The product life cycle of durable goods

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Abstract

The model presented here derives the product life cycle of durable goods. It is based on the idea that the purchase process consists of first purchase and repurchase. First purchase is determined by the market penetration process (diffusion process), while repurchase is the sum of replacement and multiple purchase. The key property of durables goods is to have a mean lifetime in the order of several years. Therefore replacement purchase creates periodic variations of the unit sales (Juglar cycles) having its origin in the initial diffusion process.
The theory suggests that there exists two diffusion processes. The first can be described by Bass diffusion and is related to the information spreading process within the social network of potential consumers. The other diffusion process comes into play, when the price of the durable is such, that only those consumers with a sufficient personal income can afford the good. We have to distinguish between a monopoly market and a polypoly/oligopoly market. In the first case periodic variations of the total sales occur caused by the initial Bass diffusion, even when the price is constant. In the latter case the mutual competition between the brands leads with time to a decrease of the mean price. This change is associated with an effective increase of the market volume, which can be interpreted as a diffusion process. Based on an evolutionary approach, it can be shown that the mean price decreases exponentially and the corresponding diffusion process is governed by Gompertz equation (Gompertz diffusion).
Most remarkable is that Gibrat's rule of proportionate growth is a direct consequence of the competition between the brands. The model allows a derivation of the lognormal size distribution of product sales and the logistic replacement of durables in competition. A comparison with empirical data suggests that the theory describes the main trend of the product life cycle superimposed by short term events like the introduction of new models.

Keywords: Product Life Cycle, Consumer Durables, Product Diffusion, Bass Diffusion, Competition, Gompertz Diffusion, Replicator Equation, Logistic Growth, Evolutionary Economics, Monopoly, Takeoff, Gibrat's Rule, Juglar Cycles,

1. Introduction

In marketing research increasingly complex models based on Bass diffusion were developed in order to explain the market penetration of goods with economic decision variables [1-4]. The presented model focuses not only on the diffusion process but on the entire sales evolution of consumer durable goods, the so called product life cycle (PLC) [5, 6].

The idea to describe the PLC is, to consider the sales evolution as consisting of a first and a repurchase process, while the repurchase is the sum of multiple and replacement purchase. As shown in the model replacement purchase causes periodic variations of the sales. These variations are coined by the initial diffusion process and occur with a periodicity given by the mean product lifetime of the consumer durable.

However, as suggested in a previous paper the market penetration of a durable good may consist of two diffusion processes, a spreading process governed by Bass diffusion, and an evolutionary Gompertz diffusion regime [7]. The first has its origin in an information spreading process within the social network. The latter is caused by an expansion of the market volume due to a decrease of the mean price. Both diffusion processes, together with the corresponding repurchase processes, form the total PLC derived in this paper.

Whether Gompertz diffusion comes into play depends on two conditions. Gompertz diffusion can be neglected, when:

1. The introduction price of the durable is less than the price even adopters with the lowest income are willing to pay.
2. When a brand covers a market segment, and the consumers repurchase a single brand, i.e. the brand can be treated as a de facto monopoly.

In these cases the PLC is determined completely by Bass diffusion and in difference to Gompertz diffusion the price may even increase. Applying statistical methods the model also allows the derivation of two stylized facts, the mutual logistic replacement of brands and the lognormal size distribution of the brands in terms of unit sales.

The paper is organized as follows. The next section is devoted to a presentation of the model, while key assumptions are numbered by lower case roman letters. In order to show the applicability of the model a comparison with empirical investigations is performed in the third section of the paper, while competitive and monopoly markets are considered, followed finally by a conclusion.

2. The Model

2.1. The Durable Market

First, we want to make some key assumption about the (static) durable market.

The demand side

The demand side can be characterized as an ensemble of agents who are interested in purchasing the consumer durable. The total number of agents is denoted as the market potential *M*, which is considered for simplicity as time independent. The market potential determines the number of all potential adopters. However for a given nominal price, *p* the number of potential adopters is limited to those consumers, who can afford the product. The corresponding number of potential adopters is denoted as the market volume, $V(p) \leq M$.

In order to estimate $V(p)$ we assume:

i) The purchase of a durable good is determined by the personal income and not by the total income of a household. In other words, the decision of a household to purchase a consumer durable is assumed to be governed by the main source of income. The market volume is therefore restricted to those agents, who have sufficient personal annual income, *h*, to afford the good.

It is an empirical fact that the annual personal income distribution, $P_I(h)$ exhibits a two-class structure [8]. The upper class can be described by a Pareto power-law distribution. The majority of the population, however, belongs to the lower class. For consumer durables we further assume:

ii) The upper class can always afford the good and is not limited by the product price.

The contribution of the upper class to the market volume is denoted M_U. Also not limited by the price are firms. Therefore, implicitly assumed is that M_U also includes industrial agents. Luxury goods can be considered as goods where assumption ii) is not fulfilled. They are not considered here.

As to evaluate $V(p)$, we take advantage form the personal income distribution of the lower class. The cumulated income distribution can be described by an exponential Boltzmann-Gibbs distribution, a lognormal distribution or a Γ-distribution (except for zero income) with an appropriate choice of the free parameters [9-11]. For mathematical simplicity we approximate the income distribution of the lower class by a Boltzmann-Gibbs distribution. In this case the relative abundance to find a representative agent having an annual income between *h* and *h+dh*, can be given by the probability density function (pdf):

$$P_I(h) = \frac{1}{I}\exp(-h/I)$$

(1)

where the average personal income can be obtained from:

$$I = \int_0^\infty dh P_I(h) h$$
(2)

In order to exclude inflation effects the model is formulated in terms of real prices determined by:

$$\mu = \frac{p}{I}$$
(3)

where I is the mean income of the lower class Eq.(2). The last assumption to determine the market volume is:

iii) The chance to find an agent of the lower class who is willing to purchase the good has a maximum at a minimum price, μ_m.

In other words, there exists a minimum mean price μ_m, at which all potential adopters purchase the good. We want to denote μ_m as the natural price. This assumption implies that for an introduction price $\mu \leq \mu_m$, the market volume must be equal to the market potential:

$$V(\mu_m) = M$$
(4)

For $\mu > \mu_m$, the market volume $V(\mu)$ is the sum of M_U and a price dependent part from the lower class. This can be written as:

$$V(\mu) = M_U + M_L \int_{z(\mu)}^{\infty} P_I(z')dz'$$
(5)

where $z'=h/I$. The integral determines the probability to find an agent with sufficient income, while the unknown function $z(\mu)$ specifies how this probability varies as a function of the price. M_L is the total amount of potential adopters from the lower part of the personal income distribution determined by:

$$M_L = M - M_U$$
(6)

The function $z(\mu)$ can be derived from two conditions.

1. The cumulative income distribution over $P_I(z')$ is normalized to one for the lower class. Hence, the function $z(\mu)$ must be zero at μ_m, in order to fulfil Eq.(4). The function $z(\mu)$ can therefore be given by a Taylor expansion close to μ_m. We obtain up to the second order:

$$z(\mu) \cong z_1(\mu - \mu_m) + z_2(\mu - \mu_m)^2$$
(7)

with the coefficients $z_1, z_2 \geq 0$.

2. The assumption iii) suggests that the chance to find a potential consumer as a function of the price has a maximum at $\mu=\mu_m$. Hence, $dV(\mu_m)/d\mu=0$. This condition implies that $z(\mu)$ must have a minimum at μ_m, and therefore, $z_1=0$ in Eq.(7).

The market volume can therefore be approximated for $\mu>\mu_m$ by:

$$V(\mu) \cong M_L \exp\left(-\frac{(\mu-\mu_m)^2}{2\Theta^2}\right)+M_U$$

(8)

where $\Theta^2=1/(2z_2)$.

We want to formulate a continuous model. Therefore the market volume is scaled by the market potential introducing the density:

$$v(\mu)=\frac{V(\mu)}{M}$$

(9)

while $0\leq v(\mu)\leq 1$. Because M is a large figure, densities are treated in this model as continuous variables.

