not only in terms of unemployment but also in terms of wages.

We also compare the results under the assumption that the proportion of skilled native workers is fixed, to those obtained when the proportion of skilled natives responds endogenously to immigration-induced changes in the relative supply of skills. This comparison allows us to make an inference regarding the short-run versus the long-run effects of immigration. We find that the presence of endogenous skill acquisition has a positive and significant impact on the overall welfare of natives, mainly because it improves the impact of unskilled immigration on unskilled natives. Specifically, due to intensified competition from unskilled immigrants, a higher share of the native population chooses to become skilled. This acts to mitigate the negative competition effect on the productivity of unskilled workers, but also to lower the positive impact on the productivity of skilled workers, thereby improving the impact on the former, but worsening that on the latter. This result suggests that, compared to the short-run, the long-run effects of unskilled immigration tend to be less negative to competing and less positive to complementary native workers.

Although there is a vast empirical literature on this topic, the number of studies that analyze immigration within theoretical frameworks is small. Most of the earlier dynamic theoretical studies employ the standard neoclassical growth model; examples include, but are not limited to, Hazari and Sgro (2003), Ben-Gad (2004), Moy and Yip (2006) and Ben-Gad (2008). To the best of our knowledge, the only other papers that analyze immigration within frameworks that allow for labor market search frictions are those of Ortega (2000) and Liu (2010). The former considers a two-country model where workers decide whether to search in their own country or immigrate. He shows that Pareto-ranked multiple steady-state equilibria may arise with or without immigration. Ortega's analysis also takes into account the positive impact of immigration on job entry due to firms anticipating that they will pay lower wages to immigrants that have higher search costs. However, the model in Ortega assumes that worker productivity is constant and therefore independent of immigration influx. Moreover, in his framework there is only one labor type. Thus, his analysis overlooks both the negative competitive effects on the marginal product of native workers and the across-skill externalities that arise when otherwise identical natives and immigrants compete for the same types of jobs.

Liu (2010) concentrates on the welfare effects of illegal immigration within a dynamic general equilibrium model with search frictions. The presence of search frictions allows him to identify a new channel through which immigration can alter domestic consumption: intensified job competition from illegal immigrants lowers the job finding rate of native workers and forces them to accept lower wages. Our model is closer to an extended version of his baseline model, where there are two types of domestic labor in constant numbers, namely skilled and unskilled, and illegal immigrants belong to the unskilled group. In this extended model, illegal immigration has a positive impact on skilled, but a negative impact on unskilled natives, both in terms of employment and wages, mainly because it raises the marginal product of the former group and lowers that of the latter.

As regards the production technology, the main difference between our model and Liu's extended model is that we employ a nested CES aggregator that allows for skilled labor to be more complimentary to capital than unskilled labor, whereas Liu assumes a Cobb-Douglas production function, which implies that the types of labor are equally complementary to capital. Furthermore, Liu's extended model assumes that immigrants and natives are perfect substitutes in production, while we also explore the case of imperfect substitutability between the two labor types. Our assumptions regarding the production technology are closer to those of Ben-Gad (2008) who analyzes a neo-classical growth model with overlapping dynasties and two types of labor, but does not allow for search frictions. He shows that, because of capital-skill complementarity, skilled immigration is far more beneficial to the economy than unskilled.

The rest of the paper is organized as follows. Section 2 presents the baseline model. Section 3 defines the steady-state equilibrium and analyzes its existence and uniqueness. In Section 4, we analyze two special cases of the model. In the first, we assume that there are no differences in search costs between otherwise identical native and immigrant workers. In the second, we assume differential search costs, but let the two labor inputs (skilled and unskilled) be perfect substitutes to each other. Considering these two cases separately allows us to identify two different channels through which immigration can affect labor market outcomes: one that comes from the impact on firms' expected cost of establishing an employment relationship and one that comes from the impact on the prices of labor inputs. In Section 5 we calibrate the model and present simulation results in the general case when both of these channels are present. In Section 6 we examine the sensitivity of our results to alternative parameterizations of the production function. We also consider the behavior of an extended version of the model where natives and immigrants of the same skill type are assumed to be imperfect substitutes. Finally, Section 7 offers some concluding remarks.

2 The Model

We construct a search and matching model with two intermediate inputs and one final consumption good. Time is continuous and begins at $t = 0$. The economy is populated by a continuum of workers and a continuum of jobs. Workers are either natives (N) or immigrants (I). The mass of natives is normalized to unity, while that of immigrants is denoted by I and is determined exogenously. The mass of jobs, on the other hand, is determined endogenously as part of the equilibrium. All agents are risk neutral and discount the future at a common rate $r > 0$, which is equal to the interest rate. The rest of this section offers a detailed description of the model; see also Figure 1 for a graphic presentation of its basic structure.

2.1 Workers and Firms

Native workers are either skilled (H) or unskilled (L). Investment in human capital/skill is a discrete choice. Before entering the labor market each agent decides whether to invest in education and become skilled or remain unskilled.[2] Native young agents differ with respect to their ability to learn, which in turn determines their cost of acquiring education. Older agents, on the other hand, face an additional cost, which is prohibitive. Thus, older workers never opt for training.

Let the cost of acquiring training be denoted by z and assume that it is distributed uniformly over the closed interval $[0, \overline{z}]$. A native young agent j will invest in education if the benefit from this decision exceeds the cost. More specifically, let J^U_{HN} and J^U_{LN} denote the discounted values associated with the state where a native unemployed worker is skilled and unskilled, respectively. Then a native young worker j will invest in education if

$$J^U_{HN} - J^U_{LN} > z^j.$$

Thus, all agents with a cost of education lower than z^* will invest in education, where z^* is given by

$$z^* = J^U_{HN} - J^U_{LN}.$$

[2] We use the terms skilled (unskilled) workers and high- (low-) skill workers interchangeably.

Let λ be the fraction of the native workers that are unskilled and $1-\lambda$ the fraction of those that are skilled. Then, in equilibrium the proportion of the skilled natives is given by

$$1-\lambda^{*}=\frac{z^{*}}{\overline{z}}. \tag{1}$$

Immigrants, on the other hand, can be either skilled or unskilled, but their numbers, denoted by I_H and I_L, respectively, are determined exogenously. Also, all workers are born and die at the rate n.

Our production side borrows some elements from Acemoglu (2001). Firms operate either in one of the two intermediate sectors or in the final sector. The two intermediate sectors produce inputs Y_H and Y_L using skilled and unskilled labor, respectively. More specifically, each of these two sectors operates a linear technology, which, through normalization of units, can be written as $Y_i = i$, $i = H, L$, that is, the output of an intermediate input is equal to the number of workers employed. These intermediate inputs are non-storable. Once produced, they are sold in competitive markets and are immediately used for the production of the final good (Y).

Next we turn to the final good sector. Motivated by a series of empirical papers (see, among others, Griliches 1969 and Krusell et al. 2000), which support the idea that skilled labor is relatively more complimentary to capital than unskilled labor, we post the following production technology for the final good

$$Y=[\alpha Y_L^{\rho}+(1-\alpha)Q^{\rho}]^{1/\rho}, \qquad \rho\leq 1, \tag{2}$$

with

$$Q=[xK^{\gamma}+(1-x)Y_H^{\gamma}]^{1/\gamma}, \qquad \gamma\leq 1, \tag{3}$$

where K denotes capital, a and x are positive parameters that govern income shares and ρ and γ drive the elasticities of substitution between capital and the unskilled input and capital and the skilled input, respectively. Thus, the production function is a two-level CES function in which capital (K) and the skilled input (Y_H) are nested together in the sub-aggregate input Q given by equation (3) and then Q and the unskilled input (Y_L) enter the main production function (equation 2). Capital-skill complementarity is defined as $\rho > \gamma$, which implies that an increase in the capital stock raises the skill premium (see, among others, Krusell et al. 2000 and Polgreen and Silos 2008). If either γ or ρ equals zero, then the corresponding nesting is Cobb-Douglas.

Since the two intermediate inputs are sold in competitive markets, their prices, p_L and p_H, will be equal to their marginal products, that is

$$p_L = \alpha Y_L^{\rho-1} Y^{1-\rho}, \tag{4}$$

and

$$p_H = (1-\alpha)(1-x) Y_H^{\gamma-1} Q^{\rho-\gamma} Y^{1-\rho}. \tag{5}$$

We assume that there exists a competitive capital market in which each firm can buy and sell capital without delay. Hence, only filled vacancies will buy capital. In addition, since the market is competitive, the marginal product of capital is equal to its rental price (p_K), which is in turn equal to the interest rate (r) plus the depreciation rate (δ). Thus,

$$p_K = (1-\alpha) x K^{\gamma-1} Q^{\rho-\gamma} Y^{1-\rho} = r + \delta. \tag{6}$$

2.2 Search and Matching

From now on we dispense with the Walrasian auctioneer and assume that in each of the two labor markets unemployed workers and unfilled vacancies are brought together via a stochastic matching technology $M(U_i, V_i)$, where U_i and V_i denote respectively the number of unemployed workers and vacancies of skill type i. This function $M(.)$ exhibits standard properties: it is at least twice continuously differentiable, increasing in its arguments, exhibits constant returns to scale and satisfies the familiar Inada conditions. Using the property of constant returns to scale, we can write the flow rate of a match for a worker as $M(U_i, V_i)/U_i = m(\theta_i)$ and the flow rate of a match for a vacancy as $M(U_i, V_i)/V_i = q(\theta_i)$, where $\theta_i = V_i/U_i = m(\theta_i)/q(\theta_i)$ is an indicator of the tightness prevailing in labor market i. Also, the above-mentioned assumptions on M imply the following properties for $m(.)$ and $q(.)$:

$$m'(\theta_i) > 0, \quad \lim_{\theta_i \to 0} m(\theta_i) = 0, \quad \lim_{\theta_i \to \infty} m(\theta_i) = \infty, \tag{7}$$

$$q'(\theta_i) < 0, \quad \lim_{\theta_i \to 0} q(\theta_i) = \infty, \quad and \quad \lim_{\theta_i \to \infty} q(\theta_i) = 0. \tag{8}$$

Firms post either high-skill vacancies, which are suited for skilled workers, or low-skill vacancies, which are suited for unskilled workers. Each firm posts at most one vacancy and the number of firms of each type is determined endogenously by free entry. Firms can choose to open either skilled or unskilled vacancies, but cannot ex-ante open vacancies suited only for natives or immigrants. A vacant firm bears a cost c_i, $i = H, L$, specific to its type. On the other hand, an unemployed worker of type i receives a flow of income b_i, which can be consider as the opportunity cost of employment. There is no cross-skill matching. High skill workers direct their search towards the high-skill sector and low-skill workers towards the low-skill sector. Also, for simplicity, we assume that creating a vacancy is costless, although this can be easily amended following, for example, Acemoglu (2001) and Laing et al. (2003).

