Detecting and forecasting business cycle turning points

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Abstract

The R word has begun to appear in the media again bringing with it three technical questions viz,

How will we know we are in recession?

How will we know when it has ended? And

How can we forecast its onset and ending?

This paper does not provide answers to these questions rather it focuses on the technical issues that we need to resolve in order to provide good answers to these questions.

The paper has three significant findings. First, the business cycle states obtained by the BBQ algorithm are complex statistical processes and it is not possible to write down an exact likelihood function for them. Second, for the classical and acceleration cycles it is possible to obtain a reasonably simple approximation to the BBQ algorithm that may permit one to write down a likelihood function. Third, when evaluating these algorithms there is a large difference between the results using US GDP as compared to UK GDP or simulated data from models fit to US GDP. Specifically, turning points are much easier to detect in US GDP than in other series. One needs to take this into account when using US based research on detecting and forecasting business cycle turning points.

Key Words: Business cycle; turning points, forecasting, peak, trough.

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1 Introduction

The possibility of recessions in several European economies, the United States and Japan means that the topic of the business cycle is very much alive in 2008 and the R word has begun to appear in the media bringing with it three technical questions viz,

How will we know we are in recession?

How will we know when it has ended? And

How can we forecast its onset and ending?

Research over the past decade means that economists are better placed than ever before to answer these questions. From that research we now have widely accepted algorithms for locating turning points in time series viz[1]

- Bry and Boschan's (1971) and formalize into algorithms aspects of the NBER procedures for locating business cycle turning points in a single time series;

- Harding and Pagan's (2002) BBQ algorithm which is a quarterly approximation to Bry and Boschan. James Engle's' modified BBQ (MBBQ) code provides a useful implementation of that algorithm that can be used in simulation.

- Harding and Pagan (2006) develop an algorithm to aggregate the turning points in several series to replicate the NBER business cycle chronology for the United States and Australia.

Although the detection of turning points is well established the modelling and forecasting of turning points is less advanced. Turning points and the business cycle chronologies constructed from them are best viewed as constructed binary variables. Harding and Pagan (2007) show that because they are constructed they can have complicated data generating processes which can be difficult to express mathematically. In general the DGP will be a binary Markov process of order greater than two something that makes standard tools from microeconometrics, such as the Probit model, inadmissible.

Against that background there are four main avenues available when forecasting turning points. The first is to ignore the method of construction and assume that the businesses cycle state S_t that takes the value one in expansions and zero in contractions is generated by the rule

[1] Throughout this paper when I talk of turning points I am referring to turning points in the sample path. There are a variety of other concepts of turning points including regime switching models. Harding and Pagan (2002) show how the two concepts are related.

$$S_t = \mathbf{1}\left(\Delta y_t > 0\right)$$

where y_t is either the logarithm of GDP or some unobserved series that represents economic activity. The next step in this method is to ignore the true DGP and proceed as if a probit model or could be applied to S_t. This is the approach followed by Estrella and Mishkin (1998), Birchenall et al (1999) and Chen et al (2000) among many others. Harding and Pagan (2007) show that this approach can result in misleading inference about the probability of recessions. But surprisingly the method seems to work reasonably well when applied to the US business cycle. But there is no evidence that it works well on any other data set.

The second approach is to use the BBQ or Bry Boschan algorithms in conjunction with an assumed model for y_t. The model is simulated and forecasts of BBQ turning points produced. This approach suffers from two main problems. In some important cases such as the NBER business cycle chronology y_t is not observed and its data generating process is unknown. Even where y_t is observed the BBQ algorithm has the feature that forecasts of y_t must be made a long way into the future so as to ensure that all of the censoring procedures in BBQ have been applied. This latter feature means that BBQ is tricky to apply in a forecasting setup.

The third approach which is available when y_t is not observed is to assume that the business cycle state S_t follows a high order binary Markov process with some forcing variable such as the yield spread. Nonparametric methods can then be applied to generate forecasts of turning points. This approach is followed in Harding and Pagan (2007) and seems to work well although it is not a fully efficient method.

