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by

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Abstract

Most retirement withdrawal rate studies are either based on historical data or use a particular assumption about portfolio returns unique to the study in question. But planners may have their own capital market expectations for future returns from stocks, bonds, and other assets they deem suitable for their clients' portfolios. These uniquely personal expectations may or may not bear resemblance to those used for making retirement withdrawal rate guidelines. The objective here is to provide a general framework for thinking about how to estimate sustainable withdrawal rates and appropriate asset allocations for clients based on one's capital market expectations, as well as other inputs about the client including the planning horizon, tolerance for exhausting wealth, and personal concerns about holding riskier assets. The study also tests the sensitivity of various assumptions for the recommended withdrawal rates and asset allocations, and finds that these assumptions are very important. Another common feature of existing studies is to focus on an optimal asset allocation, which is expected either to minimize the probability of failure for a given withdrawal rate, or to maximize the withdrawal rate for a given probability of failure. Retirement withdrawal rate studies are known in this regard for lending support to stock allocations in excess of 50 percent. This study shows that usually there are a wide range of asset allocations which can be expected to perform nearly as well as the optimal allocation, and that lower stock allocations are indeed justifiable in many cases.

***JEL* Codes:** C15, D14, G11, G17, N21, N22

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Introduction

Most retirement withdrawal rate studies are either based on the historical data parameters from Ibbotson Associates' *Stocks, Bonds, Bills, and Inflation* (SBBI) data since 1926, or on some other variant of the historical data, or they use a particular assumption about market returns unique to the study. Guidelines from such studies may not fulfill the needs of planners who develop their own capital market expectations for a wide variety of asset classes they and their clients deem suitable. These personal expectations may or may not bear close resemblance to the expectations built in to existing retirement withdrawal rate guidelines. Not only might planners hold views about future stock and bond returns that are different from historical averages, but planners may also include other assets such as TIPS, international stocks and bonds, REITs, and commodities, for instance. Lack of sufficient data often prevents such asset classes from inclusion in withdrawal rate studies.

Another common feature of existing studies is to focus on an optimal asset allocation, which is expected either to minimize the probability of failure for a given withdrawal rate, or to maximize the withdrawal rate for a given probability of failure. Studies have tended to support stock allocations for retirees in excess of 50 percent. The framework for this study also shows that usually there are a wide range of asset allocations which can be expected to perform nearly as well as the optimal allocation, and that lower stock allocations are indeed justifiable in many circumstances.

The objective of this study is to provide a framework for thinking about how to estimate sustainable withdrawal rates and appropriate asset allocations for clients based on a planner's capital market expectations and asset choices, as well as other inputs about the client including the planning horizon, tolerance for exhausting wealth, and personal concerns about holding riskier assets. The study also includes an investigation about the sensitivity of various assumptions and their impacts on the results. The study concludes by combining these elements together with a hypothetical example to translate capital market expectations into recommendations for a sustainable withdrawal rate and asset allocation strategy.

Literature Review

We can classify existing withdrawal rate studies into several categories. First are studies investigating the issue with overlapping periods from the historical data. The most common historical data are Ibbotson Associates' *Stocks, Bonds, Bills, and Inflation* (SBBI) data for total returns in U.S. financial markets since 1926. Studies of this nature tend to support a relatively high stock allocation in retirement and tend to provide the strongest support for the safety of the 4 percent rule. For instance, the seminal Bengen (1994) article concludes that retirees in all historical circumstances could safely withdraw 4 percent of their assets at retirement and adjust this amount for inflation in subsequent years for a 30-year retirement duration. Using the S&P 500 and intermediate-term government bonds (ITGB), he determines that retirees are best served with a stock allocation between 50 and 75 percent, concluding "stock allocations below 50 percent and above 75 percent are counterproductive." Several years later, Cooley, Hubbard and Walz (1998), which is more popularly known as the "Trinity Study," augment Bengen's work to show success rates for different withdrawal rates and asset allocations also using overlapping historical simulations. Recently, Cooley, Hubbard, and Walz (2011) update their earlier findings, also concluding that retirees are best served with stock allocations of at least 50 percent. From their tables, a 75 percent stock allocation maximizes the success rates for 30 years of inflation-adjusted withdrawals using a 4 or 5 percent withdrawal rate. Moving toward one of themes of this paper, Bengen (1996) did already note, however, that with rolling 30-year periods from the historical data, withdrawal rates above 4 percent could have been supported with asset allocations ranging between 35 and 90 percent. In other words, retirees fearing high stock allocations could reduce their stock allocations below 50 percent with minimal impacts on their sustainable withdrawal rates.

A second approach to studying withdrawal rates is to use Monte Carlo simulations which are parameterized to the same historical data as used in historical simulations. This can be done either by randomly drawing past returns from the historical data to construct 30-year sequences of returns in a process known as bootstrapping, or by simulating returns from a distribution (usually a normal or lognormal distribution) that matches the historical parameters for asset returns, standard deviations, and correlations. This simulation approach has the advantage of allowing for a greater variety of scenarios than the rather limited historical data can provide (between 1926 and 2010, there are only 56 rolling 30-year periods). In this regard, Monte Carlo simulation studies of this nature generally show slightly higher failure rates for stock allocations in the 50 to 75 percent range. At the same time, if past returns are not reflective of the distributions for future returns, then these Monte Carlo simulations will suffer from the same deficiency of misestimating the sustainability of various withdrawal rates. Another advantage of Monte Carlo simulations, relative to historical simulations, though, is that because of the way that overlapping periods are formed with historical simulations, the middle part of the historical record plays an overly important role in the analysis. With data since 1926, this overweighted portion (1955-1981) of the data tends to coincide with a severe bear market for bonds. Monte Carlo simulations treat each data point equally, and simulations based on the same underlying data tend to show both greater success for bond strategies and lower optimal stock allocations than historical simulations. Probably the best demonstration of this particular Monte Carlo simulation approach is Spitzer, Strieter, and Singh (2007), who provide illustrations for how withdrawal rates, asset allocations, failure probabilities, and bequest motives all interact for a 30-year retirement duration with simulations based on the historical data.

