Multiproduct pricing and the Diamond Paradox

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Multiproduct Pricing and The Diamond Paradox

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We study the pricing behavior of a multiproduct monopolist, when consumers must pay a search cost to learn its prices. Equilibrium prices are high because rational consumers understand that visiting the store exposes them to a hold-up problem. However a firm with more products attracts more consumers with low valuations, and therefore charges lower prices. We also show that when the firm advertises the price of one product, it provides consumers with some indirect information about all of its other prices. The firm can therefore build a store-wide 'low-price image' by advertising just one product at a low price.

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Most retailers sell multiple products and consumers frequently purchase several items in one shopping trip. Consumers often know a lot about a retailer's products, but they are poorly-informed about the prices of individual items unless they buy them on a regular basis. Therefore a consumer must spend time searching a retailer in order to learn its prices. The literature has mainly focused on single-product search, but many issues are more naturally analyzed in a multiproduct framework. For example it is well-documented that larger retailers with broader product ranges charge lower prices, so we would expect consumers to take this into account when searching. Some firms also use advertising to inform consumers about a small proportion of their prices. The remaining (unadvertised) prices are typically much higher, and can only be learnt by visiting the retailer. Restaurants for example advertise cheap deals on main courses but sell over-priced drinks and desserts. We would again expect advertised prices to influence a consumer's search behavior.

The aim of this paper is to build a simple model of multiproduct search, and use it to explore questions such as the following. Why do larger retailers often charge lower prices? Why might a retailer sell selected items at below-cost prices? When a retailer offers a good deal on one product, does it simply compensate by raising the prices of other goods?

To answer these questions we focus on the pricing behavior of a monopolist who sells a number of independent products. Consumers have different valuations for different products, and would like to buy one unit of each. Every consumer is privately informed about how much she values the products, but does not know their prices. The retailer can inform consumers about one of its prices by paying a cost and sending out adverts. Consumers observe whether or not an advert was sent, and form (rational) expectations about the price of every product within the store. Each consumer then decides whether or not to pay a search cost and visit the retailer. After searching, a consumer learns actual prices and makes purchases.

We think this set-up closely approximates several product markets. For instance local convenience shops and drugstores sell mainly standardized products which do not change much over time. A consumer's main reason for visiting them is not to learn whether their products are a good 'match', but instead to buy products that she already knows are suitable. On the other hand prices are hard to remember and may change over time, so consumers are not well-informed about them before they go shopping.

Our paper is closely related to Diamond (1971), which has become known as the 'Diamond Paradox'. Suppose that consumers have unit demands, that firms sell only one product and do not advertise its price, and that each search incurs a cost $s > 0$. Then the Paradox states that the market completely collapses (Stiglitz 1979). The reason is that if consumers expect retailers to charge a price p^e, only people with a valuation above $p^e + s$ will search. After paying the search cost, all these consumers will buy the product provided its actual price is below $p^e + s$. Hence there cannot be an equilibrium in which some consumers search, because retailers would always charge more than was expected. Instead in equilibrium p^e must be so high that nobody searches, no trade occurs, and the market collapses.

Many papers have suggested possible ways to overcome this 'no trade' Paradox. Firstly some consumers might enjoy shopping and have no search cost (Stahl 1989). Secondly firms might send out adverts which commit them to charging a particular price, and thereby guarantee consumers some surplus (Wernerfelt 1994, Anderson and Renault 2006). Thirdly consumers might only learn their match for a product after searching for it. Anderson and Renault (1999) show that the market does not collapse provided there is enough preference for variety.

We show that multiproduct retailers can also overcome Diamond's 'no trade' Paradox. Intuitively in the single-product case, only consumers with a high valuation decide to search, so retailers exploit this and charge a high price. However in the multiproduct case, somebody with a low valuation on one product may search because she has a high valuation on another. When the firm increases one of its prices, some consumers with a low valuation for that product stop buying it. This reduces the retailer's incentive to surprise consumers by charging more than they expected. Consequently there can exist equilibria in which consumers search, even when all prices are unadvertised and everybody has a positive search cost.

Nevertheless the logic behind the Diamond Paradox can provide new insights into how multiproduct firms set their prices. Consumers still only search if they have relatively high valuations, so unadvertised prices are also relatively high. However we show that a larger retailer (who sells more products) is searched by a broader 'mix' of consumers, who on average have lower valuations for any individual product. A larger retailer therefore charges lower prices - consumers anticipate this and incorporate it into their search decision. We also show that despite charging lower prices, larger retailers earn higher profit on each product.[1]

Whilst there is a large literature on advertising (see Bagwell 2007 for a survey) there is relatively little work on the relationship between advertised and unadvertised prices. This paper tries to fill that gap by allowing the firm to send out an advert which directly informs consumers about one of its prices. We show that when the firm cuts its advertised price, some new consumers with relatively low valuations decide to search. The firm then finds it optimal to also reduce its unadvertised prices, in order to sell more products to these new searchers. Therefore

[1]Two related papers are Villas-Boas (2009) and Zhou (2010), in which consumers search for both price and product match information. In Villas-Boas' model a monopolist sells many substitute products. It charges higher prices when it sells more products, because it provides consumers with a better product match. In Zhou's model firms sell two independent products, and a consumer's match realizations are independent across retailers. Prices are typically lower than in the single-product version of the model, because a price reduction on one product causes more consumers to stop searching and buy both products immediately.

consumers (rationally) expect a positive relationship between the firm's advertised and unadvertised prices. Whilst the firm cannot commit in advance to its unadvertised prices, it can indirectly convey information about them via its advertised price. One implication is that a low advertised price on one product can build a store-wide 'low price-image', even on completely unrelated products.

Our result on multiproduct advertising is related to but different from Lal and Matutes (1994) and Ellison (2005), in which two firms are located on a Hotelling line and sell two products but advertise only one of them. In Lal and Matutes all consumers have an identical willingness to pay H for one unit of each good. In Ellison all consumers value the advertised (base) product the same, but have either a high or a low valuation for the unadvertised (add-on) product.[2] As in Diamond's model, the unadvertised price is driven up to H in Lal and Matutes, and (typically) up to the high-types' willingness to pay in Ellison's model. Firms use their advertised price to compete for store traffic, but unlike in our model, cannot use it to credibly convey information about their unadvertised price. The difference arises because in our paper valuations are heterogeneous and continuously distributed. Therefore when a firm changes its advertised price, the pool of searchers also changes and this alters the firm's pricing incentives on its other products. Simester (1995) also finds that prices are positively correlated, but due to cost rather than preference heterogeneity. In his model a low-cost firm charges a lower unadvertised price, and may signal its cost advantage to consumers by advertising a lower price on another good as well.

We also show that sometimes the firm sells its advertised product at a loss. This can be a profitable strategy because it builds a low-price image, and this encour-

[2] More precisely consumers in Ellison's model have either a high or a low marginal utility of income (and therefore either a low or a high valuation for the add-on). This induces correlation in horizontal and vertical attributes, and raises industry profits when add-on prices are not advertised.

ages many more consumers to search and ultimately buy products. Bliss (1988) and Ambrus and Weinstein (2008) also study loss-leader pricing, but in their models consumers are fully-informed about all prices. Bliss shows that firms cover their overheads by using Ramsey pricing, and he argues that some mark-ups could be negative. However in a related paper Ambrus and Weinstein prove that loss-leading is only possible when there are very delicate demand complementarities between products.[3] Our model on the other hand can generate loss-leaders even when all products are symmetric and independent. Closer in spirit are Hess and Gerstner (1987) and Lal and Matutes (1994), both of which show that firms advertise loss-leaders to attract consumers, before charging them very high prices on other unadvertised products.[4] However none of these papers capture the idea that loss-leaders are profitable because they commit a firm to charging store-wide low prices.

Finally we extend the model in two directions. Firstly we show that many of our results generalize when consumers have downward-sloping (rather than unit) demands. Secondly we introduce competition, by studying the case where two firms sell the same two products but can advertise only one of them. We show that each firm's advertised and unadvertised prices are random and, as in the benchmark model, are positively correlated. This contrasts with McAfee (1995) and Hosken and Reiffen (2007) who find that multiproduct firms' prices are negatively correlated, and with Shelegia (2010) who finds that prices are uncorrelated. However

[3] Also related are DeGraba (2006) and Chen and Rey (2010). In DeGraba's model some consumers are more profitable than others, and firms target the more profitable consumers by offering loss-leaders on products that are (primarily) bought by them. In Chen and Rey consumers differ in terms of their shopping cost, such that a retailer may use a loss-leader on one product in order to better discriminate between multi-stop and one-stop shoppers.

[4] Konishi and Sandfort (2002) also show that a monopolist selling substitute products, often advertises a low price on one of them to attract consumers to its store. The firm hopes that once inside the store, some of these consumers will switch to a more expensive substitute.

these papers are quite different from ours because all prices are unadvertised, and instead competition is driven by the assumption that some consumers are shoppers.

The rest of the paper proceeds as follows. Section 1 outlines the model, whilst section 2 characterizes prices when the firm decides not to advertise. Section 3 then investigates how the firm can use advertising to create a low-price image. Finally section 4 extends some of the results whilst section 5 concludes.

1 Model

A single firm produces n goods, denoted by $j = 1, 2, \ldots, n$, at zero marginal cost. The n products are neither substitutes nor complements, and consumers demand at most one unit of each. There is a unit mass of consumers, and we let $(v_1, v_2, ..., v_n)$ denote a typical consumer's valuations for the n products. Each v_j is drawn independently across both products and consumers, using a distribution function $F(v_j)$. The corresponding density $f(v_j)$ is strictly positive, continuously differentiable, and logconcave on the interval $[a, b]$ (where $b > 0$).[5] In the textbook zero-search-cost model, each good's profit function is strictly quasiconcave and has a unique maximizer $p^m = \arg\max p[1 - F(p)]$. To simplify matters we focus on the case $p^m > a$, although our results also hold when $p^m = a$.

The monopolist can pay a cost c_a and advertise one of its prices. The advertisement is received by all consumers and must be truthful. Consumers observe whether or not the firm advertises (and if so, observe the advertised price) and then form expectations about all prices, denoted $\boldsymbol{p^e} = (p_1^e, p_2^e, ...p_n^e)$. Consumers must pay a search cost $s > 0$ to visit the store and learn its unadvertised prices. Once incurred, this search cost is sunk. Prior to visiting the store, consumers know their valua-

[5]Bagnoli and Bergstrom (2005) show that many common densities (and their truncations) are logconcave. Logconcavity ensures that the hazard rate $f(v_j)/(1 - F(v_j))$ is increasing.

tions $(v_1, v_2, ..., v_n)$ for each of the products. Using their expectations about price, they visit if and only if their expected surplus $\sum_{j=1}^{n} \max\left(v_j - p_j^e, 0\right)$ is greater than the search cost. After they have arrived at the store, consumers learn actual prices $(p_1, p_2, ...p_n)$ and make their purchases. All parties are risk neutral and rational.

