Why do Facebook and Twitter facilitate revolutions more than TV and radio?

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Abstract

A distinctive feature of recent revolutions was the key role of social media (e.g. Facebook, Twitter and YouTube). In a simple model we assume that while social media allow to observe all previous decisions, mass media only give aggregate information about the state of a revolt. We show, first, that when individuals' willingness to revolt is publicly known, then both sorts of media foster a successful revolution. However, when willingness to revolt is private information, only social media ensure that a revolt succeeds, with mass media multiple outcomes are possible. This suggests that social media enhance the likelihood that a revolution triumphs more than traditional mass media.

Keywords: social media, mass media, revolution, coordination game, sequential games

JEL Classification: C72, D02, D74

1 Introduction

"We use Facebook to schedule protests, Twitter to coordinate, and YouTube to tell the world." (Anonymous Cairo Activist)

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In the first months of 2011 a cascade of revolutions swept through the Arab world. A distinguishing feature of the uprisings was the omnipresence of social media (especially, Facebook, Twitter and YouTube).\(^1\) Social media affect the evolution of protests in various ways. They offer an easy, quick and inexpensive means of communication that helps to spread information and facilitates community creation (e.g. Garrett (2006)). Through dense interconnections and a decentralized structure, communication between protesters becomes robust and less susceptible to possible (and likely targeted) disruption (see Friedland and Rogerson (2009)). In spite of heavily controlled traditional mass media, social media technology helps to inform audiences around the world about the unfolding of the events, attracting international attention and provoking diplomatic pressure.\(^2\)

Though there is a diverse set of theories about political protests and social movements, most of these theories identify efficient mobilization as a key factor to achieve the goals of any movement.\(^3\) Mobilization relies on the channels of communication and the flow of information that enable protesters (both the actual and potential ones) to organize themselves and engage in collective action. Hence, communication and the technologies that make it possible play a crucial role in mobilization.

In this paper, we present a model that explains how mass and social media affect mobilization when it is known that there are enough willing individuals to overthrow the dictator. Our starting point is that willingness to participate in the protests depends on the perceived costs and benefits of participation, as proposed by the resource mobilization literature in sociology (e.g. Klandermans (1984), Opp (2009)). Arguably, heading out onto the streets implies the costs of facing tear gas, rubber bullets and potential arrest and incarceration. Benefits involve the perceived gains in participating in an uprising that may bring about a better future, provided it succeeds. The probability of success is highly related to the number of participants. Yet, when a potential protester decides whether to participate, possibly she has only a vague idea about if sufficient other people will participate. Different types of media may affect these expectations (and the resulting mobilization) in diverse ways. We study how individuals’ decisions to participate in the revolution is affected by two different communication technologies: mass and social media.

We posit that when an individual obtains information through mass media then she gets to know the actual state of the revolution in that moment, whereas when informed via social media she is able to observe

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\(^1\)Protesters in Tunisia, Egypt and other Arab countries were not the first to use Internet actively to organize themselves and inform the world. For instance, during the Ukrainian Orange Revolution in 2004 or the Iranian protest in 2009 (dubbed as the ‘Twitter revolution’) social media had a prominent role (e.g. Goldstein (2007) and Kamalipour (2010)).

\(^2\)However, it should also be noted that Internet is a value-neutral technology that is also used by the repressive regimes for surveillance purposes, for blocking and hacking websites and e-mails (see, for instance, 2009 Country Reports on Human Rights Practices (2010)).

\(^3\)Opp (2009) provides a comprehensive survey of the literature.
the sequence of decisions leading up to that state. For instance, when a search in Twitter is realized, the individual gets the last conversations about the topic, and by scrolling down the page she is able to see all previous tweets about it. In Facebook, the users may comment on the events and all previous comments can be read. We suppose that tweets and comments are informative about the individuals’ decision to join the protests or to stay at home. By contrast, when TV or radio inform about the state of a given event, the precise history remains hidden, only aggregate information about the turnout is reported.

We model the problem of revolution as a coordination problem. We suppose that there are two groups in the society: one consists of individuals who want to overthrow the dictator (willing individuals) and the another one is composed of individuals who do not want to change the regime (unwilling individuals). We assume that there are enough willing individuals to bring about a change. That is, if all of them revolt, then the dictator is overthrown. However, if the number of protesters (those willing individuals who actually head onto the streets) falls short of a critical mass, then the dictator remains in power and may punish those who participated in the failed revolt. We assume that individuals choose consecutively if they want to take part in the revolt or not, and the order of decisions is randomly selected. Before decision, each individual is informed about the state of the revolution. When this information is channeled through mass media, the individual learns how many people have already chosen to participate (i.e. the actual state of the revolution). The difference when informed through the social media is that individuals observe each of the past decisions (e.g. the precise history).

First, we show that when the type of individuals (that is, if she is willing to revolt or not) is public information and they are informed through any of the communication technologies, each willing individual takes part in the revolution and, thus, it is successful. However, it is not necessarily the case when the individuals receive no information about the state of the revolution. This result indicates that the mere existence of communication technologies that enable to spread information facilitates that social movements achieve their objective. Without the communication technologies individuals play a simultaneous game that has multiple equilibria, whereas the means of communication transforms the game into a sequential one in which the equilibrium outcome is unique.

Second, we study a more realistic setup in which the type of individuals is not observed (i.e. it is private information). We assume that individuals know that there are enough individuals to change the regime, but they do not know who the willing individuals are. Under these circumstances, the type of communication technology becomes relevant. We prove that mass media do not necessarily enable willing individuals to organize themselves efficiently. Thus, depending on the perceived costs and benefits willing individuals possibly choose not to revolt. However, successful revolution is the unique equilibrium outcome when willing
individuals use social media, independently of the severity of punishment that protesters may suffer if they fail to overthrow the dictator. This result shows that communication through social media facilitates that revolutions succeed more than when communication is channeled through traditional mass media.

Although our comparison of mass and social media suggests that they are competing communication technologies, our results can be interpreted as answering the question about how social media enhances mass media’s ability to mobilize individuals. In this respect, our findings shed light on the differential effect that social media have compared to the traditional mass media.

Section 2 presents the model and the results. In Section 3 we discuss two examples. All the proofs are relegated to the Appendix.

2 The model

We study in a simple model how different communication technologies determine the outcome of a revolt. Suppose that there is a finite set of individuals, \( N = \{1, 2, \ldots, n\} \) and a dictator. Each individual chooses an action \( a_i \in \{r, s\} \) where \( r \) means "revolt" and \( s \) "stay at home".\(^4\) We assume that each individual decides only once, and therefore decisions cannot be changed. Each person \( i \) is either of type \( \tau_i = w \) (willing to revolt) or \( \tau_i = x \) (unwilling). Changing the regime is assumed to be the socially efficient outcome (as it will be clear from the payoffs). Willing individuals are ready to participate in protests, unwilling individuals are reluctant to do so. We suppose that there is a fixed amount \( \xi \) of individuals of the willing type, \( \# \{i : \tau_i = w\} = \xi \), \( \xi \in (0, n) \) and, therefore the number of unwilling citizens is also fixed, \( \# \{i : \tau_i = x\} = n - \xi \).\(^5\) We assume that \( n \) and \( \xi \) are common knowledge.

