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The Definition, Dating and Duration of Cycles

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1 Introduction

The ultimate objective of this paper is to discuss the duration of business cycles and the related issue of the probability of recession. To reach that objective it is necessary to first agree on a definition of business cycles. It is also necessary to agree on how to define the key features of business cycles and to agree on the rules for identifying and dating these key features. Although not strictly necessary for this paper, it is also helpful to discuss why we seek to identify and date key features of the business cycles.

2 Defining business cycles

When discussing business cycles the year 1920 has particular significance. Of course, much business cycle research was conducted in the century prior to 1920. For example, Wesley Mitchell had published his *Business Cycles* in 1913. Nonetheless the year 1920 saw two births of significance to business cycle research.

First, was establishment of the National Bureau of Economic Research (NBER) in 1920. Foundation of the NBER resulted in a increase in the scale of effort devoted to collecting and

collating business cycle facts. For example, the NBER assembled in one source book the a collection of economic and social statistics for Great Britain, France, Germany, the United States and a number of other countries. The NBER also published Thorp's (1926) *Business Annals* which described 166 cycles from 17 countries. Mitchell's 85 page introduction to the *Business Annals* contains a wealth of information about the duration of cycles and the international relationship among business cycles.

The second, birth of course, was that of Ernst Boehm on 27 July 1920. For the last two decades Ernst has largely defined Australian business cycle research. He has provided much of the foundation on which the Westpac-Melbourne Institutes' modern business cycle research program is built. No serious paper on the Australian business cycle could ignore the business cycle chronologies that he established for Australia and its states.

The 1920s are also saw two other developments that are central to defining and understanding business cycles. In 1922 E.F. Clements, at the Carnegie Institution "Conference on Cycles", gave the first clear statement of what constitutes a cycle in common scientific usage:

> *"In general scientific use the word (cycle) denotes a recurrence of different phases of plus and minus departures which are often susceptible of exact measurement. It has no necessary relation to a definite time interval, though this is frequently a characteristic of astronomical cycles. Apart from the familiar cycles of the day, the lunar month, and the year, the one best known is the sun-spot recurrence, to which the term cycle is almost universally applied. This furnishes convincing evidence that the significance of the term resides in the fact of recurrence rather than in that of the time interval, since the sun-spot cycle has varied in length from 7 to 17 years since 1788,…. In consequence, it seems desirable to use cycle as the inclusive term for all recurrences that lend themselves to measurement, and period or periodicity for those with a definite time interval, recognising, however, that there is no fixed line between the two." Quote taken from Mitchell (1926).*

Mitchell (1926) states that subsequent discussion showed that the definition given by Clements, commended itself to the others.

Mitchell then uses the following argument to conclude that business cycles fall within this scientific definition of cycles.

> *Now our annals show beyond doubt "a recurrence of different phases" in business activity, and these recurrences "lend themselves to measurement". Hence we have ample warrant in the usages of other sciences than economics for applying the term "cycles" to business fluctuations.*

Mitchell recognised that given Clements' definition of cycle a key issue is whether it is appropriate to limit business cycles to periodic cycles. He suggests that the answer is no.

> *But the term "periodicity" we should not use with reference to business cycles, or with reference to crises. For the time intervals between crises are far from regular. They vary as will appear presently, even more than the length of sun-spot cycles.*

Mitchell's statement can be taken as authoritative since it was based on the data in Thorp's (1926) *Business Annals* which described 166 cycles from 17 countries (including Australia).

The second development of the 1920s was the technically brilliant paper, first published in Russian[1], in which Slutsky argued that business cycles could be explained in terms of sums of random shocks to an economy.

Slutsky went on to argue that Mitchell was incorrect in denying the periodicity of business cycles. Thus Slutsky argued that in summarising business cycle facts we could limit our focus to facts about processes that are periodic. In rejecting Mitchell, Slutsky made use of an experiment which involves a 10 period moving sum of white noise, he then claimed that the distribution of the duration of "business cycles" in this generated process was similar to the distribution of business cycle durations. However, Pagan (1997a) reports redoing Slutsky's experiment using the Bry-Boschan dating program and finds that the average length of the

[1] The English translation was published in econometrica, Slutsky (1937).

generated cycles is 32 months which is substantially shorter than the observed duration of actual business cycles. Pagan suggests that in order to generate business cycles with the length observed by real world data it is necessary for the data to be generated by infinite sums of random shocks rather than by a 10 period sum of random shocks.

Thus Pagan finding shows how Mitchell and Slutsky can be integrated to give a unified and improved understanding of business cycles. However, as Pagan recognises, this is 'old wine in new bottles'. For example Macaulay (1931) suggested that:

> *"Many economic time series seem to be of a type somewhat analagous to such cumulated chance series. Some economic series suggest chance series which have been cummulated twice. For example, each observation on a population series is not only higly correlated with the immediately preceeding observation, but the first differences are highly correlated with the preceding first differences"*

Macaulay suggests that the reason for this pattern of correlations is that

> *"The first difference are the resultant of three factors, births, deaths and migration. Births and deaths are functions of the size of the population and hence highly correlated with the same items for the preceding year. The excess of births and deaths and the amount of migration are correlated with the same items in the preceding year."*

It is a small step to think of the economy being made up of a population of firms, each of which has a population of clients and a population of employees. Each of these populations changes through births deaths and migration the dynamics of which are very similar to those described by Macaulay for the human population. At any point births, deaths and migration will be a possibly non linear function of the population. Moreover, the relationship between births, deaths migration and population will be subject to some randomness and thus the dynamics of the various populations that make up the economy will comprise both doubly and singly summed random shocks. In addition, there may well be transitory shocks and seasonal shocks that overlay the underlying dynamics.

