Uniform profit ratios

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Abstract

The equalization of profit rates as the outcome of free competition is one of the oldest tenets in theoretical economics. Being intuitively convincing its premises and implications, though, are not well defined. As Walras put it: 'To state a theory is one thing; to prove it is another.' First of all a consistent concept of profit is required. In the present paper the structural axiom set is taken as premise. Thereof the determinants of profit and the profit ratio follow. This makes it possible to definitively state the conditions for uniform profit ratios in a hierarchical market structure.

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Adam Smith's notion of the natural price is related to the existence of a single rate of profit on the capital invested in all sectors and entails the assertion that it is free competition that quite naturally produces this result, at least approximately and in the longer run (Vaggi, 2008, p. 2). Since Smith profit rate equalization is an integral part of theoretical economics. However, as Walras criticized, it was by no means clear what was meant by this intuitively convincing concept: 'the term *profit*, as they [the English economists] use it, signifies simultaneously *interest* on capital and *profit* of enterprise' (2010, p. 423, original emphasis).

To put an end to what Davidson much later lamented as endemic 'Babylonian incoherent babble' (Dow, 2005, p. 385), Walras endeavored to set the principle of free competition on firm foundations:

> To state a theory is one thing; to prove it is another. I know that in economics so-called proofs which are actually nothing more than gratuitous assertions are doled out and find acceptance again and again. And precisely for this reason, I submit that economics will not attain the status of a science until economists are compelled to demonstrate that which they have hitherto been content, in the main, merely to assert. (Walras, 2010, p. 427)

To prove an assertion means to demonstrate how it follows consistently from a small set of foundational 'hypotheses or axioms or postulates or assumptions or even principles' (Schumpeter, 1994, p. 15). General equilibrium theory rests on a set of *behavioral* axioms (Arrow and Hahn, 1991, p. v). Human behavior and proof, though, make a strange couple. The main thesis of the present paper is that human behavior does not yield to the axiomatic method, yet the axiomatization of the money economy's fundamental structure is feasible. The crucial point is not axiomatization per se but the real world content of axioms. Our objective is to make the implications of the *structural* axiom set with regard to uniform profit ratios explicit.

By choosing objective structural relationships as axioms behavioral hypotheses are not ruled out. On the contrary, the structural axiom set is open to *any* behavioral assumption and not restricted to the standard optimization calculus.

The case for structural axiomatization has been made at length elsewhere (2011a, 2011b, 2011c), thus we can proceed without further programmatic preliminaries. In the following the minimalistic formal frame that constitutes the pure consumption economy is set up in section 1. Then, in section 2, profit and the profit ratio as pivotal concepts for the analysis of the market system are derived from the axiom set. The distinction between profit and distributed profit is crucial for the analysis of the functioning of the money economy. Standard profit theory is known to be incoherent (Desai, 2008), hence a new conceptual approach is in order. In sections 3 to 5 the conditions for uniform profit ratios are formally established for two limiting cases, that is, for two firms in two different markets and for two firms in a single market.

1 Axioms

The first three structural axioms relate to income, production, and expenditures in a period of arbitrary length. For the remainder of this inquiry the period length is conveniently assumed to be the calendar year. Simplicity demands that we have at first one world economy, one firm, and one product. 'The economic system is made up of households and firms' (Arrow and Hahn, 1991, p. 3).

Total income of the household sector Y is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N.

$$Y = WL + DN \quad |\mathrm{t} \tag{1}$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |\mathrm{t} \tag{2}$$

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |\mathrm{t} \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment expenditures, no foreign trade, and no taxes or any other government activity.

2 The overall profit ratio

The business sector's financial profit Q_{fi} in period t is defined with (4) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditures C – and costs – here identical with wage income Y_W[1]:

$$Q_{fi} \equiv C - Y_W \equiv PX - WL \quad \Leftarrow \quad Y_W \equiv WL \quad |\mathrm{t} \tag{4}$$

For the business sector as a whole to make a profit consumption expenditures C have in the simplest case to be greater than wage income Y_W. So that profit comes into existence in the *pure consumption economy* the *household* sector must run a deficit at least in one period[2]. This in turn makes the inclusion of the financial sector

[1] Profits from changes in the value of financial and non-financial assets are excluded here to streamline the analysis. For the general case see (2011d, p. 4).

