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Bazhanov, Andrei

Far Eastern Federal University

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Andrei V. Bazhanov[a,b]

[a] *Far Eastern Federal University, Vladivostok, Russia*
[b] *Department of Mathematics and Statistics, Queen's University, Kingston, ON, K7L 3N6, Canada*

Abstract

An extensive literature shows the importance of investment policy for sustainability of resource-based economies. The approaches of these studies are mostly based on theoretical results that examine the role of investments in a competitive optimizing economy. This paper extends some of these results by considering the dependence of current consumption change on investment under distortions causing modification of the standard Hotelling rule (HR). This extension implies that resource policy in the presence of the distortions can be more important for sustainability than under the standard HR. The examples of the analysis for distorted resource-based economies are provided.

Key words:
nonrenewable resource; sustainable development; resource policy; genuine investment; Hotelling rule
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Email address: bazhanov@yahoo.com (Andrei V. Bazhanov)

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1. Introduction

Hamilton and Hartwick (2005, p. 615) noted that "the magnitude of 'net investment' or 'genuine savings' has become a central focus in the measurement of the sustainability of an economy." For example, Pearce and Atkinson (1993) has offered a simple indicator of weak sustainability[1] based on the assertion that "an economy is sustainable if it saves more than the *combined* depreciation on the two forms of capital" (man-made and natural). A variant of this indicator, modified for open economies, has been developed in Proops et al. (1999). These indicators were used in both papers to classify a number of countries into sustainable and unsustainable. Hamilton and Clemens (1999) developed a theory of genuine saving by adding the investment in human capital to traditional net savings and subtracting the value of resource depletion and environmental damage. The value of genuine saving was offered as an indicator of sustainability, and this indicator was used for comparing sustainability of a wide range of developing countries.

Historically, this keen attention to investment policies in empirical research of sustainability originates from importance of investment in economic theory in general; in particular, this interest stems from theoretical studies of perfectly competitive resource-based economies satisfying the standard Hotelling rule (HR) as a necessary condition of dynamic efficiency. For example, Solow (1986) showed for Weitzman's (1976) model with utilitarian criterion and a constant discount rate, coinciding with the fixed rate of interest, that investment of the resource rent into man-made capital (Hartwick investment rule) at a specific moment of time results in a constant consumption path starting from this moment.[2]

Asheim (1994) provided a counterexample, which was infeasible for Weitzman's framework, using the Dasgupta-Heal-Solow-Stiglitz (DHSS) economy (Dasgupta and Heal, 1974; Solow, 1974; Stiglitz, 1974), where the rate of interest asymptotically declines to zero. In this model, consumption declines to zero under utilitarian criterion for any fixed positive discount rate despite satisfaction of the Hartwick rule at some time. The standard Solow (1974) - Hartwick (1977) case implies for the DHSS economy that per capita con-

[1]Weak sustainability of growth (development) is defined by Pezzey (1992) as nondecreasing per capita consumption (utility).

[2]Svensson (1986) noted that the requirement of the constant discount rate is quite restrictive in this model.

sumption is constant at a maximum sustainable level if the Hartwick rule is satisfied at any time. Dixit et al. (1980) generalized the Hartwick rule by showing that net investment (increase in man-made capital minus resource depletion) that is nonnegative and constant over time in present prices is a necessary and sufficient condition for a constant path of utility.[3] Dasgupta and Heal (1979, pp. 303-306), Hamilton and Hartwick (2005), and Hamilton et al. (2006) analyzed the link between genuine investment in current prices and current change in per capita consumption. Hamilton and Withagen (2007) derived the result of Hamilton and Hartwick (2005) in a more general setting, showing that instantaneous utility increases if and only if genuine investment decreases in present prices remaining positive.

However, as Hamilton et al. (2006) fairly noted, "saving effort is ... not the whole story in sustaining development." It is known (e.g., Neumayer 2000) that the main uncertainties for sustainability are connected with the properties of production function, in particular, with technical progress and substitutability between natural and man-made forms of capital. These uncertainties depend of course on investment in R&D and, therefore, not only on the rate but also on the structure of investment.[4]

Uncertainties depend also on the paths of imperfections including such phenomena as insecure property rights, taxes, and other distortions that modify the standard HR. For example, Stollery (1998) considered an externality (climate change) that modified the HR and distorted the Solow-Hartwick path of extraction. This distorted extraction resulted in sustainable bounded growth of per capita consumption for the DHSS model under the standard Hartwick rule. Another example for the same model was provided in Bazhanov (2007), where the properties of transition paths were examined under a modified HR. The Hartwick rule also yielded bounded and unbounded consumption growth depending only on the path of extraction. The standard HR can be violated either in the optimal economy under distortions or in an inefficient economy. Besides dynamic inefficiency, the resource use in the real world can be static-inefficient, wasteful, and even counter-productive.[5]

[3] Constant investment in present prices means that investment in current prices is growing with the rate of discount.