3 Identifying key features of business cycles

Thus Pagan and Macauly are suggesting that all time series that are integrated of order greater than zero will exhibit cycles as defined by Clemments. However, such cycles will not be periodic. When analysing such integrated time series it is desirable to distinguish between different phases of the cycle. Bry and Boschan (1971) suggest that there are two reasons for drawing distinctions between the various phases of the cycle.

The first reason for distinguishing between various phases of the cycle is to generate a suite of non-parametric statistics about the business cycle. The NBER peaks and troughs of the reference cycle are the most familiar of these non-parametric statistics. However, it was once commonplace to distinguish between several stages of each cycle. Bry and Boschan (1971) describe the range of criteria for distinguishing between such phases as follows:

> *Characteristically, these schemes identify the neighborhoods of cyclical peaks and troughs, and partition the upswing and usually the downswing. This leads to sequences such as recovery - prosperity - recession - depression; upswing - boom - capital shortage - crisis - recession; or recovery - growth - contraction. In these sequences, like terms do not necessarily describe like periods. The segmentation may be determined on the basis of inflection points of fitted cyclical curves, intersection of trend and cyclical values, maximal changes in cyclical movements, attainment of prior peak levels, or by other criteria.*

These approaches have been largely discarded, seemingly, because the statistical criteria on which they based were too vague to yield reliable non-parametric statistics.

Bry and Boschan (1971) suggest that the second reason for distinguishing between the various phases of the cycle is that behaviour can be expected to vary between the phases of the cycle. The so called investment clock is one example of a device that is used and sold in the market place to suggest how investment behavior should vary over the cycle. Unfortunately, there is not much evidence of how extensively the investment clock is followed by investors.

Stephen Ellis writing in the Financial Review of 2 September 1997 provides some insight into how the financial press views economic behavior changing over the cycle.

> *"Happy days are here again — if you are in the richest 5 per cent of the population. After the early 1990s recession and a slow recovery, the wealthy are cashed-up and spending freely again in a style reminiscent of the 1980s.*
>
> *The symptoms of the bubble are much the same as during the late 1980s. French champagne sales are up 20 per cent in a year. BMW sales have grown even faster . . . . Ellis (1997).*

While I am not suggesting using Champagne or BMW sales in dating the cycle it does seem that there is scope to explore how business and household behavior interacts with the business cycle. One of the problems that arises in attempting to do this is that current economic theory does not provide much guidance and thus we need new theories of how the business cycle interacts with individual and firm behavior. One example of such a theory is provided by Ramey and Watson (1997) who introduce the notion of fragile employment contracts. They use such fragile contracts to explain the counter cyclical job destruction that is documented by Davis and Haltiwanger (1992). Another example, of a potentially useful framework is provided by Farmer (1993) who develops a theory of self fullfilling prophecies.

It also seems that theories incorporating bounded rationality may prove to be helpful in thinking about the dating of business cycles. To understand this argument it is useful to first review the rational expectations approach which remains the dominant paradigm in macroeconomics. Briefly economic models fall into the rational expectations category if the agents that populate the model are individually rational and agree in their perceptions of their economic environment. Models that fall outside this class involve either boundedly rational or 'irrational behaviour'.

Sargent (1993) observes that

> *When implemented numerically or econometrically, rational expectations models impute much* <u>*more*</u> *knowledge to the agents within the model (who use the* <u>*equilibrium*</u> *probability distributions in evaluating their Euler equations) than is possessd by an econometrician, who faces estimation and inference problems that the agents in the model have some-how solved.*

It is instructive to pause here and compare the notion of agents calmly using equilibrium probability distributions to form expectations with the exuberance and apparent myopia so clearly described in Ellis's Financial Review article — the champagne is flowing freely and the party seems like it will go on for ever. Such exuberance is matched just after a trough by a feeling of despair — there is a feeling of I haven't had champagne for months and I'll never get invited to another party. Including business and consumer confidence surveys in the dating of business cycles is one avenue for dealing with this phenomena. But those measures don't go far enough. The exuberance in the latter stages of the expansion and the despair just after a trough seem to me to be characteristic of the behavior of agents that are don't look forward or back very far. Unfortunately such agents are almost completely absent from formal theories of the business cycle.

