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Abstract. This paper contains a game-theoretic model describing the behaviour of investors at a stock exchange.

The model presented is developed to reflect the actual market microstructure.

The players constitute a non-uniform continuum, differing, among others, by the planning horizon, the external flow of money which can be invested, formation of expectations about future prices, which, briefly, divides the investors into the following groups: fundamental analysts, chartist, users of various econometric models, users of Capital Asset Pricing Model, and players observing a random exogenous signal.

Prices are determined by orders and the equilibrating mechanism of the stock exchange. The mechanism presented is the actual single-price auction system used, among others, at Warsaw Stock Exchange.

One of the main issues are self-verifying beliefs.

Results of numerical simulations of stock exchange based on the model are also included.

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1. Introduction

The stock exchange, starting from a place where buyers and sellers could face each other and even negotiate prices, evolved to a place, also in the virtual meaning, in which anonymous masses of investors buy or sell at prices dictated by the equilibrating mechanism. During this process of evolution, as the anonymity increased, various models predicting future prices were developed, among others: fundamental analysis, technical analysis, various econometric models and the Capital Asset Pricing model.

In this paper the author tries to present a model of stock exchange reflecting its actual microstructure. In this model each single player has a negligible impact on the aggregated values, such as the market demand and supply, and their functions, including the market price. Nevertheless, prices are determined by the equilibrating mechanism of the stock exchange using only players' orders. Each player has strategies depending on information about past prices and values of other available variables filtered by the prognostic technique inherent to his type of forming expectations. Such games, called games with distorted information were formally introduced by the author in Wiszniewska-Matyszkiel [42] and developed in Wiszniewska-Matyszkiel [43] in the form more applicable for modelling financial markets.

In order to make the model realistic an actual market mechanism of a real stock exchange was implemented – it is single-price auction system taken from Warsaw Stock Exchange (WSE) but similar mechanisms are used at many stock exchanges. Actual formation of prices is, as in the real life, fully deterministic: prices are determined by orders and the equilibrating mechanism of the stock exchange.

The model considered in this paper continues the research contained in Wiszniewska-Matyszkiel [40].

A continuum of players is used in order to model a "mature" stock exchange: there are many agents, each of them insignificant. Each single player is conscious that his order cannot affect prices and this reflects real situations. On the other hand, prices are effect of agents' orders.

Depending on sizes of types, even very abstract beliefs can become self-verifying at least to some extent. In the paper there are examples of such self-verifying beliefs: some of fundamental nature, technical signals of changes of trends and an absolutely abstract formation of cat. This formation has not existed by now and empirical data does not confirm it. It is explained in a quasi-psychological way which is frequently used by authors of textbooks on technical analysis. Moreover, this formation, if it was popularized among investors, would become self-verifying. This "cat" is an example of self-verifying character of some techniques of foreseeing future prices.

The paper starts by a short description of some models of price formation (subsection 1.2).

The model is formulated in section 2.

We state some results about equilibria in section 3; those concerning threshold prices and weak dominance in subgames with distorted information are in subsection 3.2. In section 4 we examine the issue of self-verification of various prognostic approaches. Some of them are self-verifying when used by a strong group of players (but not the whole population), e.g. players using fundamental analysis cause fast convergence to a price close to the fundamental value of a share (subsection 4.1), while some others are self-falsifying (subsection 4.2). The results of numerical simulations are in section 5.

1.1. Games with a continuum of players. Models with continuum of players were first introduced by Aumann [3] and [2] and Vind [26] to model competitive markets. By then it was very difficult to model insignificance of each single player.

Games with continuum of players was formally defined by Schmeidler [21], and afterwards the general theory of such games was extensively studied in, among others, Mas-Colell [17], Balder [5], Wieczorek [27] and [28], Wieczorek and Wiszniewska [29] or Wiszniewska-Matyszkiel [31]. Dynamic games with continuum of players are quite new (some examples of applications of such games are Karatzas, Shubik and Sudderth [8], Wiszniewska-Matyszkiel [30] and [32], [41], and general theory of such games in Wiszniewska-Matyszkiel [33], [34], and [36]). An interesting issue is the problem of convergence of parameters of equilibria in finitely-many-players counterparts of a dynamic game with a continuum of players to the parameters of equilibria in this game – see Wiszniewska-Matyszkiel [38] and [37].

1.2. Some models of prices of shares. In this subsection we shortly present some models and techniques used for foreseeing future prices of shares.

Fundamental analysis. The fundamental analysis approach is based on calculation of the "actual" value of a share, called its *fundamental value*. The most obvious definition is a discounted value of the infinite series of expected future dividends. Given the interest rate r and the sequence of expected (at time t_0) dividends of company i, $\{A_i^t\}_{t=t_0,t_0+1,\ldots}$, the fundamental value at time t_0 equals $F_i(t_0) = \sum_{t=t_0}^{\infty} \left(\frac{1}{1+r}\right)^{t-t_0} A_i^t$. However, at WSE most companies do not pay dividends. In such a case the fundamental value of a share should reflect the fraction of the value of the company corresponding to this share.

Investors using fundamental analysis assume that the price should be close to the fundamental value and any distortion is caused by speculations and it can prevail only in a short period – the prices on the stock exchange should reflect the fundamental value.

Technical analysis. The basic assumption of technical analysis is opposite to that of fundamental analysis: the prices move in trends. The real processes in the economy are perceived as secondary to the behaviour of prices and volumes of shares in the past. Technical analysts explain this counterintuitive assumption by saying that prices of shares contain information of future state of the economy, even this which is not explicitly known to the investors (e.g. Pring [19]).

The explanations are based on various sociological, psychological and economic terms, but in fact, technical analysis reduces to analysis of past prices and volumes. Formerly it was mainly analysis of charts, therefore its users are called *chartists*.

Although it is usually disregarded by scientists, it is taught at many departments of economic sciences and it is now the most popular way of predicting prices by private investors at WSE. Therefore it may really influence the prices (as it is described in the paper).

Probabilistic models. In this subsection we can describe various models with one common feature: all of them treat prices of shares as a realization of a stochastic process.

Portfolio analysis and Capital Assets Pricing Model (CAPM). Portfolio analysis, started by Markowitz ([13] and [15], see also [14] and [16]), was first a normative theory of investment in risky assets. It reduced the problem to an analysis of the mean and variance of the asset return.

It was converted into a description of the behaviour of investors by Lintner [12], Mossin [18], Sharpe [23], and Fama (e.g. [6]) and is known as CAPM (Capital Asset Pricing Model).

The parameters of the model (mean and the covariance matrix, and, consequently, so called β coefficients) are estimated on the basis on empirical data taken from the stock exchange.

According to this model, at equilibrium the price of an asset i should be such that the expected return fulfils the equation $R_i = r + \beta_i \cdot (R_M - r)$, where R_i is the expected return of asset i, β_i its β-coefficient, r – the interest rate of the risk free asset and R_M the expected return from the market portfolio (usually the stock exchange index).

This model is static, but after a slight modification it can be applied for predicting prices at a stock exchange.

Econometric models. This wide genre of models encompasses all prognostic methods based on data analysis using various econometric techniques, starting from the simplest – linear regression. In such models, we can consider dependence on past prices and volumes, day of the week, or some external data.

Pricing of derivative securities. A model of price formation on a stock exchange is necessary not only for ”usual” investors trying to make money on buying and selling shares, but also for financial institutions selling derivative securities based on assets sold at the stock exchange. Pricing of derivative securities requires knowledge about the form of the equation describing future prices.

It is usually a stochastic differential equation. In the model of Black and Scholes [4] it is $dP(t) = P(t)\,(b(t)dt + \sigma(t)dW(t))$, where W is a standard Wiener process. See also Karatzas [7] for an extensive description of the theory of pricing of derivatives.

Beside the prices of derivatives, the *hedging* strategies depend on the equation describing the evolution of prices of shares.

We are not going to model such investors, mainly because their strategies of buying or selling shares depend on the contract they want to hedge, which is exogenous to the model considered.

No model. There are also investors who do not form expectations about prices. They either choose a strategy from some simple investors manuals (e.g. constant sum, constant relation or constat reaction), and believing that they turn out to be fruitful, or decide at random by opening the bible or visiting a fortune teller. Both kinds of players may turn out to be successful. However, the first type cannot be nontrivially modelled by a game-theoretic model, since their strategies are fixed and no optimization takes place. The latter type can be encompassed by our model of a stock exchange. Moreover, they can improve the operation of the stock exchange.

Previous models of stock exchange based on optimization of independent agents. The model presented in this paper, as well as the earlier authors papers on financial markets [35] and [40], are not the first models, considering a microeconomic approach to the behaviour of the players. The main issue in agent-based models was the influence of players expectations about price behaviour on actual prices. There were so called models of artificial stock exchange, in which players tended to maximize their payoffs given some expectations. One of them was the model and a computer simulation programme called Santa Fe Artificial Stock Market. In this models there is a share with a stochastic dividend and a risk free asset. Player estimate the expected value of future dividends. A market clearing condition was added. Players adjust their expectations during the game. See e.g. Arthur et al. [1], LeBaron [9] and [10] or LeBaron, Arthur and Palmer [11] for more details.

2. Formulation of the model

In this section we formulate the game theoretic model of a stock exchange.

A game $\mathfrak{G}$ is defined by specifying the *set of players*, the *sets of players strategies* and the *payoff functions*.

Here we consider a dynamic game, therefore the strategy specifies choices of *decisions* at every time instant during the game and the response of the whole system to these decisions.

The first object to define is the set of players. We consider a model of a mature stock exchange, i.e. such that a single player has a negligible impact on prices – the *set of players* is the unit interval $\Omega = [0, 1]$ with the Lebesgue measure λ.

In our model of stock exchange we consider $n + 2$ types of assets. Firstly, there are shares of n companies sold at the stock exchange. Shares in our model are not assumed to pay any dividends. Secondly, there is a risk free but not fully liquid asset of positive interest rate r, for simplicity called bonds. And finally, money, which are risk free and liquid but of interest rate 0.