The supply side

The good is produced and distributed by a number of manufacturers characterizing the supply side of the market. The consumers may choose between a number of different variants of the durable good, denoted here as brands (models) having similar utility properties. They are assumed to be manufactured by business units. Note that we do not consider firms, because they have usually a number of business units. We want to indicate the brands (and the corresponding business units) with index i, while N is the total number of models. The absolute number of products of the *i-th* brand sold per unit time is denoted Y_i, while S_i indicates the number of supplied products. The corresponding purchase and supply flow densities are determined by:

$$y_i=\frac{Y_i}{M};s_i=\frac{S_i}{M}$$

(10)

The total unit sales and supply flow can be obtained from:

$$y_t=\sum_{i=1}^{N} y_i;s_t=\sum_{i=1}^{N} s_i$$

(11)

The real price of the *i-th* brand is denoted μ_i. We want to assume that the price decision of the business units is influenced by their competitors and therefore located around the mean price. In other words we assume:

iv) The frequency distribution of the price P_μ is close to the mean price, and the variance $Var(P_\mu)$ is small and considered nearly constant.

The mean price is determined by the average over the sold products:

$$\langle \mu(t) \rangle = \frac{1}{y_t(t)} \sum_{i=1}^{N} y_i(t)\mu_i(t)$$
(12)

while the brackets indicate the average over the unit sales. The variance is defined as:

$$Var(P_\mu) = \left\langle \left(\mu_i - \langle \mu \rangle\right)^2 \right\rangle$$
(13)

2.2. The Market Dynamics

The market dynamics is determined by the interaction between the supply and demand sides of the durable market. We want to establish relations that determine the market dynamics in order to derive the product life cycle of a consumer durable in terms of the total unit sales, $y_t(t)$. As a first step we specify the key processes that determine the dynamics on both sides of the durable market.

The supply side

The main process on the supply side is the production and distribution of the good by the business units. Business units can be considered as input-output systems, where the input is the financial flow (revenue) and the output is the product supply. The supply flow can therefore be viewed as a function of the unit sales $s_i=s_i(y_i)$. Expanding the supply of the *i-th* brand up to the first order we obtain:

$$s_i(y_i, t+\delta t) \cong c_1 + c_2 y_i(t)$$
(14)

where δt is a short time increment indicating the response time on an input variation. In order to keep the model simple this time shift is neglected, that means:

v) We assume a fast response of the business units on sales variations, $\delta t=0$.

If there is no input flow (proportional to the unit sales y_i) the output flow s_i will be zero, hence $c_1=0$. In order to minimize the costs per unit, it can be expected that the number of supplied products will be nearly equal to the number of sold products, $c_2 \approx 1$. Therefore we write $c_2=1+\gamma_i(t)$, where the productivity $\gamma(t)$ is denoted as reproduction coefficient, while usually $\gamma_i(t)<1$. We obtain for the supply flow:

$s_i(t) = [1+\gamma_i(t)]y_i(t)$
(15)

This relation specifies the supply side completely, while the dynamics of the individual business units is contained in the reproduction coefficient. The total supply obeys the relation: $s_t=(1+<\gamma(t)>)y_t$. The fast response assumption relates the supply side directly to the demand side dynamics, such that total supply flow follows immediately total demand, while (small) deviations are contained in $<\gamma(t)>$.

The demand side

The demand side dynamics is mainly governed by two processes, the first- and the repurchase process. These processes can be described by the total density of potential consumers:

$\psi(t) = \psi_f(t) + \psi_r(t)$
(16)

where ψ_f indicates the number of potential first purchase consumers (potential adopters) scaled by the market potential. The density of potential consumers caused by a repurchase of the good is denoted ψ_r.

How are potential consumer related to the sales? The key idea to answer this question is to consider the purchase process as consisting of statistical events, where potential consumers meet available products of the *i-th* brand (e.g. in stores, the internet etc.) and purchase it with a certain probability. The sales function y_i must be zero if there are either no potential consumers ψ or available products x_i. The sales of the brand y_i must therefore be up to the first order proportional to the product of both densities, potential consumers ψ and available products x_i. Hence purchase events occur with a frequency:

$y_i \cong \eta_i x_i \psi(\mu_i)$
(17)

where the probability rate $\eta_i>0$ characterizes the mean success of the *i-th* model and is denoted here as the preference parameter. This parameter is assumed to be essentially characterized by the product features (utility, design etc.) and the spatial distribution. We take into account that the density $\psi(\mu_i)$ is limited to those potential consumers, who are willing to pay the product price μ_i. Note that Eq.(17) expresses Say's theorem, which suggests that supply creates its own demand.

The density of available products of a brand, $x_i(t)=X_i(t)/M$, can be derived from the balance between supply and purchase flow:

$\frac{dx_i}{dt} = s_i - y_i = \gamma_i y_i$
(18)

where we used Eq.(15).

In order to establish a dynamic relation for the sales of the individual brands, Eq. (17) suggests that we have to know the number of consumers generated per unit time. For this purpose we introduce a price dependent demand rate $d(<\mu>)$. This rate determines how many agents decide per unit time to purchase the good, scaled by the market potential. The density of potential consumers is then governed by the balance:

$$\frac{d\psi}{dt} = \frac{d\psi_f}{dt} + \frac{d\psi_r}{dt} = d(\mu) - y_t$$
(19)

That means, the density of potential consumer increases with an increasing demand rate and decreases due to the purchase of the good. The stationary density of potential consumers ψ_S is determined by the condition $d\psi/dt=0$. In this state the demand rate is just equal to the sales:

$$d(\langle\mu\rangle) = y_t(\langle\mu\rangle)$$
(20)

Note that in this stationary state, total supply is related to the demand rate by $s_t(<\mu>)=(1+<\gamma(t)>)\ d(<\mu>)$. In other words, for $<\gamma(t)>=0$ demand is equal supply, which indicates a market equilibrium in standard microeconomics [12].

2.3. The Diffusion Process

The key idea to describe the PLC of a durable good is that the dynamics depends on the introduction price μ_0 of the good. If the introduction price is much less than the natural price, $\mu_0<<\mu_m$, the market is called homogeneous and the market volume must be equal to the market potential. The density of potential adopters at introduction is $\psi_f=1$. Since all potential adopters can afford the good, the first purchase process (diffusion process) is price independent and limited only by the information spreading process trough the social network. The standard theory describing this diffusion process is the Bass model (denoted here as Bass diffusion [13]).

However, in the case $\mu_0>\mu_m$ the density of potential adopters is limited to the market volume, $\psi_f=v(\mu_0)$. Since within the market volume potential adopters are not limited by the price, the information spreading process governs always the market evolution at introduction. Therefore, the initial first purchase process of consumer durables can be always described by Bass diffusion. Though, in this case the mean price plays a crucial role.

We have to distinguish between a monopoly market and a polypoly/oligopoly market. For a monopoly market, per definition the total sales are equal to the sales of the monopoly. Therefore, if the price of the good is nearly constant (hence, $v(\mu)\approx const$) the sales dynamics is completely determined by Bass diffusion.

In the case of a competitive market, however, the situation is entirely different. Those brands with a lower price have a higher market volume and have therefore higher unit sales. The corresponding business units will raise the supply of products with a lower price and as a result the corresponding sales increase. Due to this increase the mean price Eq.(12) will decrease, with the consequence that the market volume expands. The growth of the market volume in turn can be interpreted as a diffusion process.

But how fast does this price change occur? The idea is to take advantage form the competition between the brands and derive the price evolution on the basis of an evolutionary approach. As shown below the market evolution is governed by Gompertz equation. Hence, we denote the corresponding first purchase process as Gompertz diffusion. Taking into account the repurchase processes caused by Bass and Gompertz diffusion the total PLC of a durable good can be derived.