[2] It needs hardly emphasis that in the investment economy the process of profit generation appears more complex. This does not affect the essence of profit but simply removes the *formal* necessity that the households have to incur a deficit to get the economy going. This is then done by the investing business sector. It is not advisable, though, to tackle the complexities of the investment economy before the pure consumption economy is fully understood. For dissuasive examples of confusion generated by premature analytical complexity see (Schmitt and Greppi, 1996, pp. 352-356).

mandatory. An economic theory that does not include at least one bank that supports the concomitant credit expansion cannot capture the essential features of the market economy (2011d, p. 2).

From (4) and (1) follows for the relation of profit and distributed profit:

$$Q_{fi} \equiv C - Y + Y_D \quad \Leftarrow \quad Y_D \equiv DN \quad |t \tag{5}$$

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. To the definitions in (4) and (5) three structural ratios are added now. With (6) the expenditure ratio ρ_E, the sales ratio ρ_X, and the distributed profit ratio ρ_D is defined:

$$\rho_E \equiv \frac{C}{Y} \qquad \rho_X \equiv \frac{X}{O} \qquad \rho_D \equiv \frac{Y_D}{Y_W} \quad |t \tag{6}$$

From (5), the first axiom (1), and the definitions (6) results for total profits that it depends on the key ratios ρ_E and ρ_D and the absolute amount of income:

$$Q_{fi} \equiv \left(\rho_E - \frac{1}{1+\rho_D}\right) Y \quad |t \tag{7}$$

To get rid of all absolute magnitudes the profit ratio ρ_Q is defined with (8) and this gives a succinct summary of the *structural* interrelations of the profit ratio, the expenditure ratio, and the distributed profit ratio for the business sector as a whole:

$$\rho_Q \equiv \rho_E (1+\rho_D) - 1 \quad \Leftarrow \quad \rho_Q \equiv \frac{Q_{fi}}{WL} \quad |t \tag{8}$$

The overall profit ratio ρ_Q is positive if the expenditure ratio ρ_E is *>1* or the distributed profit ratio ρ_D is *>0*, or both[3]. In the pure consumption economy no profit rate exists because no capital exists. Therefore, the profit ratio is the general concept.

3 Two firms in two markets

The axioms and definitions have first to be differentiated for two firms (2011c, Appendix). For the relative prices of two products follows from (3) in combination with (2):

$$\frac{P_1}{P_2} = \frac{R_2}{R_1}\frac{L_2}{L_1}\frac{C_1}{C_2} \quad \text{if} \quad \rho_{X1} = 1; \rho_{X2} = 1 \quad |t \tag{9}$$

If the markets for both products are cleared the price ratio is inversely proportional to the ratio of productivities and the ratio of labor inputs and directly proportional to the ratio of the consumption expenditures for the two products. This implicates a soft budget constraint, that is, the sum of consumption expenditures C_1+C_2 needs in the general case not be equal to income *Y*.

[3] For the full implications of the profit definition see (2011a, pp. 16-17)

Relative prices depend according to (9) on the *objective* ratio of outputs, i.e. on supply, and on the *subjective* partitioning of consumption expenditures, i.e. on demand. The partitioning ρ_{E1}, ρ_{E2} can be taken as the realization of the consumers' optimal consumption plans (2011c, p. 6).

A straightforward result materializes as limiting case if the labor inputs of the two firms stand in the same proportion as the expenditures for both products and the markets are cleared:

$$\frac{P_1}{P_2} = \frac{R_2}{R_1} \quad \text{if} \quad \frac{L_1}{L_2} = \frac{C_1}{C_2} = \frac{\rho_{E1}}{\rho_{E2}} \quad \text{and if} \quad \rho_{X1} = 1; \rho_{X2} = 1 \quad |t \tag{10}$$

If labor input is allocated according to the consumers' preferences, which are revealed by their expenditure ratios, then relative prices are inversely proportional to the productivities in the two lines of production. Budget balancing, i.e. $\rho_{E1}+\rho_{E2}=1$, is not required. We refer to this configuration as *competitive structure*.

