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Abstract

The conventional search and matching model has been criticized for its inability to explain large cyclical volatility in the vacancy-unemployment ratio without ad hoc assumptions of wage rigidity. This paper presents a mechanism of such volatility without assuming wage rigidity by showing that households can rationally select a Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path. This type of path is generated after a time preference shock and causes a persistently large amount of extra unutilized resources. The labor market is thereby distorted and becomes more cyclically volatile. Vacancy costs are particularly affected by this Nash equilibrium. Because this Pareto inefficient path proceeds “rigidly,” that is, the Pareto inefficiency diminishes gradually, an ingredient of rigidity is introduced into the economy, and the vacancy-unemployment ratio experiences large cyclical fluctuations.

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1 INTRODUCTION

The conventional search and matching model (e.g., Pissarides, 1985; Mortensen and Pissarides, 1994) has been criticized by Shimer (2004, 2005) and Hall (2005a) for not having the power to generate sufficiently large cyclical volatility in the vacancy-unemployment (v-u) ratio. Shimer (2004, 2005), Farmer and Hollenhorst (2005), Hall (2005a), Hall and Milgrom (2008), Gertler and Trigari (2009), and Kennan (2010) suggested the necessity of modifying the mechanism of wage formation in conventional models, for example, by introducing wage rigidity, to solve this shortcoming because the wage-setting mechanism in these models (i.e., the Nash bargaining solution) has increasingly been regarded as unsatisfactory (see also Hornstein et al., 2005; Yashiv, 2007).

Introducing wage rigidity into these models may solve the problem with cyclical volatility, but a consensus on the validity of wage rigidity has not necessarily been reached even though wage rigidity, or more broadly price rigidity, has long been studied. Price rigidity has been criticized for its fragile theoretical (micro-) foundation and its inability to explain the persistent nature of inflation. Mankiw (2001) argued that the so-called new Keynesian Phillips curve is ultimately a failure and is not consistent with the standard stylized facts about the dynamic effects of monetary policy (see also, e.g., Fuhrer and Moore, 1995; Galí and Gertler, 1999).

The purpose of this paper is to offer an alternative approach to the problem associated with the conventional search and matching model. The focus is not on frictions that may exist on the shock transmission path (e.g., wage rigidity) but instead on the structure of the transmission path itself. If the transmission path is not a simple straight conduit but rather a more complicated conductor, rigidity-like phenomena may be observed. The remedy of introducing price rigidity has been used to explain observed phenomena that look like persistent deviations from Pareto efficiency. Rational agents will usually not allow Pareto inefficiency to remain for a long period, and it will disappear soon after it is generated. However, an exception is possible because a Nash equilibrium can conceptually coexist with Pareto inefficiency. If a Nash equilibrium that consists of strategies that generate Pareto inefficient payoffs is rationally selected, rigidity-like phenomena may be observed.

This paper shows that a Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path of consumption to the steady state (hereafter called a "Nash equilibrium of a Pareto inefficient path") is generated even in a frictionless economy if—and probably only if—the rate of time preference shifts. An essential reason for the generation of this path is that households are intrinsically risk averse and not cooperative. In a strategic environment, this generates the possibility that, if consumption needs to be substantially and discontinuously increased to keep Pareto optimality, a non-cooperative household's strategy to deviate from the Pareto optimal path gives a higher expected utility than the strategy of choosing the Pareto optimal path.

The Nash equilibrium of a Pareto inefficient path should not be confused with a Pareto inferior Nash equilibrium or a Nash equilibrium that is Pareto inefficient. They are conceptually quite different, although the Nash equilibrium of a Pareto inefficient path discussed in this paper is also a Pareto inferior Nash equilibrium and a Nash equilibrium that is Pareto inefficient. Multiple equilibria due to, for example, increasing returns, an externality or a complementarity in a macro-economic framework are usually Pareto ranked equilibria and include a Pareto inferior equilibrium (e.g., Morris and Shin, 2001). Such a Pareto inferior equilibrium usually indicates lower production and consumption than Pareto superior equilibria, suggesting a recession. However, if consumption is immediately adjusted completely when the economy is switched from a Pareto superior equilibrium to the inferior one, unutilized resources will not be generated as a result of the switch; therefore, merely showing the possibility of multiple Pareto ranked equilibria is not sufficient to explain the generation mechanism of persistent Pareto inefficiency. A mechanism that generates huge and persistent unutilized resources during the transition path to the new equilibrium should be also presented, and the Nash equilibrium of a Pareto inefficient path fully explains this mechanism.

If households are cooperative, they will always proceed on Pareto efficient paths because they will coordinate with each other to perfectly utilize all resources. Conversely, if they do not coordinate with each other, they may strategically not utilize all resources; that is, they may select a Nash equilibrium of a Pareto inefficient path. Such a possibility cannot be denied *a priori*, because a Nash equilibrium can coexist with Pareto inefficiency. In fact, households are intrinsically not cooperative—they act independently of one another. Suppose that an upward shift of the time preference rate occurs. All households will be knocked off the Pareto efficient path on which they have proceeded until the shift occurred. At that moment, each household must decide on a direction in which to proceed. Because they are no longer on a Pareto efficient path, households choose a path strategically on the basis of the expected utility calculated considering other households' choices; that is, each household behaves non-cooperatively in its own interest considering other households' strategies. This situation can be described by a non-cooperative mixed strategy game. In this paper, I show that there is a Nash equilibrium of a Pareto inefficient path in this game.

A weaknesses of the dynamic stochastic general equilibrium approach to macroeconomics stems from going too directly from statements about individuals to statements about the aggregate (Caballero, 2010). The Nash equilibrium of a Pareto inefficient path is an answer to this problem because this equilibrium is not derived from a simple summation of individuals' identical behaviors but is a result of strategic interactions among non-cooperative individuals.

Time preference is the source of shock in this mechanism. The rate of time preference has been naturally supposed and actually observed to be time-variable since the era of Böhm-Bawerk (1889) and Fisher (1930). This paper presents an endogenous time preference model, in which the rate of time preference is inversely proportionate to the expected steady-state consumption. Hence, the model is consistent with many observations that the rate of time preference is negatively correlated with permanent income (e.g., Lawrance, 1991) and thus escapes from the drawback of Uzawa's (1968) well-known endogenous time preference model. The model in this paper indicates that a shock to the expected steady-state consumption changes the rate of time preference.

A Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path introduces an ingredient of rigidity into various phenomena in the economy because this Pareto inefficient path proceeds "rigidly," that is, the Pareto inefficiency diminishes gradually, and the labor market is not an exception. The paper shows that this Nash equilibrium affects various parameters used in the conventional search and matching model and distorts the matching process. The separation rate rises, job-finding rate falls, vacancy costs increase, and labor productivity decreases. Vacancy costs are particularly important, which is intuitively and logically reasonable because firms should reduce the number of vacancies as the cost of vacancies increase and vice versa. Because Pareto inefficiency on this path persists, an ingredient of rigidity is introduced into the economy, and the v-u ratio experiences large cyclical fluctuations.

The paper is organized as follows. Section 2 shows that a Nash equilibrium of a Pareto inefficient path is rationally generated when the time preference rates of risk-averse and non-cooperative households shift. In addition, an endogenous time preference model is constructed, in which the rate of time preference is inversely proportionate to steady-state consumption. In Section 3, the Nash equilibrium of a Pareto inefficient path is incorporated into a conventional search and matching model. In Section 4, I argue that the explanation for economic fluctuations based on time preference shocks has many advantages over other explanations from various points of view. Finally, I offer concluding remarks in Section 5.

2 THE NASH EQUILIBRIUM OF A PARETO INEFFICIENT PATH

2.1 Model with non-cooperative households [1]

2.1.1 The shock

The model describes the utility maximization of households after an upward time preference shock. This shock was chosen because it is one of the few shocks that result in a Nash equilibrium of a Pareto inefficient path (other possible shocks are discussed in Section 2.5). Another important reason for selecting an upward time preference shock is that it shifts the steady state to lower production and consumption than before the shock, which is consistent with the phenomena actually observed in a recession.

Although the rate of time preference is a deep parameter, it has not been regarded as a source of shocks for economic fluctuations, possibly because the rate of time preference is thought to be constant and not to shift suddenly. There is also a practical reason, however. Models with a permanently constant rate of time preference exhibit excellent tractability (see Samuelson, 1937). However, the rate of time preference has been naturally assumed and actually observed to be time-variable. The concept of a time-varying rate of time preference has a long history (e.g., Böhm-Bawerk, 1889; Fisher, 1930). More recently, Lawrance (1991) and Becker and Mulligan (1997) showed that people do not inherit permanently constant rates of time preference by nature and that economic and social factors affect the formation of time preference rates. Their arguments indicate that many incidents can affect and change the rate of time preference throughout life. For example, Parkin (1988) examined business cycles in the United States, explicitly considering the time-variability of time preference rate, and showed that the rate of time preference was as volatile as technology and leisure preference. Because time preference is naturally time-variable, models of endogenous time preference have been presented, the most familiar of which is Uzawa's (1968) model. In Section 2.6, the endogeneity of time preference is examined in detail and an endogenous time preference model is presented as the mechanism of generation of the shock.

2.1.2 Households

Households are not intrinsically cooperative. Except in a strict communist economy, households do not coordinate themselves to behave as a single entity when consuming goods and services. The model in this paper assumes non-cooperative, identical and infinitely living households and that the number of households is sufficiently large. Each of them equally maximizes the expected utility

$$E_0 \int_0^\infty \exp(-\theta t) u(c_t) dt \quad , \tag{1}$$

subject to

$$\frac{dk_t}{dt} = f(A, k_t) - c_t \quad , \tag{2}$$

where y_t, c_t, and k_t are production, consumption, and capital per capita in period t respectively; A is technology and constant; u is the utility function; $y_t = f(A, k_t)$ is the production function; $\theta\ (>0)$ is the rate of time preference; and E_0 is the expectations operator conditioned on agents' period 0 information set. y_t, c_t, and k_t are monotonously continuous and differentiable

[1] The model in Section 2 is based on the model by Harashima (2009). See also Harashima (2004a, 2004b).

in t, and u and f are monotonously continuous functions of c_t and k_t, respectively. All households initially have an identical amount of financial assets equal to k_t, and all households gain the identical amount of income $y_t = f(A, k_t)$ in each period. It is assumed that $\frac{du(c_t)}{dc_t} > 0$ and $\frac{d^2u(c_t)}{dc_t^2} < 0$; thus, households are risk averse. For simplicity, the utility function is specified to be the constant relative risk aversion (CRRA) utility function

$$
\begin{aligned}
u(c_t) &= \frac{c_t^{1-\gamma}}{1-\gamma} && \text{if } \gamma \neq 1 \\
u(c_t) &= \ln(c_t) && \text{if } \gamma = 1 \ ,
\end{aligned}
$$

where $0 < \gamma < \infty$. In addition, $\frac{\partial f(A, k_t)}{\partial k_t} > 0$ and $\frac{\partial^2 f(k_t)}{\partial k_t^2} < 0$. Technology A and labor supply are assumed to be constant.

The effects of an upward shift in time preference are shown in Figure 1. Suppose first that the economy is at steady state before the shock. After the upward time preference shock, the vertical line $\frac{dc_t}{dt} = 0$ moves to the left (from the solid line to the dashed line in Fig 1). To keep Pareto efficiency, consumption needs to jump immediately from the steady state before the shock (the prior steady state) to point Z. After the jump, consumption proceeds on the Pareto efficient saddle path after the shock (the posterior Pareto efficient saddle path) from point Z to the lower steady state after the shock (the posterior steady state). Nevertheless, this discontinuous jump to Z may be uncomfortable for risk-averse households that wish to smooth consumption and not to experience substantial fluctuations. Households may instead take a shortcut and, for example, proceed on a path on which consumption is reduced continuously from the prior steady state to the posterior steady state (the bold dashed line in Fig. 1), but this shortcut is not Pareto efficient.

Choosing a Pareto inefficient consumption path must be consistent with each household's maximization of its expected utility. To examine the possibility of the rational choice of a Pareto inefficient path, the expected utilities between the two options need be compared. For this comparison, I assume that there are two options for each non-cooperative household with regard to consumption just after an upward time preference shift. The first is a jump option "***J***", in which a household's consumption jumps to Z and then proceeds on the posterior Pareto efficient saddle path to the posterior steady state. The second is a non-jump option "***NJ***", in which a household's consumption does not jump but instead gradually decreases from the prior steady state to the posterior steady state, as shown by the bold dashed line in Figure 1. The household that chose the ***NJ*** option reaches the posterior steady state in period $s(\geq 0)$. The difference in consumption between the two options in each period t is b_t ($\geq$ 0). Thus, b_0 indicates the difference between Z and the prior steady state. b_t diminishes continuously and becomes zero in period s. The ***NJ*** path of consumption (c_t) after the shock is monotonously continuous and differentiable in t and $\frac{dc_t}{dt} < 0$ if $0 \leq t < s$. In addition,

$$
\begin{aligned}
&\bar{c} < c_t < \hat{c}_t && \text{if } 0 \leq t < s \\
&c_t = \bar{c} && \text{if } 0 \leq s \leq t \ ,
\end{aligned}
$$

where $\hat{c}_t$ is consumption when proceeding on the posterior Pareto efficient saddle path and $\bar{c}$

is consumption in the posterior steady state. Therefore,

$$\begin{aligned} &b_t = \hat{c}_t - c_t > 0 && \text{if } 0 \le t < s \\ &b_t = 0 && \text{if } 0 \le s \le t \quad . \end{aligned}$$

It is also assumed that, when a household chooses the option that is different from the option the other households choose, the difference in the accumulation of financial assets resulting from the difference in consumption (b_t) before period s between the household and the other households is reflected in consumption after period s. That is, the difference in the return on financial assets is added to (or subtracted from) the household's consumption in each period after period s. The exact functional form of the addition (or subtraction) is shown in Section 2.1.4.

2.1.3 Firms

Unutilized products (b_t) are eliminated quickly in each period by firms, because holding b_t for a long period is a cost to firms. Elimination of b_t is done by discarding the goods or preemptively suspending production, leaving some capital and labor inputs idle. However, in the next period, unutilized products are generated again because the economy is not proceeding on the Pareto efficient saddle path. Unutilized products are therefore successively generated and eliminated. Faced with these unutilized products, firms dispose of the excess capital that generates b_t. Disposing of the excess capital is rational for firms, because the excess capital is an unnecessary cost for firms, but this means that parts of the firms are liquidated, which takes time and thus disposing of the excess capital will also take time. If the economy proceeds on the ***NJ*** path (that is, if all households choose the ***NJ*** option), firms dispose all of the remaining excess capital that generates b_t and adjust their capital to the posterior steady-state level in period s, corresponding to households' reaching the posterior steady state. Thus, if the economy proceeds on the ***NJ*** path, capital k_t is

$$\begin{aligned} &\bar{k} < k_t \le \hat{k}_t && \text{if } 0 \le t < s \\ &k_t = \bar{k} && \text{if } 0 \le s \le t \quad , \end{aligned}$$

where $\hat{k}_t$ is capital per capita when proceeding on the posterior Pareto efficient saddle path and $\bar{k}$ is capital per capita in the posterior steady state.

The real interest rate i_t is

$$i_t = \frac{\partial f(A, k_t)}{\partial k_t} \quad . \tag{3}$$

Because the real interest rate equals the rate of time preference at steady state, if the economy proceeds on the ***NJ*** path,

$$\begin{aligned} &\tilde{\theta} \le i_t < \theta && \text{if } 0 \le t < s \\ &i_t = \theta && \text{if } 0 \le s \le t \quad , \end{aligned} \tag{4}$$

where $\tilde{\theta}$ is the rate of time preference before the shock and θ is the rate of time preference after the shock. i_t is monotonously continuous and differentiable in t if $0 \le t < s$.