2.3.1. Bass Diffusion

The first purchase process

In order to describe the first purchase process we introduce an adopter density (market penetration):

$$n(t) = \frac{N_A(t)}{M}$$

(21)

where $N_A(t)$ is the cumulative number of potential adopters at time step t. The simplest version of Bass diffusion is given by [13]:

$$\frac{dn_B(t)}{dt} = \mathrm{A}\psi_f(t) + \mathrm{B}n_B(t)\psi_f(t)$$

(22)

where $n_B(t)$ is the density of adopters due to the Bass adoption process. The first term of the differential equation describes a spontaneous purchase by potential adopters, where A is the so-called innovation rate. The second term is due to the word-of-mouth effect, where the density of adopters increases with an imitation rate B. For a constant introduction price μ_0 the density of potential adopters can be obtained from:

$$\psi_f(t) = v(\mu_0) - n_B(t)$$

(23)

while for $\mu_0 << \mu_m$ the market volume becomes $v(\mu_0)=1$.The adopter density evolution due to the Bass diffusion has the form:

$$n_B(t) = \frac{1 - e^{-(A+B)t}}{\left(1 + \frac{B}{A}e^{-(A+B)t}\right)^2} n_{B0}$$

(24)

where we have set $n_{B0}=v(\mu_0)$. For a sufficiently long time the adopter density approaches its stationary state, $n_B \rightarrow v(\mu_0)$.

The total first purchase unit sales caused by Bass diffusion become:

$$y_{tf}^B(t) = \frac{dn_B(t)}{dt} = \frac{A(A+B)^2 e^{-(A+B)t}}{\left(A + Be^{-(A+B)t}\right)^2} n_{B0}$$

(25)

while this relation can be also derived from Eq.(19) using Eq.(23).

The repurchase process

Repurchase processes separate into replacement and multiple purchases. In order to specify the repurchase process we take advantage from the first purchase process, $y_{tf}^{B}(t)$. The idea to model replacement purchase is that the time to a replacement of a good can be given by a probability distribution, $\Gamma(t)$ of product failure over the population of units [14,15]. Replacement purchase is therefore determined by the chance of a product failure at t'', times the number of sales at $t\text{-}t''$. The integration over all possible lifetimes delivers:

$$y_{tR}^{B}(t) = R\int_{0}^{t} y_{tf}^{B}(t-t'')\Gamma(t'')dt''$$

(26)

where $R>0$ is the fraction of previous sales suffered form replacement purchase. Note that Eq.(26) implies that replacement purchase is recurrent with the product life time t_p. Confining our interest to the first fundamental of the recurrent repurchase process, the failure distribution $\Gamma(t'')$ is treated as a sharp peak around t_p, with the probability distribution $\Gamma(t'')=\delta(t_p)$, and $\delta(t_p)$ indicates a Dirac delta function. We obtain from Eq. (26) for $t \geq t_p$:

$$y_{tR}^{B}(t) \cong R y_{tf}^{B}(t-t_p)$$

(27)

else, $y_{tR}^{B}(t)=0$. Replacement purchase induces therefore periodic variations of the unit sales with a periodicity given by the average product lifetime t_p.

Any other repurchase decision, not correlated with the first purchase fundamental wave, is denoted here as multiple purchase. In difference to replacement purchase, multiple purchase must be proportional to the actual number of adopters, $n_B(t)$. Hence the sales can be approximated by:

$$y_{tm}^{B}(t) = Q n_B(t)$$

(28)

where $Q>0$ is a multiple purchase rate. The parameter Q can in general be considered to be also a function of the price. However, for durable goods multiple demand is treated as a price independent constant, because the marginal utility for the simultaneous use of many durables is usually small.

The total sales caused by Bass diffusion become:

$$y_{t}^{B}(t) = y_{tf}^{B}(t) + y_{tr}^{B}(t)$$

(29)

while repurchase sales is the sum of replacement and multiple purchase $y_{tr}^{B}=y_{tR}^{B}+y_{tm}^{B}$.

2.3.2. Gompertz Diffusion

The first purchase process

As discussed above, for $\mu_0>\mu_m$, next to Bass diffusion there is another diffusion process in action. This process is due to the competition between the brands in the case of a polypoly/oligopoly. The competition causes a decrease of the mean price associated with an increase of the market volume $v(<\mu>)$. Because new potential adopters are created by the expansion of the market volume the change of the density of new adopters $n_G(t)$ is related to the market volume according to:

$$\frac{dn_G(t)}{dt} = \frac{dv(\langle \mu(t) \rangle)}{dt}$$
(30)

The integration gives:

$$n_G(t) = v(\langle \mu(t) \rangle) - v(\mu_0)$$
(31)

where the integration constant is the market volume suffered from Bass diffusion. Applying Eq.(9) the evolution of the adopter density obeys the relation:

$$n_G(t) = n_{G0} \exp\left(-\frac{(\langle \mu(t) \rangle - \mu_m)^2}{2\Theta^2} \right)$$
(32)

while

$$n_{G0} + n_{B0} = 1$$
(33)

Obviously, the first purchase process is completely determined by the evolution of the mean price. This evolution can be given by the evolution of the price distribution $P_\mu(t)$:

$$\frac{d\langle \mu \rangle}{dt} = \int_0^\infty \frac{dP_\mu(\mu')}{dt} \mu' d\mu'$$
(34)

while the price distribution characterizes the sales frequency in the price interval μ and $\mu+d\mu$. Hence, the sales dynamics of the individual brands in competition determines the mean price evolution. In order to derive the mean price evolution we make the following assumption:

vi) The mean price is the result of the competition between the business units. However, for durable goods, price changes are rare events. Therefore, for a sufficiently small time interval the mean price can be considered to be constant.

That means we assume that the mean price evolves slowly, $d<\mu>/dt \sim \varepsilon$, with $\varepsilon<<1$. Instead studying the market evolution on the long time scale t we focus instead on a much shorter time scale indicated by the parameter τ and related to the long time scale according to:

$$t = \varepsilon\tau \tag{35}$$

(For example if t is in years and τ in weeks, $\varepsilon \approx 1/50$). On the short time scale the mean price is a constant:

$$\frac{d\langle\mu\rangle}{d\tau} \sim \varepsilon^2 \approx 0 \tag{36}$$

When the mean price is a constant on the short time scale, Eq.(20) suggest that the corresponding total sales must be also a constant, on this time scale. Though, considerable price changes can be expected when the initial Bass diffusion is over. In this period, the demand rate is known and determined by the repurchase process, $d(<\mu>) \approx y^B_{tr}$ and $n_B \approx v(<\mu>)$. In order to simplify the model, the repurchase demand is formulated as an effective multiple purchase. Neglecting periodic sales variations the demand rate can be approximated as:

$$d(\langle\mu\rangle) \cong qv(\langle\mu\rangle) \tag{37}$$

where q is a constant effective constant repurchase rate. Since on the short time scale the mean price is a constant, we conclude:

$$\frac{dqv(\langle\mu\rangle)}{d\tau} = \frac{dy_t}{d\tau} \approx 0 \tag{38}$$

This result implies that all manufacturers together cannot sell more products than y_t, which establishes a considerable competition between the products in this phase of the PLC. In order to describe this competition, we assume:

vii) On the short time scale variations of the price μ_i, the preference parameter η_i and the reproduction coefficient γ_i are rare events.