The first question is how profits are distributed between the two firms. The financial profit for each firm follows from (4) and is given by:

$$\begin{aligned} Q_{fi1} &\equiv P_1X_1 - W_1L_1 \\ Q_{fi2} &\equiv P_2X_2 - W_2L_2 \quad |t \end{aligned} \tag{11}$$

Using (10) one gets for relative profits in the competitive structure:

$$\frac{Q_{fi1}}{Q_{fi2}} = \left(\frac{1 - \dfrac{W_1}{P_1R_1}}{1 - \dfrac{W_2}{P_1R_1}} \right) \frac{C_1}{C_2} \quad \text{if} \quad \rho_{X1} = 1; \rho_{X2} = 1 \quad |t \tag{12}$$

If the wage rates in the different lines of production are equal the numerical value in the brackets is one and the ratio of profits is equal to the ratio of consumption expenditures for the two products.

Equation (12) presupposes that the wage rate for all employees, i.e. inclusive management and executives, is equal. This is normally not so; hence W has to be taken as average wage rate that is given by:

$$W_1 \equiv W_{11}\frac{L_{11}}{L_1} + \ldots + W_{1i}\frac{L_{1i}}{L_1} + \ldots + W_{1n}\frac{L_{1n}}{L_1} \quad |t \tag{13}$$

From the purely formal standpoint it suffices that the *average* wage rates W_1 and W_2 are equal. It is obvious, however, that the differentiation of wage rates within a firm affects the partitioning of consumption expenditures if the individual expenditure ratios of different employees are different. To keep things simple, this interdependency between the distribution of wage rates within each firm and the partitioning of consumption expenditures between the two firms is ruled out with the assumption that the average expenditure ratios ρ_{E1}, ρ_{E2} are, for the time being, independent of the distribution of wages within the firms or, what amounts to the same, that they are equal for the firm's employees. Changes in the partitioning of consumption expenditures can be treated separately and then combined with distributional changes.

For the comparison of firms of different size and with different absolute profits the respective profit ratios are required. The profit ratio of the business sector as a whole (8) has been directly derived from the profit definition (4) and is adapted for a single firm as follows:

$$\rho_{Q1} \equiv \frac{Q_{fi1}}{W_1 L_1} \equiv \frac{P_1 R_1}{W_1} - 1 \quad \text{if} \quad \rho_{X1} = 1 \quad |t \tag{14}$$

Combining (11) and (10) one gets for relative profit ratios:

$$\frac{\rho_{Q1}}{\rho_{Q2}} = \frac{\frac{P_1 R_1}{W_1} - 1}{\frac{P_2 R_2}{W_2} - 1} = \frac{\frac{P_1 R_1}{W_1} - 1}{\frac{P_1 R_1}{W_2} - 1} \quad \text{if} \quad \rho_{X1} = 1; \rho_{X2} = 1 \quad |t \tag{15}$$

If the average wage rates are equal the profitability of both firms is equal. There is, though, no such thing as a “law” of uniform profitability because there is nothing in the formalism that equalizes the wage rates between the two lines of production. For the classics (Mill, 2006, p. 472) and even more so for Walras (Morishima, 1977, pp. 82-83) profit equalization was self-evident. Walras gave a vivid *description* of the spontaneous working of free competition that ends with – equal – zero profits: ‘les entrepreneurs ne font ni bénéfice ne perte’ (Walras, 2010, p. 225). Adam Smith maintained that the equalization of profits rates would fall into one with the – positive – rate of interest (Smith, 2008, p. 90). In contrast to these speculations[4] the structural axiomatic approach spells out the *formal conditions* without regress to behavioral assumptions or the invocation of free competition.

