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Nathan Larson

University of Virginia

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Nathan Larson
Department of Economics
University of Virginia
Charlottesville, VA 22904
larson@virginia.edu

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Abstract

We study the incentives to acquire information from exclusive news sources versus information from popular sources in a CARA-normal asset market. Each trader is able to observe one of a finite number of news sources. Clustering on the most precise source can happen for two reasons. One is standard: traders do not care that they dilute others' profits by trading on the same information. The other reason is more novel: traders with different information sets may respond to the same news differently – when this is so, they can benefit by coordinating their attention on the same news source in order to take opposite sides of the market. News from such a source will generate abnormal volume that need not be accompanied by large price movement. Furthermore, we show that as the number of sources grows, traders concentrate their attention on a few of the best ones, leaving most information unexploited.

1 Introduction

We study a single period asset market in which there are a limited number of "news sources," each of which provides a signal (some more precise than others) about the asset value. Each agent can monitor one of these news sources, and thus observe its signal, prior to trading. In equilibrium, some news sources will be relatively popular (monitored by many traders), while others are relatively exclusive, or ignored altogether. We ask whether traders ever have incentives to cluster on popular news sources, and if so, what implications this has for aggregate market outcomes such as the informativeness of the asset price and the volume of trade.

The type of news sources that we have in mind could be analysts, brokerages, investment newsletters, company insiders, columns in the Wall Street Journal, or the like. A signal from a news source could be a revision by the analyst, a new recommendation from the brokerage or newsletter, a leak by the insider, or new information in the newspaper. Our premise is that by dedicating time and attention to monitoring a source, a trader can obtain, digest, and use new information from that source before it becomes widely known (and perfectly incorporated into the market price). The idea that time and attention are limited is captured, in a stylized way, by restricting each trader to monitoring a single news source.[1] Our market should be understood to clear at this early stage of dissemination, when only the traders who have been monitoring a news source are able to act on its new signal.

Because the model has many traders and few news sources, we will usually speak of relatively exclusive sources – that is, followed by relatively few traders – rather than private ones. There are standard reasons to expect traders to prefer more exclusive sources: all else equal, popular news should be more fully incorporated into the asset price, eroding the profits of those who try to trade on it. However, at times, casual observation seems to suggest that many traders pay attention to the same news sources. For example, a recommendation from a high profile analyst can generate dramatic movement in a stock's price and turnover. Furthermore, one sometimes hears a countervailing argument that "it is important to understand what other people know." To evaluate these arguments, we deem traders to be clustering on a news source when their actions produce an excessive impact on the asset price, trading volume, or both. Our notion of "excessive" accounts for the fact that more informative sources (those with more precise signals) *should* move the price more than less informative ones.

In equilibrium, more precise news sources are always more popular, and clustering can occur through two channels. In the first, traders who dislike popularity *per se* (for the standard reasons) accept it as the cost of acquiring a more precise signal. High quality, popular news becomes excessively incorporated into the price (relative to its precision, and the precision of other signals), essentially because individuals trading on this news are not concerned with how their actions collectively affect the informativeness of the market price. Lower quality news sources may be ignored entirely – indeed, we show that as the number of news sources grows, traders become so concentrated on the best ones that the fraction of sources that are ignored goes to one. Consequently, the price can be quite inefficient.

The second channel for clustering involves traders who, due to differences in their information sets, use the same news source in different and opposing ways. Specifically, we suppose that some traders must place market orders, and thus face price risk, while others place limit orders.[2] A market order trader is endowed with additional private information about the asset. To mitigate his price risk, he must try to assess whether this private information is good or bad news *relative to what the rest of the market knows*. One way to do this is to monitor a popular news source, as a proxy for the price, and trade *against* it. When the two types of trader follow the same news source, they have a symbiotic relationship. By selling on good news from the source, market order traders tend to reduce its correlation with the price, making it more attractive to limit order traders. Conversely, by buying on good news, limit order traders make the source a better proxy for the price, and thus more attractive to market order traders. This symbiosis can lead most traders to monitor the same news source. When this occurs, a new signal from the source will generate

[1] Allowing a trader to monitor multiple news sources would complicate the analysis, but should not change the results too substantially.

[2] Possible motivations for this setting will be discussed later.

moderately excessive price movement (because the countervailing trades cancel out) and a very large spike in trading volume. One implication of this is that two pieces of news that appear to be of roughly similar fundamental importance can generate drastically different amounts of volume.

Our asset market is in the tradition of Hellwig [22], Grossman and Stiglitz [18], and Diamond and Verrecchia [8], among others. There are risk averse (CARA) strategic traders and noise traders who provide liquidity, and signals are normally distributed. Orders are placed simultaneously, and the asset price is determined by a market clearing condition, as in Kyle [28]. As in the competitive rational expectations equilibrium literature, strategic traders form correct beliefs about the functional relationship between signals, orders, and the market price. However, we depart from that literature by assuming that only limit order traders can condition their demand on the realized price; market order traders cannot. Information acquisition in such markets has been extensively studied, but usually under the assumption that informed traders' signals are either perfectly correlated, as in Grossman and Stiglitz, or independent, as in Verrecchia [38].[3] With a few notable exceptions, there has been little study of whether agents permitted to choose *which* information to acquire will concentrate on the same information as other agents.

Market microstructure models that include both market and limit orders are a relatively recent development, and most of these models have focused on sequential trading with new orders clearing against an existing limit order book (or in some cases, against a market-maker's quotes as well). An early example with one stage of arrivals and fixed order types is Glosten [16], followed by Chakravarty and Holden [7] and Handa and Schwartz [20], where a trader is able to choose which type of order to use.[4] Goettler, Parlour, and Rajan [17] develop a fully dynamic model with limit and market orders which they solve numerically. These studies generally support the idea that market orders have the virtue of immediate execution but are exposed to price risk, while limit orders are exposed to execution risk (the chance that an order fails to execute in a timely way) and adverse selection risk (the danger that a buy order is more likely to execute if bad news about the asset arrives later, and *vice versa* for sell orders). Loosely, we would expect a trader who is impatient about execution to favor a market order; one reason for this impatience could be that he has private information that is particularly time sensitive. (And conversely for a relatively patient trader.) In our model, these trade-offs between order types will remain in the background, as unmodeled motivation for an exogenous distribution of limit and market order traders.[5]

Outside of our focus on why traders might choose to acquire the same signals, there are other ways in which financial actors may make similar decisions about information.[6] Grundy and Mc-

[3] Manzano and Vives [30], which has traders with partially correlated signals, is one recent exception.

[4] Kyle [28] permits traders to choose from an extremely flexible class of demand schedules that includes both market orders and our linear limit orders as special cases. However, in his setting, market orders are never optimal, in part because there is no execution risk for limit orders.

[5] Admittedly, there is some awkwardness in using these expressly dynamic trade-offs to motivate the order types in a static game. We discuss ways to tighten this motivation a bit later, but ultimately it would be desirable to make order choice endogenous.

[6] There are also substantial theoretical and empirical literatures examining herding in analyst and newsletter recommendations. This herding by information *providers* is orthogonal to the herding by information *acquirers* that we study. Throughout the paper we stick to the term "clustering" to avoid confusion with this other herding literature.

Nichols [19] and Manzano and Vives [30], among others, study markets in which traders' decisions about how much private information to acquire, or how aggressively to trade on that information, are strategic complements. Generally, the complementarity arises from a feedback loop between individual actions and the informativeness of the market price, and as a consequence, there may be multiple equilibria.

More closely related to us are models that allow traders to choose which information to acquire. In an influential early paper, Admati and Pfleiderer [2] let traders acquire collections of signals and provide conditions under which two different signals are complements or substitutes to each other, for a single trader. They show that the property of complementarity can be endogenous, as the standalone and combined values of two signals depend on equilibrium properties of the price. Complementary signals encourage all-or-nothing information acquisition, which generates a different sort of information concentration than the type we study: traders are either well informed or uninformed, but not moderately informed. More recently, García and Vanden [15] study the emergence of endogenous "mutual funds" when one trader can choose to buy a stake in the position taken by another trader. In a multiple asset model, Van Nieuwerburgh and Veldkamp [36] show that a trader may prefer to deepen his informational advantage on one asset rather than learn what his competition knows about the other asset; they use this to shed light on the home bias puzzle.

The idea that traders might rationally choose to acquire the same information dates to Froot, Scharfstein, and Stein [11]. Their mechanism relies traders with short term horizons and a random trade timing assumption that ensures that popular information is incorporated into the price gradually. A trader who acquires popular information can benefit from this price trend if he is fortunate enough to trade early. This illustrates how a strategic complementarity can arise in the news that traders choose to follow; in our interaction between limit and market orders, the complementarity arises from a different source, but the result is similar. In other papers the desirability of acquiring popular information arises because traders have "keeping up with the Joneses" preferences (García and Strobl [13]) or because spreading fixed costs of research across more traders makes popular information more affordable (Veldkamp [37]). In related work, Hellwig and Veldkamp [21] consider general environments in which agents acquire information prior to playing a game; they show that when the second-stage game has strategic complementarities, agents have an incentive to herd at the information acquisition stage. One key to clustering in our model is that it can be rational for differentially informed traders (in the sense that limit order traders "observe" the price, but market order traders do not) to trade on opposite sides of the same piece of new information. Based on similar logic, Dorn and Strobl [9] demonstrate how informed and uninformed traders may trade on opposite sides of new public information in a model of the disposition effect.

An advantage of using a large, static market is that we can precisely characterize how clustering on news affects the informational efficiency of the market price. In particular, like Froot, Scharfstein, and Stein, we can distinguish inefficient aggregation of the news sources that traders follow from the inefficiency that arises because some sources are simply ignored altogether. Under certain conditions, if the *potential* collective precision of all available news sources grows as N, then the number of acquired news sources, and the collective precision of *acquired* news grow no faster than $\sqrt{N}$. These results complement a body of of work on the asymptotic informativeness of prices that includes Vives [39] and [40] and García and Urošević [14].

Our model yields several novel (and potentially testable) implications. With or without market order traders, the asset price overweights information from higher quality (more precise) news sources. Under the assumption that news will eventually be priced in correctly, this suggests that the price impact of popular, high quality news will be more subject to reversals. With market order traders, the model predicts that news from a prominent source (the most informative one with the largest following) may generate trading volume that appears drastically disproportionate to its informativeness. Existing empirical studies do not address these predictions directly, but there is some suggestive indirect evidence that we will discuss after presenting the model and results.

The next section introduces the model. Section 3 characterizes equilibrium when there are only limit order traders and describes the conditions under which traders' attention is focused on a small subset of news sources. In Section 4, market order traders are added, and we show that the two order types may trade on opposite sides of the same news. Section 5 discusses how the results relate to empirical evidence, and Section 6 offers concluding remarks.

2 Model

The model has two stages; in the first stage strategic traders choose which information to acquire, and in the second stage they trade and realize payoffs. We begin with the second stage, which is conventional. There is a single asset with uncertain value θ which is traded in a single period. The asset is traded by continuum of strategic traders of size $L+M$; of these, a mass L are limit order traders, while M are market order traders. There are also noise traders who submit aggregate demand $z \sim N(0, s_z)$. All strategic traders have CARA preferences over trading profits, with initial wealth normalized to zero; if trader i buys net quantity q_i at price p, then his *ex post* realized utility is $u_i(q_i) = -e^{-\frac{1}{\gamma_i} q_i (\theta - p)}$, where γ_i is his risk tolerance. We assume that all limit (market) order traders have common risk tolerance γ_L (γ_M). A trader chooses an order strategy $q_i(\mathcal{I}_i)$ to maximize his expected utility $E(u_i(q_i) \mid \mathcal{I}_i)$, where $\mathcal{I}_i$ is his information set. The main distinction between trader types is that only the information sets of limit order traders will include the market price p, allowing them to condition their demand on it. The sequence of events is: (i) traders simultaneously submit order strategies and noise trader demand is realized, (ii) the equilibrium price p is determined by market-clearing, and (iii) trades are executed, θ is revealed, and traders realize their payoffs. If we index limit and market order traders by $l \in [0, L]$ and $m \in [0, M]$, the market-clearing condition is:

$$\int_0^L q_l \, dl + \int_0^M q_m \, dm + z \equiv 0 \tag{1}$$

The determination of the price follows from the fact that limit orders q_l will condition on p.

Next we turn to the information available to traders. First, there is a public signal $w = \theta + \varepsilon_w$, with $\varepsilon_w \sim N(0, s_w)$, that is observed by all traders; w could be considered a common prior about θ. Next, there is private information: each market order trader m is endowed with an idiosyncratic signal x_m distributed $N\left(\theta, \tau_x^{-1}\right)$. For simplicity, limit order traders are not endowed with private signals, but this is not critical. Finally, there is a finite set of N news sources, $\mathcal{Y} = \{1, 2, ..., N\}$. Each source n provides a common signal $y_n \sim N(\theta, \tau_n^{-1})$ to its subscribers. Without loss of generality, we rank the sources by precision: $0 < \tau_1 \leq \tau_2 \leq ... \leq \tau_{N-1} < \tau_N$.[7] All of the signals in the model are independent, conditional on θ, and we make the usual convention that the mean of a continuum of i.i.d. random variables (in this case, the market order traders' private signals) is equal to its expectation.

At the information acquisition stage, each trader must choose one news source $n \in \mathcal{Y}$ to subscribe to. A trader cannot see the content of the news – that is, the value y_n – before choosing, but their precisions are public. After traders simultaneously choose sources, each trader observes the realizations of the signals in his information set. For a limit order (market order) trader who acquires news source n, that information set is $\mathcal{I}_l = \{y_n, w, p\}$ ($\mathcal{I}_m = \{y_n, w, x_m\}$). While we will not attempt to endogenize order types in this paper, one could imagine these information sets arising from a trade-off in which agents with additional, time-sensitive private information x_m choose market orders to avoid delays in execution. Of course, since our model is static and all orders execute simultaneously, one must take such a motivation rather figuratively.

By imposing a capacity limit on information acquisition rather than a cost or a price, we imagine a situation in which the time and attention required to acquire, understand, and use a new signal before its value expires represents an important constraint on traders. There are no explicit financial costs to acquire a source, nor do news sources charge prices for their information. News sources are not strategic players in the model, but we have in mind a story in which free information is a loss leader that helps to bring in other types of business for the source. Such a strategy might make sense given the difficulty of limiting access to raw information, and it does not seem too distant from the approach that brokerages and investment banks take with their analysts. In Section 6 we briefly discuss how our model could be extended to incorporate price competition among news sources.

Our equilibrium concept is essentially competitive rational expectations equilibrium, but with the addition of the information acquisition stage and also the proviso that market order traders cannot condition on the realized price. To be more explicit about this, note that any profile of news source acquisition and order strategies by traders will induce some price function P over the variables $(\vec{y}, w, z, \theta)$, where $\vec{y} = (y_1, ..., y_N)$, such that the realized equilibrium price is given by $p = P(\vec{y}, w, z, \theta)$.[8] We assume that all traders correctly anticipate this functional relationship (and that limit order traders additionally observe the realization p). Let $\vec{l} = (l_1, l_2, ..., l_N)$ and $\vec{m} = (m_1, m_2, ..., m_N)$ denote the fractions of limit and market order traders choosing each news

[7] Keeping the most precise source unique (*via* the condition $\tau_N > \tau_{N-1}$) is not essential to the results, but it helps in stating some of them concisely.

[8] We omit the x_m signals because equilibrium price will depend only on the aggregate of these signals, and we have assumed $\frac{1}{L}\int_0^L x_m = \theta$.

source, and let functions $q_l^n()$ and $q_m^n()$ denote the order strategy a trader of each type who acquired news source n, with $\vec{q}_l$ and $\vec{q}_m$ the vectors of these functions. Finally, let $U_l^n = E\left(E\left(u_l\left(q_l^n\right) | \mathcal{I}_l\right)\right)$ refer to the *ex ante* expected utility anticipated by a limit order trader who acquires source n, before receiving his information. Define U_m^n similarly.

We say that order strategies are *interim optimal* (with respect to an anticipated price function P) if for all n, $q_l^n(\mathcal{I}_l)$ maximizes $E\left(u_l\left(q_l^n\right) | \mathcal{I}_l\right)$ and $q_m^n(\mathcal{I}_m)$ maximizes $E\left(u_m\left(q_m^n\right) | \mathcal{I}_m\right)$. We will say that news source choices are *ex ante optimal* if for any n such that $l_n > 0$, we have $U_l^n \geq U_l^{n'}$ for all $n' \in \mathcal{Y}$, and for any n such that $m_n > 0$, we have $U_m^n \geq U_m^{n'}$ for all $n' \in \mathcal{Y}$.

Definition 1 *A News Choice Equilibrium is a collection* $\left\{\vec{l}, \vec{m}, \vec{q}_l, \vec{q}_m, P\right\}$ *such that (i) the market-clearing price generated by* $\left\{\vec{l}, \vec{m}, \vec{q}_l, \vec{q}_m\right\}$ *satisfies* $p = P(\vec{y}, w, z, \theta)$, *(ii) order strategies are interim optimal with respect to* P, *and (iii) news source choices are ex ante optimal with respect to* P.

Because traders are free to acquire any news sources they like, optimality requires that all sources acquired by limit order traders must offer them the same *ex ante* utility, and similarly for sources acquired by market order traders. Throughout the paper, we will follow common practice by restricting attention to equilibria in linear order strategies; that is, we will look for equilibria in which q_l^n and q_m^n are linear functions of the variables in $\mathcal{I}_l$ and $\mathcal{I}_m$. In this case, the pricing function will take the form

$$P(\vec{y}, w, z, \theta) = \sum_{n=1}^{N} \lambda_n y_n + \lambda_w w + \lambda_\theta \theta + \rho z \tag{2}$$

for some coefficients $\vec{\lambda} = (\lambda_1, ..., \lambda_N)$, λ_w, λ_θ, and ρ. When this is the case, correct expectations about the form of P reduce to anticipating these coefficients correctly. In the next section, linear equilibria are characterized for the somewhat simpler case in which all traders use limit orders ($M = 0$).

