Piracy of Digital Products: A Contest Theoretical Approach

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Abstract

In the following, we examine a market of a digital consumption good with monopolistic supply. In this market, it is the ability of the consumer to bypass ("crack") the copy-protection of the monopolist which induces a lower price of the digital good, compared to an uncontested monopoly (textbook case). We analyze the complex relationship between the cracking efforts of the consumer, the copy-protection efforts and the pricing decision of the monopolist, and the welfare of the economy. We find, for example, that the monopolist will deter piracy if the (exogenous) relative effectiveness of the consumer's bypassing activity is low compared to the copy-protection technology. In this case welfare is lower than the welfare in the textbook case. On the contrary, welfare rises above the textbook case level if the relative effectiveness of cracking is sufficiently high.

Keywords: Digital Products, Contests, Security of Property Rights, Endogenous Monopoly Price
JEL classification: C72, D23, D42

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1 Introduction

Traditionally, non-price-discriminating monopolies are said to cause inefficiencies because of the differences between the marginal cost of production and the price fixed by the profit maximizing monopolist. However, given identical preferences on the consumer side and identical cost functions on the side of miscellaneous monopolists, identical monopoly prices are more often the exception than the rule. This occurs because of the different circumstances that persuade a producer to deviate from the typical textbook profit-maximizing price. For example, the fear of competition induces monopolists to set a lower price, so as to make it unprofitable for the potential intruder to enter the market.[1] In this paper, we examine a different situation, in which monopolistic behaviour changes. In our model, a monopolist produces a digital good for consumption which is demanded by n consumers. This digital good represents for example music files, copy protected pdf-files or access to pay-tv. The decisive function of the monopoly good is its ability to be in principle *replicable* by consumers, unless it is copy-protected. Thus, any non-authorized copying - which we will call pirating - produces a good which is non-discriminable from the original.[2] It is only possible to replicate a digital product with copy-protection if it is *cracked*. Cracking is defined as the attempt to remove the copy-protections inserted into software programs. A program successfully stripped of its protections is then considered cracked.[3] As cracked software programs can be downloaded free of charge via internet, they can be reproduced at no marginal cost.[4] Since any consumer is aware of the fact that he can use a cracked digital good at zero marginal costs instead of buying the good, he is willing to invest scarce resources in order to crack copy-protected goods of the monopolist. These consumer side efforts, together with the copy-protection efforts of the producer determine the probability of the monopolist retaining its control over the market. This probability is modelled by a modification of the Tullock contest success function (CFS).[5] The overall ability to bypass the copy-protection of the monopolist consists of the sum of efforts of all consumers. This takes into account the fact that once the digital good is cracked, it can be obtained through the internet free of charge. Thus, the cracking efforts of each single consumer become a *private provision of a public good*.
A related model is presented by Leidy (1994), who argues that a monopolist whose right is contested in a political market will spend lobbying efforts, and lower his price to defuse reformist opposition. The expenditure for copyright protection in

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[1] See Bain (1956), Modigliani (1958) and Sylos Labini (1962) for a classical treatment of this issue, as well as Dixit (1982) for a critical review.

[2] As in Peitz and Waelbroeck (2003), we will only consider end-user piracy.

[3] We assume that the costs of cracking are identical for all consumers, since the tools for cracking software are obtainable through internet. Well-known (illegal) software programs that bypass copy-protection are, amongst others, Hex Editor and SoftICE, which can be obtained for free.

[4] We assume that a digital product can not be sold by the cracker. This is because cracking is illegal and the cracker would be charged with illegal activities if he invoiced the product.

[5] Papers which have analyzed differences between players in Tullock contests in a rent seeking setting are Allard (1988) and Gradstein (1995). For examples of asymmetrical Tullock CSFs in the property rights literature, see Grossman and Kim (1995) and Grossman (2001).

our context are isomorphic to the lobbying efforts to retain the monopoly in Leidy (1994). Leidy finds a distinct tendency towards lower social cost of monopoly because of the lower monopoly price. The endogenously determined monopoly price in our model lies below the textbook monopoly price as long as the consumption good is challengeable. Leidy can not rule out the possibility that social cost might increase in some circumstances. In our model, there are welfare-increasing and -decreasing effects: On the one hand, the lower the probability that the monopoly continues and the lower the monopoly price, the higher the welfare. On the other hand, the higher the investment in copy-protection and appropriation, the lower the welfare. Thus, with regard to welfare, it is possible to determine the circumstances in which the combined effect is positive or negative.

Another model which is close to ours from a methodological viewpoint does not address the question of piracy at all. Epstein and Nitzan (2002) have developed a rent seeking model for monopoly rents when there is consumer opposition.[6] They have shown that consumer opposition lowers the monopolistic price. The firm trades off a lower monopoly rent against a higher probability of winning this rent because of the reduced consumer opposition. A regulation authority, i.e. an outside enforcer, is needed to ensure that the monopolist charges the price that he announced.[7]

Apart from literature on rent–seeking, there is a considerable body of literature which discusses consumers' individual decision as to whether to buy an original or to acquire a copy. The following articles on the piracy of digital goods model end-user piracy either with differences between an original and a copy or with heterogeneous consumers. Here, it is the consumer's private decision to buy an original or a copy. A survey of piracy of digital goods that focuses on these cases is contributed by Peitz and Waelbroeck (2003). In these models, the assumption of heterogeneous consumers is necessary in order to divide the consumers into buyers of the expensive original or acquirers of the low-quality copy. In our model, however, consumers are homogeneous, all investing in cracking the digital product. In our context, unlike in the previous models, a cracked product can be used by all consumers free of charge. This is due to the fact that a cracked product can be diffused through the internet. Pricing decisions of producers of information goods in the presence of copying are analyzed by Belleflamme (2002). He uses a model of vertical quality differentiation, i.e. copies are seen as lower-quality alternatives to originals. The optimal strategy for a monopolist who takes into account that consumers can acquire a lower-quality copy has analogies to the optimal strategy of an incumbent in the face of an entry threat. Unless the quality/price ratio of copies is very low (meaning that copying exerts no threat), the producer will have to modify his behavior and decide whether to set a price low enough to 'deter' copying, or to 'accommodate' copying and make up for it by extracting a higher margin from fewer consumers of originals. Belleflamme is able to show that, whatever the producer's optimal decision, copying reduces the producer's profits but more than proportionally increases consumers surplus. Thus,

[6] See also Epstein and Nitzan (2003) and Epstein and Nitzan (2004) for related models with endogenous prize.

[7] This poses the question why the regulation authority does not enforce the welfare optimizing price.

the possibility of copying provides a cheaper and lower-quality alternative to a monopolized good, thereby enhances social welfare. In our context, cracking the digital good only has an enhancing effect on social welfare under certain conditions. As expected, there is a region of cracking efforts and copy-protection where welfare is lower than in the standard monopoly case because the outlays of consumers and the monopolist are dissipated as in a rent seeking game. However, it is worth mentioning that cracking can be welfare-enhancing if consumers are relatively effective in bypassing the copy-protection of the monopolist. In this case, the rent seeking efforts of consumers cause a higher probability of procuring the good at no marginal costs, i.e. the monopoly cannot persist, which leads to the welfare optimal price.

To incorporate all significant effects, we introduce a two-stage-game: In the first stage, the monopolist makes decisions about the supply of the digital good as well as his investment in copy-protection. Depending on the outcomes of these first stage decisions, each consumer makes decision individually about his effort to crack the digital good.

The remainder of this paper is structured as follows. In the following sections the basic structure of the game is outlined as well as the solution in the second and first stage. Thereafter, there is a discussion of the full game and the welfare effects. A short concluding summary completes the discussion.

2 The basic structure of the model

Each agent has preferences which can be displayed by the following quasi–linear utility function:

$$u_i = v(y_i) + m_i, \tag{1}$$

with $i = 1, \ldots, n$. y_i represents the amount of the monopoly good, consumed by agent i. m_i represents the consumption of the numeraire good of agent i. We assume the following function to display the benefit agents get out of the monopoly good:

$$v(y_i) = -b\ e^{-y_i}, \tag{2}$$

where b is some arbitrarily chosen and positive number. Each agent faces the following budget constraint:

$$\omega_i - p\, y_i - m_i \geq 0, \tag{3}$$

where ω_i represents the income of agent i.