The game is dynamic, it starts at t_0 – initial time and terminates at $+\infty$, but each player has his own terminal time $T_\omega \leq +\infty$, identical for players of the same type. We shall denote the set of possible time instants $\{t_0, t_0+1, \ldots\}$ by $\mathbb{T}$, while the symbol $\overline{\mathbb{T}}_\omega$ denotes $\{t_0, t_0+1, \ldots, T_\omega+1\}$ if T_ω is finite, $\mathbb{T}$ otherwise.

The set of possible stock prices $\mathbb{P}$ is a discrete subset of $\mathbb{R}_+ \backslash \{0\}$.

There are some restrictions on prices – at time t it should be in the interval $[(1-h)\cdot p(t-1), (1+h)\cdot p(t-1)]$, where $h>0$ denotes the maximal rate of variability.

Beside the money earned at the stock exchange, the players can invest money from an external flow of capital (or be forced to withdraw some money). For player ω it will be represented by a function $M_\omega : \mathbb{T} \to \mathbb{R}$.

The players have to pay a commission for any transaction, but they do not have to pay additional commission for orders. For simplicity of calculations we shall assume a constant commission rate $C << 1$. The same commission is also paid for buying or selling bonds.

Portfolio of a player, denoted by x, is an $n+2$-tuple with coordinates corresponding to shares of n companies, bonds and money. Therefore $x \in \mathbb{R}_+^{n+2}$.

At the beginning of the game player ω is assigned an initial portfolio $\bar{x}^\omega$.

Players' decisions at each time instant consists of: an *order to sell* S – a pair $(p^S, q^S) \in \mathbb{P}^n \times \mathbb{R}_+^n$, two *orders to buy* BM – a pair $(p^{BM}, q^{BM}) \in \mathbb{P}^n \times \mathbb{R}_+^n$ ("*buy for money*") and BB – a pair $(p^{BB}, q^{BB}) \in \mathbb{P}^n \times \mathbb{R}_+^n$ ("*buy for bonds*"), and the part of value non invested in shares which is hold in cash: e. In each case $p^\cdot$ denotes the vector of price limits for all shares, $q^\cdot$ – the vector of amounts. Price limits (coordinates of $p^\cdot$) are in $\mathbb{P}$, amounts are nonnegative, and the ratio of liquid money $e \in [0,1]$.

Besides the general form of the orders we want to be able to illustrate the fact, that some players do not invest in some kind of companies, some players never keep cash or that some players never keep bonds. Therefore the *set of decisions of player* ω – $\mathbb{D}_\omega$ is a subset of the set $\mathbb{D} = \big\{(\mathrm{BM}, \mathrm{BB}, \mathrm{S}, e) : \mathrm{BM}, \mathrm{BB}, \mathrm{S} \in \mathbb{P}^n \times \mathbb{R}_+^n,\ e \in [0,1]\big\}$. These sets $\mathbb{D}_\omega$ have the form $\mathbb{D}_\omega = (\mathbb{P}^n \times \Gamma_\omega)^3 \times E_\omega$, where $\Gamma_\omega \subset \mathbb{R}_+^n$ is a product of real semilines starting from 0 and singletons $\{0\}$.

We also have to define the notion of physical admissibility of a decision, depending on the portfolio. The symbol $D_\omega(x^\omega) \subset \mathbb{D}_\omega$ will denote the *set of decisions of player* ω *available at his portfolio* x^ω. It is defined by the constraints $\sum_{i=1}^n (1+C)\cdot p_i^{BM} \cdot q_i^{BM} \le x_{n+2}^\omega$ (where x_{n+2}^ω denotes money; this reads as "a player cannot pay more money than he possesses"), $\sum_{i=1}^n (1+C)\cdot p_i^{BM}\cdot q_i^{BM} \le (1-C)x_{n+1}^\omega$ (where x_{n+1}^ω denotes value of bonds) and $q_i^S \le x_i^\omega$ (i.e. shortselling is forbidden) for each share $i = 1, \ldots, n$.

If $x = \{x^\omega\}_{\omega\in\Omega}$ represent a family of portfolios of the players, then any measurable function $\delta : \Omega \to \mathbb{D}$ such that for every ω $\delta(\omega) \in D_\omega(x^\omega)$ is called a *static profile available at* x. The set of all static profiles available at x will be denoted by $\Sigma(x)$, while Σ will denote the set of all static profiles.

A static profile together with the past price determines the market price as explained below.

Aggregated demand, aggregated supply and the market mechanism

Let us consider the market for shares of company i at a fixed time instant t and players porfolios x. Given a static profile available at x $\big((p^{BM}(\omega), q^{BM}(\omega)), (p^{BB}(\omega), q^{BB}(\omega)), (p^{S}(\omega), q^{S}(\omega)), e(\omega)\big)$, the market supply of share i $AS_i : \mathbb{P} \to \mathbb{R}_+$ is equal to

$$AS_i(p_i) = \int_{\Omega} q_i^S(\omega) \mathbf{1}_{p_i^S(\omega) \le p_i} d\lambda(\omega),$$

while the market demand for share i $AD_i : \mathbb{P} \to \mathbb{R}_+$ is equal to

$$AD_i(p_i) = \int_{\Omega} q_i^{BM}(\omega) \cdot \mathbf{1}_{p_i^{BM}(\omega) \ge p_i} + q_i^{BB}(\omega) \cdot \mathbf{1}_{p_i^{BB}(\omega) \ge p_i} d\lambda(\omega),$$

where $\mathbf{1}_{condition}$ is equal to 1 when the condition is fulfilled and 0 otherwise.

In order to calculate the market price fo share i, the market mechanism considered in the paper first returns returns the price maximizing a lexicographic order with criteria, starting from the most important:

1. *maximizing volume* i.e. the function $\min(AD_i(p_i), AS_i(p_i))$;
2. *minimizing disequilibrium* i.e. the function$|AD_i(p_i) - AS_i(p_i)|$;
3. *minimizing the number of shares in selling orders with price limit less then the market price and buying orders with price limits higher than the market price*;
4. *minimizing the absolute value of the difference between the calculated price and the reference price* i.e. $|p_i - p_i(t-1)|$.

The result is projected on the set $[(1-h) \cdot p(t-1), (1+h) \cdot p(t-1)] \cap \mathbb{P}$ and it constitutes the market price $p_i(t)$.

A similar procedure is used at WSE (see [20]). The differences are caused by obvious mistakes and inconsistencies of the regulations of WSE. The problem of these imperfections was studied in Wiszniewska-Matyszkiel [39].

Evolution of portfolios, strategies, and dynamic profiles

The portfolio of player ω at time t is denoted by $X^{\omega}(t)$. If player ω chooses at time t a decision $(\mathrm{BM}, \mathrm{BB}, \mathrm{S}, e) \in D_{\omega}(X^{\omega}(t))$ and the price at time t is $p(t)$, then:

$X_i^{\omega}(t+1) = X_i^{\omega}(t) + q_i^{BM} \cdot \mathbf{1}_{p_i^{BM} \ge p_i(t)} + q_i^{BB} \cdot \mathbf{1}_{p_i^{BB} \ge p_i(t)} - q_i^S \cdot \mathbf{1}_{p_i^S \le p_i(t)}$ for $t \ge t_0$, $i = 1, \ldots, n$,

$X_{n+1}^{\omega}(t+1) = (1+r) \cdot \Big(X_{n+1}^{\omega}(t) - \sum_{i=1}^{n} \frac{1+C}{1-C} \cdot q_i^{BB} \cdot p_i^{BB} \cdot \mathbf{1}_{p_i^{BB} \ge p_i(t)} +$
$+ \frac{1-e}{1+C} \cdot \Big(X_{n+2}^{\omega}(t) - \sum_{i=1}^{n} \Big((1+C) \cdot q_i^{BM} \cdot p_i^{BM} \cdot \mathbf{1}_{p_i^{BM} \ge p_i(t)} -$
$- (1-C) \cdot q_i^S \cdot p_i^S \cdot \mathbf{1}_{p_i^S \le p_i(t)}\Big)\Big)$,

$X^\omega_{n+2}(t+1) = M_\omega(t+1) + e \cdot \Big(X^\omega_{n+2}(t) - \sum_{i=1}^n \Big((1+C) \cdot q_i^{BM} \cdot p_i^{BM} \cdot \mathbf{1}_{p_i^{BM} \geq p_i(t)} -$
$- (1-C) \cdot q_i^S \cdot p_i^S \cdot \mathbf{1}_{p_i^S \leq p_i(t)}\Big)\Big).$

A *strategy* of player ω is a function defining choices of decisions at all time instants – it is a function $\Delta_\omega : \mathbb{T} \to \mathbb{D}_\omega$ with $\Delta_\omega(t) \in D_\omega(X^\omega(t))$, where X^ω denotes the *trajectory of portfolio of player* ω, which is defined by the above evolution equation with the initial condition $X^\omega(t_0) = \bar{x}^\omega$. The set of strategies of player ω will be denoted by $\mathfrak{S}$.

If for a choice of players' strategies $\Delta = \{\Delta_\omega\}_{\omega \in \Omega}$ for every t the function $\omega \mapsto \Delta_\omega(t)$ is measurable, then Δ is called a dynamic profile. The trajectory corresponding to Δ will be denoted by X^Δ and the sequence of market prices p^Δ. The set of all dynamic profiles will be denoted by $\mathbf{\Sigma}$.

Players' payoffs and expected payoffs

If T_ω is finite, then the payoff of a player, given his strategies and a sequence of market prices along the profile is defined in the obvious way as the present value of the portfolio at time $T_\omega + 1$, $\frac{V(T_\omega+1, X^\omega(T_\omega+1))}{(1+r)^{T_\omega+1-t_0}}$, where $V : \overline{\mathbb{T}}_\omega \times \mathbb{R}_+^{n+2} \to \mathbb{R}$ denotes any function representing the value of the portfolio. Here we consider $V(t,x) = x_{n+1} + x_{n+2} + \sum_{i=1}^n p_i(t) \cdot x_i$.

Elementary calculations show that the payoff can be equivalently expressed as $\sum_{t=t_0}^{T_\omega} \frac{V(t+1, X^\omega(t+1)) - (1+r) \cdot V(t, X^\omega(t))}{(1+r)^{t+1-t_0}}$, since subtracting a constant does not change choices of the players. This definition of payoff can be obviously extended to $T_\omega = +\infty$ if the sum is well defined – it can attain infinite values.