Sargent goes on to say that:

> *I interpret a proposal to build models with 'boundedly rational ' agents as a call to retreat from the second piece of rational expectations (mutual consistency of perceptions) by expelling rational agents from our model environments and replacing them with 'artifically intelligent' agents who behave like econometricians. These 'econometricians' theorize, estimate, and adapt in attempting to learn about probability distributions which, under rational expectations they already know.*

Sargent's approach to modeling bounded rationality provides a useful point of departure but it does not go far enough in questioning what we mean by rational choice. Sen (1997) provides a broad and useful discussion of maximization and the act of choice. It is instructive to compare Sen's discussion with the restrictive notions of maximization and choice that are

embodied Sargent (1993) and in real business cycle models of the type discussed in Cooley (1995).

What might be the implications of including bounded rationality in business cycle models. Basically, bounded rationality is the notion that conscious maximising involves information storage, retrieval and processing all of which are costly. Economic agents economise on these costs by developing and using rules of thumb and possibly by ignoring some information that would be used if maximising were costless. Boundedly rational agents may then be expected to exhibit certain types of behavior near peaks and troughs. First, the financial adage "the trend is your friend" suggests that peaks and troughs (periods in which the trend is not dominating) may involve more information processing than periods in which the trend is dominant. Second, and relatedly, peaks and troughs may well be times at which agents are discarding 'worn out' rules of thumb and developing new rules of thumb. If there is some learning involved with the application of these new rules of thumb then peaks and troughs may be times in which volatility and serial correlation may be stronger than usual.

This issue of how behavior interacts with the cycle is an important and largely neglected area of research. It has the potential to improve our dating of cycles and more importantly would help to make dating of cycles less mechanical.

4 Procedures for dating business cycles

Arthur Okun's simple rule that a classical recession involves two quarters of negative growth in GDP is perhaps the most well known example of procedure for dating recessions. Pagan (1997a and 1997b) shows that much can be learnt about business cycles through clever application of Okun's rule. However, as discussed in Pagan(1997a), Okun's procedure has some disadvantages. Second, Okun's procedure does not say much about when recessions end other than the implication that it must involve a period of positive growth that follows two or more periods of negative growth.

The main alternative to Okun's procedure is the NBER method, the algorithm for which is described in Bry and Boschan (1971) and is reproduced in Table 1 below.

TABLE 1: BRY BOSCHAN PROCEDURE FOR PROGRAMMED DETERMINATION OF TURNING POINTS

I.	Determination of extremes and substitution of values		
II	Determination of cycles in 12-month moving average (extremes replaced)		
		A.	Identification of points higher (or lower) than 5 months on either side
		B.	Enforcement of alternation of turns by selecting highest of multiple peakd (or lowest of multiple troughs).
III	Determination of corresponding turns in Spencer curve (extremes replaced).		
		A.	Identification of highest (or lowest) value within ±5 months of selected turn in 12-month moving average.
		B.	Enforcement of minimum cycle duration of 15 months by eliminating lower peaks and higher troughs of shorter cycles
IV	Determination of corresponding turns in short- term moving average of 3 to 6 months, depending on MCD (months of cyclical dominance).		
		A.	Identification of highest (or lowest) value within ±5 months of selected turn in Spencer curve.
V.	Determination of turning points in unsmoothed series		
		A.	Identification of highest (or lowest) value within ±4 months, or MCD term, whichever is larger, of selected turn in short-term moving average.
		B.	Elimination of turns within 6 months of beginning and end of series.
		C.	Elimination of peaks (or troughs) at both ends of series which are lower (or higher) than values closer to end.
		D.	Elimination of cycles whose duration is less than 15 months.
		E.	Elimination of phases whose duration is less than 5 months.
VI.	Statement of final turning points.		

Source: Bry and Boschan (1971) page 21.

The NBER method, however, suffers from the problem that it is not transparent and is difficult to put into the context of modern econometric techniques. Moreover, both the original Bry Boschan Fortran program and Mark Watson's GAUSS program are difficult to use in simulations.

The alternative approach that I want to discuss here involves replacing the algorithm in Table 1 with a sequence of observed events that terminates an expansion phase; and a sequence of events that terminates a contraction phase. The proposed dating algorithm is in Table 2.

TABLE 2: PROPOSED DATING ALGORITHM USING TERMINATOR SEQUENCES

1	If in expansion and an expansion terminator sequence encountered:	
	A	Mark date as a peak
	B	Switch state to contraction
2	If in a contraction and a contraction terminator sequence encountered	
	A	Mark date as a trough
	B	Switch current state to expansion
3	If neither 1 or 2 then maintain current state last observation in a contraction and current observation and no peak or trough marked	

It is useful to restrict consideration to two broad types of terminator sequence. With terminator sequences type I, an event $\{y_t > \max[y_{t+1},\ldots,y_{t+k}]\}$ terminate an expansion with t as the date of the peak and a event $\{y_t < \min[y_{t+1},\ldots,y_{t+l}]\}$ terminate a contraction with t as the date of the trough. These alternative terminator sequence simply requires that y_t be either a local peak or trough and puts no restriction on the growth rates near the peak or trough.

With terminator sequence type II, an event $\{0 > \max[\Delta y_{t+1},\ldots,\Delta y_{t+k}]\}$ terminates an expansion with t as the date of the peak and an event $\{0 < \min[\Delta y_{t+1},\ldots,\Delta y_{t+k}]\}$ terminates a contraction with t as the date of the trough. This terminator sequence requires that y_t not only be a local peak or trough but also exhibit k periods of negative growth after the peak and k periods of positive growth after the trough.