[4] Another problem of empirical sustainability evaluation, discussed in the review of Neumayer (2000), is that the real-life paths of the market price and extraction cost do not work as reliable indicators of the resource scarcity.

[5] The resource use is counter-productive when the decline in the resource stock results

The examples above show the need to further examine the roles of investment and resource policies for economies with imperfections. Straightforward applications of the results derived under the standard HR to real-world situations can form the impression that, for sustainability, it is enough to invest in a proper way into man-made and human capital regardless of the pattern of extraction. As Arrow et al. (2003) showed for imperfect economies, the accounting price of a natural resource can be considerably higher than the market price, implying that the investment of the market resource rent and even the entire marked-valued output into man-made capital can be not enough to compensate for damages in natural capital. In other words, genuine investment in accounting prices can be negative despite any effort in saving. In these cases, sustainability cannot be achieved without estimation and correction of resource extracting policies.

This paper generalizes Proposition 1 of Hamilton and Hartwick (2005) by introducing distortions modifying the standard HR. This result (Proposition 1) shows that current consumption change can be completely determined by the influence of distortion when this influence is not close to zero. The result implies a classification of resource-based economies by the importance of investment or resource policies or both for current consumption change.

Section 3 illustrates Proposition 1 using the examples of distorted economies; Section 4 discusses possible problems with using Proposition 1 for sustainability evaluation, and Section 5 concludes.

2. Investment and growth under distortions

In order to define a distorted economy, it is instructive to introduce first a "perfect" optimizing economy. Following Hamilton and Hartwick (2005, p. 618), assume that the economy is closed, time t is continuous, consumption is aggregated into a single consumption good C, labor is fixed, so that output $Q(t) = F(K, R)$ depends on man-made capital $K(t)$ and the resource flow $R(t) = -\dot{S}(t)$, where $S(t)$ is the current resource stock ($\dot{S} := dS/dt$).

A number of studies, which results were used for practical evaluation of sustainability, assume that the economy satisfies the following:

• $F(K, R)$ is a regular production function that (a) denotes the maximum output for the given K and R, and (b) satisfies the Inada conditions, in particular $F_R > 0$, where $F_R := \partial F/\partial R$ (*resource productiveness*);

in the decline of output, e.g., as a result of a wildfire or oil spill.

• output Q equals $F(K,R)$ (*static efficiency*);[6]

• the balance equation holds: $C + \dot{K} = F(K,R) - \delta K$, where $\dot{K}$ is investment and δK with $\delta = const$ is capital decay (*non-wastefulness*);

• the standard HR $\dot{F}_R = rF_R$[7] holds as a necessary condition of *dynamic efficiency*;

• the paths in the economy maximize a welfare function (*optimality*).

In the real world, however, the resource use can be

– counter-productive: $F_R \leq 0$;

– productive, but static-inefficient: $Q < F(K,R)$;

– productive, static-efficient, but wasteful: $C + \dot{K} < F(K,R) - \delta K$;

– productive, non-wasteful, efficient, but non-optimal.

This paper provides another small step towards practical evaluation of sustainability. The paper extends Hamilton and Hartwick (2005) only by assuming that the HR deviates from its standard form. The vector $\mathbf{D}(t) = (D_1(t), D_2(t), D_3(t), D_4(t))$ called *distortion* denotes possible sources of the deviation. Here, D_i are the distortions in

$$\begin{aligned} \text{production:} &\quad F = F(K,R,D_1), && (1)\\ \text{social utility:} &\quad U = U(C,D_2), && (2)\\ \text{the balance equation:} &\quad \dot{K} = F(K,R,D_1) - C - \delta K - D_3, && (3)\\ \text{the dynamics of the stock:} &\quad \dot{S} = -R + D_4. && (4) \end{aligned}$$

The distortions can include imperfections, externalities, and any effects (including favorable for sustainability) that cause violation of the standard HR.