Individuals decide in a sequence. Let the type vector \( \tau = (\tau_1, \tau_2, \ldots, \tau_n) \) denote the sequence of individuals. The set of sequences of length \( n \) with \( \xi \) willing citizens is given by

\[
\Upsilon_{n, \xi} = \{\tau : \# \{\tau_j \in \tau : \tau_j = w\} = \xi\}.
\]

There are \( \binom{n}{\xi} \) possible type vectors and any of them is selected with equal probability.

The utility of each individual \( i \) depends on her type and the actions chosen by all individuals. The amount of individuals participating in the revolution that is necessary to bring the revolt to triumph is given by the threshold \( t \). Thus, the dictator is overthrown, if and only if at least \( t \) citizens decide to revolt.\(^6\) Otherwise

\(^4\)We follow the terminology and notation of Chwe (2000) who studies the conditions of the social structure that allow successful revolts.

\(^5\)We use "individual" and "citizen" in an interchangeable manner.

\(^6\)Collective action has been studied in the literature using threshold models since Schelling (1977) and Granovetter (1978).
the dictator remains in power. Throughout the paper, it is assumed that \( t \leq \xi \) is common knowledge, so there are always sufficient willing individuals in the society to change the regime and it is known by everyone. However, although the change is achievable, it needs the coordinated action of individuals whose decision depends on the expected costs and benefits of participating in the revolution.

Let \( a_i \) be the action chosen by individual \( i \) and let \( a = (a_1, a_2, ..., a_n) \) be the profile of actions. We assume that the utility of a willing individual is

\[
\begin{align*}
    u_i(w, a : a_i = r, \# \{a_j : a_j = r, j \in N\} \geq t) &= u_{w,r,R} \\
    u_i(w, a : a_i = r, \# \{a_j : a_j = r, j \in N\} < t) &= u_{w,r,F} \\
    u_i(w, a : a_i = s) &= u_{w,s} \\
\end{align*}
\]

with \( u_{w,r,R} > u_{w,s} > u_{w,r,F} \)

In words, willing individuals’ utility is highest when they participate in a successful revolution \( (u_{w,r,R}) \). If they stay at home, they derive less utility \( (u_{w,s}) \). The lowest utility is derived from taking part in a revolution that is defeated. The payoff \( u_{w,r,F} \) can be interpreted as the punishment that the dictator imposes on protesters who participate in a revolution that fails. Suffering this punishment is the potential cost of participation. The utility of the willing individuals generates a game among them that resembles the classic stag-hunt situation.

With respect to unwilling individuals, we assume that they always prefer to stay at home:

\[
\begin{align*}
    u_i(x, a : a_i = s) &= u_{x,s} \\
    u_i(x, a : a_i = r) &= u_{x,r} \\
    u_{x,s} &> u_{x,r} \\
\end{align*}
\]

For simplicity, unwilling individuals are all those individuals who would not participate in the revolt (whatever reasons they might have). Although their choice is always the same, their presence makes coordination difficult, since a willing individual who observes somebody staying at home does not know if it is due to an unwilling citizen or a willing one who decided not to participate in the revolt.

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7 In the utilities, the first subscript refers to the type of the individual, the second to the action that she undertakes, whereas the third one indicates the outcome. \( R \) represents a successful revolution, while \( F \) denotes that it has failed.

8 The utility of staying at home may depend on whether the revolution triumphs or not. A successful revolt may bring better life to a willing individual who by staying at home avoids the costs of the revolution. Thus, there may be free-riding issues at stake as well (see for instance Lohmann (1993)). Although these are interesting questions (and promising venues of future research), we disregard them and focus on the coordination problem embedded in the above payoffs.
Notice that given these payoffs, the first best is achieved if the willing people coordinate and overthrow
the dictator. The reason is that unwilling individuals’ utility is not affected by the outcome of the revolt,
whereas willing individuals are better off if the uprising is successful.

We suppose that the index of the individual \( (i \in N) \) corresponds to her position in the sequence of
decisions. The information about past decisions that is available to individuals depends on the communication
technology. We consider three possibilities:

- **No technology**: Individuals ignore previous choices when deciding.

- **Mass media technology**: The individuals have aggregate information about the actions that have been
  already taken. We model this fact by introducing into the information set of each individual the
  number of actions carried out by the predecessors. This represents a situation in which individuals
  obtain information through radio or television about the state of the revolution before making their
  decision and observe the aggregate turnout in the protests.

- **Social media technology**: The individuals observe the individual action of each predecessor. This means
  that individual \( i \) knows exactly which action was chosen by each of its \( i - 1 \) predecessors. When deciding
  whether to participate in the revolution, through Facebook or Twitter (or any other social media) the
  individual observes the exact history of previous decisions.

Let \( \varphi_i \) denote the information set of individual \( i \). When no communication technology is available, then
\( \varphi_i = \{ \tau_i \} \). Thus, in this case individuals only know their own types, but nothing about other individuals’
decisions. Mass media technology implies \( \varphi_i = \{ \tau_i, \rho_i, i - \rho_i - 1 \} \) where \( \rho_i \) represents the number of indi-
viduals who have decided to revolt up to individual \( i \) \((\rho_i = \# \{ a_j = r, j < i \})\).\(^9\) The information set under
the social media technology becomes \( \varphi_i = \{ \tau_i, \{ a_j, \forall j < i \} \} \), so each previous decision is observed, ordered
according to the position.\(^{10}\)

When no communication technology exists, the lack of information generates a simultaneous-move game.
For the other two cases we need to specify the extent to which previous decisions are observable through the
communication technologies. When defining the information sets, we assumed implicitly that if a com-
munication technology is available to the society, then individuals are *completely* informed. As a consequence,

\(^9\) Thus, we assume that under mass media individuals know both the amount of predecessors who decided to participate and
who chose to stay at home. It is plausible if citizens may infer somehow how many individuals have already decided, that is
they may figure out their position in the sequence of decisions.

\(^{10}\) Lohmann (1993, 1994a and 1994b) addresses questions of information aggregation and political action. In her models,
individuals observe each previous action, but she does not study how different communication technologies affect coordination.
individuals may infer perfectly their position in the sequence of decisions. Mass and social media report about all previous decisions: mass media aggregate the information whereas social media present it in a disaggregated form. Our modeling choice allows us to study in a clear way the difference in the effects that mass and social media have on the evolution of revolts.\footnote{Arguably, it is unlikely that a society is completely informed about the state of the revolution. A more realistic model would have people receiving partial aggregate information, some particular information about the decisions of the personal contacts, and no information about decisions of some others in the society.}

Our aim is to determine how the different types of communication technology (or the absence of it) affect the outcome of revolutions. Moving from no technology towards social media the amount of available information increases. In principle, the effect of more information is ambiguous: more information may be good for revolution since it allows a better signaling of own actions to the subsequent individuals; but at the same time, it could also foster coordination failure, e.g. if individuals find out that too many of their predecessors have chosen not to participate in the revolution. Note that they may observe many individuals staying at home because those observed citizens were the unwilling ones.

2.1 Revolutions under different information structures

Given our environment, revolution is the socially efficient outcome. However, when information about other individuals is not available, this efficient outcome may fail to materialize as shown by our first result.