The instant a vacancy and a worker make contact, they bargain over the division of any surplus. The skill level of the worker as well as the output that will result from a match is known to both parties. We assume that wages are determined by an asymmetric Nash bargaining, where the worker has bargaining power β. After an agreement has been reached, production commences immediately. If at any point in time an employee dies, firms re-enter the labor market and search for a new employee, assuming that they find it profitable to do so. Moreover, we assume that matches dissolve at the rate s_i, specific to the type of the worker. Following a separation, the worker and the vacancy enter the corresponding market and search for new trading partners.

In addition, unemployed workers are subject to a per unit of time "search" cost, h_{ij}, which is specific to the worker's origin $j = N, I$, where N denotes "native" and I denotes "immigrant." We assume that immigrants have a search cost that is at least as high as that of natives; that is, $h_{iI} \geq h_{iN}$. There are several reasons why an immigrant may face a higher search cost or equivalently a lower income while being unemployed searching for a job. In addition to the problems that one may encounter if being in a foreign country (e.g., lack of a social network, lower language proficiency, etc.) lower income may result if immigrants do not qualify for the same unemployment insurance benefits as the natives.[3] Without loss of generality we assume that $h_{iN} = 0$, $i = H, L$. Moreover, in what follows we concentrate our attention to the impact of changes in the number of unskilled immigrants (I_L), while keeping the number of skilled immigrants (I_H) fixed. Without loss of generality we therefore also set $h_{HI} = 0$.

[3]Illegal immigrants are often not eligible for any unemployment insurance benefits. Also, in the U.S., for example, legal immigrants qualify for unemployment insurance benefits that are covered by the state governments and last for 26 weeks. Nevertheless, not all of them qualify for benefits, covered by the federal government, that extend beyond the 26-week period and are paid during times of recession (see, for example, NELP 2002).

2.3 Asset Value Functions

At any point in time a worker is either employed (E) or unemployed (U). Likewise a vacancy is either filled (F) or else is looking for a worker (V). We denote the discounted value associated with each state by J_{ij}^{κ}, where once again the subscript $i = H, L$ denotes the skill type (high- or low-skill), the subscript $j = N, I$ denotes the origin (native or immigrant), and the superscript $\kappa = V, U, F, E$, indicates the state (vacant firm, unemployed worker, filled job, employed worker). Then in steady state:

$$
\begin{aligned}
rJ_i^V &= -c_i + q(\theta_i)\left[\phi_i J_{iN}^F + (1-\phi_i)J_{iI}^F - J_i^V\right], &(9)\\
rJ_{ij}^F &= p_i - w_{ij} - (s_i + n)\left[J_{ij}^F - J_i^V\right], &(10)\\
(r+n)J_{ij}^U &= b_i - h_{ij} + m(\theta_i)\left[J_{ij}^E - J_{ij}^U\right], &(11)\\
(r+n)J_{ij}^E &= w_{ij} - s_i\left[J_{ij}^E - J_{ij}^U\right], &(12)
\end{aligned}
$$

where ϕ_i is the fraction of workers of skill type i that are natives and $h_{ij} = 0$ if $i = H$ or $j = N$. Also, w_{ij} denotes the wage rate for a worker of skill type $i = H, L$ and origin $j = N, I$. Expressions such as these have, by now, a familiar interpretation. For instance consider J_i^V. The term rJ_i^V is the flow value accrued to an unmatched vacancy of type i: it equals the loss from maintaining a vacant position plus the flow probability of becoming matched with a worker of the same type multiplied by the expected capital gain from such an event. The other asset value equations posses similar interpretation.

As there is free entry and exit on the firm side in each intermediate input market, an additional vacancy of skill type i should make expected net profit equal to zero, that is

$$J_i^V = 0. \tag{13}$$

2.4 Nash Bargaining

Since all workers and firms are risk neutral, Nash bargaining implies that the wage rate for a worker of skill type i and origin j, w_{ij}, must be such that:

$$(1-\beta)(J_{ij}^E - J_{ij}^U) = \beta(J_{ij}^F - J_i^V). \tag{14}$$

Equation (14) implies that firms get a share $1-\beta$ and workers get β of the total surplus S_{ij} generated by a match, where

$$S_{ij} = J_{ij}^F + J_{ij}^E - J_{ij}^U - J_i^V.$$

Hence,

$$J_{ij}^F - J_i^V = (1-\beta)S_{ij}, \tag{15}$$

$$J_{ij}^E - J_{ij}^U = \beta S_{ij}. \tag{16}$$

2.5 Steady-State Composition of the Labor Force

Recall that I_H and I_L denote the number of skilled and unskilled immigrants. Thus, the total number of skilled and unskilled workers in the economy is $1-\lambda+I_H$ and $\lambda+I_L$, respectively. Next by equating the flows out of unemployment to the sum of break ups and new births, we can find the steady-state employment, and hence the production of each intermediate input (see the Appendix for the details):

$$Y_H = \frac{m(\theta_H)(1-\lambda+I_H)}{n+s_H+m(\theta_H)}, \tag{17}$$

$$Y_L = \frac{m(\theta_L)(\lambda+I_L)}{n+s_L+m(\theta_L)}. \tag{18}$$

Similarly, the steady-state unemployment U_{ij} of each type $i = H, L$ and origin $j = N, I$ is given by:

$$U_{HN} = \frac{(n+s_H)(1-\lambda)}{n+s_H+m(\theta_H)}, \tag{19}$$

$$U_{LN} = \frac{(n+s_L)\lambda}{n+s_L+m(\theta_L)}, \tag{20}$$

$$U_{HI} = \frac{(n+s_H)I_H}{n+s_H+m(\theta_H)}, \tag{21}$$

$$U_{LI} = \frac{(n+s_L)I_L}{n+s_L+m(\theta_L)}, \tag{22}$$

$$U_H = U_{HN}+U_{HI} = \frac{(n+s_H)(1-\lambda+I_H)}{n+s_H+m(\theta_H)} \tag{23}$$

$$U_L = U_{LN}+U_{LI} = \frac{(n+s_L)(\lambda+I_L)}{n+s_L+m(\theta_L)}, \tag{24}$$

Moreover, as mentioned above, the probability that a type-i and unemployed worker is native is ϕ_i, and is given by

$$\phi_H = \frac{U_{HN}}{U_H} = \frac{1-\lambda}{1-\lambda+I_H}. \tag{25}$$

$$\phi_L = \frac{U_{LN}}{U_L} = \frac{\lambda}{\lambda+I_L}. \tag{26}$$

3 Steady-State Equilibrium

Consider next the definition of a steady-state equilibrium for this economy.

Definition. A steady-state equilibrium is a set $\{\theta_H^*, \theta_L^*, \lambda^*, p_H^*, p_L^*, p_K^*, w_{HN}^*, w_{LN}^*, w_{LI}^*, w_{HI}^*, Y_H^*, Y_L^*, K^*, U_{HN}^*, U_{LN}^*, U_{HI}^*, U_{LI}^*\}$ such that

(i) Individuals decide optimally whether to invest in training or not by setting the cost of training equal to its benefit (equation 1).

(ii) The intermediate input markets clear. In particular, conditions (4) and (5) are satisfied.

(iii) The capital market clears; i.e., condition (6) is satisfied.

(iv) The free entry condition (13) for each skill type i is satisfied.

(v) The Nash bargaining optimality condition (14) for each skill type i and origin j holds.

(vi) The numbers of employed and unemployed workers as well as of filled and unfilled vacancies of each type and origin remain constant; i.e., among others, conditions (17)-(24) are satisfied.

As shown in the Appendix, the steady-state equilibrium values of θ_H, θ_L, λ, are given by the following reduced system of equations:

$$\alpha\left\{\alpha+(1-\alpha)\left(\frac{A_H}{A_L\Lambda}\right)^{\rho}\left[xk^{\gamma}+(1-x)\right]^{\frac{\rho}{\gamma}}\right\}^{\frac{1-\rho}{\rho}}=B_L, \tag{27}$$

$$(1-\alpha)\left(1-x\right)\left[xk^{\gamma}+(1-x)\right]^{\frac{1-\gamma}{\gamma}}\left\{\alpha\left(\frac{A_L\Lambda}{A_H}\right)^{\rho}\left[xk^{\gamma}+(1-x)\right]^{-\frac{\rho}{\gamma}}+(1-\alpha)\right\}^{\frac{1-\rho}{\rho}}=B_H, \tag{28}$$

and

$$(1-\lambda)\overline{z}=\frac{1}{r+n}\left\{\frac{\beta m(\theta_H)p_H+(r+n+s_H)b_H}{r+n+s_H+\beta m(\theta_H)}-\frac{\beta m(\theta_L)p_L+(r+n+s_L)b_L}{r+n+s_L+\beta m(\theta_L)}\right\}, \tag{29}$$

where

$$A_i\equiv\frac{m(\theta_i)}{n+s_i+m(\theta_i)},\quad \Lambda\equiv\frac{\lambda+I_L}{1-\lambda+I_H},\quad k\equiv\frac{K}{Y_H}=(B_H)^{\frac{1}{1-\gamma}}\left[\frac{x}{(1-x)(r+\delta)}\right]^{\frac{1}{1-\gamma}},$$

and

$$B_i\equiv b_i-(1-\phi_i)h_{iI}+\frac{c_i[r+n+s_i+\beta m(\theta_i)]}{(1-\beta)q(\theta_i)}.$$

Each of equations (27) and (28) is a zero expected profit condition in the unskilled and skilled input market, respectively. The left-hand-side, which equals the price of input i, p_i, $i = L, H$, gives the revenue to the firm from employing a worker of skill type i, while the right-hand-side gives the expected cost to the firm from establishing an employment relationship. This includes the recruiting cost and the cost of being matched with a worker of skill type i. We will refer to this in short as employment cost. Also, equation (29) sets the cost to the last worker who receives training equal to the present value of the benefit from a such a decision. Obviously, all workers with a cost lower than the one given by the left-hand side of (29) invest in education. Having determined θ_H^*, θ_L^*, λ^*, we can get the equilibrium values for the other variables by substituting in the appropriate equations. In particular, the unemployment rates among the skilled and unskilled natives, and skilled and unskilled immigrants, $u_{HN}^*, u_{LN}^*, u_{HI}^*$ and u_{LI}^*, follow from equations (19) to (24)

$$u_{HN} = u_{HI} = u_H = \frac{(n + s_H)}{n + s_H + m(\theta_H)} \quad \text{and} \quad u_{LN} = u_{LI} = u_L = \frac{(n + s_L)}{n + s_L + m(\theta_L)}. \tag{30}$$

Finally, the wage rates w_{HN}^*, w_{LN}^*, w_{LI}^* and w_{HI}^* are given by (see the derivation of equation A8 in the Appendix)

$$w_{ij} = \frac{[r + n + s_i + m(\theta_i)]\beta p_i + (r + n + s_i)(1 - \beta)(b_i - h_{ij})}{r + n + s_i + \beta m(\theta_i)}. \tag{31}$$

Proposition 1. If there is no search cost, i.e., $h_{LI} = 0$ then, under certain parameter restrictions confined in the Appendix, a steady-state equilibrium exists and is unique.

