The theorem of consumer surplus and demand elasticity at equilibrium price in a monopolist competition case

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**Theorem:** Let the monopolist competitor production demand has a linear function type, and its total production cost is given by formula \( TC = F + vQ \), where \( F \) and \( v \) stand for fixed and average variable costs, and \( Q \) is a production quantity. Then, at the price of a long-run equilibrium, the consumers’ surplus is equal to a half of fixed cost value, and the price elasticity is equal to the ratio of total to fixed costs.

**The proof:**
1) In the case of monopolist competition long-run equilibrium, the average cost curve \( AC = F/Q + v \) is tangent to the demand curve \( P = g - hQ \) (see the figure below). Total consumers’ surplus value is presented, then, by the \( gEP_0 \) triangle area that is equal to 0.5\( Q_0 \) times the length of the leg \( gP_0 \); the latter being equal to \( h \cdot Q_0 \) product, where \( h = \left| \frac{dAC}{dQ} \right| = \frac{F}{Q_0} \). Finally, one gets that the surplus equals to 0.5\( Q_0 \cdot \frac{F}{Q_0} \cdot \frac{Q_0^2}{2} = 0.5F \).

2) In the long-run equilibrium, \( P = AC = F/Q + v \), hence giving \( Q = \frac{F}{P - v} \), and \( dQ/dP = -\frac{F}{(P - v)^2} \). Therefore:
\[
e^D = \frac{dQ}{dP} \cdot \frac{P}{Q} = \frac{F}{(P - v)^2} \cdot \frac{P(P - v)}{P} = \frac{P}{P - v} = \frac{AC}{AFC} = \frac{TC}{TFC}.
\]