If an equalizing mechanism exists it has to be separately identified and consistently combined with the axiom set. The equality of wage rates and profitability between the two firms is an additional *formal* property of the competitive production structure and not a law-like necessity of the economic system. Whether profit ratios are uniform or not does not affect relative prices as given by (9). For the competitive structure therefore follows that relative prices are *independent* of the equalization of profit ratios.

The structural precondition of a positive profit ratio for the economy as a whole is given with (8). If the expenditure ratio ρ_E is unity and the distributed profit ratio ρ_D is zero then the profit ratio for the business sector as a whole is zero. If wage rates are not equal in this zero-profit economy the profit of one firm is equal to the loss of the other and this is not a reproducible configuration in the longer run.

[4] “All attempts to describe formally in a consistent and verisimilar fashion how an economy adjusts from one uniform profit-rate regime to another have failed. Until such an adjustment process can be reasonably modeled, its outcome – an equalized profit rate – is an article of faith, not a scientific result” (Naples, 1988, p. 84). In (Debreu, 1959) and (Arrow and Hahn, 1991) uniform profit rates are not explicitly referred to.

When in the simplest case market clearing, i.e. $\rho_X=1$, and budget balancing, i.e. $\rho_E=1$, is assumed, the only subjectively chosen variable is the expenditure ratio for *one* product. The rest of the system is then determined by the conditions for the competitive structure. If, in addition, the average wage rates and, by consequence, the profit ratios in both firms are equal, no improvements are possible, neither for the wage earners nor for the owners of the firms.

4 Two firms in one market

When two firms operate in one market they face the same price P. Accordingly (11) changes to:

$$\begin{aligned} Q_{fi11} &\equiv P_1 X_{11} - W_{11} L_{11} \\ Q_{fi12} &\equiv P_1 X_{12} - W_{12} L_{12} \quad |t \end{aligned} \tag{16}$$

The price as dependent variable follows from the axiom set and the definitions in the general form as:

$$P = \frac{\rho_E}{\rho_X}(1+\rho_D)\frac{W}{R} \quad |t \tag{17}$$

Under the condition of market clearing, i.e. $\rho_X=1$, and budget balancing, i.e. $\rho_E=1$, the price depends alone on the distributed profit ratio and unit wage costs:

$$P = (1+\rho_D)\frac{W}{R} \quad \text{if} \quad \rho_X = 1; \rho_E = 1 \quad |t \tag{18}$$

The price comes down to unit wage costs if distributed profits are zero. This is the familiar zero profit equilibrium case (Walras, 2010, p. 225). It has to be emphasized that this case does not come about by competition but by the absence of distributed profits or, more precisely by $\rho_E=1$ and $\rho_D=0$ in combination. The assertion that competition, in the long run, brings prices down to costs and washes profits away is not well founded. Competition has nothing to do with *overall* profits. To conclude from the undeniable empirical fact that competition exerts a pressure on *individual* profits that this must hold also for overall profits is a quite ordinary fallacy of composition.

When, finally, the wage rate is taken as numéraire in (18), i.e. *W= 1 [EUR/h]* (cf. Walras, 2010, p. 189), the absolute price for the one-product economy is given as the inverse of productivity. This corresponds to (10) for relative prices. When, on the other hand, the price is taken as numéraire, i.e. *P= 1 [EUR/unit]*, then the wage rate is numerically (though not dimensionally) equal to the productivity. This entails in general terms that, in order to achieve *absolute* price stability at $P=1$, wage rate and productivity have to move *exactly* in tandem.

Since wage rates differ between firms the wage rate in (17) is the *average* wage rate that is given by:

$$W \equiv W_1 \frac{L_1}{L} + W_2 \frac{L_2}{L} \quad |t \tag{19}$$

The shares of total employment of the firms are defined as:

$$L \equiv L_1 + L_2 \quad \Rightarrow \rho_{L1} \equiv 1 - \rho_{L2} \quad \Leftarrow \quad \rho_{L1} \equiv \frac{L_1}{L}; \rho_{L2} \equiv \frac{L_2}{L} \quad |t \tag{20}$$

