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Abstract

An established result of the endogenous growth literature is that laissez-faire equilibria in expanding-varieties models are suboptimal due to the rent-effect: monopolistic pricing drives the equilibrium quantity of each intermediate input below the efficient level, implying that it is optimal to subsidize final producers. This paper shows that, if scale effects are eliminated by introducing R&D spillovers, normative prescriptions change. Since the laissez-faire economy under-invests into R&D activity, the share of resources devoted to intermediates' production increases and this reallocation effect contrasts the rent-effect. In many scenarios, including the polar case of logarithmic preferences, the reallocation effect surely dominates. The equilibrium quantity of each intermediate exceeds the optimal level and the optimal policy consists of taxing, instead of subsidizing final producers because fiscal authorities must redirect the extra-output generated by under-investment towards R&D activity.

Keywords Endogenous Growth, Scale Effects, R&D Externalities, Optimal Policy.

JEL Codes O41, O31.

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1 Introduction

An important strand of the endogenous growth literature emphasizes the role of R&D activity as a crucial source of sustained economic development. In this framework, horizontal (vertical) innovations improve the quantity (quality) of intermediate inputs, and productivity growth results from endogenous technical change. After the seminal contributions of Romer (1987; 1990), most models of R&D-based growth share a typical structure comprising three core sectors: final producers, usually assumed to be perfectly competitive and acting as price-takers; a finite mass of monopolistic firms producing differentiated intermediates; and an R&D sector, developing blueprints of new types of intermediates to be exploited by incumbent monopolists. In this framework, the role of monopolistic competition is relevant in two respects. On the one hand, the possibility of earning monopoly rents represents a crucial incentive to innovate. On the other hand, monopolistic markets generate inefficient allocations under laissez-faire conditions. The second characteristic implies that decentralizing efficient and socially-optimal paths in these market economies requires active public intervention. In order to obtain a positive mark-up, monopolists restrict supply, and the equilibrium quantity of each intermediate employed in final production is inefficiently low. This is a standard *rent-effect*, which implies that restoring efficiency requires subsidizing the purchases of intermediates of final producers.

The optimality of subsidies to final producers has been established in various contexts. Two useful references are the lab-equipment models with expanding-varieties presented in Barro and Sala-i-Martin (2004: p.285-300) and in Acemoglu (2009: p.433-444) – respectively based on Romer (1990) and Rivera-Batiz and Romer (1991). One aspect that appears neglected, however, is the robustness of this result to alternative specifications of the R&D technology that drives economic growth. In Barro and Sala-i-Martin (2004) and Acemoglu (2009), the optimality of subsidies to final producers is formally proved under the assumption that the instantaneous increase in the number of varieties of intermediate products is in fixed proportion with the absolute level of R&D expenditures. This characteristic, however, implies that the model displays pure scale effects: the equilibrium growth rate is proportional to the number of workers employed in final production, which coincides with the population size. For this reason, we will henceforth label this framework as the *Multi-sector Scale Model* (MS-model).

The presence of scale effects in endogenous growth models has been criticized on empirical grounds (e.g. Backus et al. 1992), and the subsequent literature showed that scale effects can be eliminated by means of alternative assumptions.[1] A first approach is that followed by semi-endogenous growth models (Jones, 1995; Kortum, 1997; Segerstrom, 1998), postulating a non-linear relation between the growth rate of the mass of varieties and the employment level in the R&D sector. In this case, population size only has scale effects on aggregate income levels. A second class of models, developed by Dinopoulos and Thompson (1998), Peretto (1998) and Young (1998), assumes that research can increase either productivity within a product line or the total number of available products. The mixed dimension of horizontal and vertical innovations implies that the market structure can absorb scale effects – e.g. because the increase in the number of firms makes each firm more specialized, and the higher technological distance reduces the spillovers among firms (Peretto and Smulders, 2002). A third way to eliminate scale effects is to extend the MS-model by including a linear relation between the growth rate of intermediates' varieties and the rate of R&D investment, measured by the ratio between R&D expenditures and aggregate output. For expositional clarity, we will henceforth refer to this assumption as the *linear-rate law.* This solution is mentioned in Barro and Sala-i-Martin (2004: p.300-302), and features two desirable properties. On the one hand, it eliminates scale effects since the economy's growth rate depends on the population growth rate but not on population size. On the other hand, it is consistent with the empirical observation that productivity growth appears positively related to the ratio between R&D expenditures and output with a relatively stable coefficient.

Focusing on the third approach, it may be stressed that the existing literature does not provide a detailed discussion of optimal policies in the presence of linear-rate laws. However, depending on the way in which the linear-rate law is introduced in the model, the welfare properties of the laissez-faire equilibrium are substantially modified. In particular, if the structural assumptions of the MS-model are maintained, the linear-rate law has to be reconciled with zero-profit conditions in the R&D sector, which suggests introducing externalities in R&D activity. This assumption is conceptually similar to that underlying the analysis of Lucas (1988), where human capital drives growth but does not imply scale effects because the productivity of individual knowledge depends on the average human capital in the society. In the multi-sector framework with expanding varieties, an analogous specification is that the

[1] See Jones (1999) for a detailed discussion.

marginal productivity of R&D expenditures – taken as given at the firm level – increases with the state of technology determined by previous R&D efforts. If the productivity of current research is positively affected by the results of past research, a linear accumulation law arises at the aggregate level. The crucial point is that, given the presence of dynamic externalities, the welfare properties of the decentralized equilibrium differ from those predicted by the MS-model. This paper analyzes the policy implications of the interplay between the rent-effect and the linear-rate law generated by R&D spillovers.[2] In particular, we study a *Linear-Rate Model* which maintains all the assumptions of the benchmark MS-model except for the presence of externalities in the R&D technology.

The present analysis yields three main results. First, the structure of the Linear-Rate model implies that laissez-faire equilibria exhibit a *reallocation effect* with respect to socially-optimal allocations. On the one hand, the laissez-faire economy under-invests in R&D activity, which is not surprising: since private agents do not fully internalize the positive side-effects of current research on future productivity growth, R&D activity is inefficiently low. On the other hand, this misallocation of resources has a specific consequence: a low fraction of output invested in R&D activity implies a greater share directed towards the production of intermediates. Since this mechanism tends to raise the equilibrium quantity of each intermediate input, the reallocation effect contrasts the rent-effect mentioned above. More precisely: in the laissez-faire economy of the Linear-Rate model, the equilibrium quantity of each intermediate tends to be reduced by monopolistic pricing but, at the same time, tends to be increased by the misallocation of resources in disfavor of R&D activity.

The question that naturally arises is which of the two effects dominates. In this regard, our second result is that, in the polar case with logarithmic preferences, the reallocation effect always dominates, generating overshooting in intermediates' production. This result is in contrast with the predictions of the MS-model, where the (i) rent-effect is the only market failure, (ii) equilibrium quantities of intermediates are inefficiently low, and (iii) restoring

[2] For reasons of expositional clarity, the present analysis follows the standard specification of the lab-equipment model with Cobb-Douglas technology. When R&D spillovers are sector-specific and technologies exhibit a substitution elasticity different from unity, the stability and existence properties of equilibrium paths may be altered substantially, as shown in Doi and Mino (2005). Addressing these issues is however beyond the scope of the present analysis, which focuses on the optimal taxation of final producers.

efficiency requires subsidizing final producers. The Linear-Rate model analyzed here, instead, establishes that with unit-elasticity preferences, the optimal policy consists of *taxing* final producers.

The third result of the analysis relates to the robustness of the overshooting effect and of the associated normative prescription. Relaxing the assumption of logarithmic preferences, it is shown that the reallocation effect arising in the laissez-faire economy is strengthened (weakened) by higher (lower) values of the elasticity of intertemporal substitution, denoted by $1/\sigma$. In particular, the reallocation effect surely dominates if the elasticity is above or equal to unity: when $\sigma \leq 1$, the overshooting result is reinforced and the optimal tax on final producers is strictly positive. When $\sigma > 1$, instead, it possible that the elasticity of substitution overcomes a critical threshold whereby the reallocation effect is very weak and dominated by the rent-effect. In this case, intermediates' production is inefficiently low and the final sector should be subsidized, although the optimal subsidy rate will be smaller than that predicted by the MS-model.