2.1.4 Expected utility after the shock

The expected utility of a household after the shock depends on its choice of ***J*** or ***NJ***. Let ***Jalone*** indicate that the household chooses the ***J*** option but the other households choose the ***NJ*** option, ***NJalone*** indicate that the household chooses the ***NJ*** option but the other households choose the ***J*** option, ***Jtogether*** indicate that all households choose the ***J*** option, and ***NJtogether*** indicate that all households choose the ***NJ*** option. Let $p\left(0 \le p \le 1\right)$ be the subjective probability of the household that the other households choose the ***J*** option (e.g., $p = 0$ indicates that all the other households choose option ***NJ***). With *p*, the expected utility of the household when it chooses option ***J*** is,

$$E_0(J) = pE_0(Jtogether) + (1-p)E_0(Jalone) \ , \tag{5}$$

and when it chooses option ***NJ*** is

$$E_0(NJ) = pE_0(NJalone) + (1-p)E_0(NJtogether) \ , \tag{6}$$

where $E_0(Jalone)$, $E_0(NJalone)$, $E_0(Jtogether)$, and $E_0(NJtogether)$ are the expected utilities of the household when choosing ***Jalone***, ***NJalone***, ***Jtogether***, and ***NJtogether***, respectively. With the properties of ***J*** and ***NJ*** shown in Sections 2.1.2 and 2.1.3,

$$\begin{aligned} E_0(J) = {} & pE_0\left[\int_0^s \exp(-\theta t)u(c_t + b_t)dt + \int_s^\infty \exp(-\theta t)u(\hat{c}_t)dt\right] \\ & + (1-p)E_0\left[\int_0^s \exp(-\theta t)u(c_t + b_t)dt + \int_s^\infty \exp(-\theta t)u(\bar{c} - \bar{a})dt\right] \ , \end{aligned} \tag{7}$$

and

$$\begin{aligned} E_0(NJ) = {} & pE_0\left[\int_0^s \exp(-\theta t)u(c_t)dt + \int_s^\infty \exp(-\theta t)u(\hat{c}_t + a_t)dt\right] \\ & + (1-p)E_0\left[\int_0^s \exp(-\theta t)u(c_t)dt + \int_s^\infty \exp(-\theta t)u(\bar{c})dt\right] \ , \end{aligned} \tag{8}$$

where

$$\bar{a} = \theta \int_0^s b_r \exp \int_r^s i_q \, dq \, dr \ , \tag{9}$$

and

$$a_t = i_t \int_0^s b_r \exp \int_r^s i_q \, dq \, dr \ , \tag{10}$$

and the shock occurred in the period $t = 0$. Figure 2 shows the paths of ***Jalone*** and ***NJalone***. Because there is a sufficiently large number of households and the effect of an individual household on the whole economy is negligible, then in the case of ***Jalone*** the economy almost proceeds on the ***NJ*** path, and in the case of ***NJalone*** it almost proceeds on the ***J*** path. If the other households choose the ***NJ*** option (***Jalone*** or ***NJtogether***), consumption after *s* is constant as $\bar{c}$ and capital is adjusted to $\bar{k}$ by firms in the period *s*. In addition, a_t and i_t are constant after *s* such that a_t equals $\bar{a}$ and i_s equals θ, because the economy is at the posterior steady state. Nevertheless, during the transition period before *s*, the value of i_t changes from the value of the prior time preference rate to that of the posterior. If the other households choose option ***J***

(***NJalone*** or ***Jtogether***), however, consumption after s is $\hat{c}_t$ and capital is not adjusted to $\bar{k}$ by firms in the period s and remains at $\hat{k}_t$.

As mentioned in Section 2.1.2, the difference in the returns on financial assets for the household from the returns for each of the other households is added to (or subtracted from) its consumption in each period after period s. This is described by a_t and $\bar{a}$ in equations (7) and (8), and equations (9) and (10) indicate that the accumulated difference in financial assets due to b_t increases by compound interest between the period r to s. That is, if the household takes the ***NJalone*** path, it accumulates more financial assets than each of the other ***J*** households, and instead of immediately consuming these extra accumulated financial assets after period s, the household consumes the returns on them in every subsequent period. If the household takes the ***Jalone*** path, however, its consumption after s is $\bar{c} - \bar{a}$, as shown in equation (7). $\bar{a}$ is subtracted because the income of each household $y_t = f(A, k_t)$, including the ***Jalone*** household, decreases equally by b_t. Each of the other ***NJ*** households decreases consumption by b_t at the same time, which compensates for the decrease in income; thus, its financial assets (i.e., capital per capita; k_t) are kept equal to $\hat{k}_t$. The ***Jalone*** household, however, does not decrease its consumption, and its financial assets become smaller than those of each of the other ***NJ*** households, which results in the subtraction of $\bar{a}$ after period s.

2.2 Pareto inefficient transition path [2]

2.2.1 Rational Pareto inefficient path

2.2.1.1 Rational choice of a Pareto inefficient path

Before examining the economy with non-cooperative households, I first show that, if households are cooperative, only option ***J*** is chosen as the path after the shock because it gives a higher expected utility than option ***NJ***. Because there is no possibility of ***Jalone*** and ***NJalone*** if households are cooperative, then $E_0(J) = E_0(Jtogether)$ and $E_0(NJ) = E_0(NJtogether)$. Therefore,

$$\begin{aligned}
&E_0(J) - E_0(NJ) \\
&= E_0\left[\int_0^s \exp(-\theta t) u(c_t + b_t) dt + \int_s^\infty \exp(-\theta t) u(\hat{c}_t) dt\right] - E_0\left[\int_0^s \exp(-\theta t) u(c_t) dt + \int_s^\infty \exp(-\theta t) u(\bar{c}) dt\right] \\
&= E_0\left\{\int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)] dt + \int_s^\infty \exp(-\theta t)[u(\hat{c}_t) - u(\bar{c})] dt\right\} > 0
\end{aligned}$$

since $c_t < c_t + b_t$ and $\bar{c} < \hat{c}_t$.

Next, I examine the economy with non-cooperative households. First, the special case with a utility function with a sufficiently small γ is examined.

Lemma 1: If $\gamma (0 < \gamma < \infty)$ is sufficiently small, then $E_0(Jalone) - E_0(NJtogether) > 0$.

Proof: $\lim_{\gamma \to 0} [E_0(Jalone) - E_0(NJtogether)]$

$$\begin{aligned}
&= E_0 \int_0^s \exp(-\theta t) \lim_{\gamma \to 0} [u(c_t + b_t) - u(c_t)] dt + E_0 \int_s^\infty \exp(-\theta t) \lim_{\gamma \to 0} [u(\bar{c} - \bar{a}) - u(\bar{c})] dt \\
&= E_0 \int_0^s \exp(-\theta t) b_t \, dt - E_0 \int_s^\infty \exp(-\theta t) \bar{a} \, dt \\
&= E_0 \int_0^s \exp(-\theta t) b_t dt - E_0 \theta \left[\int_0^s \left(b_r \exp \int_r^s i_q \, dq\right) dr\right] \int_s^\infty \exp(-\theta t) dt
\end{aligned}$$

[2] The idea of a rationally chosen Pareto inefficient path was originally presented by Harashima (2004b).

$$= E_0 \int_0^s \exp(-\theta t) b_t dt - E_0 \exp(-\theta s) \int_0^s \left(b_r \exp \int_r^s i_q \, dq \right) dr$$

$$= E_0 \exp(-\theta s) \int_0^s b_t \left\{ \exp[\theta(s-t)] - \exp \int_t^s i_q \, dq \right\} dt > 0 \quad ,$$

because, if $0 \le t < s$, then $i_t < \theta$ and $\exp[\theta(s-t)] > \exp \int_t^s i_q \, dq$. Hence, because $\exp[\theta(s-t)] > \exp \int_t^s i_q \, dq$, $E_0(Jalone) - E_0(NJtogether) > 0$ for sufficiently small γ. ■

Second, the opposite special case (i.e., a utility function with a sufficiently large γ) is examined.

Lemma 2: If $\gamma(0 < \gamma < \infty)$ is sufficiently large and if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}}$, then $E_0(Jalone) - E_0(NJtogether) < 0$.

Proof: Because $0 < b_t$, then $\lim_{\gamma \to \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} [u(c_t + b_t) - u(c_t)] = \lim_{\gamma \to \infty} \left[\left(\frac{c_t + b_t}{\bar{c}} \right)^{1-\gamma} - \left(\frac{c_t}{\bar{c}} \right)^{1-\gamma} \right] = 0$ for any period $t(< s)$. On the other hand, because $0 < \bar{a}$, then for any period $t(< s)$, if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$, $\lim_{\gamma \to \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} [u(\bar{c} - \bar{a}) - u(\bar{c})] = \lim_{\gamma \to \infty} \left[\left(1 - \frac{\bar{a}}{\bar{c}} \right)^{1-\gamma} - 1 \right] = \infty$. Thus, $\lim_{\gamma \to \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} [E_0(Jalone) - E_0(NJtogether)]$

$$= \lim_{\gamma \to \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} \int_0^s \exp(-\theta t) \lim_{\gamma \to \infty} [u(c_t + b_t) - u(c_t)] dt + \lim_{\gamma \to \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} \int_s^\infty \exp(-\theta t) \lim_{\gamma \to \infty} [u(\bar{c} - \bar{a}) - u(\bar{c})] dt =$$

$0 + \infty > 0$. Because $\frac{1-\gamma}{\bar{c}^{1-\gamma}} < 0$ for any $\gamma\,(1 < \gamma < \infty)$, then if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$, $E_0(Jalone) - E_0(NJtogether) < 0$ for sufficiently large $\gamma(< \infty)$. ■

The condition $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$ indicates that path ***NJ*** from c_0 to $\bar{c}$ deviates sufficiently from the posterior Pareto efficient saddle path and reaches the posterior steady state $\bar{c}$ not too late. Because steady states are irrelevant to the degree of risk aversion (γ), both c_0 and $\bar{c}$ are irrelevant to γ.

By Lemmas 1 and 2, it is proved that $E_0(Jalone) - E_0(NJtogether) < 0$ is possible.

Lemma 3: If $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$, then there is a $\gamma^*(0 < \gamma^* < \infty)$ such that if $\gamma^* < \gamma < \infty$, $E_0(Jalone) - E_0(NJtogether) < 0$.

Proof: If $\gamma(> 0)$ is sufficiently small, then $E_0(Jalone) - E_0(NJtogether) > 0$ by Lemma 1, and if $\gamma(< \infty)$ is sufficiently large and if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$, then $E_0(Jalone) - E_0(NJtogether) < 0$ by Lemma 2. Hence, if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$, there is a certain $\gamma^*(0 < \gamma^* < \infty)$ such that, if $\gamma^* < \gamma < \infty$, then $E_0(Jalone) - E_0(NJtogether) < 0$. ■

However, $E_0(Jtogether) - E_0(NJalone) > 0$ because both ***Jtogether*** and ***NJalone*** indicate that all the other households choose option ***J***; thus, the values of i_t and k_t are same as those when all households proceed on the posterior Pareto efficient saddle path. Faced with these i_t and k_t, deviating alone from the Pareto efficient path (***NJalone***) gives a lower expected utility than ***Jtogether*** to the ***NJ*** household. Opposite to ***Jtogether*** and ***NJalone***, both ***Jalone*** and ***NJtogether*** indicate that all the other households choose option ***NJ*** and i_t and k_t are not those of the Pareto efficient path. Hence, the sign of $E_0(Jalone) - E_0(NJtogether)$ varies depending on the conditions, as Lemma 3 indicates.

By Lemma 3 and the property $E_0(Jtogether) - E_0(NJalone) > 0$, the possibility of the choice of a Pareto inefficient transition path, that is, $E_0(J) - E_0(NJ) < 0$, is shown.

Proposition 1: If $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$ and $\gamma^* < \gamma < \infty$, then there is a $p^* \left(0 \le p^* \le 1\right)$ such that if $p = p^*$, $E_0(J) - E_0(NJ) = 0$, and if $p < p^*$, $E_0(J) - E_0(NJ) < 0$.

Proof: By Lemma 3, if $\gamma^* < \gamma < \infty$, then $E_0(Jalone) - E_0(NJtogether) < 0$ and $E_0(Jtogether) - E_0(NJalone) > 0$. By equations (5) and (6), $E_0(J) - E_0(NJ) = p[E_0(Jtogether) - E_0(NJalone)] + (1-p)[E_0(Jalone) - E_0(NJtogether)]$. Thus, if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$ and $\gamma^* < \gamma < \infty$, $\lim_{p \to 0}[E_0(J) - E_0(NJ)] = E_0(Jalone) - E_0(NJtogether) < 0$ and $\lim_{p \to 1}[E_0(J) - E_0(NJ)] = E_0(Jtogether) - E_0(NJalone) > 0$. Hence, by the intermediate value theorem, there is $p^* \left(0 \le p^* \le 1\right)$ such that if $p = p^*$, $E_0(J) - E_0(NJ) = 0$ and if $p < p^*$, $E_0(J) - E_0(NJ) < 0$. ■

Proposition 1 indicates that, if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$, $\gamma^* < \gamma < \infty$, and $p < p^*$, then the choice of option ***NJ*** gives the higher expected utility than that of option ***J*** to a household; that is, a household may make the rational choice of taking a Pareto inefficient transition path. The lemmas and proposition require no friction, and a Pareto inefficient transition path can be chosen even in a frictionless economy. This result is very important because it offers counter-evidence against the conjecture that households never rationally choose any Pareto inefficient transition path in a frictionless economy.

2.2.1.2 Conditions for a rational Pareto inefficient path

The proposition requires several conditions. Among them, $\gamma^* < \gamma < \infty$ may appear rather strict. If γ^* is very large, option ***NJ*** will be rarely chosen. However, if path ***NJ*** is such that consumption is reduced sharply after the shock, option ***NJ*** gives the higher expected utility than option ***J*** even though γ^* is very small. For example, for any $\gamma \left(0 < \gamma < \infty\right)$,

$$\lim_{s \to 0} \frac{1}{s}[E_0(Jalone) - E_0(NJtogether)]$$
$$= \lim_{s \to 0} \frac{1}{s} \int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)]dt + \lim_{s \to 0} \frac{1}{s} \int_s^\infty \exp(-\theta t)[u(\bar{c} - \bar{a}) - u(\bar{c})]dt$$
$$= u(c_0 + b_0) - u(c_0) - \frac{1}{\theta} \lim_{s \to 0} \frac{u(\bar{c}) - u(\bar{c} - s\theta b_0)}{s} = u(c_0 + b_0) - u(c_0) - b_0 \frac{du(\bar{c})}{d\bar{c}}$$
$$= \frac{(c_0 + b_0)^{1-\gamma} - c_0^{1-\gamma}}{1-\gamma} - b_0 \bar{c}^{-\gamma} = \bar{c}^{-\gamma} \left\{ \bar{c}^{\gamma} \left[\frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] - b_0 \right\} < 0 \quad ,$$

because $\lim_{\gamma \to 1} \bar{c}^{\gamma} \left[\frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] = \bar{c} [\ln(c_0 + b_0) - \ln(c_0)] = \bar{c} \ln\left(1 + \frac{b_0}{c_0}\right) < b_0$ and $\lim_{\gamma \to \infty} \bar{c}^{\gamma} \left[\frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] = \lim_{\gamma \to \infty} \bar{c}^{\gamma} c_0^{1-\gamma} \left[\frac{\left(1 + \frac{b_0}{c_0}\right)^{1-\gamma} - 1}{1-\gamma} \right] = 0$ due to $\bar{c} < c_0$. That is, for each combination of path ***NJ*** and γ, there is $s^*(>0)$ such that, if $s < s^*$, then $E_0(Jalone) - E_0(NJtogether) < 0$.