We will consider two definitions of clustering over news sources. The second one, which is based on excessive impact on trading volume, is deferred until later. The first definition is based on a source's price impact. In a linear equilibrium, the news sources' contribution to the price can be summarized by the normalized variable $Y = \sum_{n=1}^{N} \lambda_n' y_n$, where $\lambda_n' = \lambda_n / \sum_{n=1}^{N} \lambda_n$. Y is most informative about the asset value θ when each source has a price impact proportional to its precision; that is, when λ_n / τ_n is constant across n. When this condition fails, we will say that there is clustering on the news with higher values of λ_n / τ_n. An extreme version of this occurs if some news sources are not acquired at all and have price impacts of zero. Since this definition is rather inclusive, it is not too surprising that clustering occurs in equilibrium; what will be more interesting is the size and scope of these excessive price impacts.

2.1 Preliminaries: Orders and *Ex Ante* Utility with CARA Preferences

For now, we fix a price function P, and consider the behavior of traders. Notice that the price function in (2) is unbiased – the unconditional expectation $E(\theta - p)$ of per unit profit is zero.

Furthermore, the per-unit profit $\theta - p$ is distributed normally: $\theta - p \sim N(0, s_p)$, where

$$s_p = E\left((p-\theta)^2\right) = \sum\nolimits_{n=1}^{N} \lambda_n^2/\tau_n + \lambda_w^2/\tau_w + \rho^2/\tau_z \ . \tag{3}$$

As a preliminary step, we introduce well known general expressions for the quantity of a risky asset that a trader demands, and his *ex ante* utility, with exponential preferences and normal uncertainty. Let $R(\mathcal{I}) = E(\theta - p \,|\, \mathcal{I})$ be a trader's expectation of the per-unit return, conditional on information set $\mathcal{I}$, and let $s(\mathcal{I}) = var(\theta - p \,|\, \mathcal{I}) = E\left((R(\mathcal{I}) - (\theta - p))^2 \,|\, \mathcal{I}\right)$ be the variance of the error in his estimate. Call $\tau(\mathcal{I}) = s(\mathcal{I})^{-1}$ the precision of the trader's information; note that the values of $s(\mathcal{I})$ and $\tau(\mathcal{I})$ do not depend on the realization of the random variables in $\mathcal{I}$. If the trader has risk tolerance γ, then he chooses a quantity q to maximize the certainty equivalent wealth $R(\mathcal{I})\, q - \frac{s(\mathcal{I})}{2\gamma} q^2$ which yields an optimal order

$$q(\mathcal{I}) = \gamma \tau(\mathcal{I})\, R(\mathcal{I}) \tag{4}$$

Furthermore, prior to receiving any information, the *ex ante* expected utility of a trader who expects to acquire information set $\mathcal{I}$ is

$$U_{\mathcal{I}} = -\sqrt{s(\mathcal{I})/s(p)} \ . \tag{5}$$

For details, see Grossman and Stiglitz [18] and Admati and Pfleiderer [2]. The notable distinction in (4) and (5) is that we have not yet made any assumptions about whether $\mathcal{I}$ includes p, so the price remains inside the expectation operator in $R(\mathcal{I})$ and $s(\mathcal{I})$.[9] As a consequence of (5), the optimal information acquisition decision of a trader is simply to choose the information set $\mathcal{I}$ that gives him the most precise estimate of $\theta - p$. That is, he maximizes $\tau(\mathcal{I})$ over all available information sets. For market order traders, the wrinkle will be that a precise estimate of $\theta - p$ requires estimating both the asset value and the market-clearing price.

3 Equilibrium with Limit Order Traders

This section studies the case in which all traders submit limit orders; thus for now set $M = 0$. To characterize equilibria, we use (4) and (5) to develop detailed expressions for optimal orders and *ex ante* utility. With the latter, we can construct indifference curves over news sources that demonstrate a trade-off between a source's precision and its price impact. Finally we impose consistency (P must be generated by traders' actions) and show that equilibria exist.

3.1 Optimal Limit Order Strategies and *Ex Ante* Utility

Consider a limit order trader who observes information set $\mathcal{I}_l = \{y_n, w, p\}$ and expects that the price to satisfy (2) with $\sum\nolimits_{n=1}^{N} \lambda_n + \lambda_w + \lambda_\theta = 1$. Because this trader observes the price, his order

[9] Otherwise, (5) is just a special case of Proposition 3.1 in Admati and Pfleiderer [2], with initial wealth normalized to zero and $E(\theta - p) = 0$.

and utility hinge on his estimate of θ: $R(\mathcal{I}_l) = E(\theta \mid \mathcal{I}_l) - p$ and $s(\mathcal{I}_l) = var(\theta \mid \mathcal{I}_l)$. We will write $s_l^n = s(\mathcal{I}_l)$ and $\hat{\tau}_l^n = \tau(\mathcal{I}_l) = (s_l^n)^{-1}$ to emphasize the dependence of his estimate precision on his news source n. Because of (5), a trader's decision about which news source to follow boils down to choosing the source that maximizes $\hat{\tau}_l^n$. By standard properties of the normal distribution, $E(\theta \mid \mathcal{I}_l)$ will be a convex combination of the elements in $\mathcal{I}_l$; given (4), this implies that his demand can be written in the linear form

$$q_l^n(y_n, w, p) = \beta_y^n(y_n - p) + \beta_w^n(w - p) \tag{6}$$

for some coefficients β_y^n and β_w^n. In order to derive these coefficients and determine the trader's preferences over news sources, we must derive his estimate of θ and $\hat{\tau}_l^n$. While the errors in signals y_n and w are independent, the trader must account for the fact that both signals are correlated with the price. To deal with this, we construct a transformation of the price p to strip out the influence of the other two signals. Let

$$\zeta_n = \frac{1}{1 - \lambda_w - \lambda_n}(p - \lambda_w w - \lambda_n y_n) = \frac{1}{\sum_{n' \neq n} \lambda_{n'} + \lambda_\theta} \left(\sum_{n' \neq n} \lambda_{n'} y_{n'} + \lambda_\theta \theta + \rho z \right)$$

The set of random variables $\{w, y_n, \zeta_n\}$ is informationally equivalent to $\mathcal{I}_l$, but now those variables are also independent, conditional on θ. Furthermore, ζ_n is distributed $N\left(\theta, \tau_{\xi_n}^{-1}\right)$, with precision that can be written $\tau_{\zeta_n} = (1 - \lambda_w - \lambda_n)^2 / \left(s_p - \lambda_w^2/\tau_w - \lambda_n^2/\tau_n\right)$. Then by standard results for normal distributions, the trader's optimal estimate of θ is a precision-weighted average of w, y_n, and ζ_n, and the precision of that estimate is simply $\hat{\tau}_l^n = \tau_w + \tau_n + \tau_{\zeta_n}$. With this in hand, it is straightforward to compute the coefficients of the trader's order strategy:

$$\beta_y^n = \gamma_L \left(\tau_n - \tau_{\zeta_n} \frac{\lambda_n}{1 - \lambda_w - \lambda_n} \right) = \gamma_L \tau_n \left(1 - \frac{\lambda_n}{\tau_n} \frac{1 - \lambda_w - \lambda_n}{s_p - \lambda_w^2/\tau_w - \lambda_n^2/\tau_n} \right) \tag{7}$$

$$\beta_w^n = \gamma_L \left(\tau_w - \tau_{\zeta_n} \frac{\lambda_w}{1 - \lambda_w - \lambda_n} \right) = \gamma_L \tau_w \left(1 - \frac{\lambda_w}{\tau_w} \frac{1 - \lambda_w - \lambda_n}{s_p - \lambda_w^2/\tau_w - \lambda_n^2/\tau_n} \right) \tag{8}$$

In comparing news sources, it suffices to compare $\hat{\tau}_l^n - \tau_w = \tau_n + \tau_{\zeta_n}$, since the additional term τ_w is constant with respect to n. To make this comparison, we define a function

$$f(\tau, \lambda) = \tau + \frac{(1 - \lambda_w - \lambda)^2}{s_p - \lambda_w^2/\tau_w - \lambda^2/\tau} \tag{9}$$

such that $\hat{\tau}_l^n = \tau_w + f(\tau_n, \lambda_n)$. A limit order trader who believes that the price follows P will weakly prefer source n over source n', if and only if $f(\tau_n, \lambda_n) \geq f(\tau_{n'}, \lambda_{n'})$. Thus, level curves of f represent indifference curves over combinations of precision and price impact. As a useful hypothetical case, let us say that a news source n is *worthless* to a limit order trader if it gives him the same utility that he would earn with the information $\{w, p\}$ alone. Worthless news must satisfy $f(\tau_n, \lambda_n) = f_0$, where $f_0 = \frac{(1-\lambda_w)^2}{s_p - \lambda_w^2/\tau_w}$ and $\tau_w + f_0$ is the precision of a trader who sees $\{w, p\}$; with

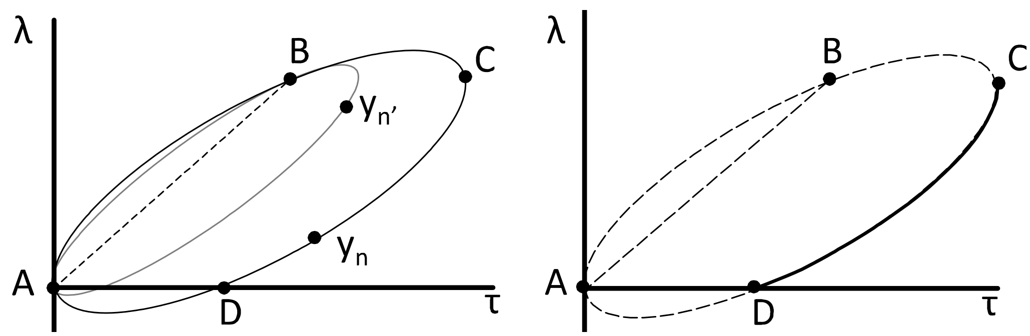

Figure 1: Limit order indifference curves

some simplification one can see that n is worthless if (τ_n, λ_n) lies along the line $\frac{\lambda}{\tau} = \frac{s_p - \lambda_w^2/\tau_w}{1-\lambda_w}$ in τ-λ space. This worthless news line marks a transition in traders' order strategies. We say that a trader with source n *buys on good news (sells on good news)* from n if β_y^n is positive (negative).

Lemma 1 *A trader with source n buys on good news from n if its price impact to precision ratio is lower than that of a worthless source: that is, if $\frac{\lambda_n}{\tau_n} < \frac{s_p - \lambda_w^2/\tau_w}{1-\lambda_w}$. Conversely, he sells on good news from n if its price impact to precision ratio is higher than that of a worthless source.*

Proof. This follows directly from (7) and basic algebra. ■

In a similar manner, one can show that this trader buys on the public signal ($\beta_w^n > 0$) if and only if $\frac{\lambda_w}{\tau_w} < \frac{s_p - \lambda_n^2/\tau_n}{1-\lambda_n}$. For limit order traders, selling on good news from a source n – or equivalently, buying on bad news – can only arise if the price severely overreacts to n. In this case, holding the price fixed, worse news from n implies either relatively strong noise trader demand or relatively good news from other (more precise) sources. It is the second possibility that, in principle, could induce trades against n. The next lemma characterizes indifference curves for signals whose value is positive (that is, $f(\tau_n, \lambda_n) > f_0$).

Lemma 2 *(Limit order indifference curves)*

i) Fix $\lambda_1, ..., \lambda_N$, λ_w, and ρ. For $K > f_0$, solutions to the equation $f(\tau, \lambda) = K$ lie on an ellipse characterized by a chord AB that does not depend on K plus two additional points C and D such that $A = (0,0)$, $B = (\frac{(1-\lambda_w)^2}{s_p - \lambda_w^2/\tau_w}, 1-\lambda_w)$, $C = (K, 1-\lambda_w)$, $D = (K - \frac{(1-\lambda_w)^2}{s_p - \lambda_w^2/\tau_w}, 0)$, and the ellipse has vertical tangencies ($d\tau/d\lambda = 0$) at A and C. Furthermore, if $K > K' > f_0$ then the ellipse $f(\tau, \lambda) = K'$ lies strictly in the interior of $f(\tau, \lambda) = K$ (except for tangencies at A and B).

ii) Thus a limit order trader strictly prefers source n to source n' if $(\tau_{n'}, \lambda_{n'})$ lies in the interior of the ellipse $f(\tau, \lambda) = f(\tau_n, \lambda_n)$. If $(\tau_{n'}, \lambda_{n'})$ lies on this ellipse, then the trader is indifferent between n and n'.

Some typical indifference curves are displayed in the left panel of Figure 1. Note that the chord AB lies along the worthless news line. In this example, a trader would prefer the less precise signal y_n over the more precise $y_{n'}$ on the inner ellipse because the price impact of the latter is too large.

Some of the possible precision-price impact combinations on these level curves seem unlikely to arise in equilibrium. For example, if source n lies along arc AB, then its price impact is so large that an agent who observes good news from n would sell on it. However, if traders sell on good news from n, its price impact will be negative, not positive. Similarly, if source n lies along arc AD, then its price impact is negative. This could only arise if traders sell on good news from n, but they would want to do the opposite. Finally, if n lies along BC, then $\lambda_w + \lambda_n > 1$, implying that some other source must have a negative price impact. In the next section, we show that these three cases are indeed incompatible with equilibrium, so for practical purposes, one can concentrate on the arc CD (as shown in the right panel of Figure 1). Arc CD looks like a conventional convex indifference curve: traders prefer news that is more accurate and less incorporated into the price. Because in equilibrium, traders must be indifferent between all news sources that are acquired, price impacts will need to adjust so that the precision-price impacts pairs for all acquired sources lie along a single curve such as CD.

3.2 Limit Order Equilibrium

Now we close the model by deriving the price function that arises from the news source choices and linear demands discussed above. If traders are distributed over sources according to $\vec{l}$ and submit orders as described in (6), the market-clearing condition becomes:

$$L \sum_{n=1}^{N} l_n \left(\beta_y^n y_n + \beta_w^n w - (\beta_y^n + \beta_w^n) p\right) + z \equiv 0 \tag{10}$$

Solving for p delivers a price function of the form (2) with coefficients

$$\lambda_n = \rho L l_n \beta_y^n \quad , \quad \lambda_w = \rho L \sum_{n=1}^{N} l_n \beta_w^n \quad , \quad \lambda_\theta = 0 \quad , \quad \rho = \left(L \sum_{n=1}^{N} l_n (\beta_y^n + \beta_w^n)\right)^{-1} \tag{11}$$

Note that the quantity on the left-hand side of (10) is the aggregate excess demand at price p. We call a equilibrium *regular* if it has the following two features.

A1. Aggregate excess demand is decreasing in p. (That is, $-L \sum_{n=1}^{N} l_n(\beta_y^n + \beta_w^n)$ is strictly negative.)

A2. $\lambda_w \in (0, 1)$

Condition A2 says that good news from the public signal has a non-negative impact on the price, but not more than one-for-one.

Proposition 1 *A regular linear equilibrium exists.*

Proof. This is a special case of Proposition 6. ■

The intuitive flavor of the proof is fairly familiar: if traders respond to their own information too aggressively, the price will be quite informative, encouraging all traders to rely more on the price and less on private information. Conversely, if all traders rely too much on the price, it will be uninformative and they will be forced to turn back to their private information. The proposition

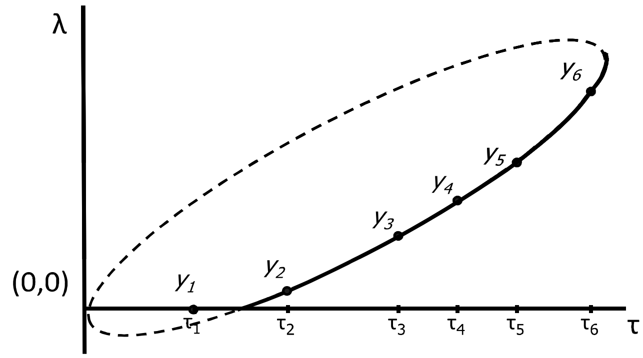

Figure 2: Equilibrium with limit order traders

is silent on uniqueness of linear equilibria satisfying A1 and A2, and furthermore, we have not ruled out the possibility of additional equilibria. (However, equilibria violating A1 or A2 seem very unlikely, and if they do exist, they would have rather perverse features.) Next we confirm that the counterintuitive possibilities mentioned in the indifference curve discussion cannot occur.

Lemma 3 *In a regular linear equilibrium, the price impact of every source is positive: $\lambda_n \geq 0$ (and consequently, $\lambda_n \leq 1 - \lambda_w$) for all n. Furthermore, no trader sells on good news from his source.*

Proof. If λ_n were strictly negative, then given $1 - \lambda_w$ positive by A2, we would have $\beta_y^n > 0$ by Lemma 1. But then (11) and A1 imply $\lambda_n > 0$, a contradiction. Furthermore, we have $\sum_{n=1}^{N} \lambda_n = 1 - \lambda_w$, so $\lambda_n > 1 - \lambda_w$ would imply that $\lambda_{n'} < 0$ for some other $n' \neq n$, which we have showed is impossible. If traders observing n were to sell on good news, then $\beta_y^n < 0$ and A1 would imply $\lambda_n < 0$, which we have ruled out. ■

A typical equilibrium is summarized concisely in Figure 2. The precision and price impact of each acquired source must lie along the CD arc of a single level curve of $f(\tau, \lambda)$; in the figure this is sources 2 through 6. Any unacquired source must lie inside this level curve, along the $\lambda = 0$ axis (source 1, in the figure). This example has the following general features:

1. There is a threshold τ, such that news sources that are less precise than this threshold are not acquired. (If this threshold is low enough, all sources are acquired.)