Importantly, both types of terminator sequences can be used for dating classical and growth cycles. When using terminator sequence type I to date classical cycles y is simply the level of the series but when dating growth cycles y is the per cent deviation from trend. Similarly when using terminator sequence type II to date classical cycles y is the log difference of the series and the point of comparison is with zero but when dating growth cycles the point of comparison is the trend rate of growth.

By choosing the length of the terminator sequences one can impose some minimum phase length and minimum cycle length conditions. For example, with terminator sequence II,

choosing k and l to both be six months automatically yields a minimum phase of 6 months and a minimum cycle duration of 12 months.

These terminator sequences provide dates that are comparable to those obtained by Ernst Boehm using the NBER Bry Boschan procedure supplemented by his judgment. Table 3 provides a comparison of Ernst's dates for the classical cycle with the dates from my proposed procedure using terminator sequence type two with k=l=6 months.

The procedures agree on the number of turns and largely agree to within a few months on the dates of the turns. For four of the seven classical cycle peaks the procedures agree on the date of the peak. For one peak they differ by one month and in two cases they differ by three months in the date of a peak. For two troughs the dates agree, in another two cases they differ by two months, in one case they differ by two months, in one case the difference is three months and the maximum difference is eighteen months for the trough following the peak in late 1955.

TABLE 3: COMPARISON OF NBER AND PROPOSED BUSINESS CYCLE DATING PROCEDURE FOR AUSTRALIAN COINCIDENT INDEX - CLASSICAL CYCLE

Dates from NBER procedure[a]		Dates from proposed algorithm	
Peak	Trough	Peak	Trough
4/51	9/52	4/51	10/52
12/55	12/57	11/55	5/56
9/60	9/61	9/60	9/61
7/74	10/75	4/74	8/75
8/76	10/77	8/76	11/77
9/81	5/83	9/81	5/83
3/90	6/92	12/89	9/92

a) Source: E.A. Boehm, "A Review of Methodological Issues in Identifying and Analysisng Business Cycles, Forthcoming.

Table 4 provides a comparison of Ernst's dates for the growth cycle with the dates from my proposed procedure using terminator sequence type two with k=l=6 months. Again the dates from Ernst's application of the NBER procedures are in close agreement with the dates from my proposed procedures. It is also encouraging that these procedures agree on the number of

growth cycle turns. As with the classical cycle there is close agreement on the dates of both peaks and troughs in the growth cycle.

TABLE 4: COMPARISON OF NBER AND PROPOSED BUSINESS CYCLE DATING PROCEDURE FOR AUSTRALIAN COINCIDENT INDEX - GROWTH CYCLE

Dates from NBER procedure[a]		Dates from proposed algorithm	
Peak	Trough	Peak	Trough
4/51	11/52	2/51	11/52
8/55	1/58	8/55	1/58
8/60	9/61	7/60	9/61
4/65	1/68	1/65	12/67
1/71	3/72	2/71	5/72
2/74	10/75	3/74	12/75
8/76	2/78	7/76	12/77
9/81	5/83	7/81	5/83
11/85	3/87	12/85	12/86
11/89	12/92	11/89	3/93
12/95		11/95	

a) Source: E.A. Boehm, "A Review of Methodological Issues in Identifying and Analysisng Business Cycles, Forthcoming.

Figure 1 which presents the smoothed coincident index overlaid with the classical cycle turns provides an illustration of how easy the proposed procedure is to use and visualise . If the economy is in expansion and smoothed coincident index falls below zero for six months or longer a classical cycle peak is signaled and vice versa for a classical cycle trough.

Figure 2 presents the data in Figure 1 with the addition of the growth rate in the trend of the Australian coincident index. When the growth rate of the smoothed coincident index moves below the trend rate of growth for six months or more a growth cycle recession is signaled. Thus, Australia appears to have entered a growth cycle recession in late 1995.

FIGURE 1: ILLUSTRATION OF THE PROCEDURE FOR DETERMINING CLASSICAL CYCLE TURNS ON THE AUSTRALIAN COINCIDENT INDEX.