Proof. All formal proofs are presented in the Appendix.

The essence of Proposition 1 can be captured with the help of Figure 2. The equilibrium values of θ_H and θ_L are given by the intersection of the two curves labeled as EP and OH. The EP curve results after combining equations (27) and (28) (it is described by equation A13 given in the Appendix). This curve comprises the set of values of θ_H and θ_L that yield *equal profit* and make firms indifferent between establishing a high-skill and a low-skill vacancy. It has a negative slope since an increase in θ_H will increase the employment cost B_H and thus will decrease the ratio (Y_H/Y_L), in order to restore the relation between p_H and B_H. The decrease in (Y_H/Y_L) will decrease the marginal product of unskilled labor

p_L. To offset this, there must be a decrease in the cost of establishing a low-skill vacancy B_L, which requires a decrease in θ_L.[4]

The curve OH, on the other hand, is the geometric locus of values of θ_H and θ_L that make the expected profit from establishing a high-skill vacancy equal to zero (it is described by equation 28).[5] It has a positive slope because an increase in θ_H leads to a higher employment cost (B_H) and a lower price (p_H) in the skilled sector. Hence, there must be an increase in θ_L, which will raise the price of the high-skill input (p_H) and restore the zero-profit condition $p_H = B_H$.

The result in Proposition 1 holds also in the case where unskilled immigrants face a search cost ($h_{LI} > 0$), but the proportion of skilled workers $(1 - \lambda)$ is exogenously determined.[6] Nevertheless, if there is a search cost and endogenous skill accumulation, then an increase in tightness in the high-skill sector θ_H affects also the employment cost in the low-skill sector B_L through its effect on the proportion of skilled workers $(1 - \lambda)$ and hence on the probability that an unskilled and unemployed worker is native (ϕ_L)(see the expression for B_L given above). Consequently, in this more general case, while the locus OH in Figure 2 remains unchanged, the locus EP may not be monotonically decreasing any more. Thus, even if an equilibrium exists, it may not be unique. However, we are able to show that

Proposition 2. If the two intermediate inputs are perfect substitutes, i.e., $\rho = 1$, then a steady-state equilibrium exists and is unique even in the case where the proportion of skilled workers $(1 - \lambda)$ is endogenous and immigrants face a search costs ($h_{LI} > 0$).

With the two inputs being perfect substitutes, their marginal products and hence prices do not depend on the ratio Y_H/Y_L. This makes the curve EP in Figure 2 disappear. The equilibrium can instead be presented using the zero profit condition for each of the two intermediate sectors. More specifically, since the marginal product of the high-skill sector is independent of Y_L, the curve OH in Figure 2 becomes vertical on the horizontal axis; this curve is relabeled HH and is shown in Figure 3. On the other hand, the zero profit condition for the low-skill sector, depicted by curve $L_\lambda L_\lambda$ in Figure 3, involves both θ_L and

[4]In general the curvature of the EP locus cannot be determined; we draw it as a straight line for simplicity.

[5]Note that we could have used instead the curve along which the expected profit of establishing a *low-skill* vacancy is zero, as described by equation (27).

[6]The proof is essentially the same as that of Proposition 1, since the curves EP and OH in Figure 2 preserve all of their relevant properties.

θ_H; an increase in the matching rate of the high-skill workers $m(\theta_H)$, due to an increase in θ_H lowers the number of unskilled native workers λ and increases the probability that an unskilled and unemployed workers is immigrant $(1-\phi_L)$. With $h_{LI} > 0$, this lowers the cost of establishing a low-skill job B_L. Hence, to restore the zero profit condition, θ_L must rise. That is why the curve $L_\lambda L_\lambda$ is drawn with a positive slope. Notice, however, that if λ is exogenously given then the zero profit condition for the low-skill sector becomes independent of θ_H. This case is represented by curve LL in Figure 3.

4 Comparative Static Resutls

In general a change in the number of unskilled immigrants (I_L), can influence the equilibrium through the impact of such a change on i) prices p_i and ii) expected employment cost B_L. Before analyzing the equilibrium in the general case, where a change in I_L is propagated through both of these channels, it is instructive to examine each case separately. Specifically, we analyze two special cases: first, we set $h_{LI} = 0$, so that there is no difference anymore between an unskilled native and immigrant worker. In other words, this assumption implies that $w_{ij} = w_i$ for each j and hence a firm is indifferent between hiring an immigrant and a native worker. In this case, a change in I_L has no impact on B_L; thus, it influences the equilibrium only through its impact on prices. The second special case that we analyze below is the one where $h_{LI} > 0$ but the two intermediate inputs are perfect substitutes $(\rho = 1)$. In this case prices are independent of I_L. Therefore, a change in I_L can affect the labor market outcomes only through its impact on B_L. Naturally, it follows that

Proposition 3. If there is no search cost $(h_{LI} = 0)$ and the two intermediate inputs are perfect substitutes $(\rho = 1)$, then the equilibrium is independent of the number of immigrants.

If $h_{LI} = 0$ and $\rho = 1$, then the equilibrium is given by the intersection of a curve that is parallel to the horizontal axis (such as LL in Figure 3) and describes the zero-profit condition in the low-skill sector, and a curve that is vertical to the horizontal axis and describes the zero-profit condition in the high-skill sector (see curve HH in Figure 3). Moreover, both curves are independent of the number of immigrants.

4.1 Imperfect Substitutes and no Search Costs

Consider next the case where $\rho < 1$ and the search cost h_{LI} is equal to zero. As mentioned above, the latter assumption implies that there is no difference between an unskilled native worker and an immigrant and in particular $w_{ij} = w_i \ \forall j$. We begin with the case where the fraction $1 - \lambda$ of the native workers that are skilled is exogenously given and time invariant. The equilibrium is then described by the system of equations (27) and (28), with λ and hence Λ being constant. Also, as shown in the Appendix, equation (31), which gives the wage rates for each group, simplifies to

$$w_i = b_i + \frac{\beta}{1-\beta}\frac{c_i}{q(\theta_i)}[r + n + s_i + m(\theta_i)], \quad i = H, L. \tag{32}$$

Proposition 4. If there is no search cost and the proportion of skilled workers is exogenously given, then

$$\frac{d\theta_H}{dI_L} > 0, \ \ \frac{d\theta_L}{dI_L} < 0, \ \ \frac{du_H}{dI_L} < 0, \ \ \frac{du_L}{dI_L} > 0, \ \ \frac{dw_H}{dI_L} > 0 \ \text{ and } \ \frac{dw_L}{dI_L} < 0.$$

An increase in the number of unskilled immigrants I_L raises the productivity of skilled labor and lowers that of unskilled labor since skilled and unskilled labor are Edgeworth complements in production. Hence, the price of skilled input p_H goes up, while the price of unskilled input p_L goes down. This induces the entry of skilled jobs and raises the matching rate for the skilled workers $m(\theta_H)$, but discourages the entry of unskilled jobs and causes the matching rate for the unskilled workers $m(\theta_L)$, to go down. In terms of Figure 2, an increase in I_L shifts the OH curve to the right (from OH to OH'), but leaves the curve EP unchanged. Given these changes in the flow probabilities, the rest of the comparative statics follow easily; namely, an increase in the probability of finding a match lowers the unemployment rate among skilled workers and raises their bargaining power and hence their wage. The opposite holds for the unskilled workers.

Next we analyze the case where there is endogenous skill acquisition and hence the fraction $1-\lambda$ of the native workers that are skilled is given by equation (29). We continue to assume that all the search costs h_{ij} are equal to zero, which implies that there is still no difference between a native worker and an immigrant of a skill type i. The equilibrium is now described by equations (27), (28) and the following simplified version of (29) (see the Appendix for the details)

$$(1-\lambda^*)\overline{z}(r+n) = b_H - b_L + \frac{\beta}{1-\beta}(c_H\theta_H - c_L\theta_L). \tag{33}$$

The wage rate of each group is still given by equation (32). Consider the following proposition.

Proposition 5. If there is no search cost and the proportion of skilled workers is determined endogenously, then

$$\frac{d\theta_H}{dI_L} > 0, \quad \frac{d\theta_L}{dI_L} < 0, \quad \frac{d\lambda}{dI_L} < 0, \quad \frac{du_H}{dI_L} < 0, \quad \frac{du_L}{dI_L} > 0, \quad \frac{dw_H}{dI_L} > 0 \text{ and } \frac{dw_L}{dI_L} < 0.$$

Moreover, the magnitude of these effects is smaller, in absolute value, compared with the case where the proportion of skilled workers is fixed.

The mechanism behind the results derived in Proposition 5 is the same as that in Proposition 4. Moreover, an increase in the number of unskilled immigrants raises the proportion of skilled workers $(1-\lambda)$, since both the increase in θ_H and the decrease in θ_L raise the benefit of education. Interestingly, starting from the same equilibrium, with endogenous changes in the skill distribution the increase in the matching rate of skilled workers $m(\theta_H)$ is lower compared to the case where λ is fixed. Similarly, the decrease in the matching rate of unskilled workers $m(\theta_L)$ is smaller when λ is allowed to adjust. This occurs because the initial effects on the prices of the two inputs are mitigated through changes in λ. More specifically, when λ is endogenous, more unskilled workers, caused by an increase in unskilled immigration, induce (discourage) entry of skilled (unskilled) jobs as well as a compositional shift in the native labor force towards skilled workers. This compositional shift acts to mitigate the initial positive (negative) impact on the price of skilled (unskilled) input. In terms of Figure 2, the curve EP remains unchanged but the shift of the curve OH to the right is smaller compared with that in Proposition 4; e.g., the curve shifts to OH' when λ is fixed but only to OH'_λ when λ is endogenously determined. This has important implications because it makes the benefits of unskilled immigration to skilled labor (i.e., the decline in the unemployment rate and the increase in the wage rate) smaller. Similarly, the losses of immigration to unskilled labor (i.e., the increase in the unemployment rate and the fall in the wage rate) are also smaller.