Assume, for simplicity, that $\mathbf{D}$ depends only on the extracted amount $S_0 - S(t)$.[8] For example, D_1 and D_2 can result from irreversible damages

[6] Conventionally, efficiency is defined via the Pareto-optimality. Some studies, e.g. Hurwicz (1960), called this notion non-wastefulness.

[7] Here, $r(t) := F_K(t) - \delta$ is the market interest rate.

[8] $\mathbf{D}$ can also depend on the rate of extraction, e.g., when damage includes the opportunity cost (Gaudet et al., 2006), or when damage is partly reversible. Then formula (6) below is more complicated, which, however, does not alter the conclusions of the paper. $\mathbf{D}$ can also depend on the amount of non-extracted resource, e.g., when the stock has an amenity value (D'Autume, Schubert, 2008). Then, if this value can be expressed in terms of utility, the problem can be reformulated by introducing the damage resulted from the resource extraction. In practice, this approach can be more precise, since the uncertainty in the extracted amount is essentially less than in the remaining stock. A review of studies with the modified HR can be found, e.g., in Gaudet (2007).

caused by economic activities (e.g., due to climate change); D_3 can stand for the growing cost of extraction (best-quality stock extracted first); D_4 can be the productivity of the stock-augmenting investment, which is, first, growing with the extraction due to learning-by-doing and eventually declining due to the scarcity of the resource. Then the following result holds.

Lemma 1. *If economy (1)-(4) is dynamically efficient, then*

$$\dot{F}_R = [v(t) + \tau(t)]\, F_R, \tag{5}$$

where $v(t) := F_K - \delta$,[9] *and* $\tau(t) = \tau\,[\mathbf{D}(t)] :=$

$$\frac{1}{F_R}\left[\frac{U_{D_2}\partial D_2/\partial(S_0 - S)}{U_C} + F_{D_1}\frac{\partial D_1}{\partial(S_0 - S)} - \frac{\partial D_3}{\partial(S_0 - S)}\right] + \frac{\partial D_4}{\partial(S_0 - S)} \tag{6}$$

is the additive HR modifier *or the influence of* $\mathbf{D}$.[10]

Proof is in Appendix.

Genuine investment is defined in Hamilton and Hartwick (2005) as

$$G(t) := \dot{K}(t) + \dot{S}(t)F_R(t). \tag{7}$$

This measure includes not only current investment into man-made capital but the value of the currently extracted resource measured in marginal resource productivity, which, with no distortion, coincides with the market price. Under distortions, the link between G and $\dot{C}$ takes the following form.

Proposition 1. *Current consumption change is*

$$\dot{C} = (v - \dot{G}/G)G + \Psi, \tag{8}$$

where $\Psi := \dot{D}_1 F_{D_1} - \dot{D}_3 - \tau[\mathbf{D}]F_R R$ *is the influence of the distortion* $\mathbf{D}$.

Proof is identical to the proof of Proposition 1 in Hamilton and Hartwick (2005). The only difference is that formula (5) is used here to substitute for $\dot{F}_R$ instead of the standard HR. Namely, equations (3), (1), and (7) give $\dot{C} = \dot{K}F_K + \dot{R}F_R + \dot{D}_1 F_{D_1} - \delta\dot{K} - \ddot{K} - \dot{D}_3 = \dot{K}F_K + \dot{R}F_R + \dot{D}_1 F_{D_1} - \delta\dot{K} - \ddot{K} -$

[9] $v(t)$ is the market interest rate only with no distortion.

[10] With no distortion, $\tau = 0$. The influence of $\mathbf{D}$ in equation (5) can be expressed in a multiplicative form: $\dot{F}_R = v\eta\,[\mathbf{D}]\,F_R$, where $\eta\,[\mathbf{D}] := 1 + \tau\,[\mathbf{D}]\,/v$.

$\dot{D}_3 + R\dot{F}_R - R\dot{F}_R \; = \; (F_K - \delta)\dot{K} - (F_K - \delta)F_R R - \dot{G} - \tau[\mathbf{D}]F_R R + \dot{D}_1 F_{D_1} - \dot{D}_3$
$= vG - \dot{G} + \dot{D}_1 F_{D_1} - \dot{D}_3 - \tau[\mathbf{D}]F_R R$ ■

With no distortion ($\Psi = 0$), Proposition 1 coincides with the result of Hamilton and Hartwick (2005) and with the result of Hamilton and Withagen (2007), expressed in present prices. Equation (8) shows, that investment (7) can indeed determine $\dot{C}$ if Ψ is relatively small. However, $\dot{C}$ can be also completely determined by $\mathbf{D}$ when the term $\left(v - \dot{G}/G\right) G$ is close to zero.

