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# Measuring regional environmental efficiency: A directional distance function approach

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## Abstract

This paper by applying a directional distance function approach measures the UK regions' municipality waste performance. In addition the paper constructs conditional stochastic kernels trying to determine nonparametrically the association of regions' GDP per capita levels with their calculated regional environmental efficiencies. There are evidences of regional environmental inefficiencies for the majority of UK regions regardless their regional GDP per capita levels.

**Keywords:** Regional environmental performance; Directional distance function; Conditional stochastic kernel

**JEL classification:** C6, O13, Q5.

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## 1. Introduction

The measurement of environmental technology has been an open challenge for researchers. The problem lies on the treatment of the pollutant<sup>1</sup> in production function framework. One of the ways that the ‘bad’ output can be modelled appeared in the pioneered work by Färe *et al.* (1989) by assuming strong (for desirable outputs) and weak (for undesirable outputs) disposability treating environmental effects as undesirable outputs in a hyperbolic efficiency measure. Generally the property of weak disposability of detrimental variables is well known and has been used in several formulations (Färe *et al.*, 1996, 2004; Chung *et al.*, 1997; Tyteca, 1996, 1997; Zofio and Prieto, 2001). But, although this approach is widely accepted among the environmental economists it has faced several criticisms (Hailu and Veeman, 2001; Färe and Grosskopf, 2003; Hailu, 2003, Kuosmanen, 2005; Färe and Grosskopf, 2009; Kuosmanen and Podinovski, 2009).

Our study applies the weak disposability assumption in a directional distance function measure in order to determine for the first time the environmental performance of UK regions.

## 2. Data and Methodology

In our analysis we use data collected we use data collected from two different regional databases (Eurostat<sup>2</sup> and OECD<sup>3</sup>) and concerning the year 2005. The two inputs used in our analysis are total regional labour force and regional gross fixed capital formation (in million Euros). In addition the two outputs used in our study are regional gross domestic product (million PPS- good output) and as ‘bad’ output

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<sup>1</sup> The pollutant is also referred to the literature of measuring environmental technology as ‘bad’ output.

<sup>2</sup> Available from: [http://epp.eurostat.ec.europa.eu/portal/page/portal/region\\_cities/introduction](http://epp.eurostat.ec.europa.eu/portal/page/portal/region_cities/introduction)

<sup>3</sup> Available from: [http://stats.oecd.org/Index.aspx?DataSetCode=REG\\_LAB\\_TL3](http://stats.oecd.org/Index.aspx?DataSetCode=REG_LAB_TL3)

municipal waste (in 1000t). The data are referring to NUTS 2 level of the UK regions<sup>4</sup>.

Therefore, following Färe and Grosskopf (2004) we let  $P(x)$  to denote an input vector  $x \in \mathfrak{R}_+^N$  which can produce a set of undesirable outputs  $u \in \mathfrak{R}_+^K$  and desirable outputs  $y \in \mathfrak{R}_+^M$ . Then in order to determine the environmental technology several assumptions are needed to be taken following Shephard (1970), Shephard and Färe (1974) and Färe and Primont (1995). We assume that the output sets are closed and bounded and that inputs are freely disposal. In addition  $P(x)$  can be an environmental output set if:

1.  $(y, u) \in P(x)$  and  $0 \leq \theta \leq 1$  then  $(\theta y, \theta u) \in P(x)$  (i.e. the outputs are weakly disposable) and
2.  $(y, u) \in P(x)$ ,  $u = 0$  implies that  $y = 0$  (i.e. the null jointness assumption of good and bad outputs).

The weak disposability assumption implies that the reduction of bad outputs are costly and therefore the reduction of bad outputs can be obtained only by a simultaneous reduction of good outputs. In addition the assumption which indicates that the good outputs are null-joint with bad outputs implies that the bad outputs are byproducts of the production process when producing good outputs. In order to formalize the environmental efficiency we use data envelopment analysis (DEA) framework. Let  $k = 1, \dots, K$  be the observations and then the environmental output can be formalized as:

