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## **Trade Models with Heterogeneous Firms: What About Importing?**

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### ABSTRACT

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Intermediate goods make up a large share of world trade. Yet trade models with heterogeneous firms focus almost exclusively on firms' export decisions rather than their import decisions. We develop an analytically solvable model of a small open economy in which heterogeneous firms make endogenous import decisions. We view decisions about importing as decisions about technology adoption. In our model, firms weigh the benefit of operating a technology that uses imported intermediate goods against the fixed cost of developing trade relationships with foreign input suppliers. Similar to the selection effect in standard export-decision models, only the most efficient firms choose to import. In addition, the model features a technology upgrading effect, where importing improves a firm's labor efficiency. The calibrated model quantifies the selection and technology upgrading effects, captures the large performance advantage associated with using imported intermediate goods, and generates large increases in trade from small decreases in tariffs.

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## Introduction

A large and increasing share of international trade is trade in intermediate goods.<sup>1</sup> Yet almost all models of trade with heterogeneous firms focus on a firm's decision to export. In these models, a firm decides whether to pay a fixed cost in order to sell its good in a foreign market. This modeling approach began with Melitz (2003) and has been extended in numerous ways.<sup>2</sup> The Melitz model has become one of the workhorse models of international trade, allowing economists to better understand firms' export decisions and the effects of trade liberalization on an economy's industrial organization.

We too develop a model with heterogeneous firms and fixed costs of trade, but our focus is on a firm's decision to import intermediate goods. Our model complements Melitz-style models that emphasize firms' export decisions. While import and export decisions may be related in some ways, they involve fundamentally different considerations by firms. In making export decisions, firms must consider the characteristics of foreign markets and the costs involved in entering those markets. In making import decisions, firms must consider how the use of imported intermediate goods will affect their production processes and weigh this against the costs of developing business relationships with foreign input suppliers.

Our model is motivated by, and consistent with, data on plants' importing behavior. Using recent plant-level data on Chilean manufacturing firms, we document a basic set of facts. Importers differ sharply from non-importers. Plants that use imported intermediate goods are much larger and more productive than plants that do not. Despite the apparent performance advantage associated with using imported inputs, only a small fraction of plants do, and, even among importers, imported intermediate goods do not make up a majority of total expenditure on intermediate goods.<sup>3</sup> We adopt a simple interpretation of these facts: most producers would prefer to use some imported intermediate inputs in production, but there are fixed costs that discourage most from

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<sup>1</sup> See, for example, Feenstra (1998) and Hummels, Ishii, and Yi (2001).

<sup>2</sup> Helpman (2006) provides a survey of this literature.

<sup>3</sup> These facts appear to be robust across countries. See, for example, Bernard, Jensen, and Schott (2009) for the United States, Kugler and Verhoogen (2009) for Colombia, Castellani, Serti, and Tomasi (2009) for Italy, and Muûls and Pisu (2009) for Belgium.

doing so. These fixed costs are the costs of developing business relationships with foreign input suppliers.

Fundamental to our modeling approach is the idea that firms' import decisions are really decisions about technology adoption. Each firm must decide how using imported intermediate goods will affect its production process and its performance. There is a growing literature providing evidence that using imported intermediate goods enhances firm or plant performance. This includes research by Amiti and Konings (2007), Kasahara and Rodrigue (2008), Halpern, Koren, and Szeidl (2009), Kugler and Verhoogen (2009), and Gibson and Graciano (2011). Our aim is to model this phenomenon in a simple way and use the model to better understand, both qualitatively and quantitatively, the effects of trade liberalizations, improvements in the terms of trade, and decreases in trade costs. We keep the model simple enough that there is an exact analytic solution for the equilibrium, yet rich enough that we can quantitatively capture some important features of the data.

We develop a general equilibrium model of a small open economy with a continuum of single-plant firms. These firms exhibit two forms of heterogeneity: they differ in their levels of efficiency and in whether or not they use imported intermediate goods. The first form of heterogeneity is the result of random draws, as in Hopenhayn (1992). The second is endogenously decided by each firm. Each firm chooses between two technologies: a technology that uses only domestic inputs and a technology that uses a combination of domestic and imported inputs. The fixed cost of operating the technology that uses imported intermediate inputs is higher, reflecting the additional costs of developing business relationships with foreign input suppliers. The technology that uses imported intermediate goods is superior, in the sense that every firm would choose it if there were no additional fixed cost of doing so. The total benefit of using this technology is increasing in a firm's scale of operation, while the operating cost is fixed.

In the model, as in the data, importers are very different from non-importers. The model provides a simple way of accounting for these differences. Because of the fixed cost of importing, only the firms with the highest efficiency draws choose to import.

This is the usual selection effect emphasized by Melitz (2003) in the context of exporting. In addition, our model has what we refer to as a technology upgrading effect: a firm that opts for the technology using some imported inputs over the technology using only domestic inputs increases its output, employment, expenditure on intermediate inputs, and variable profits. The technology upgrading effect is analogous to “learning by importing,” where the very act of importing leads to improved firm performance. Kasahara and Rodrigue (2008), among others, provide evidence of this effect.

To obtain the quantitative implications of the theory, we calibrate the model using the Chilean manufacturing data described earlier. The model does a good job of replicating the basic facts that we document, including the large performance advantage associated with importing. Importantly, the calibrated model allows us to account for the relative contributions of the selection and technology upgrading effects in accounting for this performance advantage.

We use the model to qualitatively and quantitatively analyze the effects of tariff reduction, terms-of-trade improvement, and trade-cost reduction. These sorts of changes lead to a process of reallocation across firms: the least efficient firms exit, the most efficient non-importers become importers, and aggregate technological efficiency increases. In many ways this process of reallocation resembles what Melitz (2003) finds in his export-decision model. In our model, however, the effects of the reallocation following trade liberalization are augmented through the technology upgrading effect. This leads to an improvement in labor efficiency at importing firms. This feature of the model agrees well with the evidence of, for example, Amiti and Konings (2007), who study the effects of trade liberalization on Indonesian plants.

Comparing standard trade models with the data, Yi (2003) and Kehoe (2005) stress the need to develop models that can generate large increases in trade in response to small decreases in tariffs. Our model does this. Ruhl (2004) and Chaney (2008) stress the importance of the extensive margin in Melitz-style export-decision models. We stress the importance of the extensive margin in our import-decision model. In response to trade liberalization, many non-importers switch technologies to become importers. With

this extensive margin, we can generate the sort of large aggregate Armington elasticity (the elasticity of substitution between imported and domestic goods) found in the data without assuming unusually large elasticities at the level of an individual firm.