Given these assumptions, the resulting demand function subject to any positive price is

$$y_i(p) = \begin{cases} -\ln\left[\frac{p}{b}\right] & \text{for } p < b, \\ 0 & \text{else.} \end{cases} \tag{4}$$

Aggregating this demand over all n agents delivers the following inverse demand function, subject to the aggregated supply of the monopoly good x:

$$p(x) = b\, e^{-\frac{x}{n}}, \tag{5}$$

with $x = \sum_{i=1}^{n} y_i$. Thus, given the monopoly supply x, the individual consumer rent equals

$$R_{C_i}(b, n, x) = b\left(1 - \frac{x+n}{n}\, e^{-\frac{x}{n}}\right). \tag{6}$$

In our model, agents have the opportunity to decide whether to invest scarce resources in order to appropriate the monopoly good. Appropriation here means that they seek to bypass the copy-protection of the monopolist (cracking). If they are effective in doing so, they will get access to the use of infinite units of the monopoly good at no additional costs. At first we will concentrate on the *prize*, that is the gain agents derive from the bypassing activity. The *prize* equals the extra consumer rent each agent gets by getting infinite access to the use of the monopoly good at zero cost:

$$E_{C_i}(b, n, x) = \frac{x+n}{n}\, b\, e^{-\frac{x}{n}}. \tag{7}$$

This *prize* is a continuous, decreasing and continuously differentiable function of x. We will call this *prize* the extra per-agent gross consumer rent (E_{C_i}), since the effort of agents in order to appropriate the monopoly good is not yet included.
The relations described above are represented by figure 1:

FIGURE 1 HERE

Given a monopoly price $\bar{p}$, with $\bar{p} < b$, the individual consumer rent (R_{C_i}) is displayed by array I. The additional per-agent gross consumer rent (E_{C_i}) is displayed by the other two arrays. Array II represents the revenue of the monopolist, which would disappear for the monopolist in case the agents are effective in bypassing the copy-protection. The third array (III) represents the additional individual consumer rent if the representative agent would benefit from an infinite access to the monopoly good, instead of $\frac{\bar{x}}{n}$ units.
For the display of the contest we make use of a modification of the Tullock CSF which maps the efforts of consumers and of the monopolist into a probability function.[8] The effort of each consumer (A_i) is the individual investment out of scarce ressources in order to crack the copy-protection of the monopolist. The main point is now that the effort of each single agent is a *private provision of a public good*, since all the individuals benefit from the copy-protection bypass activities of each single agent. This captures the fact, that the outcome of the copy-protection bypassing activity is not a private information, but rather displayed publicly. It is because of this feature, that the probability function ($F^C(\cdot)$) is a function of the *sum* of efforts of all agents. The effort of the monopolist is his investment in copy-protection, which we will denote as Z. If we indicate the sum of all cracking efforts of all agents with

$$A = \sum_i A_i,$$

[8]We assume that all agents and the monopolist are risk-neutral.

the probability function becomes:

$$F^C\left(A, Z, \theta\right) = \begin{cases} 0 & \text{for } Z = 0,\ A = 0, \\ \frac{\theta\, A}{\theta\, A + Z} & \text{else,} \end{cases} \tag{8}$$

with $\theta \geq 0$. F^C represents the probability that the monopoly falls, i.e., the probability that the agents are successfull in cracking the copy-protection of the monopolist. The exogenous parameter θ measures the effectiveness of resources allocated to appropriating the monopoly good, relative to resources invested by the monopolist to protect the monopoly good. It may represent the relative development status of the copy-protection technology in relation to the appropriating technology. For θ being equal to zero, appropriation is impossible (the typical monopoly textbook scenario), for θ going to infinity, the copy-protection of the monopoly good is technologically impossible (an open source digital good). In order for $F^C(A, Z, \theta)$ to be well defined, we assume that the value of $F^C(A, Z, \theta)$ is zero if the monopolist invests no ressources in copy-protection and agents invest no effort in bypassing the copy-protection, which would display the textbook-case of an uncontested monopoly good.

3 Second stage

Given this structure, the payoff function of a single consumer ($v_i(\cdot)$) is thus

$$v_i = F^C(A, Z, \theta)\, E_{C_i}(b, n, x) + R_{C_i}(b, n, x) - A_i, \tag{9}$$

where the first term represents the expected value of the extra gross consumer rent, the second term represents the typical consumer rent, and the third term represents the costs of the appropriation activity.
In the first stage the monopolist decides about the supply of monopoly good (x) and about the investment in copy-protection (Z). This does not only determine the value of E_{C_i} but also influences directly the probability function (F^C). Contingent on these first stage decisions the agents in the second stage decide about their investment in appropriating the monopoly good (A_i). Given the above function, each agent maximizes his payoff–function subject to one constraint:[9]

$$\begin{aligned} \max_{A_i} \quad & v_i \\ \text{s.t} \quad & A_i \geq 0. \end{aligned} \tag{10}$$

Partial differentiation of the payoff-function delivers the first-order-condition (FOC) for an interior solution:

$$\frac{\theta\, Z}{\left(Z + \theta\left(A_i + A_{-i}\right)\right)^2}\, E_{C_i}(b, n, x) \stackrel{!}{=} 1, \tag{11}$$

[9]The restriction is due to the fact that we do not allow A_i to be negative.

where

$$A_{-i} := \sum_{j \neq i} A_j$$

represents the efforts of all agents except agent i. On the left hand side (LHS) of (11) we see the marginal revenue of appropriation, which consists of two parts: The marginal effect on the probability function times E_{C_i}. On the right hand side (RHS) are the marginal costs for agent i. Since all agents are identical we only concentrate on the symmetric Nash-equilibrium case.[10] Given this assumption, the per-agent equilibrium level of appropriation becomes:

$$A_i^* = \begin{cases} \bar{A}_i & \text{for } Z < \widetilde{Z}, \\ 0 & \text{else }, \end{cases} \tag{12}$$

with

$$\bar{A}_i = \frac{\sqrt{\theta\, Z\, E_{C_i}(b, n, x)} - Z}{n\,\theta} \tag{13}$$

and

$$\widetilde{Z} = \quad \theta\, E_{C_i}(b, n, x). \tag{14}$$

Due to the multi-stage game, there is a level of copy-protection effort $\widetilde{Z}$ which deters the appropriation of the monopoly good by the consumers, i.e. $A_i^* = 0$. In this case, the marginal gain of appropriation (LHS of (11)) equals one for the first marginal investment in appropriation. Thus, the optimal level of appropriation is zero: Appropriation is deterred. Since E_{C_i} is a continuously decreasing function of x, the level of copy-protection sufficient to deter appropriation is also a decreasing function of x: The larger the supply of the monopoly good, the lower the investment in copy-protection necessary to deter appropriation. This is because the value of the *prize* declines with increasing monopoly supply. Figure 2 represents this finding: We see the reaction functions of a representative agent contingent on Z for various levels of x.

FIGURE 2 HERE

Suppose $\overline{x} > \underline{x}$, than as long as the level of appropriation is positive $A_i^*(\overline{x}, Z) < A_i^*(\underline{x}, Z)$ for every Z. Moreover, the necessary level of Z to deter appropriation is smaller for $x = \overline{x}$: $\widetilde{Z}(\overline{x}) < \widetilde{Z}(\underline{x})$.
The sum of all efforts of agents $A^*(x, Z)$ in the Nash-equilibrium is thus

$$A^* \quad = \quad \begin{cases} \bar{A} & \text{for } Z < \widetilde{Z} \\ 0 & \text{else }, \end{cases} \tag{15}$$

[10] All calculations can be found in the appendix which will be sent to the reader upon request. We are aware of the fact that, given the above preferences, we get a continuum of Nash-equilibria. For tractability of the model we concentrate on the symmetric case. This does not affect the basic findings of our model.

with

$$\bar{A} = n\,\bar{A}_i. \tag{16}$$

In the case of a constant prize (E_{C_i}), this reaction function is independent of the value of n due to the quasi-linear preferences.
The following proposition summarizes our main findings thus far.

Proposition 1 (*Deterrence*) *Suppose that all n consumers have identical preferences and $\theta \geq 0$.*

1. *Consumer side efforts in appropriation will be deterred if the level of copy-protection (Z) equals at least $\widetilde{Z} = \theta\, E_{C_i}(b, n, x)$.*
2. *A higher supply of the monopoly good (x) reduces the level of copy-protection necessary to deter appropriation ($\widetilde{Z}$).*

In the next section we will examine the behaviour of the profit-maximizing monopolist, contingent on the second stage decisions of the consumers.