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<sup>4</sup> For information regarding UK's regions see:  
[http://en.wikipedia.org/wiki/NUTS\\_of\\_the\\_United\\_Kingdom](http://en.wikipedia.org/wiki/NUTS_of_the_United_Kingdom)

$$\begin{aligned}
P(x) = \left\{ (y, u) : \sum_{k=1}^K z_k y_{km} \geq y_m, m = 1, \dots, M, \right. \\
\sum_{k=1}^K z_k u_{kj} = u_j, j = 1, \dots, J, \\
\sum_{k=1}^K z_k x_{kn} \leq x_n, n = 1, \dots, N, \\
\left. z_k \geq 0, k = 1, \dots, K \right\}
\end{aligned} \tag{1}$$

$z_k, k = 1, \dots, K$  indicate the intensity variables which are not negative and imply constant return to scale<sup>5</sup>. The inequality on the good outputs and the equality on the bad outputs help us to impose the weak disposability assumption and only strong disposability of good outputs. However the null-jointness is imposed by the following restrictions on bad outputs:

$$\begin{aligned}
\sum_{k=1}^K u_{kj} > 0, j = 1, \dots, J, \\
\sum_{j=1}^J u_{kj} > 0, k = 1, \dots, K.
\end{aligned} \tag{2}.$$

Furthermore, we apply the directional distance function approach as in Chung et al. (1997) and in order to be able to reduce bad and expand good outputs. In order to be able to model that in the directional distance function setting we use a direction vector  $g = (g_y, -g_u)$ , where  $g_y = 1$  and  $-g_u = -1$ . Then the efficiency score for a region  $k'$  can be obtained from:

$$\begin{aligned}
\vec{D}_o(x^{k'}, y^{k'}, u^{k'}; g) = \max \beta \\
s.t. (y^{k'} + \beta g_y, u^{k'} - \beta g_u) \in P(x)
\end{aligned} \tag{3},$$

then the linear programming problem can be calculated as:

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<sup>5</sup> Following Zelenyuk and Zheka (2006, p.149) our regional environmental efficiency measurement follows the most common assumption made in economics which is the constant returns to scale (CRS) assumption. In addition the CRS assumption provides as with greater discriminative power among the examined regions. Finally, due to the fact that we have a small sample size (37 regions) it is therefore better for our analysis to use more robust scale assumptions. However, if the variable returns are needed to be calculated the  $\sum_{k=1}^K z_k = 1$  restriction must be added to the linear programming problem (1).

$$\begin{aligned}
\vec{D}_o(x^{k'}, y^{k'}, u^{k'}; g) &= \max \beta \\
s.t. \quad \sum_{k=1}^K z_k y_{km} &\geq y_{k'm} + \beta g_{ym}, m = 1, \dots, M, \\
\sum_{k=1}^K z_k u_{kj} &= u_{k'j} - \beta g_{uj}, j = 1, \dots, J, \\
\sum_{k=1}^K z_k x_{kn} &\leq x_{k'n} \\
z_k &\geq 0, k = 1, \dots, K.
\end{aligned} \tag{4}$$

Efficiency is afterward indicated when  $\vec{D}_o(x^{k'}, y^{k'}, u^{k'}; g) = 0$  and inefficiency when  $\vec{D}_o(x^{k'}, y^{k'}, u^{k'}; g) > 0$ . Due to the fact that we are using the efficiency scores obtained in a second stage analysis we present the efficiency scores obtained in terms of Shephard's output distance function. In fact according to Chung et al. (1997) Shephard's output distance function is a special case of the directional distance function and can be calculated as:

$$D_o(x, y, u) = 1 / \left( 1 + \vec{D}_o(x^k, y^k, u^k; y^k, u^k) \right) \tag{5}$$

In addition this paper constructs estimates of conditional stochastic kernel and in order to identify how regional GDP per capita (GDPPC) used interrelates with the obtained regional environmental efficiency (REE) levels<sup>6</sup>. Following, Racine (2008) let  $f(\cdot)$  and  $\mu(\cdot)$  be the joint and marginal densities of  $(X, Y)$  and  $X$  respectively. Let  $Y$  and  $X$  be the dependent and independent variables accordingly ( $Y = REE, X = GDPPC$ ). Then the stochastic kernel (or the conditional