2. Traders cluster on more precise news sources: λ_n / τ_n is increasing in n.

Mathematically, the second point follows directly from the convexity of arc CD. The intuition for both points is also straightforward. After controlling for his own source, the amount that a trader can infer about other news n' from the price depends on the signal-to-noise ratio of $\lambda_{n'}$ relative to noise trader demand. A trader who is willing to choose a less precise source n, with $\tau_n < \tau_{n'}$, must be able to compensate by learning relatively more from the price (controlling for y_n) than he could by choosing n' instead. This turns out to require $\lambda_{n'}/\tau_{n'}$ large and λ_n/τ_n small.

ε	0	0.1	0.2	0.5	1	2	3	4
$l_{\text{sup}}^{10{,}000}$	0	0.05	0.10	0.25	0.47	0.72	0.82	0.87
$\lambda_{\text{sup}}^{10{,}000}$	0	0.05	0.11	0.29	0.59	0.87	0.94	0.97

Table 1: Approximate limiting influence ($N = 10000$) of the superior signal in Example 1. Other parameters: $\gamma = 1, L = 1, \bar{\tau} = 1, \tau_w = 0, \tau_z = 1$.

Point 1 reflects the fact that profits from a popular source never erode completely, due to the presence of noise trader demand and the cap on trading imposed by risk aversion. In some cases, the profits available from a low quality source, even a very exclusive one, cannot compete.

To explore the scope of concentration on news in this model, we present a few examples and limiting cases for which particularly sharp results are possible. One such case is the limit as the number of news sources grows large. Many market models begin from the assumption that there are many independent signals available and that each trader is endowed with a different one. This resembles the situation in our model when N is large, except that traders can decide which information to acquire.

Example 1: One superior news source

Consider a market with $N - 1$ equally precise signals, each with precision $\bar{\tau}$ and one superior signal with precision $\bar{\tau} + \varepsilon$. Models often assume that signals are identically distributed, but this is a matter of convenience – there is no particular reason to think that real-world information sources are all equally precise. This example explores the effect of a minimal amount of quality heterogeneity. The following result shows that the best source attracts disproportionate interest, and has a disproportionate influence on the price, even when there are infinitely many signals to choose from. Let λ_{sup}^N and l_{sup}^N denote the price impact and "market share" (the fraction of acquiring traders) of the superior signal in an equilibrium with N news sources.[10] As the number of news sources grows, we have:

Proposition 2 *For the model of Example 1,*

i) The price impact and market share of a non-superior source tend to zero with N.

ii) The price impact and market share of the superior source remain positive even as N grows large. That is, λ_{sup}^N and l_{sup}^N are bounded away from zero uniformly in N.)

Table 1 illustrates the excess influence of the superior signal in the large N limit for a case with no public signal and $\bar{\tau} = 1$. The results indicate that when traders can choose which information to acquire, it is not innocuous to assume away quality heterogeneity. When quality differences are relatively small, a market observer might find it puzzling and arbitrary to see news from one of these sources move the market price substantially, when similar news from other sources has a much smaller effect.

In the numerical example of Table 1, the lower quality sources have a collective market share $1 - l_{\text{sup}}^N$ that remains positive as the number of sources grows, even though the share of each

[10] We know that such an equilibrium exists; the results below do not depend on whether it is unique.

individual source vanishes. While the price does not aggregate these signals efficiently, in principle the collective information that traders acquire would reveal the asset value perfectly ($N \to \infty$), if one could only weight this information properly. However, the outcome can be starker than this: under some conditions, all news sources except for the superior one are ignored! The next result applies not just to Example 1, but for any configuration of news sources, and for any $\tau_w \geq 0$.

Proposition 3 *Suppose that the second best news source $N-1$ satisfies $D \equiv (L\gamma_L)^2 \tau_z \tau_{N-1} < 1$ and the best news source N satisfies $\tau_N \geq \frac{1}{1-D}\tau_{N-1}$. Then there is an equilibrium in which all traders acquire source N.*

The conditions for Proposition 3 tend to be met when the precision advantage of the best news source is large and opportunities to profit from noise traders are relatively large (τ_z small) and unexploited (L and γ_L small). In such an equilibrium, traders acquire information that is (endogenously) perfectly correlated, even though independent news is available.[11] The asset price will be extremely sensitive to this news: $\lambda_N = 1$. Furthermore, the necessary conditions do not depend on the number of sources, just the first and second best precisions. If we let N grow, holding τ_{N-1} and τ_N fixed, then we can easily have a situation in which potentially available information reveals θ perfectly, but the collective precision of information that traders actually acquire is bounded at τ_N. The last two results demonstrate the possibility of concentration at the very best news source. The next example shows that if we broaden the focus to the top few news sources, the phenomenon of concentration at the top is quite general.

Example 2: General quality distributions as $N \to \infty$

This example looks at the concentration of traders' news choices are in a large N setting that is more general than Example 1. The idea will be that in the $N \to \infty$ limit, there is an arbitrary cumulative distribution function Ψ over the precision of news sources. We assume that Ψ is defined over a compact interval $[\tau_L, \tau_H]$ and is Lipschitz continuous and invertible. For each $N \geq 1$, construct a market with N news sources whose precisions are spaced evenly across the percentiles of Ψ. That is, for market N, set $\tau_1 = \Psi^{-1}(1/N)$, $\tau_2 = \Psi^{-1}(2/N)$, ... , $\tau_n = \Psi^{-1}(n/N)$, and $\tau_N = \Psi^{-1}(1) = \tau_H$. As N grows, the news sources populate the interval $[\tau_L, \tau_H]$ more and more finely in such a way that their distribution approximates Ψ.[12] Notice that because the precision of each source is bounded below by τ_L, the collective precision of the information *available* to traders grows at rate N.

Proposition 4 *In the model of Example 2, as $N \to \infty$, the fraction of sources that are acquired shrinks to zero and the collective precision of acquired information grows at a rate no faster than $\sqrt{N}$.*

[11] We conjecture that the result could be strengthened to show that, when the conditions hold, a linear equilibrium *must* have all traders acquiring N. That is, the equilibrium is unique. Numerical investigation strongly suggests that this is true, but we do not have a proof.

[12] Of course, a simpler approach would be to sample N times from Ψ. The advantage of the construction we use is tractability – keeping the precisions non-stochastic avoids detours to deal with the Central Limit Theorem. However, there is no reason to expect the results to change materially under a sampling-based approach.

Proposition 4 shows that when there are many news sources and a trader can only follow one of them, clustering on a small fraction of the highest quality sources is the norm. As a consequence, the asset price will appear to be oversensitive to a small set of news items and underresponsive to the rest. Of course, whenever we say that asset-relevant information is ignored, we mean that this is true within the time horizon – a single episode of market-clearing – covered by the model. In the long run, one would expect all relevant information to become incorporated into the price; loosely interpreted, our results suggest a framework for thinking about why some news might be incorporated more slowly than others.

The results so far focus on the supply of information holding other parameters fixed, including the volume of noise trading and the level of risk tolerance. However, if traders holding a popular signal y_n are to make a profit, there must be a wedge between that signal and the price. That wedge is generated by the fact that risk aversion limits the price impact of y_n, while noise trading generates price movement that is unrelated to y_n. A reduction in noise trading or an increase in risk tolerance should tend to penalize traders with popular news, and thus encourage them to broaden the base of information acquired. This intuition is formalized in the following proposition.

Proposition 5 *For fixed N, and any sequence of equilibria as $\gamma_L \to \infty$ or $\tau_z \to \infty$, the coefficients of the equilibrium price function converge to $\lambda_w = \tau_w \left(\sum_{n=1}^N \tau_n + \tau_w\right)^{-1}$ and $\lambda_n = \tau_n \left(\sum_{n=1}^N \tau_n + \tau_w\right)^{-1}$, for all $n \in \{1, ..., N\}$. That is, every source is acquired and weighted according to its precision in the price.*

Models with exogenous information often predict that an increase in risk tolerance should encourage traders to use the information they have more aggressively; alone, or in combination with a decrease in noise trading, this should tend to boost the signal to noise ratio in the price. We extend the spirit of those results to say that because this more aggressive trading reduces the returns on shared information, traders have greater incentives to acquire more exclusive information when risk tolerance increases.

4 Equilibrium with Both Limit and Market Order Traders

Now we relax the constraint that $M = 0$ and allow market order traders in the model. The next section characterizes their order strategies and preferences over news sources. Then we examine how limit and market order traders interact in equilibrium.

4.1 Market Order Traders: Order Strategies and *Ex Ante* Utility

Consider a market order trader with information set $\mathcal{I}_m = \{y_n, w, x_m\}$. Just as with the limit order traders, his demand and *ex ante* utility are determined by (4) and (5), but because he cannot observe the price, their exact forms will be a bit different. For this trader, the precision of his estimate of $\theta - p$ is a different quantity from the precision of his estimate of θ. Write the former as $\hat{\tau}_m^n = \tau\left(\mathcal{I}_m\right)$

(with $s^n_m = s(\mathcal{I}_m) = (\hat{\tau}^n_m)^{-1}$) and the latter as t^n_m. Because the signals in $\mathcal{I}_m$ are independent, his estimate of θ, $E(\theta \mid \mathcal{I}_m)$ is just the precision-weighted average of y_n, w, and x_m, and so we have $t^n_m = \tau_n + \tau_w + \tau_x$. Then because he expects the price function to obey (2), it is straightforward to show that his estimate of the price is $E(p \mid \mathcal{I}_m) = \lambda_n y_n + \lambda_w w + (1 - \lambda_w - \lambda_n) E(\theta \mid \mathcal{I}_m)$, so we arrive at an estimate $E(\theta - p \mid \mathcal{I}_m) = (\lambda_n + \lambda_w) E(\theta \mid \mathcal{I}_m) - \lambda_n y_n - \lambda_w w$ of the per-unit profit from a purchase. Direct computations then yield the following linear order strategy.

Lemma 4 *The demand of a market order trader m who believes the price to be described by (2) and chooses information source y_n is a linear function $q^n_m(x_m, y_n, w) = \alpha^n_x x_m + \alpha^n_y y_n + \alpha^n_w w$ where*

$$\alpha^n_y = \gamma_M \hat{\tau}^n_m (\lambda_n + \lambda_w) \left(\frac{\tau_n}{t^n_m} - \frac{\lambda_n}{\lambda_n + \lambda_w} \right), \tag{12}$$

$$\alpha^n_w = \gamma_M \hat{\tau}^n_m (\lambda_n + \lambda_w) \left(\frac{\tau_w}{t^n_m} - \frac{\lambda_w}{\lambda_n + \lambda_w} \right), \text{ and} \tag{13}$$

$$\alpha^n_x = \gamma_M \hat{\tau}^n_m (\lambda_n + \lambda_w) \frac{\tau_x}{t^n_m} = -\left(\alpha^n_y + \alpha^n_w\right) \tag{14}$$

Notice that the trader always buys on good news from his idiosyncratic signal ($\alpha^n_x \geq 0$ as long as the price impacts of y_n and w are not negative), and because $\alpha^n_y + \alpha^n_w = -\alpha^n_x$ this implies that he must sell on good news either from w or y_n (or perhaps both). We will discuss intuition for this momentarily, but first we characterize $\hat{\tau}^n_m$ (and therefore, his *ex ante* utility).

Lemma 5 *For a market order trader who acquires news source n, the precision of his estimate of $\theta - p$ is given by*

$$(\hat{\tau}^n_m)^{-1} = s^n_m = s_p - \frac{\lambda_w^2}{\tau_w} - \frac{\lambda_n^2}{\tau_n} + \frac{(\lambda_n + \lambda_w)^2}{\tau_n + \tau_x + \tau_w}$$

Market order preferences over news sources can be ranked by $\hat{\tau}^n_m$.

Proof. Using the expressions above, we can write $E(\theta - p \mid \mathcal{I}_m) - (\theta - p)$ as

$$(\lambda_n + \lambda_w)(E(\theta \mid \mathcal{I}_m) - \theta) + \left(\sum_{n' \neq n} \lambda_{n'} (y_{n'} - \theta) + \rho z \right)$$

The two terms in parentheses are independent and have variances $(\lambda_n + \lambda_w)^2 / t^n_m$ and $s_p - \lambda_n^2/\tau_n - \lambda_w^2/\tau_w$ respectively, from which the result follows. ■

For reference, note that the precision of a (hypothetical) trader who cannot observe any news source reduces to $\left(\tau^0_m\right)^{-1} = s^0_m = s_p - \frac{\lambda_w^2}{\tau_w} + \frac{\lambda_w^2}{\tau_x + \tau_w}$. For a comparison across news sources, there is no harm in stripping away terms that are constant with respect to n, so the next result is immediate.

Lemma 6 *A market order trader who believes the price to be described by (2) will choose an information source that maximizes*

$$g(\tau_n, \lambda_n) \equiv \frac{\lambda_n^2}{\tau_n} - \frac{(\lambda_n + \lambda_w)^2}{t^n_m} \tag{15}$$

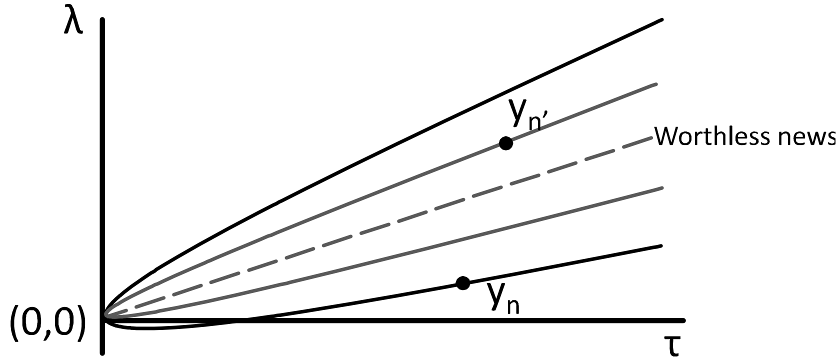

Figure 3: Market order indifference curves

across all n.

Just as with limit orders, we can treat $g(\tau, \lambda) = g(\tau_n, \lambda_n)$ as an indifference curve representing the pairs in (τ, λ) space that give a market order trader the same *ex ante* utility as (τ_n, λ_n). In this case, worthless news satisfies the condition $\tau_m^n = \tau_m^0$, or equivalently, $g(\tau_n, \lambda_n) = g_0$, where $g_0 = -\frac{\lambda_w^2}{\tau_x + \tau_w}$. Simplifying this condition, we find that source n is worthless if and only if it satisfies $\lambda_n / \tau_n = \frac{\lambda_w}{\tau_x + \tau_w}$; not coincidentally, this is also the condition under which the trader puts zero weight on y_n in his market order. If $\lambda_n / \tau_n < \frac{\lambda_w}{\tau_x + \tau_w}$ or $\lambda_n / \tau_n > \frac{\lambda_w}{\tau_x + \tau_w}$, then source n is valuable, but for different reasons: in the first case, the trader will buy on good news from n, while in the latter case he will sell on good news.

Lemma 7 *(Market order indifference curves) If source n is not worthless ($g(\tau_n, \lambda_n) > g_0$), then the set of pairs (τ, λ) satisfying $g(\tau, \lambda) = g(\tau_n, \lambda_n)$ lies on one branch of a hyperbola, with a tangency to the λ-axis at $(\tau, \lambda) = (0, 0)$. "Broader-jawed" hyperbolas are associated with higher ex ante utility.*

Some typical indifference curves are sketched in Figure 3.[13] Worthless news lies along the dashed line. A trader will buy on good news from his source if it lies below this line, such as source n in the figure. If it lies above this line, such as source n' in the figure, he will sell on good news. We will say that a source lies on the upper or lower branch respectively of an indifference curve if it lies above or below the worthless news line. As drawn, a trader would prefer source n over source n', as n lies on the higher indifference curve.

The intuition is straightforward. Because a market order trader faces execution risk (she cannot condition her order on p), her profit depends on estimating $\theta - p$. To do this, she needs signals that distinguish θ from p or *vice versa*. For this purpose, signals that are strongly correlated with θ (high τ_n) and weakly correlated with p (low λ_n) are useful as proxies for θ; these are the signals for which the trader buys on good news. Alternatively, signals that are strongly correlated with p and

[13]While all indifference curves approach the point $(0, 0)$, this point belongs only to the worthless news indifference curve. The reason that indifference curve grow close together near $(0, 0)$ relates to the fact that a signal's contribution to variation in the price is on the order of λ_n^2 / τ_n. This quantity can be quite different for two different signals, even if both have (τ_n, λ_n) approaching $(0, 0)$, and so utility can be quite different as well.

weakly correlated with θ (high λ_n and low τ_n) are valuable as proxies for p; the trader prefers to sell when these signals are high. This provides another way to think about the zero-value indifference curve: a signal is of no value precisely when a trader's optimal market order does not condition on it ($\alpha_y^n = 0$). It may be helpful to think of a trader as starting off with his private signal x_m as a proxy for θ and the public signal w as a proxy for p. If the public signal is a good price proxy, additional news y_n will be used to support x_m. However, when price risk is large – in the sense that w does not predict variation in p very well – the trader will seek out additional news that he expects to cause excess price movement in order to trade against it.

4.2 Equilibrium

We will show that equilibria with both types of trader have the following features, some of which carry over from the case with only limit order traders. The first three points are relatively self-explanatory, while for the fourth we will need to be more specific about what is meant by excess volume.

1. More precise news sources have excessive price impacts. Specifically, the ratio λ_n/τ_n is increasing in n.
2. Segmentation of trader types. If a market order trader plans to buy on good news from his source, then he follows a less precise, more exclusive source than a limit order trader would.
3. Sales on good news at the top. If any selling on good news takes place, it is done by market order sellers who observe the most precise source.
4. Excess volume at the top. If there are sales on good news, then the volume of trade associated with the most precise source y_N is greater than its precision and price impact would otherwise suggest.

