The Conflict Between General Equilibrium and the Marshallian Cross

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Abstract

This paper illustrates on a simple model of production economy how the concept of partial equilibrium can be in an unresolvable conflict with the general equilibrium.

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1 Introduction

It is generally thought that the 'partial equilibrium' (PE) framework or the Marshallian supply and demand framework is a pedagogically convenient simplification in harmony with the deeper and more complex 'general equilibrium' (GE) framework which determines equilibrium prices and quantitities in a neoclassical economy. We show on a simple model of production economy that concepts of Marshallian supply and demand are in conflict with GE and, because of certain logical contradictions, these conflicts cannot be resolved. This means that the PE ideas we use in elementary textbooks to explain formation of prices in markets are in conflict with our own ideas about the same issues in the GE framework of advanced textbooks. Since the GE framework is at least internally coherent, unlike the PE or Marshallian supply and demand framework, the latter must be very carefully used if not

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abandoned entirely. What should replace it is not clear, but we need to tell a new story, which is both internally coherent, and consistent with the more complex GE story of determination of equilibrium prices and quantities.

The organization of the paper is as follows: Section 2 introduces the model and the problems with the Marshallian analysis. Section 3 tries to give explanations for the incoherence of the Marshallian thought experiment. Finally, Section 4 contains some concluding remarks involving several conjectures about conditions under which the partial equilibrium analysis would approximate the equilibrium in one of the submarkets in a general equilibrium economy.

2 Model

We will consider a simple example of a market in which there is no equilibrium in the sense that no configuration of prices will lead to a stable configuration of decentralized decisions. We are not aware of any examples of this type, where a 'general equilibrium' cannot even be defined.

Suppose that there are 100 farmers indexed by i each of whom owns a single plot of land of equal size which is capable of growing either one unit of rice (R) or one unit of wheat (W) but not both. Let us assume for simplicity that production takes place at zero fixed cost. We will see later that this model is robust with respect to results we wish to produce; one can change the structure of the producer costs and consumer behavior substantially without affecting these results. Each of these producers is also a consumer of W or R or both and has the utility function $U(W, R) = WR$, satisfying standard assumptions.

We will now try to calculate the aggregate demand and supply functions in this economy. Let us take wheat as the numeraire, and let the price of wheat be $p_W = 1$. We will use p to denote p_R which will be the price of rice. The aggregate supply function of rice is straightforward. If $p > 1$, then all farmers will produce rice, while if $p < 1$, then all will produce wheat. If $p = 1$, then the farmers will have equal profits from producing either rice or wheat. Thus the aggregate supply curve is a step function which jumps from 0 to 100 at $p = 1$. That is, $S(R) = 0$ if $p < 1$, $S(R) \in [0, 100]$ if $p = 1$, and $S(R) = 100$ if $p > 1$.

2.1 Nonexistence of Marshallian Market Demand Function

Unlike the supply function, the thought experiment which defines the demand curve is not clear in this example. We contemplate changing the price of rice and asking consumers how much rice they would consume. However, the 'ceteris paribus' assumptions cannot be fulfilled in this example. Since each consumer is also a producer, changes in price of rice will affect his production decision and also hence income. To try to find an equivalent to the Marshallian notion, we consider two cases separately:

A wheat farmer is somehow committed to producing wheat. In this case, his income is 1, and his budget constraint is $W + pR = 1$. Subject to this budget constraint, the utility function $U(W, R)$ is maximized at $W^* = 1/2$, and $R^* = 1/(2p)$.

A rice farmer is similarly committed to producing rice. In this case, his budget constraint is $W + pR = p$, since his income is p - the thought experiment in which the price of rice is altered cannot keep the income of the rice farmer constant. Maximizing utility subject to this constraint, we find that $R^* = 1/2$ and $W^* = p/2$.

What is the Marshallian aggregate demand function? This question cannot be answered in this model, because the number of wheat and rice farmers is itself endogenous. Nonetheless, let us fix the quantity of wheat and rice farmers; perhaps this is at historical levels. Let N_W be the number of wheat farmers and N_R be the quantity of rice farmers. The demand curve for the wheat farmers is $1/(2p)$, so the aggregate demand is $N_W/(2p)$. For the rice farmers it is $1/2$ pure and simple so the total aggregate demand is $D(p) = N_R/2 + N_W/(2p)$.