The wage ratio is defined as:

$$\rho_W \equiv \frac{W_1}{W_2} \quad |t \tag{21}$$

This gives for the average wage rate:

$$W \equiv W_2\, \rho_W \left(1 - \rho_{L2}\right) + W_2 \rho_{L2} \equiv W_2 \left(\rho_W \left(1 - \rho_{L2}\right) + \rho_{L2}\right) \quad |t \tag{22}$$

Since productivities differ between firms the productivity in (17) is the *average* productivity that is given by:

$$R \equiv R_1 \frac{L_1}{L} + R_2 \frac{L_2}{L} \quad |t \tag{23}$$

The productivity ratio is defined as:

$$\rho_R \equiv \frac{R_1}{R_2} \quad |t \tag{24}$$

This gives for the average productivity:

$$R \equiv R_2\, \rho_R \left(1 - \rho_{L2}\right) + R_2 \rho_{L2} \equiv R_2 \left(\rho_R \left(1 - \rho_{L2}\right) + \rho_{L2}\right) \quad |t \tag{25}$$

For the general case the average unit wage costs that have to be substituted in (17) are then given by:

$$\frac{W}{R} \equiv \frac{W_2 \left(\rho_W \left(1 - \rho_{L2}\right) + \rho_{L2}\right)}{R_2 \left(\rho_R \left(1 - \rho_{L2}\right) + \rho_{L2}\right)} \quad |t \tag{26}$$

Profit of firm_1 (16) can now be rewritten as:

$$Q_{fi1} \equiv P R_1 L_1 - W_1 L_1 \equiv W_1 L_1 \left(\frac{P R_1}{W_1} - 1\right) \quad \text{if} \quad \rho_{X1} = 1 \quad |t \tag{27}$$

After the substitution of (17) and some reshuffling that is carried out in detail in the Appendix this yields:

$$Q_{fi1} \equiv W_1 L_1 \left(\rho_E \left(1 + \rho_D\right) \frac{1 - \rho_{L2} + \dfrac{\rho_{L2}}{\rho_W}}{1 - \rho_{L2} + \dfrac{\rho_{L2}}{\rho_R}} - 1\right) \quad \text{if} \quad \rho_{X1} = 1 \quad |t \tag{28}$$

Thereof finally follows the profit ratio of firm_1:

$$\rho_{Q1} \equiv \rho_E \left(1+\rho_D\right) \underbrace{\frac{1-\rho_{L2}+\dfrac{\rho_{L2}}{\rho_W}}{1-\rho_{L2}+\dfrac{\rho_{L2}}{\rho_R}}}_{1 \Rightarrow equalization} - 1 \quad \text{if} \quad \rho_{X1}=1 \quad |t \tag{29}$$

In the ideal case $\rho_W = \rho_R$ the profit ratio of firm$_1$ is equal to the overall profit ratio (8).

For the special case $L_1 = L_2$, i.e. both firm are of equal size, and $\rho_E = 1$, $\rho_D = 0$, i.e. the overall profit ratio is zero, the profit ratio of firm$_1$ depends alone on the wage ratio and the productivity ratio:

$$\rho_{Q1} \equiv \frac{1+\dfrac{1}{\rho_W}}{1+\dfrac{1}{\rho_R}} - 1 \quad |t \tag{30}$$

The profit ratio of firm$_1$ is zero if the wage ratio is equal to the productivity ratio, otherwise firm$_1$ makes a profit or a loss and thereby deviates from the overall profit ratio. It is worth emphasizing that the profitability does not only depend on the firm's own wage rate W_1 and productivity R_1 but also directly on the respective variables of the *other* firm. A wage increase in the other firm affects the profit ratio of firm$_1$ positively and a productivity increase negatively. This interdependency is a significant difference to standard partial analysis that fends off mutual repercussions with *cet.par.*

In the same manner the profit ratio of firm$_2$ is now derived from (16).

$$Q_{fi2} \equiv PR_2L_2 - W_2L_2 \equiv W_2L_2 \left(\frac{PR_2}{W_2} - 1\right) \quad \text{if} \quad \rho_{X2}=1 \quad |t \tag{31}$$