\textbf{Proposition 1} \textit{If no communication technology is available in the society, there exists an equilibrium in which nobody revolts and an equilibrium in which the revolution succeeds.}

This result is a straightforward consequence of the assumptions on the utilities, that imply the existence of multiple equilibria in the simultaneous case. Since \( u_{w,r,R} > u_{w,s} > u_{w,r,F} \), for the willing individuals it is optimal to participate if the other willing individuals are participating, while it is optimal to stay at home if nobody else is participating. If a willing individual believes that the other willing individuals will participate in the revolution, then she best responds to this belief by participating as well. However, if they hold the opposite beliefs, then staying at home is the best response.

The previous result does not depend on whether type is a public information or not. However, when a communication technology is available this distinction becomes relevant as shown next.

2.1.1 Type is public information

It is instructive to see how the existence of information affects the outcome of revolts in a perfect information setup in which the willingness to revolt (that is, the type of individuals) is transmitted by the communication
technology. This can be the case, for instance, when the people willing to overthrow the dictator belong
to the same social group (e.g. religious association, ethnic groups or social classes), so that the individual
knows the type of the people who have decided previously. We model this situation introducing the type of
the predecessors in the information available to each individual.

In the case of mass media the information set of individual \(i\) includes also the amount of willing indi-
viduals up to (but excluding) individual \(i\), denoted as \(\xi_i\). Thus, the information set becomes \(\varphi_i =
\{\tau_i, \rho_i, i - \rho_i - 1, \xi_i\}\). Given this information set, individual \(i\) knows the amount of willing
individuals that precede her and how many decided to stay at home. This is a valuable information since it also
reveals how many willing individuals are left to decide. For instance, if there were many willing individu-
als who abstained from participating in the protests, then it is more probable that the total number of
protesters will fall short of the threshold, so staying at home may be a best response. Regarding social me-
dia, the assumption about publicly observed types implies that the information set of individual \(i\) becomes
\(\varphi_i = \{\tau_i, \{a_j, \forall j < i\}, \{\tau_j, \forall j < i\}\}\). Hence, both the type and decision of each preceding individual are
observed.

We use Perfect Bayesian equilibrium as our solution concept. Individual \(i\)’s strategy is conditioned on
the information set. It is defined as \(\sigma_i : \varphi_i \to \{r, s\}\). Let \(\Sigma = \{r, s\}^n\) be the game’s strategy space, and let
\(\sigma \in \Sigma\) be a strategy profile, that is, \(\sigma = (\sigma_1, ..., \sigma_n)\). Let \(h_i\) be a history of the game up to \(i\), formed by a
type vector and a sequence of decision, \(h_i = \{\tau, a_1, ..., a_{i-1}\}\). Let \(\pi_i(h_i | \varphi_i)\) denote citizen \(i\)’s belief about
the true history given the available information.

**Definition 1** Strategy \(\sigma \in \Sigma\) and belief system \(\pi = \{\pi_1, ..., \pi_n\}\) are a Perfect Bayesian equilibrium (PBE)
if, and only if, for all \(i \in N\), given \(\varphi_i\) and any \(\bar{\sigma}_i \in \{r, s\}\),

\[
\sum_{h_i \in \varphi_i} \pi_i(h_i | \varphi_i)u_i(\sigma) \geq \sum_{h_i \in \varphi_i} \pi_i(h_i | \varphi_i)u_i(\bar{\sigma}_i, \sigma_{-i}),
\]

where \(\pi_i(h_i | \varphi_i)\) is consistent with Bayes’ rule whenever possible.

A strategy profile and a system of beliefs are a PBE if, and only if, the strategy is sequentially rational
for all players and beliefs are consistent with the strategy.

We find that the unique Perfect Bayesian equilibrium with the two communication technologies is that the
revolution succeeds, and every willing individual chooses to revolt. In this case, both technologies generate
the same behavior in equilibrium.
Proposition 2 If type is public information, every willing individual revolts in any Perfect Bayesian equilibrium under both communication technologies.

Proof. See Appendix A. ■

The proof of the result makes use of a backward induction argument.\textsuperscript{12} A willing individual chooses to join the revolution if she observes that already \(t - 1\) individuals have revolted. Given this fact, a willing individual who observes \(t - 2\) people participating in the revolution decides to revolt if she knows that after her there is at least one more willing individual. Since predecessors’ types are publicly observable, she can infer if there is a willing individual behind her. Iterating this reasoning, a willing individual decides to revolt when up to her sufficient willing individuals have chosen to do so and she anticipates that enough willing citizens behind her will follow suit. The conditions ensuring that this requirement is met at any position imply that all willing citizens choose to participate in the revolution.

Our assumption on predecessors’ type being public information is plausible in environments where the people willing to overthrow the dictator can be associated to particular groups. Under these circumstances, it is likely that when individuals acquire information they know both the actions and the types of those who have already decided. In this case, the existence of any of the communication technologies ensures that the revolution triumphs. If there is no technology, it is possible to find equilibria where the individuals do not coordinate and the dictator remains in power.

2.1.2 Type is private information

We study now the case where type is private information, although \(\xi\) is common knowledge. That is, individuals know that there are sufficient people willing to revolt, but they do not know who they are. Given the untrust and fear generated by dictators in repressive regimes, this setup may appear more plausible. As claimed by several authors (e.g. Ginkel and Smith (1999), Kuran (1991, 1995)), decision making in any revolution is clouded by a considerable amount of uncertainty. This uncertainty blurs the information about the public discontent due to several reasons, e.g. the lack of free press, falsified preference revelation to official public opinion polls or the presence of informants penetrating all layers of the society, among others.

When type is private information, communication technologies only transmit to individuals the actions of predecessors. For the mass media technology, the information set of individual \(i\) becomes \(\varphi_i = \{\tau_i, \rho_i, i - \rho_i - 1\}\), so the amount of citizens of each type who have already decided is not known. In the\textsuperscript{12}For the social media case, when types are known, the result is also a straightforward consequence of having a Pareto-dominant outcome in a game of perfect and complete information.

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case of social media, citizen $i$’s information set is given by $\varphi_i = \{\tau_i, \{a_j, \forall j < i\}\}$. Hence, citizen $i$ cannot distinguish perfectly the type of her predecessors although she knows the exact sequence of decisions.$^{13}$

The following proposition shows that under such circumstances the sort of communication technology matters.

**Proposition 3** Consider the case when type is private information. Under the social media technology, the revolution always succeeds because each willing individual revolts in any Perfect Bayesian equilibrium. Under the mass media technology,

- if $t < \frac{n}{n-\xi+1} + 1$, each willing individual revolts and the revolution succeeds;
- if $t \geq \frac{n}{n-\xi+1} + 1$, there are equilibria where the revolution is unsuccessful and nobody revolts for certain values of $u_{w,r,R}$; $u_{w,s}$ and $u_{w,r,F}$.

Proposition 3 states that social media technology enables revolution with certainty, when people know that there are sufficient other people willing to take part in the revolt, even if they do not know who they are. Mass media generate revolution only if the required number of participants to succeed is sufficiently low. Mass media do not ensure that the revolution triumphs when a relatively high proportion of the society is required to participate. The main difference is that social media allow to identify if individuals who have chosen to stay at home were unwilling or not. Hence, it is possible to infer exactly how many willing individuals have not decided yet and by backward induction their choices can be anticipated. These elements ensure that all willing individuals revolt. By contrast, when only mass media are available, inferring the type of previous individuals who stayed at home is generally not possible. As a consequence, it is considerably more difficult to make sure that there are enough willing individuals left in the sequence of decisions and that those individuals will revolt. Hence, with mass media the condition for successful revolts becomes quite demanding.