The fourth approach which is followed in this paper addresses the problem at its cause by exploring the feasibility of developing an approximation to the BBQ and Bry Broschan (1971) methodologies that can be used in forecasting turning points. The approximation should have five main features

- **Simplicity and parsimony**. That is it should be expressed as a simple recursive equation that lends its self to modelling and forecasting. Specifically, it should be straightforward to write down a good approximation to the data generating process of the business cycle states S_t;

- **Accuracy**. The chronology obtained by the approximation should be highly correlated with the BBQ and Bry Boshcan chronologies. This accuracy should be achieved on a wide range of data series;

- **Censoring**. The approximation should embody the main NBER censoring requirements that a) phases alternate between expansion and contraction; and b) phases have minimum duration of two quarters or five months for data recorded at the quarterly and monthly frequency respectively; c)

  – It is not essential censoring ensure completed cycles of at least five quarters or fifteen months duration 5. Nor is it essential that the censoring ensure that the next peak is higher than the previous peak or that the current trough is lower than the next trough.

- **Transferable**. The method should be readily transferable between monthly and quarterly data with relatively minor changes.

- **Transparent**. The approximation should be transparent. So that it is evident why it works And the economic content of the approximation should be evident and easily understood.

Against that background this paper has three objectives. First is to set out the various approaches to defining extreme economic events . Second, to suggest and evaluate approximations to the business cycle dating algorithms. Third, to discuss how one might use these approximations to guide the estimation of statistical models that can be used to make forecasts of business cycle turning points.

2 Approaches to defining business cycle turning points

The oldest approach to defining extreme economic events relies on the judgement of individuals and committees. For example, Willard Thorpe's *Business Annals* combined Thorpe's judgement with that of a range of scholars and writers to build business cycle chronologies for 17 countries for the period up to 1927.

Semi-official organizations such as the NBER use committee's and voting procedures to determine turning points in the business cycle chronology.[2] The advantage of the committee approach to determining the location of turning points is that it allows for human judgement to play a role thereby facilitating the consideration of factors that are difficult to model or include in formal quantitative analysis. There are two disadvantages of this approach. First it reduces the transparency of the procedures used to determine turning points and makes forecasting such turning points even more difficult. Second,

[2] The CEPR also has a business cycle dating committee for the Euro area.

the procedures of the committee may vary over time; Harding (2003) shows the procedures and variables used to construct the NBER business cycle chronology have changed markedly over time and it was only from the mid 1950s that the NBER business cycle dating procedures stabilized.

2.1 Rules for detecting turning points

It is a natural for economists to seek rules that approximate the decisions made by committees. For example, monetary policy rules in macroeconomic models are intended to approximate the complex decision processes of monetary authorities.

Rules suggested for approximate detection of business cycle turning points vary from the simple to the very complex.

A simple rule attributed to Arthur Okun, and frequently repeated in the media, defines a recession as two quarters of negative growth in GDP.[3]

The two quarters of negative growth rule, extended so that two quarters of positive growth terminates a recession, was studied by Harding and Pagan (2000) who found that it yields simple and easily detected turning points. But the two quarter rule has serious flaws. One flaw is that there can be periods of extreme economic pain in which the economy does not experience two periods of negative economic growth. Another flaw is that it does not match the NBER business cycle chronology. For example, the 2001 US recession did not exhibit two quarters of negative growth. Nonetheless it remains a popular rule for forecasting recessions and it will be one of the rules evaluated in this paper.

At the other end of the spectrum of business cycle dating rules is the calculus rule which associates recessions with periods in which GDP growth is negative for at least one quarter. This rule would classify as recessions periods in which output fell for only one quarter due to purely transitory shocks. As such it is an unsatisfactory rule for detecting recessions.

In between there are a variety of rules. For example, Zellner et al (1990) seek to define peaks and troughs in a time series as local extrema. It is useful to formalize this by using a pair of binary time series $(\wedge_t, \vee_t)$ to represent the business cycle chronology where $\wedge_t = 1$ indicates a peak at t and $\wedge_t = 0$ indicates that a peak did not occur at t. Similarly, $\vee_t = 1$ indicates a trough at t. Then the local extrema definitions of peaks and troughs are

[3] It is also the procedure adopted by the CEPR business cycle dating committee.

$$\begin{aligned}\wedge_t &= \mathbf{1}\{(y_{t-k},\cdots,y_{t-1}) < y_t > (y_{t+1},\cdots,y_{t+k})\} \\ \vee_t &= \mathbf{1}\{(y_{t-k},\cdots,y_{t-1}) > y_t < (y_{t+1},\cdots,y_{t+k})\}.\end{aligned} \tag{1}$$

The local extrema rule just given has several defects if the objective is to replicate the NBER business cycle chronology. First, it does not ensure that peaks and troughs alternate so that peaks and troughs produced according to (1) do not uniquely mark the beginning and end of recessions. Whether or not this matters depends on how the resulting turning points are being used. If the turning points are to be used to construct a business cycle chronology then the requirement that turning points alternate is essential to identifying the period between peak and trough with a contraction and the period between trough and peak as an expansion.