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<sup>6</sup> From the analysis have been excluded the two environmental efficient regions (i.e. Inner London and North Eastern Scotland, with efficiency score equal to 1) because they have significantly higher GDP per capita levels (88300 and 40400 euros) compared to the other regions. This in turn masks the visualisation effect obtained from the conditional stochastic kernel and can provide us with misleading results.

distribution function) can be estimated as:

$$\hat{g}(y|x) = \hat{f}(x, y) / \hat{f}(x) \quad (6)$$

Using a product Gaussian kernel the  $\hat{f}(x, y)$  can be estimated as:

$$\hat{f}(x, y) = \frac{1}{n} \sum_{i=1}^n \frac{1}{h_x \sqrt{2\pi}} e^{-0.5 \left( \frac{x-x_i}{h_x} \right)^2} \frac{1}{h_y \sqrt{2\pi}} e^{-0.5 \left( \frac{y-y_i}{h_y} \right)^2} \quad (7)$$

$$\text{and } \hat{f}(x) \text{ as: } \quad \hat{f}(x) = \frac{1}{n} \sum_{i=1}^n \frac{1}{h_x \sqrt{2\pi}} e^{-0.5 \left( \frac{x-x_i}{h_x} \right)^2} \quad (8)$$

where  $(h_x, h_y)$  are representing the bandwidths calculated by the least squares cross-validation data driven method as suggested by Hall et al. (2004)<sup>7</sup>.

### 3. Empirical Results and Conclusions

The empirical results (Table1) indicate that Inner London and North Eastern Scotland appear to be environmental efficient regions. In addition the last five UK regions in terms of the lowest environmental efficiencies are reported to be Tees Valley and Durham, Cumbria, West Wales and The Valleys, Cornwall and Isles of Scilly and Highlands and Islands. In addition Table 1 indicates that the average REE level is 0.7 (with standard deviation equals to 0.09). As it can be observed only fourteen UK regions are reported to have REE score above 0.7. These are reported to be Inner London, North Eastern Scotland, Berkshire, Buckinghamshire and Oxfordshire, Bedfordshire and Hertfordshire, Gloucestershire, Wiltshire and Bristol/Bath area, Outer London, Surrey, East and West Sussex, Hampshire and Isle of Wight, Cheshire, West Yorkshire, West Midlands, Leicestershire, Rutland and Northamptonshire, Greater Manchester and East Anglia.

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<sup>7</sup> For empirical applications of conditional stochastic kernels on income dynamics see Fotopoulos (2009) and Poletti Laurini and Valls Pereira (2009).

*Table 1: UK regions' environmental efficiency levels measured in Shephard's output distance functions*

<b>UK Regions-NUTS2</b>	<b>REE</b>	<b>UK Regions-NUTS2</b>	<b>REE</b>
Tees Valley and Durham	0.6423	Essex	0.6891
Northumberland and Tyne and Wear	0.6776	Inner London	1.0000
Cumbria	0.6418	Outer London	0.7409
Cheshire	0.7236	Berkshire, Buckinghamshire and Oxfordshire	0.8114
Greater Manchester	0.7053	Surrey, East and West Sussex	0.7296
Lancashire	0.6719	Hampshire and Isle of Wight	0.7249
Merseyside	0.6458	Kent	0.6829
East Yorkshire and Northern Lincolnshire	0.6591	Gloucestershire, Wiltshire and Bristol/Bath area	0.7520
North Yorkshire	0.6694	Dorset and Somerset	0.6749
South Yorkshire	0.6794	Cornwall and Isles of Scilly	0.6243
West Yorkshire	0.7130	Devon	0.6593
Derbyshire and Nottinghamshire	0.6925	West Wales and The Valleys	0.6319
Leicestershire, Rutland and Northamptonshire	0.7057	East Wales	0.6942
Lincolnshire	0.6563	Eastern Scotland	0.6835
Herefordshire, Worcestershire and Warwickshire	0.6867	South Western Scotland	0.6714
Shropshire and Staffordshire	0.6631	North Eastern Scotland	1.0000
West Midlands	0.7074	Highlands and Islands	0.6230
East Anglia	0.7027	Northern Ireland (UK)	0.6593
Bedfordshire and Hertfordshire	0.7526		
<b>Mean</b>			<b>0.7040</b>
<b>Std</b>			<b>0.0817</b>
<b>Min</b>			<b>0.6230</b>
<b>Max</b>			<b>1.0000</b>