## 3 Examples

We illustrate our results with two examples. In the first one we analyze a simple society of $n = 4$ individuals in order to clarify the mechanism why social media promote revolutions more than mass media. In the second example, we use a society of $n = 100$ in order to illustrate the quantitative difference in the effectiveness of social media versus mass media in fostering revolutions.

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$^{13}$Observing that somebody revolts indicates that she is of the willing type. However, since willing individuals may choose to stay at home, observing that someone has chosen not to participate in the revolt does not imply that she is unwilling.
Example 1  Imagine a society of four individuals, three of them are willing to overthrow the dictator. The revolution is successful if at least three individuals participate \((n = 4, \xi = 3, t = 3)\). As before, payoffs are such that \(u_{w;r,R} > u_{w;s} > u_{w;r,F}\). In Figure 1 we illustrate the differences between the communication technologies.

We have drawn a reduced extensive-form representation of the game generated by each communication technology. We name it reduced form because we have simplified the representation drawing only one of the four branches that would follow each of the type vectors. At the beginning of the game, nature selects at random one of them. In the four possible type vectors willing citizens are represented by black circles and the unwilling one by a white circle. Since individuals only observe actions but not types, we have drawn individuals as grey circles in the rest of the tree. In Appendix C we represent the complete decision tree for the social media case.

At the top, we represent the case without communication technology in which the individuals decide ignoring the decisions of their predecessors. In the middle, we depict the case of mass media. In this case, an individual only knows the number of citizens that already joined the protests. For instance, imagine that the fourth individual is informed that two individuals have chosen to revolt (and, consequently, one stayed at home). She is not able to distinguish if the sequence was \((r, r, s)\), \((r, s, r)\) or \((s, r, r)\), hence the information that the citizen in position 4 has is compatible with three possible sequences of decisions. At the bottom, we draw the social media case. Individuals are able to identify perfectly the sequence of decisions, but they do not know the type vector (even though through observed actions they may infer something about it). We show that in the case of no communication technology and mass media we may find equilibria leading to both successful revolution and to everyone staying at home. By contrast, with social media the unique equilibrium is the one where all willing individuals revolt.

Without communication technology, there are two possible equilibria (in pure strategies): the three willing individuals either choose to revolt or choose to stay at home. This is the case because for a willing individual to revolt (stay at home) is the best response when the other willing individuals choose to revolt (stay at home).

Next, consider mass media. A willing individual’s optimal decision in the last position is obvious. If she observes two people revolting \((\rho_4 = 2)\), then she revolts. Otherwise she stays at home. The same is true for a willing individual in position 3 (if \(\rho_3 = 2\), she best responds by revolting). As a consequence, a willing citizen in position 2 observing one revolting individual \((\rho_2 = 1)\) revolts as well, because she anticipates that if she decides to revolt, then the last willing individual (either in position 3 or 4) will follow suit. Thus, in any equilibrium a willing citizen revolts when \(\rho_4 = 2; \rho_3 = 2\) or \(\rho_2 = 1\), and stays at home when \(\rho_4 \in \{0,1\}\).
or $\rho_3 = 0$. In these last cases, a willing individual knows that the revolution is doomed to fail, so she does not join it.

Thus, we are left with the following $\rho_i$-s for which a willing citizen’s optimal action is not clear: $\rho_1; \rho_2 = 0$

Figure 1: Reduced extensive-form representations
and $\rho_3 = 1$. In words, we do not know yet what a willing citizen does when she is the first to decide; when she is in the second position and observes no protester and when third in the sequence of decision and observes one protester. We show an equilibrium for the case of mass media where nobody revolts on the equilibrium path for some payoffs. Then, we show that it cannot be the case for social media.

Assume the following payoffs: $u_{w,r,R} = 1$, $u_{w,s} = 0$ and $u_{w,r,F} = -10$. In this case, if nobody chooses to revolt in the previous information sets ($\rho_1; \rho_2 = 0$ and $\rho_3 = 1$) and acts optimally at the other information sets, then we end up in an equilibrium without revolution. Thus, if willing individuals stay at home when $\rho_1; \rho_2 = 0$ and $\rho_3 = 1$, nobody has a profitable unilateral deviation. Take the first individual. The deviation consists in revolting instead of staying at home. The deviation is profitable if the second individual is willing, because the first individual induces the second citizen to revolt as well by the arguments we have seen before. In this case, the revolution triumphs and the highest utility is obtained. When the second individual is unwilling (which has conditional probability $\frac{1}{3}$), then the proposed strategies imply that subsequent willing individuals will stay at home and the revolution fails. Therefore, the unilateral deviation is not profitable if and only if

$$u_{w,s} > \frac{2}{3}u_{w,r,R} + \frac{1}{3}u_{w,r,F},$$

which holds for the proposed payoffs. It is easy to check that if a willing citizen in position 2 observes that nobody has revolted yet ($\rho_2 = 0$), then given the prescribed strategy she does not have a profitable unilateral deviation. The same holds for a willing individual in position 3 when she observes that just one citizen has revolted ($\rho_3 = 1$). This is the case if the individual believes with sufficient probability that she will be followed by the unwilling citizen. Note also that the equilibrium in which each willing individual participates exists if we simply consider a strategy profile that establishes that willing individuals should revolt when $\rho_1; \rho_2 = 0$ and $\rho_3 = 1$.

We prove now that under social media there is a unique equilibrium in which all willing individuals revolt and succeed in overthrowing the dictator. For a willing individual in the last position the previous arguments apply. Thus, upon observing that two other citizens have revolted (the order does not matter) she revolts as well, otherwise she stays at home. When in position 3, a willing individual joins the protest when observing two protesters. As a consequence, a willing individual in the second position, who observes that the first citizen decided to protest, will revolt as well, anticipating that the last willing individual (either in position 3 or 4) will join the protests as well.

\footnote{This information set is off the equilibrium path. A willing individual in position 3 may assign positive probability to observe two willing individuals, and inferring that one of them deviated from the prescribed strategy. However, the revolution cannot succeed, so revolting does not pay off, yielding staying at home the optimal decision.}
As a next step, let us consider what happens if a willing citizen observes \((r, s)\), that is the first citizen revolted, whereas the second one stayed at home. By previous reasoning she can be sure that the second individual was the unwilling one (a willing individual in the second position would have joined the uprising upon observing that the first citizen revolted), so she knows that the last citizen is willing and anticipating her reaction to observing a history with two citizens revolting she decides to protest as well.

Given the previous argument, a willing individual in the first position chooses optimally to protest, since any history starting with a revolting citizen leads to a successful revolution (either if she is followed by a willing individual who protests herself or when followed by an unwilling one who stays at home, but then the next individual will join the protest and in any case the last willing individual will revolt as well). As a consequence, when observing that the first individual has stayed at home, willing citizens know that she must have been the unwilling one. Thus, willing citizens know that the type vector is \((x, w, w, w)\) and by backward induction they play the unique equilibrium of this subgame in which all of them rebel against the dictator. Hence, as the game unfolds in any information set that can be reached, willing individuals revolt and the dictator is overthrown. Notice that these arguments are valid for any payoffs such that \(u_{w,r,R} > u_{w,s} > u_{w,r,F}\). Note also that with social media the outcome is unique because individuals are able to distinguish the histories \((r, s)\) and \((s, r)\).