In other words, the short time scale is chosen such that most of the time the utility properties, the price and the output of the brands can be treated as constant, interrupted by small jumps. With this approximation the time evolution of the individual sales of the brands can be obtained from a time derivative of Eq. (17):

$$\frac{dy_i(\tau)}{d\tau} = \eta_i\psi(\mu_i)\frac{dx_i(\tau)}{d\tau} = f_i y_i(\tau) \tag{39}$$

where we used Eq.(18), neglecting the impact of the small jumps. The rate f_i becomes:

$$f_i = \eta_i \gamma_i \psi(\mu_i)$$
(40)

The constraint Eq.(38), can be satisfied by adding a constant growth rate ζ such that:

$$\frac{dy_i(\tau)}{d\tau} = \left(f_i - \zeta\right) y_i(\tau)$$
(41)

Inserting this relation into Eq.(38) we obtain:

$$\zeta = \langle f \rangle = \frac{\sum_i y_i f_i}{y_t}$$
(42)

Rewriting Eq.(41), the sales evolution of the *i-th* model is determined by the replicator equation:

$$\frac{dy_i(\tau)}{d\tau} = \left(f_i - \langle f \rangle\right) y_i(\tau) = r_i y_i(\tau)$$
(43)

where we have introduced the growth rate of the *i-th* business unit in terms of unit sales, r_i.

From evolutionary theories the parameter f_i in the replicator equation is known as the fitness [16,17]. Therefore, we want to denote f_i here as the product fitness. The result Eq.(43) is crucial, because it suggests that the brands stand in an evolutionary competition. The sales of those models which product fitness exceeds the mean fitness $<f>$ are amplified. In order to survive the business units are forced to increase the product fitness compared to their competitors. The fitness is derived explicitly here and contains elements of both sides of the market. The preference parameter η and the market volume as a function of the product price μ are related to the demand side, while the reproduction coefficient γ is linked with the supply side. They span a three-dimensional fitness space. Although the evolutionary adaptation process takes place simultaneously in the entire fitness space, we confine here to the price evolution.

The dynamics of the sales density in a price interval is governed by the same dynamics as for the individual sales [16]. Hence the price distribution is governed by the replicator equation:

$$\frac{dP_\mu}{d\tau} = \left(f(\mu) - \langle f \rangle\right) P_\mu$$
(44)

From this relation the evolution of the mean price can be derived as performed in Appendix A. The mutual competition between the brands leads to a time dependent mean price evolution of the form:

$$\langle \mu(t) \rangle = \mu_0 e^{-at} + \mu_m$$
(45)

where μ_0 is the price at $t=0$, and the parameter $a \sim \varepsilon Var(P_\mu)$ is denoted as the price decline rate. The decline rate is considered to be a constant, because the variance is treated as a constant (assumption iv).

The evolutionary approach of a competitive market therefore suggests that the average price for durable goods decays exponentially and approaches μ_m asymptotically on the long time scale. The reason for this relationship is that products with a lower price have a competitive (evolutionary) advantage. Approaching μ_m at which the market volume has a maximum this advantage, however, diminishes. As a result the price moves towards the stationary mean price μ_m. Note that the slow price evolution suggested in Eq.(22) is contained in the price decline rate, since it is proportional to ε. Note further that the price decline is the result of the competition between the brands. Since the decline rate is proportional to the variance of the price distribution, it is $a=0$ for a monopoly. This does not mean that a monopoly cannot change the price, but it is beyond the ability of the model to determine the price evolution of a monopoly market.

With the mean price evolution Eq.(45), the diffusion process Eq.(32) can be explicitly given. The model suggests that the diffusion process caused by the competition is determined by Gompertz equation:

$$n_G(t) = n_{G0} \exp\left(-k e^{-2at}\right)$$

(46)

with

$$k = \left(\frac{\mu_0}{2\Theta^2}\right)^2$$
(47)

and the evolutionary diffusion process is denoted as Gompertz diffusion. Gompertz adoption rate reads:

$$y_{tf}^G(t) = \frac{dn_G(t)}{dt} = 2akn_G(t)\exp\left(-2at\right)$$
(48)

Because Bass and Gompertz diffusion have different roots, both processes can be considered as independent. However, the competitive evolutionary process is usually delayed by Bass diffusion. The corresponding time shift Δt_0, relates the time of Gompertz diffusion to:

$$t' = t + \Delta t_0$$
(49)

Hence, the total adopter density is determined by:

$$n(t,t') = n_B(t) + n_G(t')$$
(50)

The repurchase process

Equivalently to the Bass diffusion process, Gompertz diffusion contribution to the PLC has the form:

$$y_t^{\,G}(t') = y_{tf}^{G}(t') + Q' n_G(t') + R' y_{tf}^{G}(t' - t_p')$$
(51)

where the free parameters *Q'* and *R'* indicate multiple and replacement purchase caused by the initial Gompertz diffusion. In general the product lifetime may be different for both adoption processes, $t_p \neq t_p'$.

Finally the total PLC becomes:

$$y_t(t,t') = y_t^{\,B}(t) + y_t^{\,G}(t')$$
(52)

with Eq.(29) and Eq.(51), while Bass diffusion starts at $t=0$. The PLC may exhibit two characteristic periodic waves due to the two diffusion processes. In order to describe the PLC 13 free parameters may be necessary: A, B, n_{B0}, Q, R and t_p for Bass diffusion and n_{G0}, k, a, Q', R' Δt_0 and t_p' for Gompertz diffusion.

2.3. Stylized Facts

In this section we want to show that two additional stylized empirical facts can be derive easily from the presented model.

A constant fitness advantage

We want to discuss the case of a constant fitness advantage over a long time period. For this purpose we introduce the market share:

$$m_i = \frac{y_i}{y_t}$$
(53)

The replicator equation in terms of market shares turns into:

$$\frac{dm_i(\tau)}{d\tau} = \left(f_i - \sum_j f_j m_j \right) m_i(\tau)$$
(54)

Suppose a brand with market share m_1 has a constant fitness f_1, with $f_1>f_2$, while f_2 indicates the fitness of all other products with market share m_2. For m_1 follows from Eq.(54):

$$\frac{dm_1}{d\tau} = (f_1 - f_2 m_2)m_1 - f_1 m_1^2$$
(55)

and with m_2= 1- m_1 the evolution of the market share is governed by the Fisher-Pry equation [18]:

$$\frac{dm_1}{dt} = \theta m_1(1 - m_1)$$
(56)

with the fitness advantage:

$$\theta = (f_2 - f_1)\varepsilon$$
(57)

applying a constant the fitness advantage. The time evolution of the market shares can be written as a logistic growth of the form:

$$\ln\left(\frac{m_1}{m_2}\right) = \theta t + C_m$$
(58)

while C_m is an integration constant. This result suggests that the market share relation m_1/m_2 must be a linear function plotted in a half- logarithmic diagram.

The size distribution

We want to characterize the size of the *i-th* business unit by its unit sales. The size distribution of the business units P_y, is determined by the probability to find the unit sales of a business unit in the interval y and $y+dy$. Scaling Eq.(43) by y_t, the time evolution of the relative abundance of the unit sales is governed on the short time scale by:

$$\frac{dP_y(y)}{d\tau} = r\,P_y(y)$$
(59)

As assumed in vii) parameters that determine the fitness of the brands, make small jumps. As a result the fitness of the brands fluctuate by a small amount $\delta f=r$ around the mean growth rate. From Eq. (38) and Eq. (43) follows also that the mean growth rate on the short time scale is:

$$\frac{1}{y_t}\frac{dy_t}{d\tau} = \frac{1}{y_t}\sum_i r_i y_i = \langle r \rangle \cong 0$$
(60)

Treating *r* as a random independent and identically distributed variable, Eq.(59) suggests that the size evolution of the business units is determined by a multiplicative stochastic process. Applying the central limit theorem the size distribution of the business units is therefore given for a sufficiently long time by a lognormal distribution of the form:

$$P_y(y,t) = \frac{1}{\sqrt{2\pi t}\,\omega y}\exp\left(-\frac{\left(\ln(y/y_0) - ut\right)^2}{2\omega^2 t}\right)$$

(61)

where *u* and ω are free parameters and y/y_0 is the size of the business unit scaled by the size at $t=0$.

Gibrat established the law of proportionate growth in order to derive the lognormal distribution of firm sizes [19, 20]. Most remarkable is that this law is implicitly contained in the present model as the result of the competition between the business units in form of the replicator equation. However, as discussed above, firms may consist of several business units. Therefore the size distribution of firms will deviate from the lognormal distribution for large firms.