A third category includes studies using Monte Carlo simulations based on different parameters than the historical data, either because the authors are incorporating their own expectations about future returns, or because they seek to illustrate a basic concept with simple return assumptions.

An excellent study of this nature which serves as a precursor of the present study is Blanchett and Blanchett (2008). They make the point that future returns for a 60/40 portfolio could be even one or two percentage points less than historical averages, and they show how portfolio failure rates relate to changes in both the assumed return and standard deviation.

More recently, Harlow (2011) analyzes asset allocation in retirement using a downside risk perspective, which moves beyond failure rates to also quantify the degree of failure as well. He determines that stock allocations between 5 and 25 percent work best. These low stock allocations result both from the assumed conservative nature of the retiree, but also because Harlow bases Monte Carlo simulations assuming both lower real stock returns and higher real bond and bill returns than the historical averages.

Another example from this category is Terry (2003), who argues that retirees are best served with 100 percent bonds. However, his study is invalidated by a severe computational error. Terry calculates that over a 30-year period, stocks earning an expected nominal return of 12 percent (real return of 9 percent) with a 10 percent standard deviation would only support a 1.85 percent withdrawal rate with a 10 percent chance of failure. In actuality, with these rather generous assumptions for stocks, this withdrawal rate should be closer to 6.52 percent. With the correction made, his conclusions no longer hold.

Also relevant, Athavale and Goebel (2011) investigate retirement success over 35 years using 10 different assumptions for the underlying distribution of portfolio returns that average 5.1 percent in real terms with a 12 percent standard deviation. They only use 10 simulations for each distribution, but with the limited sample they find the lowest withdrawal rate is 2.52 percent, and that 4 percent withdrawals fail 14 percent of the time over a shorter 30-year period across the 100 simulated scenarios.

Finally, Huebscher (2011) provides an example of specifically simulating the potential success for an all-TIPS strategy over a 30-year period. Real TIPS yields start from their most recent value, and results are tested for several assumptions about their volatility.

Methodology

The maximum sustainable withdrawal rate (MWR) is the highest withdrawal rate that would have provided a sustained real income over a given retirement duration. At the beginning of the first year of retirement, an initial withdrawal is made equal to the specified withdrawal rate times accumulated wealth. Remaining assets then grow or shrink according to the asset returns for the year. At the end of the year, the remaining portfolio wealth is rebalanced to the targeted asset allocation. In subsequent years, the withdrawal amount adjusts by the previous year's inflation rate and the order of portfolio transactions is repeated. Withdrawals are made at the start of each year and the amounts are not affected by asset returns, so the current withdrawal rate (the withdrawal amount divided by remaining wealth) differs from the initial withdrawal rate in subsequent years. If the withdrawal pushes the account balance to zero, the withdrawal rate was too high and the portfolio failed. The MWR is the highest rate that succeeds. Taxes are not specifically incorporated and any taxes would need to be paid from the annual withdrawals.

Monte Carlo simulations are performed using a lognormal distribution for (1 + return). 10,000 simulations are made for each of 2,156 combinations of underlying portfolio real returns and standard deviations. Average real arithmetic returns range in 0.25 percentage point increments from -2 percent to 10 percent, while standard deviations range in 0.5 percentage point increments from 0.5 percent to 22 percent. For retirement durations of 10 to 40 years in 5 year increments, the MWR is calculated for each of the 10,000 simulated return paths from each of the 2,156 return/volatility combinations. With these outcomes, failure rates can be estimated for a given withdrawal rate, or the maximum withdrawal rate can be identified for a given chance of failure. These simulated return-risk combinations provide a grid of outcomes, and I use linear interpolation between the nearest neighbors on the grid to estimate the MWR for portfolios with other returns and volatilities.

Table 1

Summary Statistics for U.S. Real Returns Data, 1926 - 2010

	Arithmetic Means	Geometric Means	Standard Deviations	Correlation Coefficients Stocks	 Bonds	 Bills
Stocks	8.70%	6.62%	20.39%	1	0.08	0.09
Bonds	2.52%	2.28%	6.84%	0.08	1	0.71
Bills	0.69%	0.61%	3.90%	0.09	0.71	1

Source: Own calculations from *Stocks, Bonds, Bills, and Inflation* data provided by Morningstar and Ibbotson Associates. The U.S. S&P 500 index represents the stock market, intermediate-term U.S. government bonds represent the bond market, and bills are U.S. 30-day Treasury bills.

To consider how capital market expectations affect withdrawal rates and asset allocation, planners can identify the expected real arithmetic returns, standard deviations, and correlations between assets which they intend to include in their clients' portfolios. For planners wishing to use historical data parameters, Table 1 provides this information for large-capitalization stocks, intermediate-term governments bonds, and Treasury bills. With the chosen inputs, the next step is to generate 100 points on the efficient frontier using standard mean-variance optimization methods. This optimization identifies the asset allocations providing the highest returns for a given risk, or the lowest risk for a given return. I assume no leverage or short selling, so each asset is bound between 0 and 100 percent of the portfolio. Table 2 provides details for 11 arbitrary portfolios taken from the efficient frontier for the asset characteristics in Table 1. The efficient frontier can then be plotted onto the grid which relates returns and risks to withdrawal rates, and the asset allocation supporting the highest withdrawal rate can then be identified from the graph. In addition to the optimal asset allocation, I will indentify asset allocations supporting withdrawal rates within 0.1 percentage points of the MWR as well.

Table 2

Characteristics of 11 Portfolios from Efficient Frontier for Historical Data

Asset Allocation			Arithmetic	Geometric	Standard
Stocks	Bonds	Bill	Means	Means	Deviations
1.8	0	98.2	0.84	0.76	3.88
8.9	12.2	79.0	1.62	1.53	4.29
14.0	32.5	53.4	2.41	2.28	5.12
19.2	52.9	27.9	3.20	3.00	6.20
24.4	73.2	2.4	3.98	3.71	7.40
36.4	63.6	0	4.77	4.37	8.91
49.1	50.9	0	5.55	4.96	10.88
61.8	38.2	0	6.34	5.48	13.09
74.6	25.4	0	7.13	5.93	15.45
87.3	12.7	0	7.91	6.31	17.89
100	0	0	8.70	6.62	20.39

Note: Results are calculated using data in Table 1 and are in real terms.