The move order of the model is summarized as follows. In the first stage the monopolist chooses whether or not to advertise, and if so picks its advertised price. It then chooses (but does not disclose) the prices of the remaining goods. In the second stage consumers observe the firm's advertising behavior, form expectations about prices, and decide whether or not to visit the store. In the third stage consumers who decided to visit then learn actual prices, and make purchase decisions.

2 No advertising

This section analyzes the benchmark case in which the firm has decided not to advertise any of its prices.

2.1 Solving for equilibrium prices

Consumers learn that the firm has not advertised, and then form expectations $\boldsymbol{p^e} = (p_1^e, p_2^e, \ldots, p_n^e)$ about the price of each product. A consumer visits the store if and only if her expected surplus $\sum_{j=1}^{n} \max\left(v_j - p_j^e, 0\right)$ exceeds the search cost s. Once inside the store, she buys product i provided that her valuation v_i exceeds the actual price p_i. Therefore demand for unadvertised product i is

$$D_i\left(p_i; \boldsymbol{p^e}\right) = \int_{p_i}^{b} f\left(v_i\right) \Pr\left(\sum_{j=1}^{n} \max\left(v_j - p_j^e, 0\right) \geq s\right) dv_i \tag{1}$$

The firm chooses the actual price p_i to maximize its profit $p_i D_i\left(p_i; \boldsymbol{p^e}\right)$ on good i. In equilibrium consumer price expectations must be correct, so we require that

$p_i^e = \underset{p_i}{\operatorname{argmax}}\, p_i D_i\left(p_i; \boldsymbol{p^e}\right)$. Imposing this condition, we have the following lemma (Note that all proofs in the paper appear in the appendices.)

Lemma 1 *The equilibrium price of unadvertised good i satisfies*

$$D_i\left(p_i = p_i^e; \boldsymbol{p^e}\right) - p_i^e f\left(p_i^e\right) \Pr\left(\sum_{j \neq i} \max\left(v_j - p_j^e, 0\right) \geq s\right) = 0 \qquad (2)$$

To understand (2), consider a small increase in p_i above the expected level p_i^e. The firm gains revenue on those who continue to buy, and they have mass equal to demand. It loses p_i^e on those who stop buying good i, and they have mass $f\left(p_i^e\right) \Pr\left(\sum_{j \neq i} \max\left(v_j - p_j^e, 0\right) \geq s\right)$. Intuitively, each consumer who stops buying the good a). has a marginal valuation $v_i = p_i^e$ for it, and b). has searched. Any consumer who is marginal for good i only searches if her expected surplus on the remaining $n-1$ goods, $\sum_{j \neq i} \max\left(v_j - p_j^e, 0\right)$, exceeds the search cost s.

Figure 1 illustrates search behavior when $n = 2$ and consumers rationally anticipate prices p_1^e and p_2^e. Consumers in the top-right corner have both a high v_1 and a high v_2, and therefore definitely search and buy both products. Consumers in the bottom-right corner have $v_2 \leq p_2^e$ and therefore do not expect to buy product 2; however they still search provided that $v_1 \geq p_1^e + s$ because they expect product 1 alone to give enough surplus to cover the search cost. If the firm considered increasing the actual price p_1 slightly above the expected level p_1^e, only those consumers on the thick line would stop buying product 1. All other marginal consumers for product 1 have $v_2 < p_2^e + s$, so they don't search and are not affected by changes in p_1. We immediately notice the following:

Example 2 (Diamond Paradox) *If $n = 1$ and the firm does not advertise, any equilibrium has $p_1^e \in (b-s, b]$ and no trade*

If the firm only sells one product, no marginal consumer pays the search cost. Therefore equation (2) can only be satisfied if $D_1\left(p_1 = p_1^e; p_1^e\right) = 0$, or equivalently

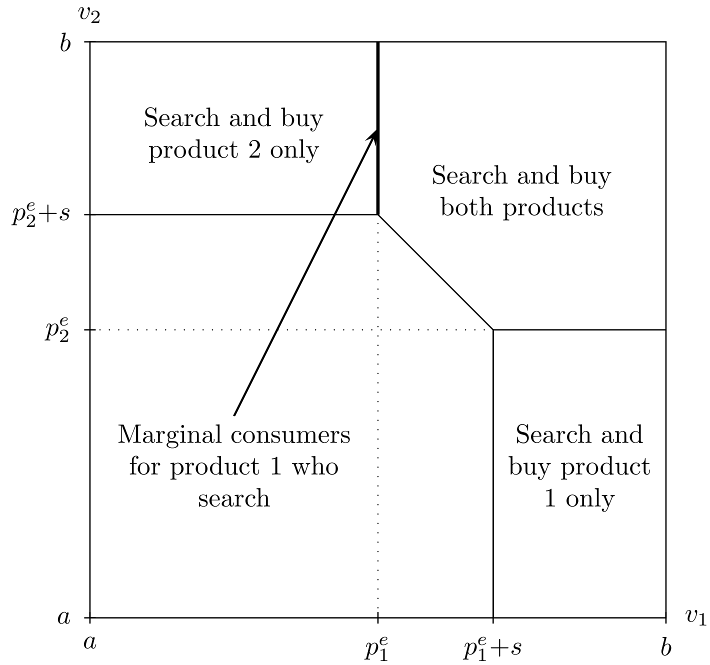

Figure 1: Search behavior when $n = 2$

if the expected price is so high that nobody finds it worthwhile to search. Intuitively consumers face the following hold-up problem: anticipating a price p_1^e, everybody who visits the store is prepared to pay at least $p_1^e + s$, so the firm exploits this and charges more than expected. The hold-up problem is only resolved when $p_1^e \in (b - s, b]$ - nobody pays the search cost so the firm is happy to set $p_1 = p_1^e$.

Proposition 3 now shows that an equilibrium with trade does exist, provided the firm sells enough products. Note that each unadvertised good has a first order condition of the form (2). A vector of unadvertised prices constitutes an equilibrium if a). the prices solve each first order condition and b). the firm's profit is quasiconcave in each unadvertised price.[6]

[6] It is not always straightforward to prove quasiconcavity in consumer search models. However sufficient conditions for $p_i D_i(p_i; \boldsymbol{p^e})$ to be quasiconcave in p_i are i). the standard monopoly profit function $p[1 - F(p)]$ is concave or ii). $\Pr\left(\sum_{j \neq i} \max\left(v_j - p_j^e, 0\right) \geq s\right) > 1/2$. The former

Proposition 3 *If n is sufficiently large, there exists at least one ('non-Diamond') equilibrium in which trade occurs*

Proposition 3 shows that multiproduct retailers can overcome the Diamond Paradox. Of course there are always equilibria in which no consumer visits the store, and no trade occurs.[7] This is also the only equilibrium outcome if $n = 1$ as illustrated in example 2. However if n is sufficiently large, there also exist 'non-Diamond' equilibria in which trade occurs. Intuitively many consumers who are marginal for a particular product, will search when n is large because there are other products which offer them positive surplus. If the firm tried to 'hold up' consumers and charge more than was expected, it would then lose a lot of demand from these marginal consumers. This deters the firm from holding people up, and so equilibria with trade can exist. We also argue in section 2.2 below that Diamond's no-trade outcome is unlikely to occur when these non-Diamond equilibria exist.

Although proposition 3 shows that a multiproduct firm can overcome the Diamond Paradox, the logic behind the Paradox can still provide insights into how unadvertised prices are determined. In order to show this, it is convenient to introduce the notation $t_j \equiv \max\left(v_j - p_j^e, 0\right)$ for the expected surplus on good j. The equilibrium price of unadvertised product i is affected by the behavior of three different groups. Consumers with $\sum_{j=1}^{n} t_j < s$ do not visit the store, and therefore do not respond to changes in p_i. Consumers with $\sum_{j=1}^{n} t_j \geq s$ do visit the store, and subdivide into two groups. We use the term '*shoppers for product i*' to denote those with $\sum_{j \neq i} t_j \geq s$. These consumers search irrespective of how much they

holds if the density $f(v_i)$ does not decrease too rapidly, and the latter if n is large enough. See lemma 19 in appendix A for further details.

[7] When $\sum_{j=1}^{n} \left(b - p_j^e\right) \leq s$ expected prices are so high that no consumer searches. The firm is then indifferent about what prices to charge, and so is happy to set $p_j = p_j^e$ for each product j.

value product i.[8] We use the term *'Diamond consumers for product i'* to denote those people with $\sum_{j=1}^{n} t_j \geq s > \sum_{j \neq i} t_j$. These consumers only search because they anticipate a strictly positive surplus on product i.

Lemma 4 *The pricing condition (2) can be rewritten as*

$$\underbrace{\Pr\left(\sum_{j \neq i} t_j \geq s\right)}_{\textit{Probability of a shopper}} \left[1 - F\left(p_i^e\right) - p_i^e f\left(p_i^e\right)\right] + \underbrace{\Pr\left(\sum_{j=1}^{n} t_j \geq s\right) - \Pr\left(\sum_{j \neq i} t_j \geq s\right)}_{\textit{Probability of a Diamond consumer}} = 0 \tag{3}$$

Lemma 4 decomposes into two parts, the change in profits caused by a small change in p_i around p_i^e. Firstly shoppers for product i search irrespective of their v_i, which therefore continues to be distributed on $[a, b]$ with the usual density $f(v_i)$. Consequently profits on shoppers are simply $p_i [1 - F(p_i)]$ (the same as in a standard zero-search-cost monopoly problem), and so small changes in p_i around p_i^e affect profits by $1 - F(p_i^e) - p_i^e f(p_i^e)$. Secondly Diamond consumers for product i have $v_i - p_i^e > 0$, so they would all buy product i even if the price were slightly more than expected. Consequently a small increase in p_i above p_i^e, causes profits on Diamond consumers to increase by 1. Just like consumers at a single-product retailer, their demand is locally perfectly inelastic. To summarize the multiproduct problem with positive search cost is really an average of the standard monopoly and Diamond problems. The next lemma is therefore very intuitive

Lemma 5 *In a non-Diamond equilibrium, $p_i^e > p^m$ for each unadvertised good i*

Although the market no longer collapses, prices are high because the firm faces a 'sample selection' problem. In particular the consumers who search are a select

[8] We use the term 'shoppers for product i' because these consumers act *as if* they have no search cost when it comes to buying good i. This terminology mimics the existing literature, in which somebody is a 'shopper' if they have no search cost. See for example Stahl (1989).

group of people who have several high valuations. Consumers understand that the firm will exploit this ex post, and therefore equilibrium prices are driven above the frictionless benchmark p^m. Mathematically equation (3) can only be satisfied when the gains on Diamond consumers (from increasing p_i above p_i^e) are offset by losses on shoppers. A small increase in p_i above p_i^e only leads to losses on shoppers if $1 - F(p_i^e) - p_i^e f(p_i^e) < 0$, which (due to the quasiconcavity of $p_i [1 - F(p_i)]$) is equivalent to $p_i^e > p^m$. Moreover prices exceed p^m even when the search cost is very small, as the next example shows.

