# A model of borrower reputation as intangible collateral

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**Abstract**

In this paper, we build a framework which can generate endogenous fluctuations in downpayment requirements. We extend the model of Kiyotaki and Moore (1997) by considering an environment, in which savers can keep their anonymity but borrowers cannot. This allows lenders to punish defaulting borrowers by excluding them from future borrowing. They cannot however stop them from saving in the anonymous financial market. We show how the possibility of such market exclusion can lead to the emergence of intangible collateral in equilibrium alongside the tangible collateral which is usually studied in the literature. Fluctuations in the value of intangible collateral are isomorphic to fluctuations in the amount of borrowing firms can secure against the value of their tangible assets.

We find that, when we combine the intangible collateral mechanism in our paper with counter-cyclical variance of idiosyncratic productivity shocks, this helps to generate realistic negative co-movement of downpayment requirements and aggregate output over the business cycle. In this case, the presence of intangible collateral increases the amplification of business cycle fluctuations relative to the standard Kiyotaki-Moore (1997) model.

JEL Classification: E21.

Key Words: Collateral constraints, Aggregate fluctuations.

# 1 Introduction

The financial boom and bust cycle of 2005-2009 was characterised by a substantial increase and subsequent fall in the permissible leverage for all sectors of the economy. Downpayment requirements on housing, capital and financial asset purchases fell during the boom and then increased sharply as the financial crisis unfolded during 2008. At the same time, asset prices and output fell sharply across the world, raising questions about the linkages between financial conditions, asset prices and real quantities during the financial crisis. And while we have a good theoretical understanding of how credit constraints affect the interaction between output and asset prices, there has been comparatively less work on downpayment requirements and other aspects of the financial conditions facing private borrowers.

In this paper, we build a framework which generates fluctuations in downpayment requirements by appealing to changes in the value of borrower's reputation for repayment. We extend the model of Kiyotaki and Moore (1997) and Kiyotaki (1998) by considering an environment, in which savers can keep their anonymity but borrowers cannot. This allows lenders to punish defaulting borrowers by excluding them from future borrowing. They cannot however stop them from saving in the anonymous financial market or by engaging in self-financed production. We show how the possibility of such market exclusion can lead to the emergence of intangible collateral in equilibrium alongside the tangible collateral which is usually studied in the literature.

The intangible collateral is essentially the value of a borrower's reputation for debt repayment. We find that this collateral form can back a very significant part of the liabilities of the private sector. One of the key contributions of this paper is to show how the financial contract in a model with tangible and intangible collateral can still be represented as a linear borrowing constraint, where a fall in the value of intangible collateral manifests itself in a higher 'haircut' (or downpayment) while a rise in the value of intangible collateral can manifest itself as a lower haircut. This result is useful because it substantially reduces the computational compexity of the model[1].

[1] The borrowing constraint is exactly linear in the steady state or during perfect foresight dynamics. Under uncertainty and risk aversion, the linearity of the borrowing constraint is only true up to an approximation,

In our numerical experiments we find that intangible collateral is large (and haircuts are low) when the cost to an entrepreneur of being excluded from borrowing is substantial. Intangible (or reputational) collateral has a non-linear relationship with the ability to collateralise tangible assets. While inefficient firms survive in equilibrium due to the tight borrowing constraints on efficient producers, expanding the ability to collateralise tangible assets boosts the excess return of efficient firms and increases the value of access to leverage. As a result, intangible collateral increases too, further amplifying firms' access to credit. Once the availability of tangible collateral becomes high enough, inefficient production disappears and further increases in tangible collateral starts to push up real interest rates, depressing the excess rate of return for high productivity entrepreneurs. From this point onwards, the value of intangible collateral declines until it reaches zero at the point at which borrowing constraints stop binding.

Finally we solve our model economy with aggregate uncertainty in order to study how intangible collateral interacts with the business cycle. We find that the model does generate endogenous fluctuations in 'haircuts'. For conventional technology shocks, these fluctuations are small and pro-cyclical. In other words, the model generates low haircuts in recessions and high haircuts in booms. The reason for this result is the following. In recessions, asset prices are low, financial constraints bind strongly and the excess return for leveraged high-productivity firms over the unleveraged low-productivity firms increases. Since recessions are expected to be persistent, this increase in excess returns leads to a rise in the value of debt market access, reducing lenders' required haircuts. In contrast, in booms, asset prices are high, financial frictions are reduced and the leveraged high productivity entrepreneurs enjoy a smaller excess return relative to unleveraged low productivity firms. Hence the value of intangible assets declines, increasing lenders' requred haircuts.

In order to replicate the counter-cyclical behaviour of downpayment requirements in the data, we augment the model by allowing pro-cyclical fluctuations in the technological gap between 'high' and 'low' productivity firms and also by allowing counter-cyclical fluctuations in the degree of uninsurable idiosyncratic production risk. This introduces a

which is very good unless the degree of risk aversion is very high.

pro-cyclical component in the value of being a leveraged high productivity producer, helping to motivate counter-cyclical haircut movements. We find that counter-cyclical variability in idiosyncratic investment risk is most promising in terms of generating a negative correlation between downpayment requirements and aggregate output. The model with counter-cyclical downpayments also features substantial business cycle amplification.

# 2 Related Literature

This paper studies the nature of dynamic borrowing contracts in an environment with permanent exclusion from credit markets. There is a large literature on dynamic optimal contracts starting with the seminal contributions of Kehoe and Levine (1993) who developed the first general equilibrium model with endogenous borrowing constraints. Subsequently, work by Alvarez and and Jermann (2000) showed how the allocation of Kehoe and Levine (1993) can be decentralised by a set of state contingent borrowing limits in a general economy with permanent exclusion from risk sharing arrangements.

Our paper is also related to the literature on the collateral amplification literature started by Kiyotaki and Moore (1997) and Kiyotaki (1998). These papers have shown that when debts are collateralised, leverage magnifies the impact of small shocks on the net worth of producers, thus amplifying and propagating impulses over time. This mechanism is central in our paper too. In the standard Kiyotaki and Moore (1997) set up, borrowers can commit to repay an exogenous fraction of project revenues or tangible asset values. In contrast, this paper explicitly models the fluctuations in such 'haircuts' as a function of the value to a borrower of being able to access credit markets in future.

There has been relatively little work on the importance of intangible collateral. Hellwig and Lorenzoni (2007) is a notable exception. They study an endowment economy with limited commitment in which there is no collateral to secure borrowing. Because the autrarkic equilibrium is dynamically inefficient and stationary bubbles on intrinsically worthless assets can exist. Hellwing and Lorenzoni show that when private borrowers can be permanently excluded from future credit market access, an equilibrium with bubbles on inside liquidity (private debt) can achieve an idential allocation as the equilibrium with bubbles on outside liquidity.

Gertler and Karadi (2010) is closer to this paper in the sense that they model banks' ability to borrow by appealing to the value of excess returns in an equilibrium with no bubbles. Their mechanism is similar to the intangible collateral studied in this paper. In Gertler and Karadi (2010) the bank is threatened with bankruptcy and the loss of the opportunity to enjoy the profits from being a banker. In our model, a defaulting entrepreneur can immediately set up a new firm and continue producing. However, she loses her access to future credit, which is costly because she can no longer lever up to maximise the returns from good business ideas (high productivity spells in the model). Finally, our paper makes the technical contribution of generalising the dynamic contracting framework to an environment of risk-averse consumer-producers while still retaining the tractability of the linear borrowing constraints of the Kiyotaki-Moore (1997) model.

We find that counter-cyclical variation in idiosyncratic production risk is one mechanism that is capable of causing counter-cyclical movements in haircuts in a way that amplifies the business cycle. Angeletos and Calvet (2006) and Perez (2006) are two papers that examine the importance of idiosyncratic production risk for the business cycle. They both show that the presence of uninsurable idiosyncratic production risk can have a profound impact on risk-taking and capital accumulation. And if the degree of idiosyncratic production risk varies in a counter-cyclical fashion (i.e. it is higher in recessions), Angeletos and Calvet (2006) show that this can amplify the business cycle by affecting entrepreneurs' investment into risky but high yielding projects. In this paper, our focus is mainly on the impact of idiosyncratic production uncertainty on haircuts. High ex post productivity variability causes the expected return from production (in utility terms) to decline and this reduces the value of borrowing. So to the extent that production uncertainty is high in recessions, this channel is capable of producing counter-cyclical downpayment requirements.

# 3 Motivating Observations

There is a lot of evidence that permissible leverage fluctuates very substantially for many private borrowers. Figure 1 below (reprinted from Geannakoplos (2009)) shows how the haircuts on securities purchases have fluctuated for the Ellington hedge fund. The chart clearly shows that haircuts average around 20% of the purchase price although they rose to 40% during the Russian default iin 1998 and during the 2008-2009 financial crisis. During the 2006-2007 credit boom haircuts were unusually low at levels just above 10%.

