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Implementation of technological breakthroughs at sector level and the technology-bias¹

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Abstract

Different goods are produced by different sectors in an economy. The fact that sectors use different production technologies is named technology-bias. The technology-bias is well documented and has important theoretical implications for economic growth and unemployment. We provide a theoretical model that explains the technology-bias and its development. We provide empirical evidence on the development of the technology-bias and explain this development by using our model-results. Last not least, we discuss the implications of our findings for the existing growth literature and structural change literature.

Keywords: sector technology, implementation of technological progress, structural change, growth, multi-sector growth models

JEL-Codes: O14, O41

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1. Introduction

Different goods are produced by different sectors in an economy. In general, we can say that each sector has its own specific production technology. That is, technology differs across sectors. This fact is named “cross-sector technology bias”. Many multi-sector models assume the existence of such a technology bias and many models imply that this bias has important impacts on the aggregate growth rate of the economy and on unemployment. (For some references, see section 8.)

The aim of the paper is to provide a long-run growth model, which can explain the existence of technology-bias endogenously. Especially, we focus on cross-sector bias regarding capital-intensities, output-elasticities of capital and factor-income-shares. That is, we analyze why output-elasticities of capital and thus capital-intensities differ across sectors and we analyze how these differences develop over time. We also provide empirical evidence on this question.

Our model is a sort of multi-sector Ramsey-Cass-Koopmans-model. General technology breakthroughs are exogenous. They have to be implemented to improve the technology of sectors. Implementation requires some creative ideas and, hence, depends upon the amount of labor employed in a sector.

To our knowledge, the following literature is related to our paper:

- 1.) There is some literature, which endogenizes the *cross-sector bias in TFP*, for example Acemoglu and Guerrieri (2006) and Ngai and Samaniego (2011). For an overview of this literature, see Ngai and Samaniego (2011). In contrast to this literature, we focus on endogenizing the *cross-sector bias in output-elasticity of capital*.²
- 2.) Zuleta and Young (2007) present a two-sector model where a “backward sector” uses labor only and a “progressive sector” uses labor and capital; the output elasticity of capital in the progressive sector can be increased by investment. Due to these assumptions the backward sector cannot catch-up (since it cannot use capital), i.e. the technologies cannot converge. In contrast, we focus on the catching-up process of the backward sector.

² To see what the exact difference between our paper and the papers, which endogenize the TFP-growth rate, consider the paper by Acemoglu and Guerrieri (2006) as an example: The implicit sectoral production functions of Acemoglu and Guerrieri’s (2006)-model are depicted on page 15 of their paper (equation (27)). In fact these functions are of type Cobb-Douglas, like in our model. Acemoglu and Guerrieri (2006) endogenize the TFP (M_1 and M_2) of these production functions on page 36 of their paper (equations (61)). Hence, we can say that the *bias of TFP* is endogenous in their model. However, the output-elasticities of inputs (α_1 and α_2) are not endogenized in their model. That is, the *bias of output-elasticities of inputs* is exogenous in their model. We focus on the *bias of output-elasticities of inputs*; i.e., we try to analyze why α_1 and α_2 are different and how this difference develops over time.

Our theoretical results imply that sectoral technologies may converge or diverge, depending upon the sort of technological implementation (labor-augmenting or capital-augmenting), structural change patterns (i.e. changes in factor-allocation across sectors), distribution of capital-production across sectors and the nature of the breakthrough. Our empirical results imply that sector-technologies converged between 1948 and 1987 in the USA, which may be explained by prevalence of capital-augmenting technological implementation in the past and/or increasing employment-shares of technologically backward sectors.

Overall, our model implies that models, which assume exogenous technology-bias, omit important dynamics of structural change and aggregate growth. The reason is that technology-bias tends to be self-reinforcing, causes structural change (cross-technology factor-reallocation) and is affected itself by cross-technology factor reallocation. For a detailed discussion of model-implications for the existing literature see section 8.

In the next section, we present the assumptions and the equilibrium of the model. In section 3 we discuss the dynamics of the model when no technological breakthroughs occur. In section 4 we study the impacts of a sequence of technology breakthroughs on the sectoral technology-bias (via implementation). In section 5 we study how general structural-change-patterns affect the implementation of breakthroughs at sector level. Section 6 provides a summary of the factors which have an impact on the development of the technology-bias in our model. In section 7 we provide some evidence on the development of the sectoral technology bias. In section 8 we discuss the implications of our results for the existing literature. In section 9 we provide some concluding remarks.

2 Model assumptions

2.1 Production

Assumption 1: Many heterogeneous goods ($i = 1, \dots, n$) are produced in the economy; each good i is produced by a subsector i .

Assumption 2: Goods/subsectors $i = 1, \dots, m$ are assigned to sector A; goods/subsectors $i = m + 1, \dots, n$ are assigned to sector B ($n > m$). This is only a simplifying assumption; in fact, our arguments work even when there are more than two sectors.

Assumption 3: Capital (K) and labor (L) are inputs in Cobb-Douglas production functions.

Assumption 4: Technology is homogenous within a sector but differs across sectors; i.e. there is technology bias across sectors but not within sectors. For example, all subsectors $i = 1, \dots, m$ have the same technology, but subsectors $i = m + 1, \dots, n$ have

not the same technology as subsectors $i = m + 1, \dots, n$. Again, this is only a simplifying assumption without implications for our main results. It helps us to model the independency assumption in the next subsection.

Assumption 5: The degree of implementation of a general technological breakthrough depends upon the number of persons who are employed in a sector. That is, the more persons are employed in a sector, the better a technological breakthrough is implemented. This is a very important assumption; it is discussed extensively in Section 2.4.

Assumption 6: The implementation of a general technological breakthrough affects the TFP-growth rate and the output elasticity of capital and labor respectively. That is, implementation affects the marginal rate of technical substitution between capital and labor and thus the optimal capital intensity of a sector. Again, this is a very important assumption, which will be discussed extensively in Section 2.4.

These assumptions imply the following production functions of subsectors i :

$$(1a) Y_i = A(l_i L)^{\alpha(l_A L)} (k_i K)^{1-\alpha(l_A L)}, \quad i = 1, \dots, m$$

where

$$(1b) 0 < \alpha(l_A L) < 1$$

$$(1c) \frac{\dot{A}}{A} = g_A(l_A L)$$

$$(1d) l_A \equiv \sum_{i=1}^m l_i$$

$$(2a) Y_i = B(l_i L)^{\beta(l_B L)} (k_i K)^{1-\beta(l_B L)}, \quad i = m + 1, \dots, n$$

where

$$(2b) 0 < \beta(l_B L) < 1$$

$$(2c) \frac{\dot{B}}{B} = g_B(l_B L)$$

$$(2d) l_B \equiv \sum_{i=m+1}^n l_i$$

where Y_i denotes the output of good i ; l_i and k_i denote respectively the fraction of labor and capital devoted to production of good i (i.e. l_i is the employment share of subsector i and k_i is the capital share of sector i); K is the aggregate capital; L is aggregate labor; A (B) is a technology parameter of sector A (B). Note that I omit here the time index. Furthermore, note that the index i denotes goods/subsectors. l_A and l_B denote respectively the employment shares of sector A and sector B . We will define additional sector-variables later.

The fact that the TFP-growth rates (g_A and g_B) and the subsectoral output-elasticities of inputs ($\alpha(l_A L)$ and $\beta(l_B L)$) depend upon sectoral employment (equations (1b,c) and (2b,c)) comes from Assumptions 5 and 6. We discuss in Section 2.4 which assumptions are reasonable for the functional forms of $\alpha(l_A L)$, $\beta(l_B L)$, $g_A(l_A L)$ and $g_B(l_B L)$.

Assumption 7: All capital and all labor are used in the production of goods $i = 1, \dots, n$:

$$(3) \quad \sum_{i=1}^n l_i = 1; \quad \sum_{i=1}^n k_i = 1$$

Assumption 8: The amount of available labor grows at exogenous rate (g_L):

$$(4) \quad \frac{\dot{L}}{L} \equiv g_L$$

Assumption 9: All goods are consumed. Furthermore, only the good m can be used as capital.
(Note, that the model could be modified such that more than one good is used as capital e.g. in the manner of Ngai and Pissarides (2007).)

This assumption implies:

$$(5) \quad Y_i = C_i, \quad \forall i \neq m$$

$$(6) \quad Y_m = C_m + \dot{K} + \delta K$$

where C_i denotes consumption of good i ; δ denotes the constant depreciation rate of capital.

Assumption 10: Each subsector consists of many identical, marginalistic and profit-maximizing producers. There are no patent-rights; productivity-improving (accidental) inventions spill over to other producers immediately. (Eventual departures from this assumption are discussed in Section 9.) Overall, there is

perfect competition in each subsector. Furthermore, there is perfect factor-mobility within and across sectors.

This assumption implies that *individual* entrepreneurs have no incentive to undertake some costly action to increase their individual ability to implement a technology breakthrough.³ They do not undertake (costly) individual research nor do they increase the amount of learning-by-doing deliberately by increasing the factor-input above the perfect-competition optimum. Rather, they behave as producers in perfect-competition environment. For some discussion and the impacts of the departure from this assumption see Sections 2.4 and 9.

These assumptions imply that entrepreneurs regard prices, factor-prices and technology-parameters as exogenous (i.e. determined by the market). That is, the entrepreneurs are price-takers and technology-takers (i.e. they consider α , β , g_A and g_B as exogenous). Hence, profit maximization and perfect factor mobility imply the following optimality conditions:

$$w = p_i \partial Y_i / \partial (l_i L), \quad \forall i$$

$$r = p_i \partial Y_i / \partial (k_i K), \quad \forall i$$

$$\frac{p_i}{p_m} = \frac{\partial Y_m / \partial (l_m L)}{\partial Y_i / \partial (l_i L)} = \frac{\partial Y_m / \partial (k_m K)}{\partial Y_i / \partial (k_i K)}, \quad \forall i$$

where w is the real wage rate, r is the real rental rate of capital and p_i is the price of good i (and correspondingly p_m is the price of good $i = m$).

By using equations (1)-(3) these optimality conditions can be reformulated as follows:

$$(7a) \quad \frac{k_i}{l_i} = \frac{k_m}{l_m} \quad \text{for } i = 1, \dots, m$$

$$(7b) \quad \frac{k_i}{l_i} = \frac{\alpha(l_A L)}{1 - \alpha(l_A L)} \frac{1 - \beta(l_B L)}{\beta(l_B L)} \frac{k_m}{l_m} \quad \text{for } i = m + 1, \dots, n$$

³ Consider the following explanations to understand why individual entrepreneurs have no incentive to undertake any costly action to improve their productivity in such an environment (immediate spill-overs and no patent rights): An individual entrepreneur, who undertakes some costly action to improve the production process, bears the costs of this action and the profit from the productivity improvement. All other entrepreneurs would have the same productivity improvements, but no costs from the costly action. Hence, they could reduce their prices more than the inventing entrepreneur could. Hence, the inventing entrepreneur would loose competitiveness and drop out of the market.

$$(7c) \frac{P_i}{P_m} = 1 \quad \text{for } i = 1, \dots, m$$

$$(7d) \frac{P_i}{P_m} = \frac{\alpha(l_A L) \left(\frac{k_m K}{l_m L} \right)^{1-\alpha(l_A L)}}{\beta(l_B L) \left(\frac{k_i K}{l_i L} \right)^{1-\beta(l_B L)}}, \quad \text{for } i = m+1, \dots, n$$

$$(7e) r = p_m A(l_A L) [1 - \alpha(l_A L)] \left(\frac{l_m L}{k_m K} \right)^{\alpha(l_A L)}$$

$$(7f) w = p_m A(l_A L) \alpha(l_A L) \left(\frac{k_m K}{l_m L} \right)^{1-\alpha(l_A L)}$$

Note that these equations were derived under the assumption that α , β , g_A and g_B are exogenous for the producers, due to Assumption 10.

2.2 Households

Now, we assume that there are many marginalistic households. We assume here that households are identical, although many dimensions of household-heterogeneity could be introduced into this model without much mathematical difficulties. However, household-heterogeneity will be a topic of a separate paper.

