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Inflation Dynamics and Labor Market Specifications: A Bayesian DSGE Approach for Japan’s Economy*

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Abstract

Which labor market specification is better able to describe inflation dynamics, a widely-used sticky wage model or a recently-investigated labor market search model? Using a Bayesian likelihood approach, we estimate these two models with Japan’s data. This paper shows that the labor market search model is superior to the sticky wage model in terms of both marginal likelihood and out-of-sample forecast performance, particularly regarding inflation. The labor market search model is better able to replicate the cross-correlation among inflation, real wages, and output in the data. Moreover, in this model, real marginal cost is determined by both hiring cost and unit labor cost that varies with employment fluctuations, which gives rise to a high contemporaneous correlation between inflation and real marginal cost as represented in the New Keynesian Phillips curve. (JEL E24, E32, E37)

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1 INTRODUCTION

Inflation dynamics have been investigated using dynamic stochastic general equilibrium (DSGE) models with sticky prices. In the models, inflation is determined by the New Keynesian Phillips curve (NKPC), where real marginal cost of producing output is the main driving force and varies with changes in labor cost. Therefore, the specification of the labor market plays a crucial role in describing inflation dynamics.

Many previous studies use sticky wage models (e.g., Christiano et al., 2005; Levin et al., 2006; Smets and Wouters, 2003, 2007). In this specification of the labor market, workers set wages on a staggered basis a la Calvo (1983) while firms employ all workers and adjust labor input by changing hours per worker (i.e., intensive margin of labor). Consequently, the cost involved to adjust employment is absent in sticky wage models.

Recently, there has been a surge of interest in the role of the extensive margin of labor (i.e., employment) for inflation dynamics. Walsh (2005), Krause and Lubik (2007), and Trigari (2009), for instance, incorporate sticky prices in a labor market search model that has been analyzed in the literature starting from Mortensen and Pissarides (1994), Merz (1995), and Andolfatto (1996). In this labor market specification, firms adjust labor input by changing employment while wages are determined through bargaining between workers and firms. These characteristics of the labor market differ radically from those in sticky wage models. Despite this fundamental difference, existing literature lacks the formal comparison of sticky wage models and labor market search models.

The present paper fills this gap. Specifically, we estimate a sticky wage model and a labor market search model with Japan’s data using a Bayesian likelihood approach, and compare these two models in terms of marginal likelihood and out-of-sample forecast performance.

The estimation results show that the labor market search model is superior to the sticky wage model in terms of both marginal likelihood and out-of-sample forecast performance, particularly regarding inflation. Why is the former model better able to fit the data and forecast inflation? Because the non-labor market part is identical between these two models, the key difference is the relationship between real wages and inflation or output. In the sticky wage

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1 For recent studies on inflation dynamics using labor market search models, see also Christoefel et al. (2006), Christoefel and Kuester (2008), Christoefel and Linzert (2010), Krause et al. (2008), Sveen and Weinke (2008), Ravenna and Walsh (2008), Gertler et al. (2008), Blanchard and Gali (2010), and Van Zandweghe (2010).
model, real wages are highly correlated with output in the presence of monopolistically competitive labor markets, and unit labor cost under full employment is identical with real marginal cost and thus drives inflation dynamics. However, the cross-correlation between real wages and output in the data is not so high as in the sticky wage model. Besides, the data of real wages and output determine the unit labor cost, which lags far behind inflation by more than three years whereas a high contemporaneous correlation between inflation and real marginal cost is represented in the NKPC. In the labor market search model, labor bargaining generates a mild cross-correlation between real wages and output. Moreover, real marginal cost is determined by both hiring cost and unit labor cost that varies with employment fluctuations, which gives rise to a high contemporaneous correlation between inflation and real marginal cost. Therefore, the labor market search model is better able to fit the data and forecast inflation.

In related literature, Rabanal and Rubio-Ramírez (2005) show that a sticky wage model matches U.S. data far better in terms of marginal likelihood than a flexible wage model. Christoffel et al. (2006), Gertler et al. (2008), and Krause et al. (2008) estimate a labor market search model with U.S. or Euro area data. To our knowledge, the present paper is the first to compare a sticky wage model and a labor market search model in terms of marginal likelihood and out-of-sample forecast performance.

As for related studies on Japan, Muto (2009) stresses that the measurement of real marginal cost plays a crucial role in estimating the NKPC and shows that the consideration of labor market frictions greatly improves the goodness of its fit to Japan’s data, in line with our result that introducing labor market search and matching frictions improves marginal likelihood. Braun et al. (2006) indicate the business cycle fact that the intensive margin plays a more important role in Japan’s labor input fluctuations than the extensive margin. This is consistent with our result that the extensive margin may be more important for inflation dynamics in Japan. Labor input adjustment at the extensive margin is very costly for firms in Japan and therefore fluctuations in hours per worker are better able to explain labor input fluctuations. For inflation dynamics, real marginal cost is the key factor and hence the costly adjustment at the extensive margin has a crucial influence on the dynamics.

The remainder of the paper proceeds as follows. Section 2 presents a sticky wage model and a labor market search model. Section 3 explains data and econometric methods for estimating these models. Section 4 shows estimation results. Section 5 conducts robustness exercises. Finally, Section 6 concludes.
2 TWO LABOR MARKET MODELS

This section presents a sticky wage model and a labor market search model. First, the common part of these two models is explained. Then, the part specific to each model is described. Note that all variables without time subscript denote steady-state values and all hatted variables represent the log-deviations from steady-state values.

2.1 Common Part of Two Models

The sticky wage model and the labor market search model contain the following common part, which describes households’ spending behavior, firms’ price-setting behavior, and a central bank’s monetary policy.\(^2\)

Under complete insurance markets, households maximize CRRA utility functions of final-good consumption with internal habit formation. The log-linearized first-order conditions for consumption and bond holdings, together with the final-good market clearing condition,\(^3\) yield

\[
\dot{\lambda}_t = \frac{1}{1-\beta}\left\{-\frac{\sigma_c}{1-\chi}(\dot{Y}_t - \chi \dot{Y}_{t-1}) + \varepsilon_{ut} - \beta \chi \left[ -\frac{\sigma_c}{1-\chi}(E_t \dot{Y}_{t+1} - \chi \dot{Y}_t) + E_t \varepsilon_{ut+1}\right]\right\},
\]

\[
\dot{\lambda}_t = E_t \dot{\lambda}_{t+1} + \dot{R}_t - E_t \dot{\pi}_{t+1},
\]

where \(\lambda_t\) is the marginal utility of consumption, \(Y_t\) is output, \(R_t\) is the gross nominal interest rate, \(\pi_t\) is the gross inflation rate, \(\varepsilon_{ut}\) is a preference shock that follows a stationary first-order autoregression process, \(E_t\) is the expectation operator conditional on information available in period \(t\), \(\beta \in (0, 1)\) is the subjective discount factor, \(\sigma_c > 0\) is the degree of relative risk aversion, and \(\chi \in [0, 1]\) is the degree of habit persistence in consumption preferences.

There are a continuum of monopolistically competitive intermediate-good firms and perfectly competitive final-good firms. The intermediate-good firms produce differentiated goods by a Cobb-Douglas production technology using labor input and a fixed capital stock that is identical among firms, and set prices of these goods on the Calvo (1983)-style staggered basis with indexation to recent past inflation and steady-state inflation. The final-good firms combine differentiated intermediate goods into homogenous goods by a CES production technology

\(^2\)For the derivation of the common part of the sticky wage model and the labor market search model, see the appendix, which is available upon request to the authors.