Formally, the *payoff function of player* ω $\Pi_\omega : \mathbf{\Sigma} \to \overline{\mathbb{R}}$ defined by $\Pi_\omega(\Delta) = \sum_{t=t_0}^{T_\omega} \frac{V(t+1, (X^\Delta)^\omega(t+1)) - (1+r) \cdot V(t, (X^\Delta)^\omega(t))}{(1+r)^{t+1-t_0}}$ for $V(t,x) = x_{n+1} + x_{n+2} + \sum_{i=1}^n p_i(t) \cdot x_i$.

This ends the definition of our "actual" game $\mathfrak{G}$.

As in the context of more general games with distorted information, defined in Wiszniewska-Matyszkiel [42] and [43], we can also define the *expected payoff* of player ω at time t given his *belief correspondence* based on his observation of the history of the game. It represents the supremum over future decisions of player ω of his payoff assuming the belief correspondence - the player assumes that in future he is going to behave optimally and considers his guaranteed payoff – the payoff corresponding to the worst future history of the system in his belief correspondence. In this paper, in order to avoid a complicated notation, we shall incorporate the belief correspondence into the expected payoff function.

While analyzing decision making processes of stock exchange investors we have to take into account what information they can use during the decision process. This information is used to estimate the behaviour of future prices of underlying assets, and, consequently, players expected payoffs.

In order to build a model we have to formalize all descriptions of formation of expectations. When this issue is concerned, we shall consider five general types of players: *fundamental*, *technical*, *econometric*, *portfolio* and *stochastic*, and the first letter will be used as a type index k. The symbol $k(\omega)$ denotes the *type of formation of expectations of player* ω.

We shall define the *expected utility function of players of type* k $U^k : \mathbb{I}_k \times \mathbb{P} \times \mathbb{D}_k \to \mathbb{R}$, where $\mathbb{I}_k$ is a specific form of processed information used by type k. The form of this function depends on type since the form and interpretation of information changes. The information used by type k during the game constitutes a function $I_k : \mathbf{\Sigma} \times \mathbb{T} \to \mathbb{I}_k$ such that $I_k(\Delta, t)$ is independent of $\Delta.(s)$ for $s \geq t$. The specific form of information, general constraints on the strategy sets and the expected payoff functions for five types of formation of expectation are as follows.

1. *Fundamental players.* Their information is a vector of fundamental values of n shares – $f \in \mathbb{R}^n_+$, which is not based on prices of share. They are the kind of players waiting for results in a long time horizon, therefore they do not keep liquid money – they invest only in bonds and shares i.e. $e \equiv 0$ (a constraint on their available decisions' set). The expected payoff is defined by

$$U^f\left(f, p, (\mathrm{BM}, \mathrm{BB}, \mathrm{S}, e)\right) = \sum_{i=1,\dots,n} \Big(\left(f_i - p_i \cdot (1+C)^2\right) \cdot q_i^{BB} \cdot \mathbf{1}_{p_i^{BB} \geq p_i} +$$
$$+ (f_i - p_i) \cdot q_i^{BM} \cdot \mathbf{1}_{p_i^{BM} \geq p_i} - \left(f_i - p_i \cdot (1-C)^2\right) \cdot q_i^S \cdot \mathbf{1}_{p_i^S \leq p_i} \Big).$$

The first part corresponds to buying-for-bonds order, therefore the commission is paid twice, the second is buying-for-money, therefore no commission is subtracted – otherwise fundamental players will also have to pay it in order to buy bonds, in the selling order commission is paid twice again since fundamental players will have to buy bonds for money: in this case for each share we get profit (compared to the fundamental value) $p_i - C \cdot p_i - C\left((1-C)p_i\right) - f_i$ which equals $-\left(f_i - p_i \cdot (1-C)^2\right)$. This explains the general rule of defining payoffs – the expected payoff of each order is the difference between this order and "doing nothing" with interpretation specific to this type.

Similarly we define the remaining payoffs.

2. *Technical players.* They use some techniques of technical analysis, based on past prices and volumes. Their information in our model will be represented as the vector $\Delta p \in \mathbb{R}^n$ of expected changes of price (of n shares) of minimal absolute value. Technical players look for short period trends, therefore in our model they do not invest in bonds (they want to have liquid money to react at once since selling bonds is costly), which is represented by $e \equiv 1$.

$$U^t\left(\Delta p, p, (\mathrm{BM}, \mathrm{BB}, \mathrm{S}, e)\right) =$$
$$= \sum_{i=1,\dots,n} \left(p_i(t-1) + \Delta p_i - p_i \cdot (1+C)\right) \cdot \left(q_i^{BM} \cdot \mathbf{1}_{p_i^{BM} \geq p_i} + q_i^{BB} \cdot \mathbf{1}_{p_i^{BB} \geq p_i}\right) +$$
$$- \left(p_i(t-1) + \Delta p_i - p_i \cdot (1-C)\right) \cdot q_i^S \cdot \mathbf{1}_{p_i^S \leq p_i}.$$

3. *Econometric players.* We do not assume that a considerable portion of stock exchange investors have economic or mathematical education sufficient to build an econometric model. This type of players use a ready programme using an econometric model, and they do not reestimate it during the game. The programme predicts prices $\widehat{P}(t+j)$ for τ periods with declared accuracy w. Econmetric players in this model treat w as the number that has to be subtracted from the estimated future price when they consider a buying order and added to the estimated price when they consider a selling order. Their information is a vector of maximal discounted prices for the prognosis period $\widehat{p}_i = \max_{j=1,\dots,\tau} \frac{\widehat{P}_i(t+j)}{(1+r)^j}$. As fundamental players they do not keep liquid money – they invest only in bonds and shares: $e \equiv 0$.

$$U^e\left(\widehat{p}, p, (\mathrm{BM}, \mathrm{BB}, \mathrm{S}, e)\right) =$$
$$= \sum_{i=1,\dots,n} \Big(\left(\widehat{p}_i - w - p_i \cdot (1+C)^2\right) \cdot q_i^{BB} \cdot \mathbf{1}_{p_i^{BB} \geq p_i} +$$
$$+ \left(\widehat{p}_i - w - p_i\right) \cdot q_i^{BM} \cdot \mathbf{1}_{p_i^{BM} \geq p_i} - \left(\widehat{p}_i + w - p_i \cdot (1-C)^2\right) \cdot q_i^{S} \cdot \mathbf{1}_{p_i^{S} \leq p_i} \Big).$$

4. *Portfolio players.* They know models of portfolio analysis, including CAPM and they try to use it for predicting prices. The problem is that in CAPM the distribution of future price is known, especially the expected return $\bar{R}_i$. In our model the players know the variance of returns as well β-coefficient for all shares, and consequently, the vector of expected returns according to CAPM, denoted by ρ. At each stage of the game they calculate the average return for last l periods $\bar{R}_i$ for each share (which constitute their information $\bar{R}$) and compare it with ρ_i. As fundamental and econometric players they do not keep liquid money – they invest only in bonds and shares: $e \equiv 0$.

$$U^p\left(\bar{R}, p, (\mathrm{BM}, \mathrm{BB}, \mathrm{S}, e)\right) =$$
$$= \sum_{i=1,\dots,n} \Big(\Big(\left(1+\bar{R}_i\right)^2 p_i(t-1) - p_i \cdot (1+C)^2 - \rho_i p_i \Big) \cdot q_i^{BB} \cdot \mathbf{1}_{p_i^{BB} \geq p_i} +$$
$$+ \Big(\left(1+\bar{R}_i\right)^2 p_i(t-1) - p_i - \rho_i p_i \Big) \cdot q_i^{BM} \cdot \mathbf{1}_{p_i^{BM} \geq p_i} +$$
$$- \Big(\left(1+\bar{R}_i\right)^2 p_i(t-1) - p_i \cdot (1-C)^2 - \rho_i p_i \Big) \cdot q_i^{S} \cdot \mathbf{1}_{p_i^{S} \leq p_i} \Big).$$

5. *Stochastic players.* In our model it will be a type describing all kinds of fortune-tellers clients. Stochastic players obtain only clear signals (*buying* $+1$, *selling* -1 or *no signal* 0) which are realization of some random variables. These random variables in common constitute a Young measure (see e.g. Valadier [25]), which implies that the set of players obtaining the same signal at each time instant is measurable.

We do not assume that the signals observed by various stochastic players are independent. We only assume that the measures of sets of players obtaining buying and selling signals are positive with probability 1 and with high probability detached from 0 and that signals obtained in different time instants are independent. Their information is the signal s they obtained. As technical players, they do not invest in bonds: $e \equiv 1$. For simplicity each type of stochastic players will invest in only one company.

$$U^s(s,p,(\mathrm{BM},\mathrm{BB},\mathrm{S},e)) =$$
$$= \sum_{i=1,\dots,n} (2\cdot h\cdot s\cdot p_i(t-1) - C\cdot p_i)\cdot \left(q_i^{BM}\cdot \mathbf{1}_{p_i^{BM}\geq p_i} + q_i^{BB}\cdot \mathbf{1}_{p_i^{BB}\geq p_i} - q_i^{S}\cdot \mathbf{1}_{p_i^{S}\leq p_i}\right).$$

For a profile Δ we introduce the symbol $\mathfrak{G}_t^\Delta$ for the game with the same set of players, players strategy sets $D_\omega\left(\left(X^\Delta\right)^\omega(t)\right)$, and payoff functions $\Pi_\omega(p,d) = U^{k(\omega)}(I_{k(\omega)}(\Delta,t),p,d)$. This game is called *subgame with distorted information* of our game $\mathfrak{G}$.

3. Results

Here we present two concepts of equilibria with applications to our model.

3.1. Nash equilibria and belief-distorted Nash equilibria. The basic concept of game theory is Nash equilibrium.

Definition 1. *A profile Δ is a Nash equilibrium if for a.e. $\omega\in\Omega$, for every profile $\widetilde{\Delta}$ such that $\widetilde{\Delta}(\nu)=\Delta(\nu)$ for $\nu\neq\omega$ we have $\Pi_\omega(\Delta)\geq\Pi_\omega(\widetilde{\Delta})$.*

However, all Nash equilibria in our game are not very interesting and they are far from reality – at a Nash equilibrium the stock exchange cannot operate.