The plan of the paper is as follows. Section 2 introduces the linear-rate law in the benchmark model with expanding varieties. Section 3 analyzes market equilibria with and without public intervention. Section 4 derives the socially-optimal allocation by solving a standard centralized problem, and clarifies the differences between the market failures arising in the present model relative to the MS-model. Section 5 derives the main results, and Section 6 concludes.

2 The Decentralized Economy

In order to facilitate the comparison with the MS-model, our set-up follows closely the most popular version of the lab-equipment model. In particular, the market structure and the assumptions regarding firms and households behavior, described in section 2.1, are identical to those made in Barro and Sala-i-Martin (2004: p.285-300) and Acemoglu (2009: p.433-444). The analysis differs in the specification of the dynamic law governing the growth rate of intermediates' varieties: this modification is introduced in section 2.2. In order to discuss optimal policies, the market economy also includes a fiscal authority that subsidizes R&D investment and may tax or subsidize the purchase of intermediate inputs by final producers.

The laissez-faire equilibrium is obtained as a special case of this more general decentralized equilibrium. For the sake of comparability, we initially assume that final producers' purchases of inputs are subsidized at rate b: the normative prediction of the MS-model is that the final sector should be subsidized due to the rent-effect, so that the optimal subsidy rate is $b^* > 0$. The present analysis shows that in the Linear-Rate model, instead, final producers should be taxed under many circumstances, so that the optimal subsidy rate b^* may well be strictly negative.

2.1 Firms and Households Behavior

Final Sector. Output consists of a single consumption good produced under constant returns to scale. The whole sector can be thus represented as a single competitive firm producing output by means of J varieties of differentiated intermediate products, indexed by $j \in [0, J]$, and labor. The technology is

$$Y(t) = L(t)^{1-\gamma} \int_0^{J(t)} x(j,t)^{\gamma} \, dj, \qquad \gamma \in (0,1), \tag{1}$$

where $t \in [0, \infty)$ is the time index, $Y(t)$ is the quantity of output, $L(t)$ is the number of workers and $x(j,t)$ is the quantity of the j-th variety of intermediate input employed (and destroyed) in production. The mass of varieties at time zero is given, $J(0) = J_0 > 0$, and may increase over time due to R&D activity that provides endogenous technological progress in the form of varieties expansion. Each household supplies one unit of labor inelastically, so that $L(t)$ equals population size. Denoting by $\ell > 0$ the constant population growth rate, we have $L(t) = L_0 e^{\ell t}$. The government subsidizes the purchase of each intermediate good $b(j,t)$: in order to focus on symmetric equilibria, we set a constant subsidy rate for each variety $b(j,t) = b$, which may be positive or negative. Denoting the wage rate by $w(t)$ and the price of the j-th intermediate by $p(j,t)$, the profit-maximizing conditions imply

$$w(t) = (1-\gamma) Y(t) / L(t), \tag{2}$$

$$p(j,t) = b + \gamma L(t)^{1-\gamma} x(j,t)^{\gamma-1}. \tag{3}$$

Each variety of intermediate input is produced by a monopolist who holds the relevant patent. As a consequence, the demand schedule (3) is taken as given by each intermediate producer.

Intermediate Sector. The j-th monopolist maximizes instantaneous profits

$$\pi(j,t) = p(j,t)\,x(j,t) - \epsilon x(j,t)$$

subject to (3), where ϵ is a constant marginal cost applying to each variety. The first-order conditions yield the pricing rule

$$p(j,t) = \frac{\epsilon - b(1-\gamma)}{\gamma} \tag{4}$$

for each $j \in [0, J]$, implying that profits and produced quantities are symmetric across varieties:

$$x(j,t) = x(t) = \left[\gamma^2/(\epsilon - b)\right]^{\frac{1}{1-\gamma}} L(t) \tag{5}$$

$$\pi(j,t) = \pi(t) = \frac{(\epsilon - b)(1-\gamma)}{\gamma} x(t). \tag{6}$$

Notice that substituting (5) in (1) we obtain

$$Y(t) = \left[\gamma^2/(\epsilon - b)\right]^{\frac{\gamma}{1-\gamma}} L(t)\,J(t), \tag{7}$$

which shows that output is linear in the number of varieties of intermediate products as well as in population size.

R&D Sector. The mass of monopolistic firms increases over time by virtue of R&D activity pursued by competitive firms. In each instant t, the number of varieties of intermediate products increases as R&D firms develop new blueprints and sell the relevant patent to an incumbent monopolist. The symmetric equilibrium in the monopolistic sector allows us to represent R&D firms as a consolidated R&D sector earning zero profits due to perfect competition and free-entry. Developing blueprints requires R&D investment and, in the aggregate, the innovation frontier is represented by the linear technology

$$\dot{J}(t) = \theta(t)\,Z(t), \tag{8}$$

where $Z(t)$ is aggregate R&D expenditure in the economy, and $\theta(t)$ is the marginal productivity of investment, taken as given at the firm level. Each R&D firm receives a subsidy to investment at constant rate $a > 0$, so that aggregate R&D expenditure consists of total expenditure of firms, denoted by $z(t)$, plus total government spending $az(t)$. We thus have

$Z(t) = z(t)(1+a)$. Denoting by $V(t)$ the value of each patent, the zero-profit condition is[3]

$$V(t) = 1/\left[\theta(t)(1+a)\right]. \tag{9}$$

The value of each patent sold to an incumbent producer equals the present value of future monopoly profits. This implies the standard no-arbitrage condition

$$i(t) = \frac{\pi(t)}{V(t)} + \frac{\dot{V}(t)}{V(t)}, \tag{10}$$

where $i(t)$ is the equilibrium interest rate yielded by private investment.

Government. The public sector finances total expenditures by means of a lump-sum tax $f(t)$ imposed on each household. Ruling out public debt, we set

$$az(t) + bJ(t)x(t) = f(t)L(t) \tag{11}$$

in order to have balanced budget in each instant.

Households. The economy is populated by $L(t)$ identical households. Individual private wealth consists of a fraction $1/L(t)$ of the $N(t)$ total assets in the economy, representing shares of owned firms. Denoting assets per capita by $n(t) \equiv N(t)/L(t)$, the individual wealth constraint reads

$$\dot{n}(t) = (i(t) - \ell)\,n(t) + w(t) - c(t) - f(t), \tag{12}$$

where $c(t)$ is individual consumption. The objective of the representative agent born in instant $t = 0$ is to maximize the present-value utility stream

$$U_0 \equiv \int_0^\infty e^{-\rho t} u(c(t))\,dt = \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\sigma} - 1}{1-\sigma} dt \tag{13}$$

where $\rho > 0$ is the time-preference rate, and $u(c)$ is the iso-elastic instantaneous utility function with $\sigma > 0$. As shown in the Appendix, the maximization of U_0 subject to (12) requires satisfying the transversality condition

$$\lim_{t\to\infty} N(t)\,e^{-\int_0^t i(s)ds} = \lim_{t\to\infty} J(t)\,V(t)\,e^{-\int_0^t i(s)ds} = 0, \tag{14}$$

[3] Aggregate profits of the R&D sector equal $V(t)\dot{J}(t) - Z(t) = V(t)\theta(t)z(t)(1+a) - z(t)$, so that condition (9) maximizes R&D profits for a given marginal productivity $\theta(t)$, and implies zero profits for each firm. The same condition is equivalently obtained assuming free entry in the R&D business for an indefinite number of firms, as in Barro and Sala-i-Martin (2004: Ch.6).

and the first-order conditions yield the usual Keynes-Ramsey rule $\dot{c}(t)/c(t) = \sigma^{-1}(i(t) - \rho - \ell)$. Aggregating across households, consumption growth is given by

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\sigma}\left[i(t) - \rho - (1-\sigma)\ell\right], \tag{15}$$

where $C(t) \equiv L(t)c(t)$ is aggregate consumption. Since the total value of assets in the economy equals the value of firms, $L(t)n(t) = J(t)V(t)$, equation (12) and the previous relations imply the aggregate constraint of the economy (see Appendix)

$$Y(t) = C(t) + Z(t) + \epsilon J(t)x(t), \tag{16}$$

which shows that total output equals aggregate consumption plus total R&D expenditures plus the cost of producing intermediates in each instant.