**Example 2**  
Mass media can also foment revolutions, for any possible payoffs, but only if just a relatively low proportion of the society is required to participate in order to triumph. Imagine a society of 100 individuals, where 76 of them want to overthrow the dictator that is achieved if at least 51 citizens revolt \((n = 100, \xi = 76, t = 51)\). If all individuals decide simultaneously (the case without communication technology), both results are possible: the revolution may succeed or fail. This occurs both when type is public or private information.

If type is public information, following Proposition 2 the unique equilibrium implies that revolution occurs for sure with either sort of communication technology.

Suppose that type is private information: it is unknown which concrete people are in favour of the revolution, although every individual knows that there are 76 people willing to revolt. Because of Proposition 3, under mass media technology, for given payoffs, it is possible that nobody takes part in the revolts, since \(t = 51 > 5 = \frac{n}{n-\xi+1} + 1\). This means that even though more than three-quarters of the citizens wish to overthrow the dictator, they succeed for sure only if 5 or less people are required to participate in the revolts. The intuition behind this result is the following. If an individual can be sure that the revolution succeeds, then she joins the protests. Whenever she may believe that with positive probability the revolution fails, it is
possible to find a punishment that is sufficiently large to deter individuals from participating in the protests. Suppose that a willing individual observes that 49 individuals have revolted. If she has 24 or less successors, then she may believe that all of them are unwilling and, therefore, she will stay at home, rendering the revolution unsuccessful. If there are at least 25 individuals left to decide, then she can be sure that one of them is of the willing type and will revolt upon observing that already 50 individuals joined the protests. Hence, a willing individual who knows that 49 individuals have already revolted and others have stayed at home, chooses to revolt for sure if she is in position 75 or before. By similar arguments, a willing individual who observes that 48 individuals joined the protests (and also knows that others stayed at home) revolts only if she knows that there are at least two other willing individuals behind her and anticipates that those individuals will revolt as well. Therefore, she will choose to revolt if she is at most in position 50. Note that if she is at any position lower than 75, she knows that there are two willing individuals behind her, but to make sure that they both will revolt, she needs to verify that the next willing individual will be sure that there is one more willing individual left to decide. If she is at most in position 50, then she knows that there will be another willing individual in any of the positions [51, 75] who by observing a revolt will join the protests as well because she knows that there is at least one more willing citizen in any of the positions [76, 100] who will do the same. This line of reasoning yields the result that given the above parameters a revolution is successful with mass media only if 5 or less individuals are required to participate. Otherwise, the revolution may fail.

In particular, suppose that payoffs are \( u_{w,r,F} = 1, u_{w,s} = 0 \) and \( u_{w,r,F} = -10^{100} \). Hence, the dictator would punish very strongly the participants of a failed revolution. We prove in Appendix D that under these payoffs it is possible that nobody revolts when people get information under mass media. However, as shown by Proposition 3, under these conditions revolution succeeds with social media.

In our example, mass media communication guarantees that the revolt succeeds only if \( t \leq 5 \) individuals’ participation is necessary to overthrow the dictator. In a society of 100 individuals in which 51 people are required to participate in the protests to change the regime, mass media facilitate a successful revolution (with all willing individuals revolting) for any payoffs only if it is known that all individuals (\( \xi = 100 \)) are in favour of the revolution. If the required threshold were \( t = 49 \), mass media would guarantee that the revolution succeeds only if it was commonly known that at least \( \xi = 99 \) individuals are of the willing type. Thus, mass media lead to a successful revolution only if there is a huge amount of people willing to participate in the protests, or if the dictator is very weak (the threshold is very low). In any other case, the dictator could implement a sufficiently high punishment \( (u_{w,r,F}) \) so that revolts may not occur in equilibrium.
4 Conclusion

In this paper we have studied the differential effect of communication technologies on the outcome of revolutions. We distinguish mass and social media by the degree of information that they provide. In particular, mass media supply an aggregate piece of information about the actual state of revolution, while when informed through social media individuals know the precise sequence of decisions that led to the actual state.

When individuals' types are public information, both communication technologies enable a successful revolution with certainty, a result that does not hold without communication technology. Hence, both mass and social media facilitate the overthrow of the dictator. When types are private information, we show that the sort of communication technology is relevant: with social media the revolution succeeds in the unique equilibrium. However, with mass media the revolution may fail except in cases when the amount of people required to succeed is sufficiently low. If the punishment for participating in a failed revolution is sufficiently high, with mass media the no-revolution equilibrium may be sustained, but it is not true for the social media technology. In this sense, we argue that social media facilitate revolts more than mass media do.

Our model relies on strong simplifications. Besides the degree of information aggregation, there are more differences between communication through social media with respect to mass media. For instance, it is likely that dictators control the information provided through mass media. Moreover, communication technologies may be used to supply false information. Another key point that we do not consider is that individuals are heterogeneous, i.e. the case of opinion leaders whose decisions may have different implications depending on the communication technology. Our assumptions about perfect knowledge of the position in the sequence of decisions and the complete information about previous decisions are also stark. All these considerations may result crucial when analyzing the effects of different sorts of communication technology and seem promising future research topics. However, our aim was to show the existence of a mechanism that can at least partly explain some of the differences that communication through social media is bringing to the society. Our results suggest that social media facilitate the efficient decisions under payoff structures that possibly generate coordination failures under mass media.

References


5 Appendix A

Proposition 2 If type is public information, every willing individual revolts in any Perfect Bayesian equilibrium under both technologies.

Proof. The existence of the equilibrium is guaranteed by standard arguments. We show that in the unique subgame perfect equilibrium each willing individual revolts. Since Perfect Bayesian equilibria are a subset of subgame perfect equilibria, the proposition follows.\footnote{We use Perfect Bayesian equilibrium for consistency with the private information case.}

Let us define $\nu_i = \# \{ \tau_j = w : j \leq i \mid \tau_i = w, \tau \}$, that is the position of a willing individual $i$ among the willing individuals in a given type vector ($\tau$). Note that $\nu_i \in \{1, 2, \ldots, \xi\}$, with $\xi > t$.

By backward induction, the last willing individual ($\nu_i = \xi$) revolts if at least $t - 1$ other willing individuals decided to revolt, because if she follows suit, then the number of protesters reaches the threshold $t$, so the revolution will be successful. Otherwise she decides to stay at home. The next to the last willing individual ($\nu_i = \xi - 1$) revolts if at least $t - 2$ willing predecessors chose to revolt anticipating that then also the last willing individual will join in. Again, if the condition is not met, then she stays at home. This argument can be repeated for all willing individuals: the individual $\nu_i$ revolts if at least $t - (\xi - \nu_i + 1)$ other individuals decided to revolt, otherwise she prefers to stay at home. As a consequence, the first willing individual ($\nu_i = 1$) revolts even if she does not observe anybody revolting ($t - \xi < 0$). Therefore, the revolution succeeds.

Notice that the previous argument to find the unique subgame perfect equilibrium works for both types of media, thus Proposition 2 holds.