Theorem 1. *Consider a game in which players have identical available strategy sets and T_ω. If $C>0$ and the maximal payoff that can be attained by the players during the game is finite, then at every Nash equilibrium for $i\in\{1,\dots,n\}$ and every $t\in\mathbb{T}$ the volume is 0.*

If, moreover, $\operatorname{esssup}_{\omega\in\Omega,q_i^{BM}(\omega,t)>0} p_i^{BM}(\omega,t)$, $\operatorname{essinf}_{\omega\in\Omega,q_i^{S}(\omega,t)>0} p_i^{S}(\omega,t)$ *and* $\operatorname{esssup}_{\omega\in\Omega,q_i^{BB}(\omega,t)>0} p_i^{BB}(\omega,t)$ *are in the interval* $[(1-h)\cdot p_i(t-1),(1+h)\cdot p_i(t-1)]$ *then*
$\operatorname{esssup}_{\omega\in\Omega,q_i^{BM}(\omega,t)>0} p_i^{BM}(\omega,t) < \operatorname{essinf}_{\omega\in\Omega,q_i^{S}(\omega,t)>0} p_i^{S}(\omega,t)$ *and*
$\operatorname{esssup}_{\omega\in\Omega,q_i^{BB}(\omega,t)>0} p_i^{BB}(\omega,t) < \operatorname{essinf}_{\omega\in\Omega,q_i^{S}(\omega,t)>0} p_i^{S}(\omega,t)$.

Proof. Let us consider a Nash equilibrium profile with a trajectory of prices p.

Let us assume that at time t player ω sells a positive amount $q_i^S(\omega,t)$ (i.e. he has $p_i^S(\omega,t)\leq p_i(t)$) while player ν buys $q_i^{BM}(\nu,t)>0$ for money (i.e. he has $p_i^{BM}(\nu,t)\geq p_i(t)$).

First let us show that at equilibrium it is impossible that a player (outside a set of measure 0) both buys and sells shares at the same time instant, i.e. that such a situation is impossible for $\nu=\omega$.

Let us assume the converse and let us denote by $\bar{q}$ the minimum of $q_i^{BM}(\omega,t)$ and $q_i^S(\omega,t)$. If player ω decreases both $q_i^{BM}(\omega,t)$ and $q_i^S(\omega,t)$ by $\bar{q}$, then he increases his instantaneous payoff at time t by $\bar{q} \cdot (1+C) \cdot p_i(t) - \bar{q} \cdot (1-C) \cdot p_i(t) = 2 \cdot C \cdot p_i(t) > 0$. At equilibrium the set of players who do not maximize their payoffs is of measure 0.

Now let consider two players ω and ν. Now let us consider a change of strategy of player ω such that instead of selling share i at time t, he repeats the part of strategy of player ν resulting from buying it, multiplied by a coefficient $\bar{q} = \frac{q_i^S(\omega,t)}{q_i^{BM}(\nu,t)}$. In order to precise what we mean, we "label" the money obtained from selling it by player ν, bonds or shares bought for this money and so on, recursively. This labelling does not have to be unique, but it exists. The part of payoff of player ν resulting from the labelled transactions (discounted for t_0), V_ν, has to fulfill $V_\nu \geq \frac{p_i(t) \cdot q_i^{BM}(\nu,t)}{(1+r)^{t-t_0}}$, since otherwise it is better for player ν not to buy share i but stay with money (if it is available in his strategy set) or buy bonds instead.

Now let us explain what we mean by repeating the labelled part of strategy of player ν by player ω. Let us consider the orders for any share j. At time t we change only $q_i^S(\omega,t)$ to 0.

For any time $s > t$ for which $p_j^S(\nu,s) > p_j(s)$, $p_j^{BM}(\nu,s) < p_j(s)$ or $p_j^{BB}(\nu,s) < p_j(s)$ we do not change the corresponding orders for share j.

Otherwise, we have the following situations.

1. The price limit in the selling order fulfils $p_j^S(\nu,s) \leq p_j(s)$. Let q' denote the labelled part of $q_j^S(\nu,s)$.

If $p_j^S(\omega,s) \leq p_j(s)$, then we change only $q_j^S(\omega,s)$ to $q_j^S(\omega,s) + q' \cdot \bar{q}$. Otherwise, we change $p_j^S(\omega,s)$ to $p_j(s)$ and $q_j^S(\omega,s)$ to $q' \cdot \bar{q}$.

2. The price limit in the BM order fulfils $p_j^{BM}(\nu,s) \geq p_j(s)$. Let q' denote the labelled part of $q_j^{BM}(\nu,s)$.

If $p_j^{BM}(\omega,s) \geq p_j(s)$, then we change only $q_j^{BM}(\omega,s)$ to $q_j^{BM}(\omega,s)+q' \cdot \bar{q}$. Otherwise, we change $p_j^{BM}(\omega,s)$ to $p_j(s)$ and $q_j^{BM}(\omega,s)$ to $q' \cdot \bar{q}$.

3. The price limit fulfils $p_j^{BB}(\nu,s) \geq p_j(s)$. Let q' denote the labelled part of $q_j^S(\nu,s)$.

If $p_j^{BB}(\omega,s) \leq p_j(s)$, then we change only $q_j^{BB}(\omega,s)$ to $q_j^{BB}(\omega,s)+q' \cdot \bar{q}$. Otherwise, we change $p_j^{BB}(\omega,s)$ to $p_j(s)$ and $q_j^{BB}(\omega,s)$ to $q' \cdot \bar{q}$.

The payoff of player ω increases by $V_\nu \cdot \bar{q}$ but decreases by the payoff corresponding to the the part of strategy resulting from selling share i at time t discounted for t_0, $V_\omega \cdot (1-C)$, which we define analogously, by labelling the part of strategy of player ω resulting from the money obtained for share i. Now we assume that player ν, instead of buying share i for money at time t repeats the labelled transactions of player ω, multiplied by $\frac{1}{\bar{q}}$, analogously to the form we have defined for player ω. By this he increases his payoff by $\frac{V_\omega}{\bar{q}}$ (without multiplying by $(1-C)$ since he does not have to pay commission) but decreases it by V_v. At equilibrium the set of players that can improve their payoffs by changing their decision is of measure 0, therefore for a.e. such ω and ν, we have both $V_\nu \cdot \bar{q} - V_\omega \cdot (1-C) \leq 0$ and $\frac{V_\omega}{\bar{q}} - V_\nu \leq 0$, which is impossible for $C \in (0,1)$.

For $q_i^{BB}(\omega,t) > 0$, the reasoning is analogous.

■

Since Nash equilibrium seems unrealistic in the context of a stock exchange, as in Wiszniewska-Matyszkiel [42] and [43], we introduce another concept of equilibrium, taking the distorted information structure into account.

Definition 2. *A profile Δ is a belief-distorted Nash equilibrium if for every $t \in \mathbb{T}$, a.e. $\omega \in \Omega$ and every $d \in D_\omega\left(\left(X^\Delta\right)^\omega(t)\right)$ we have $U^{k(\omega)}\left(I_{k(\omega)}(\Delta,t), p^\Delta(t), \Delta_\omega(t)\right) \geq U^{k(\omega)}\left(I_{k(\omega)}(\Delta,t), p^\Delta(t), d\right)$.*

Note that for a belief-distorted Nash equilibrium Δ, all static profiles $\Delta_{\cdot}(t)$ are Nash equilibria in $\mathfrak{G}_t^\Delta$, correspondingly.

Theorem 2. *If $C > 0$ and a.e. player ω is of the same type of formation of expectations, then at every belief-distorted Nash equilibrium for every t the volume is 0.*

If, moreover, $\operatorname{esssup}_{\omega\in\Omega, q_i^{BM}(\omega,t)>0} p_i^{BM}(\omega,t)$, $\operatorname{essinf}_{\omega\in\Omega, q_i^{S}(\omega,t)>0} p_i^{S}(\omega,t)$ *and* $\operatorname{esssup}_{\omega\in\Omega, q_i^{BB}(\omega,t)>0} p_i^{BB}(\omega,t)$ *are in the interval* $[(1-h)\cdot p_i(t-1), (1+h)\cdot p_i(t-1)]$ *then*
$\operatorname{esssup}_{\omega\in\Omega, q_i^{BM}(\omega,t)>0} p_i^{BM}(\omega,t) < \operatorname{essinf}_{\omega\in\Omega, q_i^{S}(\omega,t)>0} p_i^{S}(\omega,t)$ *and*
$\operatorname{esssup}_{\omega\in\Omega, q_i^{BB}(\omega,t)>0} p_i^{BB}(\omega,t) < \operatorname{essinf}_{\omega\in\Omega, q_i^{S}(\omega,t)>0} p_i^{S}(\omega,t)$.

Proof. After substituting the specific form of the expected utility function for every type of formation of expectations it becomes an easy calculation.

■

In Wiszniewska-Matyszkiel [42] and [43] equivalence theorems were stated between Nash equilibria and belief-distorted Nash equilibria along the perfect foresight path. In this paper a similar result can be proven. However, it requires an explicit formulation of the belief correspondence, omitted here for concision.

3.2. Threshold prices and weak dominance. We start our investigation of the model by defining a minimal profitable price in a selling order $\overline{\mathrm{pS}}_i^k(I)$ given information I as well as maximal profitable price in both buying orders – $\overline{\mathrm{pBM}}_i^k(I)$ for "buying for money" and $\overline{\mathrm{pBB}}_i^k(I)$ "buying for bonds".