\(^3\)Each model assumes that there is no other demand for final goods than consumption. Thus, the final-good market clearing condition shows that output supply matches consumption demand.
and sell them to households. Consequently, the inflation rate is determined by the NKPC
\[ \pi_t = \frac{\gamma_p}{1 + \beta\gamma_p} \pi_{t-1} + \frac{\beta}{1 + \beta\gamma_p} E_t \pi_{t+1} + \frac{(1 - \xi_p)(1 - \beta\xi_p)}{\xi_p(1 + \beta\gamma_p)[1 + \theta_p(1 - \alpha)/\alpha]} (mc_t + \varepsilon_{pt}), \] (3)

where \( mc_t \) is an “average” of real marginal cost (Galí et al., 2001), \( \varepsilon_{pt} \) is a price markup shock that follows a stationary first-order autoregression process, \( \xi_p \in (0, 1) \) is the probability of not reoptimizing prices, \( \gamma_p \in [0, 1] \) is the degree of price indexation to recent past inflation relative to steady-state inflation, \( \theta_p > 1 \) is the steady-state price elasticity of demand for differentiated goods, and \( \alpha \in (0, 1] \) is the labor input elasticity of output in the Cobb-Douglas production technology.

Monetary policy is conducted by following a Taylor (1993)-type interest rate rule
\[ \hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R)(\phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t) + \varepsilon_{Rt}, \] (4)

where \( \varepsilon_{Rt} \) is an i.i.d. monetary policy shock, \( \phi_R \in [0, 1] \) is the degree of interest rate smoothing, and \( \phi_\pi, \phi_Y \geq 0 \) are the degrees of policy responses to inflation and output.

### 2.2 Sticky Wage Model

In addition to the common part, the sticky wage model supposes that households supply differentiated labor services to intermediate-good firms and set wages on the Calvo (1983)-style staggered basis with indexation to steady-state inflation while the firms employ all workers and use a CES aggregate of the labor services as labor input.

In the model, log-linearized equilibrium conditions are given by Equations (1)–(4) and
\[ \hat{Y}_t = \alpha \hat{h}_t + \varepsilon_{at}, \] (5)
\[ \hat{mc}_t = \hat{w}_t - \hat{Y}_t, \] (6)
\[ \hat{w}_t = \hat{z}_t + \hat{h}_t, \] (7)
\[ \hat{\pi}_t^w = \beta E_t \hat{\pi}_{t+1}^w + \frac{(1 - \xi_w)(1 - \beta\xi_w)}{\xi_w(1 + \theta_w\sigma_h)} (\sigma_h \hat{h}_t - \hat{\lambda}_t - \hat{z}_t + \varepsilon_{wt}), \] (8)
\[ \hat{\pi}_t^w = \hat{z}_t - \hat{z}_{t-1} + \hat{\pi}_t, \] (9)

where \( h_t \) is labor hours per worker, \( w_t \) is real wages per worker, \( z_t \) is the real wage rate per worker, \( \pi_t^w \) is the gross nominal wage inflation rate, \( \varepsilon_{at} \) and \( \varepsilon_{wt} \) are a productivity shock and

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\[ ^4 \] For the derivation of log-linearized equilibrium conditions in the sticky wage model, see the appendix, which is available upon request to the authors.
a wage markup shock that follow univariate stationary first-order autoregression processes, 
\( \sigma_h \geq 0 \) is the inverse of the elasticity of labor supply, \( \xi_w \in (0,1) \) is the probability of not 
reoptimizing wages, and \( \theta_w > 1 \) is the steady-state wage elasticity of demand for differentiated 
labor services. Equation (5) is the Cobb-Douglas production technology explained above, 
Equation (6) represents firms’ cost minimization with respect to labor input and shows that 
real marginal cost is identical with unit labor cost under full employment \( (\hat{w}_t - \hat{Y}_t) \), Equation (8) 
describes households’ staggered wage setting, and Equations (7) and (9) give the definitions of 
the real wage and the nominal wage inflation rate.

2.3 Labor Market Search Model

Next turn to the labor market search model. As in Trigari (2009), employment intermediaries 
are introduced so that they make a decision on employment while intermediate-good firms 
adopt the above-mentioned Calvo-style staggered price setting. Each intermediary employs 
workers and supplies a package of their labor services to intermediate-good firms, which produce 
differentiated goods by the above-mentioned Cobb-Douglas production technology using labor 
packages \( N_t \) as labor input. Thus, as is similar to Equation (5), the production technology is 
represented by

\[ \hat{Y}_t = \alpha \hat{N}_t + \varepsilon_{at}. \]  

(10)

Letting \( Z_t \) denote the real rental rate of labor packages, firms’ cost minimization with respect 
to labor packages leads to

\[ \hat{mc}_t = \hat{Z}_t + \hat{N}_t - \hat{Y}_t, \]  

(11)

as is similar to Equations (6) and (7).

The labor market is characterized by search and matching frictions. At the beginning of 
period \( t \), \( n_{t-1} \) workers are hired by employment intermediaries, but a fraction \( \rho \in (0,1) \) of 
these workers leaves jobs and enters the pool of job searchers. Then, letting the population 
size be normalized to unity, the measure of job searchers is given by

\[ u_t = 1 - (1 - \rho)n_{t-1}. \]  

(12)

Employment intermediaries post \( v_t \) vacancies. As a consequence, \( m_t \) job searchers are newly
hired according to the constant returns to scale matching technology

\[ m_t = \xi_m u_t^\xi v_t^{1-\xi}, \quad (13) \]

where \( \xi_m > 0 \) is the matching productivity and \( \xi \in (0, 1) \) is the search elasticity of new hire. As in Ravenna and Walsh (2008) and Kurozumi and Van Zandweghe (2010), it is assumed that new hires become productive instantaneously. Thus, the measure of workers supplying labor services in period \( t \) is given by

\[ n_t = (1 - \rho)n_{t-1} + m_t. \quad (14) \]

As in Blanchard and Galí (2010) and Gertler et al. (2008), there is a hiring cost instead of a vacancy posting cost typically used in literature on labor market search (e.g., Pissarides, 2000). Hiring is costly, depending on the tightness of the labor market. As in the literature, this tightness is measured by the ratio of vacancies to job searchers,

\[ x_t = \frac{v_t}{u_t}. \quad (15) \]

Thus, the labor market tightness rises with vacancies \( v_t \) but declines when the measure of job searchers \( u_t \) increases. Employment intermediaries need \( \gamma x_t m_t \) existing workers to recruit \( m_t \) new hires. Therefore, the hiring cost is opportunity cost of these workers, who would engage in production if there were no new hire.

Employment intermediaries use a linear technology, which produces \( h_t(n_t - \gamma x_t m_t) \) units of labor packages. The intermediaries then maximize profit

\[ E_t \sum_{j=0}^{\infty} \beta_{t,t+j} [Z_{t+j}h_{t+j} (n_{t+j} - \gamma x_{t+j} m_{t+j}) - z_{t+j} h_{t+j} n_{t+j}] \]

subject to Equations (12)–(15), where \( \beta_{t,t+j} = \beta^j \lambda_{t+j}/\lambda_t \) is the stochastic discount factor. Letting \( J_t \) be the Lagrange multiplier on the law of motion of employment (14), the first-order conditions for profit maximization with respect to \( n_t \) and \( x_t \) are given by

\[ Z_t h_t = z_t h_t + J_t - (1 - \rho) E_t \beta_{t,t+1} \left[ \gamma \xi_m Z_{t+1} h_{t+1} x_{t+1}^{2-\xi} + J_{t+1} \left( 1 - \xi_m x_{t+1}^{1-\xi} \right) \right], \]

\[ J_t = \gamma \xi x_t Z_t h_t, \quad (16) \]

where \( \gamma \xi = \gamma (2 - \xi)/(1 - \xi) \). Note that the multiplier \( J_t \) represents the marginal value of employment in terms of final goods. Combining these yields