There are few other general equilibrium models that incorporate the importing decisions of firms. Ramanarayanan (2007) builds a dynamic model in which entering firms make irreversible decisions about their import status in the presence of aggregate and idiosyncratic uncertainty. He uses the model to contrast the effects of business-cycle shocks and trade liberalizations on the Armington elasticity. Kasahara and Lapham (2008) consider both the decision to export and the decision to import. They develop a dynamic model in which firms face stochastic fixed costs of importing in addition to a fixed cost of exporting. Gopinath and Neiman (2011) build a model of heterogeneous firms to analyze changes in use of imported intermediate goods during large crises. They show how changes in inputs at the firm level can affect measured productivity. By contrast to these papers, we isolate the decision to import and develop a simple, static, non-stochastic, competitive model that has an exact analytic solution. This allows for a high degree of transparency in our analysis of trade liberalization and provides a modeling framework that can be readily extended and applied.

The paper is organized as follows. In the next section we discuss our data and some basic facts on importing behavior. In the third section, we develop the model. In the fourth and fifth sections, we qualitatively and quantitatively analyze the model. The sixth section concludes.

## **Data**

Here we document a basic set of facts that we would like our model to be able to quantitatively replicate. These facts concern the extent to which producers use imported intermediate goods and how producers that use imported intermediate goods differ from those that do not. We take these facts from the annual census of manufacturing plants

conducted by Chile's Instituto Nacional de Estadísticas during the period 2001 to 2006.<sup>4</sup> The census is detailed: it includes data on plants' employment, gross output, value added, and expenditures, including expenditures on domestically produced intermediate goods and imported intermediate goods. All monetary values in the census are expressed in an inflation-adjusted unit of account, the Chilean Unidad de Fomento.

From 2001 to 2006, the census surveyed a total of 8,014 different manufacturing plants. To be consistent with our model, which is static, we omit from our sample the plants that changed their import status over this period.<sup>5</sup> We categorize the remaining 6,936 plants as follows. *Importers* are plants that purchased imported raw materials every year that they participated during the period; they are 13 percent of our sample. *Non-importers* are plants that did not purchase imported raw materials in any year that they participated during the period; they are 87 percent of our sample. When we calculate averages, we average over every relevant plant-year observation. We document four basic facts about plants' importing behavior.

**Fact 1.** Most plants do not import. Importers are only 13 percent of our sample.

**Fact 2.** Importers spend more on domestically produced intermediate goods than on imported intermediate goods. In our sample, expenditure on imported intermediate goods is 39 percent of the average importer's total expenditure on intermediate goods.

**Fact 3.** Importers are much larger than non-importers. In terms of gross output, expenditure on intermediate goods, value added, and employment, importers are, on average, 4.0 to 5.2 times larger than non-importers.

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<sup>4</sup> A previous version of this census was examined by Liu (1993), Levinsohn (1999), Pavcnik (2002), and Kasahara and Rodrigue (2008), among others.

<sup>5</sup> These plants are 13 percent of the original sample. For an empirical analysis of plants that switch import status, see Kasahara and Rodrigue (2008). Gibson and Graciano (2011) develop a quantitative dynamic general equilibrium model with switchers.

**Fact 4.** Importers are more productive than non-importers. Importers have 1.3 times higher value added per worker than non-importers.

These facts may seem contradictory. There appears to be a large performance advantage associated with importing, yet most plants do not import and those that do typically spend more on domestically produced intermediate goods than on imported intermediate goods. We next develop a model that has the potential to account for these facts.

### **Model**

There is a small open economy that competitively produces and exports a single good and imports a differentiated good from the rest of the world. Because the economy is small relative to the rest of the world, it takes the relative price of the two goods as given. The good produced by the small open economy, which serves as the numéraire, may be used in four different ways: for consumption, for export (the rest of the world has elastic demand for it), for payment of fixed costs, and as an intermediate input. The good is produced by a continuum of heterogeneous single-plant firms. The small open economy's government may impose an *ad valorem* tariff on the imported good.

#### *Consumer*

There is a representative consumer in the economy who is endowed with  $L$  units of labor. The consumer supplies labor inelastically and spends all income on consumption. The consumer's budget constraint is

$$C = wL + T, \quad (1)$$

where  $C$  is consumption,  $w$  is the wage, and  $T$  is the lump-sum rebate of tariff revenue.

#### *Firms*

There is a continuum of single-plant firms in the economy. These firms exhibit two forms of heterogeneity: they differ in their levels of efficiency and in whether or not

they use imported intermediate goods. The first form of heterogeneity is the result of random draws, while the second is endogenously decided by each firm.

A firm's actions are as follows. After paying the fixed cost of entry, the firm takes an efficiency draw from a probability distribution. The firm then has three options: not to operate, to operate using a technology that does not require imported inputs, or to operate using a technology that requires imported inputs. All firms face decreasing returns to scale, so firms with different efficiency levels can coexist, with each firm operating at its optimal scale. Each firm does, however, face a fixed cost of operating, so the firms with the worst draws may choose not to operate at all.

Fundamental to our model is the characterization of firms' technologies. Let technology  $N$  be the technology of a non-importer and let technology  $I$  be the technology of an importer. The technologies are similar, but differ along important dimensions. Technology  $N$  uses only labor and the domestically produced intermediate good as inputs, while technology  $I$  uses labor, the domestically produced intermediate good, and the imported intermediate good as inputs. For each technology, the extent of diminishing returns is determined by the parameter  $\nu$ , where  $0 < \nu < 1$ . The total factor productivity with which a firm operates technology  $N$  is given by its efficiency draw,  $a$ , while the total factor productivity with which a firm operates technology  $I$  is given by  $a\eta$ , where  $\eta > 0$ . Operating either technology requires payment of a fixed cost. We assume that the fixed cost of operating technology  $I$  is greater than the fixed cost of operating technology  $N$ . This assumption captures the idea that there are additional costs involved in developing business relationships with foreign input suppliers relative to domestic input suppliers. Next we specify the two technologies.