Ensuring that turning points alternate is also important for forecasting because business cycle turning points are conditional events — eg a business cycle peak is the event that there is a local peak at t conditional on being in expansion at t. This feature is important because (1) can produce several local peaks within an expansion something that many people seem oblivious to but which complicates both business cycle dating and the forecasting of turning points. Table 1 shows the extent to which non alternating turning points arise when the local extrema rule is applied to a range of time series that will be used throughout this paper.[4]

[4] The time series used include

- United States (US) real GDP;
- United Kingdom (UK) real GDP;
- A simulated random walk with drift

$$y_t = 0.00822 + y_{t-1} + 0.00981e_t$$

- A simulated stationary AR2 process with deterministic trend

$$y_t = 0.2736 + 0.000286t + 1.3011y_{t-1} - 0.3370y_{t-1} + .00917e_t$$

- A simulated AR1 process in growth rates.

$$y_t = 0.0056 + 1.3258y_{t-1} - 0.3258y_{t-1} + .00929e_t$$

In all cases $e_t\tilde{}N(0,1)$. The parameters above were chosen to fit the logarithm of US GDP 1947.Q1 to 2008.Q2.

Table 1: Proportion of non alternating turning points for various data generating processes

Series	Data		Simulated models		
	US GDP	UK GDP	RW	AR2 L	AR1 GR
y_t	9.4	36.4	31.7	25.4	25.9
$y_t - d_t$	16.5	31.8	29.3	22.6	23.3
$y_t - y_{t-4}$	2.0	26.1	23.4	16.0	16.5
N	246	201	$4,000,000$	$4,000,000$	$4,000,000$

The turning points in Table 1 were located using (1) with k=2. The simulated models were all calibrated to the logarithm of US GDP. Three main points emerge from Table 1.

First, non alternating (ie repeated) turning points are a problem in all of the data series considered. In some cases over thirty per cent of all turning points are repeated. This means that censoring procedures play a major role in determining the features of the business cycle. It also means that forecasting procedures based solely on (1) are doomed to failure if the objective is to forecast the NBER business cycle.

Second the frequency of non-alternating turning points is highest for the classical cycle and lowest for the acceleration cycle (ie turning points in $y_t - y_{t-4}$). This feature largely reflects the longer duration of classical cycle phases.

Third, the problem of non alternating cycles is least evident in US GDP and most evident in UK GDP. Moreover, the frequency of non alternating turning points in actual US GDP is different from the frequency for the simulated models fitted to US GDP.[5] This suggests that there may be a feature of US GDP that is not adequately captured in the simple models simulated in Table 1.

2.1.1 Ensuring phases alternate

Business cycle phases that alternate can be constructed by introducing the notion of a business cycle state S_t which takes the value one in expansions and zero in contractions. The business cycle state is related to local peaks and troughs $(\wedge_t, \vee_t)$ by the following recursion

$$S_t = S_{t-1}(1 - \wedge_{t-1}) + (1 - S_{t-1}) \vee_{t-1} \tag{2}$$

[5] An odd feature of the data is that there were no non alternating peaks in the US data. This is in strong contrast to all of the other data series.

To be feasible the recursion (2) requires a method of establishing the initial states S_1 and S_2. The simplest way of doing this is to use (1) to determine the first turning point in the data. If it is peak then the economy starts in an expansion so $S_1 = S_2 = 1$ and if the first turning point is a trough then the economy starts in a contraction so $S_1 = S_2 = 0$.

Equation (2) has the feature that when in an expansion it selects the first peak encountered and switches to a contraction phase this means that if there is a second higher peak before the next trough then it will ignore that higher peak. Similarly, when in a contraction equation (2) changes phase as soon as it encounters a trough even if that trough is higher than the next trough. This problem can be addressed in two main ways. The first is to set k relatively large (eg 4 quarters) in 1 so that the algorithm looks ahead one year when determining turning points. The issue here is the larger is k the more restrictive is the definition of a turning point and thus the more extreme economic event that is described as recession. The second solution is to add terms that look ahead to the next like turning point. This approach has not been followed because it makes the recursion unduly complicated and thus makes it hard to write a likelihood function for the business cycle states.