2.2 Spillovers in the R&D sector

All the assumptions reported in section 2.1 coincide with those made in Barro and Sala-i-Martin (2004: p.285-300) and Acemoglu (2009: p.433-444). The distinction between the Multi-sector Scale Model and the Linear-Rate Model is exclusively based on different specifications of the marginal productivity of investment - that is $\theta(t)$ in equation (8) - which is taken as given at the firm level.

If we set $\theta(t)$ equal to an exogenous constant, say $\nu > 0$, we obtain the MS-model. In this case, the free-entry condition imposes that the patent value equals the true net cost of R&D, and the mass of varieties is in fixed proportion with R&D expenditure:

$$\dot{J}(t) = \nu Z(t).$$

In this paper, we specify a different innovation frontier. Suppose that the marginal productivity of investment $\theta(t)$ is affected by spillovers whereby the productivity of past research efforts increases that of current activity. In the modern growth literature, this type of spillovers are usually formalized as knowledge-stock externalities. For example, models with human capital *à la* Lucas (1988) incorporate an un-compensated transmission across generations induced by public knowledge. The equivalent assumption in the present context is that the R&D activity of each firm is more productive the better the 'current state of technology attained by virtue of previous research'. This concept of state-of-the-art in research can be conveniently measured by the ratio between the number of existing varieties and current output levels, $J(t)/Y(t)$. Formally, we set

$$\theta(t) \equiv \phi \cdot J(t)/Y(t) \tag{17}$$

where $\phi > 0$ is a constant proportionality factor representing the intensity of the externality. Equation (17) implies that the growth rate of intermediates' varieties increases with the economy-wide rate of R&D investment: from (8), we have

$$\dot{J}(t)/J(t) = \phi \cdot (Z(t)/Y(t)). \tag{18}$$

Following the definitions given in the Introduction, equation (18) is a *linear-rate law*. As mentioned in Barro and Sala-i-Martin (2004: p.300-302), linear-rate laws like (18) generally exhibit two desirable properties. First, they eliminate scale effects by making the equilibrium growth rate of output independent of the population size. Second, they fit the data better than the MS-model since, in most industrialized countries, the growth rate of productivity appears to be positively related to the ratio between R&D expenditures and output, with a proportionality coefficient – here represented by ϕ – that is relatively stable over time. The following analysis will show that there exists a third, welfare-related implication. When the linear law (18) is obtained by postulating spillover effects in the R&D sector, as we do here, there exists a *reallocation effect* whereby a market economy under laissez-faire may over-produce each intermediate input as a result of sub-optimal R&D investment. This point has not been stressed in the literature so far but it is relevant from a policy-making perspective: despite the fact that intermediate inputs yield positive monopoly rents, the equilibrium quantity sold on the market may exceed the socially-optimal level. If this is the case, restoring efficiency requires taxing, and not subsidizing, final producers.

The remainder of the analysis proceeds in three steps. First, we characterize the decentralized equilibrium. Second, we identify the socially-optimal allocation with the solution of a standard centralized problem. Third, we characterize the optimal policy by deriving the levels of the subsidy rates that decentralize the optimum in the market economy with public intervention. The following sections analyze each point in turn.

3 Decentralized Equilibria

3.1 General Characteristics

The equilibrium quantities in the decentralized market economy will be denoted by superscript 'E'. As shown in the Appendix, the equilibrium is characterized by a constant rate of return to R&D activity, and therefore by balanced growth in each point in time:

Proposition 1 *In the decentralized equilibrium, the consumption propensity $\chi^E \equiv C^E/Y^E$, the investment rate $\psi^E \equiv Z^E/Y^E$, and the interest rate i^E are constant over time:*

$$\chi^E = 1 - \frac{\epsilon}{\epsilon - b}\gamma^2 - \frac{1}{\sigma}\left[(1+a)\,\gamma\,(1-\gamma) - (\rho/\phi)\right], \tag{19}$$

$$\psi^E = \frac{1}{\sigma}\left[(1+a)\,\gamma\,(1-\gamma) - (\rho/\phi)\right], \tag{20}$$

$$i^E = \phi\,(1+a)\,\gamma\,(1-\gamma) + \ell. \tag{21}$$

The economy follows a balanced growth path along which

$$\frac{\dot{C}^E(t)}{C^E(t)} = \frac{\dot{Z}^E(t)}{Z^E(t)} = \frac{\dot{Y}^E(t)}{Y^E(t)} = \frac{1}{\sigma}\left[\phi\,(1+a)\,\gamma\,(1-\gamma) + \sigma\ell - \rho\right], \tag{22}$$

$$\frac{\dot{J}^E(t)}{J^E(t)} = \phi\psi^E = \frac{1}{\sigma}\left[\phi\,(1+a)\,\gamma\,(1-\gamma) - \rho\right]. \tag{23}$$

in each $t \in [0, \infty)$. (Proof: see Appendix)

The absence of transitional dynamics hinges on the same mechanism of the MS-model: from (7), equilibrium output is linear in the growth rate of varieties. Differently from the MS-model, however, there are no scale effects: from (22), the equilibrium growth rate is not affected by population size $L(t)$. Expression (21) shows that the equilibrium rate of return increases with the spillover parameter ϕ, which determines the productivity of R&D expenditures, and with the associated subsidy rate a. The subsidy on the purchases of intermediate inputs, instead, does not yield growth effects: an increase in b reduces the consumption propensity (19) but does not modify expressions (22)-(23). The main role of this subsidy is to raise the equilibrium quantity of each intermediate product which, from (5), equals

$$x^E(t) = \left[\gamma^2/(\epsilon - b)\right]^{\frac{1}{1-\gamma}} L_0 e^{\ell t}. \tag{24}$$

On the basis of the above results, the laissez-faire equilibrium can be characterized as follows.

3.2 Laissez-Faire Equilibrium

Ruling out public intervention, consider the market economy previously described without taxes and subsidies, and set $a = b = f(t) = 0$ in each instant. Denoting the equilibrium quantities under laissez-faire by superscript 'F', Proposition 1 implies that the consumption propensity $\chi^F \equiv C^F/Y^F$, the investment rate $\psi^F \equiv Z^F/Y^F$, and the interest rate i^F are constant over time, and equal to

$$\chi^F = 1 - \gamma^2 - \frac{1}{\sigma}\left[\gamma(1-\gamma) - (\rho/\phi)\right], \tag{25}$$

$$\psi^F = \frac{1}{\sigma}\left[\gamma(1-\gamma) - (\rho/\phi)\right], \tag{26}$$

$$i^F = \phi\gamma(1-\gamma) + \ell, \tag{27}$$

and the economy follows a balanced growth path along which

$$\frac{\dot{C}^F(t)}{C^F(t)} = \frac{\dot{Z}^F(t)}{Z^F(t)} = \frac{\dot{Y}^F(t)}{Y^F(t)} = \frac{1}{\sigma}\left[\phi\gamma(1-\gamma) + \sigma\ell - \rho\right], \tag{28}$$

$$\frac{\dot{J}^F(t)}{J^F(t)} = \phi\psi^F = \frac{1}{\sigma}\left[\phi\gamma(1-\gamma) - \rho\right]. \tag{29}$$

in each $t \in [0, \infty)$. Moreover, from (24), the equilibrium quantity of each intermediate product equals

$$x^F(t) = \left(\gamma^2/\epsilon\right)^{\frac{1}{1-\gamma}} L_0 e^{\ell t}. \tag{30}$$

As regards the existence of the equilibrium, there are standard restrictions to be imposed on parameters. In particular, the equilibrium is well-defined if and only if parameters satisfy

$$\phi\gamma(1-\gamma) > \rho, \tag{31}$$

since otherwise the investment rate would be non-positive.[4]

As noted before, the laissez-faire equilibrium is inefficient for two independent reasons. First, monopolistic competition in the intermediate sector introduces a wedge between the price and the marginal cost of differentiated inputs. Second, spillovers in R&D activity are not internalized by atomistic agents. The interplay between the two market failures implies that the allocation achieved by the laissez-faire economy differs from the socially-optimal one – i.e., the allocation that would be chosen by a benevolent utilitarian planner endowed with perfect foresight. The social optimum is briefly described below.