Example 6 (Small search cost) *As $s \to 0$, $p_i^e \to \bar{p}_i^e$ where*

$$\frac{\bar{p}_i^e f(\bar{p}_i^e)}{1 - F(\bar{p}_i^e)} = \frac{1}{1 - \prod_{j \neq i} F\left(p_j^e\right)} \tag{4}$$

The lefthand side of equation (4) is the (absolute) elasticity of the demand curve $1 - F(p_i)$, evaluated at $p_i = \bar{p}_i^e$. It increases in $\bar{p}_i^e$ and equals 1 when $\bar{p}_i^e = p^m$. The righthand side is the inverse of the probability that a given consumer is a shopper for product i. Since the righthand side exceeds 1, the equilibrium price exceeds p^m even though the cost of searching is very small.

2.2 Equilibrium multiplicity and selection

As discussed earlier Diamond equilibria always exist, but nobody searches so the firm doesn't make any profit. By contrast proposition 3 says that when n is large enough, there exist equilibria in which consumers search and the firm does make a profit. Now if production involves a small fixed cost, the firm will only enter the market if it expects to play a non-Diamond equilibrium. Therefore using the logic of forwards induction, when the firm does enter, consumers should expect to play a non-Diamond equilibrium.

However there may be multiple non-Diamond equilibria, in which case further equilibrium selection is required. Multiple equilibria can arise because if consumers' price expectations change, the firm is searched by a different mix of people and therefore its pricing incentives also change. For example if consumers suddenly expect prices to be lower, they are more likely to search even if they have several low product valuations. The firm optimally responds by cutting its prices, and *in principle* consumers' expectations could then be correct. The following lemma is useful since it places some structure on the set of equilibrium price vectors.

Lemma 7 *In any equilibrium with trade, all products have the same price*

Lemma 7 means that we can henceforth refer to just a single 'representative' unadvertised price, and use it to rank different equilibria. Clearly consumers prefer the equilibrium with the lowest unadvertised price. More surprisingly perhaps, the next lemma shows that the monopolist does as well.

Lemma 8 *The firm prefers equilibria with lower unadvertised prices*

Equilibrium prices are so high that relatively few consumers search and make purchases. Consequently the firm's profits are actually higher in equilibria with lower prices. This can be proved using the following revealed preference argument. Suppose there are two equilibrium prices p' and p'' where $p'' < p'$. In the p' equilibrium the firm earns a profit on product i equal to

$$p' \times \int_{p'}^{b} f(v_i) \Pr\left(\sum_{j=1}^{n} \max(v_j - p', 0) \geq s\right) dv_i \tag{5}$$

When consumers expect the p'' equilibrium they search if $\sum_{j=1}^{n} \max(v_j - p'', 0) \geq s$, and then buy product i provided that it gives positive surplus. Therefore if the firm 'deviates' and charges a price p', its profit on product i is

$$p' \times \int_{p'}^{b} f(v_i) \Pr\left(\sum_{j=1}^{n} \max(v_j - p'', 0) \geq s\right) dv_i \tag{6}$$

which strictly exceeds (5). However when consumers expect the p'' equilibrium, the firm prefers to charge p'' rather than p'. So by revealed preference, profits in the p'' equilibrium must exceed (6), and therefore also exceed profits in the p' equilibrium. Lemma 8 suggests that the equilibrium with the lowest price may be salient, because it Pareto dominates any other equilibrium.[9] Therefore we assume

Assumption *When multiple equilibria exist, agents coordinate on the equilibrium with the lowest price*

2.3 Comparative statics

This section looks at how the equilibrium price - which we call p^* - is affected by changes in the search cost and product range. To do this recall that $t_j \equiv \max(v_j - p^*, 0)$ is the (equilibrium) expected surplus on good j. Substituting $p_j^e = p^*$ for $j = 1, 2, \ldots, n$ into equation (3) and then rearranging, p^* satisfies

$$1 - F(p^*) - p^* f(p^*) + \frac{\Pr\left(\sum_{j=1}^{n} t_j \geq s\right) - \Pr\left(\sum_{j=1}^{n-1} t_j \geq s\right)}{\Pr\left(\sum_{j=1}^{n-1} t_j \geq s\right)} = 0 \tag{7}$$

Following on from the previous section we focus on the lowest p^* which solves equation (7), and we assume that profit functions are quasiconcave.[10] The final

[9] Fixed costs may also lead to coordination on the lowest equilibrium. To see this let $\bar{\pi}$ and $\hat{\pi}$ be profits in the equilibria with the lowest and second-lowest prices. If the firm must pay a fixed cost between $\hat{\pi}$ and $\bar{\pi}$, it only enters the market when it expects to play the lowest equilibrium. So by forward induction when the firm does enter, consumers should believe they are playing the lowest equilibrium. Numerical examples show that $\bar{\pi} - \hat{\pi}$ can be large, so this argument may apply widely.

[10] The profit function is quasiconcave provided that $f(v_i)$ does not decrease too rapidly or that n is sufficiently large. Further details are provided in footnote 6 and in lemma 19 in the appendix.

term in equation (7) is the ratio of Diamond consumers to shoppers for a single product, and it plays an important role in subsequent proofs.

Proposition 9 *The equilibrium price is increasing in the search cost*

After controlling for product range and production costs, a multiproduct monopolist charges higher prices when search is more costly. This is a very natural result, and single-product search models such as Anderson and Renault (1999) also find a positive relationship between price and search cost. The usual intuition is that higher search costs deter consumers from looking around for a better deal, which gives firms more market power. However the mechanism which drives Proposition 9 is different. In our model a multiproduct firm faces a sample selection problem, which becomes worse when the search cost is larger. When s is small, the surplus from any single product is less pivotal in determining a consumer's search decision, so the ratio of shoppers to Diamond consumers is relatively large. Marginal pricing incentives can be close to what they would be if there were no search cost, so equilibrium prices can be relatively low. Now if s suddenly increases, some consumers with relatively low valuations no longer search, so the firm responds by increasing its prices. Consumers anticipate this and become even less willing to search - so the firm attracts an even more select sample of consumers, and (equilibrium) prices are driven up even further.

Proposition 10 *The equilibrium price is decreasing in the number of products*

There is lots of casual and empirical evidence that larger stores are cheaper (see for example Kaufman *et al* 1997), but this is usually attributed to greater buyer power and economies of scale. Proposition 10 says that even after controlling for these factors, larger stores should still be cheaper. The reason is that larger stores attract a different type of consumer and therefore have different pricing incentives.

For example compare a 'small store' with a hypothetical 'large store' (which sells everything in the small store, plus some additional products). Think about an experiment in which consumers expect every product in both stores to cost the same amount. The large store attracts some extra consumers, and they tend to have low valuations on the products sold in the small store. Consequently the large store has more incentive to cut its prices on those products, compared with the small store. Consumers anticipate this and correctly believe that larger stores charge lower prices. Moreover we can show that as $n \to \infty$, (almost) every consumer searches and the equilibrium price is arbitrarily close to p^m.[11]

Despite charging lower prices, a larger retailer still earns more profit *on each product* compared with a smaller retailer. Intuitively this is because the smaller firm is effectively 'trapped' into charging high prices, such that few consumers ever search it or buy any of its products. The following revealed preference argument can be used to prove this. If the firm sells n' products and the equilibrium price is p', the profit from any single product is

$$p' \times \int_{p'}^{b} f(v_i) \Pr\left(\sum_{j=1}^{n'} \max(v_j - p', 0) \geq s\right) dv_i \tag{8}$$

Now suppose instead that the firm sells n'' products and the equilibrium price is p'', where $n'' > n'$ and $p'' < p'$. If the firm 'deviates' and charges p', it earns

$$p' \times \int_{p'}^{b} f(v_i) \Pr\left(\sum_{j=1}^{n''} \max(v_j - p'', 0) \geq s\right) dv_i \tag{9}$$

on each product. This is because consumers expected prices to be p'' (and therefore searched on that basis), but having searched will buy any product when they value it more than its price. Notice that (9) strictly exceeds (8) since $n'' > n'$ and $p'' < p'$.

[11] We have assumed that s is independent of n, but in reality there could be a positive relationship between the two. Proposition 10 should still be robust as long as s does not increase too rapidly.

Now in fact the firm chooses to charge p'' rather than p', so by revealed preference the per-good profit when $n = n''$ must exceed (9), and therefore also exceed the per-good profit when $n = n'$.

Proposition 10 could also help explain why firms co-locate near to each other in shopping malls and highstreets. In particular we could interpret the multiproduct monopolist as a cluster of n single-product firms, each of which sells a different and unrelated product. Dudey (1990) and others have already shown that sellers of similar or identical products may cluster together. By doing this rival firms commit to fiercer competition and therefore lower prices. This makes consumers more likely to search, which increases the demand of every firm in the cluster. However proposition 10 suggests a stronger result - even single-product firms that sell completely unrelated goods, can also commit to charging lower prices (and earn higher profits) by clustering together. This is despite the absence of any competition between firms in the cluster.

3 Advertising

A retailer may sometimes use advertising to inform consumers about some of its prices. If the monopolist were able to costlessly advertise every price, then it would choose to do so. It could then commit to whatever prices it wanted to. However advertising is costly, so firms often advertise at most a small proportion of their prices. To capture this in a tractable way, we allow the firm to pay a cost c_a and advertise one of its prices. (All results in this section generalize to situations where several prices can be advertised, provided the search cost is sufficiently small.) To recap the move order is as follows. The firm first decides whether to advertise and then picks an advertised price.[12] It then chooses unadvertised prices to maximize profits, given the set of consumers that it expects will search. We therefore begin by studying how these unadvertised prices are determined in equilibrium.

3.1 Solving for equilibrium unadvertised prices

Suppose the firm sends out an advert stating that the price of good n is p_n^a. Adverts must be truthful, so consumers update their expectation of the price of good n to $p_n^e = p_n^a$. They also form an expectation about the price of every other unadvertised product. Given these price expectations $\boldsymbol{p^e} = (p_1^e, p_2^e, ...p_n^e)$, all the analysis from section 2.1 can be straightforwardly applied. In particular demand for unadvertised product i can still be written as equation (1), and the equilibrium first order condition for unadvertised good i remains

$$D_i\left(p_i = p_i^e; \boldsymbol{p^e}\right) - p_i^e f\left(p_i^e\right) \Pr\left(\sum_{j \neq i} \max\left(v_j - p_j^e, 0\right) \geq s\right) = 0 \qquad (2)$$

It is still the case that each unadvertised price (weakly) exceeds p^m - because as usual the people who search are a select sample of relatively high-valuation consumers, so the firm exploits this by charging high unadvertised prices. We can also show that in any equilibrium all unadvertised prices are the same (c.f. lemma 7), and that multiple equilibria may exist. However the lowest such equilibrium is again Pareto dominant (c.f. lemma 8), and we therefore select it for comparative statics. Since p_n^a enters equation (2) via consumer search decisions, the firm can use it to manipulate (beliefs about) equilibrium unadvertised prices.