In the following, the superfix ι denotes the corresponding variable of the individual household. For example, while E stands for consumption expenditures of the whole economy, E^ι stands for consumption expenditures of the household ι . We assume that there is an arbitrary and large number of households ($\iota = 1, \dots, x$), sufficiently large to constitute marginalistic behavior of households.

We assume the following utility function, which is quite similar to the utility function used by Kongsamut et al. (1997, 2001):

$$(8a) U^\iota = \int_0^\infty u(C_1^\iota, \dots, C_n^\iota) e^{-\rho t} dt, \quad \forall \iota, \quad \rho > 0$$

where

$$(8b) u(C_1^\iota, \dots, C_n^\iota) = \ln \left[\prod_{i=1}^n (C_i^\iota - \theta_i^\iota)^{\theta_i} \right], \quad \forall \iota$$

$$(8c) \sum_{i=1}^m \theta_i^\iota = 0 \quad \forall \iota$$

$$(8d) \sum_{i=m+1}^n \theta_i^t = 0 \quad \forall t$$

$$(8e) \sum_i \omega_i = 1$$

where C_i^t denotes the consumption of good i by household t ; ρ is the time-preference rate. θ_i^t can be interpreted as a subsistence level (if θ_i^t is positive) or as an endowment (if θ_i^t is negative) of household t regarding good i (see also e.g. Kongsamut et al. (2001)). In fact this preference structure is non-homothetic across goods in general; hence, increasing income is associated with demand-shifts across goods as shown by Kongsamut et al. (2001). Furthermore, this preference-structure features a non-unitary price elasticity of demand; hence, structural change may be caused by relative-price-changes as discussed by Ngai and Pissarides (2007).

Restrictions (8c,d) are imposed here for analytical reasons. They simplify the analysis and help us to isolate the determinants of technology-bias. They allow for the existence of a partially balanced growth path (see later), which makes our dynamic analysis traceable. Furthermore, they allow us to determine exogenously whether factor reallocation takes place across technology or not: in fact, restrictions (8c,d) prevent factor reallocation across technology A and B. Hence, later we will be able to determine whether factor reallocation across technology takes place or not, which is helpful for isolating the determinants of technology convergence/divergence, as we will see.

Furthermore, each household has the following dynamic constraint:

$$(9) \dot{W}^t = w\bar{L} + (r - \delta)W^t - E^t, \quad \forall t$$

where W^t is the wealth/assets of household t , E^t are consumption expenditures of household t and \bar{L} is the (exogenous) labor-supply of household t . The latter implies that each household supplies the same amount of labor at the market.

The dynamic constraint (A.17) is standard (compare for example Barro and Sala-i-Martin (2004), p.88). It implies that the wealth of the household increases by labor-income and by (net-) interest-rate-payments and decreases by consumption expenditures.

Note that I assume that the labor supply of each household is exogenously determined.

Consumption expenditures of a household are given by

$$(10) E^t = \sum_i p_i C_i^t, \quad \forall t$$

Each household maximizes its life-time-utility (8) subject to its dynamic constraint (9)-(10). Since this optimization problem is time-separable (due to the assumption of separable time-

preference and marginalistic household), it can be divided into two steps; see also, e.g., Foellmi and Zweimüller (2008), p.1320f:

- 1.) Intratemporal (static) optimization: For a given level of consumption-budget (E^t), the household optimizes the allocation of consumption-budget across goods.
- 2.) Intertemporal (dynamic) optimization: The household determines the optimal allocation of consumption-budget across time.

Intratemporal optimization:

The household maximizes its instantaneous utility (8b-d) subject to the constraint (9)-(10), where it regards the consumption-budget (E^t) and prices (p_i) as exogenous. (Remember that the household is price-taker.) This yields the following optimality conditions:

$$(11a) \quad C_i^t = \frac{\omega_i}{\omega_m} \frac{C_m^t - \theta_m^t}{p_i} + \theta_i^t, \quad \forall i, t$$

$$(11b) \quad E^t = \frac{C_m^t - \theta_m^t}{\omega_m}, \quad \forall t$$

Intertemporal optimization

Inserting the intratemporal optimality conditions into the instantaneous utility function yields after some algebra (where we use here equations (7c,d) as well):

$$(12a) \quad u(.) = \ln E^t - \bar{\omega}_B \ln p_B + \bar{\omega}, \quad \forall t$$

where

$$(12b) \quad \bar{\omega}_B \equiv \sum_{i=m+1}^n \omega_i$$

$$(12c) \quad \bar{\omega} \equiv \sum_i \omega_i \ln(\omega_i)$$

$$(12d) \quad p_B \equiv \frac{p_i}{p_m} = \left(\frac{\alpha}{\beta} \right)^\beta \left(\frac{1-\alpha}{1-\beta} \right)^{1-\beta} B^{-1} A^{\beta/\alpha} \left(\frac{Kk_m}{Ll_m} \right)^{\beta-\alpha}, \quad i = m+1, \dots, n.$$

Now, we have determined the instantaneous utility as function of consumption-budget (and prices). (Remember that the household is price-taker, i.e. prices are exogenous from the household's point of view.) Inserting (12) into (8) yields

$$(13) \quad U^t = \int_0^\infty \left(\ln E^t - \bar{\omega}_B \ln p_B + \bar{\omega} \right) e^{-\rho t} dt, \quad \forall t$$

Thus, the intertemporal optimization problem is to optimize (13) subject to the dynamic constraint (9). This is a typical optimal control problem, which can be solved by using a

Hamiltonian. E^t is control-variable and W^t is state variable. The prices (p_B) and factor prices (w and $r - \delta$) are regarded by the household as exogenous (since the household is marginalistic and thus price-taker.) Remember that \bar{L} is exogenous.

In this way we can obtain the following intertemporal optimality condition after some algebra:

$$(14) \frac{\dot{E}^t}{E^t} = r - \delta - \rho, \quad \forall t$$

Note that here and in the following we use use(d) good $i = m$ as numeraire; thus

$$(15) p_m = 1$$

2.3 Aggregates, equilibrium and market clearing

We define now aggregate output (Y) as follows:

$$(16a) Y \equiv \sum_{i=1}^n p_i Y_i$$

Aggregate consumption expenditures (E), aggregate consumption of good i (C_i) and aggregate subsistence needs/endowments (θ_i) are given by the sum of their individual counterparts respectively, i.e.

$$(16b) E \equiv \sum_t E^t = \sum_{i=1}^n p_i C_i$$

$$(16c) C_i = \sum_t C_i^t, \quad \forall i$$

$$(16d) \theta_i = \sum_t \theta_i^t, \quad \forall i$$

We assume that all markets are in equilibrium and there is market clearing. That is equations for the goods-market-clearing (5) and (6) hold. There is no unemployment, i.e. labor-market-clearing requires

$$(17) L = \sum_t \bar{L}$$

Last not least, since the wealth/assets can only be invested in production-capital (K), the following relation must be true if there is capital-market-clearing

$$(18) K = \sum_t W^t$$

(see also, e.g. Barro and Sala-i-Martin (2004), p.97). That is, all assets are invested in capital (capital-market-clearing).

By using these aggregate definitions, market clearing conditions and the optimality conditions from the previous sections, we can obtain the following equations describing the development of aggregates, sectors and subsectors after some algebra:

Subsectors:

$$(19) C_i = \frac{\omega_i}{\omega_m} \frac{C_m - \theta_m}{p_i} + \theta_i, \quad \forall i$$

Sectors:

$$(20) l_B = \frac{\bar{\omega}_B E}{Y} \left[\alpha + (1 - \alpha) \frac{l_m}{k_m} \right] \frac{\beta}{\alpha}$$

$$(21) l_A = 1 - l_B$$

Aggregates:

$$(22) \frac{\dot{E}}{E} = r - \delta - \rho = \left(\frac{l_m}{k_m} \right)^\alpha (1 - \alpha) AL^\alpha K^{-\alpha} - \delta - \rho$$

$$(23) \dot{K} = Y - \delta K - E$$

$$(24) Y = \left[\alpha + (1 - \alpha) \frac{l_m}{k_m} \right] \left(\frac{k_m}{l_m} \right)^{1-\alpha} AL^\alpha K^{1-\alpha}$$

$$(24) \frac{l_m}{k_m} = 1 - \frac{\beta - \alpha}{\alpha(1 - \alpha)} \bar{\omega}_n \frac{E}{\left(\frac{k_m}{l_m} \right)^{1-\alpha} AL^\alpha K^{1-\alpha}}$$

Note that in these equations α and β are still functions of sectoral employment, i.e. $\alpha = \alpha(l_A L)$ and $\beta = \beta(l_B L)$. However, we omitted the functional arguments for the sake of clarity of the formulas.

Now the remaining task is to define the functions $\alpha = \alpha(l_A L)$ and $\beta = \beta(l_B L)$. This is done in the following section.

2.4 Technological progress and its implementation

2.4.1 General breakthroughs and their implementation

We assume that some general technological breakthroughs occur during our observation period. Such breakthroughs may be rare “big breakthroughs”, which have fundamental and long-lasting impacts on the production-structure of the economy. Examples of such breakthroughs may be inventions which lead to Kondratjew-waves (e.g. steam engine, microchip). On the other hand the breakthroughs in our model may be as well some “smaller inventions” which occur more frequently.

We do not model the emergence of such breakthroughs endogenously. In fact, this has been done in endogenous growth theory (in research and development models). Why such breakthroughs occur and at which rate they occur is not in focus of our model anyway. We are rather interested in the pattern of their implementation across sectors. Studying this pattern does not necessarily require endogenous modeling of technological breakthroughs. Therefore, we keep them exogenous. However, we may imagine that such breakthroughs come from basic research.

There are two important aspects regarding the effects of such breakthroughs on production.

- 1.) General technological breakthroughs, such as electricity or microchip, do not directly improve the technologies of sectors and industries. Rather a lot of research, ideas and/or inventions are necessary to implement a general breakthrough at sector level. For example, improvements in productivity of services, which come from the invention of the microchip, required a lot of ideas and research in software programming and hardware before they increased the productivity of services. That is, general breakthroughs require further breakthroughs to be implemented. Hence, we can distinguish between general breakthroughs and “implementation breakthroughs”.
- 2.) Depending upon the industry or sector, different sorts of ideas are necessary to improve the production technology. The improvement of the productivity of a banker by using a microchip requires different sorts of knowledge/ideas in comparison to the improvement of the productivity of a car-producer by a microchip. In other words, each industry/sector requires its own sector-specific ideas/knowledge/inventions to implement a general breakthrough. Therefore, we assume that there are no spill-overs between sectors. (On the other hand, within a sector or industry strong spill-overs may exist.) Note that there is some discussion about spill-overs across sectors. For example, it is argued that technological development in the manufacturing sector has led to productivity-improvements in the services sector. This fact could be modeled as

a spill-over effect from manufacturing to services. However, at the same time these inventions in the manufacturing could be modeled as general breakthroughs, which have to be implemented in services (and manufacturing). We choose the latter way of modeling.

These two points are essential for the cross-sector technology-bias, since the need for sector specific ideas to implement breakthroughs constitutes a basis for cross-sector technology bias.

The following assumption is aimed to keep our analysis traceable:

Assumption 11: The implementation of a general breakthrough occurs within a finite period of time after the happening of the breakthrough.

Although we may also think of the implementation process as lasting forever, we have to restrict the period of time in order to keep our model solvable. In some sense, we may imagine that the relevant/important part of the implementation occurs in some finite period; the impacts of implementations, which happen long time after the breakthrough has happened, may be less relevant (in comparison to the impacts of the implementations of newer breakthroughs).

2.4.2 The relation between sector-size and degree of implementation

We assume that the degree of implementation of a general breakthrough depends upon the size of a sector. We can think of two aspects which relate sector size to the degree of implementation:

- 1.) The more labor is employed in a sector, the more creative power is concentrated on the production processes of a sector. Hence, the probability that an implementation-idea arises during the production process is higher. This aspect is closely related to learning-by-doing-models of endogenous-growth. Overall, we can assume that the more labor is employed in a sector the better the technology improvement through implementation of general breakthroughs. The ideas on implementation of a breakthrough occur accidentally in our model.
- 2.) Implementation research may require the overcoming of large fix costs and sunk-cost and may be associated with high risk. Hence, large sectors may have more financial power to overcome such costs.
- 3.) Of course, there are further aspects, which may relate market-size to degree of implementation, for example the relationship between market-size and competition

and incentives to be competitive/innovative. However, I have no clear ideas on straightforward/unambiguous chains of arguments regarding this. For some related discussion, see e.g. Klevorick et al. (1995) and Pavitt (1985). Note that “market-size” of a sector (i.e. the sector’s share in aggregate output) is not the same as sector-size (which we measure by sector’s employment share; see later Assumption 12): Even a very large market may have a very small sector-size, if, e.g., the production process is very capital-intensive (thus relatively few labor is employed to produce a large value of output).