4.2 Perfect Substitutes and Search Costs

In this subsection we analyze the other special case where $\rho = 1$ but $h_{LI} > 0$. We begin again with the case where λ is exogenously given. Consider

Proposition 6. If the two intermediate inputs are perfect substitutes, immigrants face a search cost and the proportion of skilled workers is exogenously given, then

$$\frac{d\theta_H}{dI_L} = 0, \quad \frac{d\theta_L}{dI_L} > 0, \quad \frac{du_H}{dI_L} = 0, \quad \frac{du_L}{dI_L} < 0, \quad \frac{dw_{HN}}{dI_L} = \frac{dw_{HI}}{dI_L} = 0 \text{ and } \frac{dw_{LN}}{dI_L} > 0$$

If the proportion of skilled natives is endogenous, these effects have the same sign but smaller in magnitude.

To understand the results summarized in Proposition 6 notice from equation (31) that when $h_{LI} > 0$ and $h_{LN} = 0$ the wage rate of unskilled immigrants, is lower than that of unskilled native workers; that is, $w_{LI} < w_{LN}$ because immigrants are subject to higher search costs. Intuitively, searching is costlier for immigrants, which forces them to accept lower wages. For the firm, hiring an immigrant is therefore more profitable than hiring a native, given that they are both equally productive. It follows that the increase in the immigrant's share of unskilled labor force lowers the expected employment cost in the low-skill sector B_L, by lowering the probability a firm will locate a native unskilled worker ϕ_L. This spurs low-skill job entry with a concomitant increase in the matching rate for low-skill workers. Consequently, this leads to an increase in the wage of low-skill native and immigrant workers w_{LN} and w_{LI}, given by equation (31), and a decrease in their unemployment rate, given by the second equation in (30). Finally, the matching rate for high-skill workers is given by (28). Note that if $\rho = 1$ then θ_H is independent of the number of unskilled immigrants. Consequently, the wage rate and the unemployment rate for high-skill workers will remain the same. In terms of Figure 3, the curve that depicts the locus of points along which profit is zero in the high-skill (low-skill) sector are HH (LL). An increase in the number of unskilled immigrants leaves the first curve unchanged but shifts the second curve upwards (to $L'L'$).

In the case where λ is endogenous, there will still be an increase in the matching rate and the wage rate of low-skill workers as well as a decrease in their unemployment rate. However, starting form the same equilibrium , as in the case where λ is fixed, these effects are smaller in magnitude since the increase in w_L will increase λ, which will offset partially the initial increase in the matching rate θ_L. The corresponding variables for high-skill workers will still remain unchanged. Graphically in this case the equilibrium can be presented as the intersection of the HH curve and a downward-sloping curve that depicts the zero-profit condition in the low-skill sector (such as $L_\lambda L_\lambda$ in Figure 3). An increase in the number of unskilled immigrants leaves the first curve unchanged but shifts the second curve upwards (to $L'_\lambda L'_\lambda$).

5 General Case

Next we analyze the equilibrium in the general case, where $\rho < 1$ and $h_{LI} > 0$. In this general case, a change in I_L can influence the equilibrium through the impact of such a change on both prices and expected employment costs.

From our analysis above, we can infer that in the general case the impact of an increase in the number of unskilled immigrants on skilled native workers will be unambiguously positive both in terms of wages and unemployment. However, the impact on the unskilled natives is in general ambiguous. As derived in Proposition 5, the price effect is negative, while, as derived in Proposition 6, in the presence of differential search costs the impact is positive on unskilled natives. In this section we therefore calibrate the general model with the aim to quantitatively assess the overall impact of unskilled immigration on the labor market outcomes of unskilled natives, and in turn, on the overall welfare of natives.

For the results below it is useful to characterize our measure of natives' welfare. As is the convention in these models, we measure welfare as the total steady-state output net of total costs, i.e., the total steady-state surplus of the economy. To measure the welfare of natives, we subtract from the total surplus the amount of output that accrues to immigrants. We make the assumption that all firms belong to natives so that all the profits net of the wages firms pay to immigrants accrue to natives. Thus, our measure of native's welfare is the total steady-state surplus of the economy minus the wages paid to immigrants, and is given by

$$\widetilde{Y} = Y + b_H U_{HN} + b_L U_{LN} - c_H V_H - c_L V_L - w_{HI}(I_H - U_H) - w_{LI}(I_L - U_L) \qquad (34)$$

It is equal to total flow of output, Y, plus the output-equivalent flow to native workers who are not currently working, $b_H U_{HN} + b_L U_{LN}$, minus the flow costs of job creation for skilled and unskilled vacancies, $c_H V_H$ and $c_L V_L$, respectively, minus the wages paid to currently employed skilled and unskilled immigrants, given by $w_{HI}(I_H - U_H)$ and $w_{LI}(I_L - U_L)$, respectively. In our simulation exercises below we also consider an alternative measure of natives' welfare that does not include the income enjoyed by the unemployed: $\tilde{Y} - b_H U_{HN} - b_L U_{LN}$.

In what follows we first describe the baseline calibration of the general model. The general model's quantitative predictions are then discussed in order. We then examine the sensitivity of the predictions with respect to the production parameters ρ and γ. Finally, we examine how the results change when we relax the assumption that unskilled natives and immigrants are perfect substitutes in production.

5.1 Calibration

For simplicity and realism (see Blanchard and Diamond, 1991), in our calibration we use a Cobb-Douglas matching function, $M = \xi U_i^{\epsilon} V_i^{1-\epsilon}$, which exhibits standard properties. The scale parameter ξ indexes the efficiency of the matching process.

Our model economy is fully characterized by 20 parameters. The production parameters, ρ, γ, α and x, the parameters in the matching function, ξ and ϵ, the job separation rates, s_L and s_H, the unemployment flow incomes, b_L and b_H, the vacancy costs, c_L and c_H, the depreciation rate, δ, the worker's bargaining power, β, the population growth rate, n, the upper bound of the cost of acquiring training $\bar{z}$, the numbers of skilled and unskilled immigrants, I_L and I_H, the search cost, h_{LN}, and the discount rate r. One period in the model economy represents one month, so all the parameters are interpreted monthly. A summary of our calibration is given in Table 1.

First, we adopt the standard parameter values for the monthly interest rate, $r = 0.004$. Following common practice, we set the unemployment elasticity to $\epsilon = 0.5$, which is within the range of estimates reported in Petrongolo and Pissarides (2001), and also assume $\beta = 0.5$, which internalizes the search externalities (Hosios, 1990). Shimer's (2005) data give a mean value for 1960-2004 of 0.594 for the average job finding rate and 0.036 for the average job separation rate. Moreover, the sample mean for the vacancy to unemployment ratio between 1960-2006 was 0.72.[7] We make use of the mean job finding rate, the mean separation rate and the mean vacancy to unemployment ratio to derive values for s_L, s_H, and ξ. The resulting values are $s_L = 0.040, s_H = 0.024$, and $\xi = 0.715$.

Following Krussell et al. (2000), we define as skilled a worker with at least a Bachelor's degree, and adopt their parameter estimates for the US economy, $\rho = 0.0401$ and $\gamma = -0.495$.[8] The weights in the nested CES function are set to $\alpha = 0.538$ and $x = 0.800$,

[7] This is derived in Pissarides (2009), using the Job Openings and Labor Tunover Survey (JOLTS) data since December 2000 and the Help-Wanted Index (HWI) adjusted to the JOLTS units of measurement before then.

[8] Given that we assume that there are only two distinct skill groups, the assumptions embodied in our production technology (given in (2) and (3)) may seem relatively strong. They imply that workers within each of the two skill groups are perfect substitutes. However, a variety of estimates based on US data suggest that given our partition of workers into "high-school equivalents" and "college equivalents" this simple two-skill model works. This evidence show that allowing for imperfect substitution between

which imply an output-to-capital ratio of 0.36 and a share of labor in total output of 0.68. According to D'Adda and Scorcu (2003), the US output-to-capital ratio for the period 1980-1992, ranges from 0.33 to 0.41. As for our targeted labor share, it lies within the range of available estimates: 0.65-0.75.[9] The rate of depreciation of capital is set to $\delta = 0.10$, a value that it is consistent with most of the estimates for the US.[10]

The parameter $\bar{z}$ is taken to be 91.5 so that the average share of skilled labor force in our model economy matches the average share of workers in the US labor force that have a Bachelor's degree, of about 0.25. The numbers of skilled and unskilled immigrants are set to $I_L = 0.142$ and $I_H = 0.047$ so that in our model economy 15.9% of the labor force is foreign-born and 25% of the workers in the foreign-born labor force have a Bachelor's degree. These measures come from the 2009 American Community Survey (ACS) data tables of the US Census Bureau. We follow Ben-Gad (2008) and set $n = 0.0067$, which the average US natural rate of population growth for the period 1991-2000.

Our chosen values for c_L and c_H are guided by the following observations. First, a significant part of the cost of filling a vacancy is the opportunity cost of labor effort devoted to hiring activities. Hence, the recruiting cost should be compatible with labor earnings. Since hiring is typically done by supervisors whose wage is at least as high as the wages of new hires, recruiting for skilled jobs should be more costly than recruiting for unskilled jobs. Second, recruiting costs cannot be too large relative to output. The standard upper bound in the literature is 5% of output devoted in job creation activities. Setting $c_L = 0.546$ and $c_H = 0.819$ results in 3% of output devoted to job creation activities and obeys the other criterion. Specifically, the average recruiting cost of a skilled and an unskilled job is roughly equal to the monthly wage of a skilled and an unskilled job, respectively.

We select values for b_L and b_H to match statistics from the simulated data to empirical measures of, 1) the average US employment rate, and 2) the US college-plus wage premium. We target an average employment rate of 0.93 and a skilled wage premium of 45%, consistent with the estimates reported in Goldin and Katz (2007). To match these statistics we set $b_L = 0.339$ and $b_H = 0.488$. Together with our chosen values for c_L and c_H, these values imply a replacement ratio in our model of 60%. The replacement

different age, experience, or even education groups within each of these two groups makes relatively little difference in the immigration context (see Card, 2009 for an overview of these evidence).

[9]See e.g., Gollin (2002) and Krueger (1999).

[10]See e.g., Greenwood et al. (1997) and Epstein and Denny (1980).

ratio in our model economy is larger than the 40% suggested by Shimer (2005), which does not account for the value of leisure or home production, and closer to the 71%, estimated by Hall (2006), which accounts for both the value of leisure and home production. Finally, the search cost parameter is set to $h_{LI} = 1.32$ so that each unskilled immigrant receives a wage that is 75% of that of an unskilled native. The targeted wage gap between immigrants and natives lies in the middle of estimates reported in Borjas (2000).