2.2 A Symmetric 'Equilibrium'

Because of the complete symmetry in R and W, a natural equilibrium and one with many demonstrable optimality properties is one in which there are 50 rice farmers and 50 wheat farmers, the two products have the same price and every consumer consumes (1/2,1/2) units of both wheat and rice. If $N_R = 50$ and $N_W = 50$, this desirable symmetric equilibrium somehow emerges at a Marshallian cross at which the demand curve for rice is $D(p) = 25 + 25/p$, which intersects the step function supply curve at $p^* = 1$ and $R^* = 50$.

We want to know if this historical equilibrium will perpetuate itself. We announce to the world that this is the best equilibrium - there will be 50

rice farmers and 50 wheat farmers and prices of the two products will be equal. Let every farmer maximize his profit. Then every farmer is indifferent between growing rice and wheat and therefore we cannot predict what output will emerge. That is, announcing the equilibrium prices does not decentralize production decisions in the way that economic theory predicts. Because the production decision is arbitrary, 70 farmers may decide to grow wheat and 30 to grow rice. Any set of decisions is ex-ante compatible with profit maximization. There is no way to decentralize the production decisions and produce 50 units of wheat and 50 of rice. If some arbitrary quantities are produced, like (70,30), then rice will be more expensive and wheat less so. The wheat producers will suffer welfare loss, while the rice farmers will gain. At any non-symmetric equilibrium, the gainers are fewer than the losers, so that a democratic equilibrium would be symmetric.

What will happen in the next period? If farmers are naïve and believe that these prices will continue, then they will all plant rice. This will lead to severely problematic equilibria. Some partial and probabilistic adjustment mechanisms would lead towards (50,50) the optimal equilibrium, but it can never be achieved and price fluctuations will persist since there is no decentralized way to arrive at the symmetric equilibrium. Here, we should also note that assuming rational expectations on the part of farmers will not help matters.

3 An Incoherent Thought Experiment

At some level, it is obvious that the general equilibrium story conflicts with the Marshallian supply and demand. GE tells us that all market are interdependent, while Marshall seeks to explain equilibrium in one market without reference to what is happening in others. The above model shows that the conflict is much more dramatic than is realized. The concepts used in a Marshallian model cannot even be defined within a general equilibrium framework. One cannot explain equilibrium price formation within a market in isolation. Since all our elementary textbooks rely on the Marshallian framework, and our intuitions about equilibrium price and quantities are shaped by these textbooks, some radical rethinking is required. The problem with the standard textbook argument is hidden in the income "Y". Our elementary micro models do not have money. Thus we cannot consider a thought experiment in which we fix the income in nominal terms and ask consumers

what they would purchase. But any notion of real income must go through prices, which are considered to be varying in constructing the demand.

Another explanation of the incoherence of the Marshallian thought experiment in more general terms is as follows. The Marshallian framework takes tastes and technology (represented by utility and production functions) as well as factor costs as given, and produces equilibrium prices and quantities. The demand function is based on asking the question: what would a consumer purchase if the price of the good being purchased changed? This thought experiment is incoherent, because within a Marshallian framework, a price cannot change. An endogenous variable can only change when some exogenous variable does. In a Marshallian world, if I am asked what I will do when the price changes, I must ask 'why' did the price change? Being endogenous, prices are not free to change on their own. There are three possible reasons, each of which has their own and different effects: i. prices changed because of shifts in the supply function; ii. prices changed because of shifts in the demand function; iii. there was some exogenous shock.

This last possibility is the one that is in the back of the mind of the PE economist. However, exogenous shocks are delicate and subtle and require very careful treatment. it is not enough to talk about exogenous shocks to prices; the exact and specific nature of shock must be specified before we can ask about responses. In particular, we must know whether or not this new price being quoted at us, in order to elicit a demand, is a disequilibrium price resulting from a temporary or permanent shock. To give a good response, we will need a lot more information about the nature of the disequilibrium and also the mechanisms which come into effect following disequilibrium. If it is an equilibrium price on the other hand, then ceteris paribus cannot hold, and we must know what other changes have occurred to cause this shift in equilibrium. This means that the thought experiment required to define a Marshallian demand function is internally incoherent or at least incomplete.