3. Comparison with Empirical Results

Since a number of approximations are made in order to model the complex nature of a durable goods market, the aim of the present model is not to make precise forecasts of the sales evolution. Though, we expect that the model describes the main trends of the PLC. The theory makes a number of predictions that can be tested:

I) The diffusion process of durables may consist of a Bass and a Gompertz regime. While Bass diffusion is not related to the price, Gompertz diffusion must be accompanied with an exponential decline of the mean price. The PLC is the result of first and repurchase processes. Periodic variations of the unit sales occur caused by the finite lifetime of the good.

II) A price decline is the result of the competition between brands. A market dominated by a single brand can be considered as a monopoly market. Its PLC is governed only by Bass diffusion and the corresponding repurchase process.

III) A product with a constant fitness advantage replaces competitive models such that the market share of the unit sales is governed by a logistic law.

IV) The size distribution of the product sales is lognormal.

3.1. The Diffusion Process and the Product Life Cycle

While the diffusion process can be characterized by the market penetration, the product life cycle is determined by the total unit sales. In the case of a competitive market the mean price evolution is governed by three unknown parameters and is given by Eq.(45). In order to simplify the application of the model, we introduce the following price function:

$$\mu'(t') = \frac{\mu(t') - \mu_m}{\mu_0} \cong \frac{p(t') - p_m}{p_0} \sim e^{-at'}$$

(62)

while t' is given by Eq.(49). It starts when an exponential decrease of the price evolution is evident. This price function has two advantages:

1. For short time periods, the mean income scales out, because it changes slowly. Therefore we can approximate the nominal market price $p(t')$ by the real price $\mu(t')$.

2. The scaled price function $\mu'(t')$, should be a linear function in a semi logarithmic plot. Hence, with an appropriate choice of the parameter p_m/p_0, the empirical data arrange into a linear function, while its slope determines the price decline rate a.

Empirical studies often determine the market penetration as a function of the number of households N_H in a country. The maximum market penetration in terms of households is expressed here as: $n_{max}=M/N_H$. In difference to the theoretical derivation the maximum market penetrations is therefore given by $n_{max}=n_{B0}+n_{G0}$.

As an example exhibiting Gompertz diffusion we want to discuss Black & White (B&W) TV sets in the USA, where data for the market penetration and the corresponding price evolution are given by Wang [21]. An application of the presented model was performed in [7] and is shown in Fig. 1. Displayed is the market penetration *n(t)* and the scaled price evolution $\mu'(t)$, where the solid lines are fits to empirical data with the parameters given in Table 1. As suggested by the model the price evolution forms a linear relationship in a semi-log presentation, and the market penetration can be fitted with Gompertz equation, while the price decline rate is taken from the price evolution. A similar result can be found for Colour TV sets displayed in Fig.(2) [22,7].

Fig. 3 shows the empirical sales of B&W TV sets and an application of the model (fat line) using the parameters in Table 1. Because the sales evolution is given by the derivative of the market penetration, these data are very sensitive to small fluctuations. The sales evolution exhibits periodic variations as suggested by the present model. The PLC can be understood as consisting of two periodic waves associated to Bass and Gompertz diffusion, while the Bass diffusion contribution is not evident in the aggregated market penetration data. The model suggests that the first sharp sales peak around 1950 is due to Bass diffusion. This peak appears again due to the repurchase process 1959, 1968 and probably 1977. The corresponding product lifetime can be estimated to be about, $t_p \approx 9$ years. The broad sales peak 1955 on the other can be derived directly from Gompertz diffusion (Fig.1). The sales peaks 1965 and 1973 are related according to the present theory to the repurchase process originated from Gompertz diffusion, with a product lifetime $t_p' \approx 10$ years. Although the model does not fit the sales data perfectly, the model shows the mean trend of the PLC. Note that Bass diffusion is caused by the upper part of the income distribution. The difference between the two life times is probably credited to the higher financial liquidity of higher income consumers. Additional examples for durable markets are studied in [7].

The model further suggests that a brand covering its own market segment, can be viewed as a de facto monopoly. The theory suggests that even for a constant price, periodic variations of the sales occur as a result of the finite lifetime of the brand. These variations have its origin in the initial Bass diffusion period. Examples that exhibits these features are cars brands. Since consumers have the tendency to repurchase the same brand some car brands can be viewed as de facto monopolies.

We want to study here two German car brands. Displayed in Fig.4 are the total sales of cars of the Mercedes-Benz C-Class (squares) obtained from [23]. Also displayed is the German list price of the standard model (circles) [24]. Although the brand is not designed just for the German market, the German list price gives an indication of the price evolution of the good. (Note that the price of the C-Class scaled by the mean German income is nearly constant, and hence the market volume can be considered to be constant [24].) The introduction and expiring of different models of this brand is indicated in Fig.4 by arrows. Note that although the list price (approximating the mean price) is nearly constant, considerable periodic variations of the unit sales are evident. Remarkable is that these variations are not correlated with price variations as could be expected from classic microeconomics. The presented theory, however, suggests that these variations are due to the finite lifetime of the brand. The fat line is a fit of the PLC as suggested by the model (Eq.(29)) with a mean lifetime of about 8 years. Considering the introduction of new models, it could be argued that these sales variations are the result of new models (In terms of the presented model due to a variation of the preference parameter η). And indeed in this example the introduction of a new model is accompanied with an increase of the unit sales. However, the following example shows that this is not necessarily the case.

Displayed in Fig.5 is the total output of the Mercedes-Benz S-Class [23]. Similar to the previous example the arrows indicate the model change. Applying Eq.(29) with the parameters in Table 1 we obtain a mean trend with periodic variations of the sales while the lifetime is about 16 years (fat line). Obviously the S-Class had its takeoff with the model W116 followed by W126. The Daimler Car Group introduced three models (W140, W220, W463) to the market in the decade from 1990-2000, while the first was used also for a price jump. The fat line suggests a decrease of the sales just in this decade. The introduction of the new models W140 and W220 were indeed associated with a decrease of the unit sales as suggested by the model. Only the last model W463, which was close to the predicted periodic increase of the sales, turned into a success.

This result suggest that durables have an inner mean lifetime. The repurchase process of durables creates periodic sales variations coined in the introduction period of the good. It determines the main trend. The introduction of new models (or other events) can amplify or reduce this trend to some extend. In both, the monopoly and the competition case, the presented model allows an understanding of PLC.

3.2. Stylized Facts

The first stylised fact derived is that the growth of the market share of a brand with a constant competitive advantage is governed by a logistic law. There are several examples, where a logistic growth of market shares were found in economic data [25-27]. Well known is for example the replacement of music recoding media. It was shown that records are replaced by cassettes and finally by CD's, which implies a similar replacement process of the corresponding durable goods [28]. The application of the logistic law was even generalized to logistic wavelets [28]. Discussed in [7] is the replacement of brands in the video cassette market.

Another stylized fact is the prediction of a lognormal distribution of the unit sales. Note that the present model suggests that the lognormal distribution is strictly valid only for products (business units) but not for firms, because firms consist of a number of business units. Unfortunately a consistent statistical investigation of the size distribution of durable goods is not available. However, intensive studies of the competitive market of pharmaceutical products were performed [29,30]. These studies indicate a lognormal distribution for pharmaceutical products. But for the corresponding firms they found that the lognormal distribution has a power law departure in the upper tail as suggested by the model.

4. Conclusion

The standard theory of the product life cycle suggests a number of stages of the total sales of a good [6]. Usually four stages are specified for the PLC: Introduction, Growth, Maturity and Decline. The presented model suggests that the growth phase is caused by Bass diffusion. Displayed in Fig.5 for example the model W108 indicates the introduction phase of the car brand, while W116 is related to the growth phase of the PLC. But why does the market not start immediately with Bass diffusion? In other words: Why is there an introduction phase?