5.2 Results

Using data from the US Census Bureau (Release Date: December 22, 2008) we find that over the period July 2000-July 2008 the average population growth rate, resulting from international immigration, was approximately 0.35 percent. Using this growth rate, we see that the immigration-induced change in US population over a twenty-year period would be 7 percent. In Table 2 we summarize the effects of an unskilled immigration influx of the same magnitude.[11] We report results in the general model, but for comparability, we also report results in three alternative specifications. In the first, we keep the proportion of unskilled natives fixed at $\lambda = 0.75$, as calibrated above, and set $h_{LI} = 0$. There are therefore only price effects in this case. In the second, we keep the assumption $h_{LI} = 0$, but allow for λ to adjust endogenously. Finally, in the third, we set $h_{LI} = 1.32$, as calibrated above, and keep the proportion of unskilled natives fixed by setting $\lambda = 0.75$.

In all specifications the impact of immigration on the overall welfare of natives is positive. When natives and immigrants face identical search costs, the increase in the number of unskilled immigrants raises the overall welfare of natives, but, as expected, mainly because it lowers the unemployment rate and raises the wage of skilled natives. As derived in Proposition 4, when $h_{LI} = 0$, an unskilled immigration influx has a negative impact on unskilled natives both in terms of wages and unemployment, because it deters low-skill job entry by lowering the price of the unskilled labor input. By contrast, when we allow for differential search costs, the increase in the number of unskilled immigrants raises job entry not only in the high- but also in the low-skill sector. Consequently, unemployment falls not only among skilled, but also among unskilled workers. In addition, the negative impact on the wage of unskilled natives is smaller in this case. As for the skilled natives, both the decline in their unemployment rate and the increase in their wage

[11] In conducting their simulation exercises, Borjas and Katz (2007) and Ottaviano and Peri (2010) used an immigrant influx that increased the size of the total workforce by 11.0% and 11.3%, respectively.

is larger in the presence of differential search costs.

These improvements come through the impact of search costs on the wage of unskilled immigrants. As explained above, due to their higher search costs unskilled immigrants receive lower wages than unskilled natives. For this reason, as the immigrants' share of unskilled labor force increases, firms with low-skill vacancies anticipate that they will have to pay lower wages on average. This encourages low-skill job entry. The resulting increase in the unskilled labor input (Y_L) causes the price of skilled labor input to rise (p_H), thereby also encouraging the creation of skilled jobs.

The presence of endogenous skill accumulation has a positive and significant impact on the overall welfare of natives, but mainly because it lessens the negative impact on the price of the low-skill labor input (p_L). As derived in Proposition 5, a compositional shift in the native labor force towards skilled workers acts to mitigate the negative (positive) impact of intensified competition from unskilled immigrants on the price of unskilled (skilled) input. These counteracting effects lessen the negative effect on unskilled natives, but also lessen the positive effect on skilled natives. Nevertheless, since the latter capture a smaller share of the labor force, allowing for endogenous skill accumulation improves the impact of immigration on the overall welfare of natives considerably.

It is also worth commenting on the impact of the unskilled immigration influx on the welfare of previous unskilled immigrants. Clearly, with identical search costs, immigration has the same consequences on unskilled workers, both in terms of wages and unemployment, irrespective of their origin. But with differential search costs the impact of immigration in terms of wages appears to be more positive on unskilled immigrants than natives. To understand why notice that an increase in market tightness influences the equilibrium wage through two channels: 1) through its impact on the marginal product of labor and thus the price of the labor input; an increase in tightness lowers the marginal product of labor, thereby lowering the worker's wage; 2) through its impact on the worker's value of outside option. An increase in tightness raises the value of search, thereby strengthening the worker's position in wage setting, and in turn, causing his wage to rise. Since search is much costlier for immigrants than natives, this second channel is much more important for the former, which explains why the impact of immigration on their wage is more positive. For these workers, a small increase in their chances of finding a job implies a much larger increase (in percentage terms) in their bargaining power and in turn on their wage.

6 Sensitivity Analysis

The results above are derived using the elasticities of substitution between the input factors estimated by Krusell et al. (2000) and assuming that unskilled immigrants and natives are perfect substitutes in the production of the unskilled input (Y_L). In this subsection we first examine how robust the general model's predictions are to alternative values for the elasticities of substitution between capital and the skilled and unskilled labor, respectively. Then, we employ a generalized function for the production of unskilled input, and examine the sensitivity of our results to different degrees of substitutability between native and immigrant unskilled labor.

6.1 Changing the Elasticity of Substitution between Labor and Capital

For the nested CES production function, given in equations (2) and (3), the Allen-Hicks elasticities of substitution between unskilled labor Y_L and the other two factors, skilled labor Y_H and capital K are identical and given by $\sigma_{LK} = \sigma_{LH} = \frac{1}{1-\rho}$. The Allen-Hicks elasticity of substitution between skilled labor and capital is a function of factor shares. Following Krusell et al. (2000) and Ben-Gad (2008) we employ a simplified definition of the elasticity of substitution between skilled labor and capital: $\sigma_{HK} = \frac{1}{1-\gamma}$.

In Table 3 we report results of the general model for different sets of values for the parameters ρ and γ. As in Ben-Gad (2008), we consider a set where both elasticities are low ($\sigma_{LK} = 1, \sigma_{HK} = 0.5$), a set where both elasticities are high ($\sigma_{LK} = 2, \sigma_{HK} = 1$), and two sets where one elasticity is high and the other low, ($\sigma_{LK} = 1, \sigma_{HK} = 1$) and ($\sigma_{LK} = 2, \sigma_{HK} = 0.5$). The results are qualitatively robust to our choices of σ_{LK} and σ_{LK}. In all cases the impact of immigration is positive on the welfare of natives, because it reduces the unemployment rate of both skilled and unskilled workers and raises the skilled wage.

Moreover, the effect of immigration on skilled workers becomes more positive as the degree of capital-skill complementarity increases (i.e., as γ decreases). The increase in Y_L due to the increase in unskilled immigration, raises the marginal product of capital and thus its equilibrium level. The increase in capital raises the marginal product of skilled labor and hence the price of the skilled labor input, p_H, thereby encouraging high-skill job entry and leading to higher skilled wages and smaller unemployment among skilled workers. The higher the degree of capital-skill complementarity the more positive is the impact of an increase in capital on the marginal product of skilled labor, which explains why at lower values of γ the increase in the number of unskilled immigrants benefits skilled workers by more.

Similar reasoning explains why the effect of immigration on skilled workers becomes more positive as the elasticity of substitution between skilled labor/capital and unskilled labor declines (i.e., as ρ decreases). An increase in Y_L causes a larger increase in both the marginal product of skilled labor and the marginal product of capital when ρ is small. Consequently, given that skilled labor and capital are (Hicks-Allen) complements to each other, at lower values of ρ the increase in p_H due to an immigration-induced increase in Y_L is larger.

6.2 Changing the Elasticity of Substitution between Natives and Immigrants

To permit the elasticity of substitution between unskilled-native and unskilled-immigrant labor input (Y_{LN} and Y_{LI}, respectively) to differ, we employ the following CES function

$$Y_L = [\psi Y_{LN}^{\eta} + (1-\psi) Y_{LI}^{\eta}]^{\frac{1}{\eta}} \tag{35}$$

where ψ is a positive share parameter and η determines the degree of substitutability between the two labor inputs, Y_{LN} and Y_{LI}. In particular, based on the above specification the elasticity of substitution between immigrants and natives is $\sigma_{IN} = \frac{1}{1-\eta}$. As above, inputs are sold in competitive markets. Thus, the prices p_{LN} and p_{LI} equal the marginal products of Y_{LN} and Y_{LI}, respectively

$$\begin{aligned} p_{LN} &= \alpha\psi Y^{1-\rho} Y_L^{\rho-\eta} Y_{LN}^{\eta-1} \\ p_{LI} &= \alpha(1-\psi) Y^{1-\rho} Y_L^{\rho-\eta} Y_{LI}^{\eta-1} \end{aligned} \tag{36}$$

The free entry condition in (27), the wage rates in (31) and the condition governing the human capital decision in (29) change accordingly to take into account that the price of unskilled input, p_L, is now disaggregated into p_{LN} and p_{LI}.

In Table 4 we report results in this generalized model at different values for the parameter η. As empirical basis for our choices of η we use the estimates reported in Ottavio and Peri (2010). They first partition workers into groups based on their education and experience characteristics. Then, using a CES aggregator, similar to the one in (36) they estimate the elasticity of substitution between natives and immigrants sharing similar education and experience characteristics. Based on their estimates and given our definition of unskilled workers, σ_{IN} should range from about 6.5 to about 20, meaning that η should lie somewhere between 0.85 and 0.95. In lack of a good empirical estimate that can guide our choice of value for ψ, for the results below we set $\psi = 0.6$. This value ensures that, given the other parameters of the model, the productivity of unskilled natives is greater than that of unskilled immigrants. We keep the rest of the parameter values as described above.

Our results are robust to this generalized set-up. Again, unskilled immigration raises the welfare of natives, because it lowers their unemployment rates and raises their average wage. Also, the smaller the degree of substitutability between native and immigrant unskilled labor, the larger the positive impact of immigration on both labor types. In fact, for low values of η (i.e., lower degree of substitutability) unskilled immigration has a positive impact on unskilled natives not only in terms of unemployment, but also in terms of wages. This is not surprising since a smaller degree of substitutability between unskilled natives and immigrants implies a smaller negative impact on the marginal product of unskilled natives following an increase in the number of unskilled immigrants. This also implies smaller negative impact on their price p_{LN}, and hence their wage, and larger low-skill job entry. Reasoning as above, the resulting larger increase in the unskilled labor input, Y_L, raises the price of the skilled labor input, p_H, by more, thereby improving also the consequences on skilled natives.

Notice also that as the degree of substitutability between native and immigrant unskilled workers falls, the wage effect on unskilled natives becomes more positive (turns from negative to positive), whereas that on unskilled immigrants becomes less positive (turns from positive to negative). Hence, a high degree of substitutability between immigrants and natives means that the competitive effects of additional immigrants fall more heavily on immigrants themselves, thereby lessening the burden on natives.[12] This occurs because as the degree of substitutability between immigrants and natives decreases the price effect of an increase in the number of unskilled immigrants becomes less negative on unskilled natives and more positive on existing unskilled immigrants. That is, at smaller values of η the negative effect on w_{LI} through p_{LI} is much higher in absolute value, while the negative impact on w_{LN} through p_{LN} is much smaller in absolute value.