Capital Market Expectations and Safe Withdrawal Rates

Figure 1 shows the how the frequency of failures for a 4 percent withdrawal rate over a 30-year retirement horizon relate to the underlying arithmetic average real returns and their standard deviations provided by the underlying portfolio. Naturally, higher real returns and smaller volatilities both contribute to smaller failure rates. For instance, if portfolio returns experience a standard deviation of 12, reducing the real return from just over 6 percent to just over 4 percent would increase the failure rate from 5 percent to 20 percent. Or, for instance, if the portfolio return averaged 4 percent, increasing the standard deviation from 7 percent to about 11.5 percent would also increase the failure rate from 5 percent to 20 percent. For smaller failure rates, returns must be increased at a faster pace to offset the impact of increasing standard deviations. Figure 1 also includes a red curve representing 100 points from the efficient frontier for a portfolio exhibiting the historical characteristics shown in Table 1. Though the asset allocations are not shown in the figure, we can observe for now that there will be an asset allocation which allows 4 percent to work with the lowest possible failure rate of about 5 percent.

Figure 1
Countour Map of Failure Rates for 4% Withdrawal Rate with 30-Year Retirement Horizon

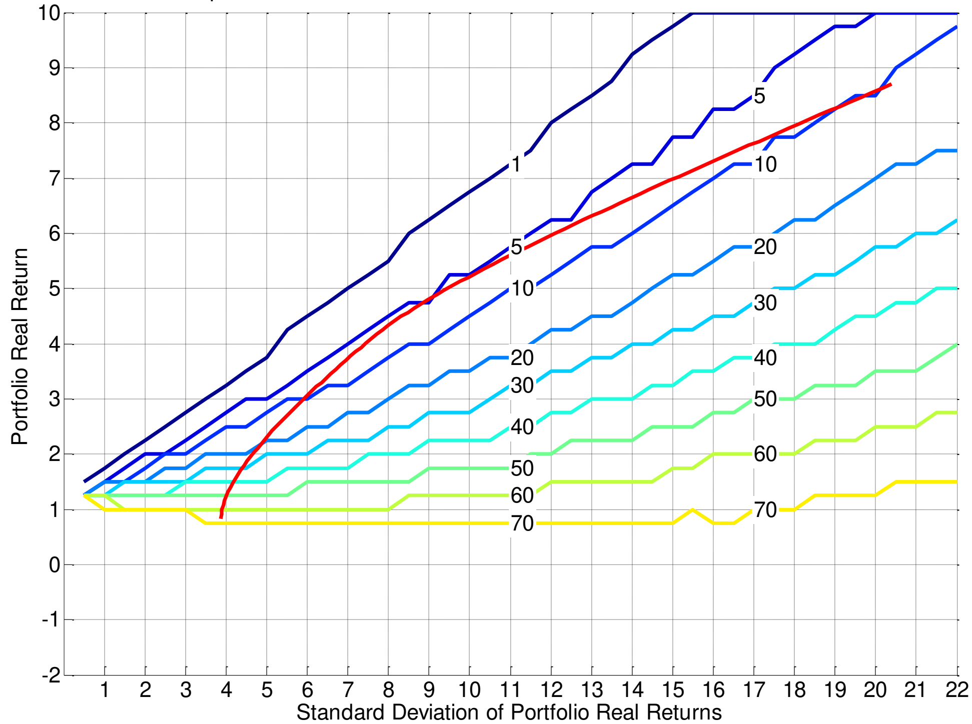
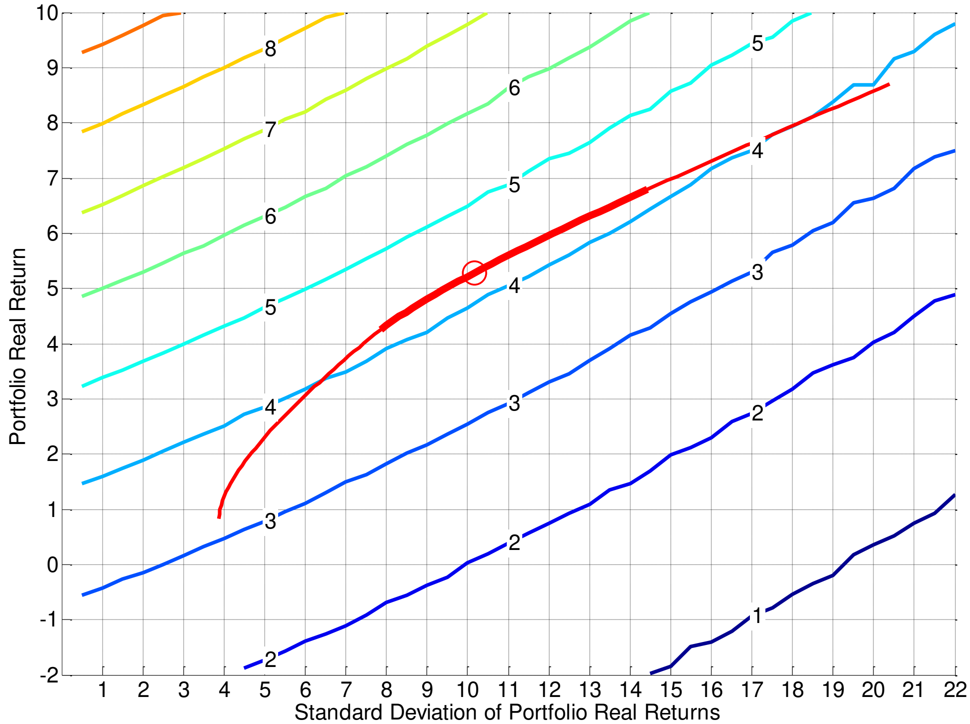