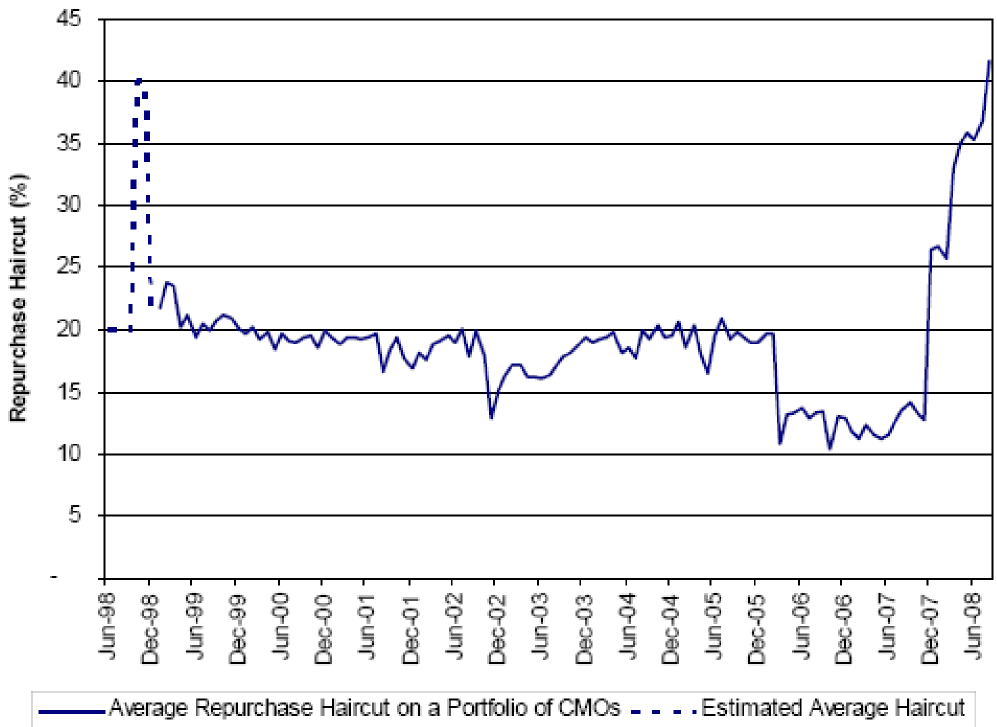


Figure 1: CMO margins at Ellington (reprinted from Geannakoplos (2009))

In housing markets, leverage fluctuations have also received a lot of recent attention. Figure 2 below shows the movement of the monthly LTV ratio for new home buyers. The chart shows that the ratio varies in a pro-cyclical fashion, with local peaks in booms (1984, 1988, 1995-1999 and 2007) and troughs in recessions (1975, 1982, 1991, 2003 and 2008).

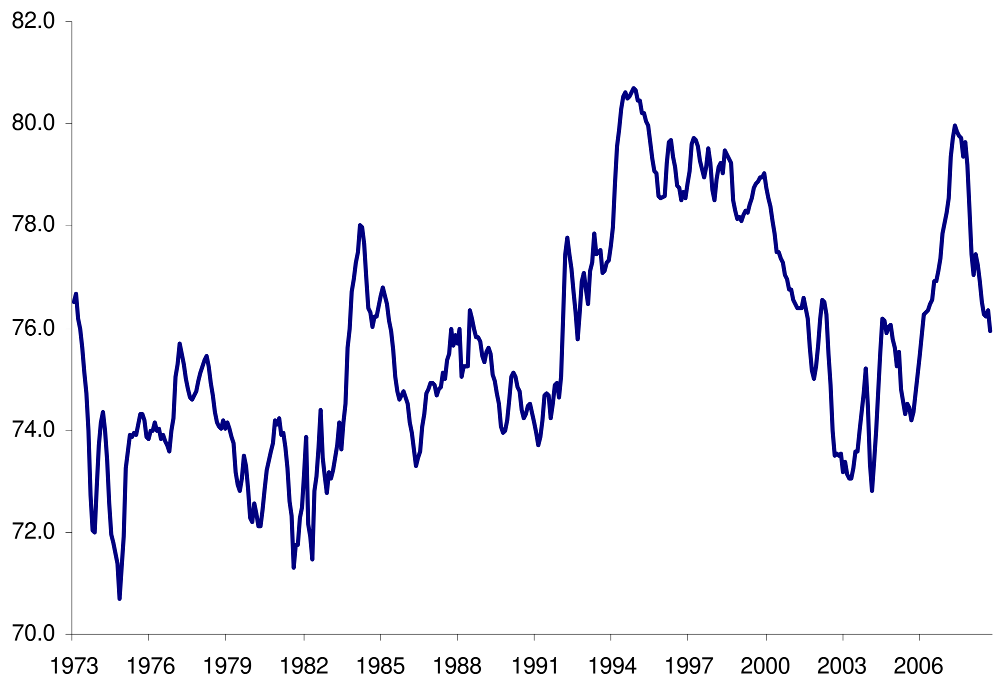


Figure 2: Loan to Value Ratios in the US: 1973-2008 (Source: FHFA)

These data show that downpayments in financial markets move in a counter-cyclical fashion and the leverage used in security or housing purchase varies in a pro-cyclical fashion. This is the feature of the data our model aims to explain.

# 4 The Model

## 4.1 The Economic Environment

The economy is populated with a continuum of infinitely lived entrepreneurs of measure 1. Each entrepreneur is endowed with a constant returns to scale production function which uses land and working capital to produce gross output $y$.

$$y_t = a_t A_t \left(\frac{k_t}{\alpha}\right)^{\alpha} \left(\frac{x_t}{1-\alpha}\right)^{1-\alpha}$$

$k$ is land (which does not depreciate and is fixed in aggregate supply) and $x$ is working capital which fully depreciates.

$a$ is the idiyosyncratic component of productivity which differs between firms both in terms of its **ex ante** expected value at the time of investment as well as in its **ex post** realised value at the time of actual production. A fraction of firms who we will refer to as 'high productivity' firms have a high idiosyncratic expected productivity:

$$E_t a_{t+1} = a^H$$

The other firms have a low expected productivity level ($a^L \equiv 1$).

Both types of firms face 'ex post' idiosyncratic risk too. If they are lucky, idiosyncratic productivity is high

$$a_{t+1} = a^i + \triangle^I$$

which happens with probability 0.5 and, if they are unlucky, idiosyncratic productivity is low

$$a_{t+1} = a^i - \triangle^I$$

which happens with probability 0.5.

The ex ante component of idiosyncratic productivity evolves according to a Markov process. Following Kiyotaki (1998) let $n\delta$ be the probability that a currently unproductive firm becomes productive and let $\delta$ be the probability that a currently productive firm becomes unproductive. This implies that the steady state ratio of productive to unproductive firms is $n$. In our baseline model, we assume that $a^H$, $a^L$ and $\triangle^I$ are constant over the business cycle. $A$ is the aggregate component of productivity (which also can be high $A^H$ or low $A^L$). The aggregate state also evolves according to a persistent Markov process.

In sensitivity analysis, we allow the possibility that the process for idiosyncratic productivity distribution varies with the aggregate state. First, we examine the possibility that the expected difference between the expected TFP of high and low productivity firms varies pro-cyclically over the business cycle. This means that high productivity agents have an expected TFP in production equal to

$$E_t a_{t+1} = a^H + \triangle^E$$

during boom periods and $a^H - \triangle^E$ during recessions. For low productivity firms, $a^L$ remains constant at unity.

Second, we examine a situation in which the 'ex post' idiosyncratic productivity component is more volatile during recessions relative to booms. In other words, the value of $\triangle^I$ is high in recessions and low in booms. This corresponds to a world in which downturns are periods of higher uncertainty for individual firms compared to booms.

## 4.2 Entrepreneurs

### 4.2.1 Preferences

Entrepreneurs are ex-ante identical and have logarithmic utility over consumption streams

$$U^e = \sum_{t=0}^{\infty} \beta^t \ln c_t$$

### 4.2.2 Flow of Funds

Agents purchase consumption ($c$), investment goods ($x$), land ($k$) at price $q_t$ and borrow using debt securities $b_{t+1}$ at price $R_t^{-1}$ where $X_t \equiv (A_t, \Gamma_t)$ is a vector describing the aggregate state of the economy. $A_t$ is aggregate TFP and $\Gamma_t$ denotes the wealth distribution.

$$c_t + q_t k_{t+1} - \frac{b_{t+1}}{R_t} = y_t + q_t k_t - b_t$$

We assume incomplete markets for idiosyncratic risk, meaning that Arrow securities contingent on the idiosyncratic state will not trade in equilibrium.