4 The first stage

In the first stage the monopolist decides about the supply of the digital good and about his investment in copy-protection. The probability that the monopoly does not fall is

$$F = 1 - F^C = \begin{cases} 1 & \text{for } Z = 0,\ A = 0 \\ \frac{Z}{Z+\theta A} & \text{else.} \end{cases} \tag{17}$$

Inserting the findings of the last subsection ($A = A^*(x, Z)$), the probability that the monopoly does not fall (i.e. retains) is a function of x and Z. Assuming zero marginal costs of production, the profit of the monopolist in the first stage becomes

$$\Pi(x, Z) \;=\; F(A^*(x, Z), Z)\, R(x) - Z, \tag{18}$$

where Z represents the efforts of the monopolist in copy-protection and $R(x)$ denotes the typical revenue[11] of the monopolist, that is

$$R(x) \;=\; b\, e^{-\frac{x}{n}}\, x. \tag{19}$$

The profit-maximizing problem for the monopolist under these circumstances is

$$\begin{aligned} \max_{x,Z} \quad & \Pi(x, Z) = F\big(Z, \bar{A}(x, Z)\big)\, R(x) - Z \\ \text{s.t.} \quad & \text{(i)} \qquad x \geq 0, \\ & \text{(ii)} \qquad Z \geq 0, \\ & \text{(iii)} \qquad \bar{A}(x, Z) \geq 0. \end{aligned} \tag{20}$$

[11] We should keep in mind that this revenue function also displays the rent of the monopolist, if $\theta = 0$, that is for the typical textbook-scenario. This is because the marginal cost of production is zero.

The Lagrangian function thus becomes:

$$\mathcal{L}(x,Z) = F\big(Z, \bar{A}(x,Z)\big)\, R(x) - Z + \lambda_1\, x + \lambda_2\, Z + \lambda_3\, \bar{A}(x,Z), \tag{21}$$

where λ_1, λ_2 and λ_3 represent the shadow prices for violating the non-negativity constraints on x, Z and $\bar{A}(x,Z)$, respectively. Given these restrictions, the FOCs are

$$\frac{\partial F(\cdot)}{\partial x}\, R(x) + F(\cdot)\, \frac{\partial R(x)}{\partial x} + \lambda_1 + \lambda_3\, \frac{\partial \bar{A}(x,Z)}{\partial x} \overset{!}{=} 0, \tag{22}$$

$$\lambda_1 = \lambda_3 = 0 \qquad \Longrightarrow \qquad -\frac{\frac{\partial F(\cdot)}{\partial x}}{F(\cdot)} = \frac{\frac{\partial R(x)}{\partial x}}{R(x)}, \tag{22$'$}$$

$$\frac{\partial F(\cdot)}{\partial Z}\, R(x) - 1 + \lambda_2 + \lambda_3\, \frac{\partial \bar{A}(x,Z)}{\partial Z} \overset{!}{=} 0, \tag{23}$$

$$\lambda_2 = \lambda_3 = 0 \qquad \Longrightarrow \qquad \frac{\partial F(\cdot)}{\partial Z}\, R(x) = 1. \tag{23$'$}$$

Equation 22 displays the FOC with respect to the optimal supply and equation $22'$ represents this FOC for an interior solution, that is, if the constraints on x and $\bar{A}(x,Z)$ are not binding. One way to view equation $22'$ is in terms of elasticities: The LHS of $(22')$ can be interpreted as the probability elasticity (a marginal increase in probability to the total probability). The RHS can be interpreted as the revenue elasticity (a marginal increase in revenue relative to the total revenue). In optimum the revenue elasticity has to equal the negative probability elasticity. Even more insight delivers a further simplified version of the first FOC in an interior solution:

$$\frac{1}{2}\, \frac{\frac{\partial E_{C_i}(x)}{\partial x}}{E_{C_i}(x)} = \frac{\frac{\partial R(x)}{\partial x}}{R(x)}. \tag{22$''$}$$

The fraction on the LHS of equation $22''$ is unambiguously negative for every positive level of x. This is due to the fact that the effect of a marginal increase in x on E_{C_i} is always negative since this decreases the possible gains of the copy-protection bypassing activity (the *prize*), and E_{C_i} itself is positive. The fraction of the right hand side is positive for $x < n$ and negative for $x > n$, since these findings apply to the marginal revenue function and the revenue function itself is always positive. Hence, for equation $22''$ to be solved, the supply of the monopoly good has to be greater than n, which would be the profit-maximizing level of output in the textbook case. Thus, in an interior solution, the monopolist will increase the supply compared to the supply in the textbook scenario. Moreover, since the revenue function ($R(x)$) and the extra per-agent gross consumer rent ($E_{C_i}(x)$) are independent of Z, we know that the profit-maximizing level of x is not contingent on the copy-protection decision of the monopolist in an interior solution. This finding does not descend from the supposed demand function.[12] It stems from the CSF and the profit function of the monopolist itself.

[12] In the mathematical appendix we define several sufficient conditions for the demand function to make sure the above findings hold in an interior solution.

Equation 23 displays the FOC with respect to the investment in copy-protection. For an interior solution, the marginal gains of copy-protection (LHS of (23′)), that is the marginal effect on the probability function (F) times the revenue ($R(x)$), has to equal the marginal costs of copy-protection (RHS of (23′)).
The resulting profit-maximizing level of x and Z are:[13]

$$Z^*(b,n,\theta) = \begin{cases} \frac{b\,\theta(2\,n-\theta)e^{\frac{-n}{n-\theta}}}{n-\theta} & \text{for } \theta \leq \widetilde{\theta}, \\ \frac{b\,e^{-\sqrt{2}}\,n^2}{2\,\theta(1+\sqrt{2})} & \text{else,} \end{cases} \tag{24.1}$$

$$x^*(n,\theta) = \begin{cases} -n\,(1+W_{-1}\,(z)) & \text{for } \theta \leq \widetilde{\theta}, \\ \sqrt{2}\,n & \text{else,} \end{cases} \tag{24.2}$$

with

$$z = \frac{e^{\frac{-2\,n+\theta}{n-\theta}}(-2\,n+\theta)}{n-\theta}.$$

We find that the restrictions on Z and x are never binding ($\lambda_1^* = \lambda_2^* = 0$) $\forall\,\theta \geq 0$. But as long as $\theta < \tilde{\theta}$, the restriction on $\bar{A}(x,Z)$ is binding ($\lambda_3^* > 0$).[14]

FIGURE 3 HERE

For $\theta = \tilde{\theta}$ the value of the shadow price (λ_3^*) equals zero (restriction on $\bar{A}(x,Z)$ is not binding) *and* $\bar{A}(x^*,Z^*) = 0$. We can determine the *maximum* level of θ to guaranty the corner solution, which is:

$$\widetilde{\theta} = \frac{n}{2+\sqrt{2}}. \tag{25}$$

Hence, the profit-maximizing combination of supply and investment in copy-protection will deter appropriation if θ is sufficiently small ($\theta \leq \widetilde{\theta}$), i.e. if consumers are sufficiently ineffective in bypassing the copy-protection of the monopolist.
For $\theta = 0$, i.e. if appropriation is technologically impossible, the typical textbook case arises. In optimum the marginal revenue equals the marginal cost of production, therefore the supply equals n which corresponds with the supply in the textbook case, and the investment in copy-protection equals zero.

FIGURE 4 HERE

[13]See appendix for calculation. $W(z)$ hereby represents the Lambert function, which is defined to be the inverse of the function $w \mapsto w\,e^w = z$. This function $W(z)$, which thus verifies $W(z)\,e^{W(z)} = z$, is a multivalued function defined in general for z complex and assuming values $W(z)$ complex. $W_{-1}(z)$ represents one of two real branches of the W function, but $W_{-1}(z)$ is the sole function satisfying $W(z) \leq -1$ and $-\frac{1}{e} \leq z < 0$, which can be shown to hold for the above z as long as $\theta \leq \tilde{\theta}$.

[14]So we only have a corner solution with respect to A. Furtheron we will call the case with $A^* > 0$ interior solution and the case with $A^* = 0$ corner solution.

For $\theta \in [0, \widetilde{\theta}]$, Z^* increases monotonically (see figure 3) and x^* increases monotonically from the level of n to $\sqrt{2}\,n$ (see figure 4).
For $\theta > \widetilde{\theta}$ the interior solution applies. Since consumers are sufficiently relatively effective in bypassing the copy-protection of the monopolist, deterring appropriation is no longer profit-maximizing. This results in a supply of the digital good of $x^* = \sqrt{2}\,n$, $\forall\, \theta > \widetilde{\theta}$. Moreover, the investment in copy-protection declines along the way, with $\lim\limits_{\theta \to \infty} Z^*(b, n, \theta) = 0$.
The following proposition summarizes the above findings.