Figure 2 provides a different perspective by showing how sustainable withdrawal rates relate to the expectations about portfolio returns and risks when the failure rate is set at 10 percent and the retirement duration is set at 30 years. Here we can see, naturally, that sustainable withdrawal rates increase as returns increase or as volatilities decrease. We can also observe the offsetting relationships between returns and standard deviations in order to maintain the same withdrawal rate. Again, the efficient frontier generated from the historical data is added to this figure. As well, the asset allocation which maximizes the sustainable withdrawal rate is identified with a circle. The MWR with a 10 percent failure rate is 4.3 percent. In addition to the optimal asset allocation, the range of points on the efficient frontier (representing a range of asset allocations) that allow for a withdrawal rate within 0.1 percentage points of the maximum are highlighted with a thicker red line. But what *is* the optimal asset allocation and the range of asset allocations performing nearly as well?

Figure 2

Sustainable Withdrawal Rates with 10% Failure Rate for 30-Year Retirement Horizon



These answers are provided in Table 3 for a wide variety of retirement durations and failure rates. For the case just discussed, the MWR of 4.3 percent is supported with a portfolio of 45 percent stocks and 55 percent bonds. Stock allocations which provide nearly as large of withdrawal rate range from 28 to 69 percent. Table 3 demonstrates, unsurprisingly, that sustainable withdrawal rates are higher both for shorter retirement durations and for higher allowable failure probabilities. As well, the optimal stock allocation tends to increase both for longer retirement durations and for higher allowed failure probabilities. There are often a wide range of asset allocations which support nearly as high of withdrawal rates as the optimal allocation. This table provides clear evidence about the viability for lower stock allocations to compete with higher stock allocations in retiree portfolios, even when the results are based on the excellent performance for stocks found in the U.S. historical record.

Table 3
Withdrawal Rate and Asset Allocation Guidelines for Retirees Using SBBI Historical Parameters

Retirement Duration (Years)	Failure Rate (%)	Withdrawal Rate (%)	Optimal Asset Allocation (%)			For Withdrawal Rates Within 0.1% of Maximum					
						Range Stocks		Range Bonds		Range Bills	
			Stocks	Bonds	Bills	Min	Max	Min	Max	Min	Max
10	1	9.1	18	47	35	7	20	6	56	25	86
10	5	9.7	28	72	0	12	31	23	75	0	66
10	10	10.1	28	72	0	19	38	53	75	0	27
10	20	10.7	46	54	0	24	63	37	75	0	4
15	1	6.1	17	43	40	11	25	19	74	1	71
15	5	6.7	28	72	0	17	37	43	75	0	40
15	10	7.1	37	63	0	22	49	51	75	0	14
15	20	7.7	58	42	0	33	76	24	67	0	0
20	1	4.7	16	41	43	14	29	33	75	0	53
20	5	5.3	25	74	1	20	46	54	75	0	22
20	10	5.7	38	62	0	23	59	41	75	0	9
20	20	6.3	64	36	0	38	81	19	62	0	0
25	1	3.9	24	72	4	15	38	35	75	0	50
25	5	4.4	28	72	0	21	50	50	75	0	17
25	10	4.8	46	54	0	25	60	40	75	0	1
25	20	5.5	64	36	0	43	91	9	57	0	0
30	1	3.4	27	73	0	16	40	39	75	0	45
30	5	3.9	37	63	0	23	51	49	75	0	12
30	10	4.3	45	55	0	28	69	31	72	0	0
30	20	4.9	64	36	0	45	94	6	55	0	0
35	1	3.0	28	72	0	18	45	47	75	0	35
35	5	3.6	46	54	0	24	59	41	75	0	7
35	10	4.0	46	54	0	31	70	30	69	0	0
35	20	4.6	69	31	0	51	99	1	49	0	0
40	1	2.8	28	72	0	20	46	54	75	0	22
40	5	3.3	46	54	0	24	60	40	75	0	4
40	10	3.7	58	42	0	33	72	28	67	0	0
40	20	4.4	90	10	0	54	100	0	46	0	0

Sensitivity Analysis for Capital Market Expectations

The next series of figures show the sensitivity of sustainable withdrawal rates to changes in some characteristics of the underlying portfolio while holding other characteristics the same as in Table 1. To provide a consistent example, these figures assume a 10 percent accepted failure rate and a 30-year retirement horizon, except for the cases when these specific characteristics are modified.

In Figure 3, real stock arithmetic averages are modified to take values between 4.5 and 10 percent, compared to the historical average of 8.7 percent. The figure shows, not surprisingly, that as stock returns increase, a higher withdrawal rate is supported for a given failure rate, and the optimal stock allocation tends to increase as well. Even if real stock returns could average 10 percent, the historical volatility of stocks is sufficiently high that still a 5 percent withdrawal rate cannot be sustained. Holding the other portfolio characteristics constant, a 4 percent withdrawal rate cannot be sustained with a sufficiently low failure rate unless real stock returns are at least 7.5 percent. With 7.5 percent returns, the optimal stock allocation falls to about 35 percent, with similarly performing allocations ranging between about 20 percent and 47 percent. If stock returns are expected to be lower, it is important to reduce the stock allocation in order to avoid an even bigger drop in sustainable withdrawal rates, suggesting that planners who are concerned that forward-looking stock returns will fall behind past averages would have good justification to reduce stock allocations.