### 4.2.3 Collateral constraints

Due to moral hazard in the credit market, agents will only honour their promises if it is in their interests to do so. We assume that an entrepreneur who borrows funds at time $t$ has the ability to default at $t+1$. Following Kiyotaki and Moore (1997) we assume that lenders can seize the entrepreneur's land holding which has value $q_{t+1}k_{t+1}$ as well as a fraction $\phi$ of the firm's revenues $y_{t+1}$. The entrepreneur keeps the rest of the firm's output ($1 - \phi$ fraction). Furthermore, we assume that, upon default, entrepreneurs can be permanently excluded from future borrowing. However, they can anonymously lend to other entrepreneurs or produce without any leverage.

Individuals will repay their debts whenever the value of repaying exceeds the value of defaulting.. Let $V(s_t, X_t)$ denote the value of an entrepreneur who has never defaulted and let $V^d(s_t, X_t)$ denote the value of an entpreneur who has defaulted in the past. $s_t \equiv (w_t, a^i)$ is the idiosyncratic state where $w_t$ is individual wealth and $a^i$ is the expected idiosyncratic level of TFP. We focus on a no-default allocation. Following Kiyotaki and Moore (1997) and Kiyotaki (1998) we assume that default (or debt renegotiation) can only occur before the realisation of the aggregate or the idiosyncratic shock at time $t+1$. So the borrowing constraint is cast in terms of expected values:

$$E_t V(s_{t+1}, X_{t+1}|s_t, X_t) \geqslant E_t V^d(s_{t+1}, X_{t+1}|s_t, X_t)$$

At this stage we conjecture that this value function comparison can be reduced to a linear collateral constraint of the following form.

$$b_{t+1} \leqslant E_t[\theta_t y_{t+1} + q_{t+1} k_{t+1}]$$

The value of intangible collateral is equal to the amount of borrowing unbacked by tangible assets which can be seized by the lender

$$(\theta_t - \phi) E_t y_{t+1}$$

We verify subsequently that this is indeed the case.

### 4.3 Entrepreneurial behaviour

The entrepreneurs in our economy have to make two types of decisions. They have to choose consumption over time optimally (the consumption problem) and they have to choose the (real and financial) assets they invest in (the portfolio problem). Fortunately, the budget constraint is linear in all the assets at the entrepreneur's disposal and as a result we can utilise the result due to Samuelson (1968), which states that we can solve separate the consumption and portfolio decisions.

### 4.3.1 The consumption problem

Due to logarithmic utility, consumption is a fixed fraction of wealth at each point in time for all entrepreneurs regardless of their level of idiosyncratic productivity. This result is proved in Appendix A and it greatly simplifies the aggregation of consumption decisions.

$$c_t = (1-\beta)\, w_t$$

### 4.3.2 The production/portfolio problem

Entrepreneurs choose their holdings of three assets (land, capital and debt) under the presence of a collateral constraint. The first order conditions for each of the three assets are given below.

The first order condition for land is:

$$-\lambda_t q_t + \beta E_t \left[ \frac{\alpha y_{t+1}}{k_{t+1}} + q_{t+1} \right] \lambda_{t+1} + \mu_t E_t \left[ \theta_t \frac{\alpha y_{t+1}}{k_{t+1}} + q_{t+1} \right] = 0 \tag{1}$$

where $\lambda_t = 1/c_t$ is the lagrange multiplier on the flow of funds constraint while $\mu_t$ is the lagrange multiplier on the collateral constraint. The first order condition for working capital investment is:

$$-\lambda_t + E_t \left[ \beta \frac{(1-\alpha)\, y_{t+1}}{x_{t+1}} \lambda_{t+1} + \theta_t \frac{(1-\alpha)\, y_{t+1}}{x_{t+1}} \mu_t \right] = 0 \tag{2}$$

Finally the first order condition for debt holdings is:

$$-\frac{\lambda_t}{R_t} + \beta E_t \lambda_{t+1} + \mu_t = 0 \tag{3}$$

Combining (1), (2) and (3) we get an expression for the optimal mix between land and working capital:

$$\frac{k_{t+1}}{x_{t+1}} = \frac{\alpha}{1-\alpha} \frac{1}{u_t^i}, i = L, H \tag{4}$$

$$u_t^H = q_t - \frac{E_t q_{t+1}}{R_t} - E_t \left( (q_{t+1} - E_t q_{t+1}) \frac{\lambda_{t+1}^H}{\lambda_t} \right) \tag{5}$$

is the user cost of land for high productivity entrepreneurs for whom borrowing constraints bind and $\mu_t > 0$.

$$u_t^L = q_t - E_t \left( q_{t+1} \frac{\lambda_{t+1}^L}{\lambda_t} \right) \tag{6}$$

is the user cost of land for low productivity entrepreneurs who are unconstrained.

### 4.4 Borrowing limit determination

Our economy is a limited commitment one. Borrowers repay their debts only if it is in their interests to do so. Upon default, a borrower loses his tangible assets but also he loses his reputation for repayment. This results in permanent exclusion from debt markets in future. As we now show, entrepreneurs will be allowed to borrow up to the value of the tangible and intangible assets they can lose when they default.

#### 4.4.1 The value of a non-defaulting entrepreneur

Let $V(s_t, X_t)$ be the value of a non-defaulting entrepreneur with idiosyncratic state $s_t$ when the aggregate state is $X_t$.

$$V(s_t, X_t) = \max_{c_t, k_{t+1}, x_{t+1}, b_{t+1}} \{\ln c_t + \beta E_t V(s_{t+1}, X_{t+1})\}$$

In Appendix B we show that the value function takes the following form

$$V(s_t, X_t) = \varphi(s_t, X_t) + \frac{\ln w_t}{1-\beta}$$

where the intercept $\varphi(s_t, X_t)$ satisfies a functional equation:

$$\varphi(s_t, X_t) = \ln(1-\beta) + \max_{k_{t+1}, x_{t+1}, b_{t+1}} \beta E_t \left[\frac{\ln \beta}{1-\beta} + \frac{\ln r^i_{t+1}}{1-\beta} + \varphi(s_{t+1}, X_{t+1})\right] \tag{7}$$

Intuitively, the value of an entrepreneur depends on his current wealth (this is the term in $\ln w_t$) as well as the rate of return the entrepreneur can earn on his wealth (this is the intercept term). Looking at (7) we can see that, if the rate of return on wealth is equal to the inverse of the rate of time preference at all times ($r^i = 1/\beta$), the intercept $\varphi(s_t, X_t)$ will be equal to zero and the value of an entrepreneur will be solely determined by his current wealth. In contrast, values of $r^i$ above $1/\beta$ would generate a positive value of $\varphi$ reflecting the net present value of 'excess returns' to the entrepreneur.

#### 4.4.2 The value of a defaulting entrepreneur

An entrepreneur who defaults experiences a large one-off wealth gain because she avoids paying some of her debt. The cost of this is that she then loses her right to borrow in future.

We guess that the value of an entrepreneur who has defaulted in the past is given as follows:

$$V^d(s_t, X_t) = \varphi^d(s_t, X_t) + \frac{\ln w_t}{1-\beta}$$

where the intercept of the value function satisfies the now familiar functional equation:

$$\varphi^d(s_t, X_t) = \ln(1-\beta) + \max_{k_{t+1}, x_{t+1}, b_{t+1}} \beta E_t \left[ \frac{\ln \beta + \ln r^{di}_{t+1}}{1-\beta} + \varphi^d(s_{t+1}, X_{t+1}) \right]$$

This guess is verified in Appendix B. Intuitively, once the entrepreneur defaults he can only lend to others (when unproductive) or produce without leverage (when productive). This is reflected in the above value function which depends on $r^{di}_{t+1}$ - the rate of return on the portfolio of an entrepreneur who has defaulted in the past.

Consider the value of an entrepreneur who defaults at time $t+1$. If state $(s_{t+1}, X_{t+1})$ realises following her decision to default is, the entrepreneur's value will be given by this now familiar expession which combines current wealth with the value of future excess returns on wealth.

$$\begin{aligned} V^d(s_{t+1}, X_{t+1}) &= \varphi^d(s_{t+1}, X_{t+1}) + \frac{\ln w^d_{t+1}}{1-\beta} \\ &= \varphi^d(s_{t+1}, X_{t+1}) + \frac{\ln[(1-\phi) y_{t+1}]}{1-\beta} \end{aligned}$$

The wealth of a defaulting entrepreneur is the $1-\phi$ fraction of output she gets to keep post default. This is higher than the wealth she would have had under repayment, because the defaulting entrepreneur gains wealth equal to $(\theta_t - \phi) y_{t+1}$ by avoiding repayments on the debt secured by intangible collateral.