Consider an example in which path ***NJ*** is such that b_t is constant as $b_t = \bar{b}$ before s (Figure 3); thus $E_0 \int_0^s b_t = s\bar{b}$. In this ***NJ*** path, consumption is reduced more sharply than it is in the case shown in Figure 2. In this case, because $\bar{a} > E_0 \theta \int_0^s b_t = \theta s \bar{b}$, $0 < \gamma$, and $c_s < c_t$ for $t < s$, then $E_0 \int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)]dt < E_0 \int_0^s \exp(-\theta t) dt [u(c_s + \bar{b}) - u(c_s)] = E_0 \frac{1 - \exp(-\theta s)}{\theta} [u(c_s + \bar{b}) - u(c_s)]$, and in addition, $E_0 \int_s^{\infty} \exp(-\theta t)[u(\bar{c} - \bar{a}) - u(\bar{c})]dt = E_0 \int_s^{\infty} \exp(-\theta t) dt [u(\bar{c} - \bar{a}) - u(\bar{c})] = E_0 \frac{\exp(-\theta s)}{\theta} [u(\bar{c} - \bar{a}) - u(\bar{c})] < E_0 \frac{\exp(-\theta s)}{\theta} [u(\bar{c} - \theta s \bar{b}) - u(\bar{c})]$. Hence,

$$
\begin{aligned}
& E_0(Jalone) - E_0(NJtogether) \\
& = E_0 \int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)]dt + E_0 \int_s^{\infty} \exp(-\theta t)[u(\bar{c} - \bar{a}) - u(\bar{c})]dt \\
& < E_0 \frac{1 - \exp(-\theta s)}{\theta} [u(c_s + \bar{b}) - u(c_s)] + E_0 \frac{\exp(-\theta s)}{\theta} [u(\bar{c} - \theta s \bar{b}) - u(\bar{c})] \\
& = E_0 \frac{1 - \exp(-\theta s)}{\theta} \left\{ [u(c_s + \bar{b}) - u(c_s)] - \frac{\exp(-\theta s)}{1 - \exp(-\theta s)} [u(\bar{c}) - u(\bar{c} - \theta s \bar{b})] \right\} .
\end{aligned}
$$

As γ becomes larger, the ratio $\frac{u(c_s + \bar{b}) - u(c_s)}{u(\bar{c}) - u(\bar{c} - \theta s \bar{b})}$ becomes smaller; thus, larger values of s can satisfy $E_0(Jalone) - E_0(NJtogether) < 0$. For example, suppose that $\bar{c} = 10$, $c_s = 10.2$, $\bar{b} = 0.3$, and $\theta = 0.05$. If $\gamma = 1$, then $s^* = 1.5$ at the minimum, and if $\gamma = 5$, then $s^* = 6.8$ at the minimum. This result implies that, if option ***NJ*** is such that consumption is reduced relatively sharply after the shock (e.g., $b_t = \bar{b}$) and $p < p^*$, option ***NJ*** will usually be chosen. It is not a special case observed only if γ is very large, but it will normally be generated when the value of γ is within usually observed values. Conditions for generating a rational Pareto inefficient transition path therefore are not strict. In a recession, consumption usually declines sharply after the shock, which suggests that households have chosen the ***NJ*** option.

2.3 Nash equilibrium

2.3.1 A Nash equilibrium consisting of *NJ* strategies

A household strategically determines whether to choose the ***J*** or ***NJ*** option, considering other households' choices. All households know that each of them forms expectations about the future values of its utility and makes a decision in the same manner. Since all households are identical, the best response of each household is identical. Suppose that there are $H\ (\in N)$ identical households in the economy where H is sufficiently large (as assumed in Section 2.1). Let $q_\eta\ (0 \le q_\eta \le 1)$ be the probability that a household $\eta\ (\in H)$ chooses option ***J***. The average utility of the other households almost equals that of all households because H is sufficiently large. Hence, the average expected utilities of the other households that choose the ***J*** and ***NJ*** options are $E_0(Jtogether)$ and $E_0(NJtogether)$, respectively. Hence, the payoff matrix of the H-dimensional symmetric mixed strategy game can be described as shown in Table 1. Each identical household determines its behavior on the basis of this payoff matrix.

In this mixed strategy game, strategy profiles

$$(q_1,q_2,\ldots,q_H) = \{(1,1,\ldots,1),\ (p^*,p^*,\ldots,p^*),\ (0,0,\ldots,0)\} \tag{12}$$

are Nash equilibria for the following reason. By Proposition 1, the best response of a household η is ***J*** (i.e., $q_\eta = 1$) if $p > p^*$, indifferent between ***J*** and ***NJ*** (i.e., any $q_\eta \in [0,1]$) if $p = p^*$, and ***NJ*** (i.e., $q_\eta = 0$) if $p < p^*$. Because all households are identical, the best-response correspondence of each household is identical such that $q_\eta = \{1\}$ if $p > p^*$, $[0,1]$ if $p = p^*$, and $\{0\}$ if $p < p^*$ for any household $\eta \in H$. Hence, the mixed strategy profiles (1, 1,…,1), ($p^*,p^*,\ldots,p^*$), and (0,0,…,0) are the intersections of the graph of the best-response correspondences of all households. The Pareto efficient saddle path solution (1,1,…,1; i.e., ***Jtogether***) is a pure strategy Nash equilibrium, but a Pareto inefficient transition path (0,0,…,0; i.e., ***NJtogether***) is also a pure strategy Nash equilibrium. In addition, there is a mixed strategy Nash equilibrium ($p^*,p^*,\ldots,p^*$).

2.3.2 Selection of equilibrium

Determining which Nash equilibrium, either ***NJtogether*** (0,0,…,0) or ***Jtogether*** (1,1,…,1), is dominant requires refinements of the Nash equilibrium, which necessitate additional criteria. Here, if households have a risk-averse preference in the sense that they avert the worst scenario when its probability is not known, households suppose very low p and select the ***NJtogether*** (0,0,…,0) equilibrium. Because

$$\begin{aligned}
&E_0(Jalone) - E_0(NJalone) \\
&= E_0\left\{\int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)]dt + \int_s^\infty \exp(-\theta t)[u(\bar{c} - \bar{a}) - u(\hat{c}_t + a_t)]dt\right\} \\
&< E_0\left\{\int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)]dt + \int_s^\infty \exp(-\theta t)[u(\bar{c} - \bar{a}) - u(\bar{c})]dt\right\} \\
&= E_0(Jalone) - E_0(NJtogether) < 0 \quad ,
\end{aligned} \tag{13}$$

by Lemma 3, then ***Jalone*** is the worst choice in the sense of the amount of payoff, followed by ***NJtogether***, and ***NJalone***, and ***Jtogether*** is the best. The outcome of choosing option ***J*** is more dispersed than that of option ***NJ***. If households have the risk-averse preference in the above-mentioned sense and avert the worst scenario when they have no information on its probability, a household will prefer the less dispersed option (***NJ***), fearing the worst situation that the household alone substantially increases consumption while the other households substantially decrease consumption after the shock. This behavior is rational because it is consistent with preferences. Since all households are identical and know inequality (13), all households will equally suppose that they all prefer the less dispersed ***NJ*** option; therefore, all of them will suppose a very low p, particularly $p = 0$, and select the ***NJtogether*** (0,0,…,0) equilibrium, which is the Nash equilibrium of a Pareto inefficient path. Thereby, unlike most multiple equilibria models, the problem of indeterminacy does not arise, and animal spirits (e.g., pessimism or optimism) are unnecessary to explain the selection.

2.4 Amplified generation of unutilized resources

A Nash equilibrium of a Pareto inefficient path successively generates unutilized products (b_t). They are left unused, discarded, or preemptively not produced during the path. Unused or discarded goods and services indicate a decline in sales and an increase in inventory for firms. Preemptively suspended production results in an increase in unemployment and idle capital. As a result, profits decline and some parts of firms need to be liquidated, which is unnecessary if the economy proceeds on the ***J*** path (i.e., the posterior Pareto efficient path). If the liquidation is implemented immediately after the shock, b_t will no longer be generated, but such a liquidation would generate a tremendous shock. The process of the liquidation, however, will take time because of various frictions, and excess capital that generates b_t will remain for a long period. During the period when capital is not reduced to the posterior steady-state level, unutilized products are successively generated. In a period, b_t is generated and eliminated, but in the next period, another, new, b_t is generated and eliminated. This cycle is repeated in every period throughout the transition path, and it implies that demand is lower than supply in every period. This phenomenon may be interpreted as a general glut or a persisting disequilibrium by some definitions of equilibrium.

2.5 Time preference shock as the exceptional shock

Not all shocks result in a Nash equilibrium of a Pareto inefficient path. If anything, this type of shock is limited because it needs to force consumption to fluctuate very jaggedly to maintain Pareto efficiency. A Pareto inefficient path is preferred, because these substantially jagged fluctuations can be averted. An upward time preference shock is one such shock, as shown in Figure 1. Other examples are rare, because shocks that do not change the steady state (e.g., monetary shocks) are not relevant. One other example is a technology regression, which would move the vertical line $\frac{dc_t}{dt} = 0$ to the left in Figure 1 and necessitate a jagged consumption path to keep Pareto efficiency. In this sense, technology and time preference shocks have similar effects on economic fluctuations. However, a technology regression also simultaneously moves the curve $\frac{dk_t}{dt} = 0$ downwards, and accordingly, the Pareto efficient saddle path also moves downwards. Therefore, the jagged consumption is smoothed out to some extent. As a result, the substantially jagged consumption that can generate a recession would require a large-scale, sudden, and sharp regression in technology, which does not seem very likely. An upward time preference shock, however, only moves the vertical line $\frac{dc_t}{dt} = 0$ to the left.

In some macro-economic models with multiple equilibra, however, changing equilibria may necessitate substantially jagged consumptions to keep Pareto optimality. There are many types of multiple equilibra models that depend on various types of increasing returns, externalities, or complementarities, but they are vulnerable to a number of criticisms (e.g., insufficient explanation of the switching mechanism; see, e.g., Morris and Shin, 2001). Examining the properties, validity, and plausibility of each of these many and diverse models is beyond the scope of this paper.

2.6 Endogenous time preference

The results in the above sections raise the question: what force drives households to shift their rates of time preference upwards? Keynes's (1936) argument suggests that an upward time preference shift is caused by a change in households' moods. Indeed, preferences may change stochastically by fluctuating moods. However, it is not compelling to accept the idea of animal spirits *ad hoc* because it implies irrationality. Before arbitrarily assuming irrationality, we should search for all possibilities of mechanisms by which an upward time preference shift is endogenously generated as a consequence of rational agents' rational behavior.

2.6.1 Endogenous time preference models

2.6.1.1 Uzawa's (1968) endogenous time preference model

The most well-known endogenous time preference model is that of Uzawa (1968). It has been applied to many analyses (e.g., Epstein and Hynes, 1983; Lucas and Stokey, 1984; Epstein, 1987; Obstfeld, 1990). However, Uzawa's model has not necessarily been regarded as a realistic expression of endogeneity of time preference because it has a serious drawback in that impatience increases as income, consumption and utility increase. The basic structure of Uzawa's model is

$$\theta_t = \theta^*[u(c_t)] \ ,$$
$$0 < \frac{d\theta_t}{du(c_t)} \ , \tag{14}$$

in which the rate of time preference θ_t in period t is time-variable and an increasing function of present utility $u(c_t)$. The problem is that $0 < \frac{d\theta_t}{du(c_t)}$ is necessary for the model to be stable. This property is quite controversial and difficult to accept *a priori*, because many empirical studies have indicated that the rate of time preference is negatively correlated with permanent income (e.g., Lawrance, 1991); thus, many economists are critical of Uzawa's model. Epstein (1987), however, argues the plausibility of increasing impatience and offers some counter-arguments. However, his view is in the minority and most economists support arguments in favor of the decreasing rate of time preference such that $\frac{d\theta_t}{du(c_t)} < 0$. Hence, although Uzawa's model attracted attention from economists such as Epstein and Hynes (1983), Lucas and Stokey (1984), and Obstfeld (1990), analysis of the endogeneity of time preference has progressed very little. Although Uzawa's model may be flawed, that does not necessarily mean that the conjecture that the rate of time preference is influenced by future income, consumption, and utility is fallacious, just that an appropriate model in which the rate of time preference is negatively correlated with income, consumption, and utility has not been presented.

2.6.1.2 Size effect on impatience

The problem of $0 < \frac{d\theta_t}{du(c_t)}$ in Uzawa's model arises because distant future levels of consumption have little influence on factors that form the rate of time preference; that is, it is formed only with the information on present consumption, and it must be revised every period in accordance with consumption growth. However, there is no *a priori* reason why information on distant future activities should be far less important than the information on the present and near future activities. Fisher (1930) argued that

> [O]ur first step, then, is to show how a person's impatience depends on the *size* of his income, assuming the other three conditions to remain constant; for, evidently, it is possible that two incomes may have the same time shape, composition and risk, and yet differ in size, one being, say, twice the other in every period of time.
>
> In general, it may be said that, other things being equal, the smaller the

> income, the higher the preference for the present over the future income. It is true of course that a permanently small income implies a keen appreciation of wants as well as of immediate wants. … But it increases the want for immediate income *even more* than it increases the want for future income.” (p. 72)

According to Fisher’s (1930) view, a force that influences time preference is a psychological response derived from the perception of the “size of the entire income or utility stream.” This view indicates that it is necessary to probe how people perceive the size of the entire income or utility stream.

Little effort has been directed towards probing the nature of the size of utility or income stream on time preference, although a large number of psychological experiments have been made with regard to anomalies of the expected utility model with a constant rate of time preference (e.g., Frederick et al., 2002). Turning to research in economics, analyses using endogenous time preference models so far have merely introduced the *a priori* assumption of endogeneity of time preference without explaining its reasoning in detail. Hence, even now, Fisher’s (1930) insights are very useful for the examination of the size effect. An important point in Fisher’s above quote is that the size of the infinite utility stream is perceived as “permanently” high or low. The size difference among the utility streams may be perceived as the permanent continuing difference of utilities among different utility streams. Anticipation of the permanently higher utility may enhance an emotional sense of well-being because people feel they have a long-lasting secure situation, which will generate a positive psychological response and make people more patient. If that is true, distant future utilities should be taken into account equally with the present utility. Otherwise, it is impossible to distinguish whether the difference of utilities continues permanently.

From this point of view, the specification that only the present utility influences the formation of time preference, as is the case of Uzawa’s model, is inadequate as the specification of the size of utility stream. Instead, a simple measure of the size where entire utilities from the present to distant future are summed with equal weight will be more appropriate as the measure of the size of a utility stream.[3]

2.6.2 Model of time preference [4]

2.6.2.1 The model

Because no strategic situation is supposed in this section unlike in Sections 2.1 - 2.5, the usual representative household is assumed for simplicity, and the representative household solves the maximization problem indicated in equations (1) and (2). Taking the arguments in Section 2.6.1 into account, the “size” of the infinite utility stream can be defined as follows.

Definition 1: The size of the utility stream W for a given technology A is

$$W = \lim_{T \to \infty} E_0 \int_0^T \rho(t) u(c_t) dt \quad , \tag{15}$$

where

$$\rho(t) = \frac{1}{T} \quad \text{if } 0 \le t \le T$$
$$\rho(t) = 0 \qquad \text{otherwise.}$$

---

[3] Das (2003) shows another stable endogenous time preference model with decreasing impatience. Her model is stable, although the rate of time preference is decreasing because endogenous impatience is almost constant. In this sense, the situation her model describes is very special.

[4] The idea of this type of endogenous time preference model was originally presented by Harashima (2004a).

The variable $\rho(t)$ indicates weights and has the same value in any period. Thus, the weights for evaluation of future utilities are distributed evenly over time, as argued in Section 2.6.1.

To this point in my argument, technology A has been assumed to be constant, but if A is time-variable (A_t) and grows at a constant rate and the economy is on a balanced growth path such that A_t, y_t, k_t, and c_t grow at the same rate, then the definition of W needs to be modified because any stream of c_t and $u(c_t)$ grows to infinity, and it is impossible to distinguish the sizes of the utility stream by simply summing up c_t with $T \to \infty$ as shown in Definition 1. Because balanced growth is possible only when technological progress is Harrod neutral, I assume a Harrod neutral production function such that

$$y_t = \omega A_t^{\varpi} k_t^{1-\varpi} \quad , \tag{16}$$

where $\varpi\,(0 < \varpi < 1)$ and $\omega\,(0 < \omega)$ are constants. To distinguish the sizes of utility stream, the following value is set as the standard stream of utility,

$$u\left(\tilde{c}\, e^{\psi t}\right) \quad ,$$

where $\tilde{c}\,(0 < \tilde{c})$ is a constant and $\psi\,(0 < \psi)$ is a constant rate of growth. Streams of utility are compared with this standard stream. Because the utility function is CRRA as shown in Section 2.1, a stream of utility in comparison with the standard stream of utility is

$$\frac{u(c_t)}{u\left(\tilde{c}\, e^{\psi t}\right)} = \frac{c_t^{1-\gamma}}{\left(\tilde{c}\, e^{\psi t}\right)^{1-\gamma}} = \frac{1-\gamma}{\tilde{c}^{1-\gamma}} u\left(\frac{c_t}{e^{\psi t}}\right) \quad .$$

By using this ratio, a stream of utility can be distinguished from the standard stream of utility. That is, the size of a utility stream W for a given stream of technology A_t that grows at the same rate ψ as y_t, k_t, and c_t can be alternatively defined as

$$W = \lim_{T \to \infty} E_0 \int_0^T \rho(t) u\left(\frac{c_t}{e^{\psi t}}\right) dt \quad . \tag{17}$$

Clearly, if $\psi = 0$, then the size (W) degenerates into the one shown in Definition 1.