\[ Z_t h_t = z_t h_t + \gamma \xi x_t Z_t h_t - (1 - \rho) E_t \beta_{t,t+1} \gamma \xi x_{t+1} Z_{t+1} h_{t+1} \left( 1 - \frac{\xi_m}{2 - \xi} x_{t+1}^{1-\xi} \right). \quad (17) \]
The real wage rate per worker \( z_t \) and labor hours per worker \( h_t \) are determined by Nash bargaining through which a joint surplus from employment is split between employment intermediaries and employed workers. Then, asset values of employed and unemployed workers, \( V_t \) and \( V_{ut} \), are given by

\[
V_t = z_t h_t - \frac{\chi_h h_t^{1+\sigma_h}}{\lambda_t(1 + \sigma_h)} + E_t \beta_{t,t+1} \left\{\left[1 - \rho(1 - p_{ut+1})\right]V_{t+1} + \rho(1 - p_{ut+1})V_{ut+1}\right\},
\]

\[
V_{ut} = b + E_t \beta_{t,t+1} \left[p_{ut+1}V_{t+1} + (1 - p_{ut+1})V_{ut+1}\right],
\]

where \( \chi_h > 0 \) is a scale parameter for labor disutility relative to consumption utility, \( p_{ut} = m_t/u_t \) is the job finding rate, and \( b = b_w w = b_w z h \) is the flow value of unemployment in terms of final goods, which includes unemployment benefits as well as other factors such as home production, and \( b_w \in (0, 1) \) is the ratio of this unemployment value to the steady-state real wage. Combining the asset values yields

\[
V_t - V_{ut} = z_t h_t - \frac{\chi_h h_t^{1+\sigma_h}}{\lambda_t(1 + \sigma_h)} - b + (1 - \rho)E_t \beta_{t,t+1}(1 - p_{ut+1})(V_{t+1} - V_{ut+1}), \tag{18}
\]

which represents workers’ net surplus from employment. Then, the real wage rate \( z_t \) and labor hours \( h_t \) are chosen so as to maximize \( (V_t - V_{ut})^{\eta_t} (J_t)^{1-\eta_t} \), where \( \eta_t = \eta \exp(\varepsilon_{\eta t})/[1 - \eta + \eta \exp(\varepsilon_{\eta t})] \in (0, 1) \) denotes workers’ share of the joint surplus and \( \varepsilon_{\eta t} \) is a labor bargaining shock that follows a stationary first-order autoregression process.\(^5\)

The first-order conditions for the wage rate and labor hours lead to

\[
V_t - V_{ut} = \frac{\eta_t}{1 - \eta_t} J_t, \tag{19}
\]

\[
\frac{\chi_h h_t^{\sigma_h}}{\lambda_t} = Z_t. \tag{20}
\]

Equation (19) implies that \( V_t - V_{ut} = \eta_t S_t \) and \( J_t = (1 - \eta_t)S_t \), where \( S_t = (V_t - V_{ut}) + J_t \) is the joint surplus, and hence \( \eta_t \) indeed shows workers’ share. From Equations (16), (18), and (19), the real wage is determined as

\[
w_t = z_t h_t
\]

\[
= b + \frac{\chi_h h_t^{1+\sigma_h}}{\lambda_t(1 + \sigma_h)} + \frac{\eta_t}{1 - \eta_t} \gamma \xi x_t Z_t h_t
\]

\[
- (1 - \rho)E_t \beta_{t,t+1} \gamma \xi x_{t+1} Z_{t+1} h_{t+1} \left(1 - \xi_m x_{t+1}^{1-\xi}\right) \frac{\eta_{t+1}}{1 - \eta_{t+1}}. \tag{21}
\]

\(^5\)The specification of workers’ share of the joint surplus has three properties: (i) \( \eta_t \to 0 \) as \( \varepsilon_{\eta t} \to -\infty \), (ii) \( \eta_t \to 1 \) as \( \varepsilon_{\eta t} \to \infty \), and (iii) \( \eta_t = \eta \) when \( \varepsilon_{\eta t} = 0 \).
Combining this and Equation (17), we have

\[ w_t = \eta_t \left\{ Z_t h_t + (1 - \rho) E_t \beta_{t+1} \gamma \varepsilon x_{t+1} Z_{t+1} h_{t+1} \left[ 1 - \frac{\xi_m}{2 - \xi} \right]^{1-\xi} - (1 - \xi_m x_{t+1}^{1-\xi}) \eta_{t+1} \frac{(1 - \eta_t)}{\eta_t (1 - \eta_{t+1})} \right\} \\
+ (1 - \eta_t) \left[ b + \frac{\chi_h h_t^{1+\sigma_h}}{\lambda_t(1 + \sigma_h)} \right]. \]

Hence, employed workers are compensated for a fraction \( \eta_t \) of employment intermediaries’ earnings and savings on future hiring cost and for the remaining fraction \( 1 - \eta_t \) of the sum of the flow values of unemployment and labor disutility.

In the labor market search model, log-linearized equilibrium conditions consist of not only Equations (1)–(4), (10), and (11) but also log-linearization of Equations (12)–(15), (17), (20), (21), and the resource constraint for labor packages\(^6\)

\[ N_t = h_t (n_t - \gamma x_t m_t). \]  

Combining Equation (11) and log-linearization of Equations (17), (21), and (22) leads to

\[ \hat{m} c_t = \hat{w}_t - \hat{Y}_t - \hat{n}_t - (1 - \frac{\varepsilon}{2}) \hat{z}_t - \left( 1 - \frac{\ln}{N} \right) (\hat{n}_t - \hat{m}_t - \hat{x}_t) \\
+ \gamma \varepsilon x (\hat{x}_t + \hat{Z}_t) - \gamma \varepsilon x / \beta (1 - \rho) (1 - \xi_m x^{1-\xi}) E_t \hat{x}_{t+1} \\
- \gamma \varepsilon x / \beta (1 - \rho) \left( 1 - \frac{\xi_m x^{1-\xi}}{2 - \xi} \right) (E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t + E_t \hat{Z}_{t+1} + E_t \hat{h}_{t+1} - \hat{h}_t). \]  

This shows that in the labor market search model, real marginal cost \( \hat{m} c_t \) is determined by not only unit labor cost \((\hat{w}_t - \hat{Y}_t + \hat{n}_t)\) but also other factors that stem from hiring cost. This real marginal cost differs radically from the one (6) in the sticky wage model, where the extensive margin of labor (i.e., employment) is absent. Therefore, if the labor market search model shows a better empirical performance than the sticky wage model, the terms related to the extensive margin are important driving forces of real marginal cost and hence inflation.

### 3 DATA AND ECONOMETRIC METHODS

Using a Bayesian likelihood approach, this paper estimates the sticky wage model and the labor market search model with Japan’s four quarterly time series as observable variables: \( \pi_t, R_t, Y_t, \) \( w_t. \) The data on \( \pi_t \) is the inflation rate of the seasonally adjusted CPI excluding fresh foods,

\(^6\)The log-linearization of these equations is given by Equations (22)–(25) and (36)–(40) of the working-paper version of this paper (Ichiue et al., 2008), where \( W_t \) (but not \( Z_t \)) denotes the real rental rate of labor packages.
and the effects of changes in the VAT rate on this inflation rate are adjusted. The data on \( R_t \) is the overnight call rate. For \( Y_t \) and \( w_t \), this paper uses real GDP per potential labor force and real wage per worker that are detrended by potential GDP per potential labor force. These potential variables are elements used by the Bank of Japan to estimate the output gap (see Hara et al., 2006), and hence the data on \( Y_t \) is completely consistent with the Bank’s estimates of the output gap. The real wage is the sum of employee income and net mixed income deflated by the CPI.\(^7\) The observation equations are given by

\[
\begin{bmatrix}
100 \log \pi_t \\
100 \log R_t \\
Y_t \\
100 \Delta \log w_t
\end{bmatrix} = \begin{bmatrix}
\frac{\hat{\pi}}{4} \\
(\hat{r} + \hat{\pi})/4 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
\hat{\pi}_t \\
\hat{R}_t \\
\hat{Y}_t \\
\hat{w}_t - \hat{w}_{t-1}
\end{bmatrix},
\]

where \( \hat{\pi} \) and \( \hat{r} \) are the annualized rates of inflation and real interest at the steady state.