### 4.4.3 Solving for the borrowing limits

Alvarez and Jermann (2000) solve for borrowing limits which are 'not too tight' as the highest possible borrowing limit consistent with repayment. In our setting this is given by the incentive compatibility constraint which equates the expected value of repayment with the expected value of defaulting.

$$E_t V(s_{t+1}, X_{t+1} | \theta) = E_t V^d\left(s^d_{t+1}, X_{t+1}\right)$$

This implies that the expected loss of reputation due to default (LHS of the expression below) exactly offsets the one-off gain from having one's debt written off (the RHS of the expression below).

$$\begin{aligned} & (1-\beta) E_t \left[ \sum_{\alpha_{t+1}} \pi \left( s_{t+1} | s_t \right) \left( \varphi \left( s_{t+1}, X_{t+1} | \theta \right) - \varphi^d \left( s_{t+1}, X_{t+1} \right) \right) \right] \\ \geqslant \; & E_t \ln \left[ (1-\phi) y_{t+1} \right] - E_t \ln \left[ y_{t+1} + q_{t+1} k_{t+1} - b_{t+1} \right] \end{aligned}$$

Using the approximation:

$$E \ln x \approx \ln Ex - \frac{1}{2} var(\ln x)$$

we get:

$$\begin{aligned} & (1-\beta) E_t \left[ \sum_{\alpha_{t+1}} \pi \left( s_{t+1} | s_t \right) \left( \varphi \left( s_{t+1}, X_{t+1} | \theta \right) - \varphi^d \left( s_{t+1}, X_{t+1} \right) \right) \right] - \Omega_t \\ \geqslant \; & \ln E_t \left[ (1-\phi) y_{t+1} \right] - \ln E_t \left[ y_{t+1} + q_{t+1} k_{t+1} - b_{t+1} \right] \end{aligned}$$

where

$$\Omega_t = \frac{1}{2} \left\{ var_t(\ln \left[ y_{t+1} + q_{t+1} k_{t+1} - b_{t+1} \right]) - var_t(\ln \left[ (1-\phi) y_{t+1} \right]) \right\}$$

is an approximate risk premium term which reflects the greater ex post wealth variability for repaying entrepreneurs. Re-arranging we have:

$$b_{t+1} \leqslant \left\{ \frac{\Delta \left( s_{t+1}, X_{t+1} | \phi \right) + \phi - 1}{\Delta \left( s_{t+1}, X_{t+1} | \phi \right)} \right\} y_{t+1} + q_{t+1} k_{t+1} \tag{8}$$

where

$$\Delta \left( s_{t+1}, X_{t+1} | \phi \right) \equiv \exp \left\{ (1-\beta) \left[ \sum_{\alpha_{t+1}} \pi \left( s_{t+1} | s_t \right) \left( \varphi \left( s_{t+1}, X_{t+1} | \theta \right) - \varphi^d \left( s_{t+1}, X_{t+1} \right) \right) \right] - \Omega_t \right\}$$

Solving for the borrowing constraints requires us to solve for the value function and for the borrowing constraints until both have converged. See Appendix B for further details on the computational procedure.

#### 4.4.4 Discussion

The entrepreneur's borrowing limit is determined by the trade off between the benefits of gaining some current wealth by defaulting against the costs of permanently losing the ability to borrow. The benefit from defaulting is determined by the size of unsecured borrowing - $(\theta_t y_{t+1} - \phi)$. The costs are dominated by the gap between the expected value of being a non-defaulting entrepreneur $(\sum_{a_{t+1}} \pi(s_{t+1}|s_t)\varphi(s_{t+1}, X_{t+1}|\theta_t))$ and the value of defaulting $(\sum_{a_{t+1}} \pi(s_{t+1}|s_t)\varphi^d(s_{t+1}, X_{t+1}))$. This gap is driven by the utility value of the entrepreneur's stream of excess returns relative to current financial wealth.

Because most of these excess returns are in the future, the discount factor is one of the main determinants of the value of repayment. A discount factor of 0.95 implies that the entrepreneur is indifferent between a 1pp increase in his rate of return on wealth in perpetuity and a 19% increase in his current financial wealth. With a discount factor of 0.9, the consumer is only willing to accept a 9.5% increase in current wealth in exchange for a 1pp increase in returns.

The other crucial determinant of the size of intangible collateral is the probability of remaining highly productive. If this probability is high, then debt access is valuable because a borrower is likely to remain productive for some time and would like therefore to keep borrowing in order to boost his return on wealth. In an environment with persistent investment opportunities, intangible collateral is high and entrepreneurs have a higher borrowing capacity than the value of their tangible assets alone.

## 4.5 Market clearing

There are three market clearing conditions in our model economy - the debt market, the land market and the goods market.

$$\int b^i_{t+1} di = 0 \tag{9}$$

$$\int k^i_{t+1} di = 1 \tag{10}$$

The total quantity of land in the economy is fixed and is normalised to unity.

$$\int c^i_t di + \int x^i_{t+1} di = \int y^i_t di \tag{11}$$

## 4.6 Behaviour of the aggregate economy

Due to the presence of binding borrowing constraints, high and low productivity entrepreneurs have different demands for assets at a given level of wealth. High productivity agents prefer to invest in production in order to take advantage of high productivity. Low productivity agents have a more balanced portfolio - they invest in production too but also lend funds to the high productivity entrepreneurs through the debt market. This implies that the wealth distribution does matter for equilibrium. But even though the individual decision rules differ according to idiosyncratic productivity, these decision rules remain linear in wealth which means that a within-groups aggregation result obtains. The economy behaves as if it is populated by two agents (a high productivity and a low productivity one). Following Kiyotaki and Moore (1997), we can concentrate on just two moments of the wealth distribution - the mean of the wealth distribution $W_t$ and the share of wealth owned by high-productivity agents $d_t$.

At any given date, the state of the aggregate economy can be summarised by the state vector

$$X_t = \{A_t, W_t, d_t\}$$

consisting of the level of aggregate productivity, the level of aggregate wealth and the share of aggregate wealth held by productive agents. $A_t$ evolves according to an exogenous two state Markov process while the evolution of the two state variables $W_t$ and $d_t$ is governed by the following relations.

$$W_{t+1} = \beta \left[ d_t R^H_{t+1} + (1 - d_t) R^L_{t+1} \right] W_t \tag{12}$$

$$d_{t+1} = \frac{(1-\delta) d_t R^H_{t+1} + n\delta (1 - d_t) R^L_{t+1}}{d_t R^H_{t+1} + (1 - d_t) R^L_{t+1}} \tag{13}$$

where $R_{t+1}$ and $r_{t+1}$ are the rates of return on wealth of, respectively, high productivity and low productivity agents.

In equilibrium, productive agents' wealth grows at state contingent rate which depends on their leverage choices

$$R^H_{t+1} = \frac{\left[ a_{t+1} A_{t+1} - \theta_t a^H E_t A_{t+1} \right] \left( u^H_t \right)^{1-\alpha} + q_{t+1} - E_t q_{t+1}}{q_t + \frac{(1-\alpha)}{\alpha} u^H_t - E_t \left( q_{t+1} + \theta_t a^H A_{t+1} \left( u^H_t \right)^{1-\alpha} \right) / R_t} \tag{14}$$

where $u_t^H$ is the user cost of capital for high productivity agents.

Unproductive agents are unconstrained and invest in their own projects as well as in the loans they make to the productive agents. This means that the wealth of low-productivity entrepreneurs grow at the following rate:

$$\begin{aligned} R_{t+1}^L &= \frac{W_{t+1}^L}{W_t^L} \\ &= \frac{Y_{t+1}^L + q_{t+1}\left(1 - K_{t+1}\right) + B_{t+1}}{X_{t+1}^L + q_t\left(1 - K_{t+1}\right) + B_{t+1}} \end{aligned} \tag{15}$$

where $W_t^L$ and $Y_t^L$, and $1 - K_t$ are, respectively, the aggregate wealth, output, and land investments of low productivity workers.

Aggregating the land demands of individual high productivity entrepreneurs yields an expression for the aggregate land purchases by productive entrepreneurs as a function of the state of the economy:

$$K_{t+1} = \frac{\beta d_t W_t}{q_t + \frac{(1-\alpha)}{\alpha} u_t^H - E_t\left(q_{t+1} + \theta_t Y_{t+1}^H\right)/R_t} \tag{16}$$

Due to log utility, individual and aggregate consumption are linear in individual and aggregate wealth. Hence goods market clearing implies:

$$(1-\beta) W_t + X_{t+1}^H + X_{t+1}^L = Y_t^H + Y_t^L$$

## 4.7 Competitive equilibrium

Recursive competitive equilibrium of our model economy is a price system $u_t^H$, $u_t^L$, $q_t$, $R_t$, household decision rules $k_{t+1}^i$, $x_{t+1}$, $b_{t+1}^i$ and $c_t^i$, $i = H, L$ and equilibrium laws of motion for the endogenous state variables (12) and (12) such that

(i) The decision rules $k_{t+1}^i$, $x_{t+1}^i$, $b_{t+1}^i$ and $c_t^i$, $i = H, L$ solve the household decision problem conditional upon the price system $u_t^H$, $u_t^L$, $q_t$, $R_t$.