Figure 3
Sustainable Withdrawal Rates with 10% Failure Rate for 30-Year Retirement Horizon
Using SBBI Historical Parameters, Except for Varying the Stock Return

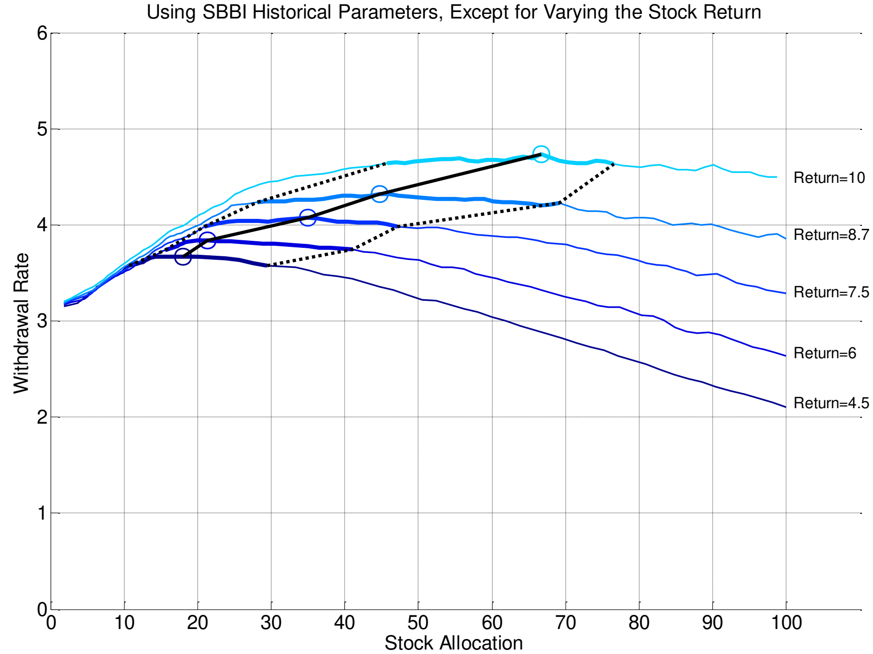


Figure 4 keeps the real stock return at 8.7 percent, but varies the stock volatility from between 12 and 22 percent, compared to the historical 20.39 percent. Higher volatilities reduce both the withdrawal rate and the stock allocation. In this case, the optimal stock allocation can change rapidly, as the optimal allocation increases from about 60 percent to 90 percent when the standard deviation declines from 18 to 16 percent. A decline in volatility to about 15 percent for stocks, holding other characteristics the same, would help push the MWR to over 5 percent and would call for a stock allocation of over 90 percent. Meanwhile, if stock volatility nudged up to 22 percent, the MWR falls to about 4.2 percent and the optimal stock allocation is less than 40 percent.

Figure 4

Sustainable Withdrawal Rates with 10% Failure Rate for 30-Year Retirement Horizon
Using SBBI Historical Parameters, Except for Varying the Stock Volatility

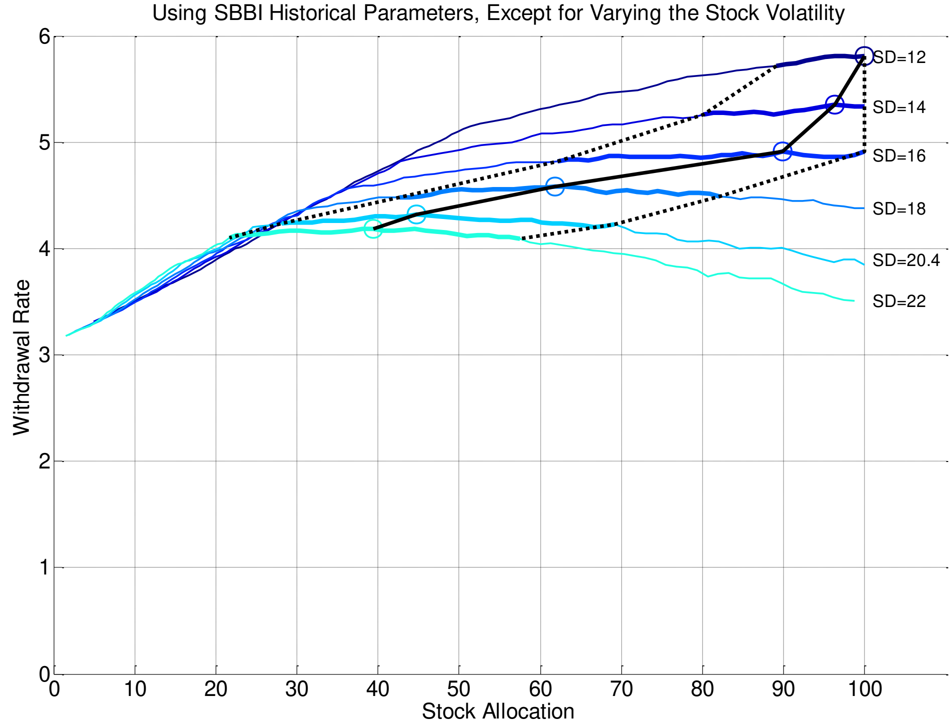


Figure 5 considers more about the combined effects of the return and volatility assumptions for stocks. I consider the impacts of reducing stock returns by 30 percent to 6.1 percent, reducing the standard deviation by 30 percent to 14.3 percent, and reducing both factors by 30 percent. This is important because even when one expects lower future stock returns, the question remains about what to assume for volatility. Would lower returns be accompanied by lower volatility, or is it more reasonable to keep volatility the same? While the author does not know the answer to this question, Figure 5 does show that how one answers it is important. If returns drop but volatility stays the same, then the withdrawal rate is pushed below 4 percent and the optimal stock allocation falls to about 20 percent. At the same time, if only standard deviations decrease, then the withdrawal rate increases to over 5 percent, supported by stock allocations of at least 60 percent. However, if both factors fall so that their ratio stays the same, the overall impact is minimal. Sustainable withdrawal rates drop slightly, and asset allocation recommendations stay about the same.