[4] If $\rho \geqslant \phi\gamma(1-\gamma)$, equation (26) implies a negative investment rate $\psi^F \leqslant 0$ and equation (29) yields a negative growth rate of varieties $\phi\psi^F \leqslant 0$.

4 Social Optimality

4.1 The Centralized Problem

Consider the social problem solved by a hypothetical central planner endowed with perfect foresight and full control over the allocation. The objective is to maximize the utilitarian social welfare function

$$W \equiv \int_0^\infty L(t)\, u(C(t)/L(t))\, e^{-(\rho+\ell)t} dt = \int_0^\infty \frac{c(t)^{1-\sigma} - 1}{1-\sigma} L(t)\, e^{-(\rho+\ell)t} dt, \tag{32}$$

where the instantaneous welfare function is the sum the utilities of all households in each point in time, and the social discount rate $(\rho + \ell)$ embodies the necessary adjustment for population growth. The maximization is subject to the aggregate constraints of the economy studied in the previous section, which can be written as

$$c(t)\, L(t) = (1 - \psi(t))\, L(t)^{1-\gamma} \int_0^{J(t)} x(j,t)^\gamma\, dj - \epsilon \int_0^{J(t)} x(j,t)\, dj, \tag{33}$$

$$\dot{J}(t) = J(t)\, \phi\psi(t). \tag{34}$$

Equation (33) is the aggregate constraint (16), where we have substituted the investment rate $\psi(t) \equiv Z(t)/Y(t)$ and technology (1): aggregate consumption equals the un-invested fraction of output minus the total cost of producing intermediates. Equation (34) is the dynamic law governing varieties' expansion (18): the use of this constraint implicitly postulates that the existence of R&D spillovers is known to the optimizer. The social planner chooses the sequence of consumption, quantities of intermediates and investment rates using $\{c(t), x(j,t), \psi(t)\}_{t=0}^\infty$ as control variables. The number of varieties $J(t)$ is the state variable, with given initial condition $J_0 > 0$. As shown in the Appendix, the optimality conditions imply balanced growth from time zero onwards. Denoting optimal quantities by superscript '$\star$', we have the following

Proposition 2 *In the social optimum, the consumption propensity $\chi^\star \equiv C^\star/Y^\star$ and the investment rate $\psi^\star(t) \equiv Z^\star/Y^\star$ are constant over time and equal to*

$$\chi^\star = (1-\gamma) \cdot \frac{\phi(\sigma-1)+\rho}{\phi(\sigma-\gamma)}, \tag{35}$$

$$\psi^\star = \frac{\phi(1-\gamma)-\rho}{\phi(\sigma-\gamma)}, \tag{36}$$

where $\sigma > \gamma$ must hold to have a well-defined equilibrium. The economy follows a balanced growth path where

$$\frac{\dot{C}^{\star}(t)}{C^{\star}(t)} = \frac{\dot{Z}^{\star}(t)}{Z^{\star}(t)} = \frac{\dot{Y}^{\star}(t)}{Y^{\star}(t)} = \frac{1}{\sigma}\left\{\phi\left[1-\gamma\left(1-\psi^{\star}\right)\right]+\sigma\ell-\rho\right\} \tag{37}$$

$$\frac{\dot{J}^{\star}(t)}{J^{\star}(t)} = \phi\psi^{\star} = \frac{\phi\left(1-\gamma\right)-\rho}{\sigma-\gamma}. \tag{38}$$

in each $t \in [0,\infty)$. (Proof: see Appendix)

The existence of the optimal path hinges on restrictions that are already satisfied if a well-defined laissez-faire equilibrium exists. In particular, given condition (31), the optimal investment rate $\psi^{\star}$ is strictly positive only if $\sigma > \gamma$. This condition is necessarily met in the limiting case of logarithmic preferences, $\sigma \to 1$.

The centralized allocation chosen by the social planner differs from the *laissez-faire equilibrium* in two respects. First, comparing Proposition 2 with expressions (25)-(29), it follows that the optimal growth rate differs from the laissez-faire growth rate in (28), and the growth gap is in favor of the centralized economy:

$$\frac{\dot{Y}^{\star}(t)}{Y^{\star}(t)} - \frac{\dot{Y}^{F}(t)}{Y^{F}(t)} = \frac{1}{\sigma}\left[\phi\left(1-\gamma\right)^{2}+\gamma\cdot\frac{\phi\left(1-\gamma\right)-\rho}{\sigma-\gamma}\right] > 0. \tag{39}$$

This result is intuitive: since the social planner internalizes the externality contained in $\theta(t)$, the market interest rate i^{F} falls short of the social return to R&D. As a consequence, the laissez-faire economy under-invests into R&D activity: from (26) and (36), the optimal investment rate is higher than the laissez-faire rate:

$$\psi^{\star}-\psi^{F} = \frac{\sigma\phi\left(1-\gamma\right)^{2}+\gamma\left[\phi\gamma\left(1-\gamma\right)-\rho\right]}{\sigma\phi\left(\sigma-\gamma\right)} > 0.$$

The second asymmetry between the social optimum and the decentralized equilibrium is that the optimal quantity of each intermediate product generally differs from the equilibrium quantity sold to final producers by monopolists under laissez-faire conditions. As shown in the Appendix, the optimal path is characterized by

$$x^{\star}(t) = \left[\frac{\gamma}{\epsilon}\cdot\frac{\phi\left(\sigma-1\right)+\rho}{\phi\left(\sigma-\gamma\right)}\right]^{\frac{1}{1-\gamma}} L_{0}e^{\ell t}. \tag{40}$$

Comparing (40) with the equilibrium quantity under laissez-faire (30), we obtain

$$\frac{x^{\star}(t)}{x^{F}(t)} = \left[\frac{1}{\gamma}\cdot\frac{\phi\left(\sigma-1\right)+\rho}{\phi\left(\sigma-\gamma\right)}\right]^{\frac{1}{1-\gamma}}. \tag{41}$$

In general, whether the right hand side of (41) is above or below unity depends on the whole set of parameters. This ambiguity does not arise in the MS-model, where the equilibrium quantities are always below the optimal level. The root of this difference in results is that, in the present model, the combination of monopolistic pricing and R&D spillovers gives rise to two contrasting effects, as clarified below.

4.2 Rent-Effect and Reallocation

The reason for the ambiguous sign in the gap $x^{\star} - x^{F}$ is as follows. On the one hand, the equilibrium quantity of each intermediate under laissez-faire tends to be reduced relative to the optimum because of the usual *rent effect*: monopolistic behavior in the intermediate sector implies a positive mark-up between prices and marginal costs, which tends to restrict supply and thereby the equilibrium quantity x^{F} relative to perfectly competitive environments – that is, the phenomenon arising in the standard MS-model. On the other hand, differently from the MS-model, the laissez-faire economy tends to under-invest in R&D activity due to the externalities in the research sector: private agents fail to recognize the linear relation between investment rates and growth rates of varieties – i.e., equation (18) – so that the laissez-faire investment rate Z^{F}/Y^{F} is inefficiently low. While going to the detriment of R&D activity, this misallocation of resources also implies a greater share of output available for consumption and for producing intermediates: this *reallocation effect* tends to increase the equilibrium quantity of each intermediate under laissez-faire, contrasting the rent effect. The bottom-line is that, in the current model, we have $x^{F} < x^{\star}$ if and only if the rent-effect dominates. When the rent-effect is dominated by the reallocation effect, instead, the laissez-faire economy displays production overshooting in the intermediate sector: x^{F} exceeds the optimal level $x^{\star}$ due to under-investment in R&D.