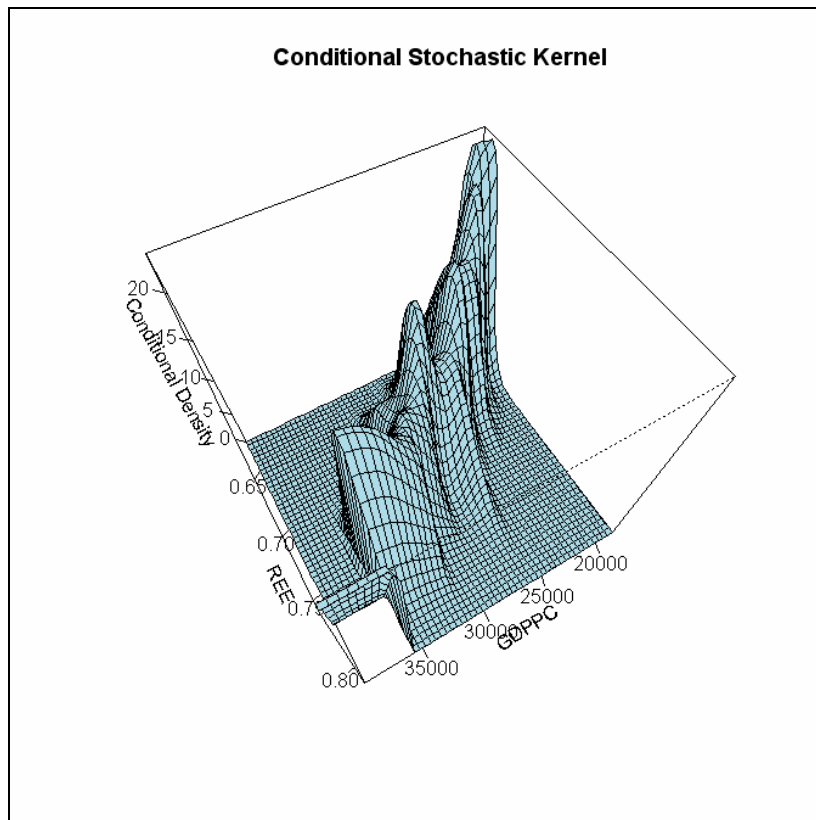
Furthermore, and in order to understand how per capita regional income levels affect regions environmental efficiencies we construct conditional stochastic kernels between REE and GDPPC variables<sup>8</sup>. This relationship is presented on Figure 1 in a conditional stochastic kernel form. When looking Figure 1 we can choose a fixed point on the axis labeled REE and then by slicing the graph from this point and moving parallel to GDPPC axis, the estimated distribution of regions' REE levels over the examined time period conditional on GDPPC levels can be traced. The graphic shows that regions in the extremes of environmental efficiency have higher probability which have been generated by the respective extremes of per capita

<sup>8</sup> The routes and theory behind the link of environmental quality and economic development stages, income disparities can be found in the works of Kuznets (1955), Grossman and Kruger (1995) and Dasgupta et al. (2002).



growth levels. That is low-environmental efficiency regions have high probability that have been generated by lower GDP per capita levels and high-environmental efficiency regions, by higher GDP per capita levels. However, for the intermediate-environmental efficient regions (with REE between 0.65-0.70), the effect of per capita income is less determinant, given the high dispersion of estimated densities. We can interpret this result as club convergence (which is conditioned on GDPPC)<sup>9</sup>.

**Figure 1: Conditional stochastic kernels of UK regions-Regional environmental efficiency (REE) conditioned on regional GDP per capita (GDPPC) levels**



Finally, in respect to the methodologies adopted the contribution of the paper is twofold: to demonstrate how directional distance functions can be applied in a regional level and how then the estimation of conditional stochastic kernels can be

<sup>9</sup> In fact regardless their GDP per capita levels twenty one (out of thirty seven) UK regions have environmental efficiency levels between 0.65-0.7.

used in order to examine the regional environmental quality-economic growth relationship.

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