6 Appendix B

Proposition 3 Consider the case when type is private information. Under the social media technology, the revolution always succeeds because each willing individual revolts in any Perfect Bayesian equilibrium. Under the mass media technology,

- if $t < \frac{n}{n-\xi+1} + 1$, each willing individual revolts and the revolution succeeds;
- if $t \geq \frac{n}{n-\xi+1} + 1$, there are equilibria where the revolution is unsuccessful and nobody revolts for certain values of $u_{w,r,R}$; $u_{w,s}$ and $u_{w,r,F}$.
Proof. Let \((\{\sigma^*\}, \{\pi^*\}) = (\{\sigma^*_i, \sigma^*_{-i}\}, \{\pi^*_i, \pi^*_{-i}\})\) be an assessment, a profile of behavioral strategies \(\sigma^*\) and beliefs \(\pi^*\) for each player in each of her information sets, that defines a Perfect Bayesian equilibrium. \(\sigma^*_i\) and \(\pi^*_i\) are, respectively, the equilibrium strategy and belief of individual \(i\). Therefore \(\sigma^*_i\) is a best response to \(\sigma^*_{-i}\) conditional on \(\pi^*_i\). We focus only on the decisions of willing individuals. First, we prove that under social media in the unique Perfect Bayesian equilibrium every willing citizen chooses to revolt, and therefore the revolution succeeds. Second, we prove that under mass media, every willing individual revolts over the equilibrium path if \(t < \frac{n}{n - 1} + 1\). Third, we construct an equilibrium in which the willing individuals fail to coordinate and do not rise up against the dictator if \(t \geq \frac{n}{n - 1} + 1\).

First. For the social media case, in which individual \(i\) observes the sequence of previous decisions, let \(\Phi_i(k, \sigma^*_{-i})\) denote as the set of information sets in which individual \(i\):

- observes \(k\) citizens who revolt and \(i - k - 1\) citizens who stay at home, with \(i - k - 1 < N - \xi\);
- individual \(i\) is willing;
- given the equilibrium strategy \(\sigma^*_{-i}\), \(\sigma^*_i(\tau_j = w, \varphi_j|\varphi_i) = r, \forall j < i\), where \(\varphi_j|\varphi_i\) indicates that \(\varphi_i\) is a continuation of \(\varphi_j\), so up to individual \(j\) the two information sets coincide (except for the type of individual \(i\)).

Let focus now on an information set \(\varphi_i \in \Phi_i(k, \sigma^*_{-i})\). Take any history \(h_i = \{\tau, a_1, ..., a_{i-1}\}\) in which there are a maximum of \(N - \xi - 1\) unwilling individuals before \(i\) and where there is at least one willing individual who has chosen to stay at home, \(\exists j < i: (\tau_j = w, a_j = s|h_i)\). Take another history \(h'_i\) which is the same as \(h_i\) but in which one of the willing individuals who chooses \(s\) in \(h_i\) is of type \(x\) in \(h'_i\). Since we use the concept of PBE, consistent beliefs for \(h_i\) require \(\pi^*(h_i|\varphi_i, \sigma^*) = 0\). It follows from our focus on the set of information sets \(\Phi_i(k, \sigma^*_{-i})\) that imply that a willing individual observing any particular information set belonging to \(\varphi_i\) chooses to revolt. Note that in those information sets we can find an alternative history \(h'_i\) where "stay at home" is chosen only by unwilling individuals (since \(i - k - 1 < N - \xi\)). Since the rest of actions have been chosen by the same type of individuals under \(h_i\) and under \(h'_i\), and under \(h'_i\) people who stay at home would choose that action with positive probability (they are unwilling individuals), Bayes rule requires to assign probability 0 to the history \(h_i\). Therefore, if an individual is observed to have stayed at home, then she cannot have been a willing one. Thus, any history with at least one willing individual choosing to stay at home in an information set \(\varphi_i \in \Phi_i(k, \sigma^*_{-i})\) has probability 0. Since \(i - k - 1\) citizens who

\[\text{Therefore, there is also an individual of type } x \text{ according to } h_i \text{ with an index higher than } i, \text{ who is of type } w \text{ according to } h'_i.\]
are observed by \( i \) stay at home, it is believed with probability 1 that after individual \( i \) there is a maximum of \( N - \xi - (i - k - 1) \) unwilling individuals and thus, a minimum of \( \xi - k - 1 \) willing individuals.\(^{17}\)

Suppose now that any willing individual \( j \) chooses to revolt in any information set \( \varphi_j \in \Phi_j(k', \sigma^*_{-j}) \), for all \( k' > k \). Then, if individual \( i \) chooses to revolt in the information set \( \varphi_i \in \Phi_i(k, \sigma^*_{-i}) \), the following willing individual \( i' \) will be in an information set of type \( \varphi_{i'} \in \Phi_{i'}(k+1, \sigma^*_{-i'}) \), and will play \( r \) (by assumption).\(^{18}\) And also the following willing individuals will be in an information set of type \( \Phi_j(k', \sigma^*_{-j}) \) and, by assumption, will revolt. Since individual \( i \) observes \( k \) citizens revolting and has \( \xi - k - 1 \) willing successors, in total there will be \( \xi > t \) individuals revolting and \( i \) receives the highest payoff by revolting (because the revolution is successful). If individual \( i \) stays at home, then she obtains the sure payoff \( (u_{w,s}) \), but has a profitable deviation. Namely, if she revolts instead of staying at home, then her successors will observe one more individual who revolts, and moreover, there is no action \( s \) observed and chosen with positive probability by a willing individual. This means that willing successors upon observing that individual \( i \) has revolted, will choose to revolt also (because they will be in an information set \( \varphi_j \in \Phi_j(k', \sigma^*_{-j}) \) and the revolution will be successful). Therefore, we have proved that if any willing individual revolts in an information set \( \varphi_j \in \Phi_j(k', \sigma^*_{-j}) \), for all \( k' > k \), a willing individual \( i \) revolts in equilibrium in the information sets \( \varphi_i \in \Phi_i(k, \sigma^*_{-i}) \).

But note that if \( k \geq t - 1 \), then willing individuals choose for sure \( r \). So, by backward induction willing individuals for \( k = t - 2, t - 3, \ldots 0 \) will do so as well. Therefore, for all \( k \), willing individuals revolt in the information sets of type \( \varphi_i \in \Phi_i(k, \sigma^*_{-i}) \).

We do not know yet if \( \Phi_i(k, \sigma^*_{-i}) \) is empty or not, although we already know that a willing individual chooses optimally to revolt if she is in an information set of that type and, moreover, any willing successor of an individual in anyone of those information sets, will be also in an information set of that type. But note that if individual 1 is willing, then she is in an information set \( \varphi_1 \in \Phi_1(0, \sigma^*_{-1}) \), and, therefore, she chooses \( r \). But then, if \( r \) or \( s \) is observed by individual 2, then she is in an information set \( \varphi_2 \in \Phi_2(1, \sigma^*_{-2}) \) or \( \varphi_2 \in \Phi_2(0, \sigma^*_{-2}) \), respectively, and so on for all information sets over the equilibrium path. Therefore, any willing citizen chooses to participate in the revolt in the social media case. Since \( \xi > t \), the revolution

\(^{17}\)Our argument applies for all histories in which there are a maximum of \( N - \xi - 1 \) unwilling individuals before \( i \). For a history with \( N - \xi \) unwilling individuals before \( i \), it is trivial that there are at least \( \xi - k - 1 \) willing individuals after \( i \).