Before addressing in detail the question of which effect dominates, it is instructive to corroborate the above reasoning by comparing the determination of optimal and laissez-faire quantities of intermediates in the MS-model and in the Linear-Rate model. First, consider the laissez-faire quantities. Since all the assumptions of section 2.1 hold in both frameworks, the laissez-faire equilibrium condition on $x\left(j\right)$ is the same: the marginal productivity of the intermediate equals the marginal cost plus the mark-up,[5]

$$\frac{\partial Y^{LF}(t)}{\partial x^{LF}(j,t)} = \varepsilon \cdot \left(1 + \frac{1-\gamma}{\gamma}\right) \text{ for each } j \in [0, J]. \tag{42}$$

Next, consider the socially optimal quantities that characterize the MS-model and the Linear-Rate model.

Socially optimal quantity in the MS-model. As shown in section 2.2, the MS-model assumes $\dot{J}(t) = \nu Z(t)$. Plugging in this expression the aggregate constraint (16) and technology (1), we have

$$\dot{J}(t)\frac{1}{\nu} = L(t)^{1-\gamma}\int_0^{J(t)} x(j,t)^{\gamma}\,dj - C(t) - \epsilon\int_0^{J(t)} x(j,t)\,dj. \tag{43}$$

Equation (43) is the dynamic constraint of the social problem in the MS-model (cf. Barro and Sala-i-Martin, 2004: p.298). It is immediately apparent that, maximizing (32) subject to (43), the first-order condition with respect to $x(j)$ implies

$$\frac{\partial Y^{MS}(t)}{\partial x^{MS}(j,t)} = \varepsilon \text{ for each } j \in [0, J], \tag{44}$$

where the superscript 'MS' indicates optimal quantities in the Multi-sector Scale Model. Equation (44) is the standard efficiency condition that would arise if intermediates were produced by perfectly competitive firms. Comparing (44) and (42), it follows that the MS-model only exhibits the rent-effect: under laissez-faire, equilibrium quantities tend to be unambiguously lower than in the optimum due to monopolistic pricing. This is the reason why final producers should be subsidized in the MS-model.

Socially optimal quantity in the Linear-Rate model. Results change in our Linear-Rate model because the planner optimizes the rate of R&D investment $\psi(t)$ on the basis of the accumulation law $\dot{J}(t)/J(t) = \phi Z(t)/Y(t)$, which differs from the one recognized by private agents. Maximizing (32) subject to (33)-(34), the first-order condition with respect to $x(j)$ becomes[6]

$$(1 - \psi^{\star}(t))\frac{\partial Y^{\star}(t)}{\partial x^{\star}(j,t)} = \epsilon \text{ for each } j \in [0, J]. \tag{45}$$

Condition (45) shows that, when the growth rate of varieties obeys the linear-rate law (18), each intermediate input should be produced up to the point where its marginal cost equals the

[5] Equation (42) directly follows from setting $b = 0$ in equations (3) and (4).

[6] Equation (45) is used in the derivation of (A11): see Appendix.

un-invested fraction of its marginal product. This result clearly differs from the one obtained in the standard MS-model – that is, condition (44) – due to the presence of the investment rate. The interpretation is the following. If private agents recognized the role of the investment rate $\psi(t)$ in enhancing *future* consumption possibilities, they would restrict the fraction of *current* output devoted to producing intermediates and set $x(j)$ below the quantity that equates the *current* marginal productivity, $\partial Y / \partial x(j)$, to the *current* production cost, ϵ. Because this internalization does not take place in the laissez-faire economy, the equilibrium quantity tends to be increased by the presence of R&D externalities.

The above analysis confirms that, in the Linear-Rate model, $x^{LF}(j)$ is generally sub-optimal for two independent reasons: the rent-effect and the reallocation effect. The fact that these mechanisms push in opposite directions is immediately evident from (42) and (45): the ratio between the marginal productivities is

$$\frac{\partial Y^{LF}}{\partial x^{LF}(j)} \Big/ \frac{\partial Y^{\star}}{\partial x^{\star}(j)} = \frac{1-\psi^{\star}}{\gamma},$$

where the right hand side determines whether $x^{LF} \gtreqless x^{\star}$. Indeed, by (36), the term $(1-\psi^{\star})/\gamma$ coincides with the term in square brackets in (41). Since this term may be above or below unity, the laissez-faire quantity x^{LF} may exceed or fall short of the optimal quantity $x^{\star}$. It must be stressed, however, that the gap $x^{LF} - x^{\star}$ does have unambiguous sign in the polar case of logarithmic preferences, as shown in the next section.

5 Optimal Policy

In the Linear-Rate model, the interplay between monopolistic pricing and R&D externalities generates contrasting effects on the equilibrium quantity of intermediate inputs. The question that naturally arises is which of the two effects is stronger. If the rent-effect dominates, we have $x^{\star}/x^{F} > 1$, and the general policy prescription is similar to that of the MS-model: as the equilibrium quantity is inefficiently low, the optimal policy consists of subsidizing final producers in order to restore efficiency. Instead, if the reallocation effect dominates, we have $x^{\star}/x^{F} < 1$, and the policy prescription is reversed: due to externalities in research, the equilibrium quantity of intermediates is inefficiently high and the optimal policy consists of *taxing* final producers in order to restrict the output share devoted to intermediates' production and to free resources for R&D investment.

For the sake of exposition, the relative magnitude of the rent-effect and the reallocation effect is firstly analyzed in the polar case of logarithmic preferences, which substantially simplifies the analysis. The interesting result is that, letting $\sigma = 1$, the reallocation effect always dominates. In the more general case $\sigma \neq 1$, there exists a critical value $\bar{\sigma} > 1$ below which the same result holds. Consequently, the rent-effect may (but does not necessarily) dominate if σ strictly exceeds unity.

5.1 Logarithmic Preferences

Notice that, from (26), the existence of a laissez-faire equilibrium with $\psi^F > 0$ requires that parameters satisfy inequality (31). Now suppose that preferences are logarithmic. When $\sigma = 1$, expression (41) reduces to $x^\star(t)/x^F(t) = \{\rho/[\phi\gamma(1-\gamma)]\}^{\frac{1}{1-\gamma}}$. In view of the existence condition (31), it follows that $x^\star(t) < x^F(t)$. Hence, logarithmic preferences imply that the reallocation effect dominates: the socially-optimal quantity of each intermediate input is lower than the equilibrium quantity attained under laissez-faire. This implies that, contrary to the prediction of the MS-model, the optimal policy consists of taxing final producers. More precisely, define the optimal policy as the set of instruments $(a^*, b^*, f^*(t))$ which decentralizes the optimal allocation – i.e., the allocation characterized by (35)-(40) – in the decentralized economy with public intervention – i.e., the economy described by (19)-(24). The comparison between Propositions 1 and 2 yields the following result:

Proposition 3 *If $\sigma = 1$, the optimal subsidy rate on final producers is strictly negative, and equal to*

$$b^* = \epsilon \cdot \frac{\rho - \phi\gamma(1-\gamma)}{\rho} < 0. \tag{46}$$

The optimal subsidy to R&D investment is

$$a^* = \frac{\phi(1-\gamma)^3 + \gamma[\phi(1-\gamma) - \rho]}{\gamma\phi(1-\gamma)^2} > 0 \tag{47}$$

(Proof: see Appendix).

As noted before, the fact that R&D activity must be subsidized – i.e., result (47) – is not surprising. The novel result of Proposition 3 is expression (46) – i.e., the fact that the purchase of intermediate goods by final producers must be taxed, not subsidized. Since $\sigma = 1$ is commonly regarded as the polar case in theoretical optimization models, Proposition 3 suggests that the reallocation effect dominates in a wider range of cases. We address this point below.