First consider a firm with efficiency  $a$  operating technology  $N$ . The firm's output is given by

$$y_N(a) = a^{1-\nu} \psi_N (\ell_N(a), d_N(a))^\nu, \quad (2)$$

where  $\psi_N(\cdot, \cdot)$  is a standard production function with constant returns to scale,  $\ell_N(a)$  is the input of labor, and  $d_N(a)$  is the input of the domestically produced intermediate good. The firm's profits are

$$\pi_N(a) = y_N(a) - w\ell_N(a) - d_N(a) - \phi_N, \quad (3)$$

where  $w$  is the wage and  $\phi_N$  is the fixed cost of operating. To maximize profits, the firm chooses  $\ell_N(a)$  and  $d_N(a)$  to solve

$$\nu a^{1-\nu} \psi_N(\ell_N(a), d_N(a))^{\nu-1} \psi_{N\ell}(\ell_N(a), d_N(a)) - w = 0 \quad (4)$$

$$\nu a^{1-\nu} \psi_N(\ell_N(a), d_N(a))^{\nu-1} \psi_{Nd}(\ell_N(a), d_N(a)) - 1 = 0, \quad (5)$$

where  $\psi_{Nk} = \partial \psi_N / \partial k$ ,  $k = \ell, d$ .

Now consider a firm with efficiency  $a$  operating technology  $I$ . The firm's output is given by

$$y_I(a) = (a\eta)^{1-\nu} \psi_I(\ell_I(a), d_I(a), f_I(a))^\nu, \quad (6)$$

where  $\psi_I(\cdot, \cdot, \cdot)$  is a standard production function with constant returns to scale,  $\ell_I(a)$  is the input of labor,  $d_I(a)$  is the input of the domestically produced intermediate good, and  $f_I(a)$  is the input of the imported intermediate good. The firm's profits are

$$\pi_I(a) = y_I(a) - w\ell_I(a) - d_I(a) - (1+\tau)pf_I(a) - \phi_I, \quad (7)$$

where  $p$  is the relative price of the imported good (taken as given by the small open economy),  $\tau$  is the country's *ad valorem* tariff on imports, and  $\phi_I$  is the fixed cost of operating. To maximize profits, the firm chooses  $\ell_I(a)$ ,  $d_I(a)$ , and  $f_I(a)$  to solve

$$\nu(a\eta)^{1-\nu} \psi_I(\ell_I(a), d_I(a), f_I(a))^{\nu-1} \psi_{I\ell}(\ell_I(a), d_I(a), f_I(a)) - w = 0 \quad (8)$$

$$\nu(a\eta)^{1-\nu} \psi_I(\ell_I(a), d_I(a), f_I(a))^{\nu-1} \psi_{Id}(\ell_I(a), d_I(a), f_I(a)) - 1 = 0 \quad (9)$$

$$\nu(a\eta)^{1-\nu} \psi_I(\ell_I(a), d_I(a), f_I(a))^{\nu-1} \psi_{If}(\ell_I(a), d_I(a), f_I(a)) - (1+\tau)p = 0, \quad (10)$$

where  $\psi_{Ik} = \partial \psi_I / \partial k$ ,  $k = \ell, d, f$ .

Given its efficiency draw, each firm decides whether to operate and, if so, which technology to use. Firms' operating decisions are as follows. For a firm with efficiency draw  $a$ , the decision rule for operating technology  $N$  is given by the indicator function

$$t_N(a) = \begin{cases} 1 & \text{if } \pi_N(a) \geq 0 \text{ and } \pi_N(a) > \pi_I(a) \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

and the decision rule for operating technology  $I$  is given by the indicator function

$$t_I(a) = \begin{cases} 1 & \text{if } \pi_I(a) \geq \max[\pi_N(a), 0] \\ 0 & \text{otherwise} \end{cases}. \quad (12)$$

The cost of firm entry is  $\phi_M$  units of output. This entitles the firm to an efficiency draw from probability distribution  $G(\cdot)$ . The expected value of entry must equal the cost of entry, so

$$\int t_N(a)\pi_N(a)dG(a) + \int t_I(a)\pi_I(a)dG(a) = \phi_M. \quad (13)$$

(Though (13) is typically referred to as a free-entry condition, it actually pins down the wage, rather than the measure of entrants, here. The expected value of entry is not decreasing in the measure of entrants, as it would be in a model with monopolistic competition.) This condition ensures that there are no aggregate profits in the economy. Let  $M$  denote the measure of entrants.

### *Market clearing*

Define aggregate use of the domestically produced intermediate good as

$$D = M \left( \int t_N(a)d_N(a)dG(a) + \int t_I(a)d_I(a)dG(a) \right). \quad (14)$$

Define aggregate use of the foreign intermediate good as

$$F = M \int t_I(a)f_I(a)dG(a). \quad (15)$$

Define aggregate output as

$$Y = M \left( \int t_N(a)y_N(a)dG(a) + \int t_I(a)y_I(a)dG(a) \right). \quad (16)$$

International balance of payments requires that

$$X = pF, \quad (17)$$

where  $X$  is aggregate exports. Tariff revenue is rebated to the consumer as a lump sum, so

$$T = \tau pF . \quad (18)$$

Clearing in the labor market requires that

$$M \left( \int l_N(a) \ell_N(a) dG(a) + \int l_I(a) \ell_I(a) dG(a) \right) = L . \quad (19)$$

Finally, clearing in the goods market requires that

$$C + D + X + M \left( \phi_M + \phi_N \int l_N(a) dG(a) + \phi_I \int l_I(a) dG(a) \right) = Y . \quad (20)$$

### *Equilibrium*

Here we define an equilibrium and specify an algorithm for calculating it.

**Definition.** A *competitive small open economy equilibrium* is a list of aggregate measures  $\hat{C}$ ,  $\hat{D}$ ,  $\hat{X}$ ,  $\hat{F}$ ,  $\hat{Y}$ , and  $\hat{M}$ ; a wage  $\hat{w}$ ; a transfer  $\hat{T}$ ; and firm decision rules  $\hat{y}_N(a)$ ,  $\hat{y}_I(a)$ ,  $\hat{\pi}_N(a)$ ,  $\hat{\pi}_I(a)$ ,  $\hat{\ell}_N(a)$ ,  $\hat{\ell}_I(a)$ ,  $\hat{d}_N(a)$ ,  $\hat{d}_I(a)$ ,  $\hat{f}_I(a)$ ,  $\hat{t}_N(a)$ , and  $\hat{t}_I(a)$  such that (1)-(20) hold.