(ii) The process governing the transition of the aggregate productivity and the household decision rules $k_{t+1}^i$, $x_{t+1}^i$ and $c_t^i$, $i = H, L$ induce a transition process for the aggregate state s given by (12) and (13).

(iii) All markets clear

# 5 Calibration

We calibrate our model economy as follows. We set $\alpha$, the share of land in output, equal to 0.2 in line with the calibration in Davis and Heathcote (2004) of the share of land in GDP. For the baseline calibration, I set $\phi$, the percentage of output that can be seized in the event of default, to zero. So any collateralisability of output in the steady state is due to the value of intangible collateral. I also set $\triangle^I$, the standard deviation of 'ex post' idiosyncratic productivity shocks, and $\triangle^E$, the standard deviation of the 'excess productivity shock' equal to zero in the baseline calibration.

Calibrating the cross-sectional dispersion of TFP is important because the quantitative importance of the pecuniary externality studied in our paper is related to the productivity gap between high and low productivity firms. Bernard et al. (2003) report an enormous cross-sectional variance of plant level value added per worker using data from the 1992 US Census of Manufactures. The standard deviation of the log of value added per worker is 0.75 in the data while their model is able to account for only around half this number. The authors argue that imperfect competition and data measurement issues can account for much of this discrepancy between model and data. In addition, the study assumes fixed labour share across plants so any departures from this assumption would lead to more variations in the measured dispersion of labour productivity.

In a comprehensive review article on the literature on cross-sectional productivity differences, Syverson (2009) documents that the top decile of firms has a level of TFP which is almost twice as high as the bottom decile. He finds that unobserved inputs such as the human capital of the labour force, the quality of management and plant level 'learning by doing' can account for much of the observed cross-sectional variation in TFP.

This model does not have intangible assets of the sort discussed in Syverson (2011) and consequently calibrating the model using the enormous productivity differentials identified in the productivity literature would overestimate the true degree of TFP differences. In addition, the Kiyotaki-Moore model would need very tight borrowing constraints or a very small number of high productivity entrepreneurs in order for credit constraints to be binding if some firms are so much more productive than others. And within the framework we have, binding credit constraints are the only mechanism for generating cross-sectional differences in productivity. Aoki et al. (2009) also consider these issues in their calibration of a small open economy version of Kiyotaki and Moore (1997). They argue that a ratio of the productivities of the two groups of 1.15 is broadly consistent with the empirical evidence and I choose this number for the baseline case.

The discount factor $\beta$, the probability that a highly productive entrepreneur switches to low productivity $\delta$, and the ratio of high to low productivity entrepreneurs $n$ are parameters I pick in order to match three calibration targets - the ratio of tangible assets to GDP, aggregate leverage and the leverage of the most indebted decile of firms.

I use data on tangible assets and GDP from the BEA National Accounts in the 1952-2008 period. The concept of tangible assets includes Business and Household Equipment and Software, Inventories, Business and Household Structures and Consumer Durables. GDP excludes government value added so it is a private sector output measure.

Aggregate leverage is defined as the average ratio of the value of the debt liabilities of the non-financial corporate sector to the total value of assets. Leverage measures can be obtained from a number of sources. In the US Flow of Funds, aggregate leverage is approximately equal to 0.5 for the 1948-2008 period. This is broadly consistent with the findings of Covas and den Haan (2011) who calculate an average leverage ratio of 0.587 in Compustat data from 1971 to 2004. Covas and Den Haan (2011) also examine the leverage of large firms and find that it is slightly higher than the average in the Compustat data set. Firms in the top 5% in terms of size have leverage of around 0.6. Covas and Den Haan (2007) have similar findings in a panel of Canadian firms. There the top 5% of firms have leverage of 0.7-0.75 compared to an average of 0.66 for the whole sample. High productivity entrepreneurs in our economy run larger firms so differences in productivity and therefore leverage could be one reason for the findings of Covas and Den Haan (2007 and 2011). But the perfect correlation of firm size and leverage that holds in our model will not hold in the data. So if we are interested in the distribution of firm leverage, the numbers in Covas and Den Haan will be an underestimate. This is why we pick a target for the average leverage of the top 10% most indebted firms to be equal to 0.75. This number is broadly consistent with the findings in Covas and Den Haan.

Finally, the high (low) realisations of the aggregate TFP shock ($\triangle^A$) are picked to to ensure that standard deviation of annual GDP in the model matches that of HP-filtered annual US GDP. (2.01% in our data sample). The probability that the economy remains in the same aggregate state it is today is equal to 0.8. Table 1 below displays a summary of the baseline calibration.

Table 1: Baseline calibration

| Parameter Name | Parameter Value |
|---|---|
| $\beta$ | 0.921 |
| $\delta$ | 0.344 |
| $n$ | 0.066 |
| $\alpha$ | 0.20 |
| $p_{HH}$ | 0.80 |
| $p_{LL}$ | 0.80 |
| $\triangle^A$ | 0.003 |
| $\triangle^E$ | 0.00 |
| $\triangle^I$ | 0.00 |
| $\phi$ | 0.00 |
| $a^H/a^L$ | 1.15 |

# 6 Numerical Results for the Baseline Economy

## 6.1 Steady state comparative statics

In this section we consider how the steady state value of intangible collateral varies with different features of the economy's production technology and nature of contract enforcement. Figure 3 below shows the value of intangible collateral as a percentage of output. We compute the value of intangible collateral as the size of firms' debts which are not secured by tangible collateral, expressed as a percentage of steady state output. The three lines on the chart correspond to three different values of $a^H/a^L$ - the ex ante productivity differential between high and low productivity entrepreneurs. In the absence of any long term punishments for defaulters, all three lines on the figure should be zero - the downpayment should be exactly pinned down by the collateralisability of the firm's capital and output. But in our framework borrowing capacity is determined by the values of a borrower's reputation for repayment as well as by the value of tangible assets.

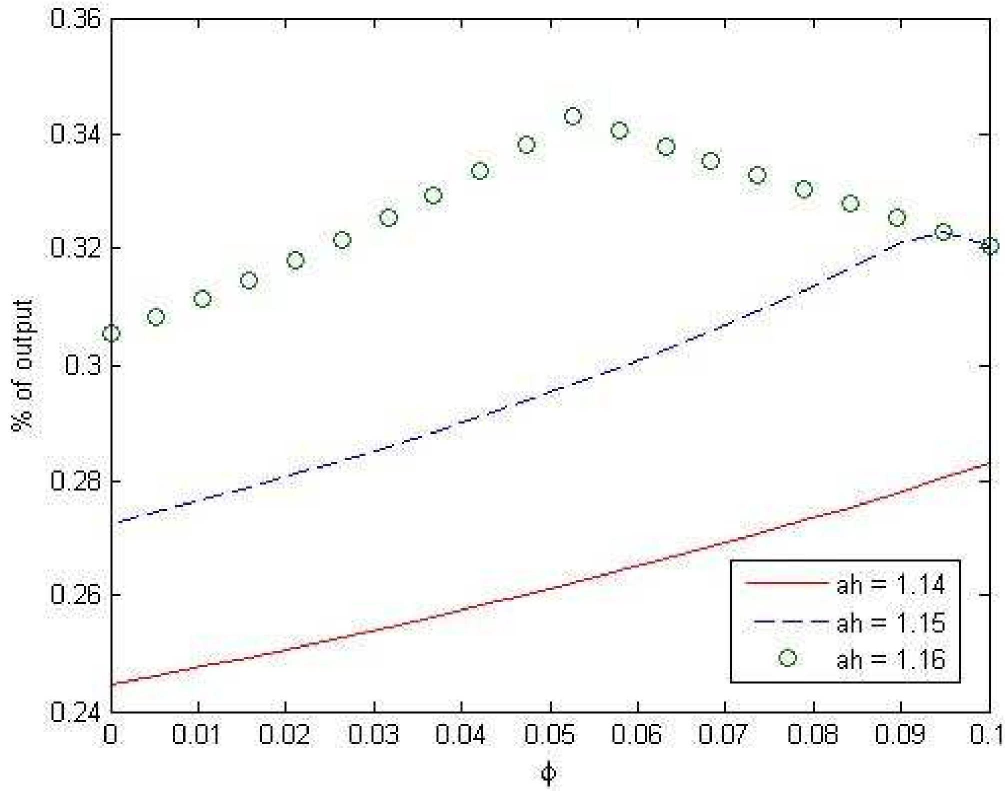


Figure 3: Intangible collateral as % of output

We can see from the figure that intangible collateral first increases with $\phi$ before declining once a critical level of $\phi$ is reached. The figure also shows that when the amount of tangible collateral  is low, a higher ex ante productivity differential $a^H/a^L$ is associated with more intangible collateral in equilibrium.