5.2 The General Case

When $\sigma \neq 1$, the optimal policy is characterized by the optimal subsidy rates (see Appendix)

$$b = b^* = \epsilon \cdot \frac{\phi(\sigma - 1) + \rho - \gamma\phi(\sigma - \gamma)}{\phi(\sigma - 1) + \rho}, \tag{48}$$

$$a = a^* = \frac{\phi(\sigma - \gamma)(1 - \gamma)^2 + \gamma[\phi(1 - \gamma) - \rho]}{\gamma(1 - \gamma)\phi(\sigma - \gamma)} > 0. \tag{49}$$

Expressions (48)-(49) show that, while the optimal subsidy to R&D firms a^* remains positive for any value of the intertemporal elasticity of substitution, the sign of the optimal subsidy to final producers b^* depends on the value of σ. In particular, the derivative

$$\frac{\partial b^*}{\partial \sigma} = \epsilon\gamma\phi \cdot \frac{\phi(1 - \gamma) - \rho}{[\phi(\sigma - 1) + \rho]^2} > 0$$

ensures that b^* is strictly increasing in σ.[7] On the basis of these results, we can prove the following

Proposition 4 *The sign of the optimal subsidy rate to final producers is determined by the intertemporal elasticity of substitution. Defining the critical level*

$$\bar{\sigma} \equiv 1 + \frac{\gamma\phi(1 - \gamma) - \rho}{\phi(1 - \gamma)} > 1, \tag{50}$$

we have $b^* < 0$ if $\sigma < \bar{\sigma}$ and $b^* \geqslant 0$ if $\sigma \geqslant \bar{\sigma}$ (Proof: see Appendix).

Proposition 4 shows that whether the reallocation effect dominates the rent effect crucially depends on the value of σ. If the elasticity parameter is below the critical level $\bar{\sigma}$, the reallocation effect dominates: the laissez-faire equilibrium quantity of intermediates exceeds the socially optimal one, and the optimal policy consists of taxing final producers. Viceversa, if σ is above the threshold value $\bar{\sigma}$, the rent effect dominates and fiscal authorities should subsidize final producers. Indeed, it can be easily verified that the special case $\sigma = \bar{\sigma}$ is associated with

[7] Recall that $\phi(1 - \gamma) - \rho > 0$ is necessary to have a positive optimal investment rate in $\psi^\star > 0$ in (36).

$x^{\star}(t) = x^{F}(t)$ (see expression (41) above). Since $\bar{\sigma} > 1$, the logarithmic case $\sigma = 1$ necessarily belongs to the scenario $\sigma < \bar{\sigma}$.

The general insight of this subsection is that the reallocation effect is weaker the higher is σ. In fact, σ determines whether, in response to a variation in the interest rate, consumers are more willing to smooth the consumption profile or to postpone consumption. When $\sigma < 1$, the reallocation effect is stronger: if agents knew that the productivity of R&D investment were higher than the level perceived by atomistic firms, they would decide to invest more into R&D activity, and the additional investment would be relatively high because $\sigma < 1$ implies that the willingness to postpone consumption overcomes the willingness to smooth the consumption profile. This explains why, in the case $\sigma < 1$, a benevolent planner would unambiguously choose to tax final producers and devote more and more resources to R&D. When $\sigma > 1$, instead, the reallocation effect arising in the laissez-faire economy is weaker: if agents knew the true rate of return they would still adjust savings and invest more into R&D activity, but the additional investment would be relatively limited because $\sigma > 1$ implies that the willingness to smooth the consumption profile dominates the willingness to postpone consumption. Given this, it is possible that the rent-effect dominates when $\sigma > 1$. If this is the case, the optimal policy is similar to the one predicted by the MS-model – i.e., subsidize final producers – although the optimal subsidy rate is still reduced by the reallocation effect, which does not exist in the MS-model.

6 Conclusion

An established result of the endogenous growth literature is that the laissez-faire equilibria arising in expanding-varieties models are sub-optimal due to the rent-effect: in order to obtain a positive mark-up, monopolists restrict the supply of intermediate inputs; consequently, the equilibrium quantity of each intermediate is inefficiently low. The policy implication is that final producers should be subsidized in order to restore efficiency. This result holds in multi-sector models displaying scale effects, where the instantaneous increase in the number of varieties is proportional to the absolute level of R&D expenditures. It is known that scale effects can be eliminated by postulating a different dynamic law, whereby the growth rate of intermediates' varieties is proportional to the investment propensity. This paper has shown that an additional consequence of assuming the linear-rate law is that the optimal subsidy to final producers becomes strictly negative in a wide range of cases. The reason is that linear-rate laws can be reconciled with zero-profits in the R&D sectors by assuming spillovers from past innovations, but this assumption substantially alters the welfare properties of decentralized equilibria. Under laissez-faire, the economy under-invests into R&D activity because agents fail to internalize research spillovers. Since under-investment in R&D implies greater shares of output devoted to consumption and to the production of intermediates, the equilibrium quantity of intermediate inputs is affected by two opposing forces: it tends to be reduced by monopolistic pricing but, at the same time, tends to be increased by the misallocation of resources in disfavor of R&D activity. Differently from the standard multi-sector model with scale effects, the equilibrium quantity of each differentiated input under laissez-faire may be higher or lower than in the optimum: if the reallocation effect dominates the rent-effect, there is overshooting in intermediates' production. Clearly, if this is the case, the policy prescription is reversed: the decentralization of the the social optimum requires final producers to be taxed, instead of being subsidized. The interesting result is that the reallocation effect surely dominates in the polar case of logarithmic preferences, as well as in all cases in which the elasticity of intertemporal substitution, $1/\sigma$, is above unity. When $\sigma \leq 1$, the overshooting result is reinforced and the optimal tax on final producers is strictly positive. When $\sigma > 1$, instead, it possible that the elasticity of substitution overcomes a critical threshold whereby the reallocation effect is weakened and dominated by the rent-effect. In this case, intermediates' production is inefficiently low and the final sector should be subsidized – although the optimal subsidy rate will be generally smaller than that predicted by the MS-model.

Appendix

The Household Problem: derivation of (14). The current-value Hamiltonian associated to the household problem is

$$H = u(c) + \lambda \left[(i - \ell) n + w - c - f \right],$$

where λ is the dynamic multiplier associated to (12). The first-order conditions $H_c = 0$ and $H_n = \rho\lambda - \dot{\lambda}$ yield $u_c = \lambda$ and $\dot{\lambda}/\lambda = \rho + \ell - i$, from which $\dot{c}/c = \sigma^{-1}(i - \rho - \ell)$. Plugging

$\dot{\lambda}/\lambda = \rho + \ell - i$ into the transversality condition

$$\lim_{t\to\infty} \lambda(t)\, n(t)\, e^{-\rho t} = 0$$

we obtain $\lim_{t\to\infty} n(t)\, e^{-\int_0^t (i(s)-\ell)ds} = 0$. Substituting $n(t) = N(t)/L(t)$, and $L(t) = L(0)\, e^{\ell t}$ together with $N(t) = J(t)\, V(t)$, we obtain $\lim_{t\to\infty} J(t)\, V(t)\, e^{-\int_0^t i(s)ds} = 0$, which implies (14).

Derivation of (16). Substituting $n(t) = J(t)\, V(t)/L(t)$ in (12) we obtain

$$\dot{J}(t)\, V(t) + \dot{V}(t)\, J(t) = i(t)\, J(t)\, V(t) + w(t)\, L(t) - C(t) - f(t)\, L(t)$$

where $C(t) \equiv L(t)\, c(t)$ is aggregate consumption. Plugging $\dot{J}(t) = \theta(t)\, Z(t)$ from (8), $V(t) = 1/[\theta(t)(1+a)]$ from (9), and $\dot{V}(t) = i(t)\, V(t) - \pi(t)$ from (10), we obtain

$$Z(t)(1+a)^{-1} = w(t)\, L(t) + J(t)\, \pi(t) - C(t) - f(t)\, L(t),$$

Substituting $\pi(t) = p(t)\, x(t) - \epsilon x(t)$ and recalling that (2)-(3) imply $Y(t) + bJ(t)\, x(t) = L(t)\, w(t) + p(t)\, J(t)\, x(t)$, we obtain

$$Z(t)(1+a)^{-1} = Y(t) - J(t)\, \epsilon x(t) - C(t) + bJ(t)\, x(t) - f(t)\, L.$$

Substituting $bJ(t)\, x(t) - f(t)\, L(t) = -az(t)$ from the government budget (11) and recalling that $z(t)(1+a) = Z(t)$, we obtain the aggregate budget constraint (16).