Definition 3. *a) A price* $\overline{\mathrm{pS}}_i^k(I)$ *is the* threshold price for selling order for players of type k at information I *if for every strategy $\bar{\delta}$ with $p_i^S = \overline{\mathrm{pS}}_i^k(I)$ and q_i^S positive, and* *a strategy* δ *differing from* $\bar{\delta}$ *only by* p_i^S *and with* $p_i^S < \overline{\mathrm{pS}}_i^k(I)$ *we have* $U^k\left(I,p,\bar{\delta}\right) > U^k\left(I,p,\delta\right)$ *for some* $p \in \mathbb{P}^n$ *and* $U^k\left(I,p,\bar{\delta}\right) \geq U^k\left(I,p,\delta\right)$ *for all* $p \in \mathbb{P}^n$.

b) A price $\overline{\mathrm{pBM}}_i^k(I)$ *is the* threshold price for buying for money order for players of type k at information I *if for every strategy* $\bar{\delta}$ *with* $p_i^{BM} = \overline{\mathrm{pBM}}_i^k(I)$ *and* q_i^{BM} *positive, and a strategy* δ *differing from* $\bar{\delta}$ *only by* p_i^{BM} *and with* $p_i^{BM} > \overline{\mathrm{pBM}}_i^k(I)$ *we have* $U^k\left(I,p,\bar{\delta}\right) > U^k\left(I,p,\delta\right)$ *for some* $p \in \mathbb{P}^n$ *and* $U^k\left(I,p,\bar{\delta}\right) \geq U^k\left(I,p,\delta\right)$ *for all* $p \in \mathbb{P}^n$.

c) A price $\overline{\mathrm{pBB}}_i^k(I)$ *is the* threshold price for buying for bonds order for players of type k at information I *if for every strategy* $\bar{\delta}$ *with* $p_i^{BB} = \overline{\mathrm{pBB}}_i^k(I)$ *and* q_i^{BB} *positive, and a strategy* δ *differing from* $\bar{\delta}$ *only by* p_i^{BB} *and with* $p_i^{BB} > \overline{\mathrm{pBB}}_i^k(I)$ *we have* $U^k\left(I,p,\bar{\delta}\right) > U^k\left(I,p,\delta\right)$ *for some* $p \in \mathbb{P}^n$ *and* $U^k\left(I,p,\bar{\delta}\right) \geq U^k\left(I,p,\delta\right)$ *for all* $p \in \mathbb{P}^n$.

Now we shall calculate the threshold prices for all types of players, given their information. In order to simplify the notation, we shall introduce two symbols: if a is a nonnegative real then by $\mathrm{succ}(a) = \min_{p\in\mathbb{P},p\geq a} p$ and by $\mathrm{pred}(a) = \max_{p\in\mathbb{P},p\leq a} p$.

Proposition 3. *Threshold prices given information of the form corresponding to the type are as follows.*

a) For fundamental players $\overline{\mathrm{pS}}_i^f(f_i) = \mathrm{succ}\left(\frac{f_i}{(1-C)^2}\right)$,
$\overline{\mathrm{pBM}}_i^f(f_i) = \mathrm{pred}\left(f_i\right)$, $\overline{\mathrm{pBB}}_i^f(f_i) = \mathrm{pred}\left(\frac{f_i}{(1+C)^2}\right)$.

b) For technical players $\overline{\mathrm{pS}}_i^t(s_i,\Delta p_i,\bar{p}_i) = \mathrm{succ}\left(\frac{\bar{p}_i+\Delta p_i}{(1-C)}\right)$,
$\overline{\mathrm{pBM}}_i^t(s_i,\Delta p_i,\bar{p}_i) = \overline{\mathrm{pBB}}_i^t(f_i) = \mathrm{pred}\left(\frac{\bar{p}_i+\Delta p_i}{(1+C)}\right)$.

c) For stochastic players $\overline{\mathrm{pS}}_i^s(s,\bar{p}_i) = \mathrm{succ}\left(\frac{\bar{p}_i+2hs}{(1-C)}\right)$,
$\overline{\mathrm{pBM}}_i^s(s,\bar{p}_i) = \overline{\mathrm{pBB}}_i^s(s) = \mathrm{pred}\left(\frac{\bar{p}_i+2hs}{(1+C)}\right)$.

d) For econometric players $\overline{\mathrm{pS}}_i^e(\widehat{p}_i) = \mathrm{succ}\left(\frac{\widehat{p}_i+w}{(1-C)^2}\right)$,
$\overline{\mathrm{pBM}}_i^e(\widehat{p}_i) = \mathrm{pred}\left(\widehat{p}_i - w\right)$, $\overline{\mathrm{pBB}}_i^e(\widehat{p}_i) = \mathrm{pred}\left(\frac{\widehat{p}_i-w}{(1+C)^2}\right)$.

e) For portfolio players $\overline{\mathrm{pS}}_i^p(\bar{R}_i,p_i(t-1)) = \mathrm{succ}\left(\frac{\left(1+\bar{R}_i\right)^2 p_i(t-1)}{(1-C)^2+\rho_i}\right)$,
$\overline{\mathrm{pBM}}_i^p(\bar{R}_i,p_i(t-1)) = \mathrm{pred}\left(\frac{\left(1+\bar{R}_i\right)^2 p_i(t-1)}{1+\rho_i}\right)$, $\overline{\mathrm{pBB}}_i^p(\bar{R}_i,p_i(t-1)) = \mathrm{pred}\left(\frac{\left(1+\bar{R}_i\right)^2 p_i(t-1)}{(1+C)^2+\rho_i}\right)$.

Proof.

We shall state the proof for fundamental players. For the remaining players it is analogous.

First let us consider the part of the expected payoff corresponding to the selling order for the i-th share $-\left(f_i - p_i \cdot (1-C)^2\right) \cdot q_i^S \cdot \mathbf{1}_{p_i^S \leq p_i}$ for positive q_i^S. It increases with p_i^S for $p_i^S \leq p_i$ and is 0 for $p_i^S > p_i$. If we restrict our attention to comparing decisions differing only by the price in this order, the remaining parts of the expected payoff do not change.

This part is nonnegative if $-\left(f_i - p_i \cdot (1-C)^2\right) \geq 0$, i.e. $p_i \geq \frac{f_i}{(1-C)^2}$. The lowest price at which it is satisfied is $\mathrm{succ}\left(\frac{f_i}{(1-C)^2}\right)$. Let us take a decision $\bar{d}$ with $p_i^S = \mathrm{succ}\left(\frac{f_i}{(1-C)^2}\right)$ and d differing from $\bar{d}$ only by $p_i^S < \mathrm{succ}\left(\frac{f_i}{(1-C)^2}\right)$. If the actual price $p_i \geq \mathrm{succ}\left(\frac{f_i}{(1-C)^2}\right)$, then both orders will be admissible and for the decision $\bar{d}$ the corresponding part of the expected payoff will be nonnegative, while for d it will be negative. If the actual price $p_i < \mathrm{succ}\left(\frac{f_i}{(1-C)^2}\right)$, then the corresponding part of the expected payoff for $\bar{d}$ will be 0, while for d it will be nonpositive.

Therefore the threshold price in selling order is $\overline{\mathrm{pS}}_i^f(f_i) = \mathrm{succ}\left(\frac{f_i}{(1-C)^2}\right)$.

To get nonnegativity of the corresponding part of the expected payoff for BM order we take $f_i - p_i \geq 0$, therefore the price limit will be $\mathrm{pred}\,(f_i)$.

For BB order, analogously, we get $\mathrm{pred}\left(\frac{f_i}{(1+C)^2}\right)$.

■

The notion of threshold price implies the following weak dominance results.

Proposition 4. *Assume that at time instant t for a past realisation of a profile Δ player ω of type k has portfolio x^ω with nonzero x_i^ω and his information is I.*

a) If $\overline{\mathrm{pS}}_i^k(I) \in [(1-h) \cdot p_i(t-1), (1+h) \cdot p_i(t-1)]$, then every strategy such that $p_i^S \neq \overline{\mathrm{pS}}_i^k(I)$ or $q_i^S < x_i^\omega$ is weakly dominated in $\mathfrak{G}_t^\Delta$.

b) If $\overline{\mathrm{pS}}_i^k(I) < (1-h) \cdot p_i(t-1)$, then every strategy such that $p_i^S > \mathrm{succ}\left((1-h) \cdot p_i(t-1)\right)$ or $q_i^S < x_i^\omega$ is weakly dominated in $\mathfrak{G}_t^\Delta$.

Proof.

a) As while calculating the threshold prices, we compare strategies in $\mathfrak{G}_t^\Delta$ differing only by the price and amount in the selling order for share i and the corresponding part of the payoff function. In all cases the payoff is constructed such that this part may be considered separately. Note that for a strategy $\bar{d}$ with $p_i^S = \overline{\mathrm{pS}}_i^k(I)$ and $q_i^S > 0$ it is always nonnegative, while for any market price higher than $\overline{\mathrm{pS}}_i^k(I)$ it is strictly positive .

For a strategy d differing only by p_i^S with $p_i^S > \overline{\mathrm{pS}}_i^k(I)$ at the market price lower than p_i^S the order will not be executed, therefore this part of the payoff will be 0 (less than for $\bar{d}$), while at the market price higher than p_i^S payoffs for d and $\bar{d}$ will be identical.

For a strategy d differing only by p_i^S with $p_i^S < \overline{\mathrm{pS}}_i^k(I)$ at the market price greater or equal to p_i^S, the corresponding part of the payoff will be negative, while for $\bar{d}$ it is nonnegative. At the market price less than p_i^S the corresponding part of the payoff for both strategies will be 0.

This completes the proof that not saying the threshold price in selling order is weakly dominated.

Now we compare $\bar{d}$ with a strategy d such that $p_i^S = \overline{\mathrm{pS}}_i^k(I)$ and $q_i^S < x_i^\omega$. The coefficient at q_i^S is always nonnegative and at some prices positive, therefore the maximum is obtained at the constraint $q_i^S = x_i^\omega$.

b) An analogous reasoning holds for the threshold price below the lower variability limit. It is the result of the fact that the market price must be at least $(1-h)\cdot p_i(t-1)$.

■

The analogous fact for buying orders does not hold. One of the reasons is that money or bonds can be used for buying all kinds of shares. Even if we assume that a player invests only in shares of one company or its money and bonds are "labeled" in the sense that the fraction of them that can be invested in shares of each company is previously defined, such a fact will not hold. The reason is the constraint: saying a lower price players can buy more shares, if the market price happens to be less or equal to the price limit. However, we have to remember the fact that our order can be not executable and we shall get nothing for this order. So we have to compare two opposite effect: moderate increase of the payoff by increasing the amount and considerable increase of risk of loosing sure profit. The profit from telling a lower price grows with the difference, and it is the highest, when we say the lower variability limit while our threshold price is equal to the upper variability limit. The threshold price is equal to the upper limit of variability when we expect a considerable growth of prices. In such a situation telling the least possible price is a nonsense, and rational investors at a stock exchange surely do not behave this way. Therefore, from now on, we add this assumption to the description of players' strategies.