The equilibrium is straightforward to calculate using the following algorithm. Taking  $w$  as given, solve for  $\hat{\ell}_N(a)$  and  $\hat{d}_N(a)$  using (4) and (5), solve for  $\hat{\ell}_I(a)$ ,  $\hat{d}_I(a)$ , and  $\hat{f}_I(a)$  using (8)-(10); calculate  $\hat{y}_N(a)$  and  $\hat{y}_I(a)$  using (2) and (6); calculate  $\hat{\pi}_N(a)$  and  $\hat{\pi}_I(a)$  using (3) and (7); and calculate  $\hat{t}_N(a)$  and  $\hat{t}_I(a)$  using (11) and (12). Solve for  $\hat{w}$  using (13). Solve for  $\hat{M}$  using (19). Calculate  $\hat{D}$ ,  $\hat{F}$ , and  $\hat{Y}$  using (14)-(16). Calculate  $\hat{X}$  and  $\hat{T}$  using (17) and (18). Finally, calculate  $\hat{C}$  using (1). By Walras's Law, (20) holds.

### **Qualitative analysis**

Here we make assumptions regarding functional forms and parameter values so that we can obtain an exact analytic solution for the equilibrium. We consider various

qualitative properties of the model and then analyze the effects of tariff reduction, terms-of-trade improvement, and trade-cost reduction.

*Further assumptions*

First, we choose functional forms for the constant-returns-to-scale components of the production technologies. We let

$$\psi_N(\ell, d) = \ell^\alpha d^{1-\alpha} \quad (21)$$

$$\psi_I(\ell, d, f) = \ell^\alpha \left( \mu d^\rho + (1-\mu) f^\rho \right)^{\frac{1-\alpha}{\rho}}. \quad (22)$$

With these Cobb-Douglas functional forms, the elasticity of substitution between labor and intermediate goods is one for each firm. This is consistent with our data, in the sense that expenditure shares for labor and intermediate goods by Chilean manufacturing plants over the period 2001 to 2006 were roughly constant. We can think of importers as using a composite intermediate good, the quantity of which is given by

$$z_I(a) = \left( \mu d_I(a)^\rho + (1-\mu) f_I(a)^\rho \right)^{1/\rho}. \quad (23)$$

Here the elasticity of substitution between domestic and imported intermediate goods is  $1/(1-\rho)$ . The price of a unit of the composite intermediate good is then

$$P = \left( \mu^{\frac{1}{1-\rho}} + (1-\mu)^{\frac{1}{1-\rho}} \left( (1+\tau)p \right)^{-\frac{\rho}{1-\rho}} \right)^{\frac{1-\rho}{\rho}}. \quad (24)$$

Second, we choose a functional form for the distribution of efficiency draws. We follow Chaney (2008) in letting the distribution be Pareto:

$$G(a) = 1 - (\theta/a)^\gamma, \quad (25)$$

$a \geq \theta$ , where  $\theta > 0$  and  $\gamma > 1$ . The size distribution of firms in the model is proportional to the distribution of efficiency draws. As Figure 1 shows, the size distribution of plants in the data is consistent with a Pareto distribution.

Third, we restrict our attention to the case where not all entering firms choose to operate and not all operating firms choose to import. This leads to a characterization of

firms' operating and technology decisions in terms of cutoff rules. We denote the *cutoff for operating* by  $\underline{a}_N$ , where  $\underline{a}_N$  satisfies

$$\pi_N(\underline{a}_N) = 0. \quad (26)$$

We assume that  $\phi_N$  is sufficiently large that  $\underline{a}_N > \theta$ . We denote the *cutoff for importing* by  $\underline{a}_I$ , where  $\underline{a}_I$  satisfies

$$\pi_I(\underline{a}_I) = \pi_N(\underline{a}_I). \quad (27)$$

We assume that  $\phi_I$  is sufficiently large that  $\underline{a}_I > \underline{a}_N$ . Under these assumptions, there is an exact analytic solution for the equilibrium, which we provide in Appendix 1. In Appendix 2, we provide the solution under the assumption that the economy is in autarky.

#### *Costs and benefits of importing*

Each operating firm weighs the cost of importing against the benefit. In our model, the net cost of importing is fixed at  $\phi_I - \phi_N$  units of output for each firm. In contrast, the total benefit of importing is increasing in the firm's scale of operation. A firm that switches from technology  $N$  to technology  $I$  increases its output, employment, expenditure on intermediate goods, and variable profits (profits gross of fixed costs) by a factor of  $B$ , where

$$B = \eta P^{\frac{(1-\alpha)\nu}{1-\nu}}. \quad (28)$$

(We are assuming that  $B > 1$ .) We refer to  $B$  as the *benefit of importing*. An implication of (28) is that, in the context of this model, it does not matter whether the benefit of importing comes from a lower price or an efficiency advantage; only the combination of these factors matters. Imported intermediates may be more expensive than domestic intermediates ( $P > 1$ ), but there will be a benefit from using them if  $\eta$  is sufficiently high. Alternatively, imported intermediates may have an undesirable effect on firm efficiency ( $\eta < 1$ ), but there will be a benefit from using them if  $P$  is sufficiently low. The findings of Kugler and Verhoogen (2009) suggest that the first case is more consistent with the data than the second.

### *Sources of differences between importers and non-importers*

In the data, importers are very different from non-importers. In the model, there are two causes of this: a selection effect and a technology upgrading effect. Because of the fixed cost, only the firms with the highest efficiency draws choose to become importers. This is the *selection effect*. If a firm chooses to use technology  $I$ , then its output, employment, expenditure on intermediate goods, and variable profits are larger by a factor of  $B$  than if the firm had chosen to use technology  $N$ . This is the *technology upgrading effect*.

We can measure the contribution of each effect to the relative size of importers (the measure of size can be gross output, employment, total expenditure on intermediate goods, or variable profits). In the absence of any selection effect, technology upgrading would result in importers being  $B$  times larger than non-importers. The selection effect determines the extent to which the size ratio is greater than  $B$ . Let  $S$  be size of the average importer relative to the size of the average non-importer. Using a logarithmic decomposition to account for this ratio, the share due to the technology upgrading effect is given by  $\log B / \log S$ ; the remaining share is due to the selection effect. Later, our calibration procedure will pin down the magnitudes of these two effects.