Proof of Proposition 1. First notice that the equilibrium relations imply[8]

$$\frac{J(t)\, x(t)}{Y(t)} = \gamma^2/(\epsilon - b), \tag{A1}$$

$$\frac{Z(t)}{Y(t)} = 1 - \frac{C(t)}{Y(t)} - \frac{\epsilon}{\epsilon - b}\gamma^2, \tag{A2}$$

$$\frac{\dot{Y}(t)}{Y(t)} = \ell + \frac{\dot{J}(t)}{J(t)}. \tag{A3}$$

Next consider (10): substituting $\pi(t)$ from (6) and $V(t) = [\theta(t)(1+a)]^{-1} = Y(t)/[\phi(1+a)\, J(t)]$ from (9) and (17), and using (A1) to eliminate $J(t)\, x(t)/Y(t)$, we have

$$i(t) = \phi(1+a)\,\gamma(1-\gamma) + \frac{\dot{V}(t)}{V(t)}. \tag{A4}$$

[8] Equation (A1) follows from (5) and (7). Plugging (A1) in (16) yields (A2). Time-differentiation of (7) implies (A3).

From (9) and (17), we have $\dot{V}/V = (\dot{Y}/Y) - (\dot{J}/J)$, and from (A3) this implies $\dot{V}/V = \ell$. Expression (A4) thus yields result (21). Setting the consumption propensity $\chi \equiv C/Y$ and the investment rate $\psi \equiv Z/Y$, equation (A2) reads $\psi(t) = 1 - \chi(t) - \frac{\epsilon}{\epsilon-b}\gamma^2$. Plugging this result in (18) yields

$$\dot{J}(t)/J(t) = \phi\psi(t) = \phi\left(1 - \chi(t) - \frac{\epsilon}{\epsilon-b}\gamma^2\right), \tag{A5}$$

which, combined with (A3), implies

$$\dot{Y}(t)/Y(t) = \ell + \phi\left(1 - \chi(t) - \frac{\epsilon}{\epsilon-b}\gamma^2\right). \tag{A6}$$

From (15) and (21), consumption growth equals

$$\dot{C}(t)/C(t) = \ell + \frac{1}{\sigma}\left[\phi(1+a)\gamma(1-\gamma) - \rho\right]. \tag{A7}$$

From (A6)-(A7), the equilibrium growth rate of the consumption propensity $\dot{\chi}/\chi = (\dot{C}/C) - (\dot{Y}/Y)$ must satisfy

$$\dot{\chi}(t)/\chi(t) = \phi\chi(t) + \frac{1}{\sigma}\left[\phi(1+a)\gamma(1-\gamma) - \rho\right] - \phi\left(1 - \frac{\epsilon}{\epsilon-b}\gamma^2\right), \tag{A8}$$

This dynamic relation has a unique fixed point

$$\chi_{ss} = 1 - \frac{\epsilon}{\epsilon-b}\gamma^2 - \frac{1}{\sigma}\left[(1+a)\gamma(1-\gamma) - (\rho/\phi)\right]. \tag{A9}$$

which is dynamically unstable[9]. Hence, the only equilibrium satisfying (A8) is $\chi^E(t) = \chi_{ss}$ in each $t \in [0,\infty)$ because $\chi(t) \neq \chi_{ss}$ at any t would generate explosive dynamics $\chi(t) \to \pm\infty$ that imply the violation in finite time of either the aggregate constraint (16) or of the non-negativity of consumption. This result proves (19). Since $\chi^E(t) = \chi_{ss}$ in each $t \in [0,\infty)$, the investment rate is constant as well: from (A2) we obtain $\psi(t)$ equal to

$$\psi^E = 1 - \frac{\epsilon}{\epsilon-b}\gamma^2 - \chi^E = \frac{1}{\sigma}\left[(1+a)\gamma(1-\gamma) - (\rho/\phi)\right] \tag{A10}$$

in each $t \in [0,\infty)$. From (A1), (A2) and (16), constant propensities to consume and to invest imply that $Y^E(t)$ grows at the same rate as $C^E(t)$ and $Z^E(t)$, given by the Keynes-Ramsey rule (A7), which proves result (22). Expression (23) can be equivalently obtained from (A3) or (A5). ■

[9]The derivative of the right hand side of (A8) with respect to $\chi(t)$ is $\phi > 0$.

Proof of Proposition 2. The Hamiltonian associated to the social problem is

$$\bar{H} = Lu(c) + \lambda \left[(1-\psi) L^{1-\gamma} \int_0^J x(j)^\gamma \, dj - \epsilon \int_0^J x(j) \, dj - Lc \right] + \mu J \phi \psi,$$

where λ is the Lagrange multiplier associated to the static constraint (33),[10] and μ is the dynamic multiplier associated to the dynamic constraint (34). Notice that the first-order conditions with respect to each $x(j)$ read

$$\gamma (1-\psi) L^{1-\gamma} x(j)^{\gamma-1} = \epsilon \tag{A11}$$

for each $j \in [0, J]$, which implies symmetry across varieties. As a consequence, the maximization is equivalently carried over by imposing symmetry ex-ante - that is, setting $x(j) = x$ for each $j \in [0, J]$ in each instant, and using the modified Hamiltonian

$$\mathcal{L} = Lu(c) + \lambda \left[(1-\psi) L^{1-\gamma} J x^\gamma - \epsilon J x - Lc \right] + \mu J \phi \psi,$$

where the control variables are (c, x, ψ), and the first-order condition with respect to x will incorporate (A11) for each $j \in [0, J]$. The necessary conditions for optimality read

$$\mathcal{L}_c = 0 \quad \rightarrow \quad u_c(t) = \lambda(t), \tag{A12}$$

$$\mathcal{L}_x = 0 \quad \rightarrow \quad \gamma (1-\psi(t)) Y(t) = \epsilon J(t) x(t), \tag{A13}$$

$$\mathcal{L}_\psi = 0 \quad \rightarrow \quad \lambda(t) Y(t) = \mu(t) J(t) \phi, \tag{A14}$$

together with the co-state equation $\mathcal{L}_J = \rho\mu - \dot{\mu}$, which implies

$$\lambda (1-\psi)(Y/J) - \lambda \epsilon x + \mu \phi \psi = (\rho + \ell) \mu - \dot{\mu}. \tag{A15}$$

The optimal dynamics of consumption are obtained as follows. Plugging (A13) in the aggregate constraint (16), and using the definitions $\chi(t) \equiv C(t)/Y(t)$ and $\psi(t) \equiv Z(t)/Y(t)$, we obtain

$$\chi(t) = (1-\psi(t))(1-\gamma). \tag{A16}$$

[10] An equivalent specification consists of eliminating λ by plugging constraint (33) directly into the instantaneous utility function as $u(c) = u\left\{ \left[(1-\psi) L^{1-\gamma} \int_0^J x(j)^\gamma \, dj - \epsilon \int_0^J x(j) \, dj - Lc \right] / L \right\}$. Obviously, results do not change.