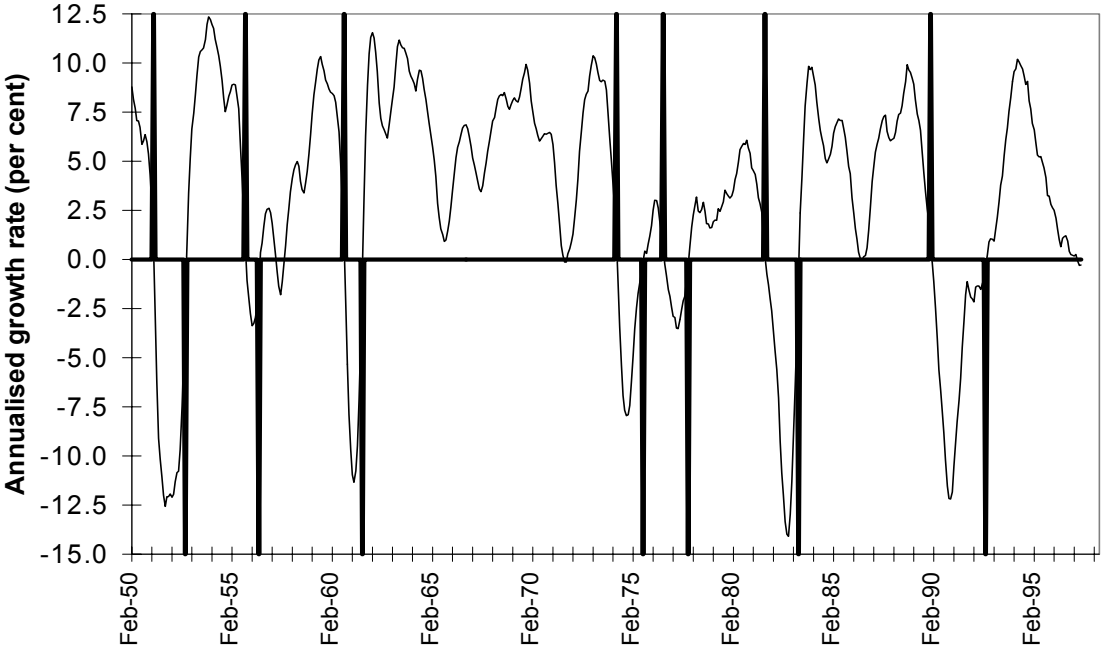
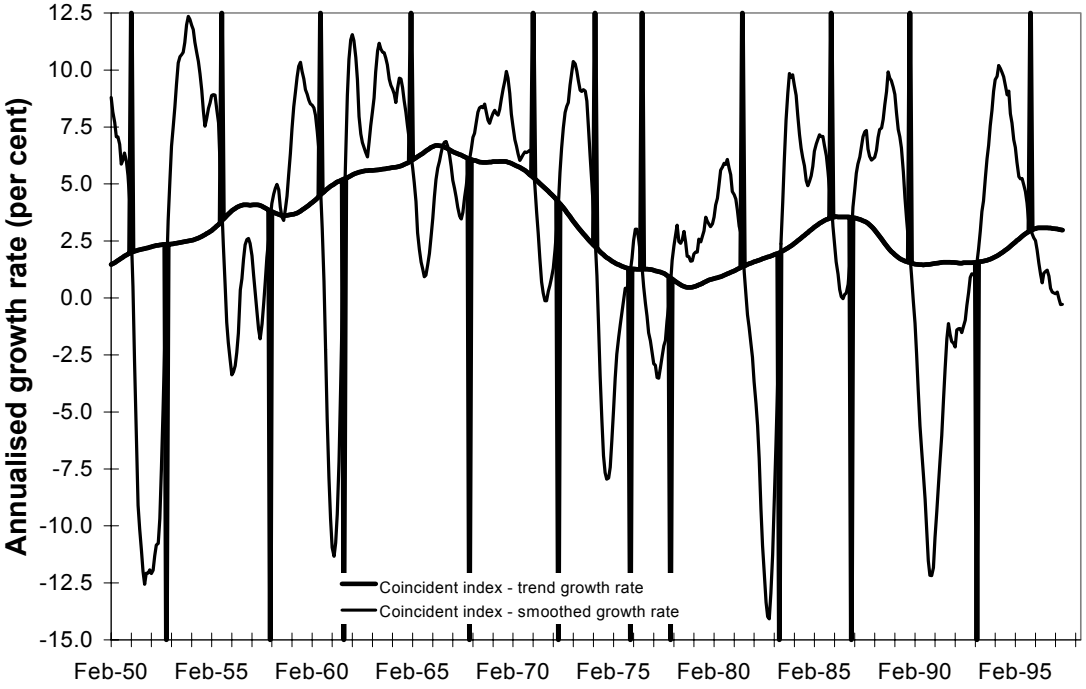


FIGURE 2: ILLUSTRATION OF THE PROCEDURE FOR DETERMINING GROWTH CYCLE TURNS ON THE AUSTRALIAN COINCIDENT INDEX.



5 The probability of turns

The procedure that I have proposed for dating business cycles has the advantage that it is easy to translate into statistical statements about the probability of peaks and troughs and the duration of expansions and contractions.

Using the following acronyms: P=Peak; T=trough; C=contraction; E=Expansion; ETS=Expansion Terminator Sequence; and CTS=Contraction Terminator Sequence, the probability of peaks and troughs are defined as follows:

Eqn 1 $$\Pr(\mathrm{P}) = \Pr(\text{in E}) * \Pr(ETS \mid \text{in E})$$

Eqn 2 $$\Pr(\mathrm{T}) = \Pr(\text{in C}) * \Pr(CTS \mid \text{in C})$$

An important part of the NBER dating rules is the requirement that the number of peaks is equal to the number of troughs. This is implemented by the condition that

Eqn 3 $$\Pr(\mathrm{P}) = \Pr(T) = \Pr(\text{Turn})$$

Trivially, it must also be the case that the economy is either in expansion or contraction, thus:

Eqn 4 $$\Pr(\text{in E}) = 1 - \Pr(\text{in C})$$

Thus after some substitution we have two equations in two unknowns.