In line with Sugo and Ueda (2008), the sample period is from 1981:1Q to 1995:4Q. The end of the sample period follows from the fact that our estimation strategy is not able to take into account the effects of the non-linearity in monetary policy rules that stems from the zero lower bound on the nominal interest rate.

Most parameters of each model are estimated, but some parameters are fixed or calculated from steady-state conditions to avoid an identification issue. In each model, this paper chooses the labor input elasticity of output in the Cobb-Douglas production technology at \( \alpha = 0.63 \) and the steady-state price elasticity of demand for differentiated goods at \( \theta_p = 6 \), which implies a steady-state price markup of 20%. The discount factor \( \beta \) is determined by the steady-state real interest rate \( \hat{r} \). In the sticky wage model, the steady-state wage elasticity of demand for differentiated labor services is set at \( \theta_w = 6 \), implying a steady-state wage markup of 20%. In the labor market search model, the present paper chooses the steady-state unemployment rate at \( 1 - n = 0.025 \), the job separation rate at \( \rho = 0.049 \), and the quarterly capital-output ratio at \( k_y = 1.24 \times 4 = 4.96 \).\(^8\) Moreover, the matching productivity and the search elasticity

\(^7\)This paper does not use data on labor hours per worker. This is because there is no data on labor hours of all workers including the self-employed. Note also that even for employed workers, labor hours may be poorly measured due to the presence of unreported labor hours.

\(^8\)The value of \( 1 - n = 0.025 \) is the sample period mean of the rate of unemployment given by subtracting the number of workers from the number of potential labor force used by the Bank of Japan. The value of \( \rho = 0.049 \) is the sample period mean of the seasonally adjusted job separation rate (industries covered, establishments with
of new hire are set at the estimates of Ishizaki and Kato (2003), \( \xi_m = 0.8 \) and \( \xi = 0.47 \). The remaining parameters and steady-state values are calculated from steady-state conditions.\(^9\)

Prior distributions of model parameters to be estimated are shown in Table 1. For the common parameters (i.e., \( \sigma_C, \chi, \xi_P, \gamma_P, \phi_R, \phi_{\pi}, \phi_{\lambda}, \bar{\pi}, \bar{r}, \rho_u, \rho_p, \sigma_a, \sigma_{\epsilon}, \sigma_R, \sigma_p \)), prior distributions are identical between the two models. These distributions and those of the other parameters in the sticky wage model (i.e., \( \sigma_h, \xi_w, \rho_w, \sigma_w \)) are chosen based on previous studies that estimate sticky wage models, such as Smets and Wouters (2003, 2007), Levin et al. (2006), Iiboshi et al. (2006), and Sugo and Ueda (2008). For the remaining parameters in the labor market search model (i.e., \( \eta, b_w, \rho_\eta, \sigma_\eta, \chi_h, \gamma \)), prior distributions are set based on recent studies that estimate labor market search models, such as Christoffel et al. (2006), Gertler et al. (2008), and Krause et al. (2008).

As in recent studies that take a Bayesian likelihood approach to estimate a DSGE model, this paper uses the Kalman filter to evaluate the likelihood function of each model and applies the Metropolis-Hastings algorithm to generate draws from a posterior distribution of model parameters.\(^10\) Based on these draws, the present paper makes inference on the parameters and obtains the Kalman smoothed estimates of unobservables.

## 4 ESTIMATION RESULTS

This section presents estimation results. We first show parameter estimates and then compare the sticky wage model and the labor market search model.

### 4.1 Parameter Estimates

For each parameter, Table 2 reports the posterior mean and the 90% HPD (Highest Posterior Density) interval.\(^11\) The estimates of the common parameters are similar between the sticky

---

30 employees or more) in the Monthly Labour Survey. Moreover, the value of \( k_y = 4.96 \) is the sample period mean of the GDP ratio of the capital stock in the Japan Industrial Productivity (JIP) Database.

\(^9\)See the working-paper version of this paper (Ichiue et al., 2008) for the steady-state relationships used in the estimation of the labor market search model.

\(^10\)In each estimation, 200,000 draws are generated and the first half of them is discarded. The scale factor for the jumping distribution in the Metropolis-Hastings algorithm is adjusted so that an acceptance rate of 24% is obtained. The Brooks and Gelman (1998) measure is used to check the convergence of model parameters.

\(^11\)In the sticky wage model, the scale parameter for labor disutility \( \chi_h \) disappears in log-linearizing the model.
wage model and the labor market search model. Moreover, most of the estimates are comparable
to those in Iiboshi et al. (2006) and Sugo and Ueda (2008), who estimate a sticky wage model
with Japan’s data, although our estimates of habit persistence $\chi$ are larger than theirs (i.e.,
0.64 in Iiboshi et al., 0.10 in Sugo and Ueda) while our estimates of price indexation to lagged
inflation $\gamma_p$ are smaller than theirs (i.e., 0.61 in Iiboshi et al., 0.86 in Sugo and Ueda).

As for the labor market parameters, our estimates of the inverse elasticity of labor supply
$\sigma_h$ and the probability of not reoptimizing wages $\xi_w$ are similar to those in Iiboshi et al. (2006)
(i.e., 2.43 and 0.37) and Sugo and Ueda (2008) (i.e., 2.15 and 0.52). Our estimate of the
steady-state worker share of $\eta = 0.50$ is comparable to the estimate of 0.58 in the case of
period-by-period labor bargaining in Gertler et al. (2008), who estimate a labor market search
model with U.S. data. Yet our estimate of the ratio of the flow value of unemployment to wages
of $b_w = 0.82$ is smaller than theirs (i.e., 0.98).

4.2 Model Comparison

Next turn to the comparison of the sticky wage model and the labor market search model in
terms of marginal likelihood and out-of-sample forecast performance.

Table 3 reports the marginal likelihood of each model. This table shows that the labor
market search model is superior to the sticky wage model in terms of marginal likelihood. The
log of Bayes factor (i.e., the difference in log of marginal likelihood) in favor of the former
model relative to the latter is $10.17 (=-277.36 - (-287.53))$. According to Jefferys (1961),
this value constitutes “decisive evidence” in favor of the labor market search model, because it
is larger than 4.61 (= log 100).

As for the out-of-sample forecast performance of each model, Table 4 reports the root mean
squared errors in the forecast of the four data series ($100 \log \pi_t$, $100 \log R_t$, $Y_t$, $100 \Delta \log w_t$)
for different forecast horizons over the period from 1991:1Q to 1995:4Q. The overall forecast
performance is measured by the log determinant of the uncentered forecast error covariance
matrix. For this exercise, each model was initially estimated over the period from 1981:1Q
to 1990:4Q, and then the estimated model was used to forecast the four series from 1991:1Q
to 1995:4Q. In this way each model was reestimated over the period until 1991:4Q, 1992:4Q,
1993:4Q, or 1994:4Q. Table 4 demonstrates that, for most of the forecast horizons, the overall
forecast performance is better (i.e., the log determinant is smaller) in the labor market search
model than in the sticky wage model. Moreover, the root mean squared errors in the out-of-sample forecast of the CPI inflation rate \(100 \log \pi_t\) and the overnight call rate \(100 \log R_t\) are smaller in the labor market search model for most of the forecast horizons.