Plugging (A16) back in (A13), we have $\frac{\gamma}{1-\gamma}\chi(t)Y(t) = \epsilon J(t)x(t)$, which can be time-differentiated to obtain

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{J}(t)}{J(t)} + \frac{\dot{x}(t)}{x(t)}. \tag{A17}$$

Since $Y = L^{1-\gamma}Jx^{\gamma}$ implies $\dot{Y}/Y = (1-\gamma)\ell + (\dot{J}/J) + \gamma(\dot{x}/x)$, we can substitute $\dot{x}/x = \gamma^{-1}\left[(\dot{Y}/Y) - (\dot{J}/J) - (1-\gamma)\ell\right]$ in (A17) to obtain

$$\frac{\dot{Y}(t)}{Y(t)} = \gamma\frac{\dot{C}(t)}{C(t)} + (1-\gamma)\frac{\dot{J}(t)}{J(t)} + (1-\gamma)\ell. \tag{A18}$$

which is useful for future reference. Using (A12) to eliminate $\lambda(t)$ from (A14) and (A15), we respectively obtain

$$u_c(t)Y(t) = \mu(t)J(t)\phi, \tag{A19}$$

$$\dot{\mu}(t)/\mu(t) = \rho + \ell - \phi[1-\gamma(1-\psi(t))]. \tag{A20}$$

Recalling that $u_c = c^{-\sigma}$, time-differentiation of (A19) and substitution of (A20) imply

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}\left\{\phi[1-\gamma(1-\psi(t))] - \rho - \ell + \frac{\dot{Y}(t)}{Y(t)} - \frac{\dot{J}(t)}{J(t)}\right\}. \tag{A21}$$

Substituting $\dot{c}/c = (\dot{C}/C) - \ell$ and using (A18) to eliminate $\dot{Y}/Y$ from (A21), we get

$$\frac{\dot{C}(t)}{C(t)}(\sigma-\gamma) = \phi[1-\gamma(1-\psi)] - \rho + (\sigma-\gamma)\ell - \gamma\frac{\dot{J}(t)}{J(t)}. \tag{A22}$$

Equation (A22) implies two possible cases, depending on whether $\sigma = \gamma$ or $\sigma \neq \gamma$. We claim that $\sigma > \gamma$ must hold in a well-defined equilibrium, and verify this claim later. Letting $\sigma > \gamma$, result (A22) implies

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\sigma-\gamma}\left\{\phi[1-\gamma(1-\psi)] - \rho - \gamma\frac{\dot{J}(t)}{J(t)}\right\} + \ell, \tag{A23}$$

which can be substituted in (A18) to obtain

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{1}{\sigma-\gamma}\left\{\phi\gamma[1-\gamma(1-\psi)] - \gamma\rho + [(1-\gamma)\sigma-\gamma]\frac{\dot{J}(t)}{J(t)}\right\} + \ell. \tag{A24}$$

Taking the difference between (A23) and (A24), we have

$$\frac{\dot{C}(t)}{C(t)} - \frac{\dot{Y}(t)}{Y(t)} = \frac{1-\gamma}{\sigma-\gamma}\left\{\phi[1-\gamma(1-\psi)] - \rho - \sigma\frac{\dot{J}(t)}{J(t)}\right\},$$

where we can substitute $\dot{\chi}/\chi$ to the left hand side and, from (34), $\dot{J}/J = \phi\psi$, obtaining

$$\frac{\dot{\chi}(t)}{\chi(t)} = \frac{1-\gamma}{\sigma-\gamma}\left\{\phi\left[1-\gamma\left(1-\psi(t)\right)\right]-\rho-\sigma\phi\psi(t)\right\}.$$

From (A16), we can replace $1-\psi(t) = \frac{\chi(t)}{1-\gamma}$ as well as $\psi(t) = 1-\frac{\chi(t)}{1-\gamma}$ in the above expression to get

$$\frac{\dot{\chi}(t)}{\chi(t)} = \phi\chi(t) + \frac{1-\gamma}{\sigma-\gamma}\left[\phi(1-\sigma)-\rho\right]. \tag{A25}$$

The fixed point of (A25) is

$$\bar{\chi} = (1-\gamma)\cdot\left[1-\frac{\phi(1-\gamma)-\rho}{\phi(\sigma-\gamma)}\right] = (1-\gamma)\cdot\frac{\phi(\sigma-1)+\rho}{\phi(\sigma-\gamma)}. \tag{A26}$$

Since $\phi > 0$, relation (A25) is dynamically unstable and can only be satisfied along optimal paths with bounded propensities by setting $\chi^{\star}(t) = \bar{\chi}$ in each $t \in [0,\infty)$, which proves (35). From (35) and (A16), we obtain (36). Since the necessary condition for an equilibrium in the laissez-faire economy is $\phi\gamma(1-\gamma) > \rho$, we have $\phi(1-\gamma) > \rho$: from (36), this implies that the condition

$$\sigma > \gamma \tag{A27}$$

must hold in order to have, also in the social optimum, a well-defined steady state with positive investment rate $\psi^{\star}(t) > 0$. Plugging (36) in (34), we have (38). A constant $\chi^{\star}(t)$ implies $\dot{C}/C = \dot{Y}/Y$ in (A18) and thus $\dot{Y}(t)/Y(t) = \dot{J}(t)/J(t) + \ell$, which can be substituted into (A21) together with $\dot{c}/c = (\dot{C}/C) - \ell$ to obtain (37). It is easy to prove that $\sigma = \gamma$ does not allow us to obtain a well-defined equilibrium: going back to (A22) and setting $\sigma = \gamma$, we obtain a growth rate of $J(t)$ equal to $\left.\frac{\dot{J}(t)}{J(t)}\right|_{\sigma=\gamma} = \frac{1}{\sigma}\left\{\phi\left[1-\gamma(1-\psi)\right]-\rho\right\}$, but combining this result with the accumulation law (34) yields an indeterminate investment rate ψ. ■

Derivation of (40). From (A13), the optimal quantity of each intermediate product is determined by

$$x^{\star}(t) = \frac{\gamma(1-\psi^{\star})}{\epsilon}\frac{Y(t)}{J(t)} = \frac{\gamma(1-\psi^{\star})}{\epsilon}L(t)^{1-\gamma}x(t)^{\gamma}.$$

Solving for $x^{\star}(t)$ we obtain $x^{\star}(t) = \left[\frac{\gamma}{\epsilon}(1-\psi^{\star})\right]^{\frac{1}{1-\gamma}}L(t)$, where we can substitute $\psi^{\star} = \frac{\phi(1-\gamma)-\rho}{\phi(\sigma-\gamma)}$ from (36) to obtain (40).

Proof of Proposition 3. In order to obtain an optimal quantity of intermediate inputs, the fiscal authority must set the subsidy rate to final producers, b, in order to make $x^{E}(t)$

coinciding with $x^{\star}(t)$. From (24) and (40), having $x^{E}(t) = x^{\star}(t)$ in each t requires setting

$$b = b^{*} = \epsilon \cdot \frac{\phi(\sigma-1)+\rho-\gamma\phi(\sigma-\gamma)}{\phi(\sigma-1)+\rho}.$$

Letting $\sigma = 1$ in the above expression, we obtain (46), where b^{*} is strictly negative from (46). In order to decentralize the optimal growth rate and the optimal propensities to invest and consume, the fiscal authorities must set $a = a^{*}$ in order to equalize the growth rates $\dot{Y}^{E}/Y^{E} = \dot{Y}^{\star}/Y^{\star}$. From (22) and (37), having $\dot{Y}^{E}(t)/Y^{E}(t) = \dot{Y}^{\star}(t)/Y^{\star}(t)$ in each t requires setting

$$a = a^{*} = \frac{\phi(\sigma-\gamma)(1-\gamma)^{2}+\gamma[\phi(1-\gamma)-\rho]}{\gamma(1-\gamma)\phi(\sigma-\gamma)} > 0.$$

Letting $\sigma = 1$ in the above expression, we obtain (47). Since $a = a^{*}$ also implies an optimal rate of return as well as an optimal propensities to consume in the market economy, i.e. $\chi^{E} = \chi^{\star}$ and $\psi^{E} = \psi^{\star}$, the optimal policy consists of $a = a^{*}$ and $b = b^{*}$, with $f^{*}(t) = (1/L(t))[a^{*}z(t)+b^{*}J(t)x(t)]$ determined by the government budget constraint (11). ■

Derivation of (48)-(49). Results (48)-(49) are derived in the Proof of Proposition 3 above.

Proof of Proposition 4. Recalling that $\phi(1-\gamma)-\rho > 0$ is necessary to have a positive optimal investment rate in $\psi^{\star} > 0$ in (36), expression (48) implies that $b^{*} = 0$ if and only if

$$\phi(\sigma-1)+\rho-\gamma\phi(\sigma-\gamma) = 0,$$

which, after some algebra, reduces to

$$\sigma = 1 + \frac{\gamma\phi(1-\gamma)-\rho}{\phi(1-\gamma)} \equiv \bar{\sigma}.$$

Since $\partial b^{*}/\partial\sigma > 0$, we necessarily have $b^{*} < 0$ for $\sigma < \bar{\sigma}$ and $b^{*} > 0$ for $\sigma > \bar{\sigma}$. ■

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