4 Extensions

Responding to earlier drafts of this article, several economists presented conjectures about conditions which would lead to the supply and demand framework holding as an approximation in one of the submarkets in a general equilibrium economy. Five conjectures are as follows:

One conjecture (due to George Judge) is that if there was a large number

of goods, and the good in question was a small proportion of the total budget, then supply and demand would be good approximation. In our model above, if we have n goods, $G_1, G_2, \ldots, G_n$ any one of which can be produced by the farmers, and the utility functions is the product of all consumptions, then the example goes through exactly as before.

A second conjecture (due to Jeffrey M. Perloff) is that additively separable utility functions were required. If $U(R, W) = W + R$, then R and W are in effect identical goods and all production and consumption decisions are equivalent. Except for this trivial special case, separable utilities do not help with the two problems that production decisions are endogenous, and that varying the prices leads to variations in the income, so one cannot define the Marshallian demand curve even conceptually.

A third conjecture (due to Debraj Ray) is that we need to separate the consumers from the producers, to get the Marshallian framework. There are several levels of separation possible, but none that we have tried could succeed in producing Marshallian supply and demand. For example, the farms are run by foreign firms which repatriate all profits to home countries (so variations in firm income do not impact on the market), do not hire domestic labor, while consumers have their own endowments which they use to purchase rice and wheat from these foreign producers.

As a fourth conjecture, one can argue at this point that the conflict between PE and GE frameworks simply arises due to the presence of production in the economy, and the problems with defining a Marshallian demand curve can be resolved if consumption, instead of consumers, is completely separated from production. To disprove this claim, take a pure exchange economy with goods $x_1, x_2, \ldots, x_k$. Suppose good one is numeraire and let $p_2, \ldots, p_k$ be the price of the others. Let $e(j) = \langle e_1(j), ..., e_k(j) \rangle$ be the endowment bundle of the j-th consumer. At any vector of prices $p = \langle 1, p_2, ..., p_k \rangle$, let $D(j, p)$ be the vector of demand of the j-th consumer. Consider the problem of defining a Marshallian demand curve for the second good.

In the first instance, this can be done by varying p_2, keeping all other prices fixed. Then we may have a well defined demand function for good 2, but it does not represent the concept of Marshallian demand since incomes of all consumers vary as p_2 is changed.

Alternatively, we may try the following. Let $I(j) = \sum_{i=1}^{k} p_i^* e_i(j)$ be the income of the consumer j at some fixed vector of prices p^*. Now ignore this origin of income and fix this income as a number for each consumer. Ask consumer j to maximize utility derived from the consumption $k-$tuple

$\langle x_1(j), x_2(j), \ldots, x_k(j)\rangle$ subject to the budget constraint $\sum_{i=1}^{k} p_i x_i(j) \leq I(j)$. This should be the Marshallian demand. The demand function is now well-defined, but does not have anything to do with consumer j's behavior because it artificially fixed the value of his endowment vector $e(j)$ at p^*.

One can finally argue that the results in this paper would vanish if the producers did not face constant returns to scale. To check this last conjecture, let us assume that each producer has a divisible labor endowment of one unit, and decreasing returns to scale technologies $f_R(.)$ and $f_W(.)$ converting labor into rice and wheat, respectively.[2] Assume that $f_R(L) = f_W(L) = f(L)$ for all $L \in [0, 1]$, where $f'(.) > 0$, $f''(.) < 0$, and $f'(0) = \infty$. As in the case of constant returns to scale, we will require that joint production of wheat and rice is not possible. To achieve that, we will assume positive fixed costs of production. Let K_W and K_R denote the fixed cost of producing rice and wheat, respectively. Then, each supplier will maximize $I_R(L)[pf(L) - K_W] + I_W(L)[f(1-L) - K_L]$ where the indicator functions $I_R(.)$ and $I_W(.)$ are such that $I_R(0) = I_W(1) = 0$, $I_R(1) = I_W(0) = 1$, and $I_R(L) = I_W(L) = 1$ if $0 < L < 1$. Now, let simply $K_W = K_R = K > 0$. It is apparent that each farmer will specialize in either rice or wheat but not both if $\max\{pf(1) - K, f(1)-K\} > \max\{pf(L)+f(1-L)-2K\} = pf(L^*(p))+f(1-L^*(p))-2K$ for all $p > 0$, where $pf'^*) = f'^*)$.[3] This is indeed the case if K is sufficiently large. Then, as in the case of constant returns to scale, the aggregate supply of rice would become a step function at $p = 1$. Hence, the problem with the existence of Marshallian market demand functions.