Overall, we can summarize this discussion in the following assumption:

Assumption 12: There is a positive relationship between sector-size and degree of implementation of breakthroughs. That is, the bigger the sector, the stronger the technology-improvement through implementation of a breakthrough. Sector-size is measured by relative employment-shares of the sectors (l_A, l_B) : The higher the employment share of a sector, the larger the size of the sector.

Note that this assumption is reflected by the fact that we have expressed our production function parameters as functions of sectoral employment, i.e. we assumed $\alpha = \alpha(l_A L)$, $\beta = \beta(l_B L)$, $g_A = g_A(l_A L)$ and $g_B = g_B(l_B L)$. That is, we measure sector-size by its employment share. This seems to be the first choice, since creativity requires the involvement of humans.

2.4.3 The impacts of implementation on sector technology

We assume that the implementation of a breakthrough does not only affect the TFP-growth rate but also the output-elasticity of inputs. That is, when a breakthrough is implemented, the marginal rate of technological substitution is affected. Thus, the optimal capital intensity of a sector changes through implementation.

Acemoglu (2002) provides a discussion and “microfoundation” of this fact. In a model, where technological progress may be labor-augmenting and capital-augmenting, he shows that technological progress leads to changes of factor-income-shares and marginal rate of technical substitution. If technological progress is labor-augmenting (capital-augmenting), the labor-income-share (capital-income-share) increases.

Since this form of microfoundation is quite complex, we omit it here in order to keep our model traceable. Instead, we simply assume that implementation affects the parameter of our Cobb-Douglas production function and we distinguish two cases:

Assumption 13:

- 1.) If implementation is *capital-augmenting*, the output-elasticity (and thus the income-share) of capital increases and the TFP-growth rate increases. That is, $\alpha(l_A L)$ and/or $\beta(l_B L)$ decreases and g_A and/or g_B increases.
- 2.) On the other hand, if implementation is *labor-augmenting*, the labor-income-share increases and the TFP-growth rate increases. That is, $\alpha(l_A L)$ and/or $\beta(l_B L)$ increases and g_A and/or g_B increases. We know from standard growth theory (e.g. Ramsey-Cass-Koopmans model) that the equilibrium growth rate of output is among others given by the TFP-growth rate divided by the output-elasticity of labor, e.g. g_A / α . Later, we will see that in our model the PBGP-growth rate is given by g_A / α . Hence, an increase in α reduces the equilibrium growth rate. In general, technological progress is not associated with a decrease in the equilibrium growth rate. Therefore, in the case of labor-augmenting technological progress we assume that the increase in g_A is stronger than the increase in α ; hence the equilibrium growth rate g_A / α increases. (The same arguments apply to sector B; i.e. we assume that labor-augmenting technological progress increases g_B / β .)

2.4.4 Summary of the impacts of a technological breakthrough

Now, we can sum up the discussion of section 2.4 as follows:

We assume that several technological breakthroughs occur over time. Each breakthrough is implemented over a finite period of time. The implementation of a breakthrough in a sector improves the technology of all subsectors which belong to this sector (within-sector spill-overs). However, the subsectors that belong to the other sector do not profit from this implementation (no cross-sector spill-overs). If the breakthrough is implemented in sector A (B), g_A (g_B) increases; the increase in g_A (g_B) is the stronger, the higher $l_A L$ ($l_B L$) is. Regarding the impacts on output-elasticities of inputs we have to consider two cases:

- 1.) Implementation is labor-augmenting: Implementation in sector A (B) increases α (β), where the increase in α (β) is the stronger, the higher $l_A L$ ($l_B L$) is. The overall-change in g_A / α (g_B / β) is positive.
- 2.) Implementation is capital-augmenting: Implementation in sector A (B) decreases α (β), where the decrease in α (β) is the stronger, the higher $l_A L$ ($l_B L$) is.

In fact, the model is now fully specified and can now analyze the development of technology-bias over time. In the next section we analyze the model-dynamics when there are no breakthroughs. Then, in section 4, we analyze how a technology break-through affects this technology-bias. In section 5, we study the impact of structural change on this relationship. Afterward, there are several sections in which we discuss and summarize our results.

3 Dynamics of the model without technological breakthroughs

In this section we assume that there are no technological breakthroughs; hence, α , β , g_A and g_B are exogenous and constant.

Definition 1: A partially balanced growth path (PBGP) is an equilibrium growth path where aggregates (Y , K and E) grow at a constant rate.

Note that this definition does require balanced growth for aggregate variables. However, it does not require balanced growth for subsectoral variables (e.g. for subsectoral outputs).

Lemma 1: When α , β , g_A and g_B are assumed to be exogenous and constant, equations (22) to (24) imply that there exists a unique PBGP, where aggregates (Y , K , and E) grow at constant rate g^* and where l_m / k_m is constant. The PBGP-growth rate is given by

$$g^* = \frac{g_A}{\alpha} + g_L.$$

Proof: is self-evident.

Lemma 2: a) A saddle-path, along which the economy converges to the PBGP, exists in the neighborhood of the PBGP. b) The PBGP is locally stable.

Proof: See APPENDIX.

Lemma 2 ensures that, if a technological breakthrough shifts the economy away from the PBGP, the economy will converge to the PBGP again, provided that the departure from the PBGP is not too strong.

Lemma 3: Along the PBGP, labor is not reallocated between sectors A and B. That is, labor is not reallocated across technology, i.e. l_A and l_B are constant.

Proof: This lemma is implied by equations (20) and (21) and by Lemma 1. **Q.E.D.**

Note, however, that it can be easily shown that labor is reallocated within each of the sectors A and B. **Furthermore, note that in section 5 we will allow for labor reallocation between sectors A and B.**

4 Impacts of a technological breakthrough: self-reinforcing technology-convergence/divergence

Now we study how a technology breakthrough affects the situation described in the previous section. We assume an initial situation where the technologies differ across sectors.

As discussed in section 2.4, we assume that a technology breakthrough induces a finite period of time where the breakthrough is implemented. During this period of implementation, the sectoral technologies change. When the implementation-period is finished, the economy converges to the new PBGP.⁴

As we will show now, the change in sectoral technologies during the implementation-period leads to a change of cross-technology factor-allocation (l_A, l_B). The change in this allocation affects the ability of the sectors to implement the next breakthrough (see e.g. section 2.4.4). Hence, over time (and over a sequence of breakthroughs) the sectoral technologies may converge or diverge over time.

We will elaborate now the factors that determine whether the technologies converge or diverge. To do so we calculate the employment shares along the PBGP. By using equations (20)/(21) and some of the equations from APPENDIX we obtain the following employment shares along the PBGP:

⁴ Note that the discussion in APPENDIX implies that the PBGP-values of the economy depend upon the technology parameters of the production functions. Hence, we know that technology-implementation leads to a departure from the old PBGP and convergence to the new PBGP (after the period of implementation is finished).

$$(25) \quad l_B^* = \beta \frac{1 - \alpha + \alpha \frac{\rho + \delta + g_L + \frac{g_A}{\alpha}}{\rho}}{(\beta - \alpha)\bar{\omega}_B + \alpha \frac{\rho + \delta + g_L + \frac{g_A}{\alpha}}{\rho}}$$

$$(26) \quad l_A^* = 1 - l_B^*$$

An asterisk denotes here the PBGP-value.

These equations imply that:

$$(27) \quad \frac{\partial l_B^*}{\partial \beta} > 0 \quad \frac{\partial l_A^*}{\partial \beta} < 0$$

$$(28) \quad \frac{\partial l_B^*}{\partial \alpha} < 0 \quad \frac{\partial l_A^*}{\partial \alpha} > 0$$

Note that in the calculation of (28), we assumed that an increase in α is associated with an increase in g_A , such that g_A/α increases if α increases, according to our discussion in point 2) of Assumption 13.

Now we can postulate the following:

Theorem 1: *Implementation of breakthroughs leads to a change in the PBGP-values of the employment-shares of sectors A and B (l_A and l_B). That is, implementation of breakthroughs leads to cross-technology labor-reallocation.*

Proof: This theorem is implied by equations (27)/(28) and Assumption 13. **Q.E.D.**

Note that this theorem holds even if implementation is such that α and β increase by the same amount. The reason is the following: a change in α and/or β leads to a change in average economy-wide output-elasticity of labor; i.e. the average capital-intensity of the economy (K/L) changes (as implied by Lemma 1). Since sector A (and especially subsector m) produces capital, a change in average capital intensity affects the employment-share of sector A, ceteris paribus. (See also equations (20) and (21) and note that aggregate investment is equal to $Y - E$).

Of course in general, the change in l_A and l_B does not only come from this change in aggregate capital demand. Additionally, the following fact determines the allocation (l_A, l_B):

Optimal factor-allocation across sectors is determined by the profitability of factor-inputs in sectors, since we assume perfect factor mobility across sectors. That is, sectors, which have relatively high output-elasticity of labor, employ more labor in comparison to sectors, which have relatively low output-elasticity of labor. (Previously, this fact has been shown by Acemogly and Guerrieri (2008) in another model.) Therefore, a change in cross-sector technology-bias ($\frac{\alpha}{\beta}$) induces an adjustment of factor-input-shares, such that optimal factor-input-shares are achieved. To understand this fact, note that equations (20) and (21) imply that a change in $\frac{\alpha}{\beta}$ induces a change in (l_A, l_B) even if we keep l_m/k_m , E and Y constant.

Hence, we can postulate the following corollary:

Corollary from Theorem 1: *The change in cross-technology labor allocation (l_A, l_B) , which has been postulated in Theorem 1, comes from two forces:*

- 1.) A change in sectoral output-elasticities of inputs affects the aggregate capital demand; since only sector A produces capital, the changes in aggregate capital demand induce factor-reallocation across sectors A and B.*
- 2.) A change in cross-sector technology-bias (α/β) changes the optimal factor-allocation across sectors. Thus, when there is free factor-mobility across sectors, cross-sector factor reallocations are induced in order to achieve the new optimal cross-sector factor allocation.*

Theorem 1 is the basis for technology convergence and divergence, since cross-sector labor-reallocation changes the sector's relative potential to implement future breakthroughs (according to Assumption 13).

According to the discussion in section 2.4.4 we distinguish between two cases.

1.) Implementation is labor-augmenting

According to our discussion in section 2.4.4 and equations (27)/(28) we can postulate the following causal chain: A breakthrough leads to implementation in sector A (B), which leads to increase in α (β), which leads to increase in $l_A L$ ($l_B L$), which leads to better implementation of the next breakthrough.

2.) Implementation is capital-augmenting

According to our discussion in section 2.4.4 and equations (27)/(28) we can postulate the following causal chain: A breakthrough leads to implementation in sector A (B), which

leads to a decrease in α (β), which leads to a decrease in $l_A L$ ($l_B L$), which leads to weaker implementation of the next breakthrough.

Hence, we can postulate the following theorem:

Theorem 2: a) *If implementation is labor-augmenting, the implementation of a breakthrough in sector A (B) increases the employment-share of sector A (B). If implementation is capital-augmenting, the implementation of a breakthrough in sector A (B) reduces the employment-share of sector A (B).*

b) *If implementation is labor-augmenting, the implementation of a breakthrough in sector A (B) increases the ability of sector A (B) to implement the next breakthrough. If implementation is capital-augmenting, the implementation of a breakthrough in sector A (B) reduces the ability of sector A (B) to implement the next breakthrough.*

Proof: This theorem is implied by equations (27)/(28) and Assumptions 12 and 13. **Q.E.D.**

Definition 2: *Technology-bias is defined as $|\alpha - \beta|$. The higher $|\alpha - \beta|$, the stronger technology-bias.*

Theorem 3: *Provided that there is a sequence of technological breakthroughs over a period of time, the following factors jointly determine whether technology-bias decreases or increases over this period of time:*

- a) *the sort of implementation of technological breakthroughs (labor-augmenting vs. capital-augmenting implementation)*
- b) *the magnitude of the increase in (productivity) growth rate of capital-goods production (g_A / α), which is induced by the implementation of the breakthroughs*
- c) *the sectors in which the breakthroughs are implemented (implementation in sector A, sector B or in both sectors)*
- d) *the correlation between sector size (l_A, l_B) and sector technology (α, β)*
- e) *the difference between sector sizes ($l_A - l_B$).*

Proof: First, we prove this theorem for a special case; then we argue that this proof can be generalized. We first assume a reference case and then show the differences to this reference case.