The model suggests that either the demand or the supply side delays self-amplification processes. On the demand side the self-amplification is contained in the word-of-mouth effect. One condition for a proper work of the word-of-mouth effect is that the market volume must cover sufficient connections in the social network. If the market is divided into separated sections, the spreading process is halted. In network theory the threshold for a continuous connection is the so called percolation threshold. If the market volume is below the percolation threshold, Bass diffusion does not work properly and the good cannot takeoff. However, the model suggests that there is an additional self-amplification process on the supply side of the market. This self-amplification is due to Say's theorem and contained in Eq.(18). As long as the business units supply just as much as demanded, the reproduction parameter is $\gamma \approx 0$. In this case supply follows demand but does not create its own demand. Only if there is an excess supply $\gamma > 0$, the self-amplification starts to work. In the case of the Mercedes Benz S-Class it took more than a decade before the brand took off, but there is already an increase of the sales in the introduction phase. The fact that the takeoff is related to the start of a new model suggests that the demand side triggers the takeoff. But the manufacturer also expanded the capacities and applied modern production technologies, which decreases the costs per unit. The latter implies also an impact of the supply side. It can be speculated that the market in the introduction phase is in a sort "metastable state", such that the preference parameter may switch from a low to a high magnitude, triggered by an event. With the data in hand, however, a definite answer cannot be given.

The model explicitly derives relations for the growth, maturity and decline phase of the PLC. The growth phase consist of Bass diffusion for a monopoly market and additionally of Gompertz diffusion in a polypoly/oligopoly market for expensive consumer durables. The maturity phase is characterized by periodic variations of the sales caused by the finite lifetime of the good (Juglar Cycles). The variations are coined by the diffusion processes and determine the main trend of the total unit sales. The manufacturers can take advantage from this trend and introduce new models with a periodicity close to the mean lifetime. Or they can act against the trend with the risk of financial losses. The decline phase of the PLC finally is caused by the replacement of the good by a new one. According to this model this replacement is related to a logistic decline of the market share compared with the new (better) good [28].

The presented theory suggests that a competitive market can be described by an evolutionary approach. In this sense the model is in line with the idea that an economy is governed by evolutionary processes [31-34]. The model suggest that an evolutionary competition is the reason for price decline of a durable good. The evolutionary theory can be applied directly, because the fitness function is explicitly known. Compared to biological evolution, the brands play the role of species and the environmental selection is performed here by the consumers. Because the fitness is not only a function of the price, but also of the preference and the reproduction parameters of the brand, it can be concluded:

1. The brand with the highest preference have a considerable competitive advantage in a free market. The brand prevail its competitors, unless other factors in the fitness prohibit this. Similar to biological evolution, the evolution of brands can be arrange in form of an

evolutionary tree, from the first invention until its present version. In difference to biological trees, however, a consumer good may have several roots.
2. A product with a higher reproduction rate prevail its competitors. However, because the reproduction parameter is not explicitly derived, a discussion of this point is not performed here (see [7]).

A competitive market can be characterized either by a brand with a dominant fitness. In this case the market dynamics is governed by the replacement of previous brands with smaller fitness. Or the market can be characterized by brands all having a similar fitness. In this case small variations in particular of the price leads to the fitness fluctuations of the brands around the mean fitness.

Most remarkable is that due to these fluctuations the replicator equation turns into a stochastic multiplicative differential equation that comprises Gibrat's rule of proportionate growth [19]. In other words, the lognormal size distribution of the brands (and implicitly of the firms) is the direct consequence of the competition between the goods. Next to the derivation of the PLC this is a crucial result, because it gives Gibrat's rule a fundament for further research.

Appendix A

The Evolution of the Mean Price for Durable Goods

Eq. (38) can be written with Eq.(46) as:

$$\frac{d\langle\mu\rangle}{d\tau} = \int_0^\infty P_\mu(\mu')\mu'\big(f(\mu') - \langle f\rangle\big)d\mu'$$
(A1)

this leads to:

$$\frac{d\langle\mu\rangle}{d\tau} = \int_0^\infty \mu' f(\mu')P_\mu(\mu')d\mu' - \langle\mu\rangle\langle f\rangle$$
(A2)

We introduce the price difference:

$$\Delta\mu = \big(\mu - \langle\mu\rangle\big)$$
(A3)

and expand the fitness of the *i-th* product as a function of the price difference:

$$f(\mu) = f\big(\langle\mu\rangle\big) + \frac{df\big(\langle\mu\rangle\big)}{d\mu}\Delta\mu$$
(A4)

where $\langle f\rangle = f(\langle\mu\rangle)$. Applying (A4) in (A2) using (A3) the mean price is governed by:

$$\frac{d\langle\mu\rangle}{d\tau} = \frac{df\big(\langle\mu\rangle\big)}{d\mu}Var(P_\mu)$$
(A5)

where the price variance is defined as:

$$Var(P_\mu) = \int P_\mu(\mu')\mu'^2\, d\mu' - \left(\int P_\mu(\mu')\mu' d\mu'\right)^2$$
(A6)

The stationary solution is determined by:

$$\frac{d\langle\mu\rangle}{d\tau} \sim \frac{df\big(\langle\mu\rangle\big)}{d\mu} = 0$$
(A7)

because the variance is always positive. It implies that the mean price is stationary at the maximum of the price dependent fitness.

The fitness at mean price can be written as:

$$f(\langle\mu\rangle) = \psi(\langle\mu\rangle)\langle\eta\gamma\rangle$$
(A8)

For the determination of $\psi(<\mu>)$, we take advantage from Eq.(39) and use the total sales:

$$y_t(\langle\mu\rangle) = \psi(\langle\mu\rangle)\sum \eta_i x_i = qv(\langle\mu\rangle)$$
(A9)

and obtain

$$\psi(\langle\mu\rangle) = \frac{q}{\sum \eta_i x_i} v(\langle\mu\rangle) = \psi_0 v(\langle\mu\rangle)$$
(A10)

Expanding the market volume $v(<\mu>)$ around μ_m up to the second order we obtain:

$$v(\langle\mu\rangle) \cong m_L \left(1 - \frac{(\langle\mu\rangle - \mu_m)^2}{2\Theta^2}\right) + m_U$$
(A11)

where we used Eq.(8) and $m_L=M_L/M$. Since only the market volume is a function of the price we get:

$$\frac{df(\langle\mu\rangle)}{d\mu} = \langle\eta\gamma\psi_0\rangle \frac{dv(\langle\mu\rangle)}{d\mu}$$
(A12)

this turns with Eq.(A11) into:

$$\frac{df(\langle\mu\rangle)}{d\mu} = -\frac{\langle\gamma\eta\psi_0\rangle m_L}{\Theta^2}(\langle\mu\rangle - \mu_m)$$

(A13)

Hence, the price evolution is governed close to μ_m by:

$$\frac{d\langle\mu\rangle}{d\tau} = -\frac{\langle\gamma\eta\psi_0\rangle m_L Var(P_\mu)}{\Theta^2}(\langle\mu\rangle - \mu_m)$$
(A14)

For the case that the pre-factor on the right hand side can be considered as time independent (iv), the integration can be carried out on the long time scale leading to Eq.(47):

$$\mu(t) = \mu_0 e^{-at} + \mu_m$$
(A15)

where μ_0 is an integration constant and the parameter

$$a = \frac{\varepsilon\langle\gamma\eta\psi_0\rangle m_L Var(P_\mu)}{\Theta^2}$$

(A16)

is denoted in this model as the price decline rate. Note that a large price variance increases the price decline rate, which is known as Fishers fundamental theorem of natural selection [34]. On the other hand for a monopoly is $a=0$. This is also the case if there is no competition, $\gamma=0$. derivation of the price distribution can be found in [7].