If there is a steady state such that

$$\lim_{t \to \infty} E_0\left[u(c_t)\right] = E_0\left[u\left(c^*\right)\right] \quad , \tag{18}$$

or for the case of expected balanced growth

$$\lim_{t \to \infty} E_0\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right] = E_0\left[u\left(c^*\right)\right] \quad , \tag{19}$$

where c^* is a constant and indicates steady-state consumption, then

$$W = E_0\left[u\left(c^*\right)\right] \tag{20}$$

for the following reason. Because $\lim_{t \to \infty} E_0[u(c_t)] = E_0[u(c^*)]$ (or $\lim_{t \to \infty} E_0\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right] =$ $E_0[u(c^*)]$), then

$$\lim_{T \to \infty} \int_0^T \rho(t)\{E_0[u(c^*)] - E_0[u(c_t)]\} dt = E_0[u(c^*)] - W$$

$$\text{(or } \lim_{T \to \infty} \int_0^T \rho(t)\left\{E_0[u(c^*)] - E_0\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right]\right\} dt = E_0[u(c^*)] - W \text{).}$$

In addition,

$$\lim_{T \to \infty} \int_0^T \rho(t)\{E_0[u(c^*)] - E_0[u(c_t)]\} dt = 0$$

$$\text{(or } \lim_{T \to \infty} \int_0^T \rho(t)\left\{E_0[u(c^*)] - E_0\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right]\right\} dt = 0 \text{) .}$$

Hence, $W = E_0[u(c^*)]$; that is, the rate of time preference is determined by steady-state consumption (c^*).

The model of time preference in this paper is constructed on the basis of this measure of *W*. An essential property that must be incorporated into the model is that the rate of time preference is sensitive to, and a function of, *W* such that

$$\theta = \theta^{**}(W) \ ,$$

where $\theta^{**}(W)$ is monotonously continuous and continuously differentiable. Because *W* is a sum of utilities, this property simply reflects the core idea of endogenous time preference. However, this property is new in the sense that the rate of time preference is sensitive not only to the present utility but also the entire stream of utility, that is, the size of utility stream represented by the utility for steady-state consumption. This property is intuitively acceptable because it is likely that people set their principles or parameters for their behaviors considering the final consequences (i.e., the steady state; see, e.g., Barsky and Sims, 2009).

Another essential property that must be incorporated into the model is

$$\frac{d\theta}{dW} < 0 \ .$$

Because $W = E_0[u(c^*)]$ and $0 < \frac{du(c_t)}{dc_t}$, the rate of time preference is inversely proportionate to c^*. This property is consistent with the findings in many empirical studies, which have shown that the rate of time preference is negatively correlated with permanent income (e.g., Lawrance, 1991).

In summary, the basic structure of the model is:

$$\theta = \theta^{**}(W) = \theta^{**}\{E_0[u(c^*)]\} \ ,$$

$$\frac{d\theta}{dW} = \frac{d\theta}{dE_0[u(c^*)]} < 0 \ . \qquad (21)$$

This model is deceptively similar to Uzawa's endogenous time preference model (14), simply replacing c_t with c^* and $0 < \frac{d\theta_t}{du(c_t)}$ with $\frac{d\theta}{dE_0[u(c^*)]} < 0$. However, the two models are completely different because of the opposite characteristics between $0 < \frac{d\theta_t}{du(c_t)}$ and $\frac{d\theta}{dE_0[u(c^*)]} < 0$.

2.6.2.2 Nature of the model

The model (21) can be regarded as successful only if it exhibits stability. In Uzawa's model, the economy becomes unstable if $0 < \frac{d\theta_t}{du(c_t)}$ is replaced with $\frac{d\theta_t}{du(c_t)} < 0$. In this section, I examine the stability of the model.

2.6.2.2.1 Equilibrium rate of time preference

In Ramsey-type models, such as equations (1) and (2), if a constant rate of time preference is given, the value of marginal product of capital (i.e., the value of the real interest rate) converges to that of the given rate of time preference as the economy approaches the steady state. Hence, when a rate of time preference is specified at a certain value, the corresponding expected steady-state consumption is uniquely determined. Given fixed values of other exogenous parameters, any predetermined rate of time preference has unique values of expected consumption and utility at steady state. There is a one-to-one correspondence between the expected utilities at steady state and the rates of time preference; therefore, the expected utility at steady state can be expressed as a function of the rate of time preference. Let c_X^* be a set of steady-state consumptions, given a set of time preference rates (θ_X) and other fixed exogenous parameters. The function $\theta \rightarrow W$ argued above can be described as

$$g(\theta) = E_0[u(c^*)](= W),$$

where $c^* \in c_X^*$ and $\theta \in \theta_X$. On the other hand, the rate of time preference is a continuous function of steady-state consumption as shown in the model (21) such that $\theta = \theta^{**}(W) = \theta^{**}\{E_0[u(c^*)]\}$. The reverse function is

$$h(\theta) = E_0[u(c^*)](= W) \ .$$

The equilibrium rate of time preference is determined by the point of intersection of the two functions, $g(\theta)$ and $h(\theta)$, as shown in Figure 4. Figure 5 shows a special but conventionally assumed $h(\theta)$, in which θ is not sensitive to W, and the rate of time preference is constant permanently. There exists a point of intersection because both $g(\theta)$ and $h(\theta)$ are monotonously continuous for $\theta > 0$. $h(\theta)$ is monotonously continuous because $\theta^{**}(W)$ is monotonously continuous. $g(\theta)$ is monotonously continuous because, as a result of utility maximization, $c^* = f(k^*)$ and $\theta = \frac{df(k^*)}{dk^*}$, where k^* is capital input per capita at steady state such that $k^* = \lim_{t\to\infty}(k_t)$. Because $f(k^*)$ and $\frac{df(k^*)}{dk^*}$ are monotonously continuous for $k^* > 0$, c^* is a monotonously continuous function of θ for $\theta > 0$. Here, because u is monotonously continuous, then $E_0[u(c^*)] = g(\theta)$ is also monotonously continuous for $\theta > 0$.

The function $g(\theta) = E_0[u(c^*)] = W$ is a decreasing function of θ because the higher rate of time preference results in the lower steady state consumption. The function $h(\theta) = E_0[u(c^*)] = W$ is also a decreasing function of θ because $\frac{d\theta}{dW} < 0$. Thus, both $g(\theta)$ and $h(\theta)$ are decreasing, but the slope of $h(\theta)$ is steeper than that of $g(\theta)$ as shown in Figure 4. This is true because $g(\theta) = W$ is the consequence of the Ramsey-type model indicated in equations (1) and (2); thus, if $\theta \to \infty$, then $g(\theta) = W \to 0$ because $\theta = i_t \to \infty$ and $k_t \to 0$, and if $\theta \to 0$, then $g(\theta) = W \to \infty$ because $\theta = i_t \to 0$ and $k_t \to \infty$. On the other hand, the function $h(\theta) = W$ indicates the endogeneity of time preference, and because the rate of time preference is usually neither zero nor infinity, then even if $h(\theta) = W \to 0$, $\theta < \infty$, and $h(\theta) = W \to \infty$, $0 < \theta$. Hence, the locus $h(\theta) = W$ cuts the locus $g(\theta) = W$ downwards from the top, as shown in Figure 4. Because the locus $h(\theta) = W$ is more vertical than $g(\theta) = W$, a permanently constant rate of time preference, as shown in Figure 5, has probably been used as an approximation of the locus $h(\theta) = W$ for simplicity.

2.6.2.2.2 Stability of the model

The rate of time preference is constant unless a shock that changes the expectation of c^* occurs. This is self-evident by $W = E_0[u(c^*)]$. W does not depend on t but on the expectation of c^*; thus, the same rate of time preference and steady state continue until such a shock hits the economy. Therefore, the endogeneity of time preference matters only when such a shock occurs. This constancy is the key for the stability of the model (21). Once the rate of time preference corresponding to the intersection is determined, it is constant and the economy converges at a unique steady state unless a shock that changes the expectation of c^* occurs. This shock is exogenous to the model, and the economy does not explode endogenously but stabilizes at the steady state. Hence, the property $\frac{d\theta}{dW} < 0$ in model (21), which is consistent with empirical findings, does not cause instability.

Model (21) therefore is acceptable as a model of endogenous time preference, which indicates that, because the rate of time preference is endogenously determined, irrationality is not necessary for determination of the time preference rate. Nevertheless, a shock on the rate of time preference is initiated by a shock on the expectation of c^*; thus, even though animal spirits are directly irrelevant to determination of the time preference rate, they may be relevant to the generation of shock on the expectation of c^*. This possibility is examined in Section 2.6.4.

2.6.3 Uncertainty and time preference

An important feature of the model (21) is that a shock on uncertainty makes the rate of time preference shift, where uncertainty means the stochastic nature of the steady-state consumption (c^*). This is not a new idea. Fisher (1930) argued that uncertainty, or risk, must naturally have an influence on the rate of time preference, and higher uncertainty tends to raise the rate of time preference. This feature is particularly important for examining the mechanism of recession, because it has been reported that uncertainty increases in a recession (e.g., Romer, 1990).

The uncertainty about c^* can be described by the stochastic dominance of the distribution of c^* in a second-degree sense or a Rothschild-Stigliz sense. Given $F\left(c^*\right)$, a subjective cumulative distribution function of c^* $\left(0 \leq a < c^* < b\right)$,

$$W = E_0\left[u\left(c^*\right)\right] = \int_a^b u\left(c^*\right) dF\left(c^*\right) . \tag{22}$$

Consider two steady-state consumptions c_1^* and c_2^*. Because $u\left(c^*\right)$ is increasing and concave in c^*, then $E_0\left[u\left(c_2^*\right)\right] \leq E_0\left[u\left(c_1^*\right)\right]$ if $F\left(c_1^*\right)$ second degree stochastically dominates $F\left(c_2^*\right)$, with strict inequality for a set of values of c^* with positive probability. If $F\left(c_1^*\right)$ stochastically dominates $F\left(c_2^*\right)$ in the Rothschild-Stigliz sense, then $E_0\left[u\left(c_2^*\right)\right] \leq E_0\left[u\left(c_1^*\right)\right]$ and the mean of c^* is preserved as well.

Suppose that a shock on the distribution of c^* occurs, which preserves the mean but makes the uncertainty increase for any θ. Because utility $u\left(c^*\right)$ is increasing and concave, this increase in uncertainty indicates a shift of the locus $g(\theta) = W$ downwards to the bold dashed line shown in Figure 4, because $W = E_0\left[u\left(c^*\right)\right]$ becomes smaller for any θ. Hence, if the uncertainty about c^* increases from $F\left(c_1^*\right)$ to $F\left(c_2^*\right)$ in the Rothschild-Stigliz sense, $W = E_0\left[u\left(c^*\right)\right]$ decreases. Even though the mean of c^* is not preserved, if the uncertainty about c^* increases from $F\left(c_1^*\right)$ to $F\left(c_2^*\right)$ in the second-degree sense, $W = E_0\left[u\left(c^*\right)\right]$ also decreases. If the mean of c^* decreases simultaneously, the locus $g(\theta) = W$ shifts further downwards to the thin dashed line in Figure 4. Therefore, the equilibrium rate of time preference increases; that is, increased uncertainty makes households more myopic. The effect of uncertainty in the model (21) is thus consistent with Fisher's (1930) argument.[5]

2.6.4 Government failure

Animal spirits may influence the generation of shocks on the expectation of c^*, but the arbitrary assumption of animal spirits is not compelling. In this section, I explore a mechanism that generates a shock on the expectation of c^* without the need to invoke animal spirits.

2.6.4.1 Policy-induced stochastic processes

2.6.4.1.1 A stochastic process with an absorbing state

Because it is not present consumption (c_t) but steady-state consumption (c^*) that matters, the factor that generates a shock on the expectation of c^* should have persistent effects on consumption. Thereby, the factor should be one of the deep parameters (e.g., total factor productivity (TFP) and preferences) that can change the steady state. In addition, since it has been reported that uncertainty increases in a recession (e.g., Romer, 1990), the factor should make c^* be expected to be random with a constant probability distribution. For the endogenous variable c^* to be expected to be random, exogenous random variables are required because, without exogenous random variables, endogenous variables are constant at steady state. Nevertheless, exogenous variables that make c^* be expected to be substantially random with a constant probability distribution are not easily found among the deep parameters. If the exogenous stochastic valuable is a stationary process with a known constant steady-state probability distribution, c^* is expected to be smoothed by the stochastic Ramsey-Euler equation and to become nearly deterministic (e.g., Brock and Mirman, 1972; Mirman and Zilcha, 1977). On the other hand, if it is a random walk, it does not have a constant probability distribution.

5 Harashima (2004a) shows that the rate of time preference and uncertainty in Japan simultaneously rose at the end of 1990s just before Japan entered a severe and persistent economic slump.

Hence, for c^* to be expected to be substantially random with a constant probability distribution, a special process of the exogenous stochastic variable is required. The following jump process with an absorbing state (Ψ_t) is such a process. For an unknown future period $\bar{t}\ (0 < \bar{t})$,

$$\Psi_t = \begin{cases} random\ (non-absorbing\ \ state) & if\ 0 \le t < \bar{t} \\ determinis\ tic\ (absorbing\ \ state) & if\ \bar{t} \le t \end{cases} , \tag{23}$$

where there are finite $\Lambda\ (\in N)$ deterministic states after the period $\bar{t}$. Which of the states becomes the absorbing state of Ψ_t after $\bar{t}$ is unknown until $\bar{t}$, but the probability distribution of the absorbing state is known for any t before $\bar{t}$. Let state $\lambda(\in \Lambda)$ take the value v_λ and its probability density function be $\tau(v_\lambda)$. Then, the present expected value of Ψ_t at steady state is $E_0\left(\lim_{t\to\infty} \Psi_t\right) = \sum_{\lambda=1}^{\Lambda} v_\lambda\ \tau(v_\lambda)$. If the value of each state is time-variable as $v_{\lambda,t}$ but converges at each constant value if $t \to \infty$, then the present expected value of Ψ_t at steady state is $E_0\left(\lim_{t\to\infty} \Psi_t\right) = \sum_{\lambda=1}^{\Lambda} \lim_{t\to\infty} v_{\lambda,t}\ \tau\left(\lim_{t\to\infty} v_{\lambda,t}\right)$ and its probability density function is $\tau\left(\lim_{t\to\infty} v_{\lambda,t}\right)$. An important feature of the process Ψ_t is that c^* is not expected to be smoothed by the stochastic Ramsey-Euler equation because it is only after $\bar{t}$ that one of the deterministic paths ($v_{\lambda,t}$) that is chosen as the absorbing state is known, and consumption proceeds solely in accordance with this unique deterministic path. Therefore c^* is expected to be random with a constant probability distribution depending on randomly distributed deterministic paths $v_{\lambda,t}$ after $\bar{t}$.

2.6.4.1.2 Policy-induced elements

An important feature of this Ψ_t-type process is that the unique future deterministic path is decided in the future. This feature is often observed in a government's policy decisions, which often take time to make. Once the government has made a decision, the path is deterministic, but before the decision, the path is uncertain. Governments sometimes postpone decisions because they are difficult (e.g., tax hike decisions). As a result, before the policy is decided, households have uncertainty with a constant probability distribution of the deterministic path. Hence, the necessity of a Ψ_t-type process for the exogenous variable that makes c^* be expected to be substantially random with a constant probability distribution suggests that the exogenous variable is policy related.