Reference case: Assume that $l_A > l_B$. Furthermore assume that there is a breakthrough which is implemented in both sectors in labor-augmenting manner. According to Assumption 13, the

increase in α ($\Delta\alpha$) is larger than the increase in β ($\Delta\beta$), i.e. $\Delta\alpha > \Delta\beta$, where $\Delta\alpha > 0$ and $\Delta\beta > 0$. Hence, if $\alpha > \beta$ before implementation, the technology-bias ($|\alpha - \beta|$) is increased by implementation.

Proof of part a): In contrast to the Reference case, assume that there is a breakthrough which is implemented in both sectors in *capital*-augmenting manner, ceteris paribus. Thus, according to Assumption 13, α decreases more strongly than β does, i.e. $|\Delta\alpha| > |\Delta\beta|$, where $\Delta\alpha < 0$ and $\Delta\beta < 0$. Hence, if $\alpha > \beta$ before implementation, the technology-bias ($|\alpha - \beta|$) is reduced by implementation, in contrast to the reference case. This fact proves part a) of Theorem 3.

Proof of part c): In contrast to the Reference case, assume that there is a breakthrough which is implemented in labor-augmenting manner in sector B only, ceteris paribus, i.e. $\Delta\alpha = 0$ and $\Delta\beta > 0$. Hence, if $\alpha > \beta$ before implementation, the technology-bias ($|\alpha - \beta|$) is reduced by implementation, in contrast to the reference case. This fact proves part c) of Theorem 3.

Proof of part d): In contrast to the Reference case, assume that $\alpha < \beta$ before implementation, ceteris paribus. In this case, technology-bias is reduced by implementation, in contrast to the Reference case. This fact proves part d) of Theorem 3.

Proof of part b): Now assume the same situation as in the reference case. As shown there, the implementation of the breakthrough increases α and β , i.e. $\Delta\alpha > 0$ and $\Delta\beta > 0$. Equations (27)/(28) imply that these increases change the labor-allocation (l_A, l_B). If this reallocation process is such that l_A becomes smaller than l_B , the implementation of the next breakthrough will yield mirror-image effects on the technology-bias in comparison to the initial reference case. That is, this second breakthrough is implemented more strongly in sector B, which would reduce the technology-bias. Now, the question is, under which circumstances is the reallocation process, which is induced by the first breakthrough, such that l_A becomes smaller

than l_B . This can only happen if $\frac{\partial l_B}{\partial \beta} > \frac{\partial l_A}{\partial \alpha}$, where $l_A + l_B = 1$.⁵ In this case it can happen that

the changes in employment shares overweight the changes in output-elasticities of capital (at least after a long sequence of many breakthroughs) such that at some point of time breakthroughs become better implementable in sector B. After some algebra it can be shown

⁵ If $\frac{\partial l_B}{\partial \beta} \leq \frac{\partial l_A}{\partial \alpha}$, l_A cannot become smaller than l_B during the implementation of the first breakthrough, since

in this phase $\Delta\alpha > \Delta\beta$. That is, if $\frac{\partial l_B}{\partial \beta} \leq \frac{\partial l_A}{\partial \alpha}$ and if $\Delta\alpha > \Delta\beta$ l_A increases more than l_B .

that $\frac{\partial l_B}{\partial \beta} > \frac{\partial l_A}{\partial \alpha}$ can be satisfied only if the change in g_A/α , which is induced by the implementation of the first breakthrough, is relatively small. (If g_A/α is relatively large $\frac{\partial l_B}{\partial \beta} < \frac{\partial l_A}{\partial \alpha}$). Note that g_A/α determines the PBGP-growth rate of capital-goods-production, since capital goods are produced in sector A only (see also Lemma 1).

Proof of part e): In the light of proof of part b) we can also postulate, that l_A does not become smaller than l_B during the first breakthrough, if the increase in g_A/α is relatively large (but not too large) and the difference between l_A and l_B is relatively large before the first breakthrough. In this case, the fact that $\frac{\partial l_B}{\partial \beta} > \frac{\partial l_A}{\partial \alpha}$ cannot overweight the relatively strong difference between $\Delta\alpha$ and $\Delta\beta$. Hence, the difference between l_A and l_B (i.e. the difference in implementability) is decisive for the development of the technology bias.

Note that it can be shown in analogous ways that Theorem 3 is not only valid in our Reference case, but in all the other cases (e.g. if $l_A < l_B$). The proof of this fact is obvious.

Q.E.D.

We discuss this theorem in Section 6.

5 The impacts of cross-technology structural change

We define here structural change as changes in employment shares of (sub)sectors. That is, we say that structural change takes place if (sub)sectoral employment shares change. It can be easily shown that structural change takes place even along the PBGP in our model.

Cross-technology structural change means here that labor is reallocated across technology. Hence, in our model, changes in (l_A, l_B) indicate that cross-technology structural change takes place.

In this section we show that cross-technology structural change can affect the technology-bias; i.e. structural change can lead to technology convergence/divergence.

Structural change is a well known fact and has been modeled by, e.g., Baumol (1967), Kongamut et al. (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008) and Foellmi and Zweimueller (2008).

The literature on structural change implies that structural change (across technology) is caused by several determinants; especially, non-homothetic preferences (as modeled by

Kongsamut et al. 2001), cross-sector-differences in TFP-growth (as modeled by Ngai and Pissarides 2007) and cross-sector-differences in capital-intensities (as modeled by Acemoglu and Guerrieri 2008). Although all these determinants are included in our model, our parameter-restriction (8c,d) prevent them from taking effect across technology; hence, l_A and l_B are constant along the PBGP. Instead of deviating from parameter-restrictions (8c,d), which would make our analysis quite complicated, we model structural change as follows: We assume that the utility-weights of technology change exogenously; i.e. $\bar{\omega}_B$ changes. This leads to changes in demand and to structural change across technology. Hence, we do not model the structural change (mentioned above) explicitly. Note that our way of modeling has, in fact, very similar dynamic implications in comparison to the implications of structural change models, which model the structural change determinants explicitly. In some sense, our way of modeling only omits the microfoundations, which are anyway provided by the previous literature.

Lemma 4: *A onetime change in $\bar{\omega}_B$ causes a deviation from the old PBGP and convergence to the new PBGP.*

Proof: This lemma is implied by equations (C.5) and Lemma 2. **Q.E.D.**

Theorem 4: *a) A onetime increase in $\bar{\omega}_B$ induces labor-reallocation from sector A to sector B, provided that $\alpha > \beta$; i.e. l_A decreases and l_B increases. b) A onetime increase in $\bar{\omega}_B$ induces labor-reallocation from sector B to sector A, provided that $\alpha < \beta$; i.e. l_A increases and l_B decreases. c) The stronger the difference between α and β , the stronger the change in l_A and l_B . d) Analogous results are obtained for the case that there is a onetime decrease in $\bar{\omega}_B$.*

Proof: This lemma is implied by equation (25).

Corollary from Theorems 2, 3 and 4: *Depending on the structural change pattern (i.e. whether $\bar{\omega}_B$ increases or decreases), structural change may lead to technology convergence or divergence between sectors A and B.*

In fact, the explanation of this corollary is simple: Structural change affects the employment share of a sector and thus sector's ability to implement a breakthrough. For example, if the income elasticity of demand of sector B is relatively high, factors are reallocated to sector B.

This fact increases the ability of sector B to implement breakthroughs. Hence, if sector B was technologically backward in the past, structural change (induced by income elasticity of demand) may lead to technology convergence over time. This development may be reflected by the reality: the backward sector B may be interpreted as the services sector (or at least as some very labor-intensive subsectors of the services sector) that may catch up technologically in future due to high income elasticity of demand.

By using similar arguments a case can be constructed where it may happen that technology divergence occurs: if, e.g., income elasticity of demand of the technologically backward sector is relatively low, technology divergence may be caused by structural change.

See also the discussion in section 8 for some further arguments.

6 Summary of factors which determine technology-bias

In our model (especially in Theorem 3 and in Corollary from Theorems 2, 3 and 4) we have shown the relevance of several factors for the explanation of technology-bias:

- a) *the sort of implementation of technological breakthroughs (labor-augmenting vs. capital-augmenting implementation)*
- b) *the magnitude of the increase in the (productivity) growth rate of capital-goods production (g_A / α), which is induced by the implementation of breakthroughs*
- c) *the sectors in which the breakthroughs are implemented (implementation in sector A, sector B or in both sectors)*
- d) *the correlation between sector size (l_A, l_B) and sector technology (α, β)*
- e) *the difference between sector sizes ($l_A - l_B$).*
- f) *structural change*

Now we provide intuitive explanations regarding the question why these factors affect the development of technology-bias:

a.) The reasons for the relevance of the sort of implementation are the following: In general, labor-augmenting technological progress reduces the optimal capital-intensity. Hence, the sectors, which implement the breakthroughs, have to increase their labor input in order to profit from the implementation. However, in contrast to capital, labor cannot be accumulated; hence, the sectors, which implement the breakthrough, pull labor from sectors, which do not implement the breakthrough. Since breakthrough-implementation is more successful in large

sectors than in small sectors⁶, labor is withdrawn from small sectors and reallocated to large sectors. This fact reduces the ability of small sectors to implement future breakthroughs (less creative potential). In contrast, capital-augmenting implementation increases the capital intensity. Hence, big sectors can release some part of their labor-force, since it can be substituted by capital (which can be accumulated endogenously). This labor-force is reallocated to small sectors, which increases their creative potential and thus ability to implement future breakthroughs. (See also the Corollaries from Theorems 1 and 2.)

b.) If productivity of capital-goods-production is increased by implementation of breakthroughs, factors are reallocated to the sector, which produces capital. This factor-reallocation affects the ability of these sectors to implement future breakthroughs. Thus, the development of the technology bias is affected by this fact. This effect arises only if capital-goods-production is not evenly distributed across all sectors. In our model this is the case: only sector A produces capital-goods. (Thus, the productivity of sector A corresponds to productivity of capital-goods-production.) Empirical evidence implies that this assumption is preferable: e.g. in the USA, capital-goods are produced primarily by the manufacturing sector and output-elasticity of capital differs across consumption-goods and capital-goods; see e.g. Valentinyi and Herrendorf (2008).

c.) This point is relatively obvious but seems to be relatively important. For example, if breakthroughs are such that they can only be implemented in the progressive sector, technology-bias increases. It may be possible that nature of the development-process is such that it features different phases, where in early phases some breakthroughs occur which can be implemented primarily in manufacturing and where in later phases breakthroughs occur, which can be implementation in some (personal) services sectors.

d.) Despite of our model results, which postulate that there are some mechanisms, which correlate the employment share with the output-elasticity of labor (see Theorem 2), it is possible to deviate from this scheme. For example, we have shown in the previous section that income elasticity of demand may cause deviations from this scheme. Hence, it is possible that large sectors (large employment shares) are associated with low output-elasticity of labor, if their income elasticity of demand is relatively large. The fact, that the correlation between

⁶ Remember that large sectors have more “creative power”; hence a large sector is feasible to increase its productivity strongly by implementation, whereas a small sector can increase its productivity hardly by implementation.

employment share and output-elasticity may be important regarding the technology-bias, is relatively obvious: If progressive sectors have high employment shares (negative correlation between output-elasticity and employment shares), implementation in progressive sectors is more successful than in implementation in backward sectors, hence technology-bias increases. Otherwise, if progressive sectors have low employment shares (positive correlation), implementation will be less successful, and backward sectors catch up (technology-bias decreases).

e.) We assume that the difference in sector size determines how strong the difference in implementation of breakthroughs across sectors is. If there are very small sectors and very large sectors in an economy, the differences in technology are large (according to our model) and may be difficult to overweight by other effects (e.g. those from point b)), as seen in the proof of Effect b) in Theorem 3.

f.) As mentioned in the previous section, structural change is caused by several determinants. Hence, all these determinants affect the cross-sector-factor allocation and thus relative abilities to implement breakthroughs. For further discussion see previous section.