### *Aggregate technological efficiency*

Along with the benefit of importing, an important statistic in the model — it shows up in the calculation of every equilibrium object — is

$$A = \frac{1}{\gamma\phi_M} \left( \int_{\underline{a}_N}^{\underline{a}_I} adG(a) + B \int_{\underline{a}_I}^{\infty} adG(a) \right). \quad (29)$$

We refer to  $A$  as an *aggregate technological efficiency index* because the expression in parentheses is a weighted average of firms' efficiency draws. The relative weight on importers,  $B$ , accounts for the benefit of using technology  $I$ . Implicitly, the weight on firms that choose not to operate is zero. Since the cutoffs  $\underline{a}_N$  and  $\underline{a}_I$  are equilibrium

objects, we can simplify (29) to express aggregate technological efficiency entirely in terms of parameters:

$$A = \theta \left( \frac{\phi_N^{1-\gamma} + (B-1)^\gamma (\phi_I - \phi_N)^{1-\gamma}}{(\gamma-1)\phi_M} \right)^{1/\gamma}. \quad (30)$$

Aggregate technological efficiency affects many important aspects of the economy. For example, aggregate output can be expressed as

$$Y = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL}{\alpha\nu}, \quad (31)$$

where  $K$  is given in Appendix 1. The wage and the measure of entrants are also proportional to  $A^{(1-\nu)/\alpha\nu}$ , as is social welfare if  $\tau = 0$ . In addition, the cutoffs for operating and importing are proportional to  $A$ :

$$\underline{a}_N = A\phi_N \quad (32)$$

$$\underline{a}_I = \frac{A(\phi_I - \phi_N)}{B-1}. \quad (33)$$

Notice that the cutoff for importing is increasing in the cost of importing and decreasing in the benefit of importing.

#### *Effects of tariff reduction, terms-of-trade improvement, and trade-cost reduction*

In our experiments, we consider the effects of trade liberalization, as given by a decrease in  $\tau$ ; an exogenous improvement in the terms of trade, as given by a decrease in  $p$ ; and a reduction in trade costs, as given by a decrease in  $\phi_I$ . Since our model is static, we view it as capturing the long-term effects of permanent changes. These three types of changes have many qualitative effects in common.

**Proposition.** Tariff reduction, an improvement in the terms of trade, or a decrease in the cost of importing has the following effects: (i) the cutoff for operating increases, (ii) the cutoff for importing decreases, (iii) the (real) wage increases, (iv) output increases, (v) firm entry increases, and (vi) social welfare increases.

Proving the proposition just involves finding the signs of various partial derivatives using Appendix 1, so we omit it here. A decrease in either  $\tau$  or  $p$  decreases the price of the composite intermediate input,  $P$ , which increases the benefit of importing,  $B$ , which leads to an increase in aggregate technological efficiency,  $A$  (see (24), (28), and (30)). A decrease in  $\phi_I$  does not change  $P$  or  $B$  but, rather, directly increases  $A$ .

As the proposition indicates, all the changes result in a reallocation of resources across firms. The least efficient firms exit because they can no longer profitably operate at the higher wage, the most efficient non-importers become importers, and technological efficiency increases. With a decrease in  $\tau$  or  $p$ , the effects of reallocation are augmented because, for a given efficiency draw, there is an increase in the optimal scale at which technology  $I$  is operated and a decrease in the optimal scale at which technology  $N$  is operated. By contrast, with a decrease in  $\phi_I$ , both technologies are operated at smaller scales due to the increase in the wage.

#### *Decomposing changes in trade volume*

Yi (2003) and Kehoe (2005) stress the need to develop trade models that can generate large increases in trade in response to small decreases in tariffs. Our model has this potential, as it has both extensive and intensive margins of importing. The change in total imports resulting from a change in  $\tau$ ,  $p$ , or  $\phi_I$  can be decomposed into changes on three margins: (i) the measure of entrants, (ii) the cutoff for importing, and (iii) use of the imported good by existing operators of technology  $I$ . Specifically,

$$d \log F = d \log M + (1 - \gamma) d \log \underline{a}_I + d \log Q, \quad (34)$$

where

$$Q = f_I(a) / a. \quad (35)$$

(As Appendix 1 shows,  $f_I(a)$  is proportional to  $a$ , so  $Q$  does not depend on  $a$ .) The percentage change in existing importers' use of the imported intermediate good is equal

to the percentage change in  $Q$ . Thus the first two margins are extensive, while the third is intensive. Importantly, with the presence of extensive margins, the Armington elasticity — the elasticity of substitution between domestic and imported goods — is greater at the macro level than at the micro level. As a result, the model has the potential to generate a large Armington elasticity at the macro level without assuming an unusually large elasticity at the micro level.

### **Quantitative analysis**

Here we calibrate the model using the data on the Chilean manufacturing sector discussed earlier. Then we use the calibrated model to perform a number of counterfactual numerical experiments.

#### *Calibration*

Our strategy for calibrating the model is as follows. First, we normalize certain parameters that do not affect the quantitative findings in which we are interested. Then we take some parameter values from the literature. Finally, we select the remaining parameter values to match important statistics on plants' importing behavior.

As normalizations, we set the labor endowment,  $L$ ; the lower bound on the Pareto distribution,  $\theta$ ; and the cost of entry,  $\phi_M$ , to one. As (28) indicates, it does not matter whether the benefit of importing comes from a lower price or an efficiency advantage. Consequently, we normalize the price of the composite intermediate good,  $P$ , to one and initially allow the size of the benefit of importing to be determined by the parameter  $\eta$ . To obtain  $P = 1$ , we set the tariff rate,  $\tau$ , to be consistent with the data and then choose the relative price of the imported intermediate good,  $p$ . The World Bank's World dataBank reports that Chile's average tariff rate in 2001 was 8 percent, so we set  $\tau = 0.08$ . Then we set  $p = 0.11$  to obtain  $P = 1$ .

We take the values of  $\rho$  and  $\nu$  from the literature. There is debate over the elasticity of substitution between domestic and imported goods, the Armington elasticity. Ruhl (2004) tries to resolve this debate and argues in favor of an elasticity of two at the

micro level (measurements at the macro level differ when an extensive margin is involved). Following this, we set  $\rho = 0.5$  so that an importer's elasticity of substitution between domestic and foreign inputs is two. The parameter  $\nu$  determines the degree of decreasing returns at the firm level. Calibrating a competitive model with heterogeneous firms operating decreasing-returns-to-scale technologies, Atkeson and Kehoe (2005) find that  $\nu = 0.85$  is consistent with data on U.S. manufacturing plants; we adopt this value here.

The values of  $\alpha$  and  $\mu$  are selected to match expenditure shares in the data. In the data, expenditure on labor as a share of expenditure on both labor and intermediate goods is 0.34, so we set  $\alpha = 0.34$ . Among plants that import, expenditure on imported intermediate goods as a share of total expenditure on intermediate goods is 0.39; we set  $\mu = 0.78$  to match this.