A Ψ_t-type process implies that, even though the exogenous variable is a stationary process, if it has break points in its process then c^* can be expected to be substantially random. The factors that break a stationary process require exogenous mechanisms. Some structural changes in the mechanism of forming TFP or preferences will be necessary. Nevertheless, the mechanisms of forming TFP and preferences do not usually change. One of the few possibilities for change is that the mechanism is policy related because policies are changed at the discretion of governments, and stationary processes will occasionally break if they are related to policies. Therefore, the necessary properties of the exogenous variable, whether it takes a Ψ_t-type process or not, suggest that the exogenous variable is policy related. The policy-induced element in TFP is particularly important, because production is substantially affected by the TFP level.[6]

2.6.4.2 A policy-induced financial element in TFP

2.6.4.2.1 Financial elements in TFP

[6] The policies on TFP related to c^* are usually policies on economic structure and do not include discretionary macro-economic (fiscal and monetary) policies.

An important element in TFP is natural science technologies and knowledge. They are usually assumed to be stochastic, primarily because of the random nature of scientific discoveries and inventions. However, that randomness implies a random walk that has no constant probability distribution and, more importantly, no steady state. Therefore, scientific technology and knowledge will not be the element in TFP that changes the expected distribution of c^*.

Elements in TFP are not limited, however, to natural science technologies and knowledge. In the production function $y_t = \omega A_t^{\varpi} k_t^{1-\varpi}$ (equation (16)), A_t usually indicates natural science technologies and knowledge, but TFP is not A_t but ωA_t^{ϖ}. If ω contains a policy-induced element, TFP is affected by the policy. Financial elements are included in this group of policy-induced elements. Economic development is proportionate to the level of financial development (e.g., Wachtel, 2003; Do and Levchenko, 2007), and wide differences of financial development have existed between developed and developing economies. Many studies have concluded that the causality is from financial development to economic activities (e.g., Levine, 1997; La Porta el al., 1998; Levine et al., 2000). In addition, the importance of financial development as a driving force of economic growth has been repeatedly emphasized (e.g., Levine, 1997; Levine et al., 2000; Temple, 2000; Easterly and Levine, 2003). Financial development reduces friction in markets, especially in capital accumulation and technological innovation (e.g., Levine, 1997), and financial systems play a critical role in allocation of resources, which is crucial for innovative activities (e.g., Schumpeter, 1912/1934; Shaw, 1973). These facts and arguments indicate that the financial element in TFP is an important determinant of the parameter ω and has significant effects on TFP. An important feature of the financial element is that it is closely related to government policies and thus has a Ψ_t-type process, because there is an important imperfection in financial markets—there is asymmetric information between borrowers (firms) and lenders (investors)—and it needs to be eliminated by government.

2.6.4.2.2 Asymmetric information

The problem of imperfection in financial markets has long been studied (e.g., Gertler, 1988; Mishkin, 1991). Lenders usually have less information than borrowers. Under this asymmetric information, lenders may lend their money to less appropriate and lower quality borrowers, which indicates that resources including technologies are not optimally allocated in the economy.[7] If there is no asymmetric information, the optimal allocation of resources in the economy will be achieved by rational activities of investors, but if there is asymmetric information, the allocation will be distorted. Non-optimal allocation of resources decreases the economy's overall efficiency, and TFP becomes lower in the long run if asymmetric information is left as it is.

Financial intermediaries mitigate the asymmetric information. Because financial intermediaries join in activities between firms and investors, the asymmetric information is separated into two parts: between firms and financial intermediaries, and between financial intermediaries and investors. The former will be reduced substantially by activities of financial intermediaries that monitor and investigate information on firms. Nevertheless, the latter is usually not easily minimized because of the principal-agent problem between investors and financial intermediaries. A financial intermediary (the agent) has an incentive to divert its behaviors from what an investor (the principal) wishes if there is asymmetric information and the investor does not know whether the contract has been satisfied. As a result, markets are distorted.

To reduce the principal-agent problem, investors must sufficiently monitor financial intermediaries. Investors, however, including individual small depositors of banks, cannot sufficiently monitor the intermediaries because such monitoring requires very complex

[7] Not all technologies are embodied in a unit of capital, and each capital embodies only a portion of technologies. The adequate allocation of technologies over capital is important for maximizing production efficiency.

processes, special skills, and a great deal of technical knowledge. More importantly, it is necessary to access perfect information on financial intermediaries and firms. If signals in financial markets contain and transmit perfect information on financial intermediaries and firms, investors may sufficiently monitor financial intermediaries, but many empirical studies have shown that the information is not perfect. For example, DeYoung et al. (2001) show that supervisors' assessments of banks contain some information that is not incorporated into prices of subordinated debts in markets. Other studies have also shown that signals from financial markets do not contain and transmit information perfectly (e.g., see Berger et al., 2000; Curry et al., 2008; Furlong and Williams, 2006). Such imperfect market signals suggest that some information—in particular, bad information—is deliberately hidden from markets.

2.6.4.2.3 The financial supervision authority

The market's inability to solve the problem of asymmetric information justifies the government's intervention to eliminate the distortion. On behalf of investors, the financial supervision authority eliminates the asymmetric information. As argued in Section 2.6.4.2.2, the problem of asymmetric information is separated into two parts. With addition of a financial supervision authority, the problem is further divided: asymmetric information between firms and financial intermediaries, between financial intermediaries and the authority, and between the authority and investors. The first two parts can be solved by financial intermediaries and the authority, respectively. The last part is not necessarily easily solved, however, because investors cannot fully monitor the authority's activities. They have to trust the authority. Hence, self-regulation is quite important for the authority.

It is very difficult to be perfect, and the supervision may occasionally fail. Such failure is more likely to occur and be more severe after regulations have been substantially changed, for example, after deregulation. In such cases, the financial supervision authority has to innovate to adapt to the new regulations. Because the authority is a monopoly, its failure is not a single negligible error among many authorities,[8] and once the supervision fails, its negative effects will spread widely through financial markets. In addition, there is also a principal-agent problem between the authority and investors. The authority has an incentive to hide its failure from investors, and if the authority deliberately hides its failure, investors cannot easily know of the failure.

If asymmetric information is unchecked because of the failure of supervision, financial intermediaries will obtain extra profits thanks to the asymmetric information. The negative effect of non-optimal allocation of resources will be recognized only by less-informed investors and households far later. Faced with the extra profits of financial intermediaries, less-informed investors and households may wrongly guess that technology is unexpectedly progressing more than it actually is. The lee-informed households will then undertake activities on the basis of this incorrect guess—activities that would be considered to be irrational if perfect information were available—and this may make the economy spuriously appear to be in a boom in the short run.

2.6.4.2.4 Revelation of the failure of supervision

Even if an authority deliberately hides its failure, it is impossible to hide it forever. Because there is a gap between the distorted expectation by less-informed households and actual economic activities, the failure will eventually be revealed, perhaps by accident. When the failure is revealed, the trust in the financial supervision authority will immediately be lost, and the expectation of future policy will change suddenly and sharply. Because the financial element in TFP is a policy-induced element and has a Ψ_t-type process, the expected probability distribution of the financial element in TFP at steady state will also immediately change.

The arguments in Section 2.6.4.2.2 indicate that the present financial element in TFP

[8] In some economies, the authority is separated across a few branches in the government, depending on the type of financial intermediary, but each branch is a monopoly authority for each of type of intermediary.

will not change suddenly and sharply, because the already allocated resources cannot change suddenly and sharply. Nevertheless, unlike the present financial element in TFP, the expected probability distribution of the financial element in TFP at steady state can change suddenly and sharply with the revelation of the failure of supervision. In addition, the failure of supervision implies that the expected distributions of the financial element in TFP and c^* were wrongly formed before the revelation of the failure; thus, the revisions of the expected distributions of the financial element in TFP and c^* resulting from the revelation will be more substantial than usual. As a result, the rate of time preference is immediately raised and a Nash equilibrium of a Pareto inefficient path will be immediately selected even though the present TFP is almost unchanged.

3 CYCLICAL VOLATILITY OF THE V-U RATIO

3.1 The Nash equilibrium of a Pareto inefficient path

Harashima (2009) shows that a Nash equilibrium of a Pareto inefficient path is generated even in a frictionless economy if—and probably only if—the rate of time preference shifts. An essential reason for the generation of this Nash equilibrium is that households are intrinsically risk averse and not cooperative. In a strategic environment, this generates the possibility that, if consumption needs to be substantially and discontinuously increased to keep Pareto optimality, a non-cooperative household's strategy to deviate from the Pareto optimal path gives a higher expected utility than the strategy of choosing the Pareto optimal path.

A weaknesses of the dynamic stochastic general equilibrium approach to macroeconomics stems from going too directly from statements about individuals to statements about the aggregate (Caballero, 2010). The Nash equilibrium of a Pareto inefficient path is an answer to this problem because this equilibrium is not derived from a simple summation of individuals' identical behaviors but is a result of strategic interactions among non-cooperative individuals.

The difference in consumption between the Pareto inefficient path in this Nash equilibrium and the conventional Pareto efficient path in each period t is b_t (≥ 0 for upward time preference shocks). b_t is successively generated and eliminated on the Pareto inefficient path.

3.1 Matching friction

The standard version of the search and matching model by Shimer (2004) is used in this paper as the base model. The model is a simplified version of the model by Pissarides (1985). The economy consists of a measure 1 of risk-neutral, infinitely lived workers and a continuum of risk-neutral, infinitely lived firms. The common discount rate of workers and firms is r. An unemployed worker gets flow utility z from non-market activity and searches for a job. An employed worker earns an endogenous wage w but may not search. The production function exhibits constant returns to scale, and for simplicity only labor inputs are used for production. Although capital inputs do not affect production, they are implicitly assumed and can affect matching friction only if they affect parameter values in the model. By employing a worker, a firm obtains profit equal to the difference between labor productivity π and the wage (i.e., $\pi - w$). Jobs end at rate σ, which leaves a worker unemployed and a firm with a vacancy. In order to hire a worker, a firm must maintain an open vacancy at flow cost κ.

Matching technology is assumed to be a Cobb–Douglas and constant returns to scale. Thereby, the rate at which unemployed workers find jobs and the rate at which vacancies are filled depend only on the endogenous v-u ratio in period t, $\varphi_t\left(=\dfrac{v_{c,t}}{u_{e,t}}\right)$, where $u_{e,t}$ is the unemployment rate in period t, and $v_{c,t}$ is the job vacancy rate in period t. Workers find jobs at rate $\mu\varphi_t^{1-\alpha}$ and vacancies are filled at rate $\mu\varphi_t^{-\alpha}$, where μ is a parameter and $\alpha\left(0<\alpha<1\right)$ is the elasticity of the matching function with respect to the unemployment rate. The unemployment rate $u_{e,t}$ increases with job destruction and decreases when workers find jobs, and thus it moves such that

$$\dot{u}_{e,t} = \sigma\left(1 - u_{e,t}\right) - \mu\varphi_t^{1-\alpha} u_{e,t} \quad . \tag{24}$$

This matching process is summarized by the following Bellman equations:

$$rU = z + \mu\varphi^{1-\alpha}\left(E - U\right) \tag{25}$$

$$rE = w + \sigma\left(U - E\right) \tag{26}$$

$$rV = -\kappa + \mu\varphi^{-\alpha}\left(F - V\right) \tag{27}$$

$$rF = \pi - w + \sigma\left(V - F\right) \quad . \tag{28}$$

Equations (25) and (26) indicate the value of a worker when unemployed (U) and employed (E), respectively. If unemployed, the worker gets current value from non-market activity z and finds a job at the rate $\mu\varphi^{1-\alpha}$. When employed, the worker earns wage w and loses the job at rate σ. Equations (27) and (28) indicate the value of a job that is vacant (V) or filled (F), respectively. A free entry condition for vacancies is assumed, and firms create job openings until

$$V = 0 \quad . \tag{29}$$

In addition, wages are assumed to be set by asymmetric Nash bargaining. At any point in time all workers are paid a common wage w. The Nash bargaining assumption amounts to

$$\frac{E - U}{\beta} = \frac{F - V}{1 - \beta} \quad , \tag{30}$$

where $\beta\left(0 < \beta < 1\right)$ represents workers' bargaining power. By equations (25)–(30),

$$\frac{r + \sigma}{\mu\varphi^{-\alpha}} + \beta\varphi = \left(1 - \beta\right)\frac{\pi - z}{\kappa} \quad . \tag{31}$$

Equation (31) determines the v-u ratio φ when the values of α, β, κ, μ, π, σ, and r are given.

3.2 The effect of successive b_t on the v-u ratio

A Nash equilibrium of a Pareto inefficient path successively generates large amounts of unutilized products b_t and creates a great deal of idle and discarded resources. As shown in Section 2, this Nash equilibrium has significant impacts on various aspects of the economy, and the matching process shown in Section 3.1 is no exception. In this section, I examine in detail how successive b_t affects the matching process.

The Nash equilibrium of a Pareto inefficient path yields an unusual path on which Pareto inefficiency (successive b_t) is persistently generated. As shown in Section 2, successive b_t distorts the economy and changes various conditions for economic activities in many aspects. In the matching process shown in Section 3.1, the conditions are represented by the values of the parameters α, β, κ, μ, π, σ, and r. These values are usually assumed to be unchanged, but the distortion may cause them to change substantially. Moreover, if any of them do change, the v-u ratio also will change from that when b_t is not generated by equation (31). Furthermore, because the generation of b_t persists as shown in Section 2, the values of the v-u ratio will stay different for a long period; thus, if time preference shocks occur, the v-u ratio will also fluctuate as a result of the successively generated b_t. The respective effects of successive b_t on the parameters σ, μ, κ, and π are examined in detail in the following sections.

Note that the rate of time preference can shift up or down. The effects of a downward time preference shock are opposite to those of an upward one, and negative b_t is generated by the same mechanism as positive b_t. If b_t is negative, the economy booms. Goods and services and resources for inputs become scarce and need to be supplemented immediately through the creation of extra resources from scratch. However, in practical terms, the immediate creation of resources is physically very difficult; thus the needed goods and services will be substituted through the increased utilization of existing resources, for example, by increasing the amount of overtime work and rates of operation. The stream of negative b_t will therefore be considerably constrained.

3.2.1 The effect of successive b_t on separation rate

As positive b_t is successively generated, it is successively eliminated through products being discarded or through the preemptive suspension of production, leaving some capital and labor inputs idle, as shown in Section 2. Firms will eliminate excess workers by either firing them or abstaining from recruiting new workers. The former measure relates to the separation rate σ, whereas the latter relates to vacancy costs κ, which will be examined in greater detail in Section 3.2.3. Firing workers usually imposes additional costs on firms. These costs are implicit in the model presented in Section 3.1, but they are naturally reflected in and constrain the separation rate σ; that is, because of these costs, the value of σ is lowered.

Successive positive b_t indicates not only that firms fire more workers but that this larger than usual job destruction will last for a long period. As a result, the job separation rate will stay at a higher level as long as positive b_t is successively generated. Hence, the separation rate σ is a function of b_t such that $\sigma = \sigma(b_t)$ and

$$\frac{d\sigma(b_t)}{db_t} > 0 \quad . \tag{32}$$

Equation (32) implies that separation rate will increase during recessions.

In the case of negative b_t, the separation rate will decrease because firms will maximize the use of existing resources and the number of forced job separations will decrease. Hence, inequality (32) also holds for negative b_t. Equation (32) therefore also implies that the separation rate will decrease during economic booms.

Notice, however, that empirical evidence of cyclical fluctuations in the separation rate is mixed. Shimer (2005) and Hall (2005b) stated that the separation rate is almost acyclic in the United States, but Fujita and Ramey (2009) and Barnichon (2009) argued that the separation rate explains no small part of fluctuations in unemployment.

3.2.2 The effect of successive b_t on the job-finding rate

Positive successive b_t will also affect the job-finding rate. More specifically, it will decrease the rate because the labor market becomes more segmented by space and skill, and mismatch is exacerbated when positive b_t is generated.