Figure 4 below examines the determinants of the value of a borrower's reputation. The evolution of reputational collateral in response to changes in $\phi$ is governed by the interplay of the impact of rising land prices and falling real interest rates on the leveraged rate of retun on wealth for high productivity entrepreneurs. While the economy is productively inefficient ($K < 1$), rising $\phi$ increases the price of land and this depresses the rate of return on production for low productivity entrepreneurs. Because low productivity savers need to be indifferent on the margin between making loans and producing using their own technology, the lower rate of return on low productivity projects also pushes down on the risk-free real interest rate.

Rising capital prices and falling real interest rates increase the leverage available to high productivity entrepreneurs, boosting the excess rate of return during high productivity spells. This in turn makes access to borrowing more attractive, driving up intangible collateral values higher and helping to increase leverage and capital prices even more. Here there is something of a multiplier effect. Higher leverage boosts excess returns and increases the value of intangible collateral, thereby securing further increases in excess returns. What caps the increase in corporate leverage is the growing immediate benefit from default which comes with a high quantity of borrowing which is not secured by tangible assets (land or pledgable future production).

The reason for the non-linearity in the relationship between tangible and intangible collateral arises due to the fact that once $\phi$ becomes high enough, high productivity entrepreneurs have enough financing capacity to purchase the entire land stock and low productivity firms stop producing. At this point, the economy achieves productive efficiency even though borrowing constraints still bind. Once the economy becomes productively efficient, further increases in $\phi$ boost demand for credit by more than they increase the supply of savings. This starts to bid up the real interest rate and reduces high productivity firms' excess return on wealth in the process. Lower excess returns, in turn, errode the value of reputational collateral. The value of the reputation for repayment reaches zero at the point at which borrowing constraints stop binding and the excess return disappears.

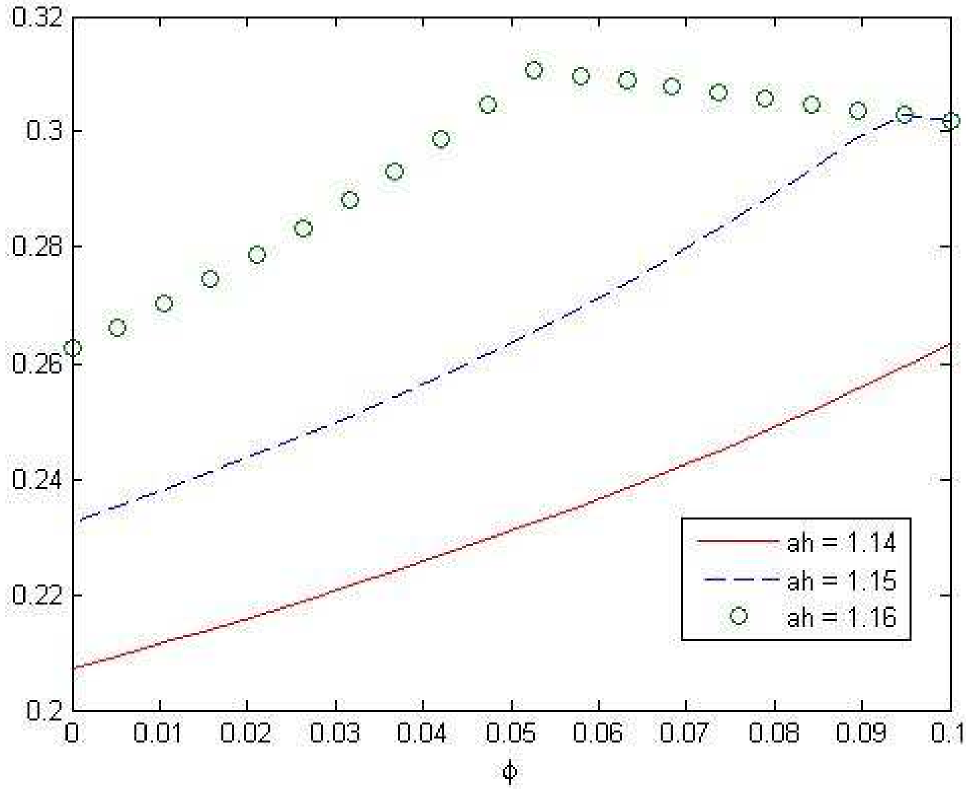


Figure 4: Excess return for high productivity entrepreneurs

Figures 3 and 4 also show that the value of repayment increases as the productivity differential $a^H/a^L$ rises. The bigger the productivity advantage the greater the benefit of leverage and therefore the greater the leverage a borrower can obtain by mortgaging his tangible assets and reputation for repayment.

## 6.2 Numerical results for the stochastic economy

### 6.2.1 The cyclical behaviour of debt limits in the model

Table 4 below shows how debt limits evolve over the business cycle for different parameterisations of the economy[2]. To produce the numbers in the table, we simulated the model economy and computed the average realisation of $\theta_t$ (the collateralisability of output) in booms and in recessions. Since we keep the value of $\phi$ (the ability to pledge future output by means of tangible collateral) fixed over the business cycle, any changes in the value of $\theta_t$ over the business cycle is due to fluctuations in intangible (reputational) collateral.

In the baseline case (the first column of the table) we can see that the debt limits $\theta_t$ are very slightly counter-cyclical. They tend to be lower in booms than in recessions though the difference is very small. The reason for this is that the net worth of high productivity entrepreneurs is low in recessions and this lowers asset prices. Low current asset prices implies higher rates of return on investment, and this is magnified by the availability of leverage. Access to debt markets is more beneficial in recessions in our economy because asset prices are low and the potential profits from leveraged investments are high.

Table 4: Borrowing limits over the economic cycle in the baseline economy

| | Baseline | $\triangle^I = 0.100$ | $\triangle^I = 0.200$ | $\triangle^E = 0.003$ |
|---|---|---|---|---|
| average $\theta$ in booms | 0.2699 | 0.2715 | 0.2777 | 0.2714 |
| average $\theta$ in recessions | 0.2670 | 0.2683 | 0.2632 | 0.2693 |

Note: average $\theta$ downpayment from a 10000 period long simulation

In columns 2-4 of Table 4 we consider a number of modifications of the basic environment in order to study their implications for the cyclical behaviour of intangible collateral. In columns 2 and 3 of the table, we show results from our model under the assumption that firms face a high degree of idiosyncratic production risk in recessions ($\triangle^I = 0.1$ in column 2 and $\triangle^I = 0.2$ in column 3) but idiosyncratic risk is zero in booms. The results show that counter-cyclical idiosyncratic production risk is capable of generating pro-cyclical borrowing limits. This is because uninsurable idiosyncratic risk reduces the ex ante value (in

[2] In each case, we re-calibrated the model to hit the targets in Table 1. Appendix D displays the calibrated parameters for each of these different parameterisations of our economy.

terms of utility) of leveraged investments in productive projects. As the value of leveraged investment falls, so does the value of an entrepreneur's reputation for repayment and this makes entrepreneurs less able to pledge it as collateral and borrow against its value in the capital market. As idiosyncratic uncertainty in the recession increases, the average value of reputational collateral falls and the gap between haircuts in the boom and the recession increases. This makes the value of intangible collateral pro-cyclical.

In column 4 of the table, we add pro-cyclical movements to the TFP differential between high and low productivity firms. This modification makes the value of $\theta$ pro-cyclical too. The bigger the productivity advantage of leveraged producers, the larger the value of taking high leverage in order to exploit this productivity advantage. Consequently, the value of intangible collateral increases too.

#### 6.2.2 Intangible collateral amplification

In the previous subsection we demonstrated that the model is capable of generating movements in firms' borrowing limits over the business cycle. Our simulations showed that, under some parameter values (e.g. the baseline calibration), the model delivers counter-cyclical movements in $\theta$ which should dampen the operation of the conventional collateral amplification mechanism which works via the effect of the price of land on firms' ability to borrow. Under other parameter values (e.g. substantial idiosyncratic investment risk in recessions), the intangible collateral mechanism generates amplification because of the positive correlation in the value of tangible and intangible collateral.

In this subsection we demonstrate the quantitative impact of the intangible collateral mechanism on the volatility of output and land prices. To do this, we compare the behaviour of the model with fluctuating intangible collateral with that of a model in which firms' ability to borrow against future output is constant over the business cycle at the same average value as in the intangible collateral model. In other words, we fix average leverage and examine the role of $\theta$ volatility on the second moments of output and asset prices. The results of this exercise are presented in Table 5 below.

Table 5: Output and asset price volatility

| | Baseline | $\triangle^I = 0.100$ | $\triangle^I = 0.200$ | $\triangle^E = 0.003$ |
|---|---|---|---|---|
| Intangible collateral | $\sigma_y = 2.01$ | $\sigma_y = 2.01$ | $\sigma_y = 2.01$ | $\sigma_y = 2.01$ |
| | $\sigma_q = 2.38$ | $\sigma_q = 2.25$ | $\sigma_q = 1.90$ | $\sigma_q = 2.62$ |
| No intangible collateral | $\sigma_y = 2.02$ | $\sigma_y = 1.83$ | $\sigma_y = 1.31$ | $\sigma_y = 1.93$ |
| | $\sigma_q = 2.39$ | $\sigma_q = 1.99$ | $\sigma_q = 0.95$ | $\sigma_q = 2.51$ |

Note: $\sigma_y$ is the standard deviation of output and $\sigma_q$ is the standard deviation of the land price. All numbers are produced from a 10000 period long simulation.