Definition 4. *We say that the set of available strategies of player ω is constrained with respect to information I if $p_i^{BM} \geq \overline{\mathrm{pBM}}_i^{k(\omega)}(I)$ and $p_i^{BB} \geq \overline{\mathrm{pBB}}_i^{k(\omega)}(I)$.*

Proposition 5. *Assume that a time instant t given the past realisation of a profile Δ player ω of type k has information I.*

a) If player's portfolio x^ω has positive x_{n+2}^ω and $\overline{\mathrm{pBM}}_i^k(I) \in [(1-h)\cdot p_i(t-1), (1+h)\cdot p_i(t-1)]$ and i is the only share considered by ω such that $\overline{\mathrm{pBM}}_j^k(I) \geq (1-h)\cdot p_j(t-1)$, *then each strategy of* ω *with* $p_i^{BM} \neq \overline{\mathrm{pBM}}_i^k(I)$ *or* $q_i^{BM} < \frac{x\omega_{n+2}}{p_i^{BM}(1+C)}$ *is weakly dominated in* $\mathfrak{G}_t^{\Delta}$ *with the set of strategies of* ω *constrained with respect to* I.

b) If player's portfolio x^ω *has positive* x_{n+2}^ω *and* $\overline{\mathrm{pBM}}_i^k(I) > (1+h) \cdot p_i(t-1)$ *and* i *is the only share considered by* ω *such that* $\overline{\mathrm{pBM}}_j^k(I) \geq (1-h) \cdot p_j(t-1)$, *then each strategy of* ω *with* $p_i^{BM} < \mathrm{pred}\,((1+h) \cdot p_i(t-1))$ *or* $q_i^{BM} < \frac{x_{n+2}^\omega}{p_i^{BM}(1+C)}$ *is weakly dominated in* $\mathfrak{G}_t^{\Delta}$ *with the set of strategies of* ω *constrained with respect to* I.

c) If player's portfolio x^ω *has positive* x_{n+1}^ω *and* $\overline{\mathrm{pBB}}_i^k(I) \in [(1-h) \cdot p_i(t-1), (1+h) \cdot p_i(t-1)]$ *and it is the only share considered by* ω *such that* $\overline{\mathrm{pBB}}_j^k(I) \geq (1-h) \cdot p_j(t-1)$, *then each strategy of* ω *with* $p_i^{BB} \neq \overline{\mathrm{pBB}}_i^k(I)$ *or* $q_i^{BB} < \frac{(1-C) \cdot x_{n+1}^\omega}{p_i^{BB}(1+C)}$ *is weakly dominated in* $\mathfrak{G}_t^{\Delta}$ *with the set of strategies of* ω *constrained with respect to* I.

d) If player's portfolio x^ω *has positive* x_{n+1}^ω *and* $\overline{\mathrm{pBB}}_i^k(I) > (1+h) \cdot p_i(t-1)$ *and it is the only share considered by* ω *such that* $\overline{\mathrm{pBB}}_j^k(I) \geq (1-h) \cdot p_j(t-1)$, *then each strategy of* ω *with* $p_i^{BB} < \mathrm{pred}\,((1+h) \cdot p_i(t-1))$ *or* $q_i^{BB} < \frac{(1-C) \cdot x_{n+1}^\omega}{p_i^{BB}(1+C)}$ *is weakly dominated in* $\mathfrak{G}_t^{\Delta}$ *with the set of strategies of* ω *constrained with respect to* I.

Proof.
Analogous to the proof of 4.
■

Proposition 6. *Assume that a time instant* t *given the past realisation of a profile* Δ *player* ω *of type* k *investing only in share* i *and having constant* e *has information* I. *If player's* ω *portfolio* x^ω *has positive* x_{n+1}^ω *and* x_{n+2}^ω, *the threshold prices* $\overline{\mathrm{pBM}}_i^k(I)$ *and* $\overline{\mathrm{pBB}}_i^k(I)$ *are greater or equal to the lower limit of variability and* $\overline{\mathrm{pS}}_i^k(I)$ *is less or equal to the upper limit of variability, than the strategy of* ω $\left((\overline{\mathrm{pBM}}_i^k(I), \frac{x_{n+2}^\omega}{p_i^{BM} \cdot (1+C)}), (\overline{\mathrm{pBB}}_i^k(I), \frac{(1-C) \cdot x_{n+1}^\omega}{p_i^{BB} \cdot (1+C)}), (\overline{\mathrm{pS}}_i^k(I), x_i^\omega), e\right)$ *is weakly dominant in* $\mathfrak{G}_t^{\Delta}$ *with the set of strategies of* ω *constrained with respect to* I.

Proof.
As of proof 4.
■

4. Implications for prediction

From now on we shall assume that players use only strategies consistent with their information. We shall answer the question, what may happen if a strong (i.e. large and having a considerable portion of assets) group of players uses the same prognostic technique and they obtain the same information.

We assume that there is at least a small group of stochastic players. The reason is that in the case when all players have identical prognostic technique, the stock exchange cannot work – we need at least a small fraction of players having expectations to some extent opposite than the majority.

4.1. Self-verifying beliefs. It is obvious from this model, but also from the real life, that the beliefs can influence prices. In this context, the most interesting thing to consider is the question, whether and to what extent the ways of predicting prices can force the prices behave according to the beliefs – we have to match the abstract "information" the players obtain with their interpretation of future prices.

Fundamental analysis. The simplest example of self-verifying beliefs is fundamental analysis. We shall consider a game starting at time t_0 with a vector of reference prices $p(t_0-1)$. Assume that there is a strong group of fundamental players with identical $\{F_i(t)\}$, and assume that there is also a small group of stochastic players investing in i, possessing i as well as bonds or money. Consider any time instant t such that reaching the fundamental value is theoretically possible, i.e. $F_i(t) \in \mathbb{P} \cap [(1-h)^{t-t_0} \cdot p_i(t-1), (1+h)^{t-t_0} \cdot p_i(t-1)]$.

First, we have to define what we understand by a *strong group of players* in $\mathfrak{G}_t^{\Delta}$ – a group that can dominate the market.

Definition 5. *We call a set of players $\bar{\Omega} \subset \Omega$* strong in $\mathfrak{G}_t^{\Delta}$

a) in share i *(for $i = 1, \ldots, n$) if* $\int_{\bar{\Omega}} (1-h) \cdot p_i(t-1) \cdot X_i^{\omega}(t) d\lambda(\omega) \geq \int_{\Omega\setminus\bar{\Omega}} X_{n+1}^{\omega}(t) \cdot (1-C) + X_{n+2}^{\omega}(t) d\lambda(\omega)$*;*

b) in bonds *if* $\int_{\bar{\Omega}} X_{n+1}^{\omega}(t) \cdot (1-C) d\lambda(\omega) \geq \sum_{i=1}^{n} \int_{\Omega\setminus\bar{\Omega}} (1+h) \cdot p_i(t-1) \cdot X_i^{\omega}(t) d\lambda(\omega)$*;*

c) in money *if* $\int_{\bar{\Omega}} X_{n+2}^{\omega}(t) d\lambda(\omega) \geq \sum_{i=1}^{n} \int_{\Omega\setminus\bar{\Omega}} (1+h) \cdot p_i(t-1) \cdot X_i^{\omega}(t) d\lambda(\omega)$*;*

d) in risk free assets $\int_{\bar{\Omega}} X_{n+1}^{\omega}(t) \cdot (1-C) + X_{n+2}^{\omega}(t) d\lambda(\omega) \geq \sum_{i=1}^{n} \int_{\Omega\setminus\bar{\Omega}} (1+h) \cdot p_i(t-1) \cdot X_i^{\omega}(t) d\lambda(\omega)$.

Definition 6. *A set of players $\bar{\Omega} \subset \Omega$ is* strong in asset(s) i *if for every $t \leq \sup_{\omega\in\bar{\Omega}} T_\omega$ and every profile Δ the set $\bar{\Omega}$ is strong in $\mathfrak{G}_t^{\Delta}$ in asset(s) i.*

Proposition 7. *Let $\bar{\Omega}$ be a set of fundamental players with identical $F_i(t)$ and let Δ be a belief distorted Nash equilibrium.*

a) If $\bar{\Omega}$ is strong in i in $\mathfrak{G}_t^{\Delta}$, then $p_i(t)$ will not exceed $\max(\overline{\mathrm{pS}}_i^f(F_i(t)), (1-h) \cdot p_i(t-1))$.

b) If $\bar{\Omega}$ is strong in money in $\mathfrak{G}_t^{\Delta}$, then $p_i(t)$ will not be less than $\min(\overline{\mathrm{pBM}}_i^f(F_i(t)), (1+h) \cdot p_i(t-1))$,

c) If $\bar{\Omega}$ is strong in bonds in $\mathfrak{G}_t^{\Delta}$, then $p_i(t)$ will not be less than $\min(\overline{\mathrm{pBB}}_i^f(F_i(t), (1+h) \cdot p_i(t-1)))$.

Proof.

The probability that a set of stochastic players possessing shares i of positive measure will get a selling signal and the probability that a set of stochastic players of positive measure possessing money or bonds will get a buying signal are equal to 1. Let us note that the threshold selling price for stochastic players getting selling signal is below $(1-h)\cdot p_i(t-1)$. Therefore we shall have some selling orders with the price limit greater or equal to lower limit of variability as well as some buying orders with the price limit greater or equal to upper limit of variability.

Therefore at each price greater or equal to $\overline{\mathrm{Ps}}_i^f(F_i(t))$ the volume is equal to the demand, which is nonincreasing.