Proposition 2 (*Monopolist's reaction*)

1. *If the relative effectiveness parameter (θ) equals zero, the marginal revenue will in optimum equal the marginal costs of production (and therefore $x^* = n$) and the investment in copy-protection is zero (textbook case).*

2. *If $\theta \in]0, \widetilde{\theta}]$ the optimal investment in copy-protection is sufficient to deter appropriation ($Z^* = \widetilde{Z}(x^*(n, \theta))$, $A^* = 0$). Moreover, the optimal level of Z is flanked by a supply of x which lies above the textbook optimal supply of a monopolist ($x^* > n$).*

3. *For $\theta > \widetilde{\theta}$, deterring appropriation is no longer profit-maximizing. The supply of the digital good becomes $x^* = \sqrt{2}\,n$ and remains unchanged $\forall\, \theta > \widetilde{\theta}$. The optimal level of Z is lower than $Z^*\big|_{\theta=\widetilde{\theta}}$, with $\lim\limits_{\theta \to \infty} Z^*(b, n, \theta) = 0$ (open source digital good).*

4. *The case-separating level of θ ($\widetilde{\theta}$) is a monotonically increasing function of the number of consumers (n).*

5 The full game

In this section we would like to describe the solution in the full game. Turning to the second stage decision of the consumer on A, it is obvious that for θ being sufficiently small, the optimal supply/copy-protection decision of the monopolist will induce deterrence of appropriation, i.e. $A^* = 0$. For $\theta > \widetilde{\theta}$, A^* will become positive:

$$A^*(b, n, \theta) = \begin{cases} 0 & \text{for } \theta \leq \widetilde{\theta}, \\ \frac{b\, e^{\sqrt{2}}\, n \left(n - \sqrt{2}(n-\theta)\right)}{2\,\theta^2} & \text{else,} \end{cases} \tag{26}$$

which is a monotonically increasing function of θ for $\theta \in]\widetilde{\theta}, \hat{\theta}[$ and a monotonically decreasing function of θ for $\theta > \hat{\theta}$, with

$$\hat{\theta} = n(2 - \sqrt{2}). \tag{27}$$

This fact is captured in figure 5.

FIGURE 5 HERE

Together with the findings of the last section, we are now able to determine the probability F in equilibrium:

$$F^*(n,\theta) = \begin{cases} 1 & \text{for } \theta \leq \widetilde{\theta}, \\ \frac{n}{(2+\sqrt{2})\theta} & \text{else.} \end{cases} \tag{28}$$

FIGURE 6 HERE

In the corner solution ($\theta \in [0,\widetilde{\theta}]$) the probability equals one since appropriation is deterred (see figure 6). Thus, property rights are *perfectly secure* in this interval. In the interior solution ($\theta > \widetilde{\theta}$) the probability becomes a monotonically decreasing function of θ, with $\lim\limits_{\theta\to\infty} F^*(n,\theta) = 0$. Hence, property rights are *insecure* for θ being sufficiently large.
Finally, we take a look at the resulting levels of the consumer rent (R_{C_i}) and the extra per-agent gross consumer rent (E_{C_i}) in equilibrium. In the corner solution, the supply of the digital good rises from n (for $\theta = 0$) to $\sqrt{2}\,n$ (for $\theta = \widetilde{\theta}$). This does not only reduce the revenue of the monopolist ($R(x^*(n,\theta))$), but also $E^*_{C_i}$ and increases $R^*_{C_i}$ (see figure 7), with

$$E^*_{C_i}(b,n,\theta) = E_{C_i}(b,n,x^*(n,\theta)), \tag{29}$$
$$R^*_{C_i}(b,n,\theta) = R_{C_i}(b,n,x^*(n,\theta)). \tag{30}$$

Since ($E^*_{C_i}$) declines along the way, the economic rent subject to appropriation (the *prize*) declines, thus reducing the marginal gain of appropriation (see equation (11)).

FIGURE 7 HERE

We should keep in mind that, since $E_{C_i} + R_{C_i} \equiv b$, the reduction of economic rent subject to appropriation induces a reallocation of the total economic rent in favour of the individual consumer rent (R_{C_i}). In the interior solution, both rents remain unchanged, since the supply of the monopoly good remains unchanged. The following proposition recapitulates our findings:

Proposition 3 (*Rents, revenue and probability in equilibrium*)

1. *If appropriation is deterred ($\theta \in [0,\widetilde{\theta}]$) in equilibrium, the probability that the monopoly retains becomes one, i.e. perfectly secure property rights emerge. Moreover, the individual extra consumer rent ($E^*_{C_i}$) and the revenue of the monopolist (R^*) declines as θ approaches $\widetilde{\theta}$. The individual consumer rent ($R^*_{C_i}$) increases along the way.*

2. *When appropriation becomes positive, the probability F^* declines in θ, $\forall\, \theta > \widetilde{\theta}$. The revenue, the individual consumer rent and individual extra consumer rent are independent from the value of θ, as long as $\theta > \widetilde{\theta}$.*

6 Welfare aspects

We now would like to turn to the welfare implications of the game. Traditionally, the welfare losses due to a monopolistic supply of a consumption good arises because of the difference between the marginal cost of production and the price required by the monopolist. Our model incorporates these inefficiencies and broadens this statement.
The profit function of the monopolist becomes:

$$\Pi^*(b,n,\theta) = F^*(n,\theta)\, R(x^*(n,\theta)) - Z^*(b,n,\theta). \tag{31}$$

Partial differentiation of the profit function subject to θ delivers the marginal profit function:

$$\frac{\mathrm{d}\,\Pi^*(\cdot)}{\mathrm{d}\,\theta} = \frac{\partial F^*(\cdot)}{\partial\theta} R(x^*(\cdot)) + F^*(\cdot)\,\frac{\partial R(x^*(\cdot))}{\partial x^*(\cdot)}\frac{\mathrm{d}\,x^*(\cdot)}{\mathrm{d}\,\theta} - \frac{\partial Z^*(\cdot)}{\partial\theta} \tag{32}$$

As long as $\theta < \tilde{\theta}$, the marginal effect of θ on the probability function is zero, therefore the first term on the RHS of (32) is zero. The second term is negative, since the supply of the monopoly good is expanded for rising θ. This has an unambiguous effect on the marginal revenue. It declines, since the marginal revenue equals zero in the unconstrained case ($\theta = 0$) and becomes negative $\forall\,\theta > 0$. Furthermore, the third term on the RHS of (32) is positive, since the investment in copy-protection rises in θ, $\forall\,\theta < \tilde{\theta}$. Thus, the profit declines in θ for $\theta \in [0, \tilde{\theta}]$.
For $\theta > \tilde{\theta}$ the marginal effect of θ on the probability function is negative (see equation (28), lower case) and the revenue function itself is positive. Thus, the first term on the RHS of (32) is negative. Since the supply of the monopoly good remains unchanged $\forall\,\theta > \tilde{\theta}$ (see (24.2), lower case), the second term equals zero. Finally, the third term on the RHS of (32) is negative, since the investment in copy-protection declines with θ, $\forall\,\theta > \tilde{\theta}$. Nevertheless, we are able to show that the combined effect is unambiguously negative. Hence, $\forall\,\theta \geq \tilde{\theta}$ the profit function declines in θ.
Turning to the consumer side, we define the consumers' surplus in optimum as the sum of all payoff functions of all n individuals:

$$\begin{aligned} & V^*(b,n,\theta) = \sum_{i=1}^{n} v_i^*(b,n,\theta) \\ \Leftrightarrow\ & V^*(b,n,\theta) = n\big[\,(1-F^*(n,\theta))\,E_{C_i}(b,n,x^*(n,\theta)) + R_{C_i}(b,n,x^*(n,\theta))\big] - A^*(b,n,\theta). \end{aligned} \tag{33}$$