3.2.2.1 Successive b_t and mismatch

Mismatch has long been studied in labor economics (e.g., Kain, 1968; Taylor, 1995; Coles and Muthoo, 1998; Hall, 2000; Shimer, 2007), including spatial mismatch (e.g., Ihlanfeldt, 1997; Brueckner and Zenou, 2003; Smith and Zenou, 2003) and skill mismatch (e.g., Thisse and

Zenou, 2000). Shimer (2007) argued that the mismatch factor can explain most of the cyclical volatility in the v-u ratio. However, his model is too simplified to be used to explain actual phenomena, and it is arguable that mismatch is always economically important because job seekers can rationally prepare for the factors that cause mismatch. If a worker can prepare for job separation, for example, by collecting information on and studying financial situations of firms located even in distant places or by obtaining new skills utilizing a relatively long period before the separation, the cost of moving from one possibly segmented labor market to another will be dispersed over time before the separation. The costs can be paid in installments in the long period before the separation. Thereby, costs per period can be reduced substantially, and the hurdle that a worker has to clear to change labor markets will be considerably lowered.

If job separations are intentional, for example, to obtain better and more interesting jobs with higher wages, workers will sufficiently prepare before deciding to leave their current jobs. Even if job separations are forced, workers can prepare if the separations can be sufficiently foreseen. More generally, if there is no unexpected disturbance in the economy, most workers can sufficiently foresee their own job separations before the separations occur. Conversely, mismatch will be economically important if an unexpected large disturbance occurs. A large enough upward time preference shock would represent this type of unexpected large disturbance and make mismatch an important element.

As discussed in Section 3.2.1, many workers are fired owing to positive successive b_t, but they will not have enough time to prepare for their job separations because these separations are not expected sufficiently prior to the actual separation. Without preparation, the costs of changing markets are high, and the high costs indicate that the labor market is substantially segmented. At the same time, successive b_t requires a larger reallocation of workers than usual across these substantially segmented labor markets, and mismatch will be exacerbated. In addition, because b_t is successively generated, the effect of the segmented labor market will last for a long period. As a result, mismatch will continue to be economically important while b_t is generated.

3.2.2.2 The effect on the job-finding rate

A conventional matching function is $m_t = M(u_{e,t}, v_{c,t})$, where m_t is the number of new matches in period t. Considering the exacerbated mismatch by positive b_t discussed in the previous section, the matching function is changed to $m_t = \hat{M}(u_{e,t}, v_{c,t}, b_t)$, and more specifically

$$m_t = \widetilde{M}(b_t) M(u_{e,t}, v_{c,t}) \ , \tag{33}$$

where $\widetilde{M}(b_t)$ is a function of b_t. The effect of successive b_t indicates that $\widetilde{M}(0) = 1$ and

$$\frac{d\widetilde{M}(b_t)}{db_t} < 0 \ . \tag{34}$$

By incorporating this matching function (equation [33]) into the model in Section 3.1, the job-finding rate $\mu\varphi^{1-\alpha}$ in equation (25) and the vacancy filled rate $\mu\varphi^{-\alpha}$ in equation (27) are changed to $\mu(b_t)\varphi^{1-\alpha}$ and $\mu(b_t)\varphi^{-\alpha}$, respectively, and

$$\frac{d\mu(b_t)}{db_t} < 0 \tag{35}$$

because of equation (34). Equation (35) implies that the job-finding rate will decrease during recessions.

In the case of negative b_t, μ will increase because firms manage to utilize currently existing resources to the greatest possible extent and the number of forced and unprepared job separations will decrease, which will result in more moderate mismatch. Hence, inequality (35) holds for negative b_t. Equation (35) therefore also implies that the job-finding rate will increase during economic booms.

3.2.3 The effect of successive b_t on vacancy costs

Vacancy costs κ are the costs that firms are obliged to pay as a result of recruitment activities. Vacancy costs therefore include not only the direct costs of recruitment activities (e.g., advertising, selection) but obligations engendered by recruitment activities. In particular, recruitment obligations include the fact that, after a new worker is hired, the firm is obliged to keep employing the worker at least in some periods because many regulations protect workers and firms usually cannot freely fire workers arbitrarily at any time. Even without such regulations, however, when engaging in recruitment activities a firm is publicly exhibiting its intention to continue to employ newly hired workers for some periods. No firm engages in recruitment activities if it has the intention of immediately firing the newly hired worker. This obligation or intention behind recruitment activities is implicitly but naturally assumed in the model presented in Section 3.1.

As shown in equation (27), information about κ is used for decision-making in the labor market at the time of recruitment. Therefore, if the above-mentioned obligation or intention is foreseen to surely cause extra losses or profits in the future, the provision of a reserve for these extra losses or profits should be added to the flow cost of vacancy κ because all available information should be utilized at the time of decision-making in order to be rational. Nevertheless, this obligation or intention usually will do no extra harm or good to firms and will not be foreseen to surely cause future extra losses or profits. Hence, κ is usually irrelevant to this obligation or intention and therefore is constant. Moreover, it is relatively small because it consists mostly of direct costs.

However, if b_t is generated, κ will no longer be irrelevant to the obligation or intention. As shown in Section 2, successively generated b_t is eliminated successively, and the elimination is implemented by discarding products or preemptively suspending production, leaving some capital and labor inputs idle. The number of excess and idle workers has to be reduced, and one way to do so is to fire workers as discussed in Section 3.2.1. Another way is to abstain from recruiting new workers. Without recruitment, the number of workers a firm employs will gradually decrease as some existing workers separate from the firm for various reasons. Because firing workers imposes sizable costs on firms from both the financial and management points of view, firms will adopt both measures to reduce the number of excess and idle workers.

Both of these measures must be correctly reflected in the model. As shown in Section 3.2.1, the measure of firing excess and idle workers can be directly introduced into the model as an increase in the separation rate σ by positive b_t. On the other hand, abstaining from recruiting workers cannot be directly introduced as a change in the separation or job-finding rates. It nevertheless can be reflected in vacancy costs κ because vacancy costs increase owing to positive b_t as shown below. When positive b_t is successively generated, the additional production that a newly employed worker contributes to should be eliminated as a part of b_t, or the new worker should be left idle to preemptively suspend production even though the worker is still paid a wage. In either case, the firm will continue to lose money because of the obligation or intention to keep employing workers. Unlike the case when b_t is not generated, the obligation or intention significantly harms firms. These additional costs do not exist unless b_t is generated. An important point is that these losses are foreseen at the time of recruitment to be surely incurred in the future because firms know that the economy is in the state of Nash equilibrium of a Pareto inefficient path. Hence, the provision of a reserve for these losses should be added to the flow cost of a vacancy. Maintaining an open vacancy while b_t is successively generated indicates that a firm has made a decision to accept these extra losses in the future, considering the increased cost of the vacancy because of the added reserve provision.

In addition, the costs caused by the obligation or intention will be far larger than the direct costs for recruitment activities because they include a part of b_t. For example, if a newly employed worker is left idle, the costs will be equivalent to the total wages w paid to the worker during idle periods. If the worker works and the products are immediately discarded, the costs become even greater because they amount to π. Therefore, it is clear that a positive b_t makes recruitment very costly.

Hence, when positive b_t is generated, vacancy costs κ will substantially increase. As long as b_t is successively generated, κ will continue to be substantially high. Therefore, vacancy costs κ are a function of b_t such that $\kappa = \kappa(b_t)$ and

$$\frac{d\kappa(b_t)}{db_t} > 0 \quad . \tag{36}$$

Equation (36) implies that the number of vacancies will substantially decrease during recessions because the cost of vacancy becomes very high.

In the case of negative b_t, vacancy costs κ will decrease because extra profits (a part of b_t) are foreseen to be surely obtained in the future as a result of recruitment activities; thus, inequality (36) holds for negative b_t. Equation (36) therefore also implies that the number of vacancies will increase during economic booms because the cost of a vacancy decreases. Notice, however, that the scale of extra profits will be far smaller than that of extra losses because extra profits generated by negative b_t are the difference between production and wages (i.e., $\pi - w$), whereas extra losses by positive b_t are w or π. A large part of b_t leaks to workers in the case of negative b_t. Notice also that, by equations (27) and (28), a lower κ makes $\pi - w$ lower, and if $\kappa \rightarrow 0$, then $\pi - w \rightarrow 0$. Thus, extra profits approach zero, which makes κ increase. That is, κ is always positive. In addition, a correlation between κ and π or w in the case of positive b_t also exists. Nevertheless, unlike the correlation with negative b_t, that with positive b_t only slightly affects the scale of the costs caused by the obligation or intention because the costs are not $\pi - w$ that can vary largely but no less than w that is far less variable. Therefore, the correlation with positive b_t is ignored in this paper for simplicity.

3.2.4 The effect of successive b_t on labor productivity

In the model presented in Section 3.1, capital inputs are implicit and have no explicit effect on matching friction. On the other hand, in the model in Section 2, a decrease of capital owing to positive successive b_t reduces labor productivity because $\frac{\partial f(A, k_t)}{\partial k_t} > 0$. This means that even though capital inputs are implicit, the effect of b_t on capital can be reflected in the model as a negative shock on labor productivity π. Hence, labor productivity π is a function of b_t such that $\pi = \pi(b_t)$ and

$$\frac{d\pi(b_t)}{db_t} < 0 \quad . \tag{37}$$

Note that the change of labor productivity π as shown in inequality (37) is not a result of a change in total factor productivity (TFP). Usually, a change in π is implicitly assumed to directly represent that of TFP in studies using search and matching models, but the change in π is irrelevant to that of TFP in this paper because conceptually b_t cannot affect TFP and thus TFP is constant.

Note also that, in the case of negative b_t, capital inputs will increase because firms manage to add extra capital inputs by utilizing currently existing resources to the greatest extent possible; thus, inequality (37) will still hold for negative b_t.

3.2.5 The combined effect of successive b_t on the v-u ratio

3.2.5.1 The combined effect

Equation (31) and inequalities (32) and (35)–(37) indicate that the v-u ratio φ is affected by b_t through changes in the separation rate, job-finding rate, vacancy costs, and labor productivity such that φ is determined by

$$\frac{r+\sigma(b_t)}{\mu(b_t)\varphi^{-\alpha}}+\beta\varphi=(1-\beta)\frac{\pi(b_t)-z}{\kappa(b_t)} \tag{38}$$

for a given value of b_t. Let $\Omega(b_t)=\dfrac{r+\sigma(b_t)}{\mu(b_t)}$ and $\Xi(b_t)=(1-\beta)\dfrac{\pi(b_t)-z}{\kappa(b_t)}$. By equation (38),

$$\Omega(b_t)=\Xi(b_t)\,\varphi^{-\alpha}-\beta\varphi^{1-\alpha} \quad . \tag{39}$$

Note that $\Xi(b_t)=(1-\beta)\dfrac{\pi(b_t)-z}{\kappa(b_t)}>0$ because naturally $\pi(b_t)>z$. The total differential of equation (39) yields

$$\frac{d\varphi}{db_t}=\varphi^{-\alpha}\left[\alpha\frac{\Xi(b_t)}{\varphi}+(1-\alpha)\beta\right]\left[\varphi^{-\alpha}\frac{\partial\Xi(b_t)}{\partial b_t}-\frac{\partial\Omega(b_t)}{\partial b_t}\right] \quad . \tag{40}$$

Because $\dfrac{d\sigma(b_t)}{db_t}>0$ and $\dfrac{d\mu(b_t)}{db_t}<0$, as shown in Sections 3.2.1 and 3.2.2,

$$\frac{d\Omega(b_t)}{db_t}=\mu^{-1}(b_t)\left\{\frac{d\sigma(b_t)}{db_t}-\frac{[r+\sigma(b_t)]}{\mu(b_t)}\frac{d\mu(b_t)}{db_t}\right\}>0 \quad . \tag{41}$$

In addition, because $\dfrac{d\kappa(b_t)}{db_t}>0$ and $\dfrac{d\pi(b_t)}{db_t}<0$, as shown in Sections 3.2.3 and 3.2.4, and because $0<\beta<1$ and $\pi(b_t)-z>0$,

$$\frac{d\Xi(b_t)}{db_t}=\frac{1-\beta}{\kappa(b_t)}\left\{\frac{d\pi(b_t)}{db_t}-\frac{[\pi(b_t)-z]}{\kappa(b_t)}\frac{d\kappa(b_t)}{db_t}\right\}<0 \quad . \tag{42}$$

Therefore, because $\varphi^{-\alpha}\left[\dfrac{\alpha\,\Xi(b_t)}{\varphi}+(1-\alpha)\beta\right]>0$ owing to $\Xi(b_t)>0$ and because $\dfrac{d\Omega(b_t)}{db_t}>0$ and $\dfrac{d\Xi(b_t)}{db_t}<0$ by inequalities (41) and (42), then by equation (40),

$$\frac{d\varphi}{db_t}<0 \quad . \tag{43}$$

Hence, when positive successive b_t is generated, unemployment increases and/or vacancies decrease even though wages are flexibly adjusted because the matching process is distorted by the Nash equilibrium of a Pareto inefficient path.

The magnitude of the effect of b_t on φ depends on the values of $\frac{d\Omega(b_t)}{db_t}$ and $\frac{d\Xi(b_t)}{db_t}$, that is, those of $\frac{d\sigma(b_t)}{db_t}$, $\frac{d\mu(b_t)}{db_t}$, $\frac{d\kappa(b_t)}{db_t}$, and $\frac{d\pi(b_t)}{db_t}$, as equations (40)–(42) indicate. The larger the values of $\left|\frac{d\sigma(b_t)}{db_t}\right|$, $\left|\frac{d\mu(b_t)}{db_t}\right|$, $\left|\frac{d\kappa(b_t)}{db_t}\right|$, and $\left|\frac{d\pi(b_t)}{db_t}\right|$ (i.e., the more σ, μ, π , and κ are affected by b_t), the more φ decreases. Furthermore, equations (40)–(42) and inequalities (32) and (35)–(37) indicate that the four factors (σ, μ, κ, and π) transmit the effect of b_t on φ in the same direction such that an increase of b_t decreases φ and vice versa.

3.2.5.2 Cyclical fluctuations

The successiveness of b_t is an important point. When a positive b_t is successively generated, φ continues to be low for a long period by inequality (43). Although b_t eventually disappears, it takes a long time. This persistent or "rigid" nature in the stream of b_t indicates that if time preference shocks occasionally occur, φ will experience large cyclical fluctuations even if wages are flexibly adjusted.

3.2.5.3 Importance of vacancy costs

Among the four factors (σ, μ, κ, and π), vacancy costs κ are particularly important. The proposition that higher prices normally reduce demand is fundamental in economics so it is intuitively and logically quite reasonable that firms reduce the number of vacancies because the cost of vacancies increases, thereby decreasing the v-u ratio. Compared with vacancy costs, the impacts of the separation and job-finding rates on firms' behaviors toward vacancies are indirect and ambiguous. Unless the cost of a vacancy is substantially changed, it seems unlikely that firms' behaviors toward vacancies are considerably affected by these rates. Decreased labor productivity may be also attributed to a fewer number of open vacancies, but Shimer (2004, 2005) and Hall (2005a) argue that a productivity shock alone cannot generate sufficient cyclical volatility in the v-u ratio. Therefore, κ has a clear advantage over σ, μ, and π because its change naturally and directly affects the number of vacancies.

Furthermore, as shown in Section 3.2.3, vacancy costs κ substantially increase because of positive b_t. If b_t is not generated, κ consists mostly of relatively small direct costs for recruitment activities, but once a positive b_t is generated, κ consists not only of the direct costs but also a part of b_t, for example, the sum of w or π during the period of obligation. Hence, if successive b_t is generated, the value of κ will increase noticeably (e.g., several times or more than the previous value), and the impact of $\frac{d\kappa(b_t)}{db_t}$ on $\frac{d\varphi}{db_t}$ will be very large. Compared with the scale of impact of b_t on κ, the impacts on σ, μ, and π appear to be relatively much smaller because the observed cyclical variances of σ and $\mu\varphi^{1-\alpha}$ are 10–20% and those of π are only about several percent. Equations (40)–(42) imply that, if the absolute values of σ, μ, κ, and π change at the same rate because of b_t, their contributions to $\frac{d\varphi}{db_t}$ have roughly the same scale. Therefore, the especially high sensitivity of κ to b_t suggests that a change in κ makes a relatively large contribution to $\frac{d\varphi}{db_t}$.

These two features of vacancy costs (i.e., κ naturally and directly affects vacancies and is extremely increased by b_t) imply that substantial changes in κ because of successive b_t are the main factor driving the large cyclical fluctuations in the v-u ratio.