Proposition 11 *If p_n^a decreases, so does the equilibrium price of all other products*

[12] For tractability we assume that the advert is received by every consumer. Bester (1994) presents a single-product search model in which a monopolist can pay to increase the reach of its advert. The firm employs a mixed strategy, randomizing over both its price and its advertising reach.

Proposition 11 shows that by advertising the price of just a single product, the firm is able to credibly transmit some information about all its other (unadvertised) prices. This is despite the fact that products are completely unrelated in terms of use, valuation, and production cost. Consequently even rational consumers, who have no interest in buying the advertised product, should nevertheless account for its price when deciding whether or not they should search. Our model also gives a new explanation for why selected low-price advertising might give retailers a store-wide 'low-price image'. As discussed earlier, proposition 11 differs from both Lal and Matutes (1994) and Ellison (2005), because they find that when a firm cuts its advertised price, it is never able to credibly convince rational consumers that its unadvertised price is any lower.

The intuition behind proposition 11 is as follows. If a group of consumers are not searching, then a reduction in p_n^a persuades some of them to do so. These new consumers must have relatively low valuations, otherwise they would have already been searching. The firm therefore reduces its unadvertised prices in order to sell more products to these new (low-valuation) visitors. Whilst consumers do not naively expect every price to be p_n^a, they do anticipate that a fall in p_n^a is accompanied by a decrease in unadvertised prices. As a comparison with Lal and Matutes and Ellison, there is one extreme situation in which a reduction in p_n^a does *not* strictly reduce unadvertised prices. In particular if $p_n^a \leq a - s$, the advertised price is so low that every consumer expects to earn s or more surplus from it alone. Since every consumer searches, there is no sample selection effect and the firm charges p^m on each unadvertised product. Even if p_n^a falls, no new consumers search so unadvertised prices are still p^m.

3.2 Optimal advertised price

When the firm advertises, it chooses p_n^a to maximize total profits across all n products. In order to characterize the optimal p_n^a, first write out the demand for advertised good n as

$$D_n\left(p_n^a;\boldsymbol{p^e}\right)=\int_{p_n^a}^{b} f\left(v_n\right)\Pr\left(\sum_{j=1}^{n-1}\max\left(v_j-p_j^e,0\right)+v_n-p_n^a\geq s\right)dv_n \tag{10}$$

which is similar to the expression for unadvertised demand in equation (1) with one crucial difference. Since consumers are able to observe the actual price p_n^a prior to visiting the store, their decision about whether or not to search is affected by it. Therefore the advertised product has a more elastic demand curve. Let $\Pi_j\left(p_n^a;\boldsymbol{p^e}\right)$ denote the profit earned on good j when the advertised price is p_n^a and unadvertised prices are at the corresponding equilibrium level. The firm chooses p_n^a to maximize $\sum_{j=1}^{n}\Pi_j\left(p_n^a;\boldsymbol{p^e}\right)$, which (assuming differentiability) gives the following first order condition

$$\frac{d\Pi_n\left(p_n^a;\boldsymbol{p^e}\right)}{dp_n^a}+\sum_{j=1}^{n-1}\frac{\partial\Pi_j\left(p_n^a;\boldsymbol{p^e}\right)}{\partial p_n^a}+\sum_{k=1}^{n-1}\sum_{l=1}^{n-1}\frac{\partial\Pi_k\left(p_n^a;\boldsymbol{p^e}\right)}{\partial p_l^e}\frac{\partial p_l^e}{\partial p_n^a}=0 \tag{11}$$

The first term is the total effect of p_n^a on the profits of the advertised good, accounting for the change in unadvertised prices. We show in the appendix that this term is negative for all $p_n^a \geq p^m$. Intuitively the search cost depresses demand for the advertised product, so the firm would like to partly offset this by reducing p_n^a below p^m. The second term is the direct effect of p_n^a on the profits of unadvertised goods, keeping unadvertised prices fixed. It is negative because an increase in p_n^a reduces the probability a consumer searches, which then reduces the probability that she buys any unadvertised product. Finally the third term accounts for the indirect effect of a change in p_n^a on profits earned from unadvertised products, and is also negative. When p_n^a increases so do unadvertised prices (proposition 11), and this is bad for profits because prices are already too high. (This can be proved using a similar revealed preference argument to those on pages 14 and 17.) To summarize, the lefthand side of (11) is negative when $p_n^a \geq p^m$, therefore:

Lemma 12 *The optimal advertised price is strictly below* p^m

According to lemma 12 the firm uses a low advertised price (below p^m) to attract consumers into the store, and then charges a high price (above p^m) on its remaining unadvertised products. In Lal and Matutes and Ellison firms also use a low advertised price to attract consumers into the store, whereupon they are sold over-priced unadvertised products. This incentive is also present in our model, and is captured by the first two sets of terms in equation (11). However in our model there is an additional reason to use low-price advertising - captured by the last set of terms in equation (11) - namely the desire to build a low-price image. We have already seen that unadvertised prices are high, and the firm would be better off if they were lower, because then more people would search and make purchases. Therefore in order to acquire such a low-price image, the firm exploits the mechanism identified in proposition 11 and further reduces its advertised price. Indeed as the following example illustrates, sometimes the firm may find it optimal to reduce the advertised price even below marginal cost:

Example 13 (Loss-leader pricing) *Suppose* $n = 15$*,* $s \to 0$ *and valuations are independently distributed on* $[-1/2, 1/2]$ *with* $f(v) = 4e^{-4v}/(e^2 - e^{-2})$*. The firm's optimal advertised price is* $p_n^a \approx -0.074$ *- which is less than its (zero) marginal cost*

When deciding whether or not to advertise, the firm compares the gain in profits against the cost c_a. The firm therefore advertises whenever c_a is sufficiently small, and the store-wide implications of this are as follows

Remark 14 *When the firm advertises, it charges strictly lower prices on every product*

When the firm does not advertise, it charges p^u (say) on every product, where $p^u > p^m$. Now when it does advertise, the firm could always choose $p_n^a = p^u$ and then by inspection of equation (2), it would still be an equilibrium for every other product to be priced at p^u. However lemma 12 shows that the optimal advertised price is below p^m, and therefore by proposition 11 the remaining unadvertised prices must also be below p^u. Therefore when the firm advertises a good deal on one product, it also offers cheaper prices on everything else - advertising consequently leads to store-wide lower prices.[13]

Finally it would be interesting to know whether there is any relationship between store size and advertising behavior. For example is a larger store more or less likely to advertise? How does product range affect the optimal advertised price? It turns out that equation (11) is not sufficiently tractable to study these questions analytically, but numerical examples show that the answers are in any case ambiguous. For instance figure 2 shows an example in which the optimal advertised price at first decreases in n and then increases rapidly. (In figure 2 valuations are uniformly distributed on $[-1, 1]$ and $s \to 0$.) Intuitively an increase in n has two opposing effects on the optimal advertised price. Firstly a larger store tends to charge lower prices

[13]As far as we are aware, only Milyo and Waldfogel (1999) have tested empirically whether or not advertising leads to store-wide lower prices. They note that after the Supreme Court overturned Rhode Island's ban on alcohol advertising, a few stores advertised some of their liquor prices. Milyo and Waldfogel show (Table 5, page 1091) that those stores cut advertised prices by around 20 percent, but did not reduce their other unadvertised prices by a statistically significant amount. However prior to the Court ruling some retailers were advertising prices of non-alcoholic items such as peanuts and potato chips; the Court ruling may have caused them to switch to advertising liquor products instead. The effect on most unadvertised prices would then be ambiguous.

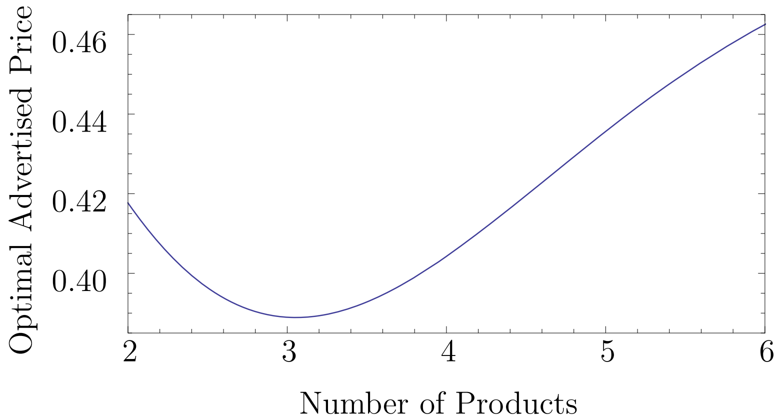

Figure 2: Non-monotonicity of the optimal advertised price

anyway, so has less need to use a low advertised price to create a low-price image. But secondly when there are more (unadvertised) products, the benefits of having a low-price image are larger. Figure 2 suggests that the latter effect dominates when n is small, and the former dominates otherwise.

4 Extensions

4.1 Downward-sloping demands

The Diamond Paradox is often presented with downward-sloping demands. It is well-known that if consumers have an *identical* downward-sloping demand curve and the search cost is not too large, there is an equilibrium in which the firm charges the standard monopoly price and everybody searches. We now show that our earlier results can be generalized to the case where consumers have *heterogeneous* downward-sloping demands.

Start with the simplest case where $n = 1$. Suppose that each consumer's demand can be written as $p_1 = \theta + P(q_1)$ where q_1 is the quantity consumed and

$P(q_1)$ is continuously differentiable, strictly decreasing and concave. θ is a random variable which is distributed on $[\underline{\theta}, \overline{\theta}]$ according to a strictly positive density function $g(\theta)$; consumers are heterogeneous and each receive an independent draw from this distribution. An important feature of this set-up is that consumers with higher θ have both a larger and a less elastic demand; the former implies that consumer surplus increases in θ, and this plays an important role. If there is no search cost and types are not too different, the firm sells to everybody and charges the standard monopoly price p^m which is defined as

$$p^m = \underset{x}{\operatorname{argmax}} \int_{\underline{\theta}}^{\overline{\theta}} g(\theta) \left\{ x P^{-1}(x-\theta) \right\} d\theta \tag{12}$$

If instead the search cost is positive and the firm doesn't advertise, there exist two critical thresholds $\underline{s}$ and $\overline{s}$ which satisfy $0 < \underline{s} < \overline{s}$. Firstly and unlike with unit demands, a small search cost need not affect the equilibrium price[14]

Lemma 15 *When $s \leq \underline{s}$ there exists an equilibrium in which the price is p^m*

The explanation behind lemma 15 is simple: since the search cost is small (and expecting to pay a price p^m) every consumer finds it worthwhile to search. The firm then faces the same problem as it does when $s = 0$, and consequently charges p^m. Secondly however when $s \in (\underline{s}, \overline{s})$ the equilibrium described in lemma 15 no longer exists. If consumers expected to pay p^m, those with a low θ would not expect to earn enough surplus to cover the search cost and therefore would not search. The firm would only attract high-θ consumers (who have relatively inelastic demands) and would therefore optimally charge more than p^m. However unlike in example 2 the market does not completely unravel, and instead

[14]As usual there are also equilibria in which consumers expect a very high price and nobody visits the store. As in section 2.2 we rule out such equilibria.