Partial differentiation of V^* subject to θ delivers

$$\begin{aligned} \frac{\mathrm{d}\,V^*(\cdot)}{\mathrm{d}\,\theta} = n\bigg[& -\frac{\partial F^*(\cdot)}{\partial\theta} E_{C_i}(x^*(\cdot)) + (1-F^*(\cdot))\frac{\partial E_{C_i}(x^*(\cdot))}{\partial x^*(\cdot)}\frac{\mathrm{d}\,x^*(\cdot)}{\mathrm{d}\,\theta} \\ & + \frac{\partial R_{C_i}(x^*(\cdot))}{\partial x^*(\cdot)}\frac{\mathrm{d}\,x^*(\cdot)}{\mathrm{d}\,\theta}\bigg] - \frac{\partial A^*(\cdot)}{\partial\theta}. \end{aligned} \tag{34}$$

For $\theta < \tilde{\theta}$ the first and second term in brackets on the RHS of equation (34) equal zero, since $F^*(n,\theta) = 1\ \forall\,\theta \in [0, \tilde{\theta}]$. The third term in brackets is unambiguously

positive, since the supply (x^*) rises in the named interval and therefore the price of the digital goods declines. And the forth term on the RHS of (34) equals zero since appropriation is deterred for $\theta < \tilde{\theta}$. Therefore V^* increases for the aforementioned interval in θ.
For $\theta > \tilde{\theta}$ the first term on the RHS of (34) is positive, since $E_{C_i} > 0$ and the probability that the monopoly retains declines in the named interval. The second and third term in brackets equal zero, since the extra gross consumer rent (E_{C_i}) and the consumer rent (R_{C_i}) remain unchanged $\forall\, \theta > \tilde{\theta}$. Finally, the last term on the RHS of (34) has an ambiguous sign. For $\theta > \hat{\theta}$, A^* declines in θ and therefore V^* rises in θ. In the case of $\theta \in [\tilde{\theta}, \hat{\theta}[$ A^* rises in θ. We are able to show that the combined effect is positive in this interval. Hence, for $\theta \geq \tilde{\theta}$, V^* rises unambiguously in θ.
If we define the welfare of the economy as the sum of the profit function plus the consumers' surplus, we get

$$\begin{aligned} & W = \Pi^*(b,n,\theta) + V^*(b,n,\theta) \\ \Longleftrightarrow\; & W = F^*(n,\theta)\, R(x^*(n,\theta)) + n\big[\,(1-F^*(n,\theta))\, E_{C_i}(b,n,x^*(n,\theta)) \\ & \qquad + R_{C_i}(b,n,x^*(n,\theta))\big] - B^*(b,n,\theta), \end{aligned} \tag{35}$$

with

$$B^*(b,n,\theta) \equiv Z^*(b,n,\theta) + A^*(b,n,\theta) \tag{36}$$

representing the sum of all efforts in the contest. Partial differentiation of W subject to θ delivers:

$$\begin{aligned} \frac{\mathrm{d}\, W}{\mathrm{d}\,\theta} = & \frac{\partial F^*(\cdot)}{\partial \theta}\; [R(x^*(\cdot)) - nE_{C_i}(x^*(\cdot))] \\ & + F^*(\cdot) \left[\frac{\partial R(x^*(\cdot))}{\partial x^*(\cdot)} \frac{\mathrm{d}\, x^*(\cdot)}{\mathrm{d}\,\theta} - n\, \frac{\partial E_{C_i}(x^*(\cdot))}{\partial x^*(\cdot)} \frac{\mathrm{d}\, x^*(\cdot)}{\mathrm{d}\,\theta} \right] - \frac{\partial B^*(\cdot)}{\partial \theta}. \end{aligned} \tag{37}$$

Given that $\theta \in [0, \widetilde{\theta}]$, the first term on the RHS of (37) equals zero, since a marginal increase in θ does not alter the probability (F^*). The sign of the second term is ambiguous, since R^* and $E^*_{C_i}$ decline with θ. The third term is unambiguously negative, since $A^* = 0$ and Z^* increases in θ in the aforementioned interval. Hence, the welfare effect is ambiguous.

FIGURE 8 HERE

For $\theta > \widetilde{\theta}$ the first term becomes positive, since $R^* < n\, E^*_{C_i}, \forall\, \theta \geq 0$ and $\partial F^*/\partial\theta < 0$ for $\theta > \widetilde{\theta}$. The second term on the RHS of (37) equals zero, since both, R^* and $E^*_{C_i}$ are independent from θ in the named interval. Finally, the sign of the third term is ambiguous. Thus, the sign of the change in welfare is ambiguous for $\theta > \widetilde{\theta}$.
It is easy to verify that

$$\left.\frac{\mathrm{d}\, W}{\mathrm{d}\,\theta}\right|_{\theta=0} = -\frac{b}{e} \quad \text{and} \tag{38}$$

$$\lim_{\theta\to\infty} W = n\, b, \tag{39}$$

i.e., an incremental increase in θ - starting from the textbook monopoly case - leads to a decrease in welfare. By continuity of the functions involved, this is also true in a neighbourhood of $\theta = 0$. Moreover, the welfare in the economy tends to the welfare in the polypol case if copy-protection of the monopoly good is technologically impossible ($\theta \to \infty$). Furthermore, in the interior solution ($\theta \geq \tilde{\theta}$) we find that there is a unique level of relative effectiveness ($\bar{\theta}$) which causes the coincidence of the standard textbook case and our model in terms of welfare:

$$\begin{aligned}\bar{\theta} = 2\,n \Big/ \Bigg(& 2 + \sqrt{2} + n(1 + \sqrt{2}) \\ & - \sqrt{6 + 4\sqrt{2} + n\left(8 + 6\sqrt{2} + 3\,n + 2\sqrt{2}\,n\right) - 8\left(1 + \sqrt{2}\right)\, e^{\sqrt{2}-1}\, n}\Bigg).\end{aligned} \tag{40}$$

In figure 8 we have plotted welfare (W) for $b = 1$ and $n = 5$. We see that welfare decreases as long as θ is sufficiently small. Then, welfare increases in the interior solution, but stays below the textbook case, for $\theta < \bar{\theta}$, and rises above this level for $\theta > \bar{\theta}$. For $\theta \to \infty$ it tends toward $b\,n$, the welfare in a competitive economy. Again, our findings are

Proposition 4 (*Welfare aspects*)

1. *The profit of the monopolist is a decreasing function of* θ.
2. *The consumers' surplus is an increasing function of* θ.
3. *The impact of* θ *on the total welfare of the economy (*W*), defined as the sum of all consumers' surplus and the profit of the producer, is ambiguous. The welfare decreases for small* θ. *If* θ *rises above* $\bar{\theta}$ *welfare is higher than in the case* $\theta = 0$ *(standard monopoly case) and tends for* $\theta \to \infty$ *to* $b\,n$, *the welfare in a competitive economy.*

7 Conclusion

Traditionally, monopolies are said to cause inefficiencies because of the differences between the marginal cost of production and the price fixed by the profit-maximizing monopolist. Given identical preferences on the consumer side and identical cost functions on the side of miscellaneous monopolists, identical monopoly prices are by far the exception more than the rule. The reason for this are the various circumstances that persuade a producer to deviate from the typical textbook profit-maximizing price. For example, the fear of competition induces monopolists to set a lower price so as to make it unprofitable for the possible intruder to enter the market. Compared to the textbook case, in our model the monopolist increases the supply in order to diminish the incentive for consumers to bypass copy-protection on the monopolist's product. Our research demonstrates that inefficiencies increase if the relative effectiveness of cracking is low in comparison to copy-protection. In this case cracking is deterred by the monopolist, due to the investment in copy-protection and a lower monopoly price, compared to the textbook case. Hence, the probability that the monopoly falls is zero. Here, the investment in copy-protection overcompensates the positive effect on welfare due to the increase in supply of the monopoly good.

If the relative effectiveness increases above a certain value ($\tilde{\theta}$) deterring cracking is no more profit-maximizing for the monopolist. Therefore the level of copy-protection decreases and the supply of the monopoly good remains unchanged. A further increase of the relative effectiveness leads to a value of welfare which lies above the standard monopoly case ($\theta > \bar{\theta}$). In this case the combined negative effect on welfare due to the unproductive investments in cracking and copy-protection is overcompensated by the positive effects on welfare due to the increased supply of the digital good and the higher probability that the monopoly falls.

Thus, we were able to show that for a wide range of relative effectiveness, "cracking" induces positive welfare effects in a challenged monopolistic market. This result depends on the quasi-linear utility function, the special form of the Tullock CSF and the Stackelberg form of strategic interaction. Relaxing these assumptions may result in ambiguous effects on all relevant endogenously determined parameters. Further research should focus its attention on generalizing our findings.