Of course, sharp changes in G can determine an *instant* sign of $\dot{C}$ despite the large values of Ψ. Formula (8) shows that even when Ψ has a large positive (negative) value for some $t = \bar{t}$, $\dot{C}(\bar{t})$ can be negative (positive) if $G(\bar{t})$ is negative (positive) and $\dot{G}(\bar{t})/G(\bar{t})$ has a large positive (negative) value. However, these cases are not relevant to sustainability due to the boundedness of investments, whereas distortions in general are less restricted. The boundedness of investment implies that the larger is Ψ, the shorter is the period of time when these cases are possible. Therefore, neglecting the short-run oscillations, it can be assumed that $\left|\dot{G}/G\right| < v$ along the long-run trends.[11] Assume also, for determinateness, that $v(t) > 0$ for all $t \geqslant 0$ and the current investment $\dot{K}$ is bounded by the current output Q. Then a *feasible investment* can be defined as follows.

Definition 1. *Investment* $\dot{K}(t) = w(t)Q(t)$ *is feasible if* $w(t) \in (0,1)$ *and* $|\dot{G}/G| < v$ *for any* $t \geqslant 0$.

Definition 1 results in the following Corollaries.

Corollary 1. *Equation (8) implies that*

$$\text{(I) } \dot{C} \gtreqless 0 \text{ iff}$$
$$\Psi \gtreqless -\left(v - \dot{G}/G\right) G \text{ or } G \gtreqless -\Psi/\left(v - \dot{G}/G\right); \tag{9}$$

(II) *a feasible investment policy can change the sign of* $\dot{C}$ *iff*

$$-\left(v - \dot{G}/G\right)(Q - RF_R) < \Psi < \left(v - \dot{G}/G\right) RF_R. \tag{10}$$

[11] The analysis can be easily complemented by the case with $\left|\dot{G}/G\right| \geqslant v$.

Corollary 2. *If* $\dot{D}_1 F_{D_1} - \dot{D}_3 = 0$, *equation (8) implies that*

(I) $\dot{C} \gtreqless 0$ *iff*

$$\tau \lesseqgtr \left(v - \dot{G}/G\right)\left[\dot{K}/\left(RF_R\right) - 1\right] \text{ or } G \gtreqless \tau RF_R/\left(v - \dot{G}/G\right);$$

(II) *a feasible investment policy can change the sign of* $\dot{C}$ *iff*

$$-\left(v - \dot{G}/G\right) < \tau < \left(v - \dot{G}/G\right)\left[Q/\left(RF_R\right) - 1\right].$$

The following examples show that the impact of distortions on the efficacy of investment depends on the level of output and the share of the resource rent in output. Assume that $v(\bar{t}) = 0.06$ and $\dot{G}(\bar{t})/G(\bar{t}) = 0.03$ at $\bar{t} \geqslant 0$.

(a) Large resource-poor economy. Let $Q(\bar{t}) = 101$ and $R(\bar{t})F_R(\bar{t}) = 1$. Then (Corollary 1) an investment policy can change the sign of $\dot{C}(\bar{t})$ iff

$$-3 < \Psi < 0.03.$$

(b) Small resource-rich economy. Let $Q(\bar{t}) = 11$ and $R(\bar{t})F_R(\bar{t}) = 10$. Then an investment policy can affect the sign of $\dot{C}(\bar{t})$ iff

$$-0.03 < \Psi < 0.3.$$

It is intuitive that a large economy has more opportunities in investment than a small one, and so the range for Ψ, in which investment is able to affect the sign of consumption change, is about ten times larger in case (a) than in case (b). Another difference between these two cases is that investment in a large resource-poor economy can change the sign of $\dot{C}$ mostly when Ψ affects $\dot{C}$ negatively. In this example, the range of negative Ψ, in which investment is able to compensate for the influence of distortion, is 100 times larger than the range of positive Ψ, which effect can be annihilated by negative G. This asymmetry is inverted in a small resource-rich economy.