Figure 5

Sustainable Withdrawal Rates with 10% Failure Rate for 30-Year Retirement Horizon
Using SBBI Historical Parameters, Except for Varying Stock Returns and Volatility

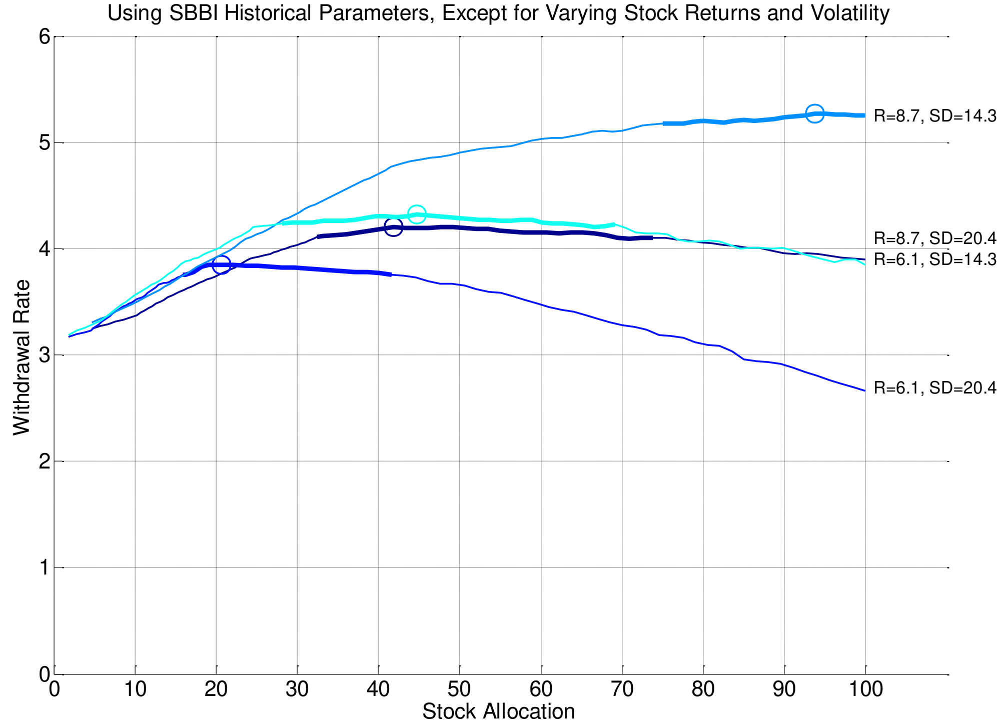
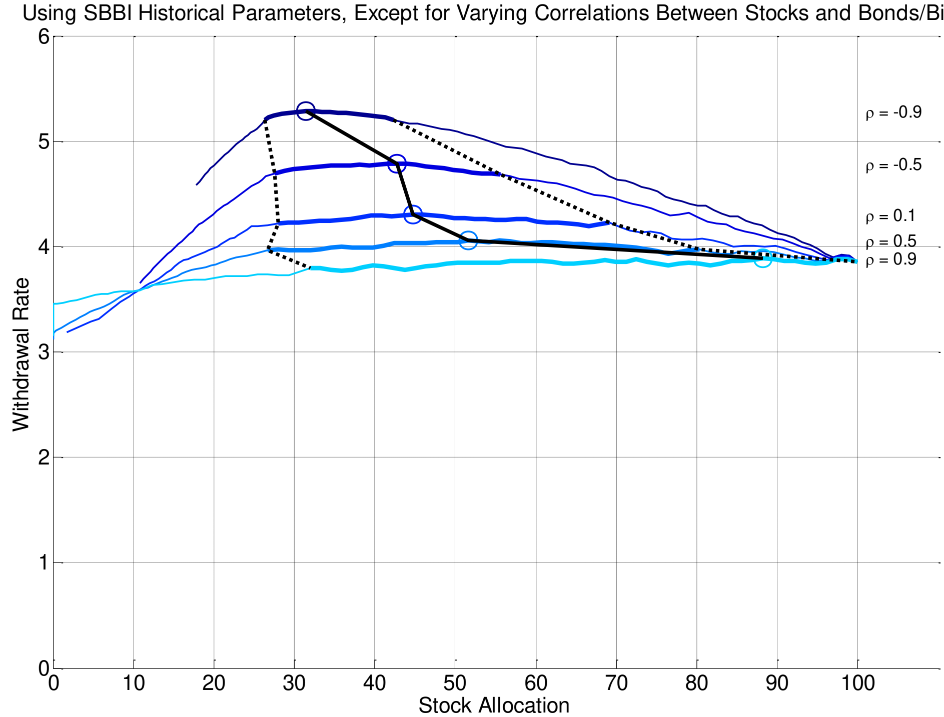


Figure 6 investigates the role of correlations. In the historical data, stocks experience a correlation of about 0.1 with both bonds and bills. Figure 6 considers the impact of changing both correlations simultaneously to various values. The lower the correlation, the larger are the diversification benefits and the higher are the withdrawal rates. As correlations increase, withdrawal rates decline, and stock allocations increase, since they offer larger potential returns and there are less benefits from owning fixed income. Holding other characteristics the same, with very high correlations 4 percent withdrawals cannot be sustained.

Figure 6
Sustainable Withdrawal Rates with 10% Failure Rate for 30-Year Retirement Horizon
Using SBBI Historical Parameters, Except for Varying Correlations Between Stocks and Bonds/Bills



With Figure 7, we observe that increasing the retirement duration results in reductions to sustainable withdrawal rates, but only a slight trend toward increasing stock allocations. A 35 percent stock allocation falls within the range of “nearly optimal” allocations for all of these retirement durations. But more broadly, this shows that choosing the appropriate retirement duration is very important, as longer planning horizons call for smaller withdrawal rates. Figure 8 also helps to make this point. This figure shows the withdrawal rates supported with different probabilities of failure for a 30-year retirement horizon. To see an example in action, consider a retiree using Figure 7 to plan for a 20-year retirement duration. The figure suggests that a 5.7 percent withdrawal rate can be supported with a 10 percent failure probability with a stock allocation of about 40 percent. But from Figure 8, if the planner misjudged the client’s lifespan and the client lived for 30 years, the probability of failure for the withdrawal rate strategy increases dramatically to about 50 percent. If such a withdrawal rate were used, the best stock allocation would have been about 90 percent, but this still leaves a failure rate of 30 percent.

