A Note on Contribution Games with Loss Functions

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Abstract

Decisions on joint funding of continuous public goods between two agents often involve heterogeneous targets. We introduce loss functions in a contribution game in order to study the effect of this conflict. Unlike Varian (1994), joint contribution occurs only if the players' targets are sufficiently close and the sequential game reduces free riding problems, while total contribution is higher in the simultaneous game.

Jel Codes: H41, H77

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1 Introduction

This model is concerned with voluntary contribution to the provision of a continuous public good. Since Warr (1982, 1983) many scholars have analysed these contribution games where free riding problems are pervasive, and public goods are underprovided in equilibrium (Cornes and Sandler, 1984; Bergstrom et al., 1986).

Our main reference is the complete information contribution game by Varian (1994) -henceforth Varian- where two contributors have quasi–linear utility, the marginal evaluations of the public good are ordered and contribution can be sequential or simultaneous. Varian shows that free riding is normal in the simultaneous and sequential frameworks, and that the equilibrium public good provision is never higher if players contribute sequentially rather than simultaneously.

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In the simultaneous case the player with higher evaluation of the public good is the only contributor, while in the sequential case the amount of public good provided is even less because the first mover could have an incentive to shift the burden of providing the public good onto the follower.

This result has been further investigated by variations of the original framework and has even been tested experimentally (Falkinger et al., 2000; Gächter et al., 2010). Buchholz et al (1997) consider the case in which the players can exchange monetary compensations after a sequential contribution. Vesterlund (2003) points out the importance of the informational value in a sequential fund raising and, more recently, Bag and Roy (2011) compare sequential and simultaneous contributions with incomplete information. Andreoni (1998) finds interesting results by introducing altruism in public good provision.

In our model we consider an aspect that has not yet been covered in the literature but is relevant in several cases. Namely the possibility that contributors have different targets for the optimal level of the public good to be provided (examples include international or federal deals on carbon emissions, immigration quotas, exchange rates, and even tax harmonization). In our model this is depicted by introducing a quadratic loss function in the contribution game.

The existence of conflicting targets in the absence of an authority able to enforce contribution is common in international negotiations.

We consider two agents (C and L) who decide whether to contribute to the joint production of a public good. C and L have different resources and different targets, and we compare the outcomes when contribution is sequential or simultaneous. Although information is complete and symmetric, we find that heterogeneity in the players' targets can easily prevent contribution. Unlike Varian, joint contribution occurs only if the players' preferences are not too different. In addition, in contrast to Varian, joint contribution is more likely under the sequential regime. However, total contribution is unambigously higher in the simultaneous game (no matter who is the leader).

The paper is organised as follows: the next section introduces our model, Section 3 presents the results for both the simultaneous and sequential setups. Section 4 is devoted to compare the equilibrium contributions under the different institutional frameworks and Section 5 concludes.

2 The model

Our model must depict some basic issues: there exist a conflict over the funding of a public good; players may have different targets and different costs in raising the resources necessary to produce the public good; nobody is forced to cooperate.

The following contribution game includes all of these points.

2.1 Production of the public good

A convenient way to summarize our idea is by describing the public good as an output produced through the resources C and L are willing to spend in order to achieve their targets.

We define g_L and g_C the contributions by L and C respectively. Let M be the quantity of public good produced according to a linear technology:

$$M = \bar{M} + d(g_L + g_C) \qquad 0 < d < 1; \tag{1}$$

Where $\bar{M} \geq 0$ is the pre-existing level of the public good. This kind of production function fits the idea that the amount of public good is proportional to the resources used[1].

2.2 Payoffs

We assume that each player has a quadratic loss function with respect to his own target, and bears a quadratic cost to collect the resources needed to produce the public good[2]. As a consequence, utilities are

$$U_j = -\frac{1}{2}(M - M_j^*)^2 - \frac{\pi_j}{2} g_j^2 \tag{2}$$

$$j = C,\ L; \qquad \pi_C = 1;\ \pi_L > 1 \tag{3}$$

where $\pi_L > 1$ means that for L it is relatively costly to gather the resources needed to produce the public good. Cost asymmetry is assumed for sake of generality.

Finally, we assume $\bar{M} < M_C^*$ and $\bar{M} < M_L^*$.