In sum, the labor market search model is superior to the sticky wage model in terms of both marginal likelihood and out-of-sample forecast performance, particularly regarding inflation.

### 4.3 Why is Labor Market Search Model Better Able to Fit Data and Forecast Inflation?

In this subsection, we address the question of why the labor market search model fits the data and forecast inflation better than the sticky wage model. Because the non-labor market part is identical between these two models, the key difference is the relationship between real wages and inflation or output.

In the sticky wage model, real wages are highly correlated with output in the presence of monopolistically competitive labor markets. Besides, unit labor cost under full employment is identical with real marginal cost, which is the main driving force of inflation dynamics as can be seen in the NKPC (3).

Figure 1 illustrates the cross-correlation between real wages and output or inflation in the data and its 90% HPD interval in each model. The cross-correlation between real wages and output in the data, however, is not so high as in the sticky wage model. Moreover, Equation (6) shows that unit labor cost in the sticky wage model is determined by the data of real wages and output, and this cost lags far behind inflation by more than three years as shown in Figure 2. This fact is hard to replicate by the NKPC (3).

In the labor market search model, labor bargaining determines real wages as in Equation (21), which gives rise to a mild cross-correlation between real wages and output as can be seen in Figure 1. Moreover, as in Equation (23), real marginal cost is determined by both hiring cost and unit labor cost that varies with fluctuations in employment. This generates a high contemporaneous correlation between inflation and real marginal cost as represented in the NKPC (3). Therefore, the labor market search model fits the data and forecasts inflation better than the sticky wage model.

The estimation results presented above are consistent with the results of previous studies on Japan’s economy. Muto (2009) stresses that the measurement of real marginal cost plays a
crucial role in estimating the NKPC and shows that the consideration of labor market frictions improves the goodness of its fit to Japan’s data. Our estimation results show that introducing labor market search and matching frictions improves marginal likelihood. Braun et al. (2006) indicate the business cycle fact that the intensive margin plays a more important role in Japan’s labor input fluctuations than the extensive margin. This is consistent with our result that the extensive margin may be more important for inflation dynamics in Japan, because labor input adjustment at the extensive margin is very costly for firms in Japan. Therefore, fluctuations in hours per worker are better able to explain labor input fluctuations. For inflation dynamics, real marginal cost is the key factor and hence the costly adjustment at the extensive margin has a crucial influence on the dynamics.\(^\text{12}\)

## 5 ROBUSTNESS EXERCISES

This section evaluates the robustness of the results mentioned above. The present paper conducts the following two robustness exercises. The first exercise estimates an associated labor market search model without the intensive margin of labor and examines whether the labor market search model is superior to the sticky wage model even in the absence of the intensive margin. The second exercise estimates the labor market search model (with both the margins) using data on the unemployment rate additionally and compares it with the sticky wage model.

### 5.1 Labor Market Search Model without Intensive Margin

So far the labor market search model has allowed for labor input adjustment at the intensive margin in addition to the extensive margin. The presence of the intensive margin may help the labor market search model fit the data and forecast inflation better than the sticky wage model. Thus, we estimate an associated labor market search model without the intensive margin and compare it with the sticky wage model in terms of marginal likelihood and out-of-sample forecast performance.

\(^\text{12}\)A similar argument can be applied to the U.S. economy. Braun et al. (2006) indicate that in U.S. the extensive margin plays a more important role in labor input fluctuations, which may imply that labor input adjustment at the extensive margin is less costly and has a minor influence on inflation dynamics. In fact, when our two models are estimated with the U.S. data provided in Smets and Wouters (2007), it is shown that the sticky wage model is superior to the labor market search model in terms of marginal likelihood.
In the absence of the intensive margin, the labor market search model is modified in the following three respects.

First, employment intermediaries now maximize profit

\[ E_t \sum_{j=0}^{\infty} \beta_{t,t+j} [W_{t+j} (n_{t+j} - \gamma x_{t+j} m_{t+j}) - w_{t+j} n_{t+j}] \]

subject to Equations (12)–(15), where \( W_t \) is the real price of labor packages. The first-order conditions for profit maximization with respect to \( n_t \) and \( x_t \) are given by

\[ W_t = w_t + J_t - (1 - \rho) E_t \beta_{t,t+1} \left[ \gamma \xi_m W_{t+1} x_{t+1}^{2-\xi} + J_{t+1} \left( 1 - \xi_m x_{t+1}^{1-\xi} \right) \right], \]

\[ J_t = \gamma \xi x_t W_t. \tag{24} \]

Combining these yields

\[ W_t = w_t + \gamma \xi x_t W_t - (1 - \rho) E_t \beta_{t,t+1} \gamma \xi x_{t+1} W_{t+1} \left( 1 - \frac{\xi_m}{2 - \xi} x_{t+1}^{1-\xi} \right). \tag{25} \]

Second, only the real wage is determined via bargaining between employment intermediaries and employed workers. The asset value of employed workers is now given by

\[ V_t = w_t + (1 - \rho) V_{t+1} \left\{ \left[ 1 - \rho (1 - p_{ut+1}) \right] V_{t+1} + \rho (1 - p_{ut+1}) V_{ut+1} \right\}. \]

Hence, the difference between the asset values of employed and unemployed workers is

\[ V_t - V_{ut} = w_t - b + (1 - \rho) E_t \beta_{t,t+1} (1 - p_{ut+1}) (V_{t+1} - V_{ut+1}), \tag{26} \]

where the flow value of unemployment \( b \) now additionally includes labor disutility. From (19), (24), and (26), the wage is determined as

\[ w_t = b + \frac{\eta_t}{1 - \eta_t} \gamma \xi x_t W_t - (1 - \rho) E_t \beta_{t,t+1} \left( 1 - \xi_m x_{t+1}^{1-\xi} \right) \frac{\eta_{t+1}}{1 - \eta_{t+1}} \gamma x_{t+1} W_{t+1}. \tag{27} \]

Combining this and Equation (25), we have

\[ w_t = \eta_t \left\{ W_t + (1 - \rho) E_t \beta_{t,t+1} \gamma \xi x_{t+1} W_{t+1} \left[ 1 - \frac{\xi_m}{2 - \xi} x_{t+1}^{1-\xi} - \left( 1 - \xi_m x_{t+1}^{1-\xi} \right) \frac{\eta_{t+1}}{\eta_t (1 - \eta_{t+1})} \right] \right\} + (1 - \eta_t) b. \]

Last, the resource constraint for labor packages is now given by

\[ N_t = n_t - \gamma x_t m_t. \tag{28} \]
Consequently, in the labor market search model without the intensive margin, log-linearized equilibrium conditions consist of not only Equations (1)–(4), (10), and (11) but also log-linearization of Equations (12)–(15), (25), (27), and (28). These equilibrium conditions are estimated using the same data and the same econometric methods as presented above.

The second and third columns of Table 5 report the posterior mean and the 90% HPD interval for each parameter. The parameter estimates are similar to those in the presence of the intensive margin presented in the last two columns of Table 2, although the estimated ratio of the flow value of unemployment to wages, $b_w = 0.99$, is higher in line with the estimate of Gertler et al. (2008) in the case of period-by-period labor bargaining (i.e., 0.98). This reflects the inclusion of labor disutility in the flow value of unemployment.