Last not least, there are several further factors, which are not explicitly modeled here, but which affect the model outcome. These factors rather depend on the nature of the breakthrough and/or on the nature sectoral output. Some discussion of the nature of sectors and its implications for the scope of technological progress can be found, e.g. in the essays by Wolfe (1955), Baumol (1967) and Klevorick et al. (1995).

1.) It may happen that some breakthroughs are rather implementable in capital-augmenting or labor-augmenting way. That is, it may depend on the nature of the breakthrough whether it is primarily labor-augmentingly or capital-augmentingly implemented.

2.) Some sectors (e.g. services) may implement a breakthrough (e.g. electricity) in a primarily labor-augmenting way and other sector (e.g. agriculture) may implement the same breakthrough (electricity) in a primarily capital-augmenting way. This could yield technology-divergence. That is, in which way the breakthroughs are primarily implemented may depend on the nature of the output of a sector.

3.) Simply speaking, it may also be accidental that over some period of time breakthroughs occur, which are implementable in only one sector; which would yield technology

convergence. As well technical feasibility may dictate the order of breakthroughs and thus development of the technology-bias.

Overall, there seems to be some accidental or technical component which determines the timely order of breakthroughs and their implementability in a sector.

7 Empirical evidence on technology convergence

In this section we construct a simple measure of cross-sector technology-bias and analyze its development over time. Since quite easy measurable, we use sectoral labor-income shares as indices of sectoral technology (output-elasticity of labor).

The measure of technology-bias, which we suggest is the following:

$$(29) \text{Var}_1 = \sum_i l_i^0 \left(\frac{1}{\alpha_i^t} - \frac{\bar{1}}{\alpha_i} \right)^2$$

where

$$(30) \frac{\bar{1}}{\alpha_i} \equiv \sum_i l_i^0 \frac{1}{\alpha_i^t}$$

or alternatively

$$(31) \text{Var}_2 = \sum_i l_i^T \left(\frac{1}{\alpha_i^t} - \frac{\bar{1}}{\alpha_i} \right)^2$$

where

$$(32) \frac{\bar{1}}{\alpha_i} \equiv \sum_i l_i^T \frac{1}{\alpha_i^t}$$

where i denotes sector; α_i^t is the labor-income share of sector i at time t (or approximately output-elasticity of labor); l_i^t is the employment-share of sector i at time t ; l_i^0 is the employment-share of sector i at the beginning of the observation period; l_i^T is the employment-share of sector i at the end of the observation period.

That is, our measure of cross-sector technology variation is simply a variance. Note that it makes sense to use the employment-shares as weighting-factors in variance-calculation, since

it is theoretically reasonable: $\sum_i l_i \frac{1}{\alpha_i} = \sum_i l_i \frac{p_i Y_i}{w l_i L} = \frac{\sum_i p_i Y_i}{w L} = \frac{Y}{w L}$. Hence, this term is equal

to the reciprocal of the aggregate labor-income-share.

Note that we do not use the actual employment-shares to calculate the technology-variance, but the employment-shares from the beginning of the observation period. (For control we also calculate the variance by using employment-shares at the end of the period.) The reason for this is that the employment shares change over time and they would eventually bias the technology development. Hence, with actual employment shares we could not say whether the technology-variance decreased or whether some changes in employment-shares led to the result.

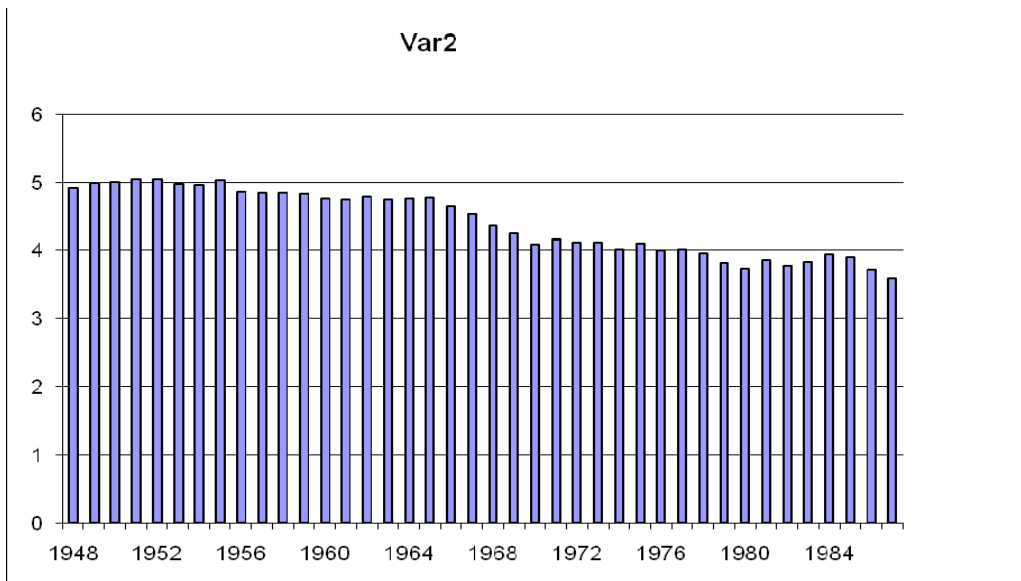
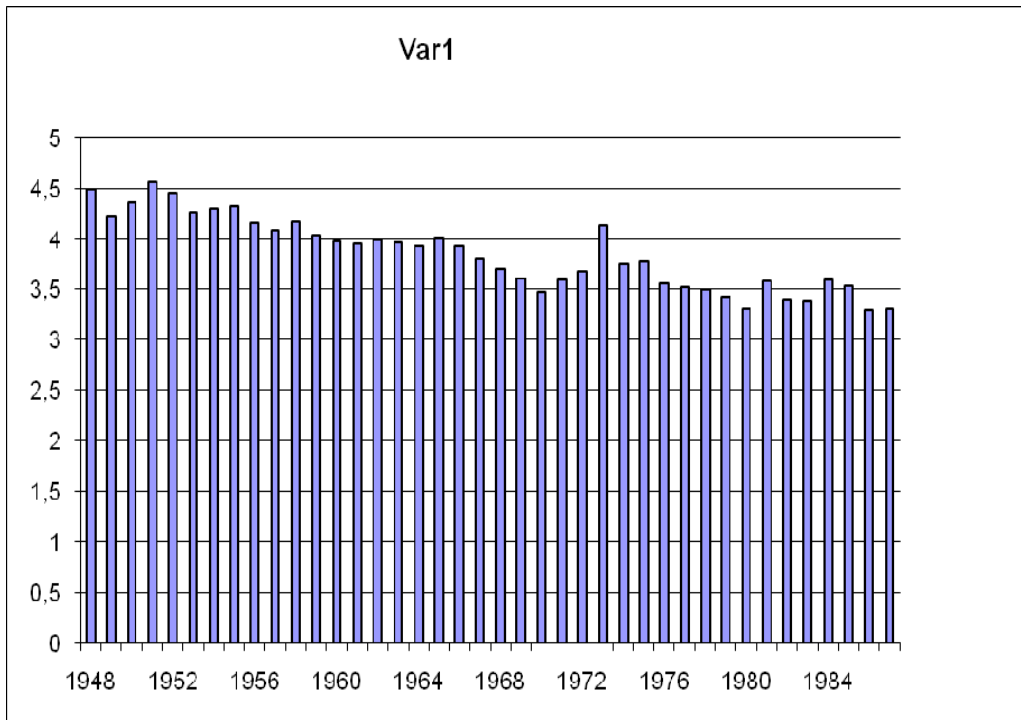
For the calculations in this section I use the data for the U.S.A., which is available at the website of the U.S. Department of Commerce (Bureau of Economic Analysis). I use the U.S.-Gross-Domestic-Product-(GDP)-by-Industry-Data, which is based on the sector-definition from the “Standard Industrial Classification System”, which defines the following sectors:

- (1) Agriculture, forestry, and fishing
- (2) Mining
- (3) Construction
- (4) Manufacturing
- (5) Transportation and public utilities
- (6) Wholesale trade
- (7) Retail trade
- (8) Finance, insurance, and real estate
- (9) Services

My calculations are based on the data for the period 1948-1987. Uniform data for longer time-periods is not available, since the “Standard Industrial Classification System” has been modified over time (hence, the sector definition after 1987 is not the same as the sector definition before 1987).

To calculate the sectoral labor-income-shares (α_i) I divide “(Nominal) Compensation of Employees” by “(Nominal) Value Added by Industry” in each sector. The sectoral employment shares (l_i) are calculated by using the sectoral data on “Full-time Equivalent Employees”. (This approach is similar to that used by Acemoglu and Guerrieri (2008)).

The following two figures depict the development of the two measures over time:



Both measures imply that the cross-sector technology-variance is decreasing. Hence, it seems that technologies converged during the observation period.

In fact, our model-explanations of this finding are:

- Over the long-run in the past, technological progress has been capital-augmenting; hence technologies are converging now.
- Structural change has increased the employment in technologically backward sectors much. Therefore, they are able to implement new breakthroughs and are catching-up.

These explanations are discussed in the next chapter.

8 Relevance of the results for existing literature and an interesting topic for further research

8.1 GDP-growth-slow-down

The results of this paper are closely related to the theoretical and empirical literature on Baumol's cost disease. Examples of this literature are the papers by Baumol (1967), Baumol et al. (1985), Ngai and Pissarides (2007), Nordhaus (2008) and Acemoglu and Guerrieri (2008). In fact this literature implies that factors are reallocated to technologically backward sectors. In part, this literature implies that therefore the real GDP-growth rate decreases over time. Our model and our empirical evidence imply that this reallocation process can lead to technological catching-up of backward sectors. Hence, the technologies may converge over the very long-run. Therefore, the slow-down of the real GDP-growth rate may cease and structural change may slow-down (since in this literature the technology-bias is a key determinant of the direction and strength of structural change).

8.2 Microfoundation and additional structural change determinants

There is some literature, which models technologically heterogeneous sectors; especially, this literature assumes that output-elasticities of inputs (i.e. the α_i 's) differ across sectors, but assumes that this technology-bias is exogenous. Examples are the models by Kongsamut et al. (1997), Echevarria (1997, 2000), Golin et al (2002), Meckl (2002), Jensen and Larsen (2004), Greenwood and Uysal (2005), Bah (2007), Golin et al. (2007), Zuleta and Young (2007), Acemoglu and Guerrieri (2008) and Buera and Kaboski (2009). Our model adds a microfoundation of the technology-bias of these models. Especially, our model and our empirical evidence imply that this technology-bias may vanish in the future and, thus, complicated modeling of this technology-bias may not be necessary. Furthermore, our model implies that the predictions of these models regarding the structural change dynamics may be biased. Our model results imply that when the technology-bias is endogenized several relationships between structural change and technology arise, which are omitted in the models with exogenous technology-bias:

- 1.) Technology implementation causes by itself structural change (Theorem 2)
- 2.) Technology-change slows-down/accelerates structural change patterns (Theorem 2 and 4c).
- 3.) Structural change affects technology-bias (Corollary from Theorems 2, 3 and 4).

In fact point 1) implies that all the factors, which determine technology implementation (see section 6), are themselves structural change determinants. Hence, they could be added to the traditional structural change determinants, which are studied in the literature (non-homothetic preferences, exogenous TFP-bias and exogenous bias in output-elasticities of inputs).

8.3 Explanation of stylized facts

The theoretical results by Acemoglu and Guerrieri (2008) imply that cross-sector-differences in factor-income-shares are not compatible with the relatively stable development of the aggregate economy, which is described by “Kaldor’s stylized facts of economic growth”. (This result is supported by Kongsamut et al. (1997)). Nevertheless, in the simulation of their model they find that the model satisfies the Kaldor-facts approximately. Our model and our empirical evidence imply the following explanation of this fact: It is possible that the sectoral technologies have already converged significantly. Hence, the cross technology-bias is sufficiently small and has therefore relatively weak impacts on the aggregate economy.