After the substitution of (17) and (26)

$$\equiv W_2L_2 \left(\rho_E \left(1+\rho_D\right) \frac{W_2 \left(\rho_W \left(1-\rho_{L2}\right)+\rho_{L2}\right)}{R_2 \left(\rho_R \left(1-\rho_{L2}\right)+\rho_{L2}\right)} \frac{R_2}{W_2} - 1\right) \tag{32}$$

and some reshuffling this yields:

$$Q_{fi2} \equiv W_2L_2 \left(\rho_E \left(1+\rho_D\right) \frac{\rho_W}{\rho_R} \frac{1-\rho_{L2}+\dfrac{\rho_{L2}}{\rho_W}}{1-\rho_{L2}+\dfrac{\rho_{L2}}{\rho_R}} - 1\right) \quad \text{if} \quad \rho_{X2}=1 \quad |t \tag{33}$$

Thereof finally follows the profit ratio of firm$_2$:

$$\rho_{Q2} \equiv \rho_E \left(1+\rho_D\right) \underbrace{\frac{\rho_W}{\rho_R} \frac{1-\rho_{L2}+\dfrac{\rho_{L2}}{\rho_W}}{1-\rho_{L2}+\dfrac{\rho_{L2}}{\rho_R}}}_{1 \Rightarrow equalization} - 1 \quad \text{if} \quad \rho_{X2}=1 \quad |t \tag{34}$$

In the ideal case $\rho_W=\rho_R$ the profit ratio of firm_2 is equal to the overall profit ratio (8).

For the special case $L_1=L_2$ and $\rho_E=1$; $\rho_D=0$ the profit ratio of firm_2 depends alone on the wage ratio and the productivity ratio:

$$\rho_{Q2} \equiv \frac{\rho_W}{\rho_R}\frac{1+\dfrac{1}{\rho_W}}{1+\dfrac{1}{\rho_R}} - 1 \quad |t \tag{35}$$

This is the complementary to (30). If firm_1 makes a profit firm_2 makes a loss and vice versa. The profit ratio of one firm depends on what happens in the other firm.

5 Conditions of profit ratio equalization

In section 3 the pure consumption economy has first been divided into two different product lines or industries as summarized in (36). Under the conditions of the competitive structure the equalization of profit ratios between the two representative firms requires that the average wage rates are equal. Total employment is allocated according to the ratio ρ_{E1}/ρ_{E2}.

$$\begin{array}{ccccccc} & & & \rho_E & & & \\ & & \swarrow & & \searrow & & \\ & \rho_{E1} & & & & \rho_{E2} & \\ \swarrow & & \searrow & & \swarrow & & \searrow \\ \rho_{E11} & & \rho_{E12} & & \rho_{E21} & & \rho_{E22} \end{array} \tag{36}$$

Now it is supposed that the productivity in industry_2 increases. Relative prices change according to (10). Let us assume at first that P_2 falls. This has no effect on the *relation* of profit ratios according to (15) as long as average wage rates in the two industries remain unchanged. The price in industry_2 declines with rising productivity such that its *own* profit ratio remains unaltered.

We have, though, a sinking price level when P_2 declines and P_1 remains unchanged. This is undesirable when we assign the price the role of the numéraire (2011b, p. 17). Hence an increase of the average wage rate in both industries is in order such that P_1 rises and P_2 falls to a lesser degree in accordance with (10). This adaptation of relative prices leaves the price level steady. The productivity gain in the second production line is evenly spread among all wage earners by the increase of the average wage rate which is equal in both industries.

The analytically separate cases of sections 3 and 4 are now integrated into a hierarchy as shown in (36). In the first production line firm_{11} and firm_{12} face the same market price. Since their production conditions are normally non-identical their profit ratios are different as long as the productivity differential is not compensated for by a wage differential according to (29) and (34).