The recursion (2) has the feature that peaks terminate expansions and troughs terminate contractions thereby ensuring that turning points $(\wedge_t^a, \vee_t^a)$ defined by (3) alternate

$$\begin{aligned} \wedge_t^a &= S_t\left(1 - S_{t+1}\right) \\ \vee_t^a &= \left(1 - S_t\right) S_{t+1} \end{aligned} \tag{3}$$

I refer to this chronology made with (1) and (2) as the Zellner chronology with alternating turning points. Let S_t^Z denote the business cycle states obtained using (1) and (2) and S_t^{MBBQ} represent those obtained via James Engle's modified BBQ algorithm.[6] We can now compare the correlation between these two chronologies to of the this chronology with the Once we have enforced alternation of peaks and troughs we can begin to compare the business cycle chronologies produced using with those produced Table 2 provides information on the correlation between the two chronologies

Table 2 clearly demonstrates that the combination of (1) and (**??**) is insufficient to match the NBER procedures as coded into the MBBQ algorithm

[6] James Engle's MBBQ Gauss code is available at http://www.ncer.edu.au/data/mbbq.g.

Table 2: Correlation between the MBBQ chronology and the alternating Zellner et al turning points for various data generating processes

Series	Data		Simulated models		
	US GDP	UK GDP	RW	AR2 L	AR1 GR
y_t	0.94	0.23	0.44	0.44	0.45
$y_t - d_t$	0.32	0.20	0.27	0.25	0.25
$y_t - y_{t-4}$	0.95	0.63	0.77	0.80	0.79
N	246	201	$4,000,000$	$4,000,000$	$4,000,000$

for UK data and simulated data from models fit to US GDP. Somewhat surprisingly, the combination of (1) and (**??**) does well in detecting the MBBQ business cycle chronology for US GDP. This is a finding that is repeated below in this paper viz, turning points in US GDP are easily detected and it does not matter very much which algorithm one uses to detect the turning points.

2.1.2 Ensuring completed phases have a minimum duration

The rules (1) and (2,3) do not enforce a minimum distance between successive peaks and troughs. So the period between a peak and the next trough could be as little as one month for monthly data or one quarter for quarterly data. Table3 shows the proportion of completed contraction and expansion phases with duration of one quarter in a range of time series. The rules (1) and (2,3) result in about 10 per cent of phases having duration of one quarter. Again US GDP is the exception as rules (1) and (2,3) when applied to that series do not detect any completed phases with duration of exactly one quarter.

Table 3: Proportion of completed phases with duration of one quarter for various data generating processes

Series	Data		Simulated models		
	US GDP	UK GDP	RW	AR2 L	AR1 GR
y_t	0	0	10.5	8.8	9.1
$y_t - d_t$	8.3	13.9	12.7	8.6	8.9
$y_t - y_{t-4}$	0	10.5	9.9	6.0	6.3
N	242	197	$4,000,000$	$4,000,000$	$4,000,000$

Does the existence of short phases matter? The answer to this question depends largely on the objectives of the analysis. In much of macroeconomics the focus of attention is on shocks that are persistent. The NBER's, procedures that eliminate certain turning points from their chronologies because they are associated with short phases can be viewed as a convenient device for focusing attention on persistent shocks. Thus, if the objective of the analysis is to focus attention on persistent shocks then censoring to achieve minimum phase length is an important part of the turning point detection methodology.

Indeed, Harding (2003) shows that there is a close relationship between censoring of turning points and band pass filtering of the original data and Harding (2008) establishes the equivalence of the permanent and transitory decompositions obtained via band pass filtering, Hodrick-Prescott filtering, Butterworth filtering, the Beveridge-Nelson decomposition and certain unobserved components models. Thus, censoring of turning points to achieve minimum phase duration has close connections with filtering procedures used in macro economics.