The first step in characterizing equilibrium is to update equation (11), which expressed the coefficients of the market-clearing price generated by traders' order strategies and distribution across news sources. When market orders are present, these coefficients become:

$$\begin{aligned} \lambda_n &= \rho(Mm_n\alpha_y^n + Ll_n\beta_y^n) \quad , \quad \lambda_w = \rho\textstyle\sum_{n=1}^N(Mm_n\alpha_w^n + Ll_n\beta_w^n) \quad , \\ \lambda_\theta &= 1 - \lambda_w - \textstyle\sum_{n=1}^N \lambda_n \quad , \quad \text{and} \quad \rho = \left(L\textstyle\sum_{n=1}^N l_n(\beta_y^n + \beta_w^n)\right)^{-1} \end{aligned} \tag{16}$$

and the market-clearing condition is

$$\sum_{n=1}^N \left\{(Mm_n\alpha_y^n + Ll_n\beta_y^n)y_n + (Mm_n\alpha_w^n + Ll_n\beta_w^n)w + Mm_n\alpha_x^n\theta - Ll_n(\beta_y^n + \beta_w^n)p\right\} + z \equiv 0 \tag{17}$$

Proposition 6 *A regular linear equilibrium with both limit and market order traders exists. Furthermore, any such equilibrium satisfies $\lambda_n \in [0, 1 - \lambda_w]$ for every news source n, and $\lambda_n > 0$ for any news source that is acquired by either type of trader.*

As earlier, a source could only become negatively correlated with the price if traders trade against its good news, but in fact this negative correlation would be an extra incentive to trade in the same direction as its good news. From this point forward, our analysis will restrict attention to equilibria that satisfy one additional restriction.

A3. In equilibrium, the price impact of the public signal satisfies $\lambda_w/\tau_w < s_p$.

We suspect that A3 is actually an implication of equilibrium, as it is satisfied in every computational example we have studied, but we do not have a proof of this. In any case, A3 has a sensible and appealing interpretation. Consider adding to the market a hypothetical limit trader whose only information is $\{w, p\}$. The optimal order of this trader can be shown (see the limit order strategies in the appendix) to be proportional to $(\tau_w s_p - \lambda_w)(w - p)$, so A3 is equivalent to the condition that this trader's demand is downward sloping in the price.

In characterizing equilibria, A3 acts like a single-crossing condition – it ensures that a trader planning to buy on good news will find high precision - high price impact news sources relatively more attractive if he can place a limit order than he would if placing a market order. A hint of this can been seen in the worthless news indifference curves. With a slight manipulation, A3 may be written as $\lambda_w/\tau_w < \left(s_p - \lambda_w^2/\tau_w\right) / (1 - \lambda_w)$, where the righthand side is the slope of the worthless news indifference curve for limit order traders. Then because $\lambda_w / (\tau_x + \tau_w) < \lambda_w/\tau_w$, A3 implies that limit order traders have a steeper worthless news indifference curve than market order traders do. This relationship is generalized in Lemma 8.

Lemma 8 *Fix an equilibrium and a news source n. Suppose that $0 \le \lambda_n \le 1 - \lambda_w$ and that, if constrained to choose source n, a limit order trader would buy on good news. Then the limit order indifference curve through (τ_n, λ_n) is strictly steeper at (τ_n, λ_n) than the market order indifference curve through the same point.*

The intuition behind this is fairly straightforward if market order traders would also prefer to buy on good news at from source n. For a market order trader, a more precise signal may not be helpful in discriminating θ from p if it is also more correlated with the price. For a limit order trader, the role of the signal is not to distinguish θ from p – after all, this trader can condition on p – but just to estimate θ. Thus, the downside of a higher price impact is less severe for the limit order trader.

As earlier, the (τ_n, λ_n) pairs for all news sources chosen by limit order traders must lie along a common indifference curve. Proposition 6 rules out the possibility that one of these sources lies along the AD or BC arc from Figure 1. Next, we rule out segment AB.

Lemma 9 *(i) If, in equilibrium, limit order traders would sell on good news at some source y_n, then market order traders would also sell on good news at that source. (ii) Consequently, limit order traders never sell on good news in equilibrium.*

Proof. (i) If limit order traders would sell on good news at y_n only if $\lambda_n > \frac{s_p - \lambda_w^2/\tau_w}{1-\lambda_w}\tau_n$ holds. A3 implies that $s_p - \lambda_w^2/\tau_w > (1-\lambda_w)\lambda_w/\tau_w$, and therefore, $\frac{\lambda_w}{\tau_w} < \frac{s_p - \lambda_w^2/\tau_w}{1-\lambda_w}$. Consequently, $\lambda_n >$

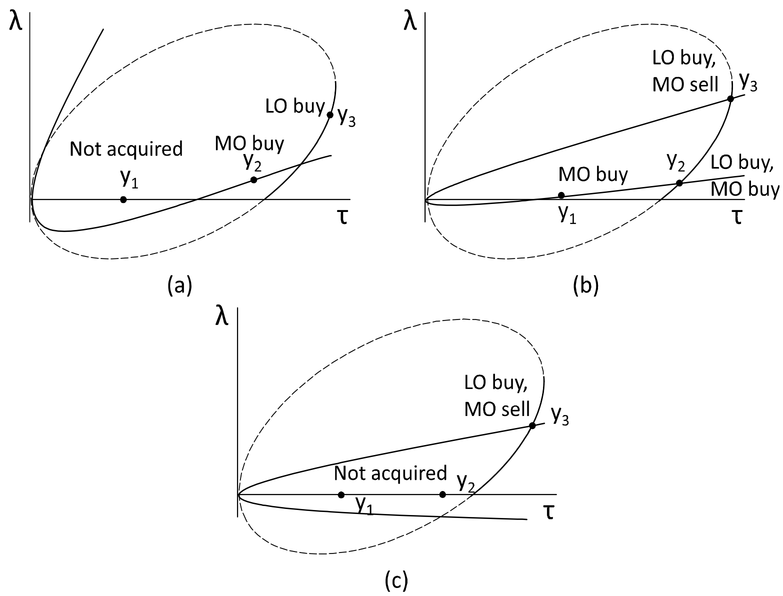

Figure 4: Examples of equilibrium news source acquisition with $N = 3$. Equilibrium limit order and market order indifference curves pictured. *LO* buy (sell) means limit order traders acquired the source and buy (sell) on good news.

$\frac{\lambda_w}{\tau_w}\tau_n > \frac{\lambda_w}{\tau_x+\tau_w}\tau_n$, so by Lemma 4, any market order trader at y_n would also sell on good news. For (ii), suppose limit order traders acquire n and sell on its good news, so $\lambda_n > \frac{s_p-\lambda_w^2/\tau_w}{1-\lambda_w}\tau_n > 0$ holds. Then $\beta_y^n < 0$ and by part (i), $\alpha_y^n < 0$. But then (16) implies $\lambda_n \leq 0$, a contradiction. ■

This result, which also has the flavor of a single-crossing property, is complementary to Lemma 8. Because market order traders need to find a proxy for p and limit order traders do not, the former tend to be more disposed to trade against any particular signal than the latter are. We are finally ready to characterize the equilibria of the model with both order types.

Proposition 7 *Suppose that news source precisions are distinct. In any regular linear equilibrium satisfying A3, there are threshold sources $\bar{n}_1$ and $\bar{n}_2$, with $\bar{n}_1, \bar{n}_2 \in \{1, 2, ..., N\}$ and $\bar{n}_2 \geq \bar{n}_1$, such that:*

i. If $n < \bar{n}_1$, then source n is not acquired.

ii. Market order traders acquire all sources in $\{\bar{n}_1, ..., \bar{n}_2 - 1\}$. They may acquire $\bar{n}_2$ or N as well.

iii. Limit order traders acquire all sources $\{\bar{n}_2 + 1, ..., N\}$. They may acquire $\bar{n}_2$ as well.

iv. All limit order traders, and all market order traders who do not acquire N, buy on good news from their sources.

v. Market order traders who acquire source N may sell on good news.

vi. The sequences $\{\lambda_n\}$ and $\{\lambda_n/\tau_n\}$ are increasing, strictly for $n \geq \bar{n}_1$.

The condition that news sources have distinct precisions is not critical; it is only imposed to avoid a verbose description of special cases.[14] Various configurations of news acquisition can arise depending on the equilibrium thresholds $\bar{n}_1$ and $\bar{n}_2$; Figure 4 illustrates some of the possibilities. Market order traders who acquire news in order to improve their estimate of the asset value focus on less precise, more exclusive sources, while limit order traders focus on more precise, less exclusive sources. This reflects the intuition discussed with Lemma 8. As earlier, the least precise news sources may not be acquired at all, as in panels (a) and (c) of Figure 4. Also as earlier, more popular news sources create price impacts that are disproportionate to the quality of their information, in the sense that λ_n/τ_n rises with n.

The more novel possibility raised by Proposition 7 is that market order traders may treat the source that is most incorporated into the price as a proxy for the price, and trade against it. For these traders, news source N is valuable precisely because it is popular, as this makes it a good bellwether of which way the price is likely to go, and its high precision is actually a disadvantage. Furthermore, by selling on good news from N, these traders tend to depress its price impact, making it possible for more limit order traders to acquire N and buy on good news. This means that there may be a disproportionate volume of trade associated with source N, a subject that we will turn to in the next section.

4.3 Complementary Clustering on News Source N

We refer to clustering by both types of trader at news source N as complementary because each type tends to absorb the other's trades, preventing the price impact of news from N from becoming either unattractively high (for limit orders) or unattractively low (for market orders). The next result shows that complementary clustering on source N will occur whenever public information about the asset value is weak or when private information is strong.

Proposition 8 *For τ_w sufficiently small or for τ_x sufficiently large, all market order traders acquire source N and sell on good news.*

While the proof is a bit involved, the intuition is quite direct. A market order trader who already has accurate private information about the asset value has little to gain from acquiring additional information about θ. He does better by acquiring additional information to help him hedge against adverse price movements – he should try to buy less when the price is likely to be undeservedly high (relative to θ), and more when it is undeservedly low. Because of its popularity and large excess price impact, news source N is the best barometer of the direction the price will move.

Alternatively, when there is a strong public consensus about the asset value (τ_w large), price risk is mitigated, as a trader can weigh his private information x_m against this consensus w. In

[14] The main issue has to do with the case in which both types acquire the threshold precision $\tau_{\bar{n}_2}$. If there were many identical sources at the threshold precision, then then the masses of limit and market traders at this threshold precision could be distributed over these identical sources in many different ways that are all essentially equivalent.

this case, if his private information is only moderately precise, he may want to acquire relatively exclusive news to supplement it. However, his price risk grows with increasing public uncertainty about the asset (declining τ_w). If public beliefs become sufficiently uncertain, he will begin to turn to popular news sources to better gauge market sentiment.

To put a sharp focus on the implications of complementary clustering, we will look at the extreme case in which public information is absent ($\tau_w = 0$) and market order traders' private information is perfect ($x_m \equiv 0$, so $(\tau_x)^{-1} = 0$).

Example 3 : Two almost identical news sources

Suppose that there are two sources with $\tau_1 = \bar{\tau}$ and $\tau_2 = \bar{\tau} + \varepsilon$. Set $\tau_w = (\tau_x)^{-1} = 0$ as discussed above, and consider limiting equilibria with $\varepsilon \to 0$, so the two news sources are of essentially identical quality. Our discussion will be kept intuitive, but all of the points could easily be made more rigorous. Notice that as ε is taken to zero, the price impacts λ_1 and λ_2 of the two sources must converge together, as a consequence of limit order indifference. Let $\bar{\lambda}$ be the common, limiting price impact. We can write the traders' orders in the $\varepsilon \to 0$ limit as $q_l^1 = \beta_y^1 (y_1 - p)$, $q_l^2 = \beta_y^2 (y_2 - p)$, and $q_m^2 = \alpha_y^2 (\theta - y_2)$. However, because (τ_1, λ_1) and (τ_2, λ_2) converge together, limit order strategies at the two sources will be identical: $\beta_y^1 = \beta_y^2$. Furthermore there is no ambiguity which source market order traders acquire, so we can simplify notation and simply write the limiting orders as $q_l^1 = \beta (y_1 - p)$, $q_l^2 = \beta (y_2 - p)$, and $q_m = \alpha (x_m - y_2)$.

Given these observations, we can work out how many limit order traders must acquire each source. From (16), the equality of price impacts requires $\lambda_1 = \rho L l_1 \beta = \rho (L l_2 \beta - M\alpha) = \lambda_2$, so we must have $l_1 = \frac{1}{2} - \frac{1}{2}\frac{M\alpha}{L\beta}$ and $l_2 = \frac{1}{2} + \frac{1}{2}\frac{M\alpha}{L\beta}$. This is not a complete characterization – α and β are endogenous quantities – but it gives the flavor of how news acquisition must look. The number of excess limit order traders at source 2, $Ll_2 - Ll_1 = \frac{M\alpha}{\beta}$, depends on how much limit order demand is required to soak up the market orders on the opposite side of the market. This excess rises the more market order traders there are (M), or the more aggressively they trade relative to limit order traders (α/β).

The expected volume of trade in this limiting equilibrium is

$$Vol = \frac{1}{2} E \left(L l_1 \left| q_l^1 \right| + L l_2 \left| q_l^2 \right| + M \left| q_m \right| + |z| \right)$$

We can partition this volume into gross trade associated with traders using each news source, $Vol_1 = E \left(L l_1 \left| q_l^1 \right| \right)$ and $Vol_2 = E \left(L l_2 \left| q_l^2 \right| + M \left| q_m \right| \right)$, and gross noise trader volume $Vol_z = |z|$. Notice that the expected size of a single limit order will not depend on which source is observed – because y_1 and y_2 appear symmetrically in the price and in q_l^1 and q_l^2, we will have $E \left(\left| q_l^1 \right| \right) = E \left(\left| q_l^2 \right| \right)$. We will drop the subscript and write $E (|q_l|)$. Our focus will be on the ratio Vol_2/Vol_1, which we will interpret as a measure of 'abnormal' trading volume generated by news from source

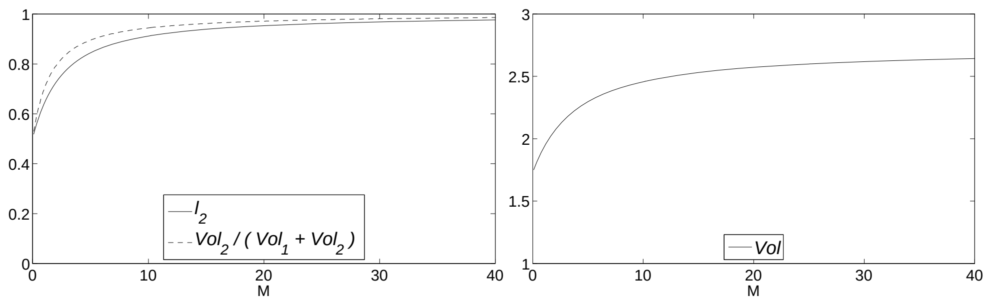

Figure 5: Two identical news sources: Concentration on Source 2 vs. M. (Parameters: $\tau_1 = \tau_2 = 1$, $\tau_w = \tau_x^{-1} = 0$, $\tau_z = 1$, $L = \gamma_L = \gamma_M = 1$. Left panel: fraction of limit order traders l_2 and of informed volume $Vol_2/(Vol_1 + Vol_2)$ associated with Source 2. Right panel: total expected volume Vol.)

2. We can write this ratio as[15]

$$\frac{Vol_2}{Vol_1} = \frac{l_2}{l_1} + \frac{M}{Ll_1}\frac{E(|q_m|)}{E(|q_l|)} = \frac{l_2}{l_1} + \frac{M}{Ll_1}\frac{\alpha}{\beta}\sqrt{\frac{var(\theta - y_2)}{var(y_2 - p)}}$$

Figure 5 presents numerical results for the limiting equilibrium when $\tau_1 = \tau_2 = 1$. (For other parameters, see the figure.) Holding the mass of limit order traders, we vary the mass of market order traders from $M = 0$ up to $M = 50$, computing a new $\varepsilon = 0$ equilibrium at each step. When market order traders are absent, the two news sources appear identical in terms of observable outcome variables: half of the traders acquire each source ($l_2 = \frac{1}{2}$), and half of the 'informed volume of trade' is associated with each source ($Vol_2/Vol_1 = 1$). As more and more market order traders enter, both types find it mutually advantageous to cluster on source 2, l_2 tends to one, and the ratio Vol_2/Vol_1 of excess volume related to source 2 tends to infinity. To outside observers, news from the two sources will appear to be of similar quality and to generate similar changes in the price, but only news from source 2 will be accompanied by a spike in volume.

Example 4 : Varying the precision of public information

To illustrate the transition to the state of weak public information where Proposition 8 applies, Figure 6 presents numerical results for a market with two news sources of quality $\tau_1 = 1$ and $\tau_2 = 1.1$. We vary τ_w from 1 down to 0; other parameters are described in the figure. Initially, for τ_w large, all market order traders acquire source 1 and buy on good news. At this point, the trading volume associated with the two sources is similar. As τ_w falls below 0.5, there is a fairly abrupt transition in which these traders all switch to source 2 and begin to sell on good news. At the same point Vol_1 and Vol_2 diverge sharply.[16] This transition is accomplished with almost no

[15] The last step follows from the fact that, unconditionally, q_l and q_m are both distributed normally with mean zero. Then it is a standard result that $E(|q_l|) = \sqrt{\frac{2}{\pi} var(q_l)}$ and similarly for q_m.

[16] Vol_1 and Vol_2 are defined as above, but the expressions for $E(|q_l^n|)$ and $E(|q_m^n|)$ become a bit more complicated than in Example 3. One must also remember that while, e.g., $E(|q_m^2|)$ is the expected order size of a market order trader holding signal y_2, now variation in w and x_m is responsible for some portion of that order.