Eqn 5 $$\Pr(\text{Turn}) = \Pr(\text{in E})\Pr(\text{ETS} \mid \text{in E})$$

Eqn 6 $$\Pr(\text{Turn}) = \{1 - \Pr(\text{in E})\}\Pr(\text{CTS} \mid \text{in C})$$

Solving equation 5 and 6 yields

Eqn 7 $$\Pr(\text{in E}) = \frac{\Pr(\text{CTS} \mid \text{in C})}{\Pr(\text{CTS} \mid \text{in C}) + \Pr(\text{ETS} \mid \text{in E})}$$

Eqn 8 $$\Pr(\text{Turn}) = \frac{\Pr(\text{CTS} \mid \text{in C}) * \Pr(\text{ETS} \mid \text{in E})}{\Pr(\text{CTS} \mid \text{in C}) + \Pr(\text{ETS} \mid \text{in E})}$$

Now once a model is specified it can be used to obtain the probabilities of the terminator sequences and distributions for the duration of expansions and contractions. For example, suppose that the statistical model is that GDP follows a random walk with drift μ and innovations that are serially independent and normally distributed with mean zero and variance σ^2. Now consider type II terminator sequences of the form ETS=(-,-) and CTS=(+,+). This is a particularly simple model where the probability of each terminator sequence is independent of the state of the economy. Let $\phi = \Pr(\Delta y_t < 0)$ then[2]:

<u>Eqn 9</u> $$\Pr(ETS|in\ E) = \Pr(-,-) = \phi^2$$,and

<u>Eqn 10</u> $$\Pr(CTS|in\ \mathrm{C}) = \Pr(+,+) = (1-\phi)^2$$

Substituting into equations 7 and 8 yields.

<u>Eqn 11</u> $$\Pr(in\ E) = \frac{(1-\phi)^2}{(1-\phi)^2 + \phi^2} = \frac{1}{1 + \frac{\phi^2}{(1-\phi)^2}}$$

<u>Eqn 12</u> $$\Pr ob(Turn) = \frac{(1-\phi)^2 * \phi^2}{(1-\phi)^2 + \phi^2}$$

Estimates from Australian quarterly GDP from the September Quarter 1959 to the March quarter 1997 suggest that $\mu = 0.0093$ and $\sigma = 0.0118$. These yield the following estimates Prob(in Expansion)=0.93, Prob(in Contraction)=0.07, Prob(Peak)=Prob(Trough)=0.043. It is useful to compare these figures with those obtained by Pagan (1997a) who obtains a probability of Trough of almost 0.1. This arises because in Pagan's formulation the probability of a trough is constrained to equal the probability that the economy is in a contraction.

[2] These probabilities sum to a number that is less than one because there are positive probabilities of obtaining ETS in C and CTS in E.

It is also useful to calculate the probability distributions for the duration of expansions and the duration of contractions. Let τ^E denote the duration of an expansion and τ^C denote the duration of an contraction. Then, the probability that the expansion lasts exactly j periods is

Eqn13

$$\begin{aligned}\Pr\left(\tau^E = j\right) &= \Pr\left(\tau^E > j-1\right)\Pr\left(\tau^E = j \mid \tau^E > j-1\right)\\ &= \left\{1-\Pr\left(\tau^E \le j-1\right)\right\}\Pr\left(\tau^E = j \mid \tau^E > j-1\right)\end{aligned}$$

Expansions start with the pattern (+,+) and must last for two or more periods. Thus the probability that an expansion lasts exactly two periods is just Pr(-,-). This provides a starting value for the recursion in eqn 13. The knowledge that the expansion lasted for more than j-1 periods tells us that the pair $\left(\Delta y_t, \Delta y_{t+1}\right)$ was in the set $\left\{(+,+),(-,+),(+,-)\right\}$. The probability that the expansion lasts exactly j periods conditional on it having lasted j-1 periods is the probability of the event that the pair $\left(\Delta y_t, \Delta y_{t+1}\right)$ has the shape $(+,-)$ and $\left(\Delta y_{t+2} \le 0\right)$. Formally,

Eqn 14

$$\Pr\left(\tau^E = j \mid \tau^E > j-1\right) = \Pr\left\{\left(\Delta y_t > 0, \Delta y_{t+1} \le 0, \Delta \mathrm{y}_{\mathrm{t+2}} \le 0\right) \mid \left(\Delta y_t, \Delta y_{t+1}\right) \; in \; \{+,-\},(-,+),(+,+)\right\}$$

For the random walk example the probability distribution for the duration of expansions is

Eqn 15 $$\Pr\left(\tau^E = 2\right) = \phi^2$$

Eqn 16 $$\Pr\left(\tau^E = j\right) = \left\{1-\Pr\left(\tau^E \le j-1\right)\right\}\frac{\phi^2}{1+\phi} \qquad \text{for } j > 2$$

Similarly, for the random walk example the probability distribution for the duration of contractions is

Eqn 17 $$\Pr\left(\tau^C = 2\right) = \left(1-\phi\right)^2$$

Eqn 18 $$\Pr\left(\tau^C = j\right) = \left\{1 - \Pr\left(\tau^C \le j-1\right)\right\}\frac{\left(1-\phi\right)^2}{2-\phi} \qquad \text{for } j > 2$$

For the case where the parameters of the random walk are set to the values for Australian quarterly GDP we obtain distributions of the durations of classical expansions and contractions in Figures 3 and 4. The expected duration of a classical cycle expansion for the random walk model with normal innovations is 27 quarters and the expected duration of the classical cycle contraction is 3.1 quarters yielding a classical cycle with expected duration of 30 quarters or 90 months. This is somewhat longer than the classical cycle measured by Boehm of some 80 months.