[12]The view that the competitive effects of additional immigrant inflows are concentrated among immigrants themselves, lessening the negative impact on competing natives due to immigrants and natives being imperfect substitutes is also supported by evidence reported in Card (2009) and Ottaviano and Peri (2010)

7 Conclusion

In this paper we have examined the effects of immigration on the native population in a search and matching model, where search frictions generate unemployment and break the link between marginal product and wages. Within this framework we have been able to explicitly account for the unemployment and wage effects that come from the impact of immigration on the availability of jobs. Most of the existing contributions to the immigration literature overlook such effects by adopting a Walrasian market-clearing determination of wages. Other features of the model we have developed that deserve attention are: heterogeneity in terms of skills, which allows for the analysis of distributional effects across different skill types; endogenous skill acquisition on behalf of natives, which gives them the opportunity to react to the negative pressure of immigration; a generalized production technology, which requires both capital and labor and accounts for the effects of immigration on input prices; and differential search costs, which can explain the equilibrium wage gap between otherwise identical native and immigrant workers.

Within the confines of our model we have shown that the inflow of unskilled immigrants has two countervailing effects on unskilled domestic labor. First, it lowers the marginal product of the unskilled labor input, thereby discouraging the creation of unskilled jobs. Second, it makes opening vacancies suited for unskilled workers more profitable to firms, because firms anticipate that they will be able to pay lower wages to immigrants that have higher search costs. In our calibrated baseline economy, where we let unskilled immigrants and natives be perfect substitutes in production, we have found that the second effect dominates leading to a higher availability of unskilled jobs and lower unemployment among unskilled native workers. The higher availability of unskilled jobs also strengthens their bargaining position in wage setting, which acts to mitigate the negative effect of the immigration-induced fall in their marginal product on their wages. We have shown that these results are robust under various choices of values for the production-function parameters that drive the elasticities of substitution between the three inputs. We have also shown that in a calibrated version of the model where unskilled natives and immigrants are imperfect substitutes in production, the inflow of unskilled immigrants benefits unskilled native workers, not only in terms of unemployment but also in terms of wages.

In all cases that we have considered, the inflow of unskilled immigrants improves the labor market outcomes of skilled native workers, because it encourages the creation of skilled jobs by raising the price of the skilled labor input. Moreover, we have found that despite the negative pressure on the wages of unskilled native workers the inflow of unskilled immigrants generates significant welfare gains to the native population overall. This suggests that a system of transfers from skilled to unskilled native workers together with a less restrictive immigration regulation can make everyone better off. However, before reaching such a conclusion, one should also take into account the fact that low-income unskilled immigrants are likely to use the programs of the welfare state at higher rates than natives and contribute less to it. In other words, immigrants may impose a net fiscal burden on the host country. We leave this as a possible extension, which we plan to undertake in the future.

Appendix

Derivation of equations (17)-(24)

The change in the number of unemployed native-skilled workers (U_{HN}) is given by the difference between the sum of new births ($n(1-\lambda)$) and break-ups ($s_H Y_{HN}$) and the sum of deaths (nU_{HN}) and matches ($m(\theta_H)U_{HN}$); that is,

$$\dot{U}_{HN} = n(1-\lambda) + s_H Y_{HN} - [nU_{HN} + m(\theta_H)U_{HN}],$$

where a dot over a variable denotes its time derivative. Likewise, the change in the number of unemployed immigrant-skilled workers (U_{HI}) is given by the difference between break-ups ($s_H Y_{HI}$) and matches ($m(\theta_H)U_{HI}$):

$$\dot{U}_{HI} = s_H Y_{HI} - m(\theta_H)U_{HI},$$

Setting $\dot{U}_{HN} = 0$ and $\dot{U}_{HI} = 0$ and using the identities $Y_{HN} + Y_{HI} = Y_H$ and $Y_H + U_{HN} + U_{HI} = 1 - \lambda + I_H$ yields equation (17). The other equations follow similarly.

Derivation of the system of equations (27)-(29)

Equation (4) can be written as

$$p_L = \alpha \left[\alpha + (1-\alpha)\left(\frac{Q}{Y_L}\right)^{\rho}\right]^{\frac{1-\rho}{\rho}},$$

or after using (3)

$$p_L = \alpha \left\{\alpha + (1-\alpha)\left[x\left(\frac{K}{Y_H}\right)^{\gamma} + (1-x)\right]^{\frac{\rho}{\gamma}}\left(\frac{Y_H}{Y_L}\right)^{\rho}\right\}^{\frac{1-\rho}{\rho}}. \tag{A1}$$

Similarly, from (5) and (3) we get

$$p_H = (1-\alpha)(1-x)\left[x\left(\frac{K}{Y_H}\right)^{\gamma} + (1-x)\right]^{\frac{1-\gamma}{\gamma}}\left\{\frac{\alpha\left(\frac{Y_H}{Y_L}\right)^{-\rho}}{\left[x\left(\frac{K}{Y_H}\right)^{\gamma} + (1-x)\right]^{\frac{\rho}{\gamma}}} + (1-\alpha)\right\}^{\frac{1-\rho}{\rho}}, \tag{A2}$$

and from (6) and (3)

$$p_K = (1-\alpha)x\left[x + (1-x)\left(\frac{K}{Y_H}\right)^{-\gamma}\right]^{\frac{1-\gamma}{\gamma}}\left\{\frac{\alpha\left(\frac{Y_H}{Y_L}\right)^{-\rho}}{\left[x\left(\frac{K}{Y_H}\right)^{\gamma} + (1-x)\right]^{\frac{\rho}{\gamma}}} + (1-\alpha)\right\}^{\frac{1-\rho}{\rho}}. \tag{A3}$$

Taking the ratio of (A2) to (A3) we have

$$\frac{p_H}{p_K} = \frac{1-x}{x}\left(\frac{K}{Y_H}\right)^{1-\gamma}, \quad \text{where} \quad p_K = r+\delta. \tag{A4}$$

Moreover, taking the ratio of equations (17) and (18), we get

$$\frac{Y_H}{Y_L} = \frac{m(\theta_H)[n+s_L+m(\theta_L)](1-\lambda^*)+I_H}{m(\theta_L)[n+s_H+m(\theta_H)](\lambda^*++I_L)}. \tag{A5}$$

Equations (10) and (13) imply that

$$J_{ij}^E = \frac{w_{ij}+s_i J_{ij}^U}{r+n+s_i}. \tag{A6}$$

Also, combining (A6), (13) and (15) we obtain

$$S_{ij} = \frac{1}{1-\beta}\frac{p_i - w_{ij}}{r+n+s_i}. \tag{A7}$$

Next, subtracting (11) from (12) and using (A7) yields yields the expression for the wage rate

$$w_{ij} = \frac{[r+n+s_i+m(\theta_i)]\beta p_i + (r+n+s_i)(1-\beta)(b_i-h_{ij})}{r+n+s_i+\beta m(\theta_i)}. \tag{A8}$$

Substitute (A8) in (A7) to get

$$S_{ij} = \frac{p_i - b_i + h_{iI}}{r+n+s_i+\beta m(\theta_i)}, \tag{A9}$$

Substituting (A9) and (15) in (9) and taking into account the free entry condition (13) yields

$$p_i = B_i \quad \text{where} \quad B_i \equiv b_i - (1-\phi_i)h_{iI} + \frac{c_i[r+n+s_i+\beta m(\theta_i)]}{(1-\beta)q(\theta_i)}. \tag{A10}$$

where it may be recalled that by assumption $h_{iN}=0$ for $i=H,L$ and $\phi_H=1$.

Next, substitute (A9) and (16) in (11) to get

$$(r+n)J_{ij}^U = \frac{\beta m(\theta_i)p_i + (r+n+s_i)(b_i-h_{ij})}{r+n+s_i+\beta m(\theta_i)}. \tag{A11}$$

Combining equations (A1), (A5) and (A10) yields (27), where the expression for k follows from (A4) and (A10). Similarly, combining (A2), (A5) and (A10) yields (28). Finally, substituting (A11) in (1) we get (29).

Proof of Proposition 1.

Combining equations (27) and (28), we arrive at the following equation:

$$\frac{\left(\frac{B_L}{\alpha}\right)^{\frac{\rho}{1-\rho}} - \alpha}{1-\alpha} = \frac{\alpha}{\left[\frac{B_H}{(1-\alpha)(1-x)}\left[xk^\gamma + (1-x)\right]^{\frac{-\gamma}{1-\gamma}}\right]^{\frac{\rho}{1-\rho}} - (1-\alpha)}, \tag{A12}$$

where B_L, B_H and k are defined in the main text. Simple differentiation shows that B_H and k are both increasing functions of θ_H. On the other hand, if $h_{LI} = 0$, then B_L is an increasing function of θ_L only. Rearranging equation (A12) we obtain

$$X = \frac{\alpha\Psi}{\Psi - (1-\alpha)}, \tag{A13}$$

where

$$X \equiv \left(\frac{B_L}{\alpha}\right)^{\frac{\rho}{1-\rho}} \quad \text{and} \quad \Psi \equiv \left[\frac{B_H}{(1-\alpha)(1-x)}\left[xk^{\gamma} + (1-x)\right]^{\frac{-\gamma}{1-\gamma}}\right]^{\frac{\rho}{1-\rho}}.$$

Equation (A13) defines a locus of θ_H and θ_L along which a firm is indifferent between opening a low-skill and a high-skill vacancy. This locus, which is labeled EP in Figure 2, has negative slope:

$$\frac{d\theta_L}{d\theta_H}|_{EP} = \frac{-\frac{\alpha(1-\alpha)}{[\Psi-(1-\alpha)]^2}\frac{d\Psi}{dB_H}\frac{dB_H}{d\theta_H}}{\frac{dX}{dB_L}\frac{dB_L}{d\theta_L}} < 0.$$

Equation (29) defines implicitly a function $\lambda = l(\theta_H,\, \theta_L)$, where $l_1 < 0$ and $l_2 > 0$. Substituting the function $\lambda = l(\theta_H,\, \theta_L)$ in equation (28) we obtain

$$p_H(\theta_H, \theta_L) = B_H(\theta_H), \tag{A14}$$

where $\partial p_H/\partial\theta_H < 0$, $\partial p_H/\partial\theta_L > 0$ and $dB_H/d\theta_H > 0$. Equation (A14) defines a locus of θ_H and θ_L along which a high-skill vacancy has zero expected profit. This locus, which is labeled as OH in Figure 2, has the following properties:

$$\lim_{\theta_H\to 0}\theta_L = 0, \quad \lim_{\theta_H\to\overline{\theta_H}}\theta_L = \infty, \text{ where } \overline{\theta_H} < \infty, \quad \frac{d\theta_L}{d\theta_H}|_{OH} = \frac{\frac{dB_H}{d\theta_H} - \frac{\partial p_H}{\partial\theta_H}}{\frac{\partial p_H}{\partial\theta_L}} > 0.$$