Assume that price of i at time t is equal to $\widetilde{p}_i > \overline{\mathrm{Ps}}_i^f(F_i(t))$. This would imply the demand is constant at the interval $[\overline{\mathrm{Ps}}_i^f(F_i(t)), \widetilde{p}_i]$, as well as the disequilibrium. Now let us check criterion 3. In our case we want to minimise the number of shares in selling order with price limit greater than the market price. The minimum cannot be attained at $\widetilde{p}_i$, only in $\overline{\mathrm{Ps}}_i^f(F_i(t))$, which contradicts our assumption.

b) and c) are proven analogously. First we assume that a lower price was chosen. In this case the volume is equal to the supply. Thus it is constant at the corresponding interval, as well as disequilibrium, but then criterion 3 is not satisfied.

■

Thus we get fast convergence to quite a narrow interval of prices.

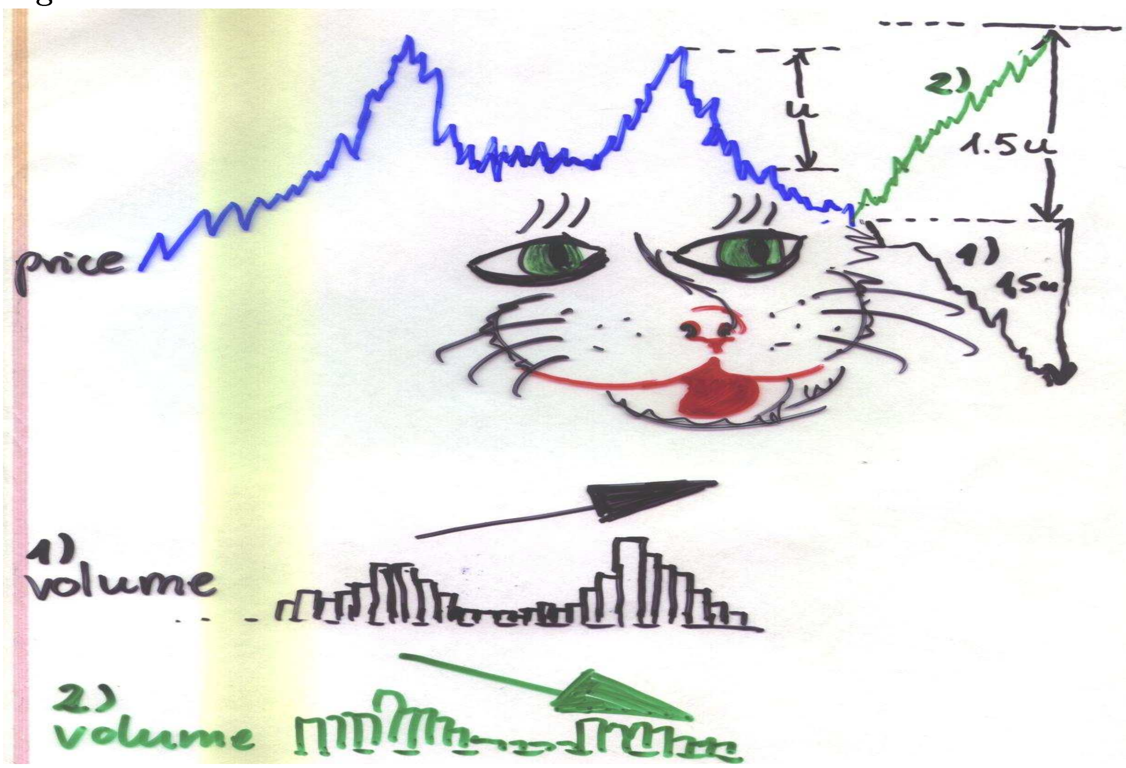
Technical analysis. Similar self-verification results can be proven for technical analysis. Nevertheless, they cannot be treated as a proof of validity of technical analysis as a cognition device.

Formation of cat. In order to show how technical analysis can make the prices behave as it predicts we shall show an abstract formation, previously defined in Wiszniewska-Matyszkiel [40], and consider the results of its popularization among investors. This formation has not existed in technical analysis and is not reflected by data. It will be formulated as in textbooks on technical analysis and "explained" by a similar quasi-sociological explanation (see e.g. Pring [19]) and it will turn out to be approximately self-verifying.

Formation of Cat starts by a moderate increase of prices of shares (back of the neck), then prices rapidly rice, and afterwards fall (left ear), then there is a flat summit (crown of the head) and the third summit similar to the first one (right ear), ending by a moderate fall of price (forehead) starting from the base of the right ear and lasting at least as long as the right ear. The volumes at the crown of the head are always low.

If the volume at the top of the right ear is less than at the top of the left ear, then *the cat is looking down*, if the converse holds, *the cat is looking up*. Since cats are contrary animals, cats looking up forecast fall of prices, while cat looking down forecast rise of prices, and the absolute value of changes is at least one and a half of the height of the ears.

Figure 1



Now we construct a quasi-sociological explanation as from textbooks on technical analysis.

A moderate but quite stable increase of prices causes an *exaggerated optimism* among players, which increases the demand. At the top of left ear *strong* (better informed) players sell their shares to *weak* (worse informed) players, constituting majority. Then there is a correction and weak players sell their shares. When the price reaches the level of the end of the *back of the neck*, players observe the market waiting for signals, therefore the volume is low. If the *optimism* wins, the *right ear* is formed. High volume at right ear means strong *distribution*: *strong* players sell their shares to *weak* players, which are prone to *panic* in the case of fall of prices. Low volumes at right ear mean that the majority of shares is in the hands of *strong* players, which usually do not panic, since by their *information* they expect *increase* of prices.

To simplify the analysis, we assume that we consider only players investing in share i. We shall denote the height of the ears by U. Assume that technical players using the cat formation either have no further signals or treat them as less important than the cat formation and that there is also a small set of stochastic players possessing shares and risk free assets.

Proposition 8. *Let Δ be a relisation of a profile and let t be a time instant at which the* cat *formed as a result of playing Δ up to t. If the set $\bar{\Omega}$ of technical players believing in cat formation is strong in risk free assets in $\mathfrak{G}_t^{\Delta}$, then at every belief distorted Nash equilibrium the* cat looking down *implies an increase of price of i at least to* pred $\left(\frac{p_i(t-1)+\frac{3}{2}U}{(1+C)}\right)$, *while if $\bar{\Omega}$ is strong in i in $\mathfrak{G}_t^{\Delta}$, then the* cat looking up *implies a decrease of prices at least to* succ $\left(\frac{p_i(t-1)-\frac{3}{2}U}{(1-C)}\right)$.

Proof.

Let us consider the cat looking down. Since technical players expect increase of price, their threshold price at each time instant is equal to pred $\left(\frac{p_i(t-1)+\frac{3}{2}U}{(1+C)}\right)$. First it can be above the upper variability limit. In each of such time instants t the price limit in buying orders of technical players will be equal to pred $(p_i(t-1)\cdot(1+h))$. As in the proof of proposition 7, we get that the market price is equal to the price limit of the strongest group of players. Finally, technical players will have the price limit equal to the threshold price pred $\left(\frac{p_i(t-1)+\frac{3}{2}U}{(1+C)}\right)$, which will be the market price.

The reasoning for the cat looking up is analogous.

■

Strong signals in technical analysis. In the case of strong signals in technical analysis, especially when technical players expect a change of the trend, they expect changes of prices of large absolute value.

Proposition 9. *Let Δ be a belief distorted Nash equilibrium and let t be a time instant at which a strong signal was observed and indentically interpreted as Δp_i by a set $\bar{\Omega}$ of technical players.*

a) Assume $\Delta p_i << -h\cdot p_i(t-1)$ (a selling signal)*. If $\bar{\Omega}$ is strong in i in $\mathfrak{G}_t^{\Delta}$ and there is a set of stochastic players of positive measure investing in this company still possessing risk free assets at t, then with probability 1 prices of share i will fall and the fall will be to at least* succ $\left(\frac{p_i(t-1)+\Delta p_i}{(1-C)}\right)$.

b) Assume $\Delta p_i >> h\cdot p_i(t-1)$ (a buying signal)*. If $\bar{\Omega}$ invests only in company i or for other companies j considered by players from $\bar{\Omega}$ $\overline{\mathrm{pBM}}_j^k(I) < (1-h)\cdot p_j(t-1)$ and if $\bar{\Omega}$ is strong in risk free assets in $\mathfrak{G}_t^{\Delta}$ and there is a set of stochastic players of positive measure still possessing i at t, then with probability 1 prices of i will grow and the increase will be to at least* pred $\left(\frac{p_i(t-1)+\Delta p_i}{(1+C)}\right)$.

Proof.

The proof is analogous to that of the cat formation.

■

4.2. Self-falsifying beliefs. Here we want to show that not all beliefs are self-verifying.

To simplify the analysis, we again consider players investing in share i only, and money or bonds, and assume that they consider strategy sets constrained with respect to information.

CAPM. Now we shall consider the case in which there is a strong group of portfolio players and a small group of stochastic players. We also assume that C is small.

The basic result in the papers about CAPM cited in the introduction, is that prices adjust such that the return of each asset is equal to its theoretical ρ_i. However, there was assumption that there is an equilibrium and no dynamics was considered. We get the result, that in the case of starting from aggregate returns differing from ρ_i, we do not have to converge to it. Conversely, rather divergence can be expected.

Proposition 10. *Let Δ be a relisation of a profile, let t be a time instant and let $\bar{\Omega}$ be a set of porfolio players.* Portfolio analysis *is* self-falsifying *in the sense, that*

a) if $\bar{R}_i$ is essentially greater than ρ_i, $\bar{\Omega}$ is strong in money in $\mathfrak{G}_t^{\Delta}$ and there is a set of stochastic players of positive measure investing in i still possessing i at t, then $R_i(t)$ will be greater than $\bar{R}_i$;

b) if $\bar{R}_i$ is greater than $\rho_i + C^2 + 2C$, $\bar{\Omega}$ is strong in risk free assets in $\mathfrak{G}_t^{\Delta}$ and there is a set of stochastic players of positive measure investing in i still possessing i at t, then $R_i(t)$ will be greater than $\bar{R}_i$;

c) if $\bar{R}_i$ is essentially less than $\rho_i + C^2 - 2C$, $\bar{\Omega}$ is strong in i in $\mathfrak{G}_t^{\Delta}$ and there is a set of stochastic players of positive measure investing in i still possessing risk free assets at t, then $R_i(t)$ will be less than $\bar{R}_i$.