Table 3 shows that the labor market search model with no intensive margin is superior to the sticky wage model in terms of marginal likelihood. The log of Bayes factor in favor of the former model relative to the latter is $6.95 = -280.58 - (-287.53)$. This value is larger than 4.61 ($= \log 100$) and hence, according to Jefferys (1961), it constitutes “decisive evidence” in favor of the labor market search model without the intensive margin. As illustrated in Figure 3.a and b, the labor market search model with no intensive margin generates the 90% HPD intervals that contain the cross-correlation between real wages and output and between real wages and inflation in the data. Hence, the labor market search model fits the data better than the sticky wage model even in the absence of the intensive margin. Moreover, we can see that embedding the intensive margin makes the labor market search model fit the data better. The log of Bayes factor of $3.22 = -277.36 - (-280.58)$ is larger than 2.30 ($= \log 10$) and hence constitutes “strong evidence” in favor of the presence of the intensive margin.

As for the out-of-sample forecast performance, the second to fifth columns of Table 6 report the root mean squared errors in the forecast of the four data series ($100 \log \pi_t$, $100 \log R_t$, $Y_t$, $100 \Delta \log w_t$) for different forecast horizons over the period from 1991:1Q to 1995:4Q. The forecast performance of the labor market search model in the absence of the intensive margin

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13The log-linearization of these equations is given by Equations (22)–(28) of the working-paper version of this paper (Ichiue et al., 2008).

14In the labor market search model with no intensive margin, the scale parameter for hiring cost $\gamma$ is calculated from steady-state conditions. See the working-paper version of this paper (Ichiue et al., 2008).

15Hagedorn and Manovskii (2008) argue that the flow value of unemployment needs to be close to wages in order for the labor market search framework to replicate the actual movements in U.S. real wages.
shown in the third to tenth rows of Table 6 is similar to that in the presence of the intensive margin presented in the last eight rows of Table 4. This suggests that the labor market search model without the intensive margin is superior to the sticky wage model in terms of out-of-sample forecast performance, particularly regarding inflation.\textsuperscript{16}

5.2 Labor Market Search Model Estimated with Unemployment Rate Data

The preceding subsection has confirmed the robustness of the result in terms of model specification. In this subsection we examine the robustness in terms of data. Because the data on the unemployment rate can be constructed from the time series of the numbers of workers and potential labor force used in building the data on output and real wages, we address the question of whether the labor market search model is superior to the sticky wage model when it is estimated using the unemployment rate data additionally. The comparison of these two models in terms of marginal likelihood is not possible, since the data set is now different between the models due to the absence of unemployment in the sticky wage model. Thus, out-of-sample forecast performance is compared.

As in Blanchard and Galí (2010), the observation equation for the unemployment rate at the end of period $t$, $U_t = 1 - n_t$, is given by

$$100U_t = \bar{U} - (1 - U)\hat{n}_t,$$

where $\bar{U} = 100U = 100(1 - n)$ is the steady-state unemployment rate, which is set at the same value as used above, i.e., $\bar{U} = 2.5$.

With the five data series ($100\log \pi_t$, $100\log R_t$, $Y_t$, $100\Delta \log w_t$, $100U_t$) and the same econometric methods as presented above, the labor market search model is estimated. The last two columns of Table 5 report the posterior mean and the 90% HPD interval for each parameter. Most of the estimates are similar to those in no use of the unemployment rate data shown in the last two columns of Table 2, although the estimated ratio of the flow value of unemployment to wages of $b_w = 0.75$ and the estimate of the bargaining shock persistence of $\rho_\eta = 0.83$ are smaller while the estimate of the technology shock persistence of $\rho_a = 0.86$ is larger.

\textsuperscript{16}Figure 3.c illustrates that the contemporaneous correlation between inflation and real marginal cost is high in the labor market search model with no intensive margin. Hence, the labor market search model forecasts inflation better than the sticky wage model even in the absence of the intensive margin.
The out-of-sample forecast performance is presented in the last eight rows of Table 6. Even when the labor market search model is estimated using the unemployment rate data additionally, the overall forecast performance is still better than that in the sticky wage model. Moreover, the root mean squared errors in the out-of-sample forecast of the CPI inflation rate \((100\log \pi_t)\) are also smaller for most of the forecast horizons.\(^{17}\)

6 CONCLUDING REMARKS

This paper has estimated a sticky wage model and a labor market search model with Japan’s data using a Bayesian likelihood approach and has examined which labor market specification is better able to describe inflation dynamics. The estimation results have shown that the labor market search model is superior to the sticky wage model in terms of both marginal likelihood and out-of-sample forecast performance, particularly regarding inflation. Compared to the sticky wage model, the labor market search model is better able to replicate the cross-correlation among inflation, real wages, and output in the data. Moreover, real marginal cost in the latter model is determined by both hiring cost and unit labor cost that varies with employment fluctuations, which generates a high contemporaneous correlation between inflation and real marginal cost as represented in the NKPC.

This paper follows previous studies such as Rabanal and Rubio-Ramírez (2005) to assume that there is no investment spending. Future work will extend the present analysis so that investment spending is incorporated in the two models. A preliminary estimation result shows that the result of the present paper still holds in the extended models.

\(^{17}\)Even when the unemployment rate data is additionally used in the model estimation, Figure 3.c illustrates that the contemporaneous correlation between inflation and real marginal cost is still high in the labor market search model, which yields a better performance regarding out-of-sample inflation forecast than that in the sticky wage model.
References


Table 1: Prior distribution of each model parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>90% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_C$ relative risk aversion</td>
<td>Gamma</td>
<td>2.00</td>
<td>[0.68, 3.88]</td>
</tr>
<tr>
<td>$\chi$ consumption habit persistence</td>
<td>Beta</td>
<td>0.60</td>
<td>[0.25, 0.90]</td>
</tr>
<tr>
<td>$\xi_p$ probability of not reoptimizing prices</td>
<td>Beta</td>
<td>0.50</td>
<td>[0.17, 0.83]</td>
</tr>
<tr>
<td>$\gamma_p$ price indexation to past inflation</td>
<td>Beta</td>
<td>0.50</td>
<td>[0.17, 0.83]</td>
</tr>
<tr>
<td>$\phi_R$ interest rate smoothing</td>
<td>Beta</td>
<td>0.75</td>
<td>[0.57, 0.90]</td>
</tr>
<tr>
<td>$\phi_\pi$ policy response to inflation</td>
<td>Normal</td>
<td>1.50</td>
<td>[1.17, 1.83]</td>
</tr>
<tr>
<td>$\phi_Y$ policy response to output</td>
<td>Normal</td>
<td>0.50</td>
<td>[0.17, 0.83]</td>
</tr>
<tr>
<td>$\bar{\pi}$ annualized steady-state inflation rate</td>
<td>Normal</td>
<td>1.00</td>
<td>[-0.64, 2.64]</td>
</tr>
<tr>
<td>$\bar{r}$ annualized steady-state real interest rate</td>
<td>Normal</td>
<td>2.00</td>
<td>[0.36, 3.64]</td>
</tr>
<tr>
<td>$\rho_a$ persistence of productivity shock</td>
<td>Beta</td>
<td>0.50</td>
<td>[0.25, 0.75]</td>
</tr>
<tr>
<td>$\rho_p$ persistence of price markup shock</td>
<td>Beta</td>
<td>0.60</td>
<td>[0.34, 0.83]</td>
</tr>
<tr>
<td>$\sigma_a$ s.d. of productivity shock innovation</td>
<td>Inv. gamma</td>
<td>0.50</td>
<td>[0.11, 1.41]</td>
</tr>
<tr>
<td>$\sigma_u$ s.d. of preference shock innovation</td>
<td>Inv. gamma</td>
<td>5.00</td>
<td>[1.05, 14.1]</td>
</tr>
<tr>
<td>$\sigma_R$ s.d. of monetary policy shock</td>
<td>Inv. gamma</td>
<td>0.50</td>
<td>[0.11, 1.41]</td>
</tr>
<tr>
<td>$\sigma_p$ s.d. of price markup shock innovation</td>
<td>Inv. gamma</td>
<td>5.00</td>
<td>[1.05, 14.1]</td>
</tr>
<tr>
<td><strong>Labor market parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_h$ inverse of elasticity of labor supply</td>
<td>Gamma</td>
<td>2.00</td>
<td>[0.68, 3.88]</td>
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<tr>
<td>$\xi_w$ probability of not reoptimizing wages</td>
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<td>0.40</td>
<td>[0.10, 0.75]</td>
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<td>$\rho_w$ persistence of wage markup shock</td>
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<td>[0.34, 0.83]</td>
</tr>
<tr>
<td>$\sigma_w$ s.d. of price markup shock innovation</td>
<td>Inv. gamma</td>
<td>5.00</td>
<td>[1.05, 14.1]</td>
</tr>
<tr>
<td>$\eta$ steady-state worker share in labor bargaining</td>
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<td>0.50</td>
<td>[0.17, 0.83]</td>
</tr>
<tr>
<td>$b_w$ flow-value-of-unemployment wage ratio</td>
<td>Beta</td>
<td>0.75</td>
<td>[0.50, 1.00]</td>
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<td>$\rho_\eta$ persistence of labor bargaining shock</td>
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<td>0.75</td>
<td>[0.47, 0.95]</td>
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<td>$\sigma_\eta$ s.d. of labor bargaining shock innovation</td>
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<td>20.0</td>
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<td>$\chi_h$ scale parameter for labor disutility</td>
<td>Gamma</td>
<td>0.10</td>
<td>[0.03, 0.19]</td>
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<tr>
<td>$\gamma$ scale parameter for hiring cost</td>
<td>Gamma</td>
<td>0.05</td>
<td>[0.02, 0.10]</td>
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Table 2: Posterior distribution of model parameters