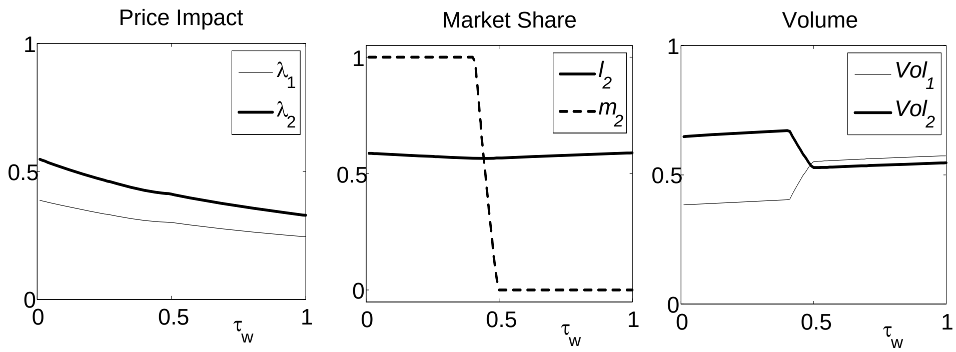

Figure 6: Varying the precision of public information. (Parameters: $\tau_w \in [0, 1]$, $\tau_1 = 1$, $\tau_2 = 1.1$, $\tau_x = 0.2$, $\tau_z = 1$, $L = M = \gamma_L = \gamma_M = 1$.)

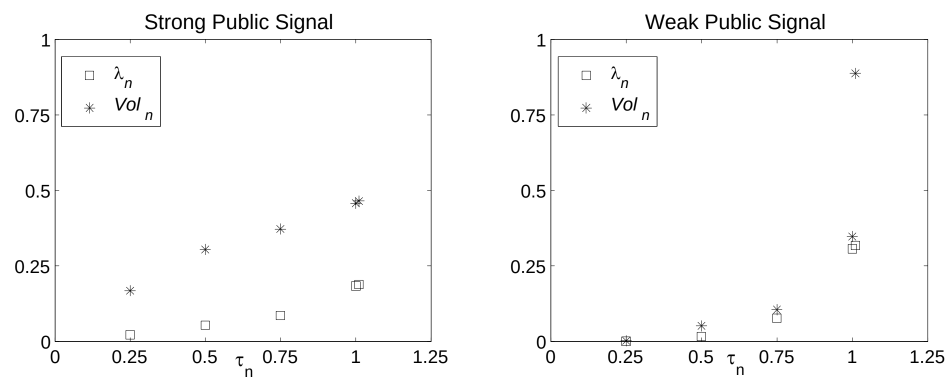

Figure 7: Excess volume at the top source. ($N = 5$ sources with precisions .25, .5, .75, 1, and 1.01. Other parameters: $L = \gamma_L = \gamma_M = 1, M = 5, \tau_x = .2, \tau_z = 1$. Left: $\tau_w = 1.5$. Right: $\tau_w = 0.2$.)

movement by limit order traders, roughly 58% of whom acquire source 2 throughout. The reason is that from the standpoint of a limit order trader, the shift of market order traders from source 1 to 2 has two immediate effects: there is new demand on the opposite side of the market at source 2, and there is less congestion on the same side of the market at source 1. These effects make both sources more attractive to limit order traders, and they happen to roughly balance. Both sources have growing impacts on the price as public information deteriorates, but λ_1 and λ_2 rise smoothly as τ_w declines, showing no evidence of the sharp transitions in market share and volume.

Example 5 : Abnormal volume at the most popular source

When there are many news sources, one can plot trend lines for the equilibrium price impact and volume associated with a source of quality τ. Then one way to define excessive, or abnormal volume associated with a news source would be to ask whether that source is an outlier from the trend. To illustrate this idea, consider an example with five news sources. For the first four, set $\tau_n = \frac{n}{4}$, and let the last source be just slightly superior to source 4: $\tau_5 = 1.01$. Figure 7 shows the equilibrium values of λ_n and Vol_n in an equilibrium with strong public information ($\tau_w = 1.5$, left panel) and with weak public information ($\tau_w = 0.2$, right panel). With strong public information, market order traders focus on the lower quality sources, and price and volume impacts are roughly linear in τ. In particular, the best two sources have almost identical price and volume impacts. However, in the equilibrium with weak public information, most market order traders shift to selling on good news from source 5, and as a result, Vol_5 becomes an outlier.

5 Empirical Evidence

The model delivers several predictions that in principle could be tested. The main predictions are: (1) higher quality news sources over-affect the price relative to their precision, while news from lower quality sources is under-incorporated or ignored altogether (in the short run); (2) new information from top quality news sources may be associated with abnormal trading volume; (3) effect (2) should be more pronounced when public information is weak relative to privately held information. As mentioned earlier, it is probably most natural to think of our static market as a stylization of short run market clearing around the arrival of new information. If we indulge this interpretation, and suppose that new information becomes efficiently incorporated into the price in the longer run, then as a counterpart to (1) we can also suggest (4): price movements generated by the most popular news sources will be prone to long run reversals. While the existing empirical literature has not addressed these predictions directly, there is a certain amount of indirect evidence which we survey below. One limitation in comparing this evidence to a static model is that natural candidates for new signals from sources, such as analyst revisions or newsletters, are generally studied as isolated events, not as an ensemble.

A common empirical approach involves identifying an information event, which could be an earnings or insider trade announcement, an analyst or newsletter recommendation, a macroeconomic surprise, or other news, and then studying an asset's abnormal returns and volume in a window around that event. In an influential series of papers, Kim and Verrecchia [24] [25] [26] laid out a theoretical framework for the price and volume effects of a public announcement. In their models, an announcement generates trade both because traders differ in the quality of their pre-announcement private information (leading them to weight the news differently in their posteriors, which leads to trade) and because they interpret the announcement differently.[17] The first effect generates a positive linear relationship between volume and the magnitude of price changes, while the latter generates excess volume unrelated to the size of price changes. On this basis, it has been suggested (see, for example, Bamber et al. [5]) to take residual volume, after controlling for that portion explained by price changes, as a proxy for differential interpretations of the same information. Our model suggests a different explanation: excess volume can arise, as in Figure 7, because traders are rationally using the same information to serve different needs.

In data there is relatively strong evidence that firm-specific announcements generate abnormal

[17] These differential interpretations arise because traders have idiosyncratic private information about the error in the announcement that only becomes useful when the announcement arrives. The operational effect is similar to Kandel and Pearson's [23] assumption that traders interpret the same news with different likelihood functions.

volume.[18] At the aggregate level, Mitchell and Mulherin [31] find that Dow Jones news announcements are strongly correlated with aggregate market volume but relatively weakly related to the sum of firms' absolute price changes. For individual stocks, Kandel and Pearson [23], find abnormal volume in a window around earnings announcements, even after controlling for a positive relationship with the magnitude of price changes in that window. They appeal to heterogeneous interpretations of news to explain how an announcement could generate substantial volume relative to a small price impact. Our model offers a complementary explanation: it can be fully rational for traders to use news in opposite ways due to differences in the 'portfolios' of other information they have access to. In this case, prominent news can be a convenient coordination device for traders who need to take opposite sides of the market. While it is outside our analysis in this paper, this coordination issue may also shed light on Stice's [33] puzzling finding that a Wall Street Journal report about a firm's 10-K or 10-Q filing sparks abnormal price and volume activity, even when the filing was submitted (and made publicly available) several days earlier. Conceivably a delayed response to news could be self supporting if traders who plan to respond in opposite ways wait until a time when they expect the other side of the market to be deep.

To apply our model more directly to data, it would be advantageous to have some exogenous proxy for a news source's precision. In studies focused on analyst reports, a common criterion for identifying top quality analysts is selection as an *Institutional Investor* All American (AA). Stickel ([34] and [35]) find evidence that AA analysts' revisions generate larger (in magnitude) short run abnormal returns than non-AA analysts' revisions do, but that this excess impact tends to be reversed in long run returns.[19] This is loosely consistent with our predictions (1) and (4). Park and Stice [32] apply Stickel's approach using a different quality measure based on an analyst's past forecasting accuracy relative to the I/B/E/S consensus. They also find that revisions by top quality analysts generate significantly larger short run price impacts than revisions by other analysts. Their results suggest that the relationship between an analyst's percentile in the quality ranking and the price impact of his revision is positive but relatively weak below the 80^{th} percentile. However, as an analyst rises above the 80^{th} or 90^{th} percentile, his price impact rises substantially. This is similar to the convex relationship that we find between τ_n and λ_n (because λ_n/τ_n is increasing in τ_n), although without knowing the cumulative distribution function for analyst precision we cannot draw any firm conclusions.

A related paper by Loh and Stultz [29] provides some suggestive evidence for prediction (2). Rather than look at the average response to an analyst's revision as an outcome variable, they study the probability that the revision generates a "large" response, as measured by abnormal returns or volume exceeding pre-defined thresholds. Among the explanatory variables are the analyst's past forecast accuracy, a continuous measure of quality, and AA status, which could be interpreted

[18] See, for example, Bamber [4] for earnings announcements, or Womack [41] for brokerage recommendations.

[19] In recent work, Fang and Yasuda [10] find that portfolios formed by following AA analyst buy recommendations (but not sell recommendations) do earn short run excess returns, but that these excess returns are not reversed in the long run. However, given the portfolio methodology, it is difficult to draw conclusions about long run price impacts for any single stock.

as an indicator variable singling out the right tail of the analyst quality distribution. Even after controlling for forecast accuracy, AA analysts' revisions are substantially more likely to have large impacts on both price and volume.

Finally, in one strand of work that touches on prediction (3), dispersion of analysts' recommendations is used to proxy for the absence of a strong consensus belief among investors. In a typical result, Atiase and Bamber [3] find that analyst dispersion is associated with higher trading volume around earnings announcements. One possible explanation, as with the Kandel and Pearson findings, is that traders interpret the same signal differently. Our model offers the complementary possibility that analyst dispersion is associated with periods when market order traders face substantial price risk, making it particularly important for them to closely monitor (and trade on) news that is expected to move the price.

6 Concluding Remarks

We show that if there are many potential sources of information about an asset but a trader must focus on one of them to use its information profitably, then most traders may concentrate on a few precise and popular sources, leaving other information under-utilized. Furthermore, differences in baseline information – in our case, market order traders who give up conditioning on the price in order to exploit additional private information – can motivate rational traders to focus on the same news but respond to it in opposite ways. Together, these effects can explain why one signal may generate substantial price movement and trading volume, while another signal of apparently similar quality does not. Below we comment on the robustness of these conclusions and suggest a few extensions.

By restricting traders to one source, we have implicitly assumed that there is a capacity limit on how much information a trader can acquire, absorb, and profit from before it becomes stale – this amounts to a highly convex cost of information. If this limit were relaxed so that a trader could follow more than one news source, we would expect concentration to be less extreme, but the results should be qualitatively similar. A capacity limit may be reasonable if inelastic time and attention are important components of the cost of information, but there is probably a degree to which both time and attention can be multiplied with additional cash. Thus it would be useful to know how our results would change if a trader can acquire more signals, or higher quality signals, by spending more money.

Of course, signal costs rising with precision might emerge endogenously if news sources set prices for their information. In a setting with a monopolist information provider, Admati and Pfleiderer [1] have shown that profit maximization affects how information will be packaged for buyers (in some cases with extra noise added), thus making the set of available signals endogenous. Less is known about oligopoly competition among information providers. In our model, if one is willing to take the news source precisions as given, it is straightforward to append a first stage with price competition among news sources. One loses the simple geometric representation of traders' indifference curves, but the model can still be solved computationally.[20] One would expect price competition to lead to less concentration by traders, since popular sources will charge relatively higher prices. Examples suggest that this is correct, but concentration is not wiped out. Moreover, if the market order - limit order complementarity gives a news source a disproportionately large market share, this does not mean that it has substantial market power. If it has a close competitor on the characteristics that matter to traders, precision and price impact, then if it charges too much, the excess market share will switch *en masse* to the competitor.[21] It appears that competition for this bloc of traders can keep information prices fairly competitive.

While a static model is useful for examining how efficiently the market aggregates dispersed information, the conceit that news sources release new signals at the same time is stylized. Within the scope of the model, one could make this more realistic by, for example, assuming that each news source fails to deliver a signal with some probability. However, ultimately it would make sense to study which news sources traders choose to follow in a dynamic model. A dynamic model could also allow traders' order types to emerge endogenously, which might yield interesting insights. Finally, an alternative to our assumption that the market clears without intermediation would be to introduce a market-maker who sets a competitive price conditional on the order flow as in Kyle [27]. If this market-maker can acquire additional information, then one can show that he and the market order traders may have incentives to coordinate on the same information, for reasons that are similar to the complementarity between market and limit order traders in our model.[22]

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[20] A price of c_n at news source n introduces an extra multiplicative factor of $e^{c_n/\gamma}$ into our expressions for the *ex ante* utility of a trader with risk tolerance γ; see Grossman and Stiglitz [18] or Admati and Pfleiderer [2].

[21] The situation would be different if a source's value to any trader were to rise with the *total* number of other traders. That type of one-sided network effect creates a wedge in value that discourages demand from switching away. The network effect here is two-sided and self-regulating: a trader is attracted to a source when the ratio of traders acquiring it favors the other order type. This prevents the value of a popular news source from running away from its competitors.

[22] Specifically, if market order traders expect the market-maker to incorporate a signal y_n into the price, then they may have an incentive to acquire y_n in order to trade against it. But then the market-maker may need to acquire y_n, not for its own sake *per se*, but in order to correct for its effect on the order flow before estimating θ.

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7 Appendix

Proof of Lemma 2

Any solution to $f(\tau,\lambda) = K$ must also satisfy $Qf(\tau,\lambda) = QK$ where $Q = \tau\left(s_p - \lambda_w^2/\tau_w - \lambda^2/\tau\right)$. With algebra, $Qf(\tau,\lambda) - QK = 0$ can be written as the quadratic form $[\tau\,\lambda\,1]\;Z\;[\tau\,\lambda\,1]' = 0$, where

$$Z = \begin{bmatrix} s_p - \lambda_w^2/\tau_w & -(1-\lambda_w) & -\frac{1}{2}(K - f_0)\left(s_p - \lambda_w^2/\tau_w\right) \\ -(1-\lambda_w) & K & 0 \\ -\frac{1}{2}(K - f_0)\left(s_p - \lambda_w^2/\tau_w\right) & 0 & 0 \end{bmatrix}$$

This quadratic form is an ellipse if $|Z| \neq 0$, $|Z_{33}| > 0$ (where Z_{33} is the leading principal minor), and the diagonal entries of Z_{33} do not have the same sign as $|Z|$. Checking these conditions, we have $|Z| = -\frac{1}{4}K(K - f_0)\left(s_p - \frac{\lambda_w^2}{\tau_w}\right)^2 < 0$, $|Z_{33}| = \left(s_p - \lambda_w^2/\tau_w\right)\left(K - \frac{(1-\lambda_w)^2}{s_p - \lambda_w^2/\tau_w}\right) = \left(s_p - \lambda_w^2/\tau_w\right)(K - f_0) > 0$, and the diagonal entries of Z_{33} are positive.

The fact that A, B, C, and D lie on this ellipse and the vertical tangencies at A and C are easy to check directly. Note that the same points A and B lie on every ellipse in the family $Qf(\tau,\lambda) = QK$ indexed by K, but as singularities of f, they do not belong to the level curve $f(\tau,\lambda) = K$ for any $K > f_0$. The sign of $f(\tau,\lambda) - K$ must be constant over the interior or exterior of ellipse $Qf(\tau,\lambda) = QK$; to confirm that this sign is negative on the interior and positive on the exterior, check the value of f at the midpoint of AB and at $(\tau, 0)$ for τ large.

Proof of Proposition 2

Let $\lambda_{\inf}^N$ be the price impact of one of the inferior sources in an equilibrium with N sources. (Indifference requires that all $N-1$ of these sources have the same price impact.) Let $l_{\inf}^N$ be the fraction of traders who acquire each inferior source. We have $\lambda_{\inf}^N \geq 0$, $\lambda_{\sup}^N \geq 0$, and $(N-1)\,\lambda_{\inf}^N + \lambda_{\sup}^N \leq 1$, so we must have $\lim_{N\to\infty} \lambda_{\inf}^N = 0$. A similar argument applies to $l_{\inf}^N$.

For the superior source, first note that we can bound ρ above and below, uniformly in N: $0 < \underline{\rho} \leq \rho \leq \bar{\rho} < \infty$. (See the proof of Proposition 6.) Next, fix an equilibrium with N sources. We either have $l_{\text{sup}}^N = 1$ or $l_{\text{sup}}^N < 1$. In the former case, only the superior source is acquired; working through (11) we find $\lambda_{\text{sup}}^N = \frac{\bar{\tau}+\varepsilon}{(\bar{\tau}+\varepsilon)+\tau_w}$. Alternatively, suppose that $l_{\text{sup}}^N < 1$, so inferior sources are acquired. Then indifference requires $f\left(\bar{\tau}, \lambda_{\text{inf}}^N\right) = f\left(\bar{\tau}+\varepsilon, \lambda_{\text{sup}}^N\right)$. Let Δ be the slope of this indifference curve where it crosses the $\lambda = 0$ axis. Convexity of the indifference curve then implies that $\lambda_{\text{sup}}^N \geq \lambda_{\text{inf}}^N + \Delta\varepsilon$. From the proof of Lemma 8 we have $\Delta = \frac{1}{2}\frac{s_p^N - \left(\lambda_w^N\right)^2/\tau_w}{1-\lambda_w} \geq \frac{\underline{\rho}^2}{2\tau_z}$, so $\lambda_{\text{sup}}^N \geq \lambda_{\text{inf}}^N + \frac{\underline{\rho}^2}{2\tau_z}\varepsilon \geq \frac{\underline{\rho}^2}{2\tau_z}\varepsilon$. Then applying (11), and using $\rho \leq \bar{\rho}$ and $\beta_y^{\text{sup}} \leq \gamma_L\left(\bar{\tau}+\varepsilon\right)$, we have $l_{\text{inf}}^N \geq \frac{\lambda_{\text{sup}}^N}{L\gamma_L\bar{\rho}(\bar{\tau}+\varepsilon)} \geq \frac{\underline{\rho}^2}{2L\gamma_L\bar{\rho}\tau_z(\bar{\tau}+\varepsilon)}\varepsilon$. To summarize, in any equilibrium with N sources, we have $\lambda_{\text{sup}}^N \geq \min\left(\frac{\bar{\tau}+\varepsilon}{(\bar{\tau}+\varepsilon)+\tau_w}, \frac{\underline{\rho}^2}{2\tau_z}\varepsilon\right)$ and $l_{\text{sup}}^N \geq \min\left(1, \frac{\underline{\rho}^2}{2L\gamma_L\bar{\rho}\tau_z(\bar{\tau}+\varepsilon)}\varepsilon\right)$. Thus λ_{sup}^N and l_{sup}^N are bounded away from zero uniformly in N, as claimed.