The remaining four parameters are the fixed cost of operating technology  $N$ ,  $\phi_N$ ; the fixed cost of operating technology  $I$ ,  $\phi_I$ ; the TFP of technology  $I$  relative to technology  $N$ ,  $\eta$ ; and the shape parameter of the Pareto distribution,  $\gamma$ . We jointly select the values of these four parameters so that the following four statistics hold in the model: (i) 13 percent of operating plants use imported intermediate goods, (ii) the average gross output of importers relative to non-importers is 4.5, (iii) the coefficient of variation for gross output is 6.0, and (iv) 90 percent of entrants choose to operate.<sup>6</sup> The resulting parameter values are  $\phi_N = 1.05$ ,  $\phi_I = 1.66$ ,  $\eta = 1.21$ , and  $\gamma = 2.02$ . To place these numbers in context, consider the following. The fixed cost of operating technology  $N$  is 10 percent of the average non-importer's gross output and 68 percent of its variable profits. The fixed cost of operating technology  $I$  is 4 percent of the average importer's gross output and 24 percent of its variable profits. Since  $B = 1.21$ , switching from technology  $N$  to technology  $I$  increases a firm's gross output, employment, expenditure on intermediate goods, and variable profits by 21 percent. Our value for the shape

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<sup>6</sup> The last statistic is not based on data (we do not observe plants that do not operate), but the quantitative results in which we are interested are not sensitive to the particular percentage chosen. We simply need the cutoff for operating to be binding.

parameter of the Pareto distribution is consistent with the findings of Del Gatto, Mion, and Ottaviano (2007). Table 1 summarizes the calibration.

The calibrated model allows us to account for the average importer being 4.5 times larger, in terms of gross output, than the average non-importer. Using the decomposition discussed in the previous section, with  $B = 1.21$  the share due to the selection effect is 87.4 percent and the share due to the technology upgrading effect is 12.6 percent.

The calibration guarantees that the model satisfies Facts 1 to 3. We did not use Fact 4 in the calibration, but the calibrated model is not far off. In the data, importers are 1.3 times more productive than non-importers, as measured by value added per worker. In the calibrated model, importers are still substantially more productive than non-importers, by a factor of 1.2. It is worth pointing out that the ratio would be one if we did not take into account fixed costs of operation (as noted by Atkeson and Kehoe (2005)). But our interpretation of the fixed costs is that they are expensed costs of setting up business relationships with input suppliers, so they must be subtracted from gross output to obtain value added. Though  $\phi_I > \phi_N$ , expenditure on fixed costs relative to average output is smaller for importers than non-importers. Consequently, measured value added per worker is higher among importers than among non-importers. Table 2 provides a comparison between the facts in the data and in the model.

### *Experiments*

We use the calibrated model to perform four numerical experiments: (i) elimination of the *ad valorem* tariff, (ii) an exogenous improvement in the terms of trade, (iii) a reduction in fixed costs, and (iv) going from autarky to free trade.

We set up the first two experiments to be of the same magnitude. We consider (i) the elimination of the 8 percent tariff in the benchmark calibration and (ii) an equivalent reduction in the terms of trade (a 7.4 percent decrease in  $p$ ). Table 3 presents the percentage changes in a number of statistics of interest. These two experiments have almost identical quantitative results. The only differences are with respect to the changes

in exports, consumption, and tariff revenue. The main difference between the two experiments is that the tariff reduction results in the loss of all tariff revenue, while the terms-of-trade improvement increases tariff revenue. Our index of social welfare is the consumption level. Because of the changes in tariff revenue, the welfare gain from tariff elimination is much smaller than the welfare gain from the terms-of-trade improvement. The terms-of-trade improvement allows the economy to import just as much as after the tariff elimination, but without exporting as much output.

Both experiments result in a large increase in trade: the quantity of imports increases by 98.8 percent. Using the decomposition given by (34), we find that 4.7 percent of the increase is due to the change in the measure of entrants, 69.7 percent is due to the change in the cutoff for importing, and 25.6 percent is due to the change in existing importers' use of the imported intermediate good. Thus the extensive margins account for the majority of the increase. In the benchmark calibration, of the firms that enter, 10 percent choose not to operate, 78 percent choose to operate technology  $N$ , and 12 percent choose to operate technology  $I$ . Following the change, entry increases by 3.3 percent and, of the firms that enter, 21 percent choose not to operate, 49 percent choose to operate technology  $N$ , and 30 percent choose to operate technology  $I$  (see Table 4).

When there are extensive margins, the Armington elasticity at the macro level is greater than the Armington elasticity at the micro level. Here the Armington elasticity at the level of an individual firm is 2, while the measured aggregate Armington elasticity is 10.5. This large aggregate Armington elasticity is consistent with the empirical findings of researchers who estimate it using data from trade liberalizations. As Ruhl (2004) notes, these researchers typically find Armington elasticities ranging from 4 to 15. This experiment makes clear that the extensive margins play an important role in generating large increases in trade from small decreases in tariffs.

In addition to reporting how the changes affect our theoretical measure of technological efficiency, we also report how they affect measured real GDP. To calculate real GDP in our model in a manner similar to the way real GDP is calculated in

the data, we use base-period prices.<sup>7</sup> If we let  $t$  denote the current period, then real GDP at period-0 prices is

$$RGDP_t = Y_t - D_t - (1 + \tau_0)p_0F_t - \phi_N M_{Nt} - \phi_I M_{It} + \tau_0 p_0 F_t, \quad (36)$$

where  $M_N$  is the measure of non-importers and  $M_I$  is the measure of importers. The first term is gross output, the next four terms are expenditure on intermediate goods and expensed costs of operation, and the last term is the rebate of tariff revenue, all valued at period-0 prices. We assume that investment in new firms is a tangible investment rather than an intermediate input, so these expenditures are not subtracted from gross output. The experiments result in a 1.5 percent increase in real GDP. In contrast to Kehoe and Ruhl (2008), who find that terms of trade shocks have no first-order effect on real GDP in a small open economy model with a representative firm, we obtain a substantial increase in GDP by modeling heterogeneous firms and an extensive margin of importing.

The first two experiments involved increasing the benefit of importing. The third experiment involves decreasing the fixed cost of importing,  $\phi_I$ . To make this experiment somewhat comparable in magnitude to the first two experiments, we select the percentage decrease in  $\phi_I$  so that the share of entrants that choose to operate technology  $I$  is the same as after the previous two experiments (30 percent). This requires a 14.2 percent decrease in  $\phi_I$ . The results are presented in Table 3. Since  $B$  does not change in this case, we can isolate the basic effects of reallocation. The increase in aggregate technological efficiency is not as large as in the previous two experiments, so the changes are mostly smaller in magnitude.

Finally, we consider the extreme case of going from autarky to free trade. In the free trade case, we take the relative price of the imported good from the benchmark calibration. This experiment puts an upper bound on the effects of tariff reduction alone. The welfare gain is 5.2 percent. Table 5 presents the changes in some statistics of interest.