Valentinyi and Herrendorf (2008) show that capital-income shares are relatively similar at low degree of disaggregation (i.e. when comparing manufacturing and services). Our model and our empirical evidence imply that this finding may have come from the technology-convergence-tendencies described in our paper.

8.4 Structural change as a determinant of technology paths

Since we study implementation of breakthroughs and not technology shocks in our model, our model postulates a much stronger importance of structural change for aggregate growth than standard structural change models do. In “general structural change models”, the current factor allocation affects the current productivity of factor use (the current “aggregate technology”). In contrast, in our model the current factor allocation determines how a future path of technology innovation is implemented. (A breakthrough in our model is associated with a change in the *growth rate* of technological progress.) Hence, e.g., if relatively high income elasticity of demand for services yields a very high employment share in services, breakthroughs are implemented in the services sector primarily. (The many possible implementations/inventions in, e.g., the agricultural or manufacturing sector are “wasted”, since only little labor is employed in the agricultural/manufacturing sector. Those inventions in the agricultural/manufacturing sector may have been the basis for further groundbreaking inventions.)

8.5 Relevance for endogenous growth theory

Last not least, neoclassical endogenous growth theory studies aggregate production functions (or: aggregate technological progress). However, aggregate production functions are some weighted averages of sectoral production functions. Hence, an endogenous growth theory without the study of the question how technologies are implemented at sector level seems to be incomplete.

In general, endogenous growth theory includes models with heterogeneous goods/”sectors”; however, the production functions of these “sectors” do not feature significant technological heterogeneity; hence, these models are not consistent with the empirical facts of technological heterogeneity of sectors; compare e.g. Krueger (2008).

Since our model is neoclassical in many aspects, our model may be regarded as a contribution to the neoclassical endogenous growth theory, especially regarding models where technological change arises from research and development. A very simplified description of such a R&D-model may be the following: factors are devoted to research activities; the “output” of these research activities is an increase in aggregate productivity. This line of arguments can be depicted as follows:

Resources \rightarrow Research \rightarrow Increase in aggregate productivity

Our model adds an additional argument in this chain of arguments:

Resources \rightarrow Research \rightarrow Implementation (\leftrightarrow Structural change) \rightarrow Increase in aggregate productivity

That is, one and the same resource-input in research activities can produce a different change in aggregate productivity, depending upon the sector structure. (This sector structure is determined by all the structural change determinants, which have been discussed in sections 6 and 7.) This sector structure determines how research output is implemented today and in future. This argument may be useful in generating a microfoundation of increasing/decreasing returns-to-scale in research. For example, a constant research input may produce smaller and smaller increases in aggregate productivity over time, if e.g. non-homothetic preferences shift demand to sectors, in which implementability is low.

8.6 Limit development over an infinite sequence of breakthroughs

Our results and especially Theorem 3 could be used to explain the development of the technology-bias in a country over a relatively long period of time. This is not difficult to do by using our results. In the proof of Theorem 3 we have elaborated several examples of possible cases; furthermore, we have elaborated there how the factors, which determine the

technology-bias, affect each other. Thus, our model-results could be used to analyze which combinations of determining factors exist in a country and to explain this country's technology-bias-development. The data, which is necessary for such an exercise, includes: sectoral employment shares, sectoral output-elasticities of inputs (and TFP-growth rates) and the sort of technological progress in sectors.

However, it may be interesting to analyze how an economy behaves over an infinite sequence of technological breakthroughs. This can be done as follows:

Assume that there is a sequence of points in time $\tau = 1, 2, \dots, \infty$. At each of these points in time a breakthrough happens. Assume, furthermore, the following:

- the technology breakthroughs are not too influential on the economy; i.e., only little implementation is possible; i.e. when breakthroughs are implemented, the change in α and β is relatively small and/or
- the periods of time between points $\tau = 1, 2, \dots, \infty$ are relatively large.

These assumptions imply that we can assume that at each point $\tau = 1, 2, \dots, \infty$ the economy is very close to the PBGP. Hence, we can approximate the employment shares by using the PBGP-employment shares (equations (25)/(26)).

As an example assume that implementation of breakthroughs is always capital-augmenting. Furthermore, assume that the implementability of the breakthroughs is not sector-dependent, but is only dependent on the employment-shares of the sector.

By using these assumptions we can postulate the following difference equation system:

$$(33) \quad \alpha_{\tau+1} = \alpha_{\tau} - \Delta(1 - l_{B\tau}^*)$$

$$(34) \quad \beta_{\tau+1} = \beta_{\tau} - \Delta(l_{B\tau}^*)$$

where it follows from equation (25) that

$$(35) \quad l_{B\tau}^* = \beta_{\tau} \frac{1 - \alpha_{\tau} + \alpha_{\tau} \frac{\rho + \delta + g_L + \frac{g_A}{\alpha_{\tau}}}{\rho}}{(\beta_{\tau} - \alpha_{\tau})\bar{\omega}_B + \alpha_{\tau} \frac{\rho + \delta + g_L + \frac{g_A}{\alpha_{\tau}}}{\rho}}$$

Furthermore, it follows from equation (26) that $1 - l_{B\tau}^* = l_{A\tau}^*$.

Note that in both difference equations the functional form $\Delta(\cdot)$ is the same; this fact is due to assumption that implementability does not depend upon sector but only upon sectoral employment.

Reasonable assumptions for the function Δ are:

$$(36) \quad \Delta(\cdot) \geq 0, \quad \frac{\partial \Delta(l_{B\tau}^*)}{\partial l_{B\tau}^*} > 0, \quad \frac{\partial \Delta(1-l_{B\tau}^*)}{\partial (1-l_{B\tau}^*)} > 0, \quad \Delta(0) = 0$$

The first of these assumptions is due to our assumption that implementation is only capital-augmenting; hence, α and β are *reduced* at every point of time τ (see also Assumption 13).

The second, third and fourth assumptions in (36) are resulting from Assumption 12.

Since the analysis of difference equation systems is relatively difficult (because results of phase diagrams are not unambiguous), and since anyway we are interested only in the development of the technology bias ($\alpha - \beta$), the difference-equation-system (33)-(34) could be transformed into a single difference-equation as follows:

$$(37) \quad D_{\tau+1} = D_{\tau} + G(l_{B\tau}^*)$$

where $D_{\tau} \equiv \alpha_{\tau} - \beta_{\tau}$ and $G(l_{B\tau}^*) \equiv -\Delta(1-l_{B\tau}^*) + \Delta(l_{B\tau}^*)$

The analysis of this differential equation is relatively comfortable: it can be done in a phase diagram. The preliminary analysis of this system, which we have done by now and which will be published in a separate paper, implies that the dynamic path of the technology-bias D_{τ} may have very different features depending upon the parameters, which determine the savings rate (g_A, ρ, δ, g_L), and factors, which are elaborated in Theorem 3. For example, there may be cycles in the development of the bias. A detailed discussion of this analysis will be provided in a separate paper.

9 Concluding remarks

We have presented a model of technology-implementation at sector level. We have elaborated several aspects of this process, which can explain and predict the development of the technology-bias at sector level. The focus of our model has been on the technology-bias regarding capital-intensities, output-elasticities of inputs and factor-income-shares. That is, we have analyzed why output-elasticity of capital (and thus capital-intensity) is different across sectors and how this technology-bias develops over time. These questions are important, since many models assume the existence of such a technology-bias and since endogenous technology-bias would affect their results. These aspects among others have been discussed in section 8.

As discussed in section 6, our model provides several factors which determine whether technologies converge/diverge at sector level:

- the sort of implementation of technological breakthroughs (labor-augmenting vs. capital-augmenting implementation)

- the magnitude of the increase in (productivity) growth rate of capital-goods production, which is induced by the implementation of the breakthroughs
- the nature of the breakthrough (In which sector can the breakthrough be implemented?)
- structural change (Structural change determines the (relative) ability of sectors to implement breakthroughs.)

Of course, it is possible to elaborate which combinations of these factors exist over some (restricted) period of time in a country and then to explain the technology-bias-development of the country. As discussed in section 8.6, by using our results this exercise can be done without many difficulties.

Furthermore, it is possible to analyze the limit development of the economy. That is, we could assume that there is an infinite sequence of (similar) breakthroughs and then we could analyze how the economy develops over this sequence. In Section 8.6 we have elaborated an approach to such an analysis. However, the benefit of such an analysis may be questionable: As shown in Section 8.6, such limit analysis can be done only if we assume a specific combination of “determining factors”. (“Determining factors” means here factors which determine the technology-bias; they are elaborated in Theorem 3.) For example, we would have to assume that from now to infinity breakthroughs are implemented only in capital-augmenting manner. However, we have no reason to assume that these determining factors are constant over a very long period of time. We guess that a key-aspect of technological development is frequent change in determining factor-combinations, e.g. progress is sometimes labor-augmenting and sometimes capital-augmenting. Therefore, we guess that the most practicable way of using our results for explaining the development of technology-bias over a given (past) period of time is the one, which is discussed in the previous paragraph and in the beginning of Section 8.6: use the results of Theorem 3 to elaborate the existing combinations of “determining factors” over a period of time; then, explain the development of technology-bias by using the Theorems of our paper, like we did in the examples of the proof of Theorem 3.

Overall, we can say that our model implies that it is difficult to extrapolate the *future* development of technology-bias from past development. In fact, we show that many factors determine the technology-bias, and there is no reason to assume that these factors will not change in future.

It may seem that many of our results are “obvious”, i.e. they can be derived by verbal analysis. However, this is not true and maybe the most important result of our paper is the

proof that such “obvious” verbal arguments hold when using a microfoundation. In general, a change in model-parameters (e.g. a technological breakthrough) may shift a modeled system to a qualitatively very different growth path. The effects of this shift may outweigh all the obvious verbally derived results. Hence, we can never say a priori (i.e. without microfoundation) whether such verbally derived results hold. In our paper we have shown that (locally) stable equilibriums exist and that transition-paths retain their qualitative features even if there is a technology change. That is, we have shown that development paths exist along which all the results of “obvious” verbal analysis hold. However, a very interesting question for further research is to analyze whether in alternative multi-sector models technology-implementation can shift the economy to very different growth paths and thus overturn all the “obvious” verbal results.

Our empirical evidence (in section 7) implies that technologies have converged in the USA between 1948 and 1987. Our model explanation of this fact is that technological breakthroughs have been implemented in capital-augmenting manner in the past and that structural change (increasing factor-shares of technologically-backward sectors) has increased the ability of backward-sectors to implement general technological breakthroughs.

Furthermore, we have discussed several implications of our model for the growth and structural change literature in the previous section. Especially, we have discussed the following topics: relevance of our results for the literature on the slow-down of GDP-growth and for the endogenous growth-literature, relevance of our model as a microfoundation of structural change literature, structural change determinants implied by our results, implications of our results for the explanation of Kaldor-facts and the relevance of structural change as a determinant of technology-paths.

Note that our model may be regarded as a baseline model only; it shows how technology implementation and cross-sector technology-bias can be modeled. It could be used to study several further questions, which are important for cross-sector technology convergence. Especially, we think that it could be interesting to study the question of how do patent rights (monopolistic competition) and implementation-research-costs affect technological convergence. In part, this fact has been studied by Acemoglu and Guerrieri (2006), in a model where TFP-bias is endogenized. Introducing patent rights and research costs into our model is a quite challenging task and it would change the quantitative results. In fact, employment

shares and capital-intensities would be different in comparison to our baseline model; however, we do not believe that this modification could change our key qualitative results (regarding the determinants of technology bias).

We assumed that preferences are independent of technology. It would be interesting (although very challenging) to see what happens if, e.g., income elasticity of demand is correlated with output-elasticity of capital. On the other hand, we have “approximated” such dependency in section 5; furthermore, it seems to be difficult to defend such a dependency assumption in long run modeling.

Note that we assumed that breakthroughs are not foreseen by individual households; however, since in our analysis we compared different stationary points (PBGPs), and not transitional dynamics, this simplifying assumption does not change our results significantly. (If the households foresaw the breakthroughs, the consumption paths would be smoother, but the equilibriums (PBGPs) would not change qualitatively.)

Furthermore, it would be interesting to analyze which specification of our model depicts the reality of different country groups (developing countries, industrialized countries,...) by using empirical data.