Figures (Place indicated in text)

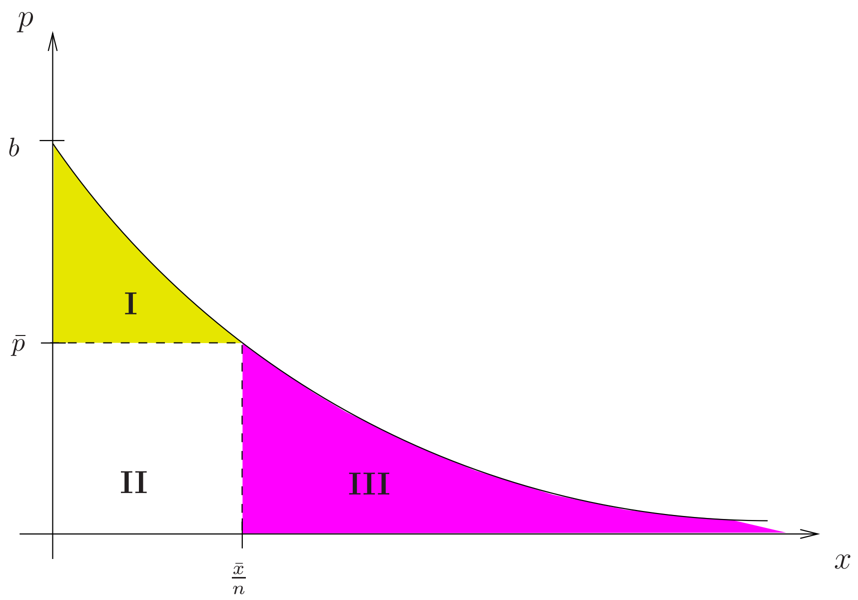

Figure 1: The demand function of agent i ($y_i(p)$), the individual consumer rent (R_{C_i}) and the extra per-agent gross consumer rent (E_{C_i}).

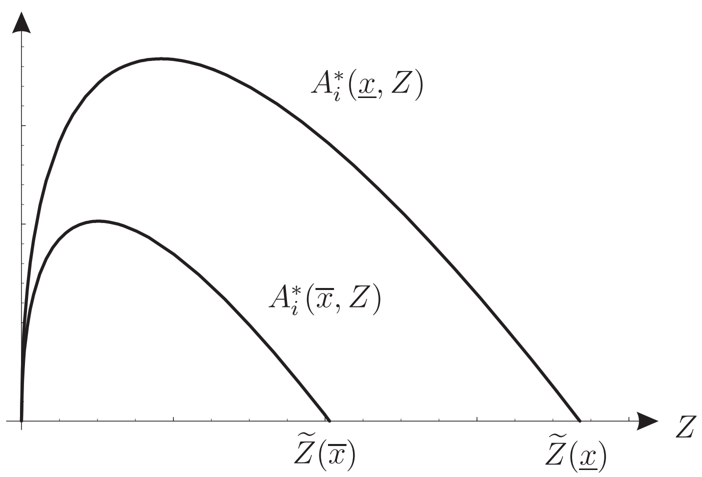

Figure 2: The reaction function $\bar{A}_i(x, Z)$ for various levels of x

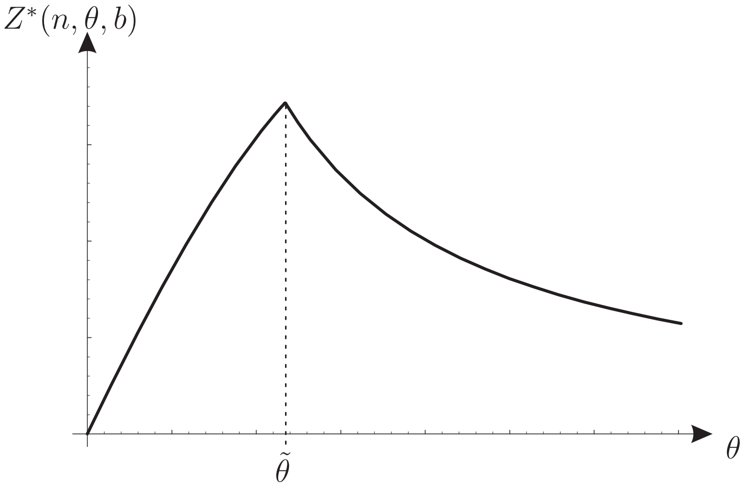

Figure 3: Investment in copy-protection contingent on θ.

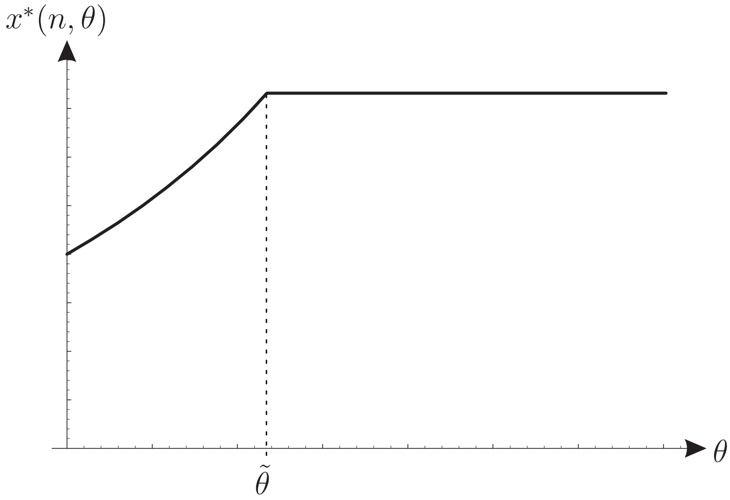

Figure 4: The supply of the monopoly good contingent on θ.

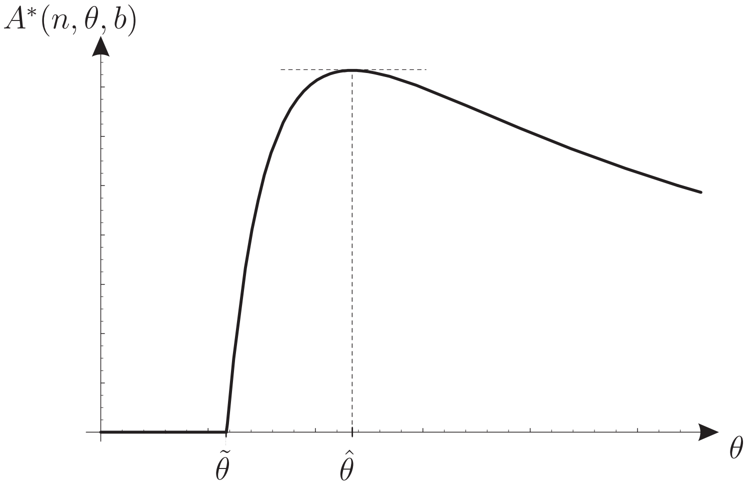

Figure 5: The effort of consumers in the copy-protection bypass activity contingent on θ

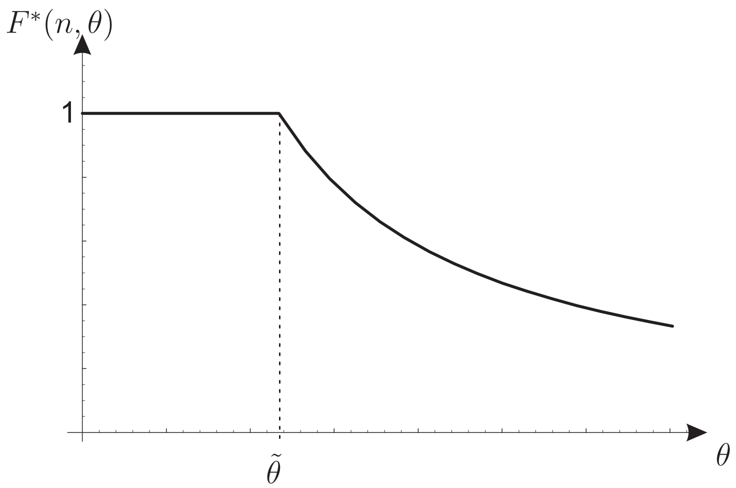

Figure 6: The probability that the monopoly retains contingent on θ.