By substituting (1) into (2) and (3) we can rewrite the payoffs:

$$U_j = -\frac{1}{2}(\bar{M} + d(g_L + g_C) - M_j^*)^2 - \frac{\pi_j}{2} g_j^2 \tag{4}$$

In Table 1 we present the equilibrium contributions under sequential and simultaneous decisions. In the sequential case both C and L could have the leadership.

Table 1

C leader	*L leader*	Simultaneous
$g_C^* = \frac{\Delta_C(\pi_L+d^2)\pi_L d - \pi_L d^3 \Delta_L}{\pi_L^2 d^2 + (\pi_L+d^2)^2}$	$g_C^{**} = \frac{\Delta_C(d^2+\pi_L+\pi_L d^2)d - d^3\Delta_L}{d^2+\pi_L(1+d^2)^2}$	$\tilde{g}_C = \frac{\Delta_C(\pi_L+d^2)d - d^3\Delta_L}{d^2+\pi_L+\pi_L d^2}$
$g_L^* = \frac{\Delta_L(\pi_L+d^2+\pi_L d^2)d - \pi_L d^3\Delta_C}{\pi_L^2 d^2+(\pi_L+d^2)^2}$	$g_L^{**} = \frac{\Delta_L(1+d^2)d - d^3\Delta_C}{d^2+\pi_L(1+d^2)^2}$	$\tilde{g}_L = \frac{\Delta_L(1+d^2)d - d^3\Delta_C}{d^2+\pi_L+\pi_L d^2}$

where $\Delta_C \equiv (M_C^* - \bar{M})$ and $\Delta_L \equiv (M_L^* - \bar{M})$.

[1] Linearity is useful to obtain closed-form solutions with no loss of generality.

[2] Gathering real resources always generates costs: for a country they can be the political costs of raising taxes, or even the opportunity costs of diverting funds from alternative projects.

By observing the outcomes summarized in Table 1, it is evident that in all cases the contribution of C is decreasing with respect to Δ_L, and the contribution of L is decreasing with respect to Δ_C.

Intuitively, if L prefers more public good relative to C, the latter could have an incentive to free ride should L produce enough for both players. This conveys the essential insight that, in order to achieve joint contribution, the targets M_C^* and M_L^* must not be too different. This result has crucial consequences that we are going to discuss in the rest of the paper.

3 Positivity conditions for the equilibrium contributions

In Table 2 we report the conditions under which equilibrium contributions are positive.

Table 2

C leader	*L leader*	Simultaneous
$\frac{\Delta_C}{\Delta_L} > \frac{d^2}{\pi_L + d^2}$	$\frac{\Delta_C}{\Delta_L} > \frac{d^2}{d^2 + \pi_L + \pi_L d^2}$	$\frac{\Delta_C}{\Delta_L} > \frac{d^2}{d^2 + \pi_L}$
$\frac{\Delta_C}{\Delta_L} < \frac{\pi_L + d^2 + \pi_L d^2}{\pi_L d^2}$	$\frac{\Delta_C}{\Delta_L} < \frac{1+d^2}{d^2}$	$\frac{\Delta_C}{\Delta_L} < \frac{1+d^2}{d^2}$

The conditions that assure a positive contribution by *both* players in the different games are summarized in Figure 1.

By simple inspection of the previous conditions we can write the following proposition:

Proposition 1 *(Positivity conditions for the individual contributions): equilibrium contributions are both positive if and only if the individual targets are not too different. The admissible difference is broader in the sequential game.*

Proof. see the appendix. ■

These results depart from Varian in two respects. Firstly, Varian argues that free riding occurs when preferences are similar. Secondly, in Varian sequentiality can exacerbate free riding problems -a leader with higher marginal utility from the public good might be better off by not contributing and free riding on the follower. Both these outcomes are reversed in our model.

The first departure happens because the initial level of public good is below the target for both players, and free riding is convenient only if one player's contribution saturates the other player's utility. This is possible when the targets M_C^* and M_L^* are quite different. When targets are close, a single player's contribution is never sufficient to achieve the other player's target.

We see the second departure because the leader does not contribute if and only if his target is sufficiently low relative to the follower. If this is not the case, the only way to exploit the leadership is to try to set the contribution at a level that pushes the follower to add his own contribution.

As a consequence, contributions will be both positive under a wider difference in the individual targets (see Figure 1).