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Tables

Parameter	*Colour TV*	*B&W TV*	*MB C-Class*	*MB S-Class*
t_0	1954	1948	1979	1964
Δt*[years]*	1/2	0	0	0
p_m/p_0	0	0.33	-	-
a[1/year]	0.103	0.2	-	-
k	27	8.5	-	-
n_{G0}	0.97	0.77	-	-
n_{B0}	0.01	0.18	1	1
A	0.001	0.02	0.004	0.02
B	1.8	2.5	0.58	0.5
R	-	0.3	1.2	1
Q	-	0.06	0.05	0.15
R'	-	0.65	-	-
Q'	-	0.06	-	-
t_p*[years]*	-	9.2	-	-
t'_p*[years]*	-	10.2	-	-
$M\ 10^6$	-	~53	~1.1	~0.27

Table 1. Characteristic parameters of the studied examples.

Figures

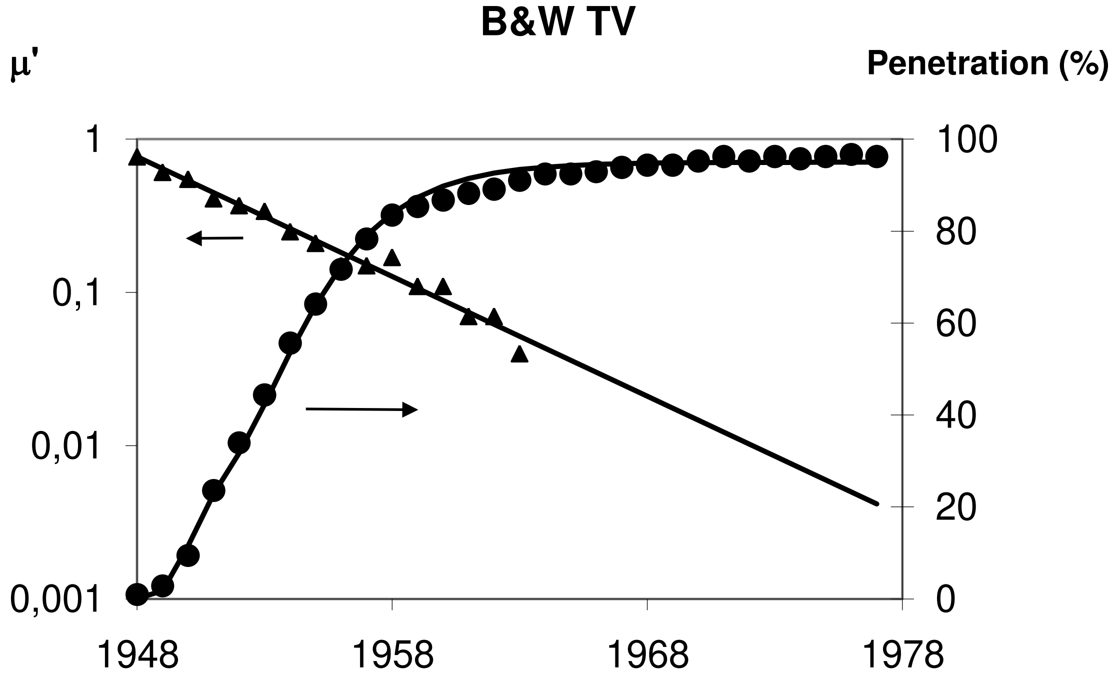

Figure 1: Evolution of the price function μ' (triangles) and market penetration (dots) of Black & White TV sets in the USA [7].

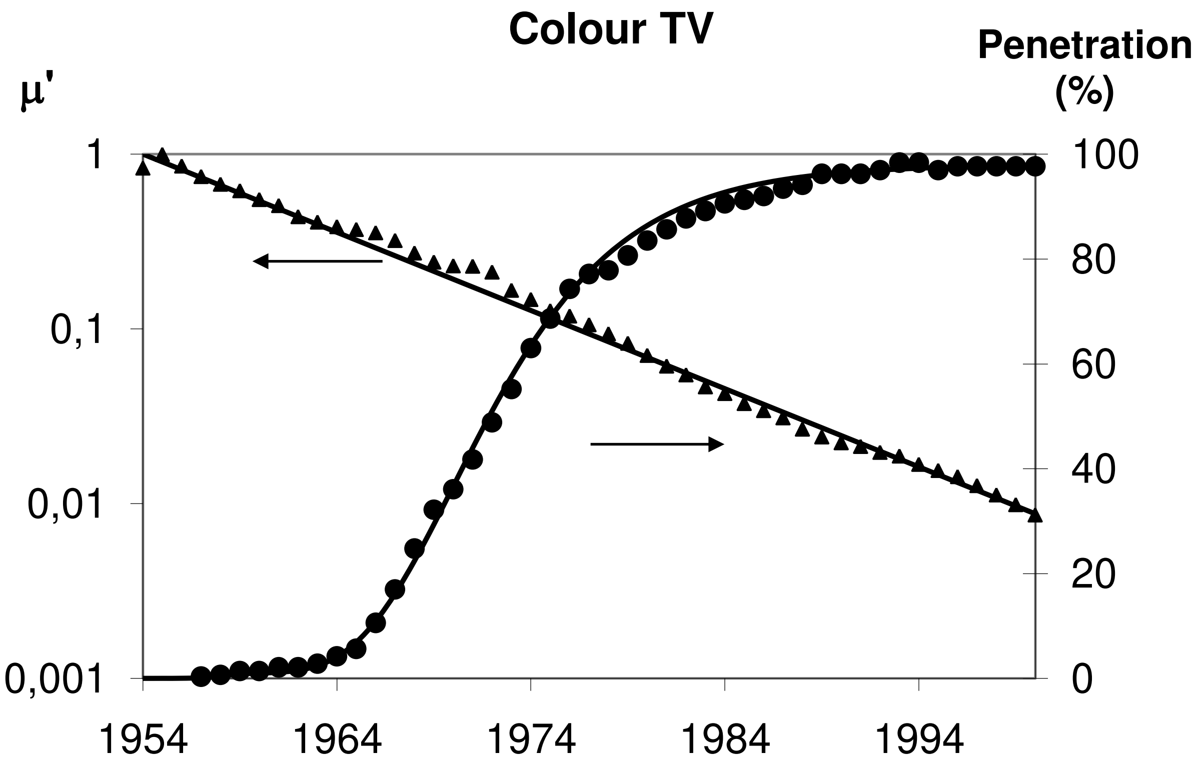

Figure 2: Evolution of the price μ'(triangles) and market penetration (dots) of Colour TV sets in the USA [7].