<table>
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<tr>
<th>Parameter</th>
<th>Sticky wage</th>
<th>Labor market search</th>
<th>Sticky wage</th>
<th>Labor market search</th>
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<td>$\sigma_C$</td>
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<td>1.90 [0.42, 3.36]</td>
<td>1.64 [0.41, 2.73]</td>
<td>1.90 [0.42, 3.36]</td>
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<td>0.90 [0.80, 0.98]</td>
<td>0.86 [0.77, 0.96]</td>
<td>0.90 [0.80, 0.98]</td>
<td>0.86 [0.77, 0.96]</td>
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<td>$\xi_p$</td>
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<td>0.73 [0.65, 0.80]</td>
<td>0.72 [0.66, 0.79]</td>
<td>0.73 [0.65, 0.80]</td>
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<td>$\gamma_p$</td>
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<td>0.17 [0.03, 0.30]</td>
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<td>$\phi_x$</td>
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<td>1.49 [1.23, 1.77]</td>
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<td>1.49 [1.23, 1.77]</td>
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<td>0.16 [0.06, 0.25]</td>
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<td>1.78 [0.97, 2.63]</td>
<td>1.67 [0.84, 2.59]</td>
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<td>2.61 [1.90, 3.30]</td>
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<td>0.37 [0.15, 0.57]</td>
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<td>0.52 [0.40, 0.65]</td>
<td>0.54 [0.28, 0.76]</td>
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<tr>
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<td>0.14 [0.12, 0.16]</td>
<td>0.14 [0.11, 0.16]</td>
<td>0.14 [0.12, 0.16]</td>
</tr>
<tr>
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<td>3.10 [1.34, 4.84]</td>
<td>4.25 [1.80, 6.38]</td>
<td>3.10 [1.34, 4.84]</td>
<td>4.25 [1.80, 6.38]</td>
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<td>$\sigma_h$</td>
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<td>2.55 [1.10, 3.87]</td>
<td>1.87 [0.70, 2.89]</td>
<td>2.55 [1.10, 3.87]</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>0.54 [0.36, 0.74]</td>
<td>- -</td>
<td>0.54 [0.36, 0.74]</td>
<td>- -</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>0.82 [0.73, 0.93]</td>
<td>- -</td>
<td>0.82 [0.73, 0.93]</td>
<td>- -</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>7.35 [1.90, 13.0]</td>
<td>- -</td>
<td>7.35 [1.90, 13.0]</td>
<td>- -</td>
</tr>
<tr>
<td>$\eta$</td>
<td>- -</td>
<td>0.50 [0.26, 0.75]</td>
<td>- -</td>
<td>0.50 [0.26, 0.75]</td>
</tr>
<tr>
<td>$b_w$</td>
<td>- -</td>
<td>0.82 [0.71, 0.94]</td>
<td>- -</td>
<td>0.82 [0.71, 0.94]</td>
</tr>
<tr>
<td>$\rho_\eta$</td>
<td>- -</td>
<td>0.96 [0.93, 0.99]</td>
<td>- -</td>
<td>0.96 [0.93, 0.99]</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>- -</td>
<td>33.3 [17.0, 50.5]</td>
<td>- -</td>
<td>33.3 [17.0, 50.5]</td>
</tr>
<tr>
<td>$\chi_h$</td>
<td>- -</td>
<td>0.10 [0.02, 0.17]</td>
<td>- -</td>
<td>0.10 [0.02, 0.17]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>- -</td>
<td>0.04 [0.01, 0.08]</td>
<td>- -</td>
<td>0.04 [0.01, 0.08]</td>
</tr>
</tbody>
</table>
Table 3: Marginal likelihood of each model

<table>
<thead>
<tr>
<th>Model</th>
<th>Log marginal likelihood</th>
</tr>
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<tbody>
<tr>
<td>Sticky wage</td>
<td>-287.53</td>
</tr>
<tr>
<td>Labor market search</td>
<td>-277.36</td>
</tr>
<tr>
<td>Labor market search (no intensive margin)</td>
<td>-280.58</td>
</tr>
</tbody>
</table>

Note: The marginal likelihood is computed based on the Geweke (1999) modified harmonic mean estimator.
Table 4: Out-of-sample forecast performance of each model

<table>
<thead>
<tr>
<th>Quarter</th>
<th>100 log $\pi_t$</th>
<th>100 log $R_t$</th>
<th>$Y_t$</th>
<th>100 log $\Delta w_t$</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.927</td>
<td>0.545</td>
<td>0.404</td>
<td>1.145</td>
<td>-4.145</td>
</tr>
<tr>
<td>2</td>
<td>1.036</td>
<td>0.989</td>
<td>0.716</td>
<td>1.154</td>
<td>-1.515</td>
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<tr>
<td>3</td>
<td>1.006</td>
<td>1.417</td>
<td>0.997</td>
<td>1.125</td>
<td>0.422</td>
</tr>
<tr>
<td>4</td>
<td>1.265</td>
<td>1.766</td>
<td>1.290</td>
<td>1.150</td>
<td>0.510</td>
</tr>
<tr>
<td>5</td>
<td>1.305</td>
<td>2.061</td>
<td>1.529</td>
<td>1.188</td>
<td>1.265</td>
</tr>
<tr>
<td>6</td>
<td>1.325</td>
<td>2.370</td>
<td>1.795</td>
<td>1.223</td>
<td>1.869</td>
</tr>
<tr>
<td>7</td>
<td>1.328</td>
<td>2.567</td>
<td>2.038</td>
<td>1.182</td>
<td>1.337</td>
</tr>
<tr>
<td>8</td>
<td>1.331</td>
<td>2.788</td>
<td>2.228</td>
<td>1.230</td>
<td>1.404</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quarter</th>
<th>100 log $\pi_t$</th>
<th>100 log $R_t$</th>
<th>$Y_t$</th>
<th>100 log $\Delta w_t$</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.886</td>
<td>0.522</td>
<td>0.419</td>
<td>1.068</td>
<td>-4.351</td>
</tr>
<tr>
<td>2</td>
<td>0.974</td>
<td>0.908</td>
<td>0.770</td>
<td>1.171</td>
<td>-1.589</td>
</tr>
<tr>
<td>3</td>
<td>1.023</td>
<td>1.264</td>
<td>1.097</td>
<td>1.133</td>
<td>0.237</td>
</tr>
<tr>
<td>4</td>
<td>1.207</td>
<td>1.530</td>
<td>1.410</td>
<td>1.144</td>
<td>0.603</td>
</tr>
<tr>
<td>5</td>
<td>1.248</td>
<td>1.743</td>
<td>1.675</td>
<td>1.199</td>
<td>1.264</td>
</tr>
<tr>
<td>6</td>
<td>1.238</td>
<td>1.977</td>
<td>1.951</td>
<td>1.228</td>
<td>1.701</td>
</tr>
<tr>
<td>7</td>
<td>1.112</td>
<td>2.134</td>
<td>2.194</td>
<td>1.185</td>
<td>1.013</td>
</tr>
<tr>
<td>8</td>
<td>1.097</td>
<td>2.318</td>
<td>2.388</td>
<td>1.228</td>
<td>1.010</td>
</tr>
</tbody>
</table>
Table 5: Posterior distribution of model parameters: robustness exercises