4 Sequential decisions vs. simultaneous decisions

4.1 Total contribution

In this section we restrict our attention to the cases in which both contributions are positive in equilibrium. It is straightforward to conclude that total contribution is higher in the simultaneous regime.

Proposition 2 *(Total contribution ranking): when both contributions are positive, total contribution is higher in the simultaneous game.*

Proof. See the appendix. ■

The proposition states that the simultaneous game dominates the sequential game in terms of total contribution *no matter who is the leader.* Unlike proposition 1, this result is in line with Varian [3].

Proposition 1 and proposition 2 convey our most important result, namely that the simultaneous game increases total contribution, but it requires more stringent conditions in order to obtain joint contribution.

In other words, the simultaneous framework is successful in increasing total contribution *given that players are willing to contribute*, while the sequential framework is successful in *inducing contribution.*

In addition, we must stress that the simplest way to obtain some contribution from a reluctant player is to have it act as a follower in the sequential game. In fact, from Proposition 1 we know that the leader tries to set his own contribution at a level that pushes the follower to contribute as well.

4.2 Individual contribution

We now compare the individual contributions within the different regimes. Again, we consider only the case of joint contribution.

Our first conclusion is summarized in the following proposition:

Proposition 3 *(Equilibrium contributions in the sequential game): in the sequential game the leader contributes more than the follower.*

Proof. See the appendix. ■

This result is important because it allows us to know unambigously who provides the higher contribution.

In the next proposition we compare the individual contributions in the simultaneous game:

[3] We also have $(g_C^* + g_L^*) \geq (g_C^{**} + g_L^{**})$ when $\frac{\Delta_C}{\Delta_L} \leq \frac{\pi_L(1+d^2)+d^2(2+d^2)}{\pi_L(1+2d^2)+d^2(1+d^2)}$.

Proposition 4 *(Equilibrium contributions in the simultaneous game): in the simultaneous game C contributes more than L if* $\frac{\Delta_C}{\Delta_L} > \frac{1+2d^2}{\pi_L+2d^2}$ *and viceversa.*

Proof. See the Appendix. ■

To understand this proposition, suppose that costs are symmetric, i.e. $\pi_L = 1$. In such a case, the condition $\frac{\Delta_C}{\Delta_L} > \frac{1+2d^2}{\pi_L+2d^2}$ boils down to $\Delta_C > \Delta_L$. Hence, when the cost of gathering the resources necessary to produce the public good is the same, the player who desires more public good contributes more. When π_L is larger than unity this condition is relaxed: we have $\tilde{g}_C > \tilde{g}_L$ if $\Delta_C > \left(\frac{1+2d^2}{\pi_L+2d^2}\right)\Delta_L$, with $\left(\frac{1+2d^2}{\pi_L+2d^2}\right) < 1$.

In other words, C observes that L bears a higher cost, and, if π_L is sufficiently high C is going to contribute more than L even though $\Delta_C < \Delta_L$.

5 Conclusions

Our simple model has several implications for public goods provision in the presence of different targets. In contrast to Varian, loss functions imply that joint contribution occurs only if players' targets are close enough, and that heterogeneity in the preferences over the optimal level of the public good is a major cause of free riding. The sequential framework reduces the free rider problem because the leader tries to push the follower to contribute. However, total contribution is higher in the simultaneous game.

We conclude that when payoffs can be conveniently represented through a loss function there exists a trade-off -a simultaneous framework obtains higher total contribution when players have similar targets, but a sequential framework may obtain joint contribution when targets are very different.

References

[1] Andreoni J. (1998), "Toward a Theory of Charitable Fund-Raising", Journal of Political Economy 106, 1186-1203.

[2] Bag P.K., Roy S., (2011), "On Sequential and Simultaneous Contributions under Incomplete Information" , International Journal of Game Theory, 40, 119-145.

[3] Bergstrom T., Blume, L., Varian, H., (1986), "On the Private Provision of Public Good", Journal of Public Economics 29, 25-49.

[4] Buchholz W., Konrad K., Lommerud, K.E. (1997), "Stackelberg leadership and transfers in private provision of public goods", Review of Economic Design, 3, 2 -43.

[5] Cornes R., Sandler T., (1984), "Easy Riders, Joint Production and Public Goods", Economic Journal 94, 580-598.