Lemma 16 *When $s \in (\underline{s}, \overline{s})$ there exists an equilibrium in which the price strictly exceeds p^m but some consumers do search and buy the product*

There is a parallel between lemma 16 and lemma 5. In both cases the firm is searched by a select sample of high-type consumers, and this pushes the equilibrium price above the frictionless benchmark p^m. Thirdly and finally, we can also prove that when $s \geq \bar{s}$ the search cost is too high to support an equilibrium with trade.

We now move to the general case where $n \geq 1$ and some prices may be advertised. Each consumer again receives an independent draw from the θ distribution, and her demand for product j can be written as $p_j = \theta + P(q_j)$ where q_j is the quantity consumed of good j.[15] We can again find two thresholds $\underline{s}'$ and $\overline{s}'$ which satisfy $0 < \underline{s}' < \overline{s}'$. Analogous to lemma 15 if $s \leq \underline{s}'$ there exists an equilibrium in which each unadvertised good is priced at p^m. Analogous to lemma 16 if $s \in (\underline{s}', \overline{s}')$ there is an equilibrium in which some consumers search and each unadvertised price strictly exceeds p^m. Moreover:

Proposition 17 *Suppose that $s \in (\underline{s}', \overline{s}')$. The (Pareto dominant) equilibrium unadvertised price is increasing in s, decreasing in n, and increasing in any advertised price*

There is a parallel between proposition 17 and earlier results in the unit demand model. Intuitively if n increases, some new consumers with a relatively low θ decide to search. These new consumers have relatively elastic demands, so the firm optimally reduces its unadvertised prices. Consumers anticipate this and again correctly believe that a larger store charges lower prices. Similarly a fall in the search cost or a decrease in advertised prices also encourage low-θ consumers to visit the

[15] We could add more heterogeneity and write demand as $p_j = \theta_j + P(q_j)$ with the θ_j differing across consumers and products. However setting $\theta_j = \theta$ is enough to make the main point.

store, and therefore also reduce equilibrium unadvertised prices. Consequently as in the unit demand model, the firm can again use a low advertised price on one product to build a low-price image.

4.2 Competition

While a full analysis of competition is beyond the scope of the current paper, we show how our results on advertising can be applied to a duopoly. We suppose that two firms A and B sell the same two products. Consumers search the firms sequentially with perfect recall, and search randomly when indifferent. Each retailer can pay a cost c_a and advertise the price that it charges for product 2.[16] The move-order is otherwise similar to section 1. In the first stage firms simultaneously choose whether or not to advertise, and pick prices for the two products. In the second stage consumers observe advertised prices, form price expectations, and decide which firm (if any) to search first. In the final stage consumers learn unadvertised price(s) at the firm that they searched, and then decide whether to search the other firm as well, before finally making purchase decisions.

It is convenient to introduce the following notation. If the market were controlled by a non-advertising monopolist, it would charge $p^u < b$ on both products and earn profit $\pi^u > 0$. If instead the monopolist advertised a price p_2^a, it would charge $\phi(p_2^a)$ on unadvertised product 1 and earn total profit $\pi(p_2^a) - c_a$. (Where $\phi(p_2^a)$ is the unique p_1^e which solves equation (2) for the case $n = 2$ and $p_2^e = p_2^a$.) We assume that $\pi(p_2^a)$ is continuous and quasiconcave in p_2^a, and reaches its maximum value of π^a when $p_2^a = p^* \, (< p^u)$. Returning to the duopoly case we have the

[16] We do not consider the case where firms advertise different products. In practice competing retailers often advertise similar or even identical products, because certain products are more salient and important to consumers than others. We could incorporate this by allowing v_1 and v_2 to have different distributions, and the equilibrium in proposition 18 would not be qualitatively different.

following proposition.

Proposition 18 *If $c_a < \pi^a - \pi^u/2$ there is a symmetric equilibrium in which*
(a) Each firm advertises with probability $\frac{\pi^a - \pi^u/2 - c_a}{\pi^a - \pi^u/2}$. When a firm advertises, the distribution of its advertised price is given by the c.d.f

$$G\left(p_2^a\right) = \frac{\pi\left(p_2^a\right)\left[\pi^a - \pi^u/2\right] - c_a \pi^a}{\pi\left(p_2^a\right)\left[\pi^a - \pi^u/2 - c_a\right]}$$

defined on the interval $\left[\underline{p}, p^*\right]$ where $\underline{p} = \pi^{-1}\left(\frac{c_a \pi^a}{\pi^a - \pi^u/2}\right)$.
(b) A non-advertising firm charges p^u for both products.
A firm that advertises a price p_2^a on product 2, charges $\phi\left(p_2^a\right)$ for product 1
(c) Consumers expect a non-advertising firm to charge p^u for both products, and an advertising firm to charge $\phi\left(p_2^a\right)$ on product 1

The first part of proposition 18 shows that provided the advertising cost c_a is not too large, duopolists randomize between not advertising, or posting an advertised price which is drawn from an atomless distribution $G\left(p_2^a\right)$. The second part shows that given its advertising decision (either no advert, or a particular advertised price p_2^a) a duopolist charges the same unadvertised price(s) as would a monopolist. Consequently prices are always strictly higher at a firm that does not advertise. Moreover the lower a firm's advertised price, the lower also is its unadvertised price. Therefore if a consumer searches, she should first visit the firm with the lowest advertised price (and choose randomly if nobody advertises).[17] In equilibrium consumers have correct price expectations, and therefore never search twice.

The reason why firms randomize over their advertising decision (part (a)) is similar to Baye and Morgan's (2001) paper on internet gatekeepers, and is as follows.

[17]In particular $\phi\left(p_2^a\right)$ is strictly increasing in p_2^a when $p_2^a \geq a - s$, and $\phi\left(p_2^a\right) = p^m$ when $p_2^a \leq a - s$. Therefore any consumer who searches, must expect to earn strictly higher surplus by searching the firm with the lowest advertised price.

If a firm doesn't advertise, it is only searched (and makes a sale) when its competitor also doesn't advertise. By posting an advert the firm becomes more likely to make a sale but also has to incur the cost c_a. In equilibrium the cost and benefit of posting an advert are identical, so the firm is indifferent about whether to advertise. Moreover when it does advertise, the firm randomizes over its choice of p_2^a in order to prevent its competitor from undercutting it and stealing the entire market.

The explanation behind the pricing strategies in part (b) of the proposition is also straightforward. First consider a non-advertising firm. The firm is only searched if its competitor is also not advertising, in which case both are expected to charge p^u. In terms of (v_1, v_2) space, each firm is then searched by the *same* mix of consumers as a monopolist. Therefore each firm maximizes its profits by charging p^u as expected. Secondly if a firm posts an advertised price p_2^a, it is always searched unless its rival has advertised a lower price. Conditional upon being searched the firm again attracts the *same* mix of consumers as a monopolist - and therefore also maximizes its profits by charging a price $\phi\left(p_2^a\right)$ on product 1.

Combining parts (a) and (b) of proposition 18, prices are random but positively correlated. A firm's advertised price still conveys information about its unadvertised price, so consumers should always check the adverts before deciding which firm (if any) they should search.

5 Conclusion

Consumers are often poorly-informed about prices and have to spend time and effort searching for them. This paper presents a simple model that analyzes how these search costs affect the pricing behavior of a multiproduct retailer. We first demonstrate that although multiproduct firms can overcome the Diamond Paradox, unadvertised prices still exceed what a monopolist would charge in the frictionless benchmark. Intuitively consumers only search if they have relatively high valuations, and the firm exploits this by charging high prices. We then show that a retailer with a broader product selection is searched by consumers who, on average, have a lower valuation for any single product. A larger store therefore has less incentive to hold-up consumers and so, in equilibrium, charges lower prices but earns more profit on each individual product.

Firms sometimes use adverts to directly inform consumers about a small proportion of their prices. We show that when consumers observe a low advertised price on one product, they rationally anticipate that all other (unadvertised) prices will be somewhat lower as well. Therefore by advertising a low price on one product, a firm can build a store-wide low-price image and significantly increase the number of consumers who search it.

One aspect that our model does not explicitly address is bundling. It is well-known that in theory a firm can sometimes increase its profits by bundling, but few retailers choose to do this in practice. It turns out that in our model, the ability to use bundling causes a two-product retailer to earn zero profits. With pure bundling the firm (effectively) sells only one product, so Diamond's no-trade result applies. One can also show that mixed bundling similarly causes the market to collapse. This suggests that when consumers have search costs, retailers may (where possible) commit to not use bundling.

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A Appendix

Proof of lemma 1. Differentiate $p_i D_i(p_i; \boldsymbol{p^e})$ with respect to p_i to get

$$D_i(p_i; \boldsymbol{p^e}) - p_i f(p_i) \Pr(T + \max(p_i - p_i^e, 0) \geq s) \tag{13}$$

where $T = \sum_{j \neq i} \max\left(v_j - p_j^e, 0\right)$. Substitute $p_i = p_i^e$ and set (13) to zero. ■

Lemma 19 *A necessary condition for $p_i D_i(p_i; \boldsymbol{p^e})$ to be quasiconcave in p_i is that $p[1 - F(p)]$ is concave in $p \forall p \in [a, p_i^e)$. Sufficient conditions are i). $p[1 - F(p)]$ is concave in $p \forall p \in [a, b]$ or ii). $\Pr(T \geq s) \geq 1/2$.*

Proof of lemma 19. **Necessity** When $p_i < p_i^e$ the derivative of (13) with respect to p_i is proportional to $-2f(p_i) - p_i f'(p_i)$ which is the second derivative of

$p_i [1 - F(p_i)]$. We can show that if $p[1-F(p)]$ is *not* concave for all $p < p_i^e$, then there exists a $\tilde{p} < p_i^e$ such that $p[1-F(p)]$ is convex $\forall p \in (\tilde{p}, p_i^e)$.[18] But then $p_i D_i(p_i; \boldsymbol{p^e})$ is also convex $\forall p_i \in (\tilde{p}, p_i^e)$ and therefore not quasiconcave.