FIGURE 3: DISTRIBUTION OF THE DURATION OF CLASSICAL CYCLE EXPANSION FOR THE RANDOM WALK MODEL CALIBRATED TO AUSTRALIAN DATA

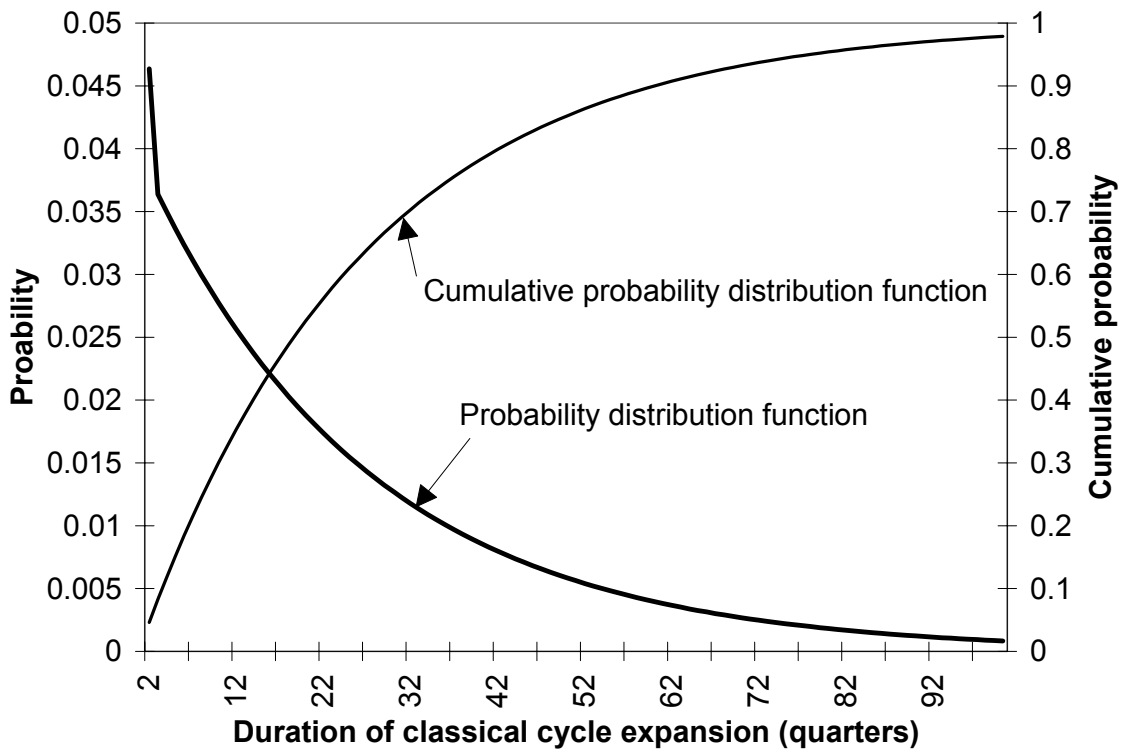
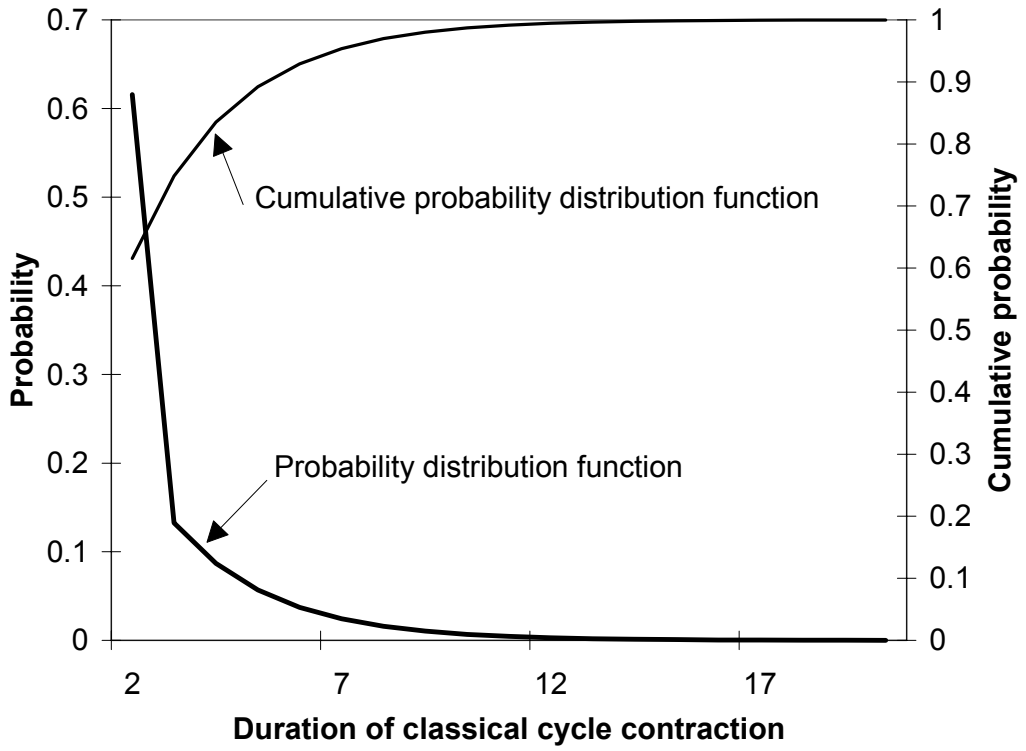
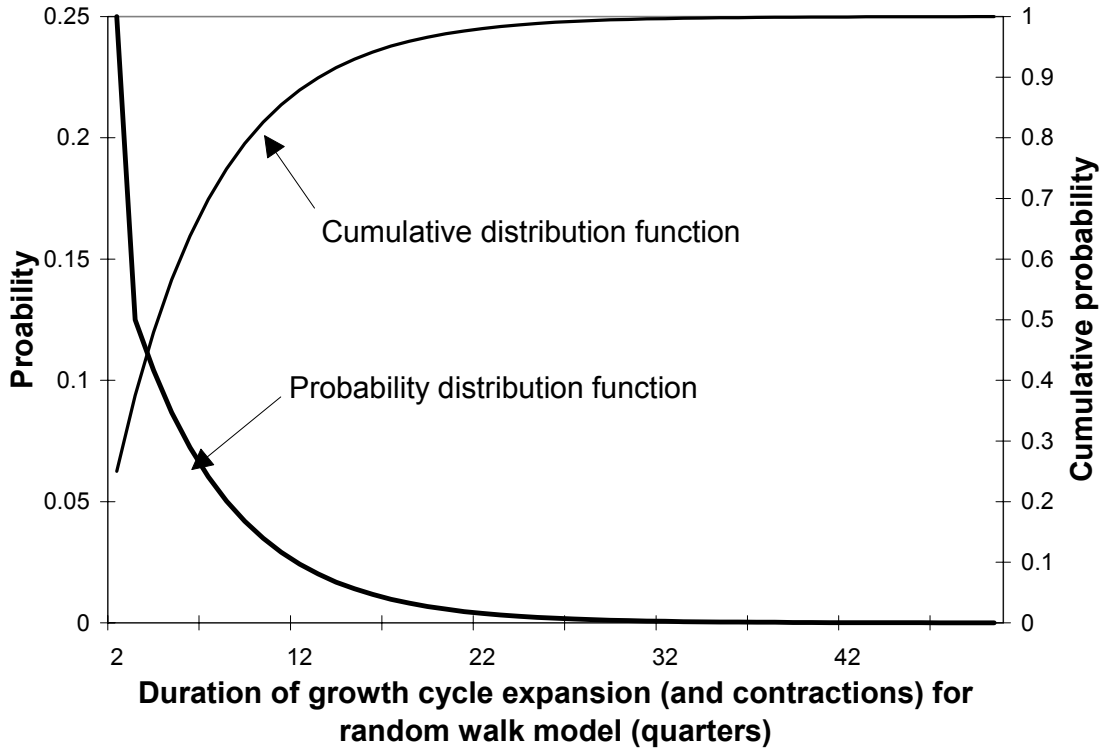