[6] Falkinger J., Fehr E., Gächter S., Winter-Ebmer R. (2000), "A Simple Mechanism for the Efficient Provision of Public Goods: Experimental Evidence", The American Economic Review 90, 247-264

[7] Gächter S., Nosenzo D., Renner E., Sefton M., (2010), "Sequential vs. simultaneous contributions to public goods: Experimental evidence", Journal of Public Economics, 94, 515–522.

[8] Varian H. (1994), "Sequential Contributions to Public Goods", Journal of Public Economics, 53 (2), 165 - 186.

[9] Vesterlund L. (2003), "The Informational Value of Sequential Fundraising", Journal of Public Economics 87, 627– 657.

[10] Warr, P.G. (1983), "The Private Provision of a Public Good Independent of the Distribution of Income", Economics Letters 13, 207-211

[11] Warr P.G. (1982), "Pareto Optimal Redistribution and Private Charity", Journal of Public Economics 19, 131-138.

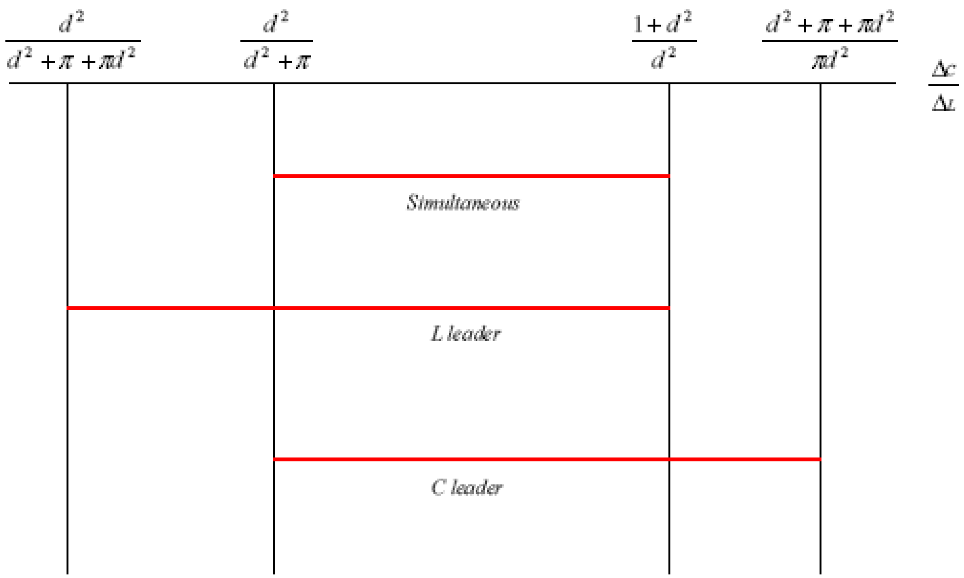

Figure 1: *positivity conditions for the equilibrium contributions*

6 Appendix

Proof of Proposition 1

In the simultaneous game both contributions are positive when

$$\frac{d^2}{\pi_L + d^2} < \frac{\Delta_C}{\Delta_L} < \frac{1 + d^2}{d^2}. \tag{5}$$

In the sequential game (C leader) both contributions are positive when

$$\frac{d^2}{\pi_L + d^2} < \frac{\Delta_C}{\Delta_L} < \frac{d^2 + \pi_L + \pi_L d^2}{d^2} \tag{6}$$

since $\frac{1+d^2}{d^2} < \frac{d^2+\pi_L+\pi_L d^2}{d^2}$, the interval of $\frac{\Delta_C}{\Delta_L}$ under which both contributions are positive is wider in the sequential game.

When L is the leader both contributions are positive when

$$\frac{d^2}{d^2 + \pi_L + \pi_L d^2} < \frac{\Delta_C}{\Delta_L} < \frac{1 + d^2}{d^2} \tag{7}$$

since $\frac{d^2}{d^2+\pi_L+\pi_L d^2} < \frac{d^2}{\pi_L+d^2}$, the interval of $\frac{\Delta_C}{\Delta_L}$ under which both contributions are positive is wider in the sequential game.