Boundedness of investments implies that sustainability evaluation should include the analysis of distortions. Depending on the influence of distortions Ψ, the states of an economy can be subdivided into the following four types.

(A) $\Psi \leqslant -\left(v - \dot{G}/G\right)\left(Q - RF_R\right)$: consumption declines regardless of saving; sustainability can be improved only by reduction of the distortion if it is still possible.

(B) $-\left(v-\dot{G}/G\right)(Q-RF_R)\ <\ \Psi\ <\ 0$: consumption growth can be achieved by investment policy alone; the optimal saving rate can be higher than under $\Psi=0$ in order to compensate not only for the exhausting natural capital but for the negative effect of distortions. Without a policy reducing this effect, the level of consumption can be lower than under $\Psi=0$.

(C) $0<\Psi<\left(v-\dot{G}/G\right)RF_R$: the optimal saving rate can be lower than under $\Psi=0$ due to the positive effect of Ψ; decline in consumption is still possible when $G<0$.

(D) $\Psi>\left(v-\dot{G}/G\right)RF_R$: consumption grows regardless of investments; investment policy is important as a determinant of the level of consumption along the growing path.

Condition (9) shows that, for $\Psi<0$, the minimum investment G, providing non-declining consumption, can be essentially higher than zero. The following section illustrates that in some cases the sustainable minimum of G does not exist.

3. Examples of distortions

3.1. Insecure property rights

Following Arrow et al. (2003, p. 664), assume that the owner i $(i=1\ldots N; N\geqslant 2)$ extracts a liquid resource from the pool with the stock S_i. All N owners are identical, non-cooperative, and the pools are separated by porous barriers. The resource diffuses from larger pools to smaller ones with the same rate $\lambda>0$.[12] Then the depletion equations are $\dot{S}_i=\lambda\sum_{j\neq i}(S_j-S_i)-R_i, i=1...N$, where R_i is the rate of extraction of the owner i. The necessary conditions for PV-maximization of the each owner's utility yield equation (5) with $\tau=(N-1)\lambda>0$. This distortion negatively affects consumption (Proposition 1), and causes socially inefficient paths. Let D_1 and D_3 are constants, $v=0.06$, and $\dot{G}/G=-0.04$. Then (Corollary 2) a feasible investment policy can change the sign of $\dot{C}$ iff

$$(N-1)\lambda<0.1\left[Q/\left(RF_R\right)-1\right].$$

When this condition is not satisfied, only institutional changes or resource policies can prevent decline in consumption. Consumption is not declining

[12] No barriers corresponds to $\lambda\to\infty$.

here if

$$w \geqslant [10(N-1)\lambda + 1]\,(RF_R/Q) \quad \text{or} \quad G \geqslant 10(N-1)\lambda RF_R > 0,$$

which is very restrictive for $N > 1$.

It is illustrative to consider two cases.

(a) Large resource-poor economy ($Q = 101$; $RF_R = 1$). In this case, $\dot{K}$ can change the sign of $\dot{C}$ iff $(N-1)\lambda < 10$, which means, e.g., that, for $\lambda = 1$, consumption declines for any investment (type A) if $N \geqslant 11$. Let $N = 5$. Then the saving rate, compensating for the shrinking resource and inefficiency, should be no less than $w_{\min} = \frac{41}{101}$ (or $G \geqslant 40$), whereas with no distortion ($N = 1$), consumption grows for any $w > \frac{1}{101}$ (or $G > 0$).

(b) Small resource-rich economy ($Q = 11$; $RF_R = 10$). $\dot{K}$ can change the sign of $\dot{C}$ iff $(N-1)\lambda < 0.01$, i.e., for $\lambda \geqslant 0.01$ and $N \geqslant 2$, consumption declines regardless of any feasible investment. Let $\lambda = 0.009$ and $N = 2$. Then not declining consumption is possible when almost all output is being invested, namely, $w \geqslant \frac{10.9}{11}$ (or $G \geqslant 0.9$), although, for this resource-dependent economy, even with no distortion, the saving rate yielding at least constant consumption must be very high, namely, $w_{\min} = RF_R/Q \approx 0.91$.