\(^{18}\)Given \( \varphi_i \in \Phi_i(k, \sigma^*_{-i}) \), note that the individual \( j \) that follows \( i \) observes the same sequence of decisions than \( i \) plus the action of \( i \). If she is willing, then she is in an information set \( \varphi_j \in \Phi_j(k+1, \sigma^*_{-i}) \) if individual \( i \) chooses \( r \). This is the case because individual \( j \) observes \( k + 1 \) revolts and she is in position \( i + 1 \), so that \( j - (k + 1) - 1 < N - \xi \) and all the actions of "stay at home" that she observes are chosen with probability 0 according to \( \sigma^* \). If citizen \( i + 1 \) is unwilling and individual \( i + 2 \) is willing, it is easy to check that \( \varphi_{i+2} \) still is an information set of type \( \varphi_{i+2} \in \Phi_{i+2}(k+1, \sigma^*_{-1-2}) \), and so on. If individual \( i \) is in an information set of type \( \varphi_i \in \Phi_i(k, \sigma^*_{-i}) \), the willing successor who follows will be with probability 1 in an information set of the same type, given that individual \( i \) plays \( r \).
succeeds. Q.E.D.

Second. Suppose that \( t < \frac{n}{n-\xi+1} + 1 \).

Let define as \( \Phi_i(k) \) the set of information sets of individual \( i \) in which

- individual \( i \) is willing;
- \( n - i \geq (t - k - 1)(n - \xi + 1) \).

Note that if individual \( i \) is in an information set \( \varphi_i \in \Phi_i(k) \) and chooses \( r \), then the following willing individual \( j \) is also in an information set \( \varphi_j \in \Phi_j(k+1) \). Let be \( j \) the following willing individual after \( i \), so that we know that \( j \leq i + (n - \xi + 1) \). If \( i \) chooses \( r \), then \( k_j = k + 1 \). Note that

\[
  n - j \geq n - i - (n - \xi + 1) = \\
  = \xi - 1 - i \geq \xi - 1 - n + (t - k_i - 1)(n - \xi + 1) = \\
  = (t - k - 2)(n - \xi + 1) = (t - k_j - 1)(n - \xi + 1)
\]

Thus, if \( \varphi_i \in \Phi_i(k) \) and \( a_i = r \), then we have that \( \varphi_j \in \Phi_j(k+1) \), where \( j \) is the following willing individual. But note that, by forward induction, this means that all willing successors of \( i \) are in information set of type \( \varphi \in \Phi(k') \), if all of them choose \( r \).

Suppose now that any willing citizen \( j \) chooses \( r \) if she is in an information set \( \varphi_j \in \Phi_j(k') \), \( \forall k' > k \). This means that \( \sigma^*_j(\varphi_j \in \Phi_j(k')) = r, \forall k' > k \). If it is the case, note that individual \( i \) best responds by revolting if she is in an information set \( \varphi_i \in \Phi_i(k) \), since she has at least \( (t - k - 1)(n - \xi + 1) \) successors from which a maximum of \( n - \xi \) are unwilling and she needs that \( (t - k + 1) \) willing individuals choose to revolt, which is true by assumption. But note that \( \sigma^*_j(\varphi_j \in \Phi_j(k')) = r, \forall k' > (t - 2) \) holds trivially. Thus, that by backward induction we have that individual \( i \) chooses optimally the action \( r \) in any information set \( \varphi_i \in \Phi_i(k), \forall k \geq 0 \).

But since we have that \( t < \frac{n}{n-\xi+1} + 1 \), we have also that \( n > (t - 1)(n - \xi + 1) \), implying \( (n - 1) \geq (t - 1)(n - \xi + 1) \). This means that individual 1, if willing, is in an information set of type \( \varphi_1 \in \Phi_1(0) \). Hence, she chooses \( r \) in equilibrium and also their successors do the same, if they are of the willing type.

Suppose that the first willing individual is not in position 1, but \( 1 < i \leq (n - \xi + 1) \). The second individual, if she observes that the first one has chosen to stay at home, knows with probability 1 that she is unwilling, and therefore her first willing successor \( j \) will be in position \( j \leq (n - \xi + 2) \). Thus, if she chooses \( r \), the successor \( j \) will be in an information set \( \varphi_j \in \Phi_j(1) \) and will best respond by revolting, as well as all the other willing successors. So that she best responds by revolting. Iterating this reasoning, if the first willing individual to choose observes a sequence of people staying at home, then she knows with probability 1 that
it is due to unwilling individuals, and she knows that there is a willing individual among the first \((n - \xi + 2)\) individuals who will observe the action \(r\) of the first willing individual, and will be in an information set of type \(\varphi_j \in \bar{\Phi}_j (1)\). Therefore, the first willing individual chooses \(r\) and all the successors choose \(r\) since they are in an information set \(\varphi_i \in \bar{\Phi}_i (k)\). As a consequence, the revolution succeeds. \(Q.E.D.\)

**Third.** Now we construct an equilibrium with the coordination failure of no revolution under the mass media communication when \(t \geq \frac{n}{n-\xi+1} + 1\). Let denote as \(\varphi_i (\rho_i)\) the information set \(\varphi_i = \{w, \rho_i, i - \rho_i - 1\}\). Define the following strategy for each willing individual:

\[
\sigma_i (\varphi_i (\rho_i)) = \begin{cases} 
  r & \text{if } i \leq n - (n - \xi + 1) (t - \rho_i - 1) \\
  s & \text{otherwise}
\end{cases}
\]

We show that \((\{\bar{\sigma}\}, \{\bar{\pi}\}) = (\{\bar{\sigma}_i, \bar{\bar{\sigma}}_{-i}\}, \{\bar{\pi}_i, \bar{\bar{\pi}}_{-i}\})\) defines an equilibrium assessment where nobody revolts over the equilibrium path, for certain values of \(u_{w,r,F}\) and \(u_{w,s}\).

Note that this strategy implies that nobody chooses the action \(r\) over the equilibrium path. This is the case because following \(\bar{\sigma}_i\) a willing individual that does not observe any revolt chooses to participate if \(i \leq n - (n - \xi + 1) (t - 1)\). However, since \(t \geq \frac{n}{n-\xi+1} + 1\), we have that \(i \leq n - (n - \xi + 1) (t - 1) \leq n - (n - \xi + 1) \frac{n}{n-\xi+1} = 0\). Therefore, \(\bar{\sigma}\) implies that nobody chooses to revolt when observing no other action \(r\). We show now that this is a Perfect Bayesian equilibrium for some payoffs.

For those information sets in which individuals revolt, the strategy is a best response to the rest of individuals’ strategies and implies consistent beliefs because of the same argument as those in part Second of the proof. We prove now that individuals do not have any profitable deviation when choosing \(s\), for certain values of \(u_{w,r,F}\) and \(u_{w,s}\). Note that the action \(s\) is a best response in the information sets \(\varphi_i (\rho) : i \leq n - (n - \xi + 1) (t - (\rho - 2) - 1) - 1\). This is the case because if individual \(i\) chooses \(r\) in such information sets, individual \(i+1\) will be in an information set where \(i+1 > n - (n - \xi + 1) (t - (\rho - 1) - 1)\), and therefore she and her successors will stay at home. Thus, \(s\) is a best response for \(i\).