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<sup>7</sup> See Gibson (2010), Kehoe and Ruhl (2008), and Bajona, Gibson, Kehoe, and Ruhl (2010) for further discussion of measured productivity and GDP in trade models.

## **Conclusion**

Most trade models with heterogeneous firms emphasize firms' export decisions. We have developed a complementary approach that emphasizes firms' import decisions. Our model has desirable qualitative properties, is straightforward to calibrate, and quantitatively captures many important features of plants' importing behavior in the data. In terms of policy implications, the welfare gains from unilateral trade liberalization may not be large enough to persuade policymakers in countries that depend on tariffs for revenue. Individuals associated with small non-importing firms are also unlikely to support liberalization. Improvements in the terms of trade generate large welfare gains, but are external. The simple modeling framework developed here allows for many interesting extensions. These include the roles of dynamics, uncertainty, multiple countries, monopolistic competition, and joint import-export decisions.

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**Table 1. Summary of the calibration**

<b>Parameters</b>	<b>Values</b>	<b>Explanations</b>
$L$	1	Normalization
$\theta$	1	Normalization
$\phi_M$	1	Normalization
$p$	0.11	Chosen so that $P = 1$
$\rho$	0.5	Ruhl (2004)
$\nu$	0.85	Atkeson and Kehoe (2005)
$\tau$	0.08	World Bank's World dataBank
$\alpha$	0.34	Expenditure on labor as a share of total expenditure on labor and intermediate goods
$\mu$	0.78	Importers' expenditure on imported intermediate goods as a share of total expenditure on intermediate goods: 0.39
$\phi_N, \phi_I, \eta, \gamma$	1.05, 1.66, 1.21, 2.02	Jointly chosen to match 4 statistics: (i) importers are 13 percent of operating plants, (ii) gross output of average importer relative to gross output of average non-importer is 4.5, (iii) coefficient of variation for gross output is 6.0, and (iv) 90 percent of entrants choose to operate

**Table 2. Data vs. model**

<b>Statistic</b>	<b>Data</b>	<b>Model</b>
Importers as a share of operating plants (%)	13	13
Average importer's expenditure on imported intermediate goods as a share of total expenditure on intermediate goods (%)	39	39
Gross output of average importer relative to average non-importer	4.5	4.5
Value added per worker of importers relative to non-importers	1.3	1.2

**Table 3. Three experiments**

<b>Statistic</b>	<b>Tariff elimination (% change)</b>	<b>Terms-of-trade improvement (% change)</b>	<b>Fixed-cost reduction (% change)</b>
Welfare	1.0	5.1	2.4
Wage	3.3	3.3	1.1
Measure of entrants	3.3	3.3	1.1
Output	3.3	3.3	1.1
Tariff revenue	-100.0	84.0	59.7
Real GDP	1.5	1.5	2.2
Price of composite intermediate	-3.0	-3.0	0.0
Imports	98.8	98.8	59.7
Technological efficiency	6.4	6.4	2.2
Benefit of importing	12.2	12.2	0.0

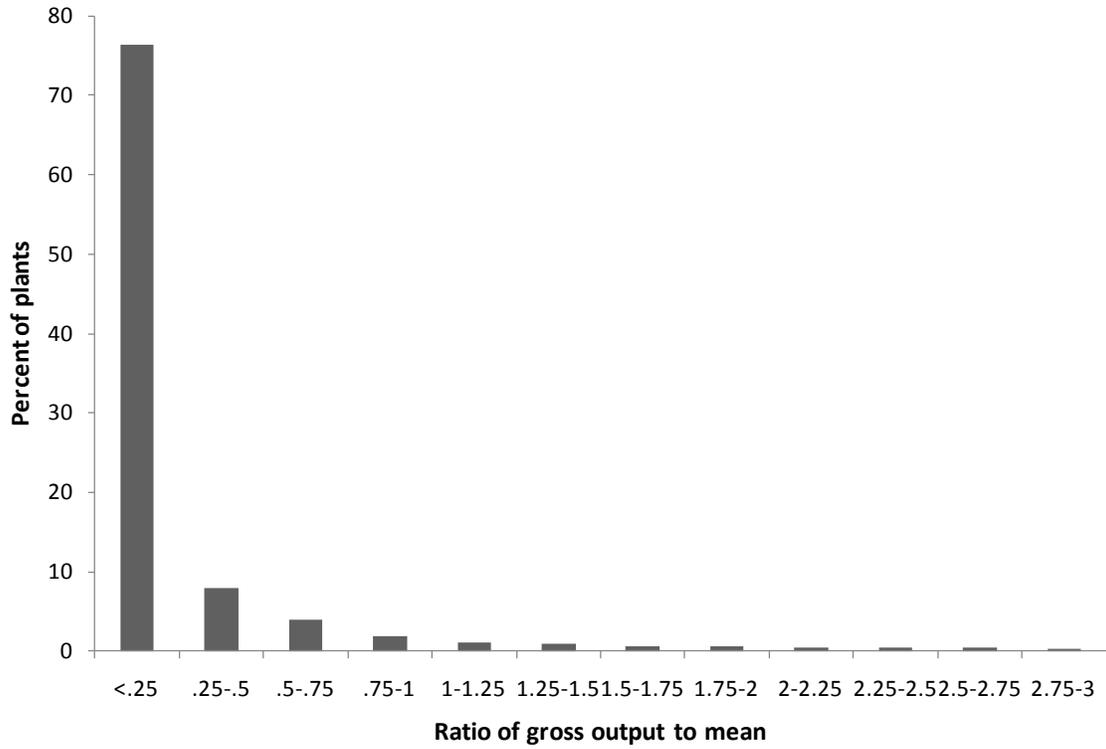
**Table 4. Entrants' operating decisions (percentage shares)**

<b>Decision</b>	<b>Benchmark</b>	<b>Tariff elimination</b>	<b>Terms-of- trade improvement</b>	<b>Trade-cost reduction</b>	<b>Autarky</b>
Not operate	10	21	21	14	3
Operate technology <i>N</i>	78	49	49	56	97
Operate technology <i>I</i>	12	30	30	30	0

**Table 5. Experiment: Going from autarky to free trade**

<b>Statistic</b>	<b>Percentage change</b>
Welfare	5.2
Wage	5.2
Measure of entrants	5.2
Output	5.2
Technological efficiency	10.3