Two features of the results stand out. In the baseline, the intangible collateral mechanism has a very mild dampening effect on the business cycle. This is because tangible and intangible collateral are negatively correlated in response to aggregate technology shocks. In downturns, when the value of land is low, excess returns are high and this boosts the value of intangible collateral. Entrepreneurs can therefore to some extent substitute intangible for tangible collateral, which helps to moderate fluctuations in their credit access. This dampens the business cycle.

Once we add counter-cyclical idiosyncratic investment risk ($\triangle^I > 0$ in recessions), the intangible collateral mechanism delivers amplification. Under this assumption, firms face a high degree of uninsurable investment risk in recessions. This investment risk hurts mainly highly leveraged borrowers whose returns are very sensitive to the output from their productive projects. Therefore, in recessions, idiosyncratic investment risk diminishes the value of being highly leveraged and consequently the intangible collateral falls too. This correlates positively with the falling value of tangible collateral in recessions, thereby delivering the amplification effect we see in the second and third columns of Table 5. The standard deviation of output in the intangible collateral model is 10% higher relative to the model without intangible collateral when $\triangle^I = 0.1$ and almost 50% higher when $\triangle^I = 0.2$. The volatility of the land price is also boosted by the intangible collateral channel. When $\triangle^I = 0.2$ the standard deviation of the land price is double in the intangible collateral model relative to the framework with tangible collateral only.

# 7 Conclusions

This paper extends the collateral amplification framework of Kiyotaki and Moore (1997) and Kiyotaki (1998) by adding intangible collateral. Intangible collateral arises due an assumption that although lending can be done anonymously in this economy, borrowing cannot. Consequently, a defaulting entrepreneur not only loses a fraction of her tangible assets but also permanently loses her ability to borrow. When credit constraints bind, leveraged high productivity entrepreneurs have a rate of return on investments which exceeds the market interest rate. Leveraged production can boost low productivity agents' rate of return on wealth and consequently exclusion from debt markets is costly to borrowers. This generates a value for intangible collateral - in our model this is a borrower's reputation for repayment.

We study the way such intangible collateral varies with the nature of technology and contract enforcement in the economy both in steady state and over the business cycle. Steady state intangible collateral is higher the larger the excess return of leveraged production relative to saving. This is the case when the productivity differential between the high and low efficiency technology is large and when the collateralisability of tangible assets is high.

When we introduce aggregate uncertainty we find that the baseline model predicts that intangible collateral is mildly counter-cyclical. This is because credit constraints are tighter in recessions and the excess return of leveraged high productivity entrepreneurs is higher, increasing the value of intangible collateral. We find that the realistic addition of counter-cyclical variability of 'ex post' idiosyncratic productivity shocks helps to introduce pro-cyclical movements in the value of intangible collateral. The amplification of the Kiyotaki-Moore framework is substantially increased by adding intangible collateral with counter-cyclical idiosyncratic productivity shocks.

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# 9 Appendix A: Optimal consumption

Suppose the entrepreneur has optimally chosen her investments in land, goods investment and debt securities. This means that she can earn a state contingent rate of return on invested wealth of $R(a_t^i, X_{t+1})$ where $a_t^i$ is the ex ante idiosyncratic TFP component of the agent. The first order condition for optimal consumption becomes:

$$\frac{1}{c_t} = \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right) \pi\left(a_{t+1}|a_t\right) R\left(a_t^i, X_{t+1}\right) \frac{1}{c\left(a_{t+1,}X_{t+1}\right)}$$

We guess that the entrepreneur consumes a fixed fraction of her available resources:

$$c_t = (1-\beta) z_t$$

This means that

$$z_{t+1} = \beta R\left(a_t^i, X_{t+1}\right) z_t$$

Substituting into the consumption Euler equation we have:

$$\begin{aligned}
\frac{1}{(1-\beta) z_t} &= \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right) \pi\left(a_{t+1}|a_t^i\right) R\left(a_t^i, X_{t+1}\right) \frac{1}{(1-\beta) z_{t+1}} \\
&= \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right) \pi\left(a_{t+1}|a_t^i\right) R\left(s_t, X_{t+1}\right) \frac{1}{(1-\beta) \beta R\left(a_t^i, X_{t+1}\right) z_t} \\
&= \frac{1}{(1-\beta) z_t}
\end{aligned}$$

This confirms our initial guessed consumption function.

# 10 Appendix B: Computing value functions

## 10.1 The value function of a non-defaulting entrepreneur

We now combine the optimal consumption and portfolio choices of entrepreneurs to derive the value function that characterises their maximum lifetime utility. Let $V\left(a_t^i, X_t\right)$ be the value of a non-defaulting entrepreneur with idiosyncratic state $s_t$ when the aggregate state is $X_t$.

$$V\left(a_t^i, X_t\right) = \max_{c_t, k_{t+1}, x_{t+1}, b_{t+1}} \left\{ \ln c_t + \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right) \pi\left(a_{t+1}|a_t^i\right) V\left(a_{t+1}, X_{t+1}\right) \right\}$$

We guess a solution of the form:

$$V\left(a_t^i, X_t\right) = \varphi\left(a_t^i, X_t\right) + \varsigma\left(a_t^i, X_t\right)\ln w_t$$

Hence the value function equals:

$$\begin{aligned}
& \varphi\left(a_t^i, X_t\right) + \varsigma\left(a_t^i, X_t\right)\ln w_t && (17) \\
= & \max_{k_{t+1}, x_{t+1}, b_{t+1}} \left\{ \begin{array}{c} \ln(1-\beta) + \ln w_t + \\ \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a_t^i\right) \\ \left[\varphi\left(a_{t+1}, X_{t+1}\right) + \varsigma\left(a_{t+1}, X_{t+1}\right)\ln w_{t+1}\right] \end{array} \right\} && (18) \\
= & \max_{k_{t+1}, x_{t+1}, b_{t+1}} \left\{ \begin{array}{c} \ln(1-\beta) + \ln w_t + \\ \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a_t^i\right) \\ \left[\varphi\left(a_{t+1}, X_{t+1}\right) + \varsigma\left(a_{t+1}, X_{t+1}\right)\left(\ln\beta + \ln R\left(a_t^i, X_{t+1}\right) + \ln w_t\right)\right] \end{array} \right\}
\end{aligned}$$

Equating coefficients we have:

$$\varsigma\left(a_t^i, X_t\right) = 1 + \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a_t^i\right)\varsigma\left(a_{t+1}, X_{t+1}\right) \tag{19}$$

and

$$\begin{aligned}
\varphi\left(a_t^i, X_t\right) = & \ln(1-\beta) && (20) \\
& + \max_{k_{t+1}, x_{t+1}, b_{t+1}} \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a_t\right) \\
& \left[\varsigma\left(a_{t+1}, X_{t+1}\right)\left(\ln\beta + \ln R\left(a_t^i, X_{t+1}\right)\right) + \varphi\left(a_{t+1}, X_{t+1}\right)\right]
\end{aligned}$$

Equation (19) implies that

$$\varsigma\left(a_t^i, X_t\right) = \frac{1}{1-\beta}$$

Plugging this into (20) we have

$$\begin{aligned}
\varphi\left(a_t^i, X_t\right) = & \ln(1-\beta) && (21) \\
& + \max_{k_{t+1}, x_{t+1}, b_{t+1}} \frac{\beta}{1-\beta} \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a_t^i\right) \\
& \left[\ln\beta + \ln R\left(a_t^i, X_{t+1}\right) + (1-\beta)\varphi\left(a_{t+1}, X_{t+1}\right)\right]
\end{aligned}$$

## 10.2 The value function of a defaulting entrepreneur

An entrepreneur who has defaulted in the past is excluded from borrowing. When such an entrepreneur has low productivity, he has the same portfolio as other low productivity entrepreneurs with a clean repayment record. This is because low productivity agents do not use leverage and lend to other agents.

When the defaulting entrepreneur is in a high productivity state, he cannot use leverage and must self-finance. This implies that he faces a higher user cost of land equal to:

$$\begin{aligned} u_t^{H,d} &= q_t - E_t\left\{q_{t+1}\frac{\lambda_{t+1}^{H,d}}{\lambda_t}\right\} \\ &= q_t - E_t\left\{\frac{q_{t+1}}{R_{t+1}^H}\right\} \end{aligned}$$

In the absence of borrowing opportunities, the defaulting entrepreneur faces a shadow cost of funds equal to his own valuation of future wealth in terms of current wealth. This will tend to be higher compared to those who have some access to debt markets because under a binding collateral constraint, high productivity agents value future wealth less than the market relative price.