Now we focus on those information sets where \(\varphi_i (\rho) : n - (n - \xi + 1) (t - (\rho - 2) - 1) - 1 \leq i \leq n - (n - \xi + 1) (t - \rho-1)\). Any of these information sets is off the equilibrium path, and, therefore, reached with probability 0. Given \(\bar{\sigma}\), all the individuals previous to individual \(i\) should play \(s\) for both types. Consequently, consistent beliefs may assign positive probability to a sequence of predecessors formed by \(\max\{i - 1, \xi\}\) willing individuals. In this case, with positive probability \(p\) either there is no subsequent willing citizen or the following willing successor is an individual \(j > n - (n - \xi + 1) \cdot (t - \rho)\) and she and all her successors will best respond by choosing \(s\), a case in which the revolution fails. Therefore, with positive probability \(p\) the individual would obtain a payoff \(u_{w,r,F} < u_{w,s}\) by choosing \(r\). For \(u_{w,r,F}\) sufficiently low with respect to \(u_{w,s}\), \(\bar{\sigma}_i\) is also a best response to \(\bar{\sigma}_{-i}\) conditional on consistent \(\bar{\pi}_i\). Hence, \((\{\bar{\sigma}\}, \{\bar{\pi}\})\) defines
an equilibrium assessment where nobody revolts.

7 Appendix C

Figure 2 represents the complete decision tree for the case of social media. Individuals are able to distinguish the sequence of actions but they do not know the type vector. For simplicity, the information nodes that belong to the same information set for individuals in position 3 and 4 have been marked with the same geometrical shape.
Figure 2: Decision tree of the social media case in Example I
8 Appendix D

Example 2: Suppose that $n = 100$, $\xi = 76$, $t = 51$, and consider the following payoffs: $u_{w,r,R} = 1$, $u_{w,s} = 0$ and $u_{w,r,F} = -10^{100}$. We study the case of mass media. Then the following strategy for willing individuals

$$\bar{\sigma}_i(\varphi_i, \bar{\pi}_i) = \begin{cases} r & \text{if } \rho_i = 50, \forall i \\ \rho_i = 49, \forall i \leq 75 \\ \rho_i = 48, \forall i \leq 50 \\ s & \text{otherwise} \end{cases}$$

where $\rho_i$ is the amount of participants that have chosen $r$ before individual $i$, defines a Perfect Bayesian equilibrium for consistent beliefs $\bar{\pi}$.

Proof: We focus on the willing individuals. For simplicity, from now on we denote as $\varphi_i(\rho_i)$ the information sets $\varphi_i(\rho_i) = \{w, \rho_i, i - \rho_i - 1\}$.

For the information sets $\varphi_i(50)$ it is obvious that the above strategy is optimal. Given this fact, an individual $i$ in an information set $\varphi_i(49)$, $\forall i \leq 75$, best responds by choosing $r$. It is the case because she has 25 successors, and at most 24 of them are unwilling. Therefore, if she chooses $r$, with probability 1 there is a willing successor $j$, and she will move in an information set $\varphi_j(50)$. Thus, the proposed strategy is a best response in the above information sets. The same reasoning applies in the information sets $\varphi_i(48)$, $\forall i \leq 50$, since there will be with probability 1 at least one willing individual in an information set $\varphi_i(49)$, $i \leq 75$.

Note that any individual $j$ that observes $\rho_j < 47$ best responds to the strategy $\bar{\sigma}_{-j}$ by choosing $s$. This is the case, because in such an information set, if the individual chooses $r$ the strategy $\bar{\sigma}_{-j}$ implies with probability 1 that no further individual chooses it and the revolution fails, yielding the payoff $u_{w,r,F} = -10^{100}$. The same argument applies for $\varphi_i(47)$ when $i \in [50, 100]$ and for $\varphi_i(48)$ when $i \in [75, 100]$.

Finally, we have to prove that the strategy is optimal in the rest of information sets. We start with the information sets where $\varphi_i(47)$ when $i \in [1, 50]$. This can be the case only if $i \in [48, 49]$. Since according to the proposed strategy nobody should choose $r$ in any of these information sets (there were no previous information set with $\rho = 48$), the beliefs can be assigned arbitrary. We use, however, the reasonable belief of assigning probability 1 to the fact of having 47 willing predecessors and $i - 48$ unwilling ones. Therefore, the individual believes that she has 28 willing successors and 23 or 24 unwilling ones, depending on if she is individual 48 or 49. For the individual $i = 49$, with probability $p = \frac{26}{51}$ the individual 50 would be of the willing type and would best respond with $r$ if individual 49 chooses $r$, and the revolution would succeed. With probability $p = \frac{25}{51}$ the follower will be of the unwilling type, and if she chooses $r$ she would get the
lowest payoff. Therefore, her expected utility is

\[ u_{49}(r, \varphi_{49}(47), \sigma_{-49}, \pi_{49}) = \frac{26}{51} u_{w,r,R} + \frac{25}{51} u_{w,r,F} = \]

\[ = \frac{27}{51} \cdot 1 + \frac{24}{51} \cdot (-10^{100}) < 0 = u_{w,s} = \]

\[ = u_{49}(s, \varphi_{49}(47), \sigma_{-49}, \pi_{49}) \]

Therefore, \( \sigma_{49}(\varphi_{49}(47)) \) is a best response conditioned on \( \pi_i \). The same reasoning holds in \( \varphi_{48}(47) \).

We show now that the strategy is optimal for \( \varphi_i(48) \) when \( i \in [51, 100] \). Note that in the information sets \( \varphi_i(48), i = [51, 100] \) individual \( i = 51 \) is the one with the highest likelihood of being followed by sufficient willing individuals so that a successful revolution arises. We construct the beliefs of individual \( i \) assuming that she assigns probability 1 to the event of having 50 willing predecessors. This is consistent with the proposed strategy \( \sigma \): since all individuals should choose \( s \) in any possible previous information set, basically any belief is equally likely. Under that belief, individual \( i = 51 \) is followed with probability 1 by 24 unwilling individuals and 25 willing individuals. With probability \( p = \prod_{i=1}^{24} \frac{25-i}{50-i} = 1.58 \cdot 10^{-14} \) the following 24 successors of individual 51 are unwilling (and the last 25 individuals are willing). If individual 51 chooses \( r \) then with probability \( (1 - 1.58 \cdot 10^{-14}) \) a willing individual will be in an information set \( \varphi_i(49), \forall i \leq 75 \), and she will choose the action \( r \) and also the rest of willing individuals, and the revolution succeeds. But with probability \( 1.58 \cdot 10^{-14} \) the following 24 individuals will be unwilling, no willing individual will choose the action \( r \) and the revolution fails. Therefore, the expected utility of individual 51 is

\[ E(u_{51}(r, \varphi_{51}(48), \sigma_{-51}) | \pi_{51}) = (1 - 1.58 \cdot 10^{-14}) \cdot u_{w,r,R} + 1.58 \cdot 10^{-14} \cdot u_{w,r,F} = \]

\[ = (1 - 1.58 \cdot 10^{-14}) \cdot 1 + 1.58 \cdot 10^{-14} \cdot (-10^{100}) < 0 = u_{w,s} = \]

\[ = E(u_{51}(s, \varphi_{51}(48), \sigma_{-51}) | \pi_{51}) \]

and choosing \( s \) is a best response in the information set \( \varphi_{51}(48) \). The same argument can be applied in the rest of information sets.