Sufficiency If $p_i \geq p_i^e + s$ then $p_i D_i(p_i; \boldsymbol{p^e}) = p_i[1-F(p_i)]$, which is decreasing in p_i because $p_i^e \geq p^m$ (lemma 5) and $p_i[1-F(p_i)]$ is quasiconcave. So we must check that $p_i D_i(p_i; \boldsymbol{p^e})$ strictly increases in p_i when $p_i \in [a, p_i^e)$, and strictly decreases in p_i when $p_i \in (p_i^e, p_i^e + s)$. To do this, rewrite (13) as

$$[D_i(p_i; \boldsymbol{p^e}) - p_i f(p_i) \Pr(T \geq s)] \\ + p_i f(p_i)[\Pr(T \geq s) - \Pr(T + \max(p_i - p_i^e, 0) \geq s)]$$

The second term is 0 when $p_i \leq p_i^e$, and negative otherwise. The first term is by definition 0 when $p_i = p_i^e$. Therefore it is sufficient to show that the first term is strictly positive when $p_i \in [a, p_i^e)$ and strictly negative when $p_i \in (p_i^e, p_i^e + s)$. i). if $p[1-F(p)]$ is concave, the first term strictly decreases in p_i and so has the required signs. ii). if we divide the first term by $f(p_i)$, it also strictly decreases in p_i and so has the correct signs, provided $\Pr(T \geq s) > 1/2$.[19] *Finally* we can also verify that if $\Pr(T \geq s) > 1/2$, then p_i^e lies on the concave part of $p[1-F(p)]$. ■

Proof of proposition 3. Let $\tilde{p}$ be the unique solution to $1 - F(p) - pf(p)/2 = 0$, and $\tilde{n}$ be the smallest n such that $\Pr\left(\sum_{j=2}^{n} \max(v_j - \tilde{p}, 0) \geq s\right) > 1/2$. It is sufficient to look for a symmetric equilibrium in which $p_j^e = p^e \forall j$ (although lemma

[18] Note that $-2f(p) - pf'(p)$ is proportional to $-2/p - f'(p)/f(p)$ which increases in p since $f(p)$ is logconcave. So if there exists $\tilde{p}$ such that $-2f(\tilde{p}) - \tilde{p}f'(\tilde{p}) = 0$ then $p[1-F(p)]$ is convex $\forall p > \tilde{p}$. (It is also simple to prove that always $\tilde{p} > p^m$.)

[19] In more detail, the derivative is $-\Pr(T + \max(p_i - p_i^e, 0) \geq s) - \Pr(T \geq s) - D_i(p_i; \boldsymbol{p^e}) f'(p_i)/f(p_i)^2$. This is negative if $\Pr(T \geq s) \geq 1/2$ because $-f'(p_i)/f(p_i)^2 \leq 1/[1-F(p_i)]$ (due to logconcavity of $1 - F(p_i)$) and because $D_i(p_i; \boldsymbol{p^e}) \leq 1 - F(p_i)$.

7 later shows this is also necessary). Using equation (2) p^e satisfies

$$D_i\left(p_i = p^e; \boldsymbol{p^e}\right) - p^e f\left(p^e\right) \Pr\left(\sum_{j \neq i} \max\left(v_j - p^e, 0\right) \geq s\right) = 0 \tag{14}$$

The lefthand side of (14) is strictly positive when evaluated at $p^e = p^m$, because $D_i\left(p_i = p^e; \boldsymbol{p^e}\right) \geq \left[1 - F\left(p^e\right)\right] \Pr\left(\sum_{j \neq i} \max\left(v_j - p^e, 0\right) \geq s\right)$ and $1 - F\left(p^m\right) - p^m f\left(p^m\right) = 0$. Assuming $n \geq \tilde{n}$, the lefthand side of (14) is strictly negative when evaluated at $p^e = \tilde{p}$, because $D_i\left(p_i = p^e; \boldsymbol{p^e}\right) \leq 1 - F\left(p^e\right)$ and $1 - F\left(\tilde{p}\right) - \tilde{p} f\left(\tilde{p}\right)/2 = 0$. Therefore since the lefthand side is continuous in p^e, there exists (at least one) $p^e \in \left(p^m, \tilde{p}\right)$ which solves (14). According to lemma 19 profit is quasiconcave because $\Pr\left(\sum_{j=2}^{n} \max\left(v_j - p^e, 0\right) \geq s\right) > 1/2$. ■

Proof of lemma 4. Add and subtract $\Pr\left(\sum_{j \neq i} t_j \geq s\right)\left[1 - F\left(p_i^e\right)\right]$ from equation (2). Then note that $\Pr\left(\sum_{j=1}^{n} \max\left(v_j - p_j^e, 0\right) \geq s\right)$ can be rewritten as

$$\int_{p_i^e}^{b} f\left(v_i\right) \Pr\left(\sum_{j=1}^{n} \max\left(v_j - p_j^e, 0\right) \geq s\right) dv_i + F\left(p_i^e\right) \Pr\left(\sum_{j \neq i} \max\left(v_j - p_j^e, 0\right) \geq s\right)$$

■

Proof of lemma 7. Suppose not - then without loss of generality suppose $p_1^e < p_2^e$ instead. Lemma 19 says that $p\left[1 - F\left(p\right)\right]$ must be concave for all $p < p_2^e$, so $1 - F\left(p_2^e\right) - p_2^e f\left(p_2^e\right)$ is less than $1 - F\left(p_1^e\right) - p_1^e f\left(p_1^e\right)$, which is itself negative since lemma 5 says that $p_1^e > p^m$. Also $\Pr\left(\sum_{j \neq 2} t_j \geq s\right) > \Pr\left(\sum_{j \neq 1} t_j \geq s\right)$, so if equation (3) holds for $i = 1$, it cannot hold for $i = 2$ - a contradiction. ■

Proof of propositions 9 and 10. Equation (7) is copied here for convenience

$$\phi\left(p^*; s, n\right) = 1 - F\left(p^*\right) - p^* f\left(p^*\right) + \frac{\Pr\left(\sum_{j=1}^{n} t_j \geq s\right)}{\Pr\left(\sum_{j=1}^{n-1} t_j \geq s\right)} - 1 = 0$$

where $t_j \equiv \max\left(v_j - p^*, 0\right)$. **Firstly** fix n and decrease s from s_0 to $s_1 < s_0$. Let $p_{s_0}^*$ denote the lowest solution to $\phi\left(p^*; s_0, n\right) = 0$. Lemma 20 (below) shows that $\phi(p^*; s, n)$ increases in s, so we conclude that $\phi\left(p^*_{s_0}; s_1, n\right) \leq 0$. Since $\phi(p^m; s_1, n) > 0$ and $\phi(p^*; s_1, n)$ is continuous in p^*, this implies that when $s = s_1$ the lowest equilibrium price is (weakly) below $p^*_{s_0}$. So the lowest equilibrium price increases in s. **Secondly** fix s and increase n from n_0 to n_1. Let $p^*_{n_0}$ be the lowest solution to $\phi(p^*; s, n_0)$. Lemma 21 (below) shows that $\phi(p^*; s, n)$ decreases in n, so we conclude that $\phi\left(p^*_{n_0}; s, n_1\right) \leq 0$. This again implies that when $n = n_1$, the lowest equilibrium price is (weakly) below $p^*_{n_0}$, so the lowest equilibrium price falls in n. ■

Lemma 20 $\phi(p^*; s, n)$ *increases in* s

Proof of lemma 20. Define $\tilde{v}_j = v_j - p^*$, and note that $\tilde{v}_j$ has a logconcave density function $\tilde{f}(\tilde{v}_j)$ defined on the interval $\left[\tilde{a}, \tilde{b}\right]$ where $\tilde{a} = a - p^*$ and $\tilde{b} = b - p^*$. Recall that the $\tilde{v}_j$ are iid and that $t_j \equiv \max(\tilde{v}_j, 0)$.
Define $\Omega(s, n) = \Pr\left(\sum_{j=1}^{n} t_j \geq s\right) \Big/ \Pr\left(\sum_{j=2}^{n} t_j \geq s\right)$. $\phi(p^*; s, n)$ increases in s if and only if $\Omega(s, n)$ increases in s. We now prove that $\Omega(s, n)$ increases in s.
Begin with $n = 2$ Consider $\Pr(t_1 + t_2 \geq x)$ for some $x > 0$. If $\tilde{v}_1 \geq x$ then $t_1 + t_2 \geq x$ since $t_2 \geq 0$; if $\tilde{v}_1 \in (0, x)$ then $t_1 + t_2 \geq x$ if and only if $t_2 = \tilde{v}_2 \geq x - \tilde{v}_1$; if $\tilde{v}_1 \leq 0$ then $t_1 + t_2 \geq x$ if and only if $t_2 = \tilde{v}_2 \geq x$. Therefore

$$\begin{aligned} \frac{\Pr(t_1 + t_2 \geq x)}{\Pr(t_2 \geq x)} &= \frac{\Pr(\tilde{v}_1 \leq 0)\Pr(\tilde{v}_2 \geq x) + \int_0^x \tilde{f}(z)\Pr(\tilde{v}_2 \geq x - z)\,dz + \Pr(\tilde{v}_1 \geq x)}{\Pr(\tilde{v}_2 \geq x)} \qquad (15) \\ &= \Pr(\tilde{v}_1 \leq 0) + \int_0^x \tilde{f}(z)\frac{\Pr(\tilde{v}_2 \geq x - z)}{\Pr(\tilde{v}_2 \geq x)}dz + 1 \end{aligned}$$

which increases in x. This is because $\tilde{v}_2$ is logconcave and so has an increasing hazard rate, which means that $\Pr(\tilde{v}_2 \geq x - z) / \Pr(\tilde{v}_2 \geq x)$ increases in x.
Now proceed by induction We show that if $\Omega(w, n-1)$ increases in w, then

$\Omega(s,n)$ increases in s. To prove this, let $k>1$ and write:

$$\Omega(s,n)-k=\frac{\Pr\left(\sum_{j=1}^{n}t_j\geq s\right)}{\Pr\left(\sum_{j=2}^{n}t_j\geq s\right)}-k=\frac{\Pr\left(\sum_{j=1}^{n}t_j\geq s\right)-k\Pr\left(\sum_{j=2}^{n}t_j\geq s\right)}{\Pr\left(\sum_{j=2}^{n}t_j\geq s\right)} \tag{16}$$

Then using the same principles used to derive equation (15), expand *only the top* of equation (16) to get

$$\Omega(s,n)-k=\Pr(\tilde{v}_n\leq 0)\left[\frac{\Pr\left(\sum_{j=1}^{n-1}t_j\geq s\right)-k\Pr\left(\sum_{j=2}^{n-1}t_j\geq s\right)}{\Pr\left(\sum_{j=2}^{n}t_j\geq s\right)}\right]$$
$$+\frac{\int_0^s\tilde{f}(z)\left[\Pr\left(\sum_{j=1}^{n-1}t_j\geq s-z\right)-k\Pr\left(\sum_{j=2}^{n-1}t_j\geq s-z\right)\right]dz+\int_s^{\tilde{b}}\tilde{f}(z)\left[1-k\right]dz}{\Pr\left(\sum_{j=2}^{n}t_j\geq s\right)} \tag{17}$$

Since t_1 and t_n are iid, the first term in (17) simplifies to $\Pr(\tilde{v}_n\leq 0)\left[1-k/\Omega(s,n-1)\right]$, which is weakly increasing in s because of the inductive assumption that $\Omega(w,n-1)$ increases in w. The second term in (17) is proportional to

$$\int_0^s\tilde{f}(z)\left[\Pr\left(\sum_{j=1}^{n-1}t_j\geq s-z\right)-k\Pr\left(\sum_{j=2}^{n-1}t_j\geq s-z\right)\right]dz+\int_s^{\tilde{b}}\tilde{f}(z)\left[1-k\right]dz$$