Last not least, there is the limit-analysis, which we have presented in Section 8.6. We will provide the results of this analysis in a separate paper.

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APPENDIX

First, I show by using linear approximation that the saddle-path-feature of the PBGP is given (Lemma 2a). Then I prove local stability by using a phase diagram (Lemma 2b).

Existence of a saddle-path (Lemma 2a)

First I rearrange the aggregate equation system (22)-(24) as follows:

$$\dot{\hat{K}} = \left(\frac{l_m}{k_m}\right)^{\alpha-1} (\alpha + (1-\alpha)\frac{l_m}{k_m})\hat{K}^{1-\alpha} - \hat{E} - (\delta + g_L + \frac{g_A}{\alpha})\hat{K} \quad (C.1)$$

$$\frac{\dot{\hat{E}}}{\hat{E}} = (1-\alpha)\left(\frac{l_m}{k_m}\right)^\alpha \hat{K}^{-\alpha} - \delta - \rho - g_L - \frac{g_A}{\alpha} \quad (C.2)$$

$$\frac{l_m}{k_m} = 1 - \frac{\beta - \alpha}{\alpha(1-\alpha)} \bar{\omega}_B \frac{\hat{E}}{\hat{K}^{1-\alpha} \left(\frac{l_m}{k_m}\right)^{\alpha-1}} \quad (C.3)$$

where aggregate variables are expressed in “labor-efficiency units”, i.e. they are divided by

$$LA^{\frac{1}{\alpha}}; \text{ hence } \hat{K} \equiv \frac{K}{LA^{\frac{1}{\alpha}}} \text{ and } \hat{E} \equiv \frac{E}{LA^{\frac{1}{\alpha}}}.$$

These equations imply that \hat{K} , \hat{E} and l_m/k_m have the following values along the PBGP

$$\hat{K}^* = \sigma^{\frac{1}{\alpha}} \left(\frac{l_m}{k_m}\right)^* \quad (C.4)$$

$$\hat{E}^* = \alpha \sigma^{\frac{1-\alpha}{\alpha}} + \rho \sigma^{\frac{1}{\alpha}} \left(\frac{l_m}{k_m}\right)^* \quad (C.5)$$

$$\left(\frac{l_m}{k_m}\right)^* = \frac{\alpha}{1-\alpha} \frac{1-\alpha - (\beta-\alpha)\bar{\omega}_B}{\alpha + (\beta-\alpha)\bar{\omega}_B} \frac{\rho}{1-\alpha} \sigma \quad (C.6)$$

$$\text{where } \sigma \equiv \frac{1-\alpha}{\delta + \rho + g_L + \frac{g_A}{\alpha}}$$

where an asterisk denotes the PBGP-value of the corresponding variable.

The proof of local saddle-path-stability of the PBGP is analogous to the proof by Acemoglu and Guerrieri (2008) (see there for details and see also Acemoglu (2009), pp. 269-273, 926).

First, I have to show that the determinant of the Jacobian of the differential equation system (C.1)-(C.2) (where l_m/k_m is given by equation (C.3)) is different from zero when evaluated at

the PBGP (i.e. for $\hat{K}^*, \hat{E}^*, \left(\frac{l_m}{k_m}\right)^*$ from equations (C.4)-(C.6)). This implies that this differential equation system is hyperbolic and can be linearly approximated around $\hat{K}^*, \hat{E}^*, \left(\frac{l_m}{k_m}\right)^*$ (Grobman-Hartman-Theorem; see as well Acemoglu (2009), p. 926, and Acemoglu and Guerrieri (2008)). The determinant of the Jacobian is given by:

$$|J| = \begin{vmatrix} \frac{\partial \dot{\hat{K}}}{\partial \hat{K}} & \frac{\partial \dot{\hat{K}}}{\partial \hat{E}} \\ \frac{\partial \dot{\hat{E}}}{\partial \hat{K}} & \frac{\partial \dot{\hat{E}}}{\partial \hat{E}} \end{vmatrix} = \frac{\partial \dot{\hat{K}}}{\partial \hat{K}} \frac{\partial \dot{\hat{E}}}{\partial \hat{E}} - \frac{\partial \dot{\hat{E}}}{\partial \hat{K}} \frac{\partial \dot{\hat{K}}}{\partial \hat{E}} \quad (\text{C.7})$$

The derivatives of equations (C.1)-(C.2) are given by:

$$\begin{aligned} \frac{\partial \dot{\hat{K}}}{\partial \hat{K}} &= (1-\alpha)\hat{K}^{-\alpha} \left(\alpha \left(\frac{l_m}{k_m}\right)^{\alpha-1} + (1-\alpha) \left(\frac{l_m}{k_m}\right)^{\alpha} \right) \\ &\quad + \hat{K}^{1-\alpha} \left(-(1-\alpha)\alpha \left(\frac{l_m}{k_m}\right)^{\alpha-2} + \alpha(1-\alpha) \left(\frac{l_m}{k_m}\right)^{\alpha-1} \right) \frac{\partial \left(\frac{l_m}{k_m}\right)}{\partial \hat{K}} - \left(\delta + g_L + \frac{g_A}{\alpha} \right) \\ \frac{\partial \dot{\hat{K}}}{\partial \hat{E}} &= \hat{K}^{1-\alpha} \left(-(1-\alpha)\alpha \left(\frac{l_m}{k_m}\right)^{\alpha-2} + \alpha(1-\alpha) \left(\frac{l_m}{k_m}\right)^{\alpha-1} \right) \frac{\partial \left(\frac{l_m}{k_m}\right)}{\partial \hat{E}} - 1 \\ \frac{\partial \dot{\hat{E}}}{\partial \hat{K}} &= (1-\alpha)\hat{E} \left(-\alpha\hat{K}^{-\alpha-1} \left(\frac{l_m}{k_m}\right)^{\alpha} + \alpha\hat{K}^{-\alpha} \left(\frac{l_m}{k_m}\right)^{\alpha-1} \frac{\partial \left(\frac{l_m}{k_m}\right)}{\partial \hat{K}} \right) \\ \frac{\partial \dot{\hat{E}}}{\partial \hat{E}} &= \left((1-\alpha) \left(\frac{l_m}{k_m}\right)^{\alpha} \hat{K}^{-\alpha} - \delta - \rho - g_L - \frac{g_A}{\alpha} \right) \\ &\quad + (1-\alpha)\hat{E}\hat{K}^{-\alpha} \alpha \left(\frac{l_m}{k_m}\right)^{\alpha-1} \frac{\partial \left(\frac{l_m}{k_m}\right)}{\partial \hat{E}} \end{aligned} \quad (\text{C.8})$$

where the derivatives of equation (C.3) are given by

$$\begin{aligned}
\frac{\partial \left(\frac{l_m}{k_m} \right)}{\partial \hat{E}} &= - \frac{\frac{\beta - \alpha}{\alpha(1 - \alpha)} \bar{\omega}_B \left(\frac{l_m}{k_m} \right)^{1 - \alpha}}{\left(1 + \frac{\beta - \alpha}{\alpha} \bar{\omega}_B \frac{\hat{E}}{\hat{K}^{1 - \alpha} \left(\frac{l_m}{k_m} \right)^\alpha} \right)} \\
\frac{\partial \left(\frac{l_m}{k_m} \right)}{\partial \hat{K}} &= \frac{\frac{\beta - \alpha}{\alpha} \bar{\omega}_B \frac{\hat{E}}{\hat{K}^{2 - \alpha} \left(\frac{l_m}{k_m} \right)^{\alpha - 1}}}{\left(1 + \frac{\beta - \alpha}{\alpha} \bar{\omega}_B \frac{\hat{E}}{\hat{K}^{1 - \alpha} \left(\frac{l_m}{k_m} \right)^\alpha} \right)} \quad (C.9)
\end{aligned}$$

Inserting the derivatives (C.8) and (C.9) into (C.7) and inserting the PBGP-values from equations (C.4)-(C.6) yields after some algebra the following value of the determinant of the Jacobian evaluated at the PBGP:

$$|J|^* = \frac{-\alpha(\beta - \alpha)\bar{\omega}_B \frac{\hat{E}^*}{\hat{K}^*} \left[\rho + \frac{\alpha}{(\beta - \alpha)\bar{\omega}_B} \frac{1 - \alpha}{\sigma} \right]}{\alpha + (\beta - \alpha)\bar{\omega}_B \frac{\hat{E}^*}{(\hat{K}^*)^{1 - \alpha} \left[\left(\frac{l_m}{k_m} \right)^* \right]^\alpha}} \quad (C.10)$$

This equation can be transformed further by using equations (C.4)-(C.6):

$$|J|^* = \frac{-\frac{\hat{E}^*}{\hat{K}^*} \frac{\alpha}{\sigma} [1 - \alpha - (\beta - \alpha)\bar{\omega}_B]}{\alpha \left(\frac{l_m}{k_m} \right)^* + 1 - \alpha} \quad (C.11)$$

Note that $\frac{\hat{E}^*}{\hat{K}^*}$ and $\left(\frac{l_m}{k_m} \right)^*$ are positive and are given by equations (C.4)-(C.6).

We can see that the determinant evaluated at PBGP is different from zero. Hence, the PBGP is hyperbolic. Furthermore, equations (C.10) and (C.11) imply that $|J|^* < 0$. (Equation (C.10) implies that $|J|^* < 0$, if $\beta - \alpha > 0$; equation (C.11) implies that $|J|^* < 0$, if $\beta - \alpha < 0$ as well.) Our differential equation system consists of two differential equations ((C.1) and (C.2)) and of two variables (\hat{E} and \hat{K}), where we have one state and one control-variable. Hence, saddle-

path-stability of the PBGP requires that there exist one negative (and one positive) eigenvalue of the differential equation system when evaluated at PBGP (see also Acemoglu and Guerrieri (2008) and Acemoglu (2009), pp. 269-273). Since $|J|^* < 0$ we can be sure that this is the case. ($|J|^* < 0$ can exist only if one eigenvalue is positive and the other eigenvalue is negative. If both eigenvalues were negative or if both eigenvalues were positive, the determinant $|J|^*$ would be positive.) Therefore, the PBGP is locally saddle-path-stable, i.e. Lemma 2a is proved. *Q.E.D.*

Local stability (Lemma 2b)

I study here only the case where output-elasticity of capital in investment goods industries ($i=m$) is relatively low in comparison to the output-elasticity of capital in the consumption goods industries ($\forall i \neq m$), i.e. I assume $\beta < \alpha$. This is consistent with the empirical evidence presented and discussed in Valentinyi and Herrendorf (2008) (see there especially p.826). Note, however, that the qualitative stability results for the other case (i.e. $\beta > \alpha$) are the same.

To show the stability-features of the PBGP, the three-dimensional system (C.1)-(C.3) has to be transformed into a two dimensional system, in order to allow me using a phase-diagram.

By defining the variable $\kappa \equiv \frac{\hat{K}k_m}{l_m}$, the system (C.1)-(C.3) can be reformulated as follows

(after some algebra):

$$(C.12) \quad \frac{\dot{\hat{E}}}{\hat{E}} = (1-\alpha)\kappa^{-\alpha} - \left(\delta + \rho + g_L + \frac{g_A}{\alpha} \right)$$

$$(C.13) \quad \frac{\dot{\kappa}}{\kappa} = \frac{\kappa^{-\alpha} - (\delta + g_L + \frac{g_A}{\alpha}) - \frac{\hat{E}}{\kappa} \left(1 - \frac{\alpha - \beta}{\alpha(1-\alpha)} \bar{\omega}_B \rho \kappa^\alpha \right)}{1 + \frac{\alpha - \beta}{1-\alpha} \bar{\omega}_B \frac{\hat{E}}{\kappa^{1-\alpha}}}$$

I can focus attention on showing that the stationary point of this differential equation system is stable: κ and \hat{E} are jointly in steady state only if \hat{K} , \hat{E} and k_m/l_m are jointly in steady state; furthermore, \hat{K} , \hat{E} and k_m/l_m are jointly in steady state only if κ and \hat{E} are jointly in steady state. Therefore, the proof of stability of the stationary point of system (C.12)-(C.13)

implies stability of the stationary point of system (C.1)-(C.3). Hence, in the following I will prove stability of the stationary point of system (C.12)-(C.13).