Proof of Proposition 3

Consider a prospective equilibrium in which all traders acquire N. If we set aside the *ex ante* utility maximization condition for the moment, by standard results we can find an order strategy $\left(\beta_y^N, \beta_y^w\right)$ for traders and a price function P that are mutually consistent (in the sense that $\left(\beta_y^N, \beta_y^w\right)$ is interim optimal with respect to the P, and the P is generated by $\left(\beta_y^N, \beta_y^w\right)$). If in addition to this, no trader would prefer to deviate by acquiring some other source instead, then $\left\{l_N = 1, \left(\beta_y^N, \beta_y^w\right), P\right\}$ is an equilibrium. Because unacquired sources will have identical price impacts $\lambda_1 = ... = \lambda_{N-1} = 0$ in this prospective equilibrium, it will suffice to check for deviations to the next most precise source, $N-1$.

Next consider a few facts about the prospective equilibrium. It is straightforward to show that $\lambda_N + \lambda_w = 1$. Because the price provides no additional information after controlling for y_n and w, the posterior precision of a trader's estimate of θ is just $\hat{\tau}_l^N = \tau_N + \tau_w$. For the order strategy, we have (consulting (7) and (8)) $\beta_y^N = \gamma_L\tau_N$ and $\beta_w^N = \gamma_L\tau_w$, so by (11) we have $\lambda_N = \tau_N/\left(\tau_N+\tau_w\right)$, $\lambda_w = \tau_w/\left(\tau_N+\tau_w\right)$, and $\rho^{-1} = L\gamma_L\left(\tau_N+\tau_w\right)$. The variance of the price will be $s_p = \lambda_N^2/\tau_N + \lambda_w^2/\tau_w + \rho^2/\tau_z$.

Now turn to a potential deviation to source $N-1$. Using $\lambda_{N-1} = 0$, (9), and the results above, we have

$$\hat{\tau}_l^{N-1} = \tau_{N-1} + \tau_w + \frac{\lambda_N^2}{s_p - \lambda_w^2/\tau_w} = \tau_{N-1} + \tau_w + \frac{1}{\frac{1}{\tau_N} + \frac{1}{\tau_z}\left(\frac{\rho}{\lambda_N}\right)^2}$$

The condition that rules out a deviation to $N-1$ is $\hat{\tau}_l^N \geq \hat{\tau}_l^{N-1}$, or, using $\rho/\lambda_N = 1/\left(L\gamma_L\tau_N\right)$:

$$\tau_N - \tau_{N-1} \geq \left(\frac{1}{\tau_N} + \frac{1}{\tau_z}\frac{1}{\left(L\gamma_L\tau_N\right)^2}\right)^{-1}$$

With rearranging, this condition becomes: $\left(\tau_N - \tau_{N-1}\right)(1-D) \geq D\tau_{N-1}$. The condition cannot hold if $D \geq 1$, since we have $\tau_N > \tau_{N-1}$. In this case, there cannot be an all-N equilibrium. If $D < 1$, then the prospective equilibrium is valid if $\tau_N - \tau_{N-1} \geq \tau_{N-1}D/(1-D)$. Note that no part of this argument depends on the number of signals N.

Proof of Proposition 4

The style is similar to the proof of Proposition 2. For each market N, let n_1^N be the worst signal that is acquired in equilibrium. We will use the constraint that $\sum_{n_1^N}^N \lambda_n \leq 1$ to show that the fraction of signals acquired, $\frac{N-n_1^N}{N}$, shrinks like $\frac{1}{\sqrt{N}}$ as N grows. Follow the proof of Proposition 2 to show

$$\lambda_n^N \geq \lambda_n^N - \lambda_{n_1^N}^N \geq \frac{\underline{\rho}^2}{2\tau_z}\left(\tau_n - \tau_{n_1^N}\right) = \frac{\underline{\rho}^2}{2\tau_z}\left(\Psi^{-1}(\frac{n}{N}) - \Psi^{-1}(\frac{n_1^N}{N})\right)$$

for all $n \in \{n_1^N, ..., N\}$. Let C be a Lipschitz constant for Ψ, so that we have $\Psi^{-1}(b) - \Psi^{-1}(a) \geq \frac{1}{C}|b-a|$

for all $a, b \in [\tau_L, \tau_H]$. Thus we have $\lambda_n^N \geq \frac{\rho^2}{2\tau_z CN}\left(n - n_1^N\right)$. Adding up over all acquired signals, we have

$$\textstyle\sum_{n_1^N}^N \lambda_n \geq \frac{\underline{\rho}^2}{2\tau_z CN} \sum_{n_1^N}^N \left(n - n_1^N\right) = \frac{\underline{\rho}^2}{4\tau_z CN} d^N (d^N - 1)$$

where $d^N = N - n_1^N + 1$ is the number of acquired signals. Applying $\sum_{n_1^N}^N \lambda_n \leq 1$, we have

$$\frac{\left(d^N\right)^2}{N} \leq \frac{4\tau_z C}{\underline{\rho}^2} + 1$$

so d^N grows no faster than $\sqrt{N}$ with the total number of signals. Thus, $d^N/N \to 0$ and the total precision of all the information acquired by traders is bounded above by $\tau_H d^N$, proving the proposition.

Proof of Proposition 5

We prove the risk tolerance case; the argument for $\tau_z \to \infty$ is essentially the same. Consider a sequence $\{(\gamma_L)_k\}_{k\geq 1} \to \infty$, and for each risk tolerance level in the sequence, fix an equilibrium. We write $(r_n)_k = (\lambda_n)_k / \tau_n$ and $(r_w)_k = (\lambda_w)_k / \tau_w$, and let $\bar{n}_k$ be the lowest numbered source that is acquired in equilibrium. Henceforth we drop the subscripts k to avoid clutter. Note that with rearranging we can write

$$B^n \equiv \beta_y^n / \gamma_L = \tau_n \frac{\rho^2/\tau_z + \sum_{n' \neq n} (r_{n'} - r_n) r_{n'} \tau_{n'}}{\rho^2/\tau_z + \sum_{n' \neq n} r_{n'}^2 \tau_{n'}} \quad \text{and} \quad B_w^n \equiv \beta_w^n / \gamma_L = \tau_w \frac{\rho^2/\tau_z + \sum_{n' \neq n} (r_{n'} - r_w) r_{n'} \tau_{n'}}{\rho^2/\tau_z + \sum_{n' \neq n} r_{n'}^2 \tau_{n'}}$$

<u>Claim 1</u> $\gamma_L \to \infty$ implies $\rho^2/\tau_z \to 0$.

Equilibrium B^n and B_w^n are weakly positive for all n. Furthermore, because r_n is weakly increasing in n, we have $B^{\bar{n}} \geq \tau_{\bar{n}} \left(\rho^2/\tau_z\right) \left(\rho^2/\tau_z + \sum_{n' \neq n} r_{n'}^2 \tau_{n'}\right)^{-1}$. Suppose toward a contradiction that $\rho^2/\tau_z \nrightarrow 0$. Then $B^{\bar{n}}$ must be bounded away from zero, but then (11) and $\gamma_L \to \infty$ imply that $\rho^2/\tau_z \to 0$.

<u>Claim 2</u> $\rho^2/\tau_z \to 0$ implies $r_n - r_{n'} \to 0$, $B^n \to 0$, and $B^{n'} \to 0$ for all n, n' such that $\lim_{\gamma_L \to \infty} r_n > 0$ and $\lim_{\gamma_L \to \infty} r_{n'} > 0$.

Consider the numerator of B^N. Because r_n is weakly increasing in n, we have $\sum_{n' \neq N} (r_{n'} - r_N) r_{n'} \tau_{n'} = -\sum_{n' \neq N} |r_N - r_{n'}| r_{n'} \tau_{n'} \leq 0$. Thus, because $B^N \geq 0$, we have $\sum_{n' \neq N} |r_N - r_{n'}| r_{n'} \tau_{n'} \leq \rho^2/\tau_z$. But then, for any n' such that $\lim_{\gamma_L \to \infty} r_{n'} > 0$, we must have $r_N - r_{n'} \to 0$. The first claim follows. Then in the numerator of B^n, each term in the sum satisfies $(r_{n'} - r_n) r_{n'} \to 0$, so we have $B^n \to 0$.

<u>Claim 3</u> $r_N - r_w \to 0$

Write the numerator of B_w^n as $\rho^2/\tau_z + \sum_{n' \neq n} (r_{n'} - r_N) r_{n'} \tau_{n'} + \sum_{n' \neq n} (r_N - r_w) r_{n'} \tau_{n'}$. Then because the first two terms tend to 0, the weak positivity of B_w^n implies that $\lim_{\gamma_L \to \infty} (r_N - r_w) \geq 0$. Suppose toward a contradiction that $\lim_{\gamma_L \to \infty} (r_N - r_w) > 0$. This would imply B_w^n eventually strictly positive and bounded away from 0, and therefore, by Claim 2, that $B^N / B_w^N \to 0$. However, from (11) we have $r_N / r_w \leq \frac{\tau_N}{\tau_w} \frac{B^N}{B_w^N} \to 0$, a contradiction. Thus, $\lim_{\gamma_L \to \infty} (r_N - r_w) > 0$.

<u>Claim 4</u> Suppose $\lim_{\gamma_L \to \infty} \bar{n} = \bar{n}^\infty$, and let $\tau_p = \sum_{n=\bar{n}^\infty}^N \tau_n + \tau_w$. The coefficients of the equilibrium price function satisfy $\lambda_w \to \tau_w / \tau_p$ and $\lambda_n \to \left\{ \begin{array}{ll} \tau_n / \tau_p & \text{if } n \geq \bar{n}^\infty \\ 0 & \text{if } n < \bar{n}^\infty \end{array} \right.$.

This follows from the earlier claims and the adding up constraint $\sum_{n=1}^N \lambda_n + \tau_w = 1$.

<u>Claim 5</u> $\bar{n} \to 1$ (All sources are acquired.)

Suppose toward a contradiction that $\lim_{\gamma_L \to \infty} \bar{n} = \bar{n}^\infty > 1$. The limiting price function is sufficient for $\{y_{\bar{n}^\infty}, y_{\bar{n}^\infty+1}, ..., y_N, \tau_w\}$, so the posterior precision of a trader choosing source $\bar{n}^\infty$ or higher tends to τ_p. But then the posterior precision of a trader who chose source 1 and observed $\{y_1, w, p\}$ would tend to $\tau_1 + \tau_p$, so source 1 must be observed in the limit after all.

Proof of Lemma 7

Let $\bar{g} = g(\tau_n, \lambda_n)$, so the curve of interest is $g(\tau, \lambda) = \bar{g} > g_0$. We proceed just as for Lemma 2. Let $\tilde{Q} = \tau (\tau + \tau_w + \tau_x)$ and clear denominators, noting that any solution to $g(\tau, \lambda) = \bar{g}$ must also solve $\tilde{Q} (g(\tau, \lambda) - \bar{g}) = 0$. This last equation can be written as the quadratic form $[\tau \, \lambda \, 1] \, \tilde{Z} \, [\tau \, \lambda \, 1]' = 0$, where

$$\tilde{Z} = \begin{bmatrix} \bar{g} & \lambda_w & \frac{1}{2}\left(\bar{g}(\tau_w + \tau_x) + \lambda_w^2\right) \\ \lambda_w & -(\tau_w + \tau_x) & 0 \\ \frac{1}{2}\left(\bar{g}(\tau_w + \tau_x) + \lambda_w^2\right) & 0 & 0 \end{bmatrix}$$

To show this curve is a hyperbola, it suffices to show $\left|\tilde{Z}\right| \neq 0$ and $\left|\tilde{Z}_{33}\right| < 0$. We have $\left|\tilde{Z}\right| = \frac{1}{4}\left(\tau_w+\tau_x\right)^2\left(\bar{g}+\frac{\lambda_w^2}{\tau_w+\tau_x}\right)^2 = \frac{1}{4}\left(\tau_w+\tau_x\right)^2\left(\bar{g}-g_0\right)^2 > 0$, and $\left|\tilde{Z}_{33}\right| = -\bar{g}\left(\tau_w+\tau_x\right)-\lambda_w^2 = -\left(\tau_w+\tau_x\right)\left(\bar{g}-g_0\right) < 0$. The tangency at $(0,0)$ is an easy application of the implicit function theorem, and the ranking of indifference curves is straightforward.

Proof of Proposition 6

We overload notation by writing $\vec{P} = (\lambda_1, ..., \lambda_N, \lambda_w, \lambda_\theta, \rho)$ for a vector of price coefficients. Let $D = D_l \times D_m \times D_P \subset \Re^{3N+3}$ be a compact set that we will define presently. Let $\Gamma = (\Gamma_l, \Gamma_m, \Gamma_P) : D \rightarrow\rightarrow \Re^{3N+3}$ be a correspondence over triples of vectors $\left(\vec{l}, \vec{m}, \vec{P}\right)$. Intuitively, we will have $\Gamma_P\left(\vec{l}, \vec{m}, \vec{P}\right)$ return the price function that would arise if traders respond optimally to $\left(\vec{l}, \vec{m}, \vec{P}\right)$, and $\Gamma_l\left(\vec{l}, \vec{m}, \vec{P}\right)$ and $\Gamma_m\left(\vec{l}, \vec{m}, \vec{P}\right)$ return the sets of market shares $\vec{l}$ and $\vec{m}$ that are consistent with optimal news choice with respect to $\vec{P}$. Then we will prove that Γ has a fixed point.

Formally, define $\Gamma_P = (\Gamma_{\lambda_1}, ..., \Gamma_{\lambda_N}, \Gamma_{\lambda_w}, \Gamma_{\lambda_\theta}, \Gamma_\rho) : D \rightarrow\rightarrow \Re^{N+3}$ as follows. Given an input $\left(\vec{l}_0, \vec{m}_0, \vec{P}_0\right)$, let $\left(\vec{\beta}_1, \vec{\alpha}_1\right)$ be the order strategies induced by $\vec{P}_0$ according to (3) and (7), (8), (12), and (13). Let $\Gamma_\rho\left(\vec{l}_0, \vec{m}_0, \vec{P}_0\right)$ be the ρ price coefficient induced, according to (16), by market shares $\left(\vec{l}_0, \vec{m}_0\right)$ and order strategies $\left(\vec{\beta}_1, \vec{\alpha}_1\right)$. Similarly, let $\Gamma_{\lambda_w}\left(\vec{l}_0, \vec{m}_0, \vec{P}_0\right)$ be the price coefficient induced on w, and $\Gamma_{\lambda_n}\left(\vec{l}_0, \vec{m}_0, \vec{P}_0\right)$ the price coefficient induced on source n, according to (16), by market shares $\left(\vec{l}_0, \vec{m}_0\right)$ and order strategies $\left(\vec{\beta}_1, \vec{\alpha}_1\right)$. Finally, let $\Gamma_{\lambda_\theta}\left(\vec{l}_0, \vec{m}_0, \vec{P}_0\right) = 1 - \Gamma_{\lambda_w}\left(\vec{l}_0, \vec{m}_0, \vec{P}_0\right) - \sum_{n=1}^{N} \Gamma_{\lambda_n}\left(\vec{l}_0, \vec{m}_0, \vec{P}_0\right)$ be the residual. In passing, notice that Γ_P is a function.

Next, the market shares. For brevity let us write $\vec{P}_1 = \Gamma_P\left(\vec{l}_0, \vec{m}_0, \vec{P}_0\right)$ for the new price function generated from $\left(\vec{l}_0, \vec{m}_0, \vec{P}_0\right)$. Let $f_n = f\left(\tau_n, \lambda_n\right)|_{\vec{P}_1}$ and $g_n = g\left(\tau_n, \lambda_n\right)|_{\vec{P}_1}$ be the values of source n to limit and market order traders respectively when evaluated according to $\vec{P}_1$. Let $\bar{f} = \max_{n\in\{1,...,N\}} f_n$ and $\bar{g} = \max_{n\in\{1,...,N\}} g_n$. Now let $\Gamma_l\left(\vec{l}_0, \vec{m}_0, \vec{P}_0\right) = \left\{\vec{l} \in \Delta_N \mid \left(\bar{f} - f_n\right) l_n = 0\ \forall n\right\}$. In other words, take the set of all distributions over news sources that put zero weight on strictly suboptimal sources. Similarly, set $\Gamma_m\left(\vec{l}_0, \vec{m}_0, \vec{P}_0\right) = \{\vec{m} \in \Delta_N \mid \left(\bar{g} - g_n\right) m_n = 0\ \forall n\}$.

The domains for $\vec{l}$ and $\vec{m}$ are straightforward; set $D_l = D_m = \Delta_N$. For $\vec{P}$, set $D_P = \Delta_{N+2} \times \left[\underline{\rho}, \bar{\rho}\right]$. Notice that any $\vec{P} \in D_P$ satisfies $\lambda_w \in [0,1]$, a relaxed version of A2, as well as $\lambda_n \geq 0$ and $1 - \lambda_w - \sum_{n=1}^{N} \lambda_n \geq 0$. The bounds $\underline{\rho}$ and $\bar{\rho}$ are defined by $\underline{\rho}^{-1} = L\gamma_L\left(\tau_w + \tau_N\right)$ and $\bar{\rho}^{-1} = \bar{r}$, where $\bar{r}$ is the positive solution to the quadratic equation $r = L\gamma_L\left(\tau_w + \tau_1 - \tau_z r^2\right)$. (To confirm that $\underline{\rho} < \bar{\rho}$, note that $\underline{\rho}^{-1} > L\gamma_L\left(\tau_w + \tau_1\right) > \bar{\rho}^{-1}$.)