Using the change of variables $y=s-z$, this can be rewritten as

$$\int_{s-\tilde{b}}^{0}\tilde{f}(s-y)\left[1-k\right]dy+\int_0^s\tilde{f}(s-y)\left[\Pr\left(\sum_{j=1}^{n-1}t_j\geq y\right)-k\Pr\left(\sum_{j=2}^{n-1}t_j\geq y\right)\right]dy$$

and then written more compactly as

$$\int_{-\infty}^{\infty}\mathbf{1}_{0\leq s-y\leq\tilde{b}}\tilde{f}(s-y)\,\Gamma(y)\,dy \tag{18}$$

where $\mathbf{1}_{0\leq s-y\leq\tilde{b}}$ is an indicator function taking value 1 when $0\leq s-y\leq\tilde{b}$, and 0 otherwise; and where

$$\Gamma(y)=\begin{cases} 1-k & \text{if} \quad y\le 0 \\ \Pr\left(\sum_{j=1}^{n-1} t_j \ge y\right) - k\Pr\left(\sum_{j=2}^{n-1} t_j \ge y\right) & \text{if} \quad y>0 \end{cases}$$

Now let S be an interval in $\mathbb{R}_{++}$ such that $\Pr\left(\sum_{j=1}^{n-1} t_j \ge s\right) > 0 \forall s \in S$, and choose $s_0, s_1 \in S$ where $s_1 > s_0$. Also choose a constant k_0 such that (18) is zero when evaluated at $s = s_0$ and $k = k_0$. (Zero terms being discarded) $\Gamma(y)$ has one sign-change from negative to positive: this follows directly from the definition of $k_0\,(>1)$ and the inductive assumption that $\Omega(w, n-1)$ increases in w.

Using the definition in Karlin (1968) §1.1, $\mathbf{1}_{0\le s-y\le\tilde{b}}$ is totally positive of order 2 (TP_2) in (s,y). Since $\tilde{f}(z)$ is logconcave, $\tilde{f}(s-y)$ is also TP_2 in (s,y). Therefore $\mathbf{1}_{0\le s-y\le\tilde{b}}\tilde{f}(s-y)$ is also TP_2 in (s,y).

Applying Karlin's Variation Diminishing Property (Karlin §1.3, Theorem 3.1), (18) changes sign once in s (and from negative to positive) when it is evaluated at $k = k_0$. Since by assumption (18) is zero when evaluated at $s = s_0$ and $k = k_0$, it is positive when evaluated at $s = s_1$ and $k = k_0$. Therefore returning to equation (17) it follows that $\Omega(s_1, n) - k_0 \ge \Omega(s_0, n) - k_0$, or equivalently that $\Omega(s,n)$ increases in s. In summary we have shown that if $\Omega(w, n-1)$ increases in w, then $\Omega(s,n)$ increases in s. Since $\Omega(w,n)$ increases in w for $n = 2$, $\Omega(s,n)$ increases in s for any integer-valued n. ■

Lemma 21 $\phi(p^*; s, n)$ *decreases in* n

Proof of lemma 21. $\phi(p^*; s, n)$ decreases in n if and only if $\Omega(s,n)$ decreases in n. To show that $\Omega(s,n)$ decreases in n, write out $\Omega(s,n)$ as

$$\frac{\Pr(\tilde{v}_n \ge s) + \int_0^s \tilde{f}(\tilde{v}_n)\Pr\left(\sum_{j=1}^{n-1} t_j \ge s-\tilde{v}_n\right)d\tilde{v}_n + \Pr(\tilde{v}_n \le 0)\Pr\left(\sum_{j=1}^{n-1} t_j \ge s\right)}{\Pr(\tilde{v}_n \ge s) + \int_0^s \tilde{f}(\tilde{v}_n)\Pr\left(\sum_{j=2}^{n-1} t_j \ge s-\tilde{v}_n\right)d\tilde{v}_n + \Pr(\tilde{v}_n \le 0)\Pr\left(\sum_{j=2}^{n-1} t_j \ge s\right)}$$

This is less than $\Omega(s, n-1)$ because $\Pr\left(\sum_{j=1}^{n-1} t_j \ge x\right) \Big/ \Pr\left(\sum_{j=2}^{n-1} t_j \ge x\right)$ exceeds 1 and (from lemma 20) is increasing in x. Therefore we know that $\Omega(s,n) \le$

$\Omega(s, n-1)$ for any n, or alternatively $\Omega(s, n)$ decreases in n. $\blacksquare$

Proof of proposition 11. Let p' be the equilibrium unadvertised price. Let $t_j = \max(v_j - p', 0)$ for $j < n$, and $t_n = \max(v_n - p_n^a, 0)$. Then using equation (7) again, p' satisfies

$$\Phi(p'; p_n^a) = 1 - F(p') - p' f(p') + \frac{\Pr\left(\sum_{j=1}^{n-1} t_j + t_n \geq s\right)}{\Pr\left(\sum_{j=2}^{n-1} t_j + t_n \geq s\right)} - 1 = 0$$

We focus on the smallest p' which solves $\Phi(p'; p_n^a) = 0$, and assume that the conditions for quasiconcavity in lemma 19 are satisfied. Note that $\Phi(p'; p_n^a)$ is continuous in p', and that $\Phi(p^m; p_n^a) \geq 0$. Therefore *if* $\Pr\left(\sum_{j=1}^{n} t_j \geq s\right) \Big/ \Pr\left(\sum_{j=2}^{n-1} t_j \geq s\right)$ increases in p_n^a, the lowest p' (that solves $\Phi(p'; p_n^a) = 0$) also increases in p_n^a. To prove that $\omega(p_n^a) = \Pr\left(\sum_{j=1}^{n} t_j \geq s\right) \Big/ \Pr\left(\sum_{j=2}^{n-1} t_j \geq s\right)$ increases in p_n^a, write:

$$\omega(p_n^a) - k = \frac{\Pr\left(\sum_{j=1}^{n} t_j \geq s\right) - k \Pr\left(\sum_{j=2}^{n} t_j \geq s\right)}{\Pr\left(\sum_{j=2}^{n} t_j \geq s\right)} \tag{19}$$

Just as in the proof of lemma 20, the numerator of (19) can be rewritten as

$$\int_a^{p_n^a} f(v_n) \left[\Pr\left(\sum_{j=1}^{n-1} t_j \geq s\right) - k \Pr\left(\sum_{j=2}^{n-1} t_j \geq s\right)\right] dv_n$$
$$+ \int_{p_n^a}^{p_n^a + s} f(v_n) \left[\Pr\left(\sum_{j=1}^{n-1} t_j \geq s + p_n^a - v_n\right) - k \Pr\left(\sum_{j=2}^{n-1} t_j \geq s + p_n^a - v_n\right)\right] dv_n$$
$$+ \int_{p_n^a + s}^{b} f(v_n) \left[1 - k\right] dv_n \tag{20}$$

Using the change of variables $y = p_n^a - z$ (20) can be written as

$$\int_{-\infty}^{\infty} \mathbf{1}_{a \leq p_n^a - y \leq b} f(p_n^a - y) \gamma(y) \, dy \tag{21}$$

where $\gamma(y) = \begin{cases} 1 - k & \text{if } \; y \leq -s \\ \Pr\left(\sum_{j=1}^{n-1} t_j \geq s + y\right) - k \Pr\left(\sum_{j=2}^{n-1} t_j \geq s + y\right) & \text{if } \; y \in (-s, 0) \\ \Pr\left(\sum_{j=1}^{n-1} t_j \geq s\right) - k \Pr\left(\sum_{j=2}^{n-1} t_j \geq s\right) & \text{if } \; y \geq 0 \end{cases}$

and $\mathbf{1}_{a \leq p_n^a - y \leq b}$ is again an indicator function that is TP_2 in (p_n^a, y). Now choose $p_n^{a,0}, p_n^{a,1} \in [a-s, b]$ such that $p_n^{a,0} < p_n^{a,1}$. Also define k_0 such that (21) is zero when evaluated at $p_n^a = p_n^{a,0}$ and $k = k_0$. (Zero terms being discarded) $\gamma(y)$ is piecewise continuous and changes sign once from negative to positive: this follows from the definition of k_0 (> 1), and because $\Pr\left(\sum_{j=1}^{n-1} t_j \geq x\right) \Big/ \Pr\left(\sum_{j=2}^{n-1} t_j \geq x\right)$ increases in x (from lemma 20). Karlin's variation diminishing property says that (21) is single-crossing from negative to positive in p_n^a. Therefore $\omega(p_n^{a,1}) - k_0 \geq \omega(p_n^{a,0}) - k_0$, or equivalently $\omega(p_n^a)$ increases in p_n^a as required. ■

Proof of lemma 12. Note that lemma 12 does not depend on differentiability. Most of the proof was sketched in the text, so we just show that $d\Pi_n(p_n^a; \boldsymbol{p^e})/dp_n^a < 0$ when $p_n^a \geq p^m$. Firstly when p_n^a increases, so do unadvertised prices - less people search so demand (and profit) for good n falls. Secondly keeping unadvertised prices fixed, $\partial \Pi_n(p_n^a; \boldsymbol{p^e})/\partial p_n^a < 0$ too. To show this, write $T = \sum_{j=1}^{n-1} t_j$: note that $T \geq 0$, that in general $\Pr(T = 0) > 0$, and that T is atomless on $(0, \infty)$. Let $\mu(T)$ be the density of T for $T \in (0, \infty)$. Then $\Pi_n(p_n^a; \boldsymbol{p^e})$ can be expressed as

$$\Pr(T=0)\, p_n^a \left[1 - F(p_n^a + s)\right] + \int_0^s \mu(z)\, p_n^a \left[1 - F(p_n^a + s - z)\right] dz + \int_s^\infty \mu(z)\, p_n^a \left[1 - F(p_n^a)\right] dz \quad (22)$$

To interpret (22) note that somebody with $T \in [0, s)$ only searches if $v_n - p_n^a \geq s - T$ - and after searching buys good n since they have $v_n > p_n^a$. Also somebody with $T \geq s$ searches irrespective of their v_n, and then buys good n if $v_n \geq p_n^a$. Since $f(v)$ is logconcave so is $1 - F(v)$, therefore $p[1 - F(p + y)]$ is quasiconcave in p. Using steps on page 111 of Anderson and Renault (2006), $p[1 - F(p + y)]$ is also maximized at a price below p^m. It then follows that $p[1 - F(p + y)]$ is decreasing in p for all $p > p^m$. Therefore using (22), $\partial \Pi_n(p_n^a; \boldsymbol{p^e})/\partial p_n^a$ whenever $p_n^a \geq p^m$. ■

B Omitted proofs

B.1 Proofs for Section 4.1 (first extension)