Figure 7
Sustainable Withdrawal Rates with 10% Failure Rate for Various Retirement Horizons
Using SBBI Historical Parameters

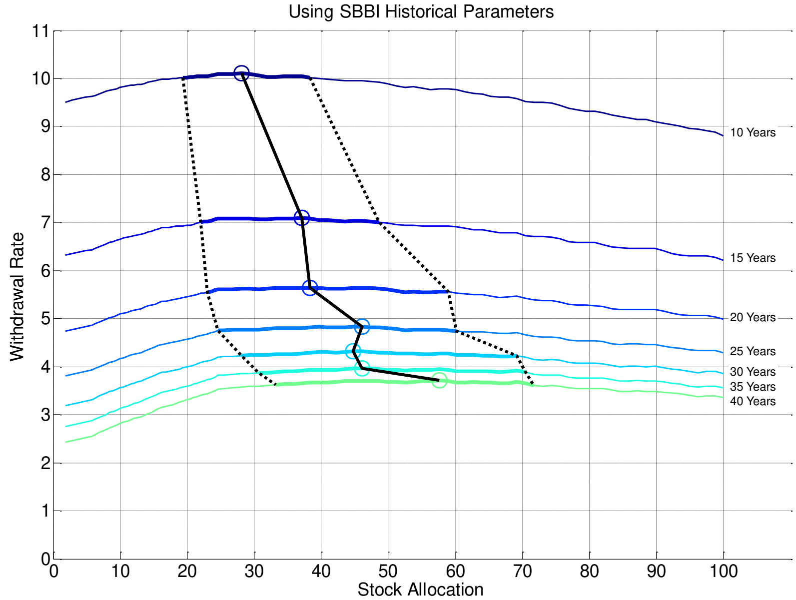
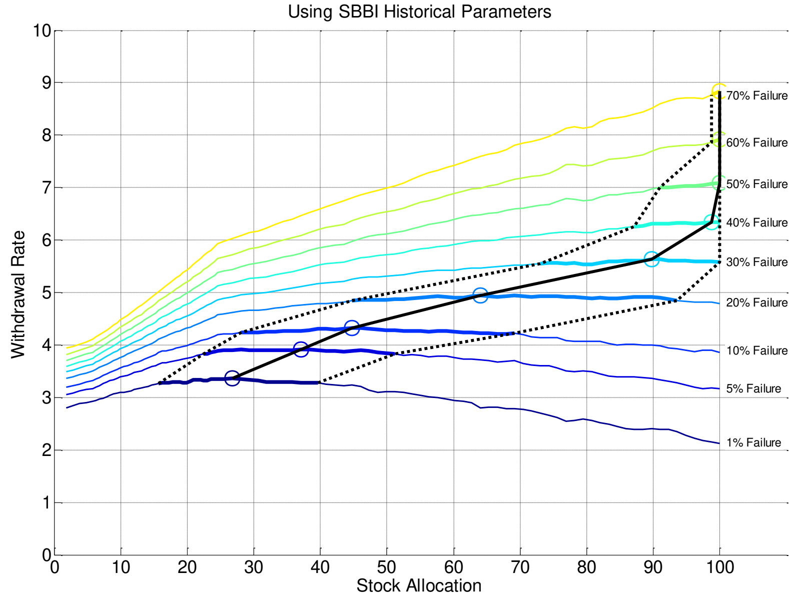
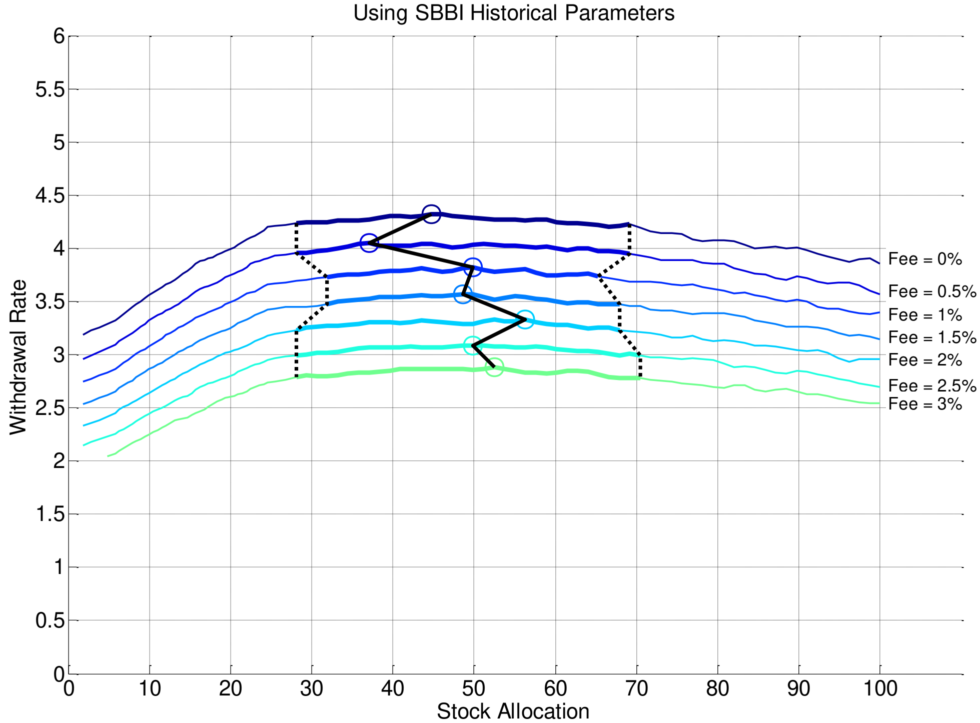


Figure 8
Sustainable Withdrawal Rates with Various Failure Rates for 30-Year Retirement Horizon
Using SBBI Historical Parameters



Finally, Figure 9 shows the impact of fees. This figure could also be interpreted as portfolio underperformance against the benchmark indices. The figure shows that asset allocation is not impacted much by fees which effect each asset equally. As well, fees reduce sustainable withdrawal rates, as a 3 percent fee reduces the MWR from 4.3 percent to 2.9 percent. Despite common misconceptions, there is not a one-to-one tradeoff between fees and withdrawal rates. As the portfolio decreases in size, fee amounts decline while real withdrawal amounts do not change.