Equations (A13) and (A14) determine the equilibrium values of θ_H and θ_L. To ensure an intersection of the EP and OH curves in the positive orthant we must impose conditions that guarantee that the intercept of the EP curve is positive. Let

$$\Psi_0 = \lim_{\theta_H\to 0}\Psi(\theta_H) = \left[\frac{b_H}{(1-\alpha)(1-x)}\left[x\left[\frac{x}{(1-x)(r+\delta)}b_H^{\frac{\gamma}{1-\gamma}}\right] + (1-x)\right]^{-\frac{\gamma}{1-\gamma}}\right]^{\frac{\rho}{1-\rho}}.$$

Notice also that

$$\lim_{\theta_L\to 0}X(\theta_L) = \left(\frac{b_L}{\alpha}\right)^{\frac{\rho}{1-\rho}}, \quad \lim_{\theta_L\to\infty}X(\theta_L) = \begin{cases} \infty & \text{if } \rho > 0 \\ 0 & \text{if } \rho < 0 \end{cases} \quad \text{and} \quad \frac{dX(\theta_L)}{d\theta_L} = \begin{cases} > 0 & \text{if } \rho > 0 \\ < 0 & \text{if } \rho < 0 \end{cases}.$$

Given these properties, existence and uniqueness is ensured if

$$\left(\frac{b_L}{\alpha}\right)^{\frac{\rho}{1-\rho}} < \frac{\alpha\Psi_0}{\Psi_0 - (1-\alpha)}.$$

Proof of Proposition 2. Setting $\rho = 1$ in equations (27), (28) we get

$$\alpha = B_L, \tag{A15}$$

and

$$(1-\alpha)(1-x)\left[xk^{\gamma} + (1-x)\right]^{\frac{1-\gamma}{\gamma}} = B_H, \tag{A16}$$

where where B_L, B_H and k are defined in the main text. Thus, the equilibrium is described by equations (A15), (A16) and (29). Equation (29) defines implicitly a function $\lambda = l(\theta_H, \theta_L)$, where $l_1 < 0$ and $l_2 > 0$. Substituting in (A15) and (A16) we obtain the two loci of θ_H and θ_L along which a low- and a high-skill vacancy have zero expected profit, respectively. These curves are labeled as $L_\lambda L_\lambda$ and HH in Figure 3. The curve $L_\lambda L_\lambda$ has the following properties

$$\lim_{\theta_H \to \infty} \theta_L = \overline{\theta}_L, \text{ where } \overline{\theta}_L < \infty, \quad and \quad \frac{d\theta_L}{d\theta_H}|_{L_\lambda L_\lambda} = -\frac{\frac{dB_L}{d\theta_H}}{\frac{dB_L}{d\theta_L}} > 0.$$

The curve HH, on the other hand, is independent of θ_L and hence vertical on the horizontal axis. Given these properties, both existence and uniqueness of the equilibrium pair (θ_H, θ_L) are immediate.

Proof of Proposition 3. If $h_{LI} = 0$ and $\rho = 1$ then the price of the unskilled input is $p_L = \alpha$ and equation (27) simplifies to $\alpha = B_L$, which is independent of θ_H and I_L. This equation is depicted by curve LL in Figure 3. Also, equation (28) simplifies to $(1-\alpha)(1-x)\left[xk^{\gamma} + (1-x)\right]^{\frac{1-\gamma}{\gamma}} = B_H$ and is independent of θ_L and I_L (see the curve HH in Figure 3). It follows that the equilibrium pair of (θ_H, θ_L) is independent of I_L.

Derivation of equation (32)

If $h_{ij} = 0$ then equation (31) implies that $w_{ij} = w_i$ and equation (10) that $J^F_{ij} = J^F_i \; \forall j$. It follows then from equations (9) and (13) that

$$J^F_i = \frac{c_i}{q(\theta_i)}.$$

On the other hand, (10) and (13) imply

$$J^F_i = \frac{p_i - w_i}{(r + n + s_i)}.$$

Combining the last two equations yields

$$w_i = p_i - (r + n + s_i)\frac{c_i}{q(\theta_i)}, \quad i = H, L,$$

and, after using (A10), (32).

Proof of Proposition 4. Differentiating equations (28) and (A13) we obtain

$$\frac{d\theta_H}{dI_L} = \frac{-\frac{1}{\rho}\frac{dX}{dB_L}\frac{dB_L}{d\theta_L}\frac{\partial p_H}{\partial \Lambda}\frac{\partial \Lambda}{\partial I_L}}{D_1} > 0 \text{ and } \frac{d\theta_L}{dI_L} = \frac{\frac{\alpha(1-\alpha)}{[\Psi-(1-\alpha)]^2}\frac{d\Psi}{dB_H}\frac{dB_H}{d\theta_H}\frac{\partial p_H}{\partial \Lambda}\frac{\partial \Lambda}{\partial I_L}}{D_1} < 0,$$

where

$$D_1 = \frac{1}{\rho}\frac{dX}{dB_L}\frac{dB_L}{d\theta_L}\left(\frac{dp_H}{d\theta_H} - \frac{\partial B_H}{\partial \theta_H}\right) - \frac{\alpha(1-\alpha)}{[\Psi-(1-\alpha)]^2}\frac{d\Psi}{dB_H}\frac{dB_H}{d\theta_H}\frac{\partial p_H}{\partial \theta_L} < 0.$$

The results regarding the unemployment variables and the wage rates follow immediately from equations (30) and (32).

Derivation of equation (33)

Substituting (A10) in (A11) yields

$$(r+n)J^U_{ij} = b_i + \frac{\beta}{1-\beta}c_i\theta_i. \tag{A17}$$

Next substitute (A17) in (1) to get (33).

Proof of Proposition 5. Differentiating (33) we obtain

$$\frac{d\lambda}{d\theta_H} = -\frac{1}{\overline{z}(r+n)}\frac{\beta}{1-\beta}c_H < 0 \text{ and } \frac{d\lambda}{d\theta_L} = \frac{1}{\overline{z}(r+n)}\frac{\beta}{1-\beta}c_L > 0.$$

Next differentiate equations (28) and (A13) to get

$$\frac{d\theta_H}{dI_L} = \frac{-\frac{1}{\rho}\frac{dX}{dB_L}\frac{dB_L}{d\theta_L}\frac{\partial p_H}{\partial \Lambda}\frac{\partial \Lambda}{\partial I_L}}{D_2} > 0 \text{ and } \frac{d\theta_L}{dI_L} = \frac{\frac{\alpha(1-\alpha)}{[\Psi-(1-\alpha)]^2}\frac{d\Psi}{dB_H}\frac{dB_H}{d\theta_H}\frac{\partial p_H}{\partial \Lambda}\frac{\partial \Lambda}{\partial I_L}}{D_2} < 0,$$

where

$$D_2 = \frac{1}{\rho}\frac{dX}{d\theta_L}\left(\frac{\partial p_H}{\partial \theta_H} + \frac{\partial p_H}{\partial \lambda}\frac{d\lambda}{d\theta_H} - \frac{\partial B_H}{\partial \theta_H}\right) - \frac{\alpha(1-\alpha)}{[\Psi-(1-\alpha)]^2}\frac{d\Psi}{d\theta_H}\left(\frac{\partial p_H}{\partial \theta_L} + \frac{\partial p_H}{\partial \lambda}\frac{d\lambda}{d\theta_H}\right) < 0.$$

Comparing these derivatives with the ones derived in Proposition 4, it follows that, starting from the same equilibrium, the effect of a change in I_L is smaller, in absolute value, on both θ_H and θ_L when λ is endogenously determined. The other results follow immediately from equations (30) and (32).

Proof of Proposition 6. If $\rho = 1$ then equation (27) simplifies to $\alpha = B_L$. If λ is exogenous (endogenous) then this equation involves θ_L and I_L (θ_H, θ_L and I_L). Similarly, equation (28) simplifies to $\alpha = B_H$, which involves just θ_H, i.e., it is independent of θ_L and I_L. Simple differentiation shows that

$$\frac{d\theta_L}{dI_L}|_{\lambda \text{ fixed}} = \frac{\frac{\lambda}{(\lambda+I_L)^2}}{D} > \frac{d\theta_L}{dI_L}|_{\lambda \text{ variable}} \frac{\frac{\lambda}{(\lambda+I_L)^2}}{\frac{I_L}{(\lambda+I_L)^2}\frac{d\lambda}{d\theta_L} + D} > 0.$$

where

$$D = \frac{c_L}{1-\beta}\frac{\beta m'(\theta_L)q(\theta_L) - q'(\theta_L)[r+n+s_L+\beta m(\theta_L)]}{[q(\theta_L)]^2} > 0$$

The results regarding the unemployment variables (u_H and u_{LN}) and the wage rates (w_H and w_L) follow immediately from equations (30) and (31).

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Table 1: Parameterization of the baseline model: general case

	Technology
$\rho = 0.0401, \gamma = -0.495$	Krusell et al. (2000)
$\alpha = 0.538, x = 0.800$	The US average share of labor in total output and capital-output ratio.
$\delta = 0.10$	Consistent with estimates for the US.
$r = 0.004$	The monthly interest rate.
	Population
$\bar{z} = 91.5$	Average share of US labor force with a BA degree. †
$I_L = 0.142, I_H = 0.047$	The US share of foreign born labor force and share of foreign-born labor force with a BA degree. ‡
$n = 0.0067$	The average US natural rate of population growth: 1991-2000, Ben-Gad (2007).
	Matching and Separations
$s_L = 0.040, s_H = 0.024,$ $M = 0.715$	Average US job finding rate, separation rate and vacancy to unemployment ratio.
$\epsilon = 0.5$	Standard, within the range of estimates in Petrongolo and Pissarides (2001).
	Vacancies and Unemployment
$c_L = 0.546, c_H = 0.819$	Less than 5% of output devoted to job creation activities and proportional to unskilled and skilled wages, respectively.
$b_L = 0.339, b_H = 0.488$	Average US employment rate and college-plus wage premium.
$h_{LN} = 1.32$	The wage of an unskilled immigrant is 75% of that of an unskilled native, Borjas (2000).
$\beta = 0.5$	Internalizes the search externalities.

† Bureau of Labor Statistics, Current Population Survey.

‡ U.S. Census Bureau, 2009 American Community Survey.