Define $Q(z) \equiv P^{-1}(z)$ and note that since $P', P'' < 0$, then $Q', Q'' < 0$ as well. Also define $\Pi(p;\theta) = pQ(p-\theta)$, define $p^*(\theta) = \arg\max_x xQ(x-\theta)$, and let $CS(p_j;\theta)$ be the consumer surplus of a θ-type on one product when its price is p_j. A preliminary result is:

Remark 22 *$p^*(\theta)$ and $CS(p^*(\theta);\theta)$ are both strictly increasing in θ*

Proof. Differentiate $pQ(p-\theta)$ with respect to p to get a first order condition $Q(p-\theta)+pQ'(p-\theta)=0$. Then totally differentiate with respect to θ to get

$$\frac{\partial p^*(\theta)}{\partial\theta} = \frac{Q'(p^*(\theta)-\theta)+p^*(\theta)Q''(p^*(\theta)-\theta)}{2Q'(p^*(\theta)-\theta)+p^*(\theta)Q''(p^*(\theta)-\theta)} \in (0,1)$$

Since $\partial p^*(\theta)/\partial\theta \in (0,1)$ and (by definition) $p^*(\theta) = \theta + P(q^*(\theta))$, it follows that $\partial P(q^*(\theta))/\partial\theta < 0$ or equivalently $\partial q^*(\theta)/\partial\theta > 0$ (because $P' < 0$). We can also write

$$CS(p^*(\theta);\theta) = \int_0^{q^*(\theta)} [p_j - p^*(\theta)]\, dq_j = \int_0^{q^*(\theta)} [P(q_j) - P(q^*(\theta))]\, dq_j$$

which is strictly increasing in θ because $P' < 0$ and $\partial q^*(\theta)/\partial\theta > 0$. ■

For simplicity assume that all prices are unadvertised. It is clear that each unadvertised price must be the same - so let p be the actual price and p^e the expected price. We will prove lemmas 15 and 16 for a general $n \geq 1$. All consumers with $\theta \geq \tilde{\theta}$ will search, where $\tilde{\theta}$ is the minimum $\theta \in [\underline{\theta},\overline{\theta}]$ such that $n \times CS(p^e;\theta) \geq s$. So profit on good j is $\int_{\tilde{\theta}}^{\overline{\theta}} g(\theta)\{pQ(p-\theta)\}\, d\theta$ - differentiating this with respect to p and imposing $p = p^e$, we find that p^e satisfies[20]

$$\int_{\tilde{\theta}}^{\overline{\theta}} g(\theta)\,\Pi'(p^e;\theta)\, d\theta = \int_{\tilde{\theta}}^{\overline{\theta}} g(\theta)\{Q(p^e-\theta)+p^eQ'(p^e-\theta)\}\, d\theta = 0 \qquad (23)$$

[20] Since $Q', Q'' < 0$ profit is quasiconcave in p provided that types are not too different.

It is useful to note that $\partial\Pi'(p^e;\theta)/\partial\theta > 0$, therefore using equation (23) $\Pi'\left(p^e;\tilde{\theta}\right) < 0$. This then further implies that $\Pi'(p^e;\theta) < 0 \forall\theta \leq \tilde{\theta}$. In particular $\Pi'(p^e;\underline{\theta}) < 0$ so since $\Pi(p,\underline{\theta})$ is concave in p, we then know that $p^e > p^*(\underline{\theta})$.

Remark 23 *In any equilibrium with trade,* $p^e \geq p^m$

Proof. Suppose not and that actually $p^e < p^m$. Then since $\tilde{\theta} \geq \underline{\theta}$ and (from above) $\Pi'(p^e;\theta) < 0\forall\theta \leq \tilde{\theta}$, we know that $\int_{\tilde{\theta}}^{\overline{\theta}} g(\theta)\Pi'(p^e;\theta)\,d\theta \geq \int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\Pi'(p^e;\theta)\,d\theta$. Also $\int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\Pi(p;\theta)\,d\theta$ is concave and maximized at $p = p^m$, therefore since by assumption $p^e < p^m$, we know that $\int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\Pi'(p^e;\theta)\,d\theta > 0$. Connecting these two inequalities, we conclude that $\int_{\tilde{\theta}}^{\overline{\theta}} g(\theta)\Pi'(p^e;\theta)\,d\theta > 0$ which contradicts equation (23) and therefore rules out $p^e < p^m$ as an equilibrium. ■

Now define $\underline{s}' = n \times CS(p^m;\underline{\theta})$ and $\overline{s}' = n \times CS\left(p^*\left(\overline{\theta}\right);\overline{\theta}\right)$. We need to check that $\overline{s}' > \underline{s}'$. Using remark 22 we know that $CS\left(p^*\left(\overline{\theta}\right);\overline{\theta}\right) > CS(p^*(\underline{\theta});\underline{\theta})$. Also by definition $\int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\Pi'(p^m;\theta)\,d\theta = 0$, so since $\partial\Pi'(p;\theta)/\partial\theta > 0$, this implies that $\Pi'(p^m;\underline{\theta}) < 0$. Since $\Pi(p;\underline{\theta})$ is concave, this further implies that $p^m > p^*(\underline{\theta})$. Therefore $CS(p^*(\underline{\theta});\underline{\theta}) > CS(p^m;\underline{\theta})$. Putting these results together gives $CS\left(p^*\left(\overline{\theta}\right);\overline{\theta}\right) > CS(p^m;\underline{\theta})$ as required.

Proof of lemmas 15 and 16. Again define $\underline{s}' = n \times CS(p^m;\underline{\theta})$ and $\overline{s}' = n \times CS\left(p^*\left(\overline{\theta}\right);\overline{\theta}\right)$. Lemma 15 then follows immediately. Now prove Lemma 16. Firstly $s > \underline{s}'$ so we know that $\tilde{\theta} > \underline{\theta}$. Then using the proof of remark 23, the lefthand side of equation (23) is strictly positive when evaluated at $p^e = p^m$. Secondly consider $p^e = p^*\left(\overline{\theta}\right)$. $s < \overline{s}'$ so consumers with $\theta = \overline{\theta}$ and (by continuity) types with $\theta \approx \bar{\theta}$ also search. Therefore $\tilde{\theta} < \overline{\theta}$. Also $\partial\Pi'(p;\theta)/\partial\theta > 0$ and (by definition) $\Pi'\left(p^*\left(\overline{\theta}\right);\overline{\theta}\right) = 0$, therefore $\Pi'\left(p^*\left(\overline{\theta}\right);\theta\right) < 0$ for all $\theta < \overline{\theta}$. Hence the lefthand side of equation (23) is strictly negative when evaluated at $p^e = p^*\left(\overline{\theta}\right)$. Thirdly since the lefthand side of (23) is continuous in p^e, strictly positive when $p^e = p^m$ and strictly negative when $p^e = p^*\left(\overline{\theta}\right)$, there exists at least one $p^e \in \left(p^m, p^*\left(\overline{\theta}\right)\right)$ such that (23) holds. ■

Proof of proposition 17. We only prove this for an increase in n on the assumption that all prices are unadvertised. If there are multiple equilibria, the one with the lowest p^e is again Pareto dominant (the proof is the same as with unit demand). Let p_0^e be the lowest equilibrium price when $n = n_0$, and $\tilde{\theta}_{0,0}$ be the lowest type that searches when $n = n_0$ and price p_0^e is expected. Now suppose that when $n = n_1$ but price p_0^e is *still* expected, the lowest type that searches is $\tilde{\theta}_{0,1}$.

Suppose that $n_1 > n_0$, in which case $\tilde{\theta}_{0,1} < \tilde{\theta}_{0,0}$. Adapting earlier arguments we know that a). $\tilde{\theta}_{0,0} > \underline{\theta}$ and b). $\Pi'\left(p_0^e;\theta\right) < 0$ for all $\theta < \tilde{\theta}_{0,0}$. Therefore the lefthand side of (23) when evaluated at $p^e = p_0^e$ *but* $n = n_1$ is

$$\int_{\tilde{\theta}_{0,1}}^{\overline{\theta}} g\left(\theta\right)\Pi'\left(p_0^e;\theta\right)d\theta < \int_{\tilde{\theta}_{0,0}}^{\overline{\theta}} g\left(\theta\right)\Pi'\left(p_0^e;\theta\right)d\theta = 0$$

i.e. strictly negative. At the same time (23) is still weakly positive when evaluated at $p^e = p^m$ and $n = n_1$. Therefore (by continuity) the lowest solution to (23) when $n = n_1$, must be strictly lower than the lowest solution when $n = n_0$.

The proof for comparative statics in s is very similar, as is the proof of comparative statics in any advertised price (except that the thresholds $\underline{s}'$ and $\overline{s}'$ are defined slightly differently to reflect the fact that not all prices are unadvertised). ■

B.2 Proof for section 4.2 (second extension)

Consider part (a). A non-advertising firm only sells when its competitor also doesn't advertise - and then splits the market equally and earns $\pi^u/2$. So profit from not advertising is:

$$\frac{\pi^u}{2}\frac{c_a}{\pi^a - \pi^u/2} \tag{24}$$

Advertising a price p^*, earns $-c_a$ if the competitor advertises and $\pi^a - c_a$ otherwise, or in total

$$\pi^a \frac{c_a}{\pi^a - \pi^u/2} - c_a = \frac{\pi^u}{2} \frac{c_a}{\pi^a - \pi^u/2} \tag{25}$$

Advertising some $p_2^a \in [\underline{p}, p^*]$ gives profit $-c_a$ if the competitor advertises a lower price, and $\pi^a - c_a$ otherwise. In total this amounts to

$$\left[1 - \frac{\pi^a - \pi^u/2 - c_a}{\pi^a - \pi^u/2} G\left(p_2^a\right)\right] \pi\left(p_2^a\right) - c_a = \frac{\pi^u}{2} \frac{c_a}{\pi^a - \pi^u/2} \tag{26}$$

Since (24), (25) and (26) are equal the firm is indifferent between not advertising or advertising any price in $[\underline{p}, p^*]$. Now check that a firm cannot benefit by posting some other advertised price. If a firm advertises $p_2^a < \underline{p}$, it earns less profit than if it advertised $p_2^a = \underline{p}$, because $\pi\left(p_2^a\right)$ is increasing in this region and in both cases the firm captures the entire market. If a firm advertises $p_2^a \in (p^*, p^u]$, it does better by advertising $p_2^a = p^*$ instead - it is searched (weakly) more often, and profit per customer $\pi\left(p_2^a\right)$ decreases in this region. Finally advertising $p_2^a > p^u$ yields zero profit since the firm is never searched. The argument behind part (b) was already sketched in the text[21], whilst consumers' beliefs in part (c) are consistent on the equilibrium path.

[21] One small point which we omitted in the text is the following. Suppose a firm has advertised p_2^a and this is the lowest price. If consumers expected a price $\phi\left(p_2^a\right)$ but the duopolist's actual price exceeded $\phi\left(p_2^a\right) + s$, some people might search (and eventually buy from) the competitor. Hence a duopolist has even less incentive than a monopolist does, to deviate from charging $\phi\left(p_2^a\right)$.