It is supposed now that, starting from equal productivities, the productivity in firm_{11} improves. The adaptation of the wage rates to the productivity differential has to take place such that the wage rate in the less productive firm falls and in the more productive firm rises. The adaptation must leave the average wage rate of the first production line unchanged and equal to the wage rate in the second production line. The relation of W_{11} and W_{12} is determined by:

$$W_1 \equiv W_{11}\frac{L_{11}}{L_1} + W_{12}\frac{L_{12}}{L_1} = W_2 \quad |t \tag{37}$$

If the average wage rate in industry_1 remains constant and the average productivity R_1 increases then the market clearing price has to fall:

$$P_1 = \rho_{E1}\left(1+\rho_D\right)\frac{WL_1}{R_1 L} \quad \text{if} \quad W_1 = W_2 = W;\, \rho_{X1} = 1 \quad |t \tag{38}$$

This, again, calls for an increase of the average wage rates in *both* industries to keep the price level steady.

If the profit ratios are uniform the allocation of the labor input between the two firms plays no role for the firms provided they are geared *exclusively* to the profit ratio and not to absolute profit. In this case the firms can be indifferent with regard to employment because the profit ratio is independent of it. Hence the distribution of labor input between the firms is indeterminate. Of importance is only that the total employment in the first production line remains in proportion ρ_{E1}/ρ_{E2} to that of the second according to (10). While this proportion is determined by the households' preferences the relation ρ_{E11}/ρ_{E12} depends on the allocation of labor input between the two firms. If not only the profit ratio but also absolute profit figures in the target function of both firms this gives rise to competition about market shares and the behavioral situation is unstable. It becomes stable not before one firm is left over as monopolist. Hence the equalization of profit ratios does *not* suspend competition and does *not* put the system behaviorally at rest. The tendency towards monopolization is still at work.

The equalization of wage ratio and productivity ratio is behaviorally stable only under the condition that the employees in the firm with the lower productivity accept that their average wage rate is lower compared to the average wage rate of the firm with the higher productivity. As soon as employees start to migrate the high productivity firm grows and the low productivity firm shrinks. Behavioral stability of employees presupposes the equalization of productivities and wage rates, that means, both firms have to become identical. Yet even in this limiting case competition about market shares continues.

From the outside perspective the voluntary migration that is induced by wage differentials is preferable to the situation with equal wage rates in both firms because the latter configuration moves the less productive firm closer to the brink of bankruptcy. Bankruptcy and the temporary (in the best case) unemployment that comes along with it is an inefficient way to carry through the migration and to arrive ultimately at the same destination.

6 Conclusion

The equalization of profit ratios in the competitive structure requires an equal average wage rate between two industries and a wage ratio that is equal to the productivity ratio between two firm in the same market. Uniform profit ratios, though, are not necessarily conductive to a behaviorally stable state of economic affairs.

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Appendix

$$Q_{fi1} \equiv W_1 L_1 \left(\rho_E \left(1+\rho_D\right) \frac{W_2 \left(\rho_W \left(1-\rho_{L2}\right)+\rho_{L2}\right)}{R_2 \left(\rho_R \left(1-\rho_{L2}\right)+\rho_{L2}\right)} \frac{R_1}{W_1} - 1 \right) \tag{39}$$

$$\equiv W_1 L_1 \left(\rho_E \left(1+\rho_D\right) \frac{W_2 \left(\rho_W \left(1-\rho_{L2}\right)+\rho_{L2}\right)}{R_2 \left(\rho_R \left(1-\rho_{L2}\right)+\rho_{L2}\right)} \frac{\rho_R R_2}{\rho_W W_2} - 1 \right) \tag{40}$$

$$\equiv W_1 L_1 \left(\rho_E \left(1+\rho_D\right) \frac{\rho_R}{\rho_W} \frac{\left(\rho_W \left(1-\rho_{L2}\right)+\rho_{L2}\right)}{\left(\rho_R \left(1-\rho_{L2}\right)+\rho_{L2}\right)} - 1 \right) \tag{41}$$

$$Q_{fi1} \equiv W_1 L_1 \left(\rho_E \left(1+\rho_D\right) \frac{1-\rho_{L2}+\dfrac{\rho_{L2}}{\rho_W}}{1-\rho_{L2}+\dfrac{\rho_{L2}}{\rho_R}} - 1 \right) \tag{42}$$

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