Now we can introduce a version of the Browder fixed point theorem due to Border [6].

Theorem 1 *Let $D \subset \Re^k$ be compact and convex and let $\Gamma : D \rightarrow\rightarrow \Re^k$ be an upper hemicontinuous correspondence with non-empty closed convex values. Define ∂D to be the boundary of D. If for every point $d \in \partial D$, there exists some $d' \in \Gamma(d)$ and some $\kappa > 0$ such that $(1-\kappa)d + \kappa d' \in D$, then Γ has a fixed point in D.*

Loosely, this says that if the "gradient set" $\Gamma(d) - d$ has an element that points into D at every point on D's surface, then there must be a point within D at which some member of the set $\Gamma(d) - d$ is equal to zero. To apply the theorem to Γ, note that Γ_P is a continuous function (and so its values are trivially closed and convex). Furthermore, the values of Γ_l constitute a simplex $\Delta_{\tilde{n}} \subseteq \Delta_N$, where $1 \leq \tilde{n} \leq N$ is the number of ex ante optimal sources, so these values are also closed, convex, and non-empty. Γ_l is defined by weak inequalities on the "quasi-utilities" f_n, which are continuous functions of $\vec{P}$, so Γ_l is uhc. (And similarly for Γ_m.)

Next note that we do not need to worry about the boundary condition for Γ_l and Γ_m because $\Gamma_l\left(\vec{l}, \vec{m}, \vec{P}\right) \subseteq D_l$ and $\Gamma_m\left(\vec{l}, \vec{m}, \vec{P}\right) \subseteq D_m$ by construction. Then, because Γ_P is a function, it will suffice to that for all

$\left(\vec{l}, \vec{m}, \vec{P}\right) \in \partial D$, there is some $\kappa > 0$ such that $(1-\kappa)\vec{P} + \kappa \Gamma_P \left(\vec{l}, \vec{m}, \vec{P}\right) \in D_P$. Note that ∂D is defined by the linear constraints $\lambda_n \geq 0$, $\lambda_w \geq 0$, $\lambda_\theta \geq 0$, and $\underline{\rho} \leq \rho \leq \bar{\rho}$. We start by considering boundary points where exactly one of these constraints binds and the others are slack. For these cases, it suffices to show that the binding constraint is satisfied at $\Gamma_P \left(\vec{l}, \vec{m}, \vec{P}\right)$, since κ may be chosen small enough that none of the constraints that are slack at $\vec{P}$ become violated at $(1-\kappa)\vec{P} + \kappa \Gamma_P \left(\vec{l}, \vec{m}, \vec{P}\right)$.

$\underline{\lambda_n = 0\text{, other constraints are slack.}}$

Suppose that $\lambda_n = 0$ in the vector $\vec{P}$. Then by (12) and (7), $\vec{P}$ generates order strategies of $\alpha_y^n = \frac{\gamma_M}{s_m^n} \frac{\tau_n}{\tau_x + \tau_n + \tau_w} \lambda_w \geq 0$ and $\beta_y^n = \gamma_L \tau_n > 0$ at source n. With (16) and $m_n \geq 0$ and $l_n \geq 0$, this implies that $\Gamma_{\lambda_n} \left(\vec{l}, \vec{m}, \vec{P}\right) \geq 0$.

$\underline{\lambda_w = 0\text{, other constraints are slack.}}$

Suppose that $\lambda_w = 0$ in the vector $\vec{P}$. Then (13) and (8) imply that $\vec{P}$ generates order strategies of $\alpha_w^n = \frac{\gamma_M}{s_m^n} \frac{\tau_w}{\tau_x + \tau_n + \tau_w} \lambda_n \geq 0$ and $\beta_w^n = \gamma_L \tau_w > 0$ for all n. Consulting (16), we have $\Gamma_{\lambda_w} \left(\vec{l}, \vec{m}, \vec{P}\right) > 0$.

$\underline{\lambda_\theta = 1 - \lambda_w - \sum_{n=1}^N \lambda_n = 0\text{, other constraints are slack.}}$

Suppose this is true at $\vec{P}$. Note that we can write (16) as $\lambda_\theta = \rho M \sum_{n=1}^N m_n \alpha_x^n$. By (14), $\vec{P}$ generates the order strategy $\alpha_x^n = \frac{\gamma_M}{s_m^n} \frac{\tau_x}{\tau_x + \tau_n + \tau_w} (\lambda_n + \lambda_w) \geq 0$ for each n, so with $m_n \geq 0$ we have $\Gamma_{\lambda_\theta} \left(\vec{l}, \vec{m}, \vec{P}\right) \geq 0$.

$\underline{\rho = \underline{\rho}\text{, other constraints are slack.}}$

By (7) and (8), $\vec{P}$ generates order strategies satisfying $\beta_y^n \leq \gamma_L \tau_n$ and $\beta_w^n \leq \gamma_L \tau_w$ for all n. Then using (16) we have $\left(\Gamma_\rho \left(\vec{l}, \vec{m}, \vec{P}\right)\right)^{-1} \leq L \gamma_L \left(\tau_w + \sum_{n=1}^N l_n \tau_n\right) \leq L \gamma_L \left(\tau_w + \tau_N\right)$, so we have $\Gamma_\rho \left(\vec{l}, \vec{m}, \vec{P}\right) \geq \underline{\rho}$.

$\underline{\rho = \bar{\rho}\text{, other constraints are slack.}}$

By (7) and (8), $\vec{P}$ generates order strategies satisfying

$$\beta_y^n + \beta_w^n = \gamma_L(\tau_n + \tau_w) - \gamma_L \frac{(\lambda_n + \lambda_w)(1 - \lambda_w - \lambda_n)}{\sum_{n' \neq n} \lambda_{n'}^2 / \tau_{n'} + \bar{\rho}^2 / \tau_z} > \gamma_L(\tau_1 + \tau_w) - \gamma_L \tau_z \left(\frac{1}{\bar{\rho}}\right)^2$$

Thus, using (16) we have $\left(\Gamma_\rho \left(\vec{l}, \vec{m}, \vec{P}\right)\right)^{-1} \geq L \gamma_L \left(\tau_1 + \tau_w - \tau_z \left(1/\bar{\rho}\right)^2\right) = 1/\bar{\rho}$. Thus $\Gamma_\rho \left(\vec{l}, \vec{m}, \vec{P}\right) \leq \bar{\rho}$.

From these arguments, it is straightforward to see that if some combination of these constraints binds at $\vec{P}$, while others are slack, then $\Gamma_P \left(\vec{l}, \vec{m}, \vec{P}\right)$ satisfies all the conditions binding at $\vec{P}$, so by taking $\kappa > 0$ small enough, we have $(1-\kappa)\vec{P} + \kappa \Gamma_P \left(\vec{l}, \vec{m}, \vec{P}\right) \in D_P$ for all $\left(\vec{l}, \vec{m}, \vec{P}\right) \in \partial D$. Applying the theorem, we conclude that Γ has a fixed point in D. Note that A1, which amounts to $\rho > 0$, is satisfied by construction. Also by construction, we have $\lambda_w \in [0, 1]$, but for A2 we need to show that $\lambda_w = 0$ and $\lambda_w = 1$ cannot be part of a fixed point. First consider some price function $\vec{P}$ with $\lambda_w = 1$. This implies $\lambda_\theta = 0$ and $\lambda_n = 0$ for all n, so in particular, $\sum_{n=1}^N \lambda_n = 0$. But then (following the argument for the λ_n case above) $\vec{P}$ would generate order strategies satisfying $\alpha_y^n \geq 0$ and $\beta_y^n > 0$ for all n. But this, using (16), would imply $\sum_{n=1}^N \Gamma_{\lambda_n} \left(\vec{l}, \vec{m}, \vec{P}\right) > 0$, so $\vec{P}$ cannot be part of a fixed point. Next consider a price function $\vec{P}$ with $\lambda_w = 0$. Then the argument above for the $\lambda_w = 0$ constraint applies directly: $\Gamma_{\lambda_w} \left(\vec{l}, \vec{m}, \vec{P}\right) > 0$, so $\vec{P}$ cannot be part of a fixed point. Thus there exists an equilibrium satisfying A2 as well. The condition that $\lambda_n \leq 1 - \lambda_w$ is implied by the constraints that $\lambda_n = 1 - \lambda_w - \lambda_\theta - \sum_{n' \neq n} \lambda_{n'}$ and the positivity of λ_w, λ_θ, and $\lambda_{n'}$. Finally, for the strict positivity of λ_n if n is acquired, use $l_n + m_n > 0$ and $\lambda_w > 0$ in the argument for the $\lambda_n = 0$ constraint, to show that $\Gamma_{\lambda_n} \left(\vec{l}, \vec{m}, \vec{P}\right) > 0$.

Proof of Lemma 8

Families of indifference curves are completely characterized by the equilibrium values of s_p and λ_w and the precisions τ_w and τ_x. We will show, *a fortiori*, that for any point $\left(\bar{\tau}, \bar{\lambda}\right)$ with $0 \leq \bar{\lambda} \leq 1 - \lambda_w$, the limit order indifference curve through $\left(\bar{\tau}, \bar{\lambda}\right)$ is steeper than the market order indifference curve at that point, if a

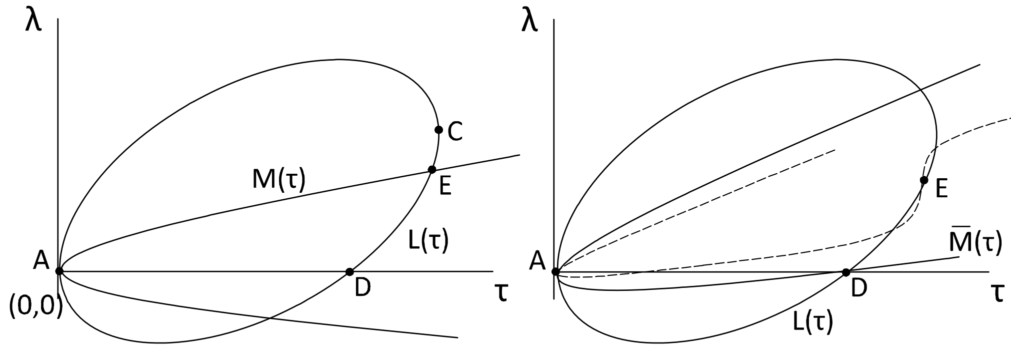

Figure 8: Relative slopes of limit and market order indifference curves

limit order trader would buy on good news at a (hypothetical) signal $(\bar{\tau}, \bar{\lambda})$. First we show that the lemma holds on the $\lambda = 0$ axis, then we will extend to $\lambda > 0$. For a point $(\bar{\tau}, 0)$, and for the indifference curves $g(\tau, \lambda) = g(\bar{\tau}, 0)$ and $f(\tau, \lambda) = f(\bar{\tau}, 0)$, by direct computation we have

$$\left.\frac{d\lambda}{d\tau}\right|_{MO} = -\left.\frac{\partial g/\partial \tau}{\partial g/\partial \lambda}\right|_{(\tau,0)} = \frac{1}{2}\frac{\lambda_w}{\tau_x + \tau_w + \bar{\tau}} \quad \text{and} \quad \left.\frac{d\lambda}{d\tau}\right|_{LO} = -\left.\frac{\partial f/\partial \tau}{\partial f/\partial \lambda}\right|_{(\tau,0)} = \frac{1}{2}\frac{s_p - \lambda_w^2/\tau_w}{1-\lambda_w}$$

But then A3 and the positivity of τ_x and $\bar{\tau}$ imply $\left.\frac{d\lambda}{d\tau}\right|_{LO} > \left.\frac{d\lambda}{d\tau}\right|_{MO}$ as claimed.

For $(\bar{\tau}, \bar{\lambda})$ with $\bar{\lambda} > 0$, first observe that the conditions $\bar{\lambda} \leq 1 - \lambda_w$ and that a limit order trader would buy on good news at $(\bar{\tau}, \bar{\lambda})$ imply that $(\bar{\tau}, \bar{\lambda})$ lies on the increasing, convex CD portion of a limit order indifference curve as in Figure 1.

Case 1: $(\bar{\tau}, \bar{\lambda})$ is on the upper branch of $g(\tau, \lambda) = g(\bar{\tau}, \bar{\lambda})$.

Refer to the left panel of Figure 8 where $(\bar{\tau}, \bar{\lambda})$ is labeled point E. Let $M(\tau)$ denote the upper branch of $g(\tau, \lambda) = g(\bar{\tau}, \bar{\lambda})$ and let $L(\tau)$ denote that portion of $f(\tau, \lambda) = f(\bar{\tau}, \bar{\lambda})$ that lies along the arc ADC, where $A = (0, 0)$, $D = (\tau_D, 0)$ is defined by $f(\tau_D, 0) = f(\bar{\tau}, \bar{\lambda})$, and $C = (\tau_C, 1 - \lambda_w)$ is defined by $f(\tau_C, 1 - \lambda_w) = f(\bar{\tau}, \bar{\lambda})$. Note that $M(\tau)$ is concave and $L(\tau)$ is convex. Furthermore, because of the vertical tangency at $(0, 0)$, we have $M(\tau) > L(\tau)$ for small τ. Thus, $M(\tau) - L(\tau)$ crosses zero at most once, and only from above, and so we must have $M'(\bar{\tau}) < L'(\bar{\tau})$.

Case 2: $(\bar{\tau}, \bar{\lambda})$ is on the lower branch of $g(\tau, \lambda) = g(\bar{\tau}, \bar{\lambda})$.

Refer to the right panel of Figure 8. $E = (\bar{\tau}, \bar{\lambda})$ is labeled as in Case 1, as are D, C, and and the limit order indifference curve through $E = (\bar{\tau}, \bar{\lambda})$, $L(\tau)$. Let $\bar{M}(\tau)$ be the lower branch of the market order indifference curve through point D. The market order indifference curve through $(\bar{\tau}, \bar{\lambda})$ is not pictured. Note that by condition A3 and the results above for the $\bar{\lambda} = 0$ case, at point D we have $L'(\hat{\tau})| > \bar{M}'(\hat{\tau})$ as depicted. Now consider the market order indifference curve $g(\tau, \lambda) = g(\bar{\tau}, \bar{\lambda})$. The hyperbola that it lies on intersects the ellipse that $L(\tau)$ lies on at points E and A, with a tangency at the latter. Suppose toward a contradiction that $g(\tau, \lambda) = g(\bar{\tau}, \bar{\lambda})$ is steeper than $L(\tau)$ at point E, as suggested in the "candidate" curve represented by the dashed line. This implies that the market order indifference curve crosses into the interior of the ellipse at E. But then it must cross the ellipse twice more, as pictured – once because the hyperbola must exit the ellipse as τ and λ tend to infinity, and once more because the hyperbola must connect E to A without crossing indifference curve $\bar{M}(\tau)$. Thus, there must be at least four points of intersection for the ellipse and hyperbola that $f(\tau, \lambda) = f(\bar{\tau}, \bar{\lambda})$ and $g(\tau, \lambda) = g(\bar{\tau}, \bar{\lambda})$ lie on respectively, including one point of tangency. But this is impossible, as an ellipse and a hyperbola can intersect at most three times, if one of those intersections is a tangency. Nor can the two indifference curves be tangent at E; if they were, we would be able to find a slightly perturbed point $\tilde{E}$ along $g(\tau, \lambda) = g(\bar{\tau}, \bar{\lambda})$, with corresponding limit order indifference curve $\tilde{L}(\tau)$, for which the contradictory four intersections reemerges. Thus the limit order indifference curve must be strictly steeper than the market order indifference curve at E.

Proof of Proposition 7

Suppose that in equilibrium, sources acquired by limit order traders lie on the curve $f(\tau, \lambda) = \bar{f}$, while sources acquired by market order traders lie on $g(\tau, \lambda) = \bar{g}$. For market order traders, let $\lambda_{MO}^{LB}(\tau)$ and

$\lambda_{MO}^{UB}(\tau)$ denote the lower and upper branches of the hyperbola $g(\tau,\lambda) = \bar{g}$. The equilibrium condition $0 \leq \lambda_n \leq 1 - \lambda_w$ and Lemma 9 imply that if limit order traders acquire n, then (τ_n, λ_n) lies along the DC arc of $f(\tau,\lambda) = \bar{f}$ (as designated in Figure 1). With this in mind, let $\lambda_{LO}^{LB}(\tau)$ denote the portion of this ellipse lying along ADC. We will proceed through a series of claims.

Claim 1 If for some n, $m_n > 0$ and market order traders sell on good news at n, then $l_n > 0$ and limit order traders buy on good news at n.

Proof A1, positivity of λ_n by Proposition 6, and (16) imply that $Mm_n\alpha_y^n + Ll_n\beta_y^n \geq 0$, so if $m_n\alpha_y^n < 0$, then $l_n\beta_y^n > 0$.

Claim 2 There is at most one source n' such that $m_{n'} > 0$ and $l_{n'} > 0$ and both types buy on good news.

Proof Suppose both types acquire both n' and $n'' > n'$ and buy on good news. Define the continuous function $\delta(\tau) = \lambda_{MO}^{LB}(\tau) - \lambda_{LO}^{LB}(\tau)$. We have $\delta(\tau_{n'}) = \delta(\tau_{n''}) = 0$ and by Lemma 8, $\delta'(\tau_{n'})$ and $\delta'(\tau_{n''})$ both strictly negative. Then there must be some $\tau^* \in [\tau_{n'}, \tau_{n''}]$ such that $\delta(\tau^*) = 0$ and $\delta'(\tau^*) \geq 0$. Let $\lambda^* = \lambda_{MO}^{LB}(\tau^*) = \lambda_{LO}^{LB}(\tau^*)$. But then (τ^*, λ^*) lies on both indifference curves with λ_{MO}^{LB} weakly steeper than λ_{LO}^{LB} at (τ^*, λ^*), a contradiction of Lemma 8.