Table 2. The Effects of a 7% Unskilled Immigration-induced Increase in Labor Supply

(Percentage Changes)

	No Human No Cost	**Human No Cost**	**No Human Cost**	**General (Human-Cost)**
		Unskilled Natives		
w_{LN}	-2.1	-0.6	-2.0	-0.5
u_{LN}	2.3	0.7	-7.3	-8.4
		Unskilled Immigrants		
w_{LI}	same as natives	same as natives	-1.1	0.8
u_{LI}	same as natives	same as natives	same as natives	same as natives
θ_L	-4.9	-1.5	17.7	20.7
		Skilled		
w_H	4.5	1.3	5.0	1.6
u_H	-4.8	-1.5	-5.2	-1.8
θ_H	11.1	3.2	11.9	3.9
		Overall Unskilled		
w_L	same as natives	same as natives	-2.9	-1.4
u_L	same as natives	same as natives	same as natives	same as natives
		Overall Natives		
w_N	0.1	0.6	0.3	0.8
u_N	1.0	-0.2	-6.8	-6.9
λ	–	-2.0	–	-2.0
Welfare1	1.1	2.3	1.5	2.7
Welfare2	1.2	2.4	1.8	3.0
		Overall		
w	-0.8	-0.2	-0.7	-0.2
u	1.6	0.4	-6.7	-6.9
Welfare1	-1.5	-0.4	-2.0	-0.9
Welfare2	-1.6	-0.5	-1.9	-0.8

Notes: Human (No Human) means that there is (not) endogenous skill acquisition. Cost (No Cost) means that there are differential search costs between unskilled immigrants and natives. The variable w indicates the wage rate, u the unemployment rate, θ the tightness in the labor market. The subscript L stands for unskilled, H for skilled, N for native and I for immigrant. The term "welfare" refers to the welfare per member in the corresponding group. The measure "Welfare1" includes the unemployment benefits, whereas the measure "Welfare2" does not.

Table 3. Sensitivity of the Calibration Results with respect to Production Parameters in the General Model (Human-Cost)

(Percentage Changes)

	$\rho = 0 \quad \gamma = -1$ ($\sigma_{LK} = \sigma_{LH} = 1$, $\sigma_{HK} = 0.5$)	$\rho = 0.5 \quad \gamma = 0$ ($\sigma_{LK} = \sigma_{LH} = 2$, $\sigma_{HK} = 1$)	$\rho = 0.5 \quad \gamma = -1$ ($\sigma_{LK} = \sigma_{LH} = 2$, $\sigma_{HK} = 0.5$)	$\rho = 0 \quad \gamma = 0$ ($\sigma_{LK} = \sigma_{LH} = 1$, $\sigma_{HK} = 1$)
	Unskilled Natives			
w_{LN}	-0.7	-0.5	-0.5	-0.0
u_{LN}	-10.2	-4.7	-10.1	-5.7
	Unskilled Immigrants			
w_{LI}	1.5	-0.2	1.7	0.5
u_{LI}	same as natives	same as natives	same as natives	same as natives
θ_L	26.0	10.7	25.9	13.1
	Skilled Natives			
w_H	2.0	1.2	1.7	1.2
u_H	-2.5	-0.9	-2.1	-1.2
θ_H	5.6	1.8	4.8	2.6
	Overall Unskilled			
w_L	-1.8	-0.9	-1.6	-0.5
u_L	same as natives	same as natives	same as natives	same as natives
	Overall Natives			
w_N	0.9	0.7	0.7	0.3
u_N	-8.3	-3.9	-8.2	-5.1
λ	-2.2	-2.8	-1.8	-1.1
Welfare1	2.5	4.4	2.0	3.4
Welfare2	2.9	4.5	2.4	3.5
	Overall			
w	-0.1	-0.2	-0.2	0.0
u	-8.4	-3.6	-8.3	-5.1
Welfare1	-1.0	-0.5	-1.3	-0.2
Welfare2	-0.7	-0.5	-1.0	-0.2

Notes: See Table 2.

Table 4. Sensitivity of the Calibration Results with respect to the Degree of Substitutability between Unskilled Natives and Unskilled Immigrants in the General Model (Human-Cost)

(Percentage Changes)

	$\eta = 0.85$ $(\sigma_{IN} = 6.7)$	$\eta = 0.90$ $(\sigma_{IN} = 10)$	$\eta = 0.95$ $(\sigma_{IN} = 20)$
	Unskilled Natives		
w_{LN}	0.3	0.0	-0.3
u_{LN}	-13.4	-13.1	-12.8
	Unskilled Immigrants		
w_{LI}	-3.1	-0.2	3.0
u_{LI}	same as natives	same as natives	same as natives
θ_L	36.9	35.9	35.0
	Skilled Natives		
w_H	2.0	1.8	1.6
u_H	-3.6	-3.2	-2.8
θ_H	8.5	7.4	6.5
	Overall Unskilled		
w_L	-3.7	-4.0	-4.3
u_L	same as natives	same as natives	same as natives
	Overall Natives		
w_N	1.3	1.1	0.8
u_N	-10.9	-10.6	-10.4
λ	-1.4	-1.4	-1.3
Welfare1	2.9	2.7	2.4
Welfare2	3.6	3.3	3.0
	Overall		
w	-1.2	-1.5	-1.8
u	-11.1	-10.8	-10.5
Welfare1	-2.5	-2.8	-3.0
Welfare2	-2.0	-2.3	-2.6

Notes: See Table 2.

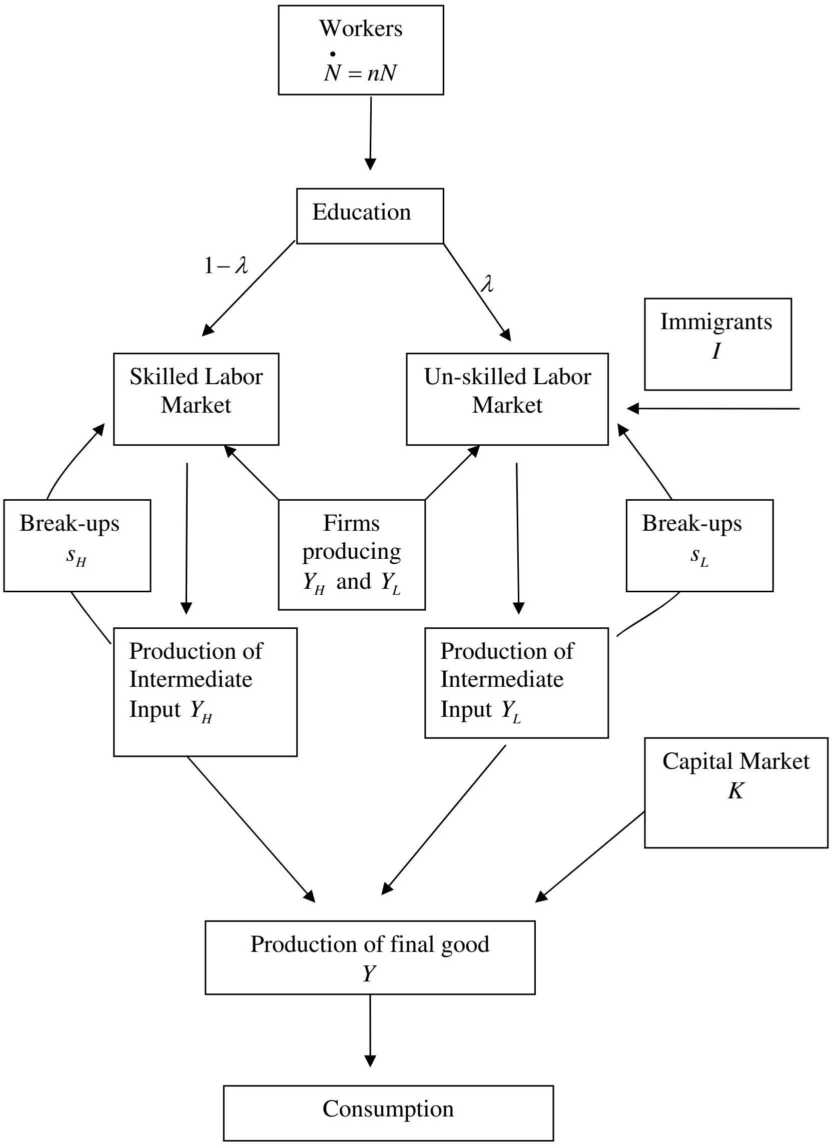

Figure 1. The Structure of the Model

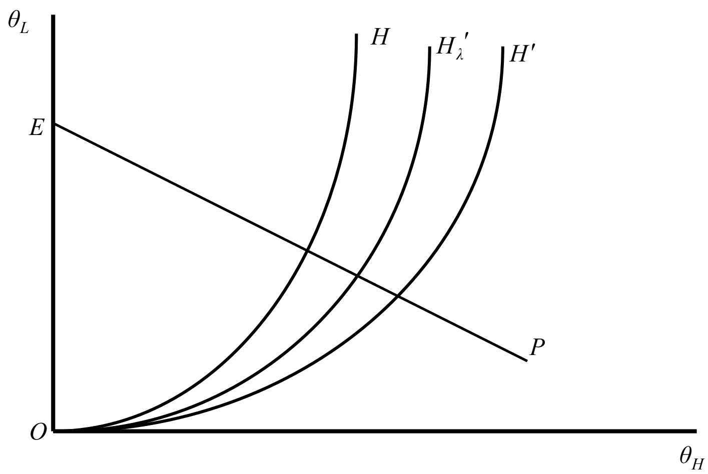

Figure 2. Steady-State Equilibrium without Search Costs

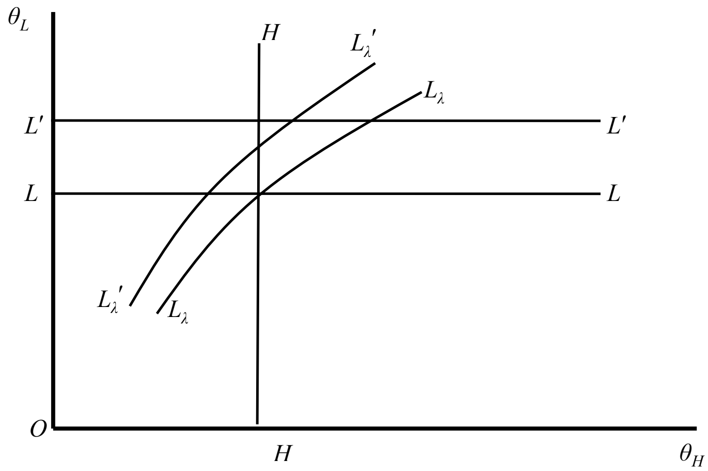

Figure 3. Steady-State Equilibrium with Search Costs and Perfect Substitutability