The high shadow cost of land investments implies that defaulted high productivity entrepreneurs will economise on land investments. Because there are decreasing returns to working capital, such an input mix will earn a lower rate of return on wealth compared to those entrepreneurs with access to debt markets.

## 10.3 Value function iterations

Let $\widetilde{R}\left(a^i, X_{t+1}\right)$ and $\widetilde{R}^d\left(a^i, X_{t+1}\right)$ denote, respectively, the rates of return on wealth for non-defaulting and defaulting entrepreneurs. We are now ready to compute the value functions by iterating on the functional equation below.

$$\begin{aligned} &\varphi\left(a^H, X_t\right) \\ =\ &\ln\left(1-\beta\right) + \frac{\beta}{1-\beta}\sum \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a^H\right) \\ &\left[\ln\beta + \ln\widetilde{R}\left(a^H, X_{t+1}\right) + \left(1-\beta\right)\varphi\left(a_{t+1}, X_{t+1}\right)\right] \end{aligned} \tag{22}$$

$$
\begin{aligned}
& \varphi\left(a^L, X_t\right) & (23) \\
= \; & \ln(1-\beta) + \frac{\beta}{1-\beta} \sum \pi\left(X_{t+1}|X_t\right) \pi\left(a_{t+1}|a^L\right) \\
& \left[\ln\beta + \ln\widetilde{R}\left(a^L, X_{t+1}\right) + (1-\beta)\varphi\left(a_{t+1}, X_{t+1}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& \varphi^d\left(a^H, X_t\right) & (24) \\
= \; & \ln(1-\beta) + \frac{\beta}{1-\beta} \sum \pi\left(X_{t+1}|X_t\right) \pi\left(a_{t+1}|a^H\right) \\
& \left[\ln\beta + \ln\widetilde{R^d}\left(a^H, X_{t+1}\right) + (1-\beta)\varphi^d\left(a_{t+1}, X_{t+1}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& \varphi^d\left(a^L, X_t\right) & (25) \\
= \; & \ln(1-\beta) + \frac{\beta}{1-\beta} \sum \pi\left(X_{t+1}|X_t\right) \pi\left(a_{t+1}|a^L\right) \\
& \left[\ln\beta + \ln\widetilde{R^d}\left(a^L, X_{t+1}\right) + (1-\beta)\varphi^d\left(a_{t+1}, X_{t+1}\right)\right]
\end{aligned}
$$

where

$$
\widetilde{R}\left(a^H, X_{t+1}\right) = \frac{\left[a_{t+1}A_{t+1} - \theta_t a^H E_t A_{t+1}\right]\left(u_t^H\right)^{1-\alpha} + q_{t+1} - E_t q_{t+1}}{q_t + \frac{(1-\alpha)}{\alpha} u_t^H - E_t\left(q_{t+1} + \theta_t a^H A_{t+1}\left(u_t^H\right)^{1-\alpha}\right)/R_t}
$$

is the rate of return on wealth for a non-defaulting high-productivity entrepreneur and

$$
\widetilde{R}^d\left(a^H, X_{t+1}\right) = \frac{a_{t+1}A_{t+1}\left(u_t^{H,d}\right)^{1-\alpha} + q_{t+1}}{q_t + \frac{(1-\alpha)}{\alpha} u_t^{H,d}}
$$

is the rate of return on wealth for a high-productivity defaulting entrepreneur. The value of intangible collateral $\theta_t$ can be computed from (8).

For given state contingent land price functions, we compute the value functions as well as the borrowing limit $\theta_t$ as follows:

(i) Pick a starting value of $\theta_t$ and solve (22) - (25) by value function iteration.

(ii) Update the value of $\theta_t$ from (8).

(iii) Return to the value function step (i) above.

(iv) Iterate until value functions and borrowing limits have converged up to a pre-specified tolerance level.

# 11 Appendix C: Computing aggregate equilibrium

From market clearing in the capital and the debt markets we can pin down the state contingent growth rate of the low productivity household without solving an explicit portfolio problem:

$$R_{t+1}^{L} = \frac{\left[a_{t+1} A_{t+1} \frac{\left(u_t^L\right)^{1-\alpha}}{\alpha} + q_{t+1}\right] (1 - K_{t+1}) + E_t\left[\theta_t Y_{t+1} + q_{t+1} K_{t+1}\right]}{\left[q_{t+1} + \frac{1-\alpha}{\alpha} u_t^L\right] (1 - K_{t+1}) + E_t\left[\theta_t Y_{t+1} + q_{t+1} K_{t+1}\right] / R_t} \tag{26}$$

where

$$\begin{aligned} u_t^L &= q_t - E_t\left(q_{t+1} \frac{\lambda_{t+1}^L}{\lambda_t^L}\right) \\ &= q_t - E_t\left(\frac{q_{t+1}}{R_{t+1}^L}\right) \end{aligned}$$

High productivity entrepreneurs invest the following fraction of their wealth in land.

$$K_{t+1} = \frac{\beta d_t W_t}{q_t + \frac{1-\alpha}{\alpha} u_t^H - E_t\left[\theta_t Y_{t+1}/K_{t+1} + q\left(X_{t+1}\right)\right]} \tag{27}$$

Their rate of return is given by:

$$R_{t+1}^{H} = \frac{\left(a_{t+1} A_{t+1} - \theta_t a^H E_t A_{t+1}\right) \frac{\left(u_t^H\right)^{1-\alpha}}{\alpha} + q_{t+1} - E_t q_{t+1}}{q_t + \frac{1-\alpha}{\alpha} u_t^H - E_t\left[\theta_t Y_{t+1}/K_{t+1} + q_{t+1}\right]} \tag{28}$$

where the user cost of land is given by

$$u_t^H = q_t - \frac{E_t q_{t+1}}{R_t} - E_t\left(\frac{q_{t+1} - E_t q_{t+1}}{R_{t+1}^H}\right)$$

The real interest rate on debt securities is given by the consumption euler equation:

$$\begin{aligned} R_t &= \beta E_t\left(\frac{\lambda_{t+1}^L}{\lambda_t^L}\right) \\ &= \beta E_t\left(\frac{1}{R_{t+1}^L}\right) \end{aligned}$$

Finally, goods market clearing implies that:

$$(1-\beta) W_t + \frac{1-\alpha}{\alpha}\left[u_t^L\left(1 - K_{t+1}\right) + u_t^H K_{t+1}\right] = W_t - q_t$$

Using a zero-finding routine, solve for the values of $\left\{R_t, K_{t+1}, q_t, R^H_{t+1}, R^L_{t+1}, u^H_t, u^L_t, K_{t+1}\right\}$ at which these conditions are satisfied up to an error tolerance level. I use Matlab's own fsolve.m routine.

3. Use the state evolution equations to compute next period's state vector:

$$W_{t+1} = \left[d_t R^H_{t+1} + (1-d_t)\, R^L_{t+1}\right] \beta W_t \tag{29}$$

$$d_{t+1} = \frac{(1-\delta) d_t R^H_{t+1} + n\delta\,(1-d_t)\, R^L_{t+1}}{d_t R^H_{t+1} + (1-d_t)\, R^L_{t+1}} \tag{30}$$

4. Repeat steps (1)-(3) for a large number of periods. Using the simulated data, update the price and forecasting function coefficients using linear regression.

5. Re-compute a simulated time series of the endogenous variables in our model economy under the new forecasting rule. Repeat steps (1)-(4) until the coefficients on the forecasting rule have converged up to an error tolerance level.

# 12 Appendix D: Parameter values under different calibrations

In section 6 we examined the sensitivity of the intangible collateral mechanism to different parameterisations of the variability of 'ex post' idiosyncratic TFP shocks ($\triangle^I$) as well as the variability of the 'ex ante' idiosyncratic TFP shock ($\triangle^E$). Changing $\triangle^I$ and $\triangle^E$affects the behaviour of the model. $\triangle^E$has a large impact on the volatility of the economy while $\triangle^I$affects desired leverage.

When changing $\triangle^I$ and $\triangle^E$we recalibrated $\phi$ (the fraction of output which can be seized by creditors) and $\triangle^A$ (the variability of the aggregate component of technology) in order to hit the two moments of the model which were most affected - the standard deviation of output and average corporate leverage. Table D below shows the values we picked in order to make sure that each of the model variants we considered matched the target moments.

Table D: Parameter values under different calibrations

| Parameter Name | Baseline | $\triangle^I = 0.100$ | $\triangle^I = 0.200$ | $\triangle^E = 0.003$ |
|---|---|---|---|---|
| $\phi$ | 0.000 | 0.0021 | 0.0092 | 0.0002 |
| $\triangle^A$ | 0.0030 | 0.0034 | 0.0042 | 0.0016 |