**Figure 1. Firm size distribution**



## Appendix 1. Analytic solution with trade

Let

$$P = \left( \mu^{\frac{1}{1-\rho}} + (1-\mu)^{\frac{1}{1-\rho}} \left( (1+\tau)p \right)^{-\frac{\rho}{1-\rho}} \right)^{\frac{1-\rho}{\rho}} \quad (37)$$

$$B = \eta P^{\frac{(1-\alpha)\nu}{1-\nu}} \quad (38)$$

$$A = \theta \left( \frac{\phi_N^{1-\gamma} + (B-1)^\gamma (\phi_I - \phi_N)^{1-\gamma}}{(\gamma-1)\phi_M} \right)^{\frac{1}{\gamma}} \quad (39)$$

$$K = \nu^{\frac{1}{\alpha}} (1-\nu)^{\frac{1-\nu}{\alpha\nu}} \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} . \quad (40)$$

Then

$$d_N(a) = \frac{a\nu(1-\alpha)}{A(1-\nu)} \quad (41)$$

$$\ell_N(a) = \frac{a\alpha\nu}{A^{\frac{1-\nu+\alpha\nu}{\alpha\nu}} K(1-\nu)} \quad (42)$$

$$y_N(a) = \frac{a}{A(1-\nu)} \quad (43)$$

$$\pi_N(a) = \frac{a}{A} - \phi_N \quad (44)$$

$$d_I(a) = \frac{aB\nu(1-\alpha)\mu^{\frac{1}{1-\rho}} P^{\frac{\rho}{1-\rho}}}{A(1-\nu)} \quad (45)$$

$$\ell_I(a) = \frac{aB\alpha\nu}{A^{\frac{1-\nu+\alpha\nu}{\alpha\nu}} K(1-\nu)} \quad (46)$$

$$f_I(a) = \frac{aB\nu(1-\alpha)(1-\mu)^{\frac{1}{1-\rho}} \left( (1+\tau)p \right)^{-\frac{1}{1-\rho}} P^{\frac{\rho}{1-\rho}}}{A(1-\nu)} \quad (47)$$

$$y_I(a) = \frac{aB}{A(1-\nu)} \quad (48)$$

$$\pi_I(a) = \frac{aB}{A} - \phi_I \quad (49)$$

$$w = A^{\frac{1-\nu}{\alpha\nu}} K \quad (50)$$

$$\underline{a}_N = A\phi_N \quad (51)$$

$$\underline{a}_I = \frac{A(\phi_I - \phi_N)}{B-1} \quad (52)$$

$$M = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL(1-\nu)}{\alpha\nu\gamma\phi_M} \quad (53)$$

$$Y = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL}{\alpha\nu} \quad (54)$$

$$D = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL(1-\alpha) \left( \phi_N^{1-\gamma} + \left( \mu^{\frac{1}{1-\rho}} P^{\frac{\rho}{1-\rho}} B - 1 \right) (B-1)^{\gamma-1} (\phi_I - \phi_N)^{1-\gamma} \right)}{\alpha \left( \phi_N^{1-\gamma} + (B-1)^\gamma (\phi_I - \phi_N)^{1-\gamma} \right)} \quad (55)$$

$$F = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL(1-\alpha)(1-\mu)^{\frac{1}{1-\rho}} \left( (1+\tau)p \right)^{-\frac{1}{1-\rho}} P^{\frac{\rho}{1-\rho}} B(B-1)^{\gamma-1} (\phi_I - \phi_N)^{1-\gamma}}{\alpha \left( \phi_N^{1-\gamma} + (B-1)^\gamma (\phi_I - \phi_N)^{1-\gamma} \right)} \quad (56)$$

$$X = \frac{A^{\frac{1-\nu}{\alpha\nu}} KLp(1-\alpha)(1-\mu)^{\frac{1}{1-\rho}} \left( (1+\tau)p \right)^{-\frac{1}{1-\rho}} P^{\frac{\rho}{1-\rho}} B(B-1)^{\gamma-1} (\phi_I - \phi_N)^{1-\gamma}}{\alpha \left( \phi_N^{1-\gamma} + (B-1)^\gamma (\phi_I - \phi_N)^{1-\gamma} \right)} \quad (57)$$

$$T = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL\tau p(1-\alpha)(1-\mu)^{\frac{1}{1-\rho}} \left( (1+\tau)p \right)^{-\frac{1}{1-\rho}} P^{\frac{\rho}{1-\rho}} B(B-1)^{\gamma-1} (\phi_I - \phi_N)^{1-\gamma}}{\alpha \left( \phi_N^{1-\gamma} + (B-1)^\gamma (\phi_I - \phi_N)^{1-\gamma} \right)} \quad (58)$$

$$C = A^{\frac{1-\nu}{\alpha\nu}} KL \left( 1 + \frac{\tau p(1-\alpha)(1-\mu)^{\frac{1}{1-\rho}} \left( (1+\tau)p \right)^{-\frac{1}{1-\rho}} P^{\frac{\rho}{1-\rho}} B(B-1)^{\gamma-1} (\phi_I - \phi_N)^{1-\gamma}}{\alpha \left( \phi_N^{1-\gamma} + (B-1)^\gamma (\phi_I - \phi_N)^{1-\gamma} \right)} \right). \quad (59)$$

## Appendix 2. Analytic solution under autarky

Let

$$A = \theta \left( \frac{\phi_N^{1-\gamma}}{(\gamma-1)\phi_M} \right)^{\frac{1}{\gamma}} \quad (60)$$

$$K = \nu^{\frac{1}{\alpha}} (1-\nu)^{\frac{1-\nu}{\alpha\nu}} \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}}. \quad (61)$$

Then

$$d_N(a) = \frac{a\nu(1-\alpha)}{A(1-\nu)} \quad (62)$$

$$\ell_N(a) = \frac{a\alpha\nu}{A^{\frac{1-\nu}{1-\nu+\alpha\nu}} K(1-\nu)} \quad (63)$$

$$y_N(a) = \frac{a}{A(1-\nu)} \quad (64)$$

$$\pi_N(a) = \frac{a}{A} - \phi_N \quad (65)$$

$$w = A^{\frac{1-\nu}{\alpha\nu}} K \quad (66)$$

$$\underline{a}_N = A\phi_N \quad (67)$$

$$M = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL(1-\nu)}{\alpha\nu\gamma\phi_M} \quad (68)$$

$$Y = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL}{\alpha\nu} \quad (69)$$

$$D = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL(1-\alpha)}{\alpha} \quad (70)$$

$$C = A^{\frac{1-\nu}{\alpha\nu}} KL. \quad (71)$$