It is interesting to establish that the very problem studied in this paper does not hinge upon the type of production technology that farmers are employed with. The essence of the matter is a coordination problem, and would generalize far beyond the simple context discussed. At equilibrium, all activities bring equal profits, so entrants can choose arbitrarily among different activities. But this leads to quantities being indeterminate. An artificial solution can be created by allowing for joint production under decreasing returns to scale. Under suitable hypotheses, this eliminates the coordination

[2]We simply disregard the case of increasing returns to scale in which the general equilibrium usually does not exist.

[3]Note that the inequality, $\max\{pf(1) - K, f(1) - K\} > \max\{pf(L) + f(1 - L) - 2K\}$, ensuring to avoid joint production simply holds true (for all $p > 0$) under constant returns to scale for any positive K, however small. This is indeed why the fixed costs of production are without loss of generality set to zero in Section 2.

problem created by the indeterminacy of the profit maximizing production levels. In the situation described in the first conjecture above, all farmers would produce all goods in identical proportions. This does not seem like a realistic scenario, and many plausible assumptions (such as fixed costs, or constant returns to scale) can be used to rule it out. Thus the conflict between the PE and GE models would appear to be generic and widespread.

5 Conclusions

Our intuitions as economists, and our policy advice, are based on intuitions generated by the models we learn in universities. One of the simplest of these is that there are competitive equilibria, and that markets achieve these equilibria. Our very simple model does not have an equilibrium. There is no configuration of decentralized decisions which is self-replicating and self-sustaining.

A second widely believed and strong intuition is that fixing prices above equilibrium values would lead to a reduction in trades. The controversy about the Card and Kreuger (1997) findings that minimum wages did not lead to increased unemployment arose because it conflicts with this intuition. In an interview, Card stated that he thought that elementary supply and demand models did not apply to the labor market, as our analysis also suggests. He felt that more complex search theoretic frameworks could explain their findings. The emotional attachment of economists to "supply and demand" framework is reflected in Card's statement[4] that "... (economists) became very angry or disappointed. They thought that in publishing our work we were being traitors to the cause of economics as a whole."

A third intuition is that planning leads to inefficiencies. Suppose that in the model described in Section 2 the government designates 60 people to be wheat farmers and the rest to be rice farmers. This will be inefficient (as discussed earlier) but will lead to a stable configuration with predictable incomes. Suppose Jeffrey Sachs comes in and advises that transition to a free market will produce better results. Since farmers cannot coordinate production decisions, chaos is likely to result. Different models of farmers

[4] "Interview with David Card", Federal Reserve Bank of Minneapolis, The Region, December 2006, download from (accessed 12 September 2009). http://www.irle.berkeley.edu/faculty/card/research.html

expectations regarding price formation will lead to different results, but rational expectations cannot prevent substantial random variation in decision making driven solely by maximization of profits. The uncertainty and random variations in incomes generated by decentralized decision making would lead to substantial loss of efficiency relative to the planned economy.

All the above discussions along with the extensions discussed in Section 4 not only establish that the problem with the Marshallian demand analysis is not restricted to the purposefully simple setup in Section 2, but also lead us to propose a challenging problem, which appears to have stumped the intuitions of some of our leading economists: what are conditions which can be imposed upon a submarket of a general equilibrium model such that Marshallian supply and demand emerges as a rough approximation?

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