Proof.

a) Here $\bar{R}_i > \rho_i$ and portfolio players are strong in money. In this case we shall calculate their return in the case when the market price equals their threshold price $\overline{\mathrm{pBM}}_i^p(\bar{R}_i, p_i(t-1))$.

Then the return at time t fulfills $R_i(t) = \dfrac{\mathrm{pred}\left(\frac{\left(1+\bar{R}_i\right)^2 p_i(t-1)}{1+\rho_i}\right) - p_i(t-1)}{p_i(t-1)} \geq \frac{\left(1+\bar{R}_i\right)^2}{1+\rho_i} - 1 - \frac{\varepsilon}{p_i(t-1)}$, where ε is a small number defining the precision of price representation in the part of $\mathbb{P}$ under consideration, i.e. such a number that for $p_i = \frac{\left(1+\bar{R}_i\right)^2 p_i(t-1)}{1+\rho_i}$ $\mathrm{pred}\,(p_i) \geq p_i - \varepsilon$. If the difference between $\bar{R}_i$ and ρ_i is large enough, then $\frac{\left(1+\bar{R}_i\right)^2}{1+\rho_i} - \frac{\varepsilon}{p_i(t-1)} > \frac{\left(1+\bar{R}_i\right)^2}{1+\bar{R}_i} = \bar{R}_i$, therefore $R_i(t) > \bar{R}_i$.

In the buying for money orders of portfolio players the price limit is equal to the threshold price.

As in the proof of proposition 7, we get that the market price is greater or equal to the price limit in the buying orders of the strongest group of players, in this case the threshold price $\overline{\mathrm{pBM}}_i^p(\bar{R}_i, p_i(t-1))$ for the porfolio players.

b) Now let us assume a greater difference $\bar{R}_i > \rho_i + C^2 + 2C$ and let us assume that portfolio players are strong in bonds.

If the market price equals the threshold price $\overline{\mathrm{pBB}}_i^p(\bar{R}_i, p_i(t-1))$, then $R_i(t) = \dfrac{\mathrm{pred}\left(\frac{\left(1+\bar{R}_i\right)^2 p_i(t-1)}{(1+C)^2+\rho_i}\right) - p_i(t-1)}{p_i(t-1)} \geq \frac{\left(1+\bar{R}_i\right)^2}{1+C^2+2C+\rho_i} - 1 - \frac{\varepsilon}{p_i(t-1)}$. If the difference between $\bar{R}_i$ and $\rho_i + C^2 + 2C$ is large enough, then $\frac{\left(1+\bar{R}_i\right)^2}{1+C^2+2C+\rho_i} - \frac{\varepsilon}{p_i(t-1)} > \frac{\left(1+\bar{R}_i\right)^2}{1+\bar{R}_i} = \bar{R}_i$, therefore $R_i(t) > \bar{R}_i$.

The market price will be greater or equal either to $\overline{\mathrm{pBB}}_i^p(\bar{R}_i, p_i(t-1))$ or $\overline{\mathrm{pBM}}_i^p(\bar{R}_i, p_i(t-1))$ (if $\int_{\bar{\Omega}} X_{n+2}^{\omega}(t) d\lambda(\omega) > 0$), for which we have already proven the inequality.

c) Now let us consider the case when $C^2 - 2C + \rho_i > \bar{R}_i$ and $\bar{\Omega}$ is strong in i.

The threshold price $\overline{\mathrm{pS}}_i^p(\bar{R}_i, p_i(t-1))$ is $\mathrm{succ}\left(\frac{\left(1+\bar{R}_i\right)^2 p_i(t-1)}{((1-C)^2+\rho_i)}\right)$, therefore if the market price is equal to this threshold price, the return fulfills $R_i(t) = \dfrac{\mathrm{succ}\left(\frac{\left(1+\bar{R}_i\right)^2 p_i(t-1)}{(1-C)^2+\rho_i}\right) - p_i(t-1)}{p_i(t-1)} \leq \frac{\left(1+\bar{R}_i\right)^2}{(1-C)^2+\rho_i} - 1 + \frac{\varepsilon}{p_i(t-1)} = \frac{\left(1+\bar{R}_i\right)^2}{1+C^2-2C+\rho_i} - 1 + \frac{\varepsilon}{p_i(t-1)}$, for ε such that for $p_i = \frac{\left(1+\bar{R}_i\right)^2 p_i(t-1)}{(1-C)^2+\rho_i}$ $\mathrm{succ}\,(p_i) \leq p_i + \varepsilon$. If the difference between $\rho_i + C^2 - 2C$ and $\bar{R}_i$ is large enough, then $\frac{\left(1+\bar{R}_i\right)^2}{1+C^2-2C+\rho_i} + \frac{\varepsilon}{p_i(t-1)} < \frac{\left(1+\bar{R}_i\right)^2}{1+\bar{R}_i} = \bar{R}_i$, therefore $R_i(t) < \bar{R}_i$. Analogously to the reasoning for the buying orders, the market price is less or equal to the price limit of selling order of portfolio players $\overline{\mathrm{pS}}_i^p(\bar{R}_i, p_i(t-1))$.

■

The facts stated in proposition may lead to trends of accelerating increases or accelerating decreases of prices.

Econometric models. We cannot state anything precise about econometric models in general. Depending on the specific type of the model they can be either approximately self-verifying or self-falsifying. If we treat them literally, they will be usually self-falsifying: increases and decreases of prices are prior to the moment they were prognosed for. Nevertheless, econometric models used as tools to foresee general tendencies are approximately self-verifying.

5. Numerical simulations

The following simulations were made with the programme SGPW [22]. In each of them we assumed existence of a small group of stochastic players with constant flow of money and possessing a small fraction of shares considered.

5.1. Convergence to the fundamental value. The figures below illustrate convergence to the fundamental value (given the initial price of a share from WSE) in the game with a large group of fundamental analysts.

Figure 2

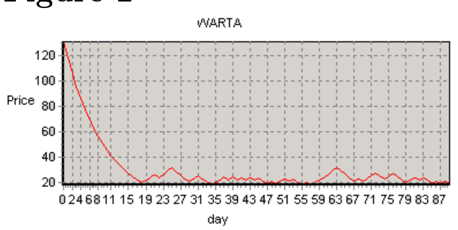
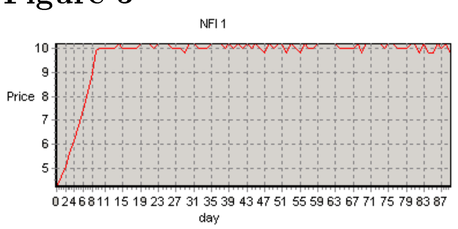


Figure 3



5.2. Trends caused by chartists. A group of chartist and trends caused by them given various initial values form WSE:

Figure 4

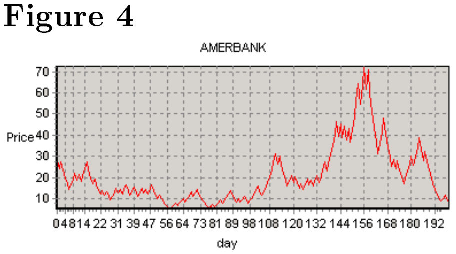
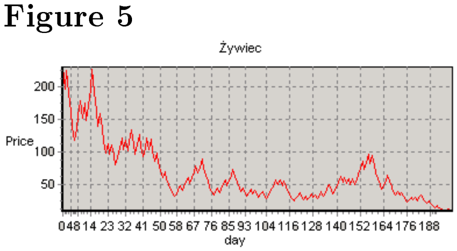


Figure 5



For comparison, if we consider stochastic players only, we get something similar to a random walk: at each time instant we either go up the upper variability limit if the measure of the set of players obtaining selling signal is less than the measure of the set of players obtaining the buying signal or to the lower variability limit if the measure of the set of players obtaining selling signal is greater than the measure of the set of players obtaining the buying signal.

5.3. Trends caused by portfolio players. For the case of a strong group of portfolio players the results are exactly as stated in the model – either an exponential growth of the prices or an exponential decrease.

5.4. Some econometric models. In this case we present two econometric models: one of them considering linear trend and sinusoidal weekly periodicity and length of prognosis 2, and the other one with the average of some of past prices. The former one is approximately self-verifying only because the linear trend dominates. However, the oscillations are translated. The latter one becomes self-verifying after a period of transition.

Figure 6

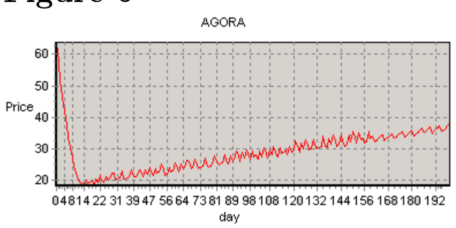
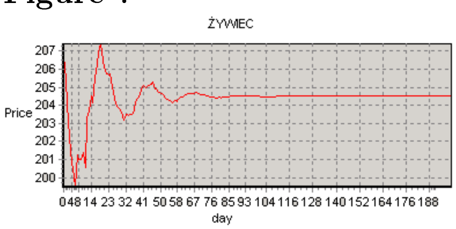


Figure 7



6. Conclusions

The paper presents a model of stock exchange as a game with a continuum of players taking into account various prognostic techniques. The continuum was used to model insignificance of any single player, while prices, and consequently, players payoffs are solely a result of players decisions. One of the results of the paper is that usually the strategies of telling the actual threshold prices are weakly dominant, while strategies of not telling the actual threshold prices are weakly dominated in a sequence of subgames with distorted information along the profile, therefore they constitute a belief distorted Nash equilibrium.

One of the consequences of that is the problem of self-verification of various prognostic techniques used by strong (i.e. large and possessing a large portion of assets) groups of players at presence of a small group of stochastic players and, possibly, other types. This is the feature of fundamental analysis and technical analysis. Taking this into account, learning about many, even absolutely senseless, techniques may turn out to be useful if they are used by many players.

The technique based on CAPM does not have this property, it is self-falsifying, while techniques based on various econometric models may be either self-verifying or self-falsifying.

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