3.3 The Beveridge curve

Whether unemployment and vacancies are negatively correlated (i.e., whether the Beveridge curve is observed) depends on the value of $\frac{d\varphi}{db_t}$. By equation (24), u_e converges at

$$u_e = \frac{\sigma}{\sigma + \mu\varphi^{1-\alpha}} \quad . \tag{44}$$

When positive successive b_t is generated, u_e increases because

$$\frac{du_e}{db_t} = \frac{\sigma\mu\varphi^{1-\alpha}}{\left(\sigma + \mu\varphi^{1-\alpha}\right)^2}\left[\sigma^{-1}\frac{d\sigma}{db_t} - \mu^{-1}\frac{d\mu}{db_t} - (1-\alpha)\varphi^{-1}\frac{d\varphi}{db_t}\right] > 0 \tag{45}$$

owing to $\frac{d\sigma(b_t)}{db_t} > 0$, $\frac{d\mu(b_t)}{db_t} < 0$, and $\frac{d\varphi}{db_t} < 0$ by inequalities (32), (35), and (43). On the other hand, the sign of $\frac{dv_c}{db_t}$ is not as simple. By equation (44) and $\varphi = \frac{v_c}{u_e}$,

$$v_c = \left[\frac{\sigma(1-u_e)}{\mu u_e^{\alpha}}\right]^{\frac{1}{1-\alpha}} \quad . \tag{46}$$

By the total differential of equation (46),

$$dv = \frac{1}{1-\alpha}\left[\frac{\sigma(1-u_e)}{\mu u_e^{\alpha}}\right]^{\frac{1}{1-\alpha}}\left[\frac{d\sigma}{\sigma} - \frac{d\mu}{\mu} + \left(1-\alpha-\frac{1}{1-u_e}\right)\frac{du_e}{u_e}\right] \quad . \tag{47}$$

Therefore, if $\frac{d\sigma}{\sigma} - \frac{d\mu}{\mu} + \left(1-\alpha-\frac{1}{1-u_e}\right)\frac{du_e}{u_e} < 0$ when positive b_t is generated, then $dv_c < 0$ and thereby u_e and v_c are negatively correlated because $\frac{du_e}{db_t} > 0$ by inequality (45). In this case, the Beveridge curve (the negative correlation between the unemployment and vacancy rates) will be observed.

Equation (47) indicates that the direct effect of an increase in σ on v_c, $\frac{1}{1-\alpha}\left[\frac{\sigma(1-u_e)}{\mu u_e^{\alpha}}\right]^{\frac{1}{1-\alpha}}\frac{d\sigma}{\sigma}$, is an increase in v_c , which is consistent with Shimer's (2005) arguments. In addition, the direct effect of a decrease in μ on v_c, $-\frac{1}{1-\alpha}\left[\frac{\sigma(1-u_e)}{\mu u_e^{\alpha}}\right]^{\frac{1}{1-\alpha}}\frac{d\mu}{\mu}$, is also an increased v_c. Therefore, the direct effects of σ and μ on v_c when positive successive b_t is generated is an increased v_c because $\frac{d\sigma(b_t)}{db_t} > 0$ and $\frac{d\mu(b_t)}{db_t} < 0$. This result conversely implies that the Beveridge curve can be observed only if $\left|\frac{d\varphi}{db_t}\right|$ is sufficiently large because $\left(\frac{d\sigma}{\sigma}-\frac{d\mu}{\mu}\right)\left[\frac{1}{1-u_e}-(1-\alpha)\right]^{-1}<\frac{du_e}{u_e}$ is necessary for the Beveridge curve by equation (47) and $\frac{du_e}{db_t}$ is inversely proportional to $\frac{d\varphi}{db_t}$, as shown in equation (45). In addition, equation (47) implies that the shape of the Beveridge curve will shift or become complicated depending on how the rate of time preference shifts and what shape the stream of b_t takes.

Suppose, for example, that $\alpha = 0.72$, $\sigma = 0.035$, and $\mu\varphi^{1-\alpha} = 0.45$ (see e.g., Shimer, 2005). Thereby, u_e is 0.072 by equation (44). Suppose also that σ increases to 0.038, and μ decreases at the rate of 0.03 because the economy fell into a recession. In this case, $\frac{d\sigma}{\sigma}-\frac{d\mu}{\mu}=\frac{0.003}{0.035}+0.03=0.116$, and because $\frac{1}{1-u_e}-(1-\alpha)=0.798$, if u_e increases more than 14.5% (i.e., from 0.072 to more than 0.083), then $dv_c < 0$ by equation (47). If $\left|\frac{d\varphi}{db_t}\right|$ is large enough such that $\left|\frac{d\varphi}{db_t}\right|\varphi^{-1} > 0.118$ (an 11.8% decrease of φ by b_t), the condition $dv_c < 0$ is satisfied by equation (44). This level of $\left|\frac{d\varphi}{db_t}\right|$ will not be difficult to satisfy by the combined effects of $\frac{d\sigma(b_t)}{db_t}$, $\frac{d\mu(b_t)}{db_t}$, $\frac{d\kappa(b_t)}{db_t}$, and $\frac{d\pi(b_t)}{db_t}$. In particular, the effect of $\frac{d\kappa(b_t)}{db_t}$ is important, as discussed in Section 3.2.5.3. Because $\frac{d\kappa(b_t)}{db_t}$ is substantially large, $\frac{d\kappa(b_t)}{db_t}$ will make $\left|\frac{d\varphi}{db_t}\right|$ sufficiently large and the condition $dv_c < 0$ will be easily satisfied.

4 CYCLICAL FLUCTUATIONS DRIVEN BY CHANGES IN TIME PREFERENCE

4.1 The source of rigidity

Shimer (2004, 2005) and Hall (2005a) showed that, in conventional search and matching models with the Nash bargaining wage formation mechanism, the v-u ratio does not change substantially because the effect of a productivity change is largely absorbed by flexible wage adjustments. This property is a natural consequence of the assumption that disturbances in the labor market are smoothly adjusted by wages (and more broadly prices). Unutilized resources other than those owing to matching friction are fully exploited by rational agents through flexible prices. Because the price mechanism is working, the magnitude of matching friction (e.g., the levels of unemployment and vacancies) cannot be substantially affected by productivity shocks, and if anything, should be almost constant. This result implies that it is unreasonable to think that labor market variables experience large cyclical fluctuations as a result of productivity shocks in an economy in which matching friction is the only friction. This conjecture has led to the idea that another friction is needed to explain large cyclical fluctuations in the labor market.

Shimer (2004, 2005) and Hall (2005a) argued that, if a friction in wage dynamics (i.e., wage rigidity) is introduced into search and matching models, the models match well with observed data. The cushion of the flexible wage is removed in this case, and the v-u ratio can thus be more substantially volatile because wages are not flexible (i.e., the price mechanism does not work well) and the impact of productivity shock cannot be sufficiently absorbed. However, unlike matching friction, the validity of frictions in price and wage dynamics has not necessarily been widely accepted among economists, not a few of whom still regard this as an ad hoc assumption, even though voluminous research has been conducted on this subject since the era of Keynes (e.g., Fuhrer and Moore, 1995; Galí and Gertler, 1999; Mankiw, 2001). The difficulty in presenting a rationale for price (wage) rigidity is easily recognizable. It is very difficult to show why rational agents deliberately refrain from changing prices (wages) even though they are fully aware they will otherwise lose a great deal of money.

In contrast to productivity shocks, time preference shocks can make the v-u ratio fluctuate largely and cyclically without introducing the controversial concept of wage rigidity because they generate a Nash equilibrium of a Pareto inefficient path (i.e., successive b_t). As shown previously, successive b_t naturally generates the possibility of some variables appearing to move rigidly. The stream of b_t persists and b_t changes gradually, that is, it proceeds rigidly. This type of movement makes the economy appear to have a rigid nature, which implies that the rigidity in economic fluctuations originally stems from the successive b_t, not from the long-disputed and still controversial notion of price rigidity. As shown in Section 2, the Nash equilibrium of a Pareto inefficient path has a clear microfoundation and thus is not an ad hoc assumption. In this sense, the Nash equilibrium of a Pareto inefficient path appears to be superior to wage rigidity as an explanation of the source of rigidity. In addition, because time preference shocks naturally generate this Nash equilibrium, they appear to be a more likely source of cyclical fluctuations that exhibit a rigid nature than productivity shocks.

4.2 Economic importance of time preference shocks

Time preference shocks are not only reasonable as a mechanism of generating economic fluctuations and as a source of rigidity, but they are also economically important. Suppose that the production function is the conventional one such that $y_t = f(A,k_t) = Ak_t^{1-\upsilon}$. At steady state, $\frac{df(A,k_t)}{dk_t} = \theta$, and thereby

$$y_t = A^{\frac{1}{\upsilon}} \left[\frac{(1-\upsilon)}{\theta} \right]^{\frac{1-\upsilon}{\upsilon}} . \tag{48}$$

By equation (48), for example, if $\upsilon = 0.7$ and the rate of time preference θ shifts upward from 0.04 to 0.06, y_t at steady state diminishes by about 16%. Table 2 shows the result of a sensitivity analysis, which indicates that a one percentage point upward shift in the time preference rate diminishes steady state production by about 10%.

A 10% change in GDP is very large compared with the scale of a productivity shock. The variance of GDP per capita owing to a productivity shock is at most 2% in most industrialized economies. Furthermore, it is well known that the TFP's variance is very small if various cyclical factors that contaminate TFP data are carefully removed (e.g., King and Rebelo, 1999), which implies that the true variance of GDP per capita owing to a productivity shock is even smaller than 2%. Hence, compared with productivity shocks, the impacts of time preference shocks appear to be very large and economically quite important.

4.3 Validity of cyclical fluctuations driven by time preference

shocks

The advantage of time preference shock over other shocks as the source of cyclical fluctuations can be seen not only in the labor market. Time preference shocks have the following important economy-wide properties:

(a) Pareto inefficient paths are rationally chosen.
(b) Effects of shocks persist.
(c) Both positive and negative shocks can occur.
(d) Various scales of economic fluctuations are possible.
(e) Uncertainty can lead to economic fluctuations.
(f) Some financial indicators can be used to predict economic fluctuations.

Property (a) is the most remarkable one. During recessions, large amounts of unutilized products and resources are usually and persistently observed, suggesting that the economy has fallen into a Pareto inefficient state. However, it is difficult to theoretically show the generation mechanism of persistent Pareto inefficiency. This difficulty has made the ad hoc assumption of rigidity compelling. However, studies on rigidity have not necessarily come to fruition, although voluminous research has been done on this subject. Some economists therefore have shifted attention from rigidity to multiple equilibria because these equilibria are usually Pareto ranked and include a Pareto inferior equilibrium (e.g., Morris and Shin, 2001). However, as argued in Section 1, merely showing the possibility of multiple Pareto-ranked equilibria is not sufficient to explain the generation mechanism of persistent Pareto inefficiency. In contrast, the Nash equilibrium of a Pareto inefficient path as a consequence of a time preference shock naturally generates a persistent Pareto inefficient state.

Property (b) covers the main subject in this paper. Section 3 indicates that the factor that makes the economy appear to be rigid is derived from successive b_t. Rigidity has been reported in fluctuations not only in the labor market but in many other markets. Keynesian economics has regarded these phenomena as an essential element in economic fluctuations and emphasized the importance of price rigidity as the main source of these phenomena. However, it is not an easy task to theoretically show the mechanism of price rigidity if rational agents are assumed, as is discussed in Section 4.1. In contrast, time preference shocks and successive b_t do not rely on ad hoc price rigidity to explain the observed nature of rigidity in economic fluctuations. Successive b_t naturally generates conditions that make the economy appear rigid.

Property (c) exhibits a clear advantage of time preference shocks over productivity shocks. The explanation of economic fluctuation based on productivity shocks has been criticized for having two serious problems. First, as discussed in Section 4.2, the cyclical volatility of TFP has been estimated to be very small, particularly if cyclical factors are carefully removed from data (e.g., King and Rebelo, 1999). In addition, it is difficult to envision a large negative productivity shock because knowledge accumulation is basically irreversible. In contrast, both positive and negative time preference shocks can naturally occur.

Property (d) is important, particularly as compared to models of multiple equilibria. In these models, there are multiple production states (usually two, high and low), and economic fluctuations are depicted as a shift between the two states. Consequently, the scale of fluctuation is always the same, even though the scales of economic booms and recessions have actually varied widely. In contrast, time preference shocks can naturally generate a very wide range fluctuation, depending on the size of the increase or decrease in the time preference rate.

Properties (e) and (f) answer the important theoretical question of why measures of uncertainty and some financial indicators can be used to predict economic fluctuations (see e.g., Romer, 1990; Estrella and Mishkin, 1998). If the source of fluctuation is external to people (e.g., productivity shocks), people cannot anticipate the events that arise from the shock until it occurs. Therefore, in the case of external shocks, uncertainty should not lead to fluctuations. Most financial indicators generally should not lead to fluctuations either, because financial indicators primarily move after people anticipate the events. However, time preference shocks are not external to people but rather are internal because a time preference, by definition, represents a person's preference. Section 2 shows that time preference shocks can be initiated by a change in the level of uncertainty people feel. People first anticipate a shock on their own time preferences, and as a result, the economy begins to change. Hence, uncertainty and some financial indicators can be naturally used to predict economic fluctuations. Of course, not all fluctuations can be predicted by these indicators because some fluctuations will be initiated by external factors, for example, an oil price hike or an outbreak of war.

Keynes emphasized the importance of "animal spirits" in economic activities (Keynes, 1936). The concept of animal spirits is vague and various interpretations exist because Keynes did not clearly define the term. Nevertheless, proponents of this idea commonly maintain that economic activities are largely governed by people's mood (e.g., optimistic or pessimistic). Similarly, the anticipation of uncertainty and time preference rates are internally governed and can be a driving force of economic fluctuations as shown in Section 2. Animal spirits as a driving force of economic fluctuations may therefore be reinterpreted as people's changing perception of uncertainty and of time preference rates, both of which in turn initiate economic fluctuations.

5 CONCLUDING REMARKS

The standard search and matching model has been criticized for not having enough power to generate sufficiently large cyclical fluctuations in the v-u ratio. One solution to this problem has been the introduction of the concept of wage rigidity into the model. However, friction on price adjustments has been criticized for its fragile theoretical foundation, and skepticism about its economic importance still exists. This paper offers an alternative approach to the explanation of the observed large cyclical volatility of the v-u ratio.

The paper argues that these large fluctuations can be explained by a mechanism that includes a Nash equilibrium of a Pareto inefficient path. Such a Nash equilibrium exists because households are risk averse and non-cooperative. On this Pareto inefficient path, unutilized products and resources are persistently generated. A Nash equilibrium of a Pareto inefficient path is generated even in a frictionless economy if, and probably only if, the rate of time preference shifts. The situation can be described by a non-cooperative mixed strategy game in which a strategy profile consisting of strategies of choosing a Pareto inefficient transition path of consumption is a Nash equilibrium.

When a Nash equilibrium of a Pareto inefficient path is generated after a time preference shock, the economy appears to be rigid. The Nash equilibrium distorts the matching process, and by affecting various factors in the labor market, sufficiently large and cyclical fluctuations in the v-u ratio occur because the Nash equilibrium successively or "rigidly" generates Pareto inefficiency. Among the affected factors, vacancy costs are particularly important, which is intuitively and logically reasonable because firms should reduce the number of vacancies as the cost of vacancies increase and vice versa. Because of the gradual or "rigid" movement on this Pareto inefficient path, the v-u ratio can fluctuate largely and cyclically if time preference shocks occur.

The advantages of time preference shocks as an explanation for economic fluctuations can be seen not only in the labor market but in many other markets as well because Pareto inefficient paths are rationally chosen, the effects of shocks persist, both positive and negative shocks can occur, various scales of fluctuations can occur, and uncertainty can lead to economic fluctuations.