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Intensive Margin</th>
<th>Unemployment Rate Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean 90% HPD interval</td>
<td>Mean 90% HPD interval</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>1.69 [0.47, 2.83]</td>
<td>1.74 [0.44, 2.92]</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.86 [0.74, 0.96]</td>
<td>0.87 [0.81, 0.95]</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>0.81 [0.76, 0.86]</td>
<td>0.75 [0.69, 0.81]</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>0.17 [0.03, 0.30]</td>
<td>0.22 [0.05, 0.38]</td>
</tr>
<tr>
<td>$\phi_R$</td>
<td>0.88 [0.82, 0.93]</td>
<td>0.83 [0.78, 0.88]</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>1.48 [1.19, 1.79]</td>
<td>1.57 [1.29, 1.86]</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.24 [0.10, 0.39]</td>
<td>0.09 [0.01, 0.18]</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>1.73 [0.86, 2.61]</td>
<td>1.67 [1.01, 2.34]</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>2.28 [1.33, 3.20]</td>
<td>2.75 [2.05, 3.41]</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.44 [0.21, 0.66]</td>
<td>0.86 [0.80, 0.92]</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>0.62 [0.43, 0.82]</td>
<td>0.56 [0.37, 0.72]</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>0.67 [0.49, 0.83]</td>
<td>0.64 [0.45, 0.84]</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.53 [0.27, 0.82]</td>
<td>0.63 [0.48, 0.76]</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>7.90 [2.64, 13.8]</td>
<td>7.86 [3.14, 12.6]</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.13 [0.11, 0.15]</td>
<td>0.14 [0.12, 0.16]</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>7.98 [2.70, 13.4]</td>
<td>5.81 [2.56, 8.60]</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>--</td>
<td>1.53 [0.70, 2.25]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.56 [0.28, 0.83]</td>
<td>0.56 [0.33, 0.81]</td>
</tr>
<tr>
<td>$b_w$</td>
<td>0.99 [0.98, 0.99]</td>
<td>0.75 [0.64, 0.86]</td>
</tr>
<tr>
<td>$\rho_\eta$</td>
<td>0.94 [0.90, 0.98]</td>
<td>0.83 [0.74, 0.93]</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>48.7 [28.0, 70.9]</td>
<td>22.8 [14.8, 29.9]</td>
</tr>
<tr>
<td>$\chi_\pi$</td>
<td>--</td>
<td>0.10 [0.02, 0.17]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>--</td>
<td>0.08 [0.03, 0.12]</td>
</tr>
</tbody>
</table>
Table 6: Out-of-sample forecast performance: robustness exercises

<table>
<thead>
<tr>
<th>Quarter</th>
<th>100 log $\pi_t$</th>
<th>100 log $R_t$</th>
<th>$Y_t$</th>
<th>100 log $w_t$</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.883</td>
<td>0.510</td>
<td>0.430</td>
<td>1.115</td>
<td>-4.354</td>
</tr>
<tr>
<td>2</td>
<td>0.933</td>
<td>0.910</td>
<td>0.809</td>
<td>1.172</td>
<td>-1.293</td>
</tr>
<tr>
<td>3</td>
<td>0.859</td>
<td>1.279</td>
<td>1.163</td>
<td>1.146</td>
<td>-0.222</td>
</tr>
<tr>
<td>4</td>
<td>1.053</td>
<td>1.556</td>
<td>1.483</td>
<td>1.156</td>
<td>1.008</td>
</tr>
<tr>
<td>5</td>
<td>1.034</td>
<td>1.760</td>
<td>1.746</td>
<td>1.206</td>
<td>1.456</td>
</tr>
<tr>
<td>6</td>
<td>1.001</td>
<td>1.980</td>
<td>2.021</td>
<td>1.234</td>
<td>1.734</td>
</tr>
<tr>
<td>7</td>
<td>0.899</td>
<td>2.134</td>
<td>2.257</td>
<td>1.187</td>
<td>1.376</td>
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<tr>
<td>8</td>
<td>0.894</td>
<td>2.313</td>
<td>2.457</td>
<td>1.233</td>
<td>1.192</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quarter</th>
<th>100 log $\pi_t$</th>
<th>100 log $R_t$</th>
<th>$Y_t$</th>
<th>100 log $w_t$</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.877</td>
<td>0.560</td>
<td>0.408</td>
<td>1.112</td>
<td>-4.329</td>
</tr>
<tr>
<td>2</td>
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<td>1.016</td>
<td>0.733</td>
<td>1.199</td>
<td>-1.744</td>
</tr>
<tr>
<td>3</td>
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<td>1.456</td>
<td>1.030</td>
<td>1.180</td>
<td>-0.660</td>
</tr>
<tr>
<td>4</td>
<td>1.208</td>
<td>1.823</td>
<td>1.320</td>
<td>1.160</td>
<td>0.162</td>
</tr>
<tr>
<td>5</td>
<td>1.253</td>
<td>2.139</td>
<td>1.564</td>
<td>1.211</td>
<td>0.943</td>
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<tr>
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<td>1.237</td>
<td>1.566</td>
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<td>2.933</td>
<td>2.246</td>
<td>1.224</td>
<td>1.022</td>
</tr>
</tbody>
</table>
Figure 1: Cross-correlation between real wages and output or inflation

a. cross-correlation between real wages and output

b. cross-correlation between real wages and inflation

Note: The solid line shows the correlation between output or inflation in period $t$ and real wages in period $t + k$, $k = 0, \pm 1, \ldots, \pm 6$ in the data and the dashed line shows the model-implied 90% HPD interval.
Figure 2: Cross-correlation between inflation and real marginal cost

Note: This figure shows the correlation between inflation in period $t$ and real marginal cost in period $t + k$, $k = 0, \pm 1, \ldots, \pm 15$. 
Figure 3: Cross-correlation in labor market search model: robustness exercises

a. cross-correlation between real wages and output: no intensive margin

b. cross-correlation between real wages and inflation: no intensive margin

c. cross-correlation between inflation and real marginal cost

Notes: In Panels a and b, the solid line shows the correlation between output or inflation in period $t$ and real wages in period $t + k$, $k = 0, \pm 1, \ldots, \pm 6$ in the data and the dashed line shows the model-implied 90% HPD interval. Panel c shows the correlation between inflation in period $t$ and real marginal cost in period $t + k$, $k = 0, \pm 1, \ldots, \pm 15$. 