3.2. Resource-augmenting technical change

Assume that the dynamics of the resource stock is $\dot{S} = -R + S\phi(L_R/L)$, where L_R/L is the share of the resource-augmenting research sector and $\phi(\cdot) \geqslant 0$ is the rate of growth of the resource stock due to research (Takayama, 1980). Under the constant D_1 and D_3, this problem yields condition (5) with $\tau = -\phi$, which corresponds to the economy of type C or D depending on the behavior of ϕ. Consumption is growing if $G > -\phi RF_R/\left(v - \dot{G}/G\right)$.

This form of the distortion D_4 can describe the policy of a resource-rich country that enjoys a high level of non-declining consumption, despite negative G, due to the reserve-expanding research. However, for a nonrenewable resource, this temporary prosperity does not relate to sustainability, since the return of the stock-augmenting investment is, in reality, eventually declining function of the extracted resource.

3.3. Irreversible climate change

Stollery (1998) examined a problem with the damage $D_1 = D_2 = D$ caused by climate change,[13] resulting from oil use ($D_{S_0 - S(t)} > 0$). D nega-

[13] For the various forms of damage function in this case, see Weitzman (2010).

tively affects social utility and production: $U_D \leqslant 0, F_D \leqslant 0, U_C > 0$. This problem yields equation (5) with $\tau = (F_D + U_D/U_C)D_{S_0-S(t)}/F_R$. The influence of D on $\dot{C}$ is $\Psi = \dot{D}F_D - (F_D + U_D/U_C)RD_{S_0-S(t)}$. Since $\dot{D} = -D_{S_0-S(t)}\dot{S} = RD_{S_0-S(t)} > 0$,[14] then $\Psi = -RD_{S_0-S(t)}U_D/U_C$.

Stollery used the Bellman-Jacobi-Hamilton equation to show that $G \equiv 0$ maximizes a welfare function $W := \int_0^\infty \overline{U}\delta e^{-\delta t}dt \equiv \overline{U} = const.$[15] Proposition 1 yields a more general result. Namely, the current utility change is $\dot{U} = U_C\dot{C} + U_D\dot{D}$, and

$$\dot{C} = \left(v - \dot{G}/G\right) G - \dot{D}U_D/U_C. \tag{11}$$

Then

$$\dot{U} = \left(v - \dot{G}/G\right) GU_C, \tag{12}$$

i.e., the result of Hamilton and Hartwick (2005) is valid in this economy.

Formula (11) shows that this efficient and optimal economy corresponds to type C or type D when $U_D < 0$. Consumption grows here even with $G < 0$, when $G > \dot{D}U_D/\left[U_C\left(v - \dot{G}/G\right)\right]$, due to a reallocation of the part of the resource from the present to the future in comparison with the Solow-Hartwick case (no damage). This reallocation results from a positive declining tax. The tax causes the lower rate of the initial extraction (lower initial consumption) and the slower rate of the decline in the rates of extraction or the thicker tail of the distribution of the resource among generations. However, growing consumption with $G < 0$ results in declining utility when $\dot{G}/G < v$, which means that the damage can override the benefits from growth, and that consumption is not always a good proxy for utility.

4. Multiple resources and distortions

Proposition 1 can be generalized in a straightforward way for n resources and m types of distortions.[16] Then formula (7) becomes

$$G(t) := \dot{K}(t) - \sum_{i=1}^{n} R_i(t)F_{R_i}(t), \tag{13}$$

[14] Damage depends on time here only via the path of extraction.

[15] This representation of the maximin was offered by Leonard and Long (1992).

[16] For simplicity, m is the same for $D_1, \ldots, D_4$.

and, applying the same approach for the proof, the combined influence of the distortions $\mathbf{D}_1, ..., \mathbf{D}_m$ on consumption can be defined as

$$\Psi := \sum_{j=1}^{m} \left(\dot{D}_{j1} F_{D_{j1}} - \dot{D}_{j3} \right) - \sum_{i=1}^{n} \tau_i F_{R_i} R_i, \tag{14}$$

where τ_i is the influence of distortions on the HR (5) for the resource R_i.

Equations (8) and (14) show that the combined effect of all distortions on the current change of aggregate consumption can be positive despite the unsustainable extraction of some resources. The problem originates from the assumption that the components of consumption are substitutes. This assumption implies a specific way of aggregation of all the factors that can influence consumption and utility. Then, for example, according to formula (14), a common pool situation ($\tau > 0$) for one resource can be compensated by the resource-augmenting investment ($\tau < 0$) for another resource. A sustainability indicator using this aggregation will show "total sustainability" despite the known problems in the future.