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Figure 1: A time preference shock

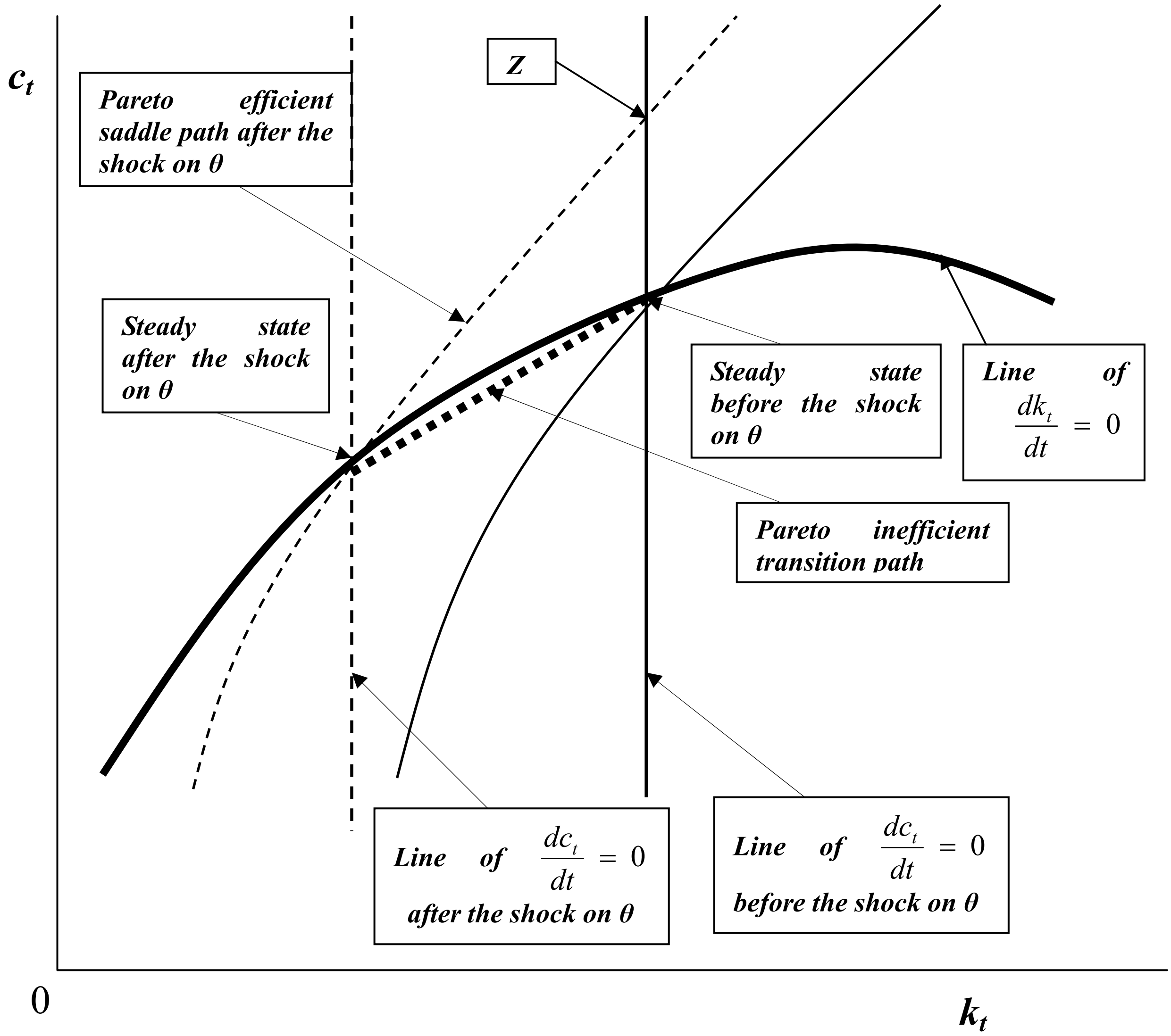

Figure 2: The paths of *Jalone* and *NJalone*

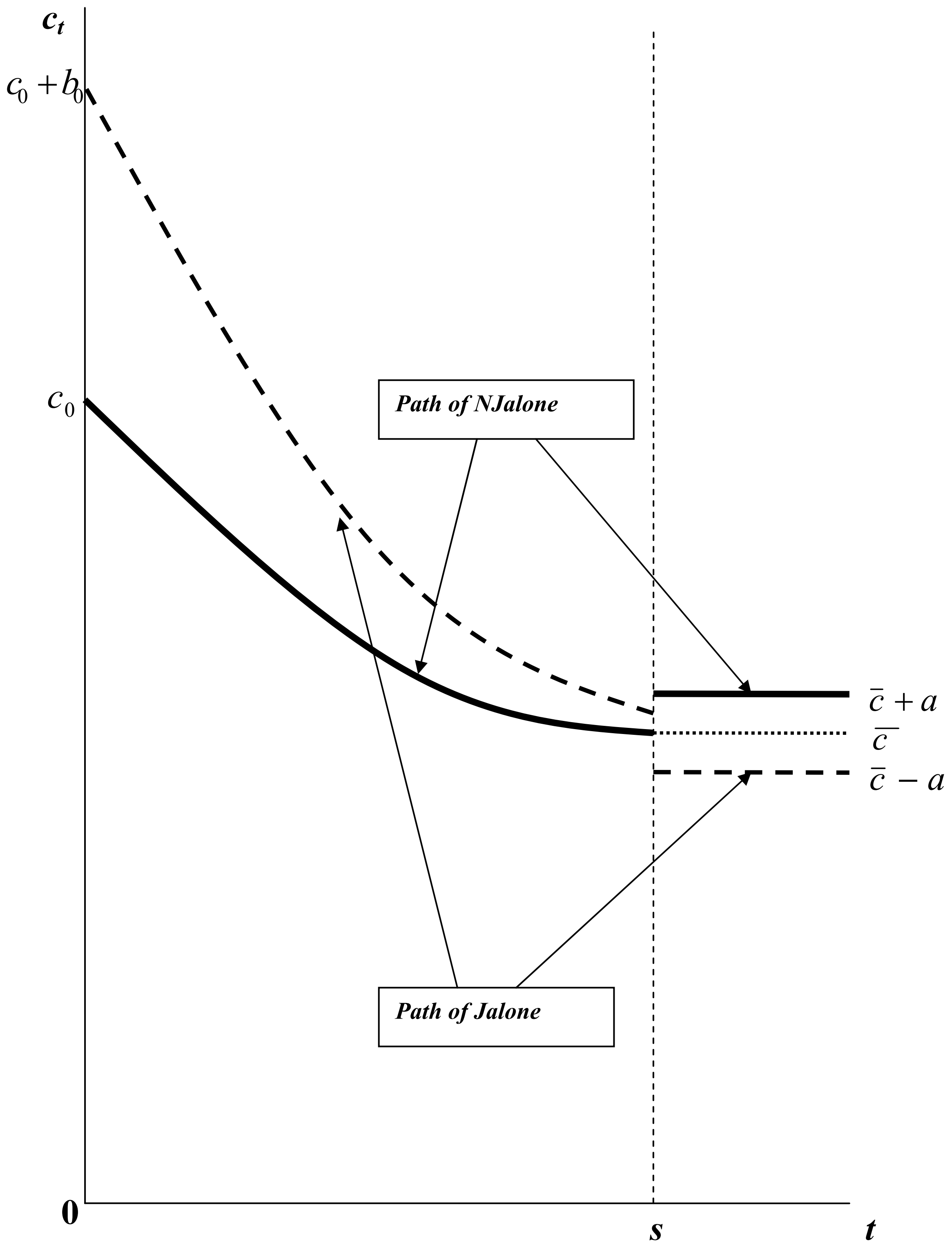

Figure 3: A Pareto inefficient transition path

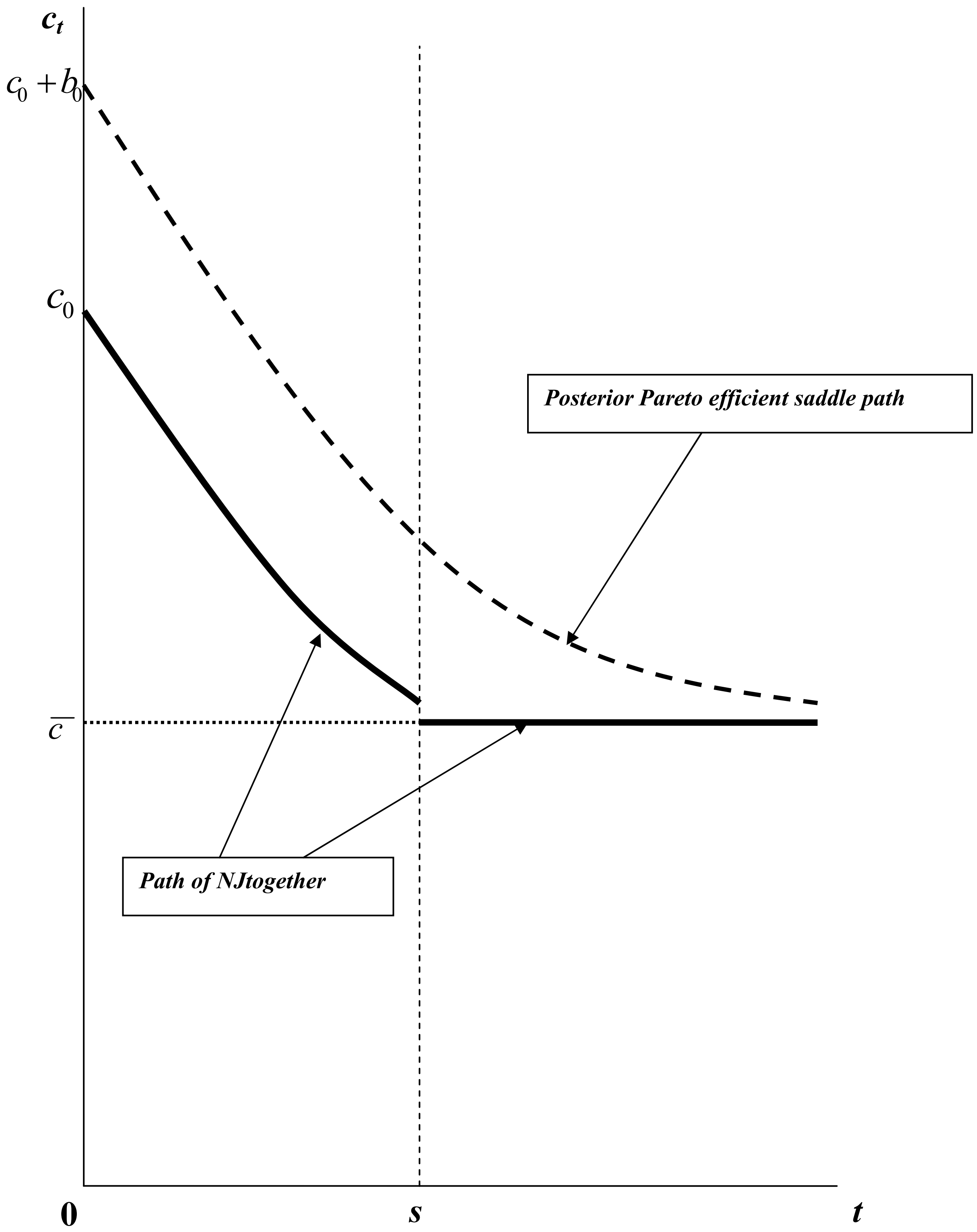

Figure 4: Endogenous time preference

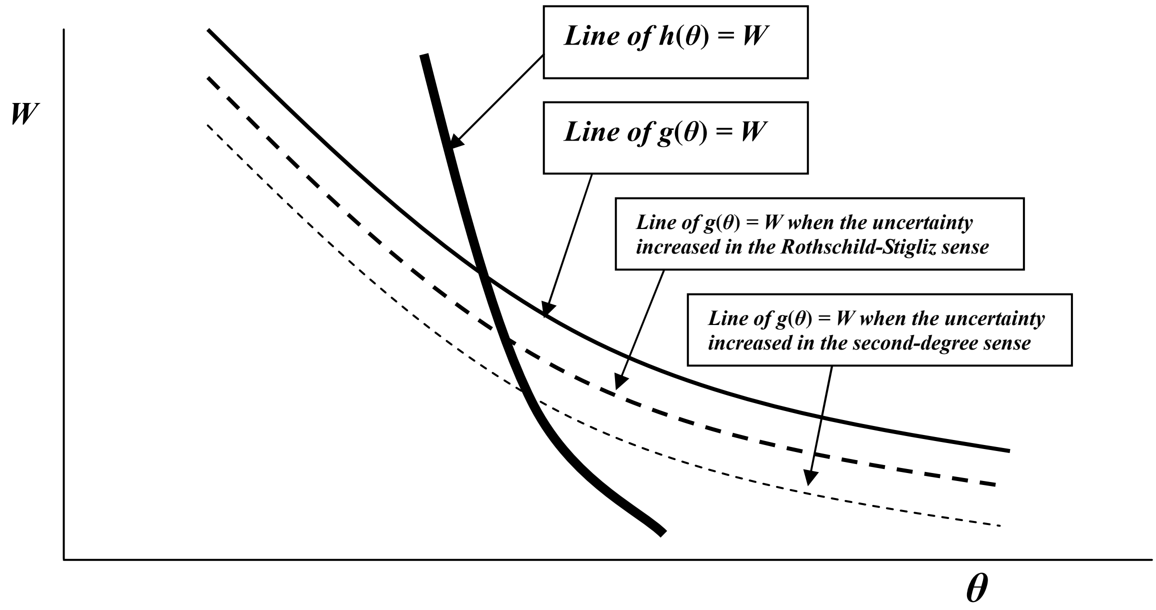

Figure 5: Permanently constant time preference

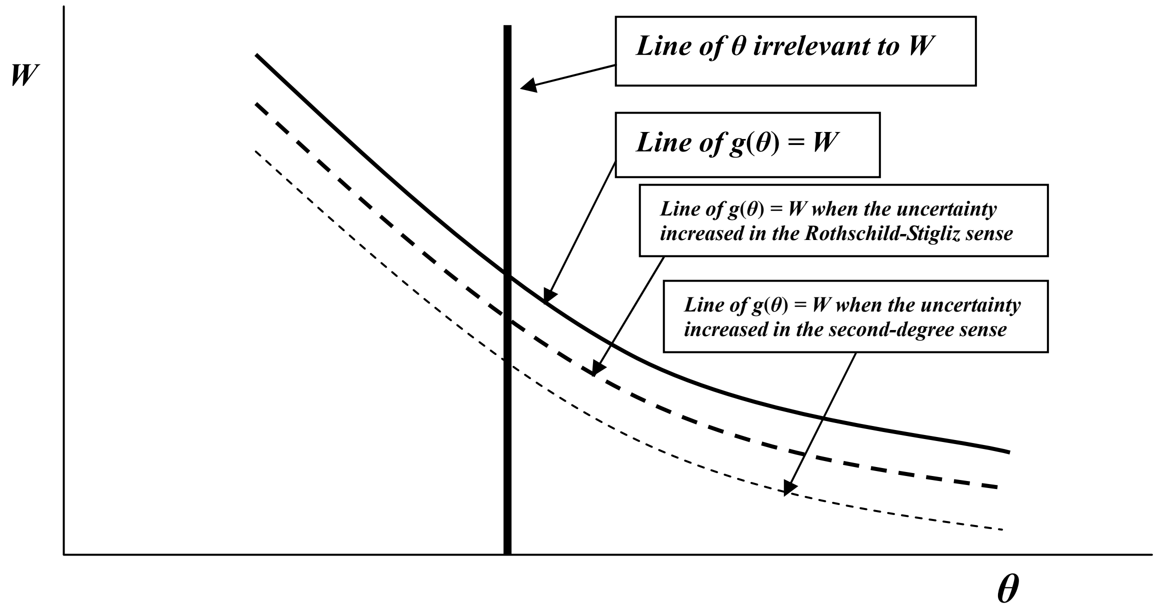

Table 1 The payoff matrix

		Any other household	
		J	***NJ***
A household	***J***	$E_0(Jtogether), E_0(Jtogether)$	$E_0(Jalone), E_0(NJtogether)$
	NJ	$E_0(NJalone), E_0(Jtogether)$	$E_0(NJtogether), E_0(NJtogether)$

Table 2 The impact of time preference shocks on production

Time preference rate		Change of steady state production (%)
Initial	**After shock**	
0.03	0.04	–11.6
0.04	0.05	–9.1
0.05	0.06	–7.5
0.03	0.05	–19.7
0.04	0.06	–16.0
0.05	0.07	–13.4
0.03	0.06	–25.7
0.04	0.07	–21.3
0.05	0.08	–18.2