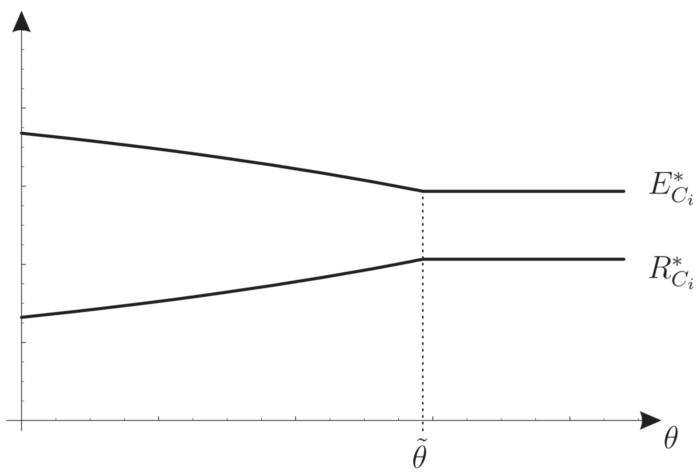

Figure 7: The individual consumer rent ($R^*_{C_i}$) and the individual extra consumer rent ($E^*_{C_i}$) in equilibrium, contingent on θ.

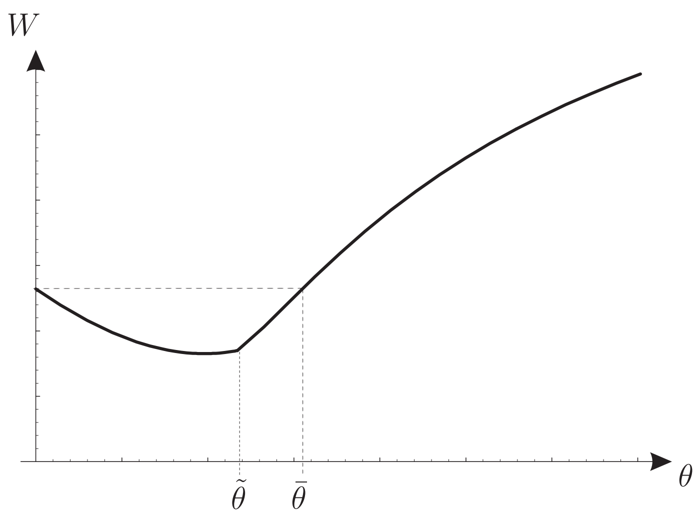

Figure 8: The welfare contingent on θ.

Mathematical Appendix (not for publication)

A Second-stage optimizing

In the second stage each consumer decides about the time he wishes to sacrifice in order to appropriate the monopoly good. The payoff-maximizing consumer has to consider certain constraints:

$$\max_{A_i} \quad v_i(A, Z, x) \tag{41}$$

$$\text{s.t.} \quad A_i \geq 0. \tag{42}$$

Partial differentiation of the payoff function subject to A_i delivers in the symmetric case, i.e. $A_{-i} = (n-1)A_i$:

$$\frac{\partial v_i(A, Z, x)}{\partial A_i} = \frac{E_{C_i}(x)\, Z\, \theta}{(Z + A_i\, n\, \theta)^2} - 1. \tag{43}$$

The solution to this FOC is a strictly concave function of Z which equals zero for $Z = \widetilde{Z}(x)$, with

$$\widetilde{Z}(x) = \theta\, E_{C_i}(x). \tag{44}$$

This yields

$$A_i^*(Z, x) = \begin{cases} \bar{A}_i(Z, x) & \text{for } Z < \widetilde{Z}(x), \\ 0 & \text{else }, \end{cases} \tag{45}$$

with

$$\bar{A}_i(Z, x) = \frac{\sqrt{\theta\, Z\, E_{C_i}(x)} - Z}{n\, \theta}. \tag{46}$$

B Profit maximizing in stage 1 (Interior solution)

The profit of the monopolist in the first stage becomes in the interior solution ($x > 0, A > 0, Z > 0$):

$$\Pi(x, Z) = F\big(Z, \bar{A}(x, Z)\big)\, R(x) - Z, \tag{47}$$

with

$$F\big(Z, \bar{A}(x, Z)\big) = \sqrt{\frac{Z}{\theta\, E_{C_i}(x)}}.$$

Partial derivation of the profit function subject to x delivers

$$\frac{1}{2}\, \frac{\frac{\partial E_{C_i}(x)}{\partial x}}{E_{C_i}(x)} = \frac{\frac{\partial R(x)}{\partial x}}{R(x)}. \tag{48}$$

If we generally describe the extra per-agent gross consumer rent ($E_{C_i}(x)$) we find that

$$E_{C_i}(\bar{x}) = \int_{\bar{x}}^{\infty} p(x)\,\mathrm{d}\frac{x}{n} + p(\bar{x})\frac{\bar{x}}{n},$$

where $\bar{x}$ represents the supply of the monopolist. The first term on the RHS represents the additional rent due to the fact that each agent gets infinite access to the monopoly good. The second term represents the expenditure per-agent. Partial derivation of $E_{C_i}(x)$ subject to x delivers

$$\left.\frac{\partial E_{C_i}(x)}{\partial x}\right|_{\bar{x}} = \frac{\bar{x}}{n}\left.\frac{\partial p(x)}{\partial x}\right|_{\bar{x}}.$$

If we display the above FOC (equation (48)) in terms of elasticity, we find that

$$\left.\varepsilon_{p,x}\right|_{x^*} = \frac{x^{*2}}{2\,n\,E_{C_i}(x^*)}\left.\frac{\partial p(x)}{\partial x}\right|_{x^*} - 1, \tag{49}$$

where

$$\varepsilon_{p,x} = \frac{\partial p(x)}{\partial x}\,\frac{x}{p(x)},$$

represents the inverse of the price elasticity of demand. Since the first term of the RHS of equation (49) is negative, $\varepsilon_{p,x}$ has to be smaller -1 in optimum, or, in other words, the monopolist's optimal supply ($x^*(\theta, n)$) lies on the inelastic part of the demand function. To guarantee that $x^*(\theta, n) \in]0, \infty[$ for the constant supplied quantity of the monopolist in the interior solution (i.e. $\bar{A}(x, Z) > 0$), the inverse demand function $p(x)$ has to satisfy the following *sufficient* conditions:

- $p(x)$ has to be continuously differentiable,
- $\lim\limits_{x\to\infty} p(x) = 0$,
- $p'(x) < 0$,
- The elasticity $\varepsilon_{p,x}$ has to satisfy $\lim\limits_{x\to 0} \varepsilon_{p,x} \geq -1$ and $\lim\limits_{x\to\infty} \varepsilon_{p,x} < \left.\varepsilon_{p,x}\right|_{x^*}$.

C Profit maximizing in stage 1 (Corner solution)

The monopolist is aware of the fact that he can deter the challenge of his monopoly, i.e. he can set Z high enough to enforce $\bar{A} = 0$. It is optimal for the monopolist to switch from the interior solution with $\bar{A} > 0$ to the corner solution with $\bar{A} = 0$ if θ is low enough. To see this, we maximize the monopolist's profit on condition that the efforts of the agents are nonnegativ. The Lagrangian function is

$$\begin{aligned}\mathcal{L}(x, Z) &= \Pi(x, Z) + \lambda_3\,\bar{A}(x, Z) &(50)\\ &= \sqrt{\frac{Z\;n\;b\;e^{-\frac{x}{n}}}{\theta\;(n+x)}}\;x - Z + \lambda_3\frac{-Z + \sqrt{Z\;\theta\;b\;e^{-\frac{x}{n}}\;(1+\frac{x}{n})}}{n\;\theta}, &(51)\end{aligned}$$

with Lagrange parameter λ_3.
Solving the Lagrangian yields the FOCs

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x} &= \left(\sqrt{b\,e^{-\frac{x}{n}}\,(1+\frac{x}{n})}\left(-\sqrt{Z}\,\lambda_3\;x\,(n+x)\,\sqrt{b\,e^{-\frac{x}{n}}\,Z\,\theta\,(1+\frac{x}{n})}\right.\right.\\
&\qquad \left.\left.+Z\;n^2\,\sqrt{\theta}\,\sqrt{b\,e^{-\frac{x}{n}}\,(1+\frac{x}{n})}\,(2n^2-x^2)\right)\right)\Big/ \qquad (52)\\
&\qquad \left(2\,n^2\,\sqrt{\theta}\,(n+x)^2\,\sqrt{b\,e^{-\frac{x}{n}}\,Z\,\theta\,(1+\frac{x}{n})}\right)\\
\frac{\partial \mathcal{L}}{\partial Z} &= \left(-2\,Z\,(\lambda_3+n\,\theta)\;(n+x)\;+\;\sqrt{Z}\,\lambda_3\sqrt{\theta}\,(n+x)\,\sqrt{b\,e^{-\frac{x}{n}}\,(1+\frac{x}{n})}\right. \qquad (53)\\
&\qquad \left.+\,n^2\,x\,\sqrt{b\,e^{-\frac{x}{n}}\,Z\,\theta\,(1+\frac{x}{n})}\right)\Big/\left(2\,Z\,n\,\theta\,(n+x)\right)\\
\frac{\partial \mathcal{L}}{\partial \lambda_3} &= \frac{-Z+\sqrt{Z\,\theta\,b\,e^{-\frac{x}{n}}\,(1+\frac{x}{n})}}{n\,\theta} \qquad (54)
\end{aligned}$$