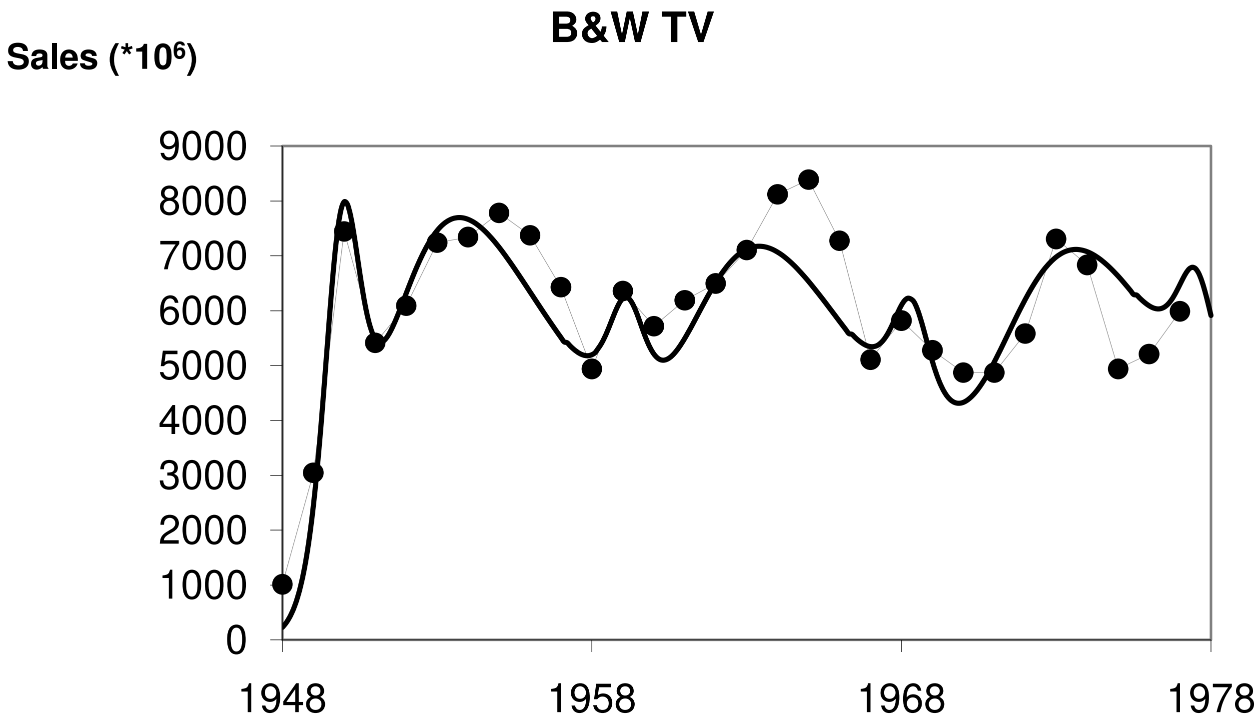

Figure 3: of Black & White TV sets in the USA, while the fat line is of the presented model. The narrow peaks are caused by the initial Bass diffusion and the broad peaks by Gompertz diffusion [7].

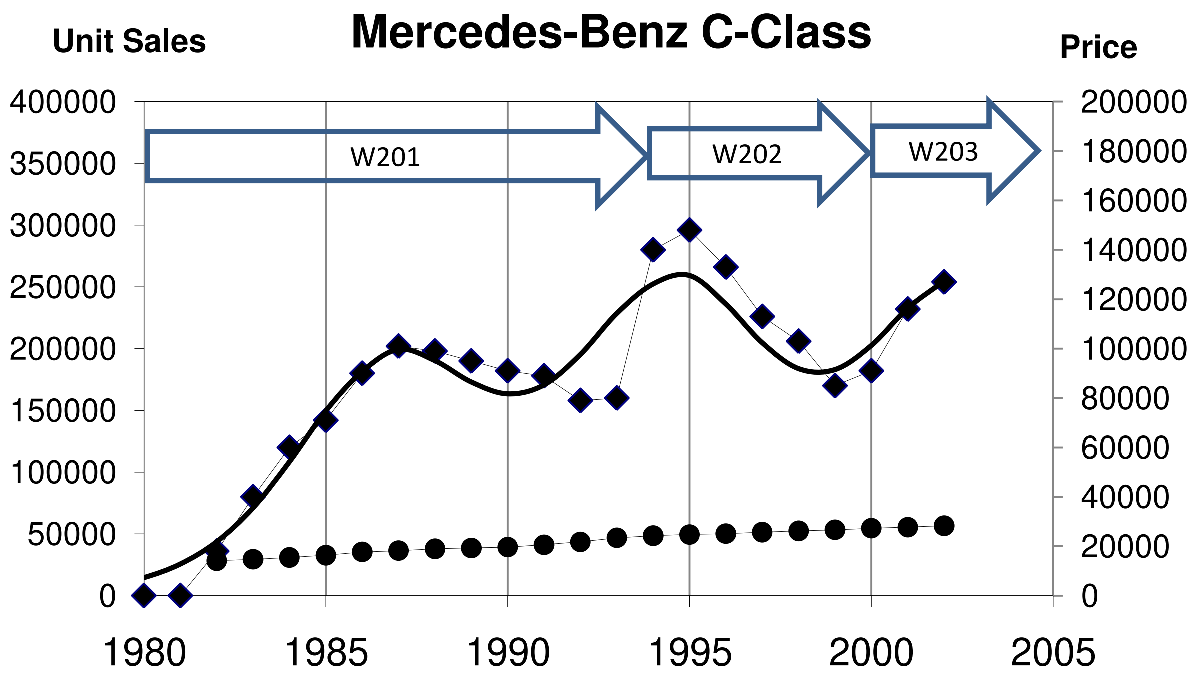

Figure 4: The PLC of the Mercedes Benz C-Class. The squares indicate the empirical sales and the fat line is a fit according to the model with the parameters in Table 1 [23,24]. The circles indicate the list price in € and the arrows the manufactured models.

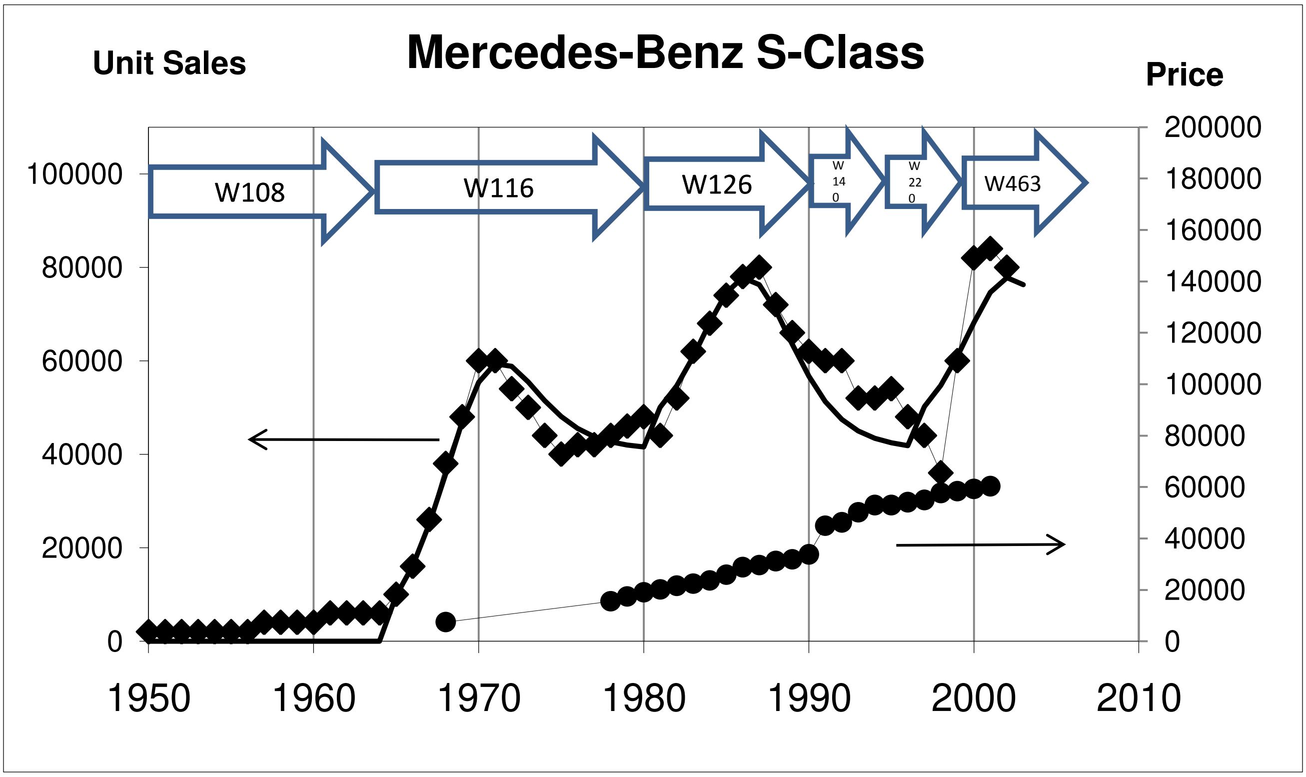

Figure 5: The PLC of the Mercedes Benz S-Class. The squares indicate the empirical sales and the fat line is a fit according to the model with the parameters in Table 1 [23,24]. The circles indicate the list price in € and the arrows the manufactured models.