A natural way of solving this problem is a disaggregation of consumption, for example, by separating some factors (e.g., transport, food, or health) that are complements in real life. Then the change in utility could be $\dot{U} = \frac{d}{dt} [\min \{f_1(C_1), ..., f_l(C_l)\}]$, where $C_1, ..., C_l$ are complements in consumption and $f_1, ..., f_l$ are monotonically increasing functions. If $f_k(C_k) = \min \{f_1(C_1), ..., f_l(C_l)\}$, then the bottleneck factor C_k determines the current consumption and utility changes implying the reinterpretations of formulas (13) and (14), where capital and production function would relate to the industry, producing C_k, rather than to the whole economy. This approach leads to policies similar to the maximin principle – the main effort is concentrated on the improvements in the "weakest" industry.

However, this replacement of one scalar indicator by another cannot solve the problem in general because the concentration on improvement in the most vulnerable sector can be accompanied by increasing unsustainability in another. Then, in order to verify sustainability of all vital aspects of consumption, sustainability should be evaluated independently for each aspect.

Unfortunately, even in the case of disaggregated evaluation, the use of formulas (8), (13), and (14) in sustainability indicators cannot prevent troubles in the future. For example, negative values of τ_i can result in positive values of sustainability indicators. But $\tau_i < 0$ is associated with the slower decline in the rates of extraction (Sections 3.2 and 3.3) and even with the growing rates,[17] implying that government can stimulate resource-extracting industries in order to increase the value of the indicator. It is known that this policy, indeed, provides the short-run benefits but eventually ends up in overextraction followed by a sharp decline in the extraction rates and consumption. The sharp decline in the extraction rates is linked to the switch to positive values of τ_i, and this switch cannot be prevented by government interventions due to the limitedness of the resource.[18] This sharp decline in resource input will result in declining consumption contradicting the definition of sustainability.

The influence of distortions on sustainability is similar to the one of investment policies due to the boundedness of the resource stock. Indeed, formula (8) with $\Psi \equiv 0$ shows that consumption can grow in the short run due to declining investments even when $G < 0$ (when $\dot{G}/G > v$), although this case has nothing to do with sustainability, since the boundedness of the saving rate and the resource reserve will eventually result in $\dot{G}/G < v$ and in $\dot{C} < 0$. Future growth in consumption is possible, but only after a period of decline. The case with $G > 0$ is different because this "reserve of investment" can be used during the infinite period of time by maintaining this level of G or asymptotically diminishing it to zero, which will positively affect consumption change during this period. However, a positive G by itself cannot guarantee sustainability, because consumption will decline when $\dot{G}/G > v$.

In the same way, the instant sign of Ψ shows only current influence of distortions on $\dot{C}$, whereas sustainability depends on the trends. Hence, Proposition 1 and an indicator based on formula (8) can, of course, provide useful policy recommendations for underinvesting and overextracting economies; however, such an indicator, calibrated for a specific economy, cannot guarantee even theoretically the existence of at least one economic program with non-declining consumption during a long period of time, and it cannot show if the ability of the economy to maintain non-declining consumption is improving.[19]

[17] See, e.g., Bazhanov (2008, formula (2)) and Bazhanov (2011).

[18] To be more precise, the limitedness of the resource implies that the switch to the rates of extraction declining slower ($\tau_i < 0$) than in the Solow-Hartwick case ($\tau_i \equiv 0$) after a period of overextraction is possible but only after a period of faster decline ($\tau_i > 0$), causing decline in consumption.

[19] E.g., in an overconsuming economy with $\dot{C} > 0$, this ability is declining.

5. Concluding remarks

The paper has extended the result of Hamilton and Hartwick (2005) regarding the role of genuine investment in current consumption change by introducing distortions modifying the Hotelling rule (HR). Proposition 1, Corollary 1, and Corollary 2 have shown that current consumption change can be determined by genuine investment, measured in the marginal resource productivity, only when the influence of the distortions is close to zero. These results entail a classification of the status of a resource-based economy by the importance of investment or resource policies or both for current consumption change. It was shown that the distortions asymmetrically affect the ability of investment to control current consumption change, and this asymmetry is different for capital-rich-resource-poor and for capital-poor-resource-rich countries.