Figure 9
Sustainable Withdrawal Rates with 10% Failure Rate for 30-Year Retirement Horizon
Using SBBI Historical Parameters



Putting it All Together

This framework can be used to estimate sustainable withdrawal rates for a given failure rate and retirement duration for most any kind of capital market expectations. Though the author claims no particular skill at forecasting asset returns, the purpose here is to demonstrate how planners can translate their own expectations into an understanding about how to choose a withdrawal rate and asset allocation strategy for their retired clients. Planners could help their clients choose suitable asset classes and develop expectations about the returns, standard deviations, and correlations among the asset classes. Table 4 provides a set of hypothetical expectations for real returns in U.S. dollars for 5 different asset classes. This information can then be used as inputs to a mean-variance optimizer to find the efficient frontier of asset allocations that provide the most return for a given risk or the most risk for a given return. The efficient frontier can then be plotted onto the grid framework relating withdrawal rates to returns and volatilities for an acceptable failure probability and retirement duration. From here, the point in which the efficient frontier touches the highest withdrawal rate can be determined. Returning to the mean-variance optimization results, we find the asset allocation for this optimal point. This optimal asset allocation can then be compared to the client's investment constraints and risk tolerance to determine its acceptability. If not acceptable, other points on the efficient frontier which support progressively lower withdrawal rates can be investigated until a suitable asset allocation is found.

Table 4

Hypothetical Long-term Capital Market Expectations (in real USD terms)

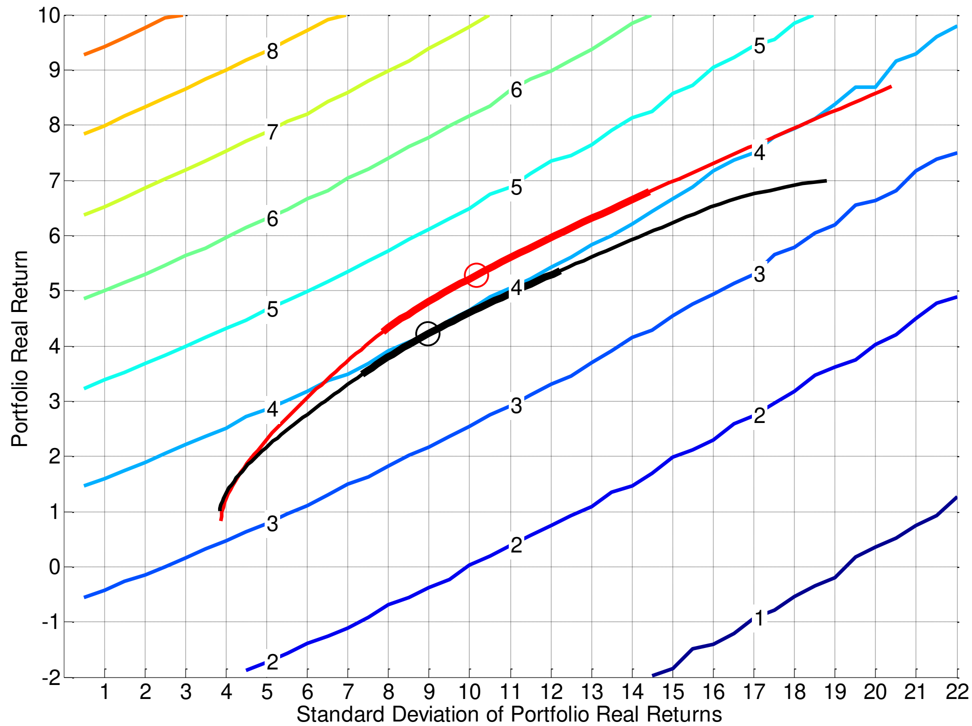
				Correlation Coefficients				
	Arithmetic Means	Geometric Means	Standard Deviations	U.S. Stocks	U.S. REITS	MSCI EAFE	U.S. Bonds	U.S. Bills
U.S. Stocks	7%	5.00%	20%	1	0.7	0.6	0.1	0.1
U.S. REITS	6%	4.72%	16%	0.7	1	0.5	0.3	0.4
MSCI EAFE	7%	4.58%	22%	0.6	0.5	1	-0.1	-0.1
U.S. Bonds	2%	1.76%	7%	0.1	0.3	-0.1	1	0.7
U.S. Bills	0.7%	0.62%	4%	0.1	0.4	0	0.7	1

Source: These are hypothetical numbers used to illustrate the framework. They do not represent actual forecasts.

Figure 10 shows the efficient frontier with these assumptions in black, along with the efficient frontier for the historical data in red. For this particular hypothetical example, the asset allocation which supports the highest withdrawal rate over 30 years with a 10 percent failure rate is 12.5 percent U.S. stocks, 16.7 percent U.S. REITS, 18.4 percent international stocks, 52.4 percent U.S. bonds, and no Treasury bills. This asset allocation supports a 4.0 percent withdrawal rate. If deemed acceptable, this could serve as the client's withdrawal rate and asset allocation strategy.

Pfau (2011) provides additional figures showing the sustainable withdrawal rates based on returns and standard deviations for different retirement durations and failure rates, allowing readers to consider how this framework may apply to a wide variety of situations.

Figure 10
Sustainable Withdrawal Rates with 10% Failure Rate for 30-Year Retirement Horizon



Conclusion

In conclusion, sustainable retirement withdrawal rates depend on capital market expectations, retirement durations, asset allocations, and acceptable failure probabilities. This article has attempted to combine these aspects to provide a more complete framework for understanding the relationships between these factors. Planners can use this framework to translate their own forecasts for capital markets into withdrawal rate and asset allocation recommendations for clients.

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