FIGURE 4: DISTRIBUTION OF THE DURATION OF CLASSICAL CYCLE CONTRACTION FOR THE RANDOM WALK MODEL CALIBRATED TO AUSTRALIAN DATA



Similar calculations can be performed for growth cycles yielding a Prob(in Expansion)=0.5, Prob(Peak)=Prob(Trough)=0.125, expected duration of expansion=expected duration of contraction=6.5 quarters giving a growth cycle of 13 quarters or 39 months. The distribution of the duration of the growth cycle expansion is shown in figure 5 for the random walk model.

FIGURE 5: DISTRIBUTION OF DURATION OF GROWTH CYCLE EXPANSION (AND CONTRACTION) FOR RANDOM WALK MODEL WITH NORMAL INOVATIONS



Conclusion

In this paper I have shown that the procedures for dating business cycles can be greatly simplified. This simplification comes at little cost in terms of the accuracy of dates when the NBER procedure is taken as the norm for comparisons. I have also shown that the simplified method for dating business cycles has the advantage that it fits neatly into modern statistical theory. This latter feature means that it is particularly simple to obtain the probability of a turn the probability of being in expansion and the probability of being in contraction from statistical models. I have also demonstrated that it is straightforward to obtain probability distributions for the duration of expansions and contractions. Finally, unlike the Okun procedure I am able to handle both classical and growth cycles.

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