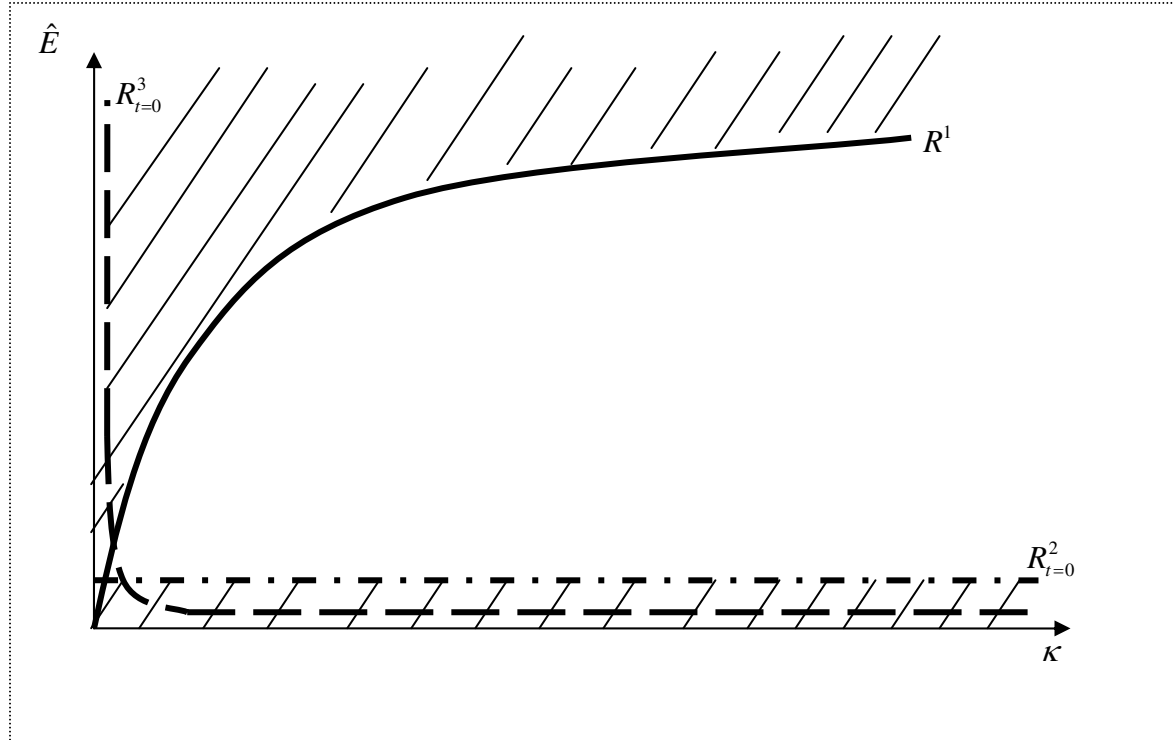
It follows from equations (C.12) and (C.13) that the steady-state-loci of the two variables are given by

$$(C.12a) \quad \frac{\dot{\hat{E}}}{\hat{E}} = 0: \quad \kappa^* = \left(\frac{1-\alpha}{\delta + \rho + g_L + \frac{g_A}{\alpha}} \right)^{\frac{1}{\alpha}}$$

$$(C.13a) \quad \frac{\dot{\kappa}}{\kappa} = 0: \quad \hat{E}_{\kappa=0} = \frac{\kappa^{-\alpha} - (\delta + g_L + \frac{g_A}{\alpha})}{1 - \rho \frac{\alpha - \beta}{\alpha(1-\alpha)} \bar{\omega}_B \kappa^\alpha} \kappa$$

Now, I could depict the differential equation system (C.12)-(C.13) in the phase space (\hat{E}, κ) . Before doing so, I show that not the whole phase space (\hat{E}, κ) is economically meaningful. The economically meaningful phase-space is restricted by three curves (R^1, R_t^2, R_t^3) , as shown in the following figure and as derived below:

Figure C.1: Relevant space of the phase diagram



Only the space below the R^1 -line is economically meaningful, since the employment-share of at least one sub-sector i is negative in the space above the R^1 -line. This can be seen from the following fact:

It follows from equations (3) and (7a,b) after some algebra that

$$(C.14) \quad \frac{l_m}{k_m} = 1 - \left(\frac{\beta - \alpha}{\beta(1 - \alpha)} \right) \sum_{i=m+1}^n l_i$$

Since, l_i cannot be negative (hence, $0 \leq \sum_{i=m+1}^n l_i \leq 1$) this equation implies that

$$(C.15) \quad \frac{l_m}{k_m} < \frac{\alpha(1 - \beta)}{\beta(1 - \alpha)}$$

Inserting equation (C.3) into this relation yields

$$(C.16) \quad R^1: \hat{E} < \frac{\alpha}{\beta} \frac{1}{\omega_B} \kappa^{1-\alpha}$$

Hence, the space above R^1 is not feasible. When the economy reaches a point on R^1 , no labor is used in sub-sectors $i=1, \dots, m$. If I impose Inada-conditions on the production functions, as usual, this means that the output of sub-sectors $i=1, \dots, m$ is equal to zero, which means that the consumption of these sectors is equal to zero. Our utility function implies that life-time utility is infinitely negative in this case. Hence, the household prefers not to be at the R^1 -curve. Note that actually the R^1 -curve is only an outer limit: Since we have existence-minima in our utility function, the utility function becomes infinitely negative when the consumption of one of these goods falls below its subsistence level. Hence, even when the consumption of all goods is positive, it may be the case that the utility function is infinitely negative due to violation of some existence minima. Therefore, the actual constraint is somewhere below the R^1 -curve. However, this fact does not change the qualitative results of the stability analysis.

Now I turn to the R_i^2 and R_i^3 -curves. I have to take account of the non-negativity-constraints on consumption ($C_i > 0 \forall i$), since our Stone-Geary-type utility function can give rise to negative consumption. By using equations (7c), (11b), (12d), (15), (16b) and (19) the non-negativity-constraints ($C_i > 0 \forall i$) can be transformed as follows:

$$(C.17) \quad \hat{E} > \frac{-\theta_i}{\omega_i} \frac{1}{LA^{\frac{1}{\alpha}}} \quad i = 1, \dots, m$$

$$(C.18) \quad \hat{E} > \frac{-\theta_i}{\omega_i} \left(\frac{\alpha(1 - \beta)}{\beta(1 - \alpha)} \right)^\beta \frac{1 - \alpha}{1 - \beta} \frac{1}{LBA^{\frac{2(1-\beta)-(1-\alpha)}{\alpha}}} \frac{1}{\kappa^{\alpha-\beta}} \quad i = m + 1, \dots, n$$

This set of constraints implies that at any point of time only two constraints are binding, namely those with respectively the largest $\frac{-\theta_i}{\omega_i}$. Hence, the set (C.17), (C.18) can be reduced

to the following set:

$$(C.19) R_t^2: \hat{E} > \frac{-\theta_j}{\omega_j} \frac{1}{LA^\alpha}$$

$$\text{where } \frac{-\theta_j}{\omega_j} > \frac{-\theta_i}{\omega_i} \quad i = 1, \dots, m$$

and $1 \leq j \leq m$.

$$(C.20) R_t^3: \hat{E} > \frac{-\theta_x}{\omega_x} \left(\frac{\alpha(1-\beta)}{\beta(1-\alpha)} \right)^\beta \frac{1-\alpha}{1-\beta} \frac{1}{LBA} \frac{1}{\kappa^{\alpha-\beta}}$$

$$\text{where } \frac{-\theta_x}{\omega_x} > \frac{-\theta_i}{\omega_i} \quad i = m+1, \dots, n$$

and $m+1 \leq x \leq n$

These constraints are time-dependent. It depends upon the parameter setting whether R_t^2 or whether R_t^3 is binding at a point of time. In Figure C.1 I have depicted examples for these constraints for the initial state of the system. Only the space above the constraints is economically meaningful, since below the constraints the consumption of at least one good is negative. Last not least, note that equations (C.19)/(C.20) imply that the R_t^2 -curve and the R_t^3 -curve converge to the axes of the phase-diagram as time approaches infinity.

Now, I depict the differential equation system (C.12)-(C.13) in the phase space (\hat{E}, κ) .

show that the system will be on the saddle-path if $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$. Furthermore, I have to discuss what happens if κ_0 is not within this range.

Every trajectory, which starts *above the saddle-path or left from $\underline{\kappa}_0$* , reaches the R^1 -curve in finite time. As discussed above, the life-time utility becomes infinitely negative if the household reaches the R^1 -curve. These arguments imply that the representative household will never choose to start above the saddle path if $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$, since all the trajectories above the saddle-path lead to a state where life-time-utility is infinitely negative.

Furthermore, all initial points that are situated *below the saddle-path or right from $\bar{\kappa}$* converge to the point T. If the system reaches one of the constraints (R_t^2, R_t^3) during this convergence process, it moves along the binding constraint towards T. However, the transversality condition is violated in T. Therefore, T is not an equilibrium. To see that the transversality condition is violated in T consider the following facts: The transversality condition is given by $\lim_{t \rightarrow \infty} \psi K e^{-\rho t} > 0$, where ψ is the costate variable in the Hamiltonian function (shadow-price of capital). This transversality condition can be reformulated such that

we obtain: $\lim_{t \rightarrow \infty} (1 - \alpha) \kappa^{-\alpha} - \delta - g_L - \frac{g_A}{\alpha} > 0$, which is equivalent to: $\lim_{t \rightarrow \infty} \kappa < \left(\frac{1 - \alpha}{\delta + g_L + \frac{g_A}{\alpha}} \right)^{\frac{1}{\alpha}}$.

However, equation (C.13a) implies that in point T in Figure C.2 $\kappa = \left(\frac{1}{\delta + g_L + \frac{g_A}{\alpha}} \right)^{\frac{1}{\alpha}}$. Hence,

the transversality condition is violated if the system converges to point T.

Overall, we know that, if $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$, the household always decides to be on the saddle-path. Hence, we know that for $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$ the economy converges to the PBGP. In this sense, **the PBGP is locally stable (within the range $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$)**.

If the initial capital is too small ($\kappa_0 < \underline{\kappa}_0$), the economy converges to a state where some existence minima are not satisfied (curve R^1) and thus utility becomes infinitely negative. This may be interpreted as a **development trap**. For example, Malthusian theories imply that in this case some part of the population dies, which would yield an increase in per-capita-capital (and hence an increase in κ_0).

On the other hand, **if initial capital-level is too large** ($\kappa_0 > \bar{\kappa}$), all trajectories violate the transversality condition. Therefore, in this case, **the representative household must waste a part of its initial capital** to come into the feasible area ($\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$). This case may be interpreted as a **“development trap of the rich”**.

Furthermore, note that there are always some happenings that reduce the capital stock, e.g. wars (like the Second World War) or natural catastrophes. These happenings could shift the economy into feasible space ($\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$).

Note that equation (C.3) and definition $\kappa \equiv \frac{\hat{K}k_m}{l_m}$ imply that κ is a strictly monotonously

increasing function of K in the relevant space of the phase diagram (ceteris paribus).

Hence, the κ -ranges which determine the ranges of the “development trap”, the local stability and the “development trap of the rich” can be directly transformed into K -ranges. That is, if initial capital (K_0) is relatively low, the economy is in the “development trap”; if K_0 is relatively high, the economy is in the “development trap of the rich”; if K_0 is somewhere inbetween the economy is in the space of local stability.

Note that Figure C.2 depicts the phase diagram for parameter constellations, which satisfy the

condition $\frac{\alpha(1-\alpha)}{\bar{\omega}_B \rho(\alpha-\beta)} < \frac{1}{\delta + g_L + \frac{g_A}{\alpha}}$. For parameter constellations, which satisfy the

condition $\frac{\alpha(1-\alpha)}{\bar{\omega}_B \rho(\alpha-\beta)} > \frac{1}{\delta + g_L + \frac{g_A}{\alpha}}$, the discussion and the qualitative results are nearly the

same. The only difference is that the $\dot{\kappa} = 0$ -locus is humpshaped (concave) for $\kappa < \kappa^{pole}$.

However, all the qualitative results remain the same (local stability of PBGP for some range

$\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$ and “infeasibility” for $\kappa_0 < \underline{\kappa}_0$ and $\kappa_0 > \bar{\kappa}$). **Q.E.D.**