Claim 3 If $m_{n^m} > 0$ and $l_{n^l} > 0$ for distinct n^m and n^l, and market order traders buy on good news at y_{n^m}, then $n^m < n^l$.

Proof Suppose instead that $n^m > n^l$. Then we have $\lambda_{MO}^{LB}(\tau_{n^m}) = \lambda_{n^m}$ and $\lambda_{LO}^{LB}(\tau_{n^l}) = \lambda_{n^l}$, with $\lambda_{n^m} > 0$ and $\lambda_{n^l} > 0$ by Proposition 6, and $\tau_{n^l} < \tau_{n^m}$. Furthermore, we must have $\lambda_{n^l} \geq \lambda_{MO}^{LB}(\tau_{n^l})$ (or else MO traders would strictly prefer to buy on good news at n^l instead) and $\lambda_{n^m} \geq \lambda_{LO}^{LB}(\tau_{n^m})$ (or else LO traders would strictly prefer to acquire n^m instead). Define $\delta(\tau)$ as in Claim 2 and observe that the last two inequalities imply $\delta(\tau_{n^l}) \leq 0$ and $\delta(\tau_{n^m}) \geq 0$. Then there must be some $\tau^* \in [\tau_{n^l}, \tau_{n^m}]$ such that $\delta(\tau^*) = 0$ and $\delta'(\tau^*) \geq 0$. Let $\lambda^* = \lambda_{MO}^{LB}(\tau^*) = \lambda_{LO}^{LB}(\tau^*) > 0$. But then (τ^*, λ^*) lies on both indifference curves, with the market order indifference curve weakly steeper at (τ^*, λ^*), a contradiction of Lemma 8. Thus $n^m < n^l$.

Claim 4 Let $\bar{n}_1$ be the least precise source acquired by any trader. If $\bar{n}_1 < N$, then every source between $\bar{n}_1$ and N is also acquired by some trader.

Proof One of the order types must buy on good news at $\bar{n}_1$. (If $l_{\bar{n}_1} > 0$, then $\beta_y^{\bar{n}_1} > 0$ by Lemma 9. If $l_{\bar{n}_1} = 0$ and $m_{\bar{n}_1} > 0$, then $\alpha_y^{\bar{n}_1} > 0$ by Claim 1.) Suppose there were some $n' > \bar{n}_1$ that was not acquired by any trader. But then we would have $\tau_{n'} > \tau_{\bar{n}_1}$ and $\lambda_{n'} = 0 \leq \lambda_{\bar{n}_1}$, so the order type that buys on good news at $\bar{n}_1$ could do strictly better by switching to the more precise, more exclusive source n'.

Claim 5 If market order traders acquire and sell on good news at a source y_n, then $n = N$.

Proof Suppose that market order traders sell on good news at some $y_{n'}$ with $n' < N$. By Claim 1, $L_{n'} > 0$ and $(\tau_{n'}, \lambda_{n'})$ must be an intersection of $\lambda_{MO}^{UB}(\tau)$ and $\lambda_{LO}^{LB}(\tau)$. Since the former is concave and the latter is convex, there is at most one such intersection, so market order traders do not sell on good news at any other source. Furthermore, Claims 3 and 4 imply $L_N > 0$. (Claim 4 implies either $L_N > 0$ or $M_N > 0$. If the latter, then market order traders would have to buy on good news at y_N but then Claim 3 applies.) But by the same convexity-concavity argument, this means that $(\tau_N, \lambda_{LO}^{LB}(\tau_N))$ lies above $\lambda_{MO}^{UB}(\cdot)$, so market order traders would prefer to acquire y_N instead.

Claims 2, 3, and 4 imply parts i-iv of the proposition. Claim 5 implies part v. Parts i-v imply that $\lambda_{n+1} - \lambda_n$ equals either $\lambda_{MO}^{LB}(\tau_{n+1}) - \lambda_{MO}^{LB}(\tau_n)$, $\lambda_{LO}^{LB}(\tau_{n+1}) - \lambda_{LO}^{LB}(\tau_n)$, or $\lambda_{LO}^{LB}(\tau_{n+1}) - \lambda_{MO}^{LB}(\tau_n)$ for $n \geq \bar{n}_1$. In the first two cases, the fact that $\lambda_{MO}^{LB}(\cdot)$ and $\lambda_{LO}^{LB}(\cdot)$ are increasing functions (where they are positive) suffices to show $\{\lambda_n\}$ increasing. The third case only applies if market order traders acquire y_n but do not acquire y_{n+1} in which case their weak preference for y_n implies $\lambda_{LO}^{LB}(\tau_{n+1}) \geq \lambda_{MO}^{LB}(\tau_{n+1})$, and this together with $\lambda_{MO}^{LB}(\cdot)$ suffices. The result that $\{\lambda_n/\tau_n\}$ is increasing follows by a similar argument from the convexity of $\lambda_{MO}^{LB}(\cdot)$ and $\lambda_{LO}^{LB}(\cdot)$ and the fact that both approach the origin as $\tau \to 0$.

The following lemmas are used in proving Proposition 8.

Lemma 10 *If A3 holds and for some source n, $\lambda_n/\tau_n < \lambda_w/\tau_w$, then $\beta_w^n > 0$.*

Proof. β_w^n strictly positive is equivalent to

$$\frac{\lambda_w}{\tau_w} < \frac{s_p - \lambda_w^2/\tau_w - \lambda_n^2/\tau_n}{1 - \lambda_w - \lambda_n}$$

Clearing denominators (note that $1-\lambda_w-\lambda_n>0$) and simplifying, this is equivalent to

$$\frac{\lambda_w}{\tau_w}<s_p+\lambda_n\left(\frac{\lambda_w}{\tau_w}-\frac{\lambda_n}{\tau_n}\right)$$

But $\lambda_w/\tau_w<s_p$ by A3, so the inequality holds. ■

Lemma 11 *(Lower bound on the total price impact of news sources and w) In equilibrium,* $\lambda_w+\sum_{n=1}^N\lambda_n=1-\lambda_\theta>\bar{\varepsilon}>0$, *where* $\bar{\varepsilon}=\frac{1}{2LM\gamma_L\gamma_M\tau_z(\tau_N+\tau_w)+1}$.

Proof. Write $V=\lambda_w+\sum_{n=1}^N\lambda_n$. Using (11), we have $V=1-\rho\sum_{n=1}^N Mm_n\alpha_x^n$, where $\alpha_x^n=\frac{\gamma_M}{s_m^n}(\lambda_n+\lambda_w)\frac{\tau_x}{t_m^n}\geq 0$. Using expressions for s_m^n and s_p, we have $s_m^n\geq\rho^2/\tau_z$. Furthermore $\tau_x/t_m^n=\tau_x/\left(\tau_x+\tau_w+\tau_n\right)<1$, so we have $\alpha_x^n\leq\frac{\gamma_M\tau_z}{\rho^2}(\lambda_n+\lambda_w)$. Finally, let $\underline{\rho}=\left(L\gamma_L\left(\tau_N+\tau_w\right)\right)^{-1}$ and note that $\rho\geq\underline{\rho}$. Combining these, we have

$$V\geq 1-\frac{M\gamma_M\tau_z}{\underline{\rho}}\sum_{n=1}^N m_n(\lambda_n+\lambda_w). \tag{18}$$

Now suppose that $V<\varepsilon$, for some $\varepsilon>0$. *A fortiori*, we have $\lambda_w<\varepsilon$ and $\lambda_n<\varepsilon$ for all n. Then by (18) we have $\varepsilon>V\geq 1-\frac{2M\gamma_M\tau_z}{\underline{\rho}}\varepsilon$. But this is impossible for $\varepsilon\leq\bar{\varepsilon}=\frac{\underline{\rho}}{2M\gamma_M\tau_z+\underline{\rho}}$. We conclude that $V>\frac{\underline{\rho}}{2M\gamma_M\tau_z+\underline{\rho}}>0$. Expand $\underline{\rho}$ to get the result. ■

Lemma 12 *If A3 holds in equilibrium, then the following are true: (i)* $\sum_{n=1}^N\frac{\lambda_n}{\tau_n}\geq\frac{\lambda_w}{\tau_w}$, *and (ii)* $\frac{\lambda_N}{\tau_N}\geq\frac{1}{N}\frac{\lambda_w}{\tau_w}$.

Proof. Part (ii) follows from part (i) and λ_n/τ_n increasing in n. For part (i), observe that if $\lambda_N/\tau_N\geq\lambda_w/\tau_w$, then the claim holds trivially, so assume $\lambda_N/\tau_N<\lambda_w/\tau_w$. Then we must have $\lambda_n/\tau_n<\lambda_w/\tau_w$ for all $n<N$ as well. By (11), we have

$$\left(\sum_{n=1}^N\frac{\lambda_n}{\tau_n}\right)-\frac{\lambda_w}{\tau_w}=\rho\sum_{n=1}^N\left(Ll_n\left(\frac{\beta_y^n}{\tau_n}-\frac{\beta_w^n}{\tau_w}\right)+Mm_n\left(\frac{\alpha_y^n}{\tau_n}-\frac{\alpha_w^n}{\tau_w}\right)\right)$$

Examine a typical term in the summation. We have

$$\frac{\beta_y^n}{\tau_n}-\frac{\beta_w^n}{\tau_w}=\gamma_L\left(\frac{\lambda_w}{\tau_w}-\frac{\lambda_n}{\tau_n}\right)S_n>0\quad\left(\text{where } S_n=\frac{1-\lambda_n-\lambda_w}{s_p-\lambda_w^2/\tau_w-\lambda_n^2/\tau_n}>0\right)$$

and

$$\frac{\alpha_y^n}{\tau_n}-\frac{\alpha_w^n}{\tau_w}=\frac{\gamma_M}{s_m^n}\left(\frac{\lambda_w}{\tau_w}-\frac{\lambda_n}{\tau_n}\right)>0$$

But then because each term in the summation is strictly positive, part (i) follows. ■

Lemma 13 *(Lower bound on* λ_N*) If A3 holds in equilibrium, then* $\lambda_N>\frac{1}{N}\frac{\tau_N}{\tau_N+\tau_w}\bar{\varepsilon}>0$, *where* $\bar{\varepsilon}$ *is the constant from Lemma 11.*

Proof. We have $\lambda_N\geq\lambda_n$ for all n, so by Lemma 11, $N\lambda_N+\lambda_w>\bar{\varepsilon}$ holds. By Lemma 12, we also have $\lambda_w\leq N\frac{\tau_w}{\tau_N}\lambda_N$, combining the two yields $N\lambda_N\left(1+\frac{\tau_w}{\tau_N}\right)>\bar{\varepsilon}$. Rearranging yields the result. ■

Proof of Proposition 8

For each case, τ_w small and τ_x large, we must show that (i) $\frac{\lambda_N}{\tau_N}>\frac{\lambda_w}{\tau_x+\tau_w}$, so by Lemma 4 a market order trader holding N would sell on its good news, and (ii) a market order trader would choose source N. Consider point (i) first. Fix τ_x and τ_w such that $\frac{\tau_x}{\tau_w}>N\frac{\tau_N}{\tau_1}$, and suppose toward a contradiction that $\frac{\lambda_N}{\tau_N}\leq\frac{\lambda_w}{\tau_x+\tau_w}$ in equilibrium. Consider the following implications.

(1) $\lambda_n/\tau_n<\lambda_w/\tau_w$ for all n. This follows from $\tau_x>0$ and λ_n/τ_n increasing in n.
(2) $\beta_w^n>0$ for all n by Lemma 10.

(3) $\beta_y^n > \frac{\tau_n}{\tau_w}\beta_w^n > 0$ for all n. This follows from the fact that $\beta_y^n/\beta_w^n = \frac{\tau_n}{\tau_w}\left(\frac{1-(\lambda_n/\tau_n)S_n}{1-(\lambda_w/\tau_w)S_n}\right)$ and $\lambda_n/\tau_n < \lambda_w/\tau_w$ (where $S_n = \tau_{\xi_n}/(1-\lambda_w-\lambda_n) > 0$).

(4) $\lambda_N/\tau_n \geq \frac{1}{N}R$, where $R = \frac{1}{\lambda_w}\sum_{n=1}^N \lambda_n$. (This follows from $\lambda_n \leq \lambda_N$ for all n.)

(5) $\alpha_y^n \geq 0$ for all n. For N, this follows from our assumption that (i) fails; for $n < N$ this follows from Proposition 7.

(6) $\alpha_w^n \leq 0$ for all n. This follows from point (5) and the weak positivity of $\alpha_x^n = -\left(\alpha_y^n + \alpha_w^n\right)$.

(7) $R \geq \left(\sum_{n=1}^N l_n \beta_y^n\right) / \left(\sum_{n=1}^N l_n \beta_w^n\right)$. From (16) and points (5) and (6).

(8) $R \geq \left(\sum_{n=1}^N l_n \beta_w^n (\tau_1/\tau_w)\right) / \left(\sum_{n=1}^N l_n \beta_w^n\right) = \frac{\tau_1}{\tau_w}$. From points (2), (3), and $\tau_1 \leq \tau_n$ for all n.

(9) $\lambda_N/\tau_n \geq \frac{1}{N}\frac{\tau_1}{\tau_w}$. From (4) and (8).

(10) $\frac{\lambda_w}{\tau_x+\tau_w} < \frac{1}{N}\frac{\tau_1}{\tau_w}$. From $\frac{\tau_x}{\tau_w} > N\frac{\tau_N}{\tau_1}$.

(11) But by hypothesis, $\frac{\lambda_N}{\tau_N} \leq \frac{\lambda_w}{\tau_x+\tau_w} < \frac{1}{N}\frac{\tau_1}{\tau_w}$, contradicting (9).

Thus, $\frac{\tau_x}{\tau_w} > N\frac{\tau_N}{\tau_1}$ implies $\frac{\lambda_N}{\tau_N} > \frac{\lambda_w}{\tau_x+\tau_w}$, and therefore, a market order trader holding N will sell on good news. Point (i) follows for τ_x sufficiently large or τ_w sufficiently small.

Now consider point (ii). Let $g(\tau,\lambda\,;(\tau_x,\tau_w)) = g(\tau_N,\lambda_N\,;(\tau_x,\tau_w))$ be the market order indifference curve through (τ_N,λ_N) at an equilibrium given τ_x and τ_w. Write $\lambda^{UB}(\tau\,;(\tau_N,\lambda_N),(\tau_x,\tau_w))$ and $\lambda^{LB}(\tau\,;(\tau_N,\lambda_N),(\tau_x,\tau_w))$ for the upper and lower branches of this indifference curve. We make two claims, deferring a proof of the first one until later:

Claim 1: For τ_x sufficiently large, or for τ_w sufficiently small, $\lambda^{LB}(\tau\,;(\tau_N,\lambda_N),(\tau_x,\tau_w)) < 0$ for all $\tau > 0$.
Claim 2: For any particular τ_x and τ_w, if there is an equilibrium in which some market order traders choose source $n < N$, then $\lambda_n \leq \lambda^{LB}(\tau_n\,;(\tau_N,\lambda_N),(\tau_x,\tau_w))$.

Proof: Optimality of n requires that (τ_n,λ_n) lie on a (weakly) broader-jawed hyperbola than (τ_N,λ_N); thus (τ_n,λ_n) lies above λ^{UB} or below λ^{LB}. However, if (τ_n,λ_n) lay above λ^{UB}, then the traders would sell on good news at n, which Proposition 7 rules out. Thus we must have $\lambda_n \leq \lambda^{LB}(\tau_n\,;(\tau_N,\lambda_N),(\tau_x,\tau_w))$.

We also have $\lambda_n \geq 0$ for all n, in any equilibrium, so Claims 1 and 2 establish that for τ_x sufficiently large or for τ_w sufficiently small, market order traders do not choose any source except N.

Proof of Claim 1: One can readily show that the lower branch of an indifference curve $g(\tau,\lambda\,;(\tau_x,\tau_w)) = \bar{g}$ lies below the $\lambda = 0$ axis if the slope of its asymptote is negative. A necessary and sufficient condition for this is $\bar{g} > 0$. Thus it will suffice to show that $g(\tau_N,\lambda_N\,;(\tau_x,\tau_w)) > 0$ for τ_x sufficiently large or τ_w sufficiently small. We have

$$g(\tau_N,\lambda_N\,;(\tau_x,\tau_w)) = \frac{\frac{\lambda_N^2}{\tau_N}(\tau_x+\tau_w) - \lambda_w(2\lambda_N+\lambda_w)}{\tau_N(\tau_N+\tau_x+\tau_w)}$$

Now we make Claim 3: There is a constant $K > 0$, independent of τ_x and τ_w, such that $\frac{\lambda_N^2}{\tau_N} \geq K$ for τ_x sufficiently large or τ_w sufficiently small. We defer a proof. Claim 3 plus the boundedness of λ_w and λ_N imply that $g(\tau_N,\lambda_N\,;(\tau_x,\tau_w)) > 0$ for τ_x sufficiently large. Finally, it is not hard to show that λ_w must tend to zero with τ_w. Thus for $\tau_w \to 0$, we have $g(\tau_N,\lambda_N\,;(\tau_x,\tau_w)) \to \frac{\lambda_N^2}{\tau_N}\frac{\tau_x}{\tau_N(\tau_N+\tau_x)} > 0$.

Proof of Claim 3: By Lemmas 11 and 13, we have $\lambda_N^2/\tau_N > \bar{\varepsilon}^2/\tau_N > 0$. For the case of $\tau_x \to \infty$, we are done, as $\bar{\varepsilon}$ does not depend on τ_x. For the case of $\tau_w \to 0$, observe that $\bar{\varepsilon}$ increases as τ_w declines, with $\bar{\varepsilon}^\infty = \lim_{\tau_w \to 0}\bar{\varepsilon} > 0$, and $\bar{\varepsilon}^\infty$ not dependent on τ_w. Then, for all τ_w sufficiently small, we have $\lambda_N > \frac{1}{2}\bar{\varepsilon}^\infty$ and thus $\lambda_N^2/\tau_N > (\bar{\varepsilon}^\infty)^2/(4\tau_N) > 0$.