The results imply the additional challenges for empirical evaluation of sustainability: except trade effects, like in Proops et al. (1999), the influence of the distortions should be taken into account. Besides, a resource policy, based on these results, should be more conservative than it can be prescribed by expression (8) in Proposition 1, since this proposition extends the previous studies only by introducing the modified HR, not considering other real-life imperfections. Another problem with expression (8) is that it does not show the change in the ability of the economy to maintain non-declining consumption (utility) during a long period of time. Arrow et al. (2003) showed that, in order to estimate current sustainability change, the accounting prices can be used for measuring genuine investment, when these prices are observable. Thus a sustainability indicator presumably should contain not only the values of current investment and the rate of extraction, but the amounts of capital, resource reserve, and the information about moderate (preferably underestimating) assumptions concerning the paths of production possibilities.

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7. Appendix. Proof of Lemma 1

Since the optimal paths are efficient, a necessary condition of dynamic efficiency for economy (1)-(4) can be obtained from optimality conditions, e.g., in the problem of PV-maximization[20] of $\int_0^\infty U(C,D_2)e^{-\rho t}dt$ with a constant discount rate ρ. The Hamiltonian of this problem is $H = U(C,D_2)e^{-\rho t} + \mu_K(F - C - \delta K - D_3) + \mu_S(D_4 - R)$, and the Pontryagin-type necessary conditions are

$$H_C = U_C e^{-\rho t} - \mu_K = 0, \tag{15}$$

$$H_R = \mu_K F_R - \mu_S = 0, \tag{16}$$

$$\dot{\mu}_K = -\frac{\partial H}{\partial K} = -\mu_K (F_K - \delta), \tag{17}$$

$$\begin{aligned} \dot{\mu}_S &= -\frac{\partial H}{\partial S} = -e^{-\rho t}U_{D_2}\frac{\partial D_2}{\partial (S_0 - S)}\frac{\partial (S_0 - S)}{\partial S} - \\ &\mu_K \left(F_{D_1}\frac{\partial D_1}{\partial (S_0 - S)}\frac{\partial (S_0 - S)}{\partial S} - \frac{\partial D_3}{\partial (S_0 - S)}\frac{\partial (S_0 - S)}{\partial S}\right) - \\ &\mu_S \frac{\partial D_4}{\partial (S_0 - S)}\frac{\partial (S_0 - S)}{\partial S}. \end{aligned} \tag{18}$$

Equation (18) with μ_K from (15) becomes

$$\begin{aligned} \dot{\mu}_S &= e^{-\rho t}U_{D_2}D_{2_{(S_0-S)}} + \\ &U_C e^{-\rho t}\left(F_{D_1}D_{1_{(S_0-S)}} - D_{3_{(S_0-S)}}\right) + \mu_S D_{4_{(S_0-S)}}. \end{aligned} \tag{19}$$

The time derivative of equation (16) is $\dot{\mu}_S = \dot{\mu}_K F_R + \mu_K \dot{F}_R$, which, combined with (19), results in

$$\begin{aligned} \dot{\mu}_K F_R + \mu_K \dot{F}_R &= e^{-\rho t}\left[U_{D_2}D_{2_{(S_0-S)}} + U_C\left(F_{D_1}D_{1_{(S_0-S)}} - D_{3_{(S_0-S)}}\right)\right] + \\ &\mu_S D_{4_{(S_0-S)}}. \end{aligned}$$

The last equation after dividing through by F_R and substitutions for $\dot{\mu}_K$ (from (17)) and μ_S (from (16)) becomes

$$\begin{aligned} -\mu_K (F_K - \delta) + \mu_K \frac{\dot{F}_R}{F_R} &= \frac{e^{-\rho t}}{F_R}\left[U_{D_2}D_{2_{(S_0-S)}} + U_C\left(F_{D_1}D_{1_{(S_0-S)}} - D_{3_{(S_0-S)}}\right)\right] + \\ &\mu_K D_{4_{(S_0-S)}}, \end{aligned}$$

[20] The maximin, formulated as $\max_{r,c}\int_0^\infty \overline{U}\delta e^{-\delta t}dt \equiv \overline{U} = const(r,c)$ with the additional constraint $U(C,D_2) = \overline{U}$, yields the same result.

which, divided through by μ_K with substitution for μ_K from (15), yields $\dot{F}_R/F_R = F_K - \delta + \tau(\mathbf{D})$, where $\tau(\mathbf{D})$ is defined by formula (6).

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