Proof of Proposition 2

We want to prove that $(\tilde{g}_C + \tilde{g}_L) > (g_C^* + g_L^*)$. Thus, we have to verify that

$$\underbrace{\frac{d(\pi_L \Delta_C + \Delta_L)}{\pi_L + d^2 + \pi_L d^2}}_{\text{simultaneous}} > \underbrace{\frac{d(\pi_L^2 \Delta_C + \Delta_L(\pi_L + d^2))}{\pi_L^2 d^2 + (\pi_L + d^2)^2}}_{\text{sequential, } C \text{ leader}} \tag{8}$$

Condition (8) boils down to

$$\frac{\Delta_C}{\Delta_L} > \frac{d^2}{\pi_L + d^2}.$$

We want to prove that $(\tilde{g}_C + \tilde{g}_L) > (g_C^{**} + g_L^{**})$. We have to verify that

$$\underbrace{\frac{d(\pi_L \Delta_C + \Delta_L)}{\pi_L + d^2 + \pi_L d^2}}_{\text{simultaneous}} > \underbrace{\frac{d\Delta_C(\pi_L + \pi_L d^2) + d\Delta_L}{d^2 + \pi_L(1 + d^2)^2}}_{\text{sequential, } L \text{ leader}} \tag{9}$$

which boils down to

$$\frac{\Delta_C}{\Delta_L} < \frac{1 + d^2}{d^2}.$$

hence, $(\tilde{g}_C + \tilde{g}_L) > (g_C^* + g_L^*)$ when $\frac{\Delta_C}{\Delta_L} > \frac{d^2}{\pi_L + d^2}$ and $(\tilde{g}_C + \tilde{g}_L) > (g_C^{**} + g_L^{**})$ when $\frac{\Delta_C}{\Delta_L} < \frac{1+d^2}{d^2}$. These conditions coincide with the values of $\frac{\Delta_C}{\Delta_L}$ assuring the positivity of both contributions in the simultaneous framework. Thus, the proposition holds.

Proof of Proposition 3

We want to prove that the leader contributes more than the follower. With C leader, we set $g_C^* \geq g_L^*$:

$$\frac{\Delta_C(\pi_L + d^2)\pi_L d - \pi_L d^3 \Delta_L}{\pi_L^2 d^2 + (\pi_L + d^2)^2} \geq \frac{\Delta_L(\pi_L + d^2 + \pi_L d^2)d - \pi_L d^3 \Delta_C}{\pi_L^2 d^2 + (\pi_L + d^2)^2} \tag{10}$$

by rearranging condition (10) we obtain

$$\frac{\Delta_C}{\Delta_L} \geq \frac{\pi_L + d^2 + 2\pi_L d^2}{\pi_L + 2\pi_L d^2}$$

since

$$\frac{d^2}{\pi_L + d^2} < \frac{\pi_L + d^2 + 2\pi_L d^2}{\pi_L + 2\pi_L d^2} < \frac{\pi_L d^2 + d^2 + \pi_L}{\pi_L d^2}$$

we conclude that, when both contributions are positive, $g_C^* > g_L^*$.

With L leader, we set $g_L^{**} \geq g_C^{**}$:

$$\frac{\Delta_L(1 + d^2)d - d^3 \Delta_C}{d^2 + \pi_L(1 + d^2)^2} \geq \frac{\Delta_C(d^2 + \pi_L + \pi_L d^2)d - d^3 \Delta_L}{d^2 + \pi_L(1 + d^2)^2}. \tag{11}$$

By rearranging condition (11) we obtain

$$\frac{\Delta_C}{\Delta_L} \leq \frac{1 + 2d^2}{\pi_L + \pi_L d^2 + 2d^2};$$

since

$$\frac{d^2}{d^2 + \pi_L + \pi_L d^2} < \frac{1 + 2d^2}{\pi_L + \pi_L d^2 + 2d^2} < \frac{1 + d^2}{d^2}$$

we conclude that, when both contributions are positive, $g_L^{**} > g_C^{**}$.

Proof of Proposition 4

To compare the individual contributions we set $\tilde{g}_C \geq \tilde{g}_L$:

$$\frac{\Delta_C(\pi_L + d^2)d - d^3 \Delta_L}{d^2 + \pi_L + \pi_L d^2} \geq \frac{\Delta_L(1 + d^2)d - d^3 \Delta_C}{d^2 + \pi_L + \pi_L d^2}. \tag{12}$$

By rearranging condition (12) we obtain

$$\frac{\Delta_C}{\Delta_L} \geq \frac{\pi_L + 2d^2}{1 + 2d^2}.$$