with

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x} &\leq 0 \qquad x\geq 0 \qquad x\,\frac{\partial \mathcal{L}}{\partial x}=0, \qquad (55)\\
\frac{\partial \mathcal{L}}{\partial Z} &\leq 0 \qquad Z\geq 0 \qquad Z\,\frac{\partial \mathcal{L}}{\partial Z}=0, \qquad (56)\\
\frac{\partial \mathcal{L}}{\partial \lambda_3} &\geq 0 \qquad \lambda_3\geq 0 \qquad \lambda_3\,\frac{\partial \mathcal{L}}{\partial \lambda_3}=0, \qquad (57)
\end{aligned}$$

where the third column represents the complementary slackness conditions.
As we have already solved the interior solution with $\lambda_3 = 0 \ \wedge \ x > 0 \ \wedge \ Z > 0$ we are now seeking the corner solution with $\lambda_3 > 0 \ \wedge \ x > 0 \ \wedge \ Z > 0$, i. e. $\bar{A} = 0$. Therefore we can concentrate on the case $\frac{\partial\,\mathcal{L}}{\partial\,x} = 0 \ \wedge \ \frac{\partial\,\mathcal{L}}{\partial\,Z} = 0 \ \wedge \ \frac{\partial\,\mathcal{L}}{\partial\,\lambda_3} = 0$.
Solving these equations yields

$$Z^*(b,n,\theta) = \frac{b\,\theta(2\,n-\theta)e^{\frac{-n}{n-\theta}}}{n-\theta} \qquad (58)$$

and

$$x^*(n,\theta) = -n\,(1+W_{-1}\,(z)) \qquad (59)$$

with

$$z = \frac{e^{\frac{-2\,n+\theta}{n-\theta}}(-2\,n+\theta)}{n-\theta}. \qquad (60)$$

The change of $x^*(n,\theta)$ and $Z^*(b,n,\theta)$ in the corner solution subject to a change in θ is as follows: First, we show that $\frac{\mathrm{d}\,x^*(n,\theta)}{\mathrm{d}\,\theta} > 0$:

$$\frac{\mathrm{d}\,x^*(n,\theta)}{\mathrm{d}\,\theta} = \frac{\partial x^*(\cdot)}{\partial W_{-1}(z)}\,\frac{\partial W_{-1}(z)}{\partial z}\,\frac{\mathrm{d}\,z}{\mathrm{d}\,\theta}, \qquad (61)$$

where it is easy to show, that the first part is unambigiously negative. The second term becomes

$$\frac{W_{-1}(z)}{z(1+W_{-1}(z))}, \tag{62}$$

which is unambiguously negative, for $z \in [-\frac{1}{e}, 0[$ and therefore $W_{-1}(z) \leq -1$. The third term becomes

$$\frac{\mathrm{d}\,z}{\mathrm{d}\,\theta} = \frac{e^{\frac{-2\,n+\theta}{n-\theta}}\,n^2}{(n-\theta)^3}, \tag{63}$$

which is positive for $\theta \leq \tilde{\theta}$ (corner solution).

Second, we show that $\frac{\mathrm{d}\,Z^*(b,n,\theta)}{\mathrm{d}\,\theta} > 0$:

$$\frac{\mathrm{d}\,Z^*(b,n,\theta)}{\mathrm{d}\,\theta} = \frac{b\,e^{-\frac{n}{n-\theta}}(2\,n^3 - 6\,n^2\,\theta + 4\,n\,\theta^2 - \theta^3)}{(n-\theta)^3}, \tag{64}$$

with the denominator being positive for $\theta \leq \tilde{\theta}$. The nominator is positive as long as

$$2\,n(n^2 - 3\,n\,\theta + 2\,\theta^2) > \theta^3 \tag{65}$$

$$\Leftrightarrow \qquad 2\,n^3 > \theta(2\,n^2 + (2\,n-\theta)^2), \tag{66}$$

where the RHS of inequation (66) is smaller than $\theta\,6\,n^2$ - which is smaller than the LHS of equation (66) for $\theta \leq \tilde{\theta}$ (corner solution).

D Intersection of corner solution and interior solution

At the intersection of the corner solution and the interior solution both factors of the complementary slackness condition ($\lambda_3 \frac{\partial \mathcal{L}}{\partial \lambda_3} = 0$) equal zero, i.e. the constraint is not binding ($\lambda_3 = 0$), but consumers' bypassing efforts are zero ($\frac{\partial \mathcal{L}}{\partial \lambda_3} = \bar{A} = 0$). Thus, $\frac{\partial \mathcal{L}}{\partial \lambda_3} = 0$ becomes

$$\theta = \frac{Z^*(b,n,\theta)}{E_{C_i}(b,n,x^*(n,\theta))}. \tag{67}$$

As we know $\lambda_3 = 0$, we can use the FOCs of the interior solution (equations (22′) and (23′)) to derive easily $Z^*(b,n,\theta)$ and $x^*(n,\theta)$. Now we can insert this into (67) and solve for the value of θ, where the corner solution and the interior solution applies:

$$\widetilde{\theta} = \frac{n}{2+\sqrt{2}}. \tag{68}$$

For $\theta \to 0$ we are in the monopoly textbook-case. Therefore, $Z^* = 0$ and $x^* = n$, which corresponds with the upper cases of equations (24.1) and (24.2), i.e. the corner solution. Since there is only one unique border between corner and interior solution ($\widetilde{\theta}$) we find that the corner solution applies for $\theta \in [0, \tilde{\theta}]$, by continuity of the functions involved.

E Change of profit and consumer rent for certain values of θ

E.1 $\frac{\mathbf{d}\Pi^*(b,n,\theta)}{\mathbf{d}\theta} < 0$ for $\theta > \tilde{\theta}$

In this case we find that

$$\begin{aligned} R(x^*(n,\theta)) &= \sqrt{2}\,n\,b\,e^{-\sqrt{2}}, \\ \frac{\partial Z^*(b,n,\theta)}{\partial\theta} &= -\frac{b\,e^{-\sqrt{2}}n^2}{2\,\theta^2(1+\sqrt{2})} \text{ and} \\ \frac{\partial F(b,n)}{\partial\theta} &= -\frac{n}{(2+\sqrt{2})\,\theta^2}. \end{aligned}$$

Therefore, equation (32) becomes negative for $\theta > \tilde{\theta}$ as long as

$$\frac{\sqrt{2}}{2+\sqrt{2}} > \frac{1}{2(1+\sqrt{2})},$$

which is clearly true.

E.2 $\frac{\mathbf{d}\,V^*}{\mathbf{d}\,\theta} > 0$ for $\theta[\tilde{\theta},\hat{\theta}[$.

V^* rises in θ if

$$\begin{aligned} && n\left(-\frac{\partial F^*}{\partial\theta}E_{C_i}\right) - \frac{\partial A^*}{\partial\theta} &> 0 \\ \Leftrightarrow && \frac{e^{-\sqrt{2}}((3+2\sqrt{2})b\,n\,\theta + b\,n^2(-2-\sqrt{2}+(6+4\sqrt{2})\theta))}{2(4+3\sqrt{2})\theta^3} &> 0, \end{aligned}$$

which is true for

$$(3+2\sqrt{2})\theta(1+2\,n) \;>\; n(2+\sqrt{2}), \tag{69}$$

with the infimum of the LHS of this inequation being $(3+2\sqrt{2})\frac{n}{2+\sqrt{2}}(1+2\,n)$. Thus, the inequation (69) holds, as long as $n > \frac{1}{2}$, which is true in our model. Therefore V^* rises unambiguously in θ for $\theta > 0$.

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