The problem of short business cycle phases can be overcome by linking peaks and troughs and the censored business cycle states S_t in the recursions (4) for quarterly data and (5) for monthly data.

$$S_t = S_{t-1}\left(1 - S_{t-2}\right) + S_{t-1}S_{t-2}\left(1 - \wedge_{t-1}\right) + \left(1 - S_{t-1}\right)\left(1 - S_{t-2}\right)\vee_{t-1} \quad (4)$$

The first term in (4) is $S_{t-1}\left(1 - S_{t-2}\right)$ which ensures that expansions last at least two periods. The second term $S_{t-1}S_{t-2}\left(1 - \wedge_{t-1}\right)$ only operates if the expansion is two or more quarters old and terminates that expansion if y_{t-1} is a local peak. The third term $\left(1 - S_{t-1}\right)\left(1 - S_{t-2}\right)\vee_{t-1}$ only operates if the contraction is two or more quarters old and terminates that contraction if y_{t-1} is a local trough. Thus (4) ensures that turning points alternate and that phases have minimum duration of two quarters. The recursion can be initialized in the same way that (2) was initialized.

Equation (5) provides the recursion that ensures minimum phase duration for monthly data. The first two lines of (5) ensure that expansions have a minimum duration of five months. The third line allows continuation of expansions more than five months duration provided a peak is not encountered. If a peak is encountered the phase is switched to a contraction. The fourth line terminates contractions of more than five months duration if a trough is encountered.

$$
\begin{aligned}
S_t &= S_{t-1}\left(1-S_{t-2}\right)+S_{t-1}S_{t-2}\left(1-S_{t-3}\right)+S_{t-1}S_{t-2}S_{t-3}\left(1-S_{t-4}\right) \\
&\quad +S_{t-1}S_{t-2}S_{t-3}S_{t-4}\left(1-S_{t-5}\right) \\
&\quad +S_{t-1}S_{t-2}S_{t-3}S_{t-4}S_{t-5}\left(1-\wedge_{t-1}\right) \\
&\quad +\left(1-S_{t-1}\right)\left(1-S_{t-2}\right)\left(1-S_{t-3}\right)\left(1-S_{t-4}\right)\left(1-S_{t-5}\right)\vee_{t-1}
\end{aligned} \tag{5}
$$

Binary series $(\wedge_t^c, \vee_t^c)$ that denote whether there are censored peaks and troughs respectively at date t can be obtained from S_t^c via equations (6) and (7).

$$\wedge_t^c = S_t\left(1-S_{t+1}\right) \tag{6}$$

$$\vee_t^c = \left(1-S_t\right)S_{t+1} \tag{7}$$

Table 4 provides information on how censoring to achieve alternation of turning points and minimum phase length affects the statistical properties of the business cycle chronology. It is evident from this table that adding the minimum phase requirement actually results in a lower correlation with the MBBQ states than was obtained in Table 2. Indeed for the UK growth cycle the correlation becomes negative.

Table 4: Correlation between the MBBQ chronology and the chronology with k=2 and turning points censored so they allternate and have minimum phase length, various generating processes

Series	Data		Simulated models		
	US GDP	UK GDP	RW	AR2 L	AR1 GR
y_t	0.94	0.23	0.37	0.36	0.37
$y_t - d_t$	0.13	-0.14	0.23	0.22	0.22
$y_t - y_{t-4}$	0.95	0.60	0.73	0.75	0.74
N	246	201	$4,000,000$	$4,000,000$	$4,000,000$

It is sometimes claimed, see for example Harvey et al (2007), that the use of $k = 2$ for quarterly data in (1) is main innovation in Harding and Pagan (2002) allowing them to apply that rule to GDP and achieve a good approximation to the NBER business cycle chronology. In fact, as is made clear in Harding and Pagan (2006), and in the discussion above it is the simplification of the rules that make turning points alternate and ensure minimum phase and completed cycle lengths that was the main contribution of the BBQ algorithm. What has been shown in the discussion above is that although the simple recursive rules (1) and (4) can implement two of the three main NBER censoring objectives viz alternating turning points and phases with minimum duration, they do not achieve a good match to the MBBQ algorithm. Further work is required to yield a good approximation to the MBBQ and Bry Boschan algorithms.

3 Approximating BBQ and Bry Boschan algorithms

3.1 Design of the approximate BBQ rule

I explored two approaches to obtaining good approximations to the Bry Boschan and BBQ algorithms. The first is to use binary series to represent the censoring procedures that enforce minimum completed cycle length. After much experimentation I have concluded that this approach is not feasible. The reason is that this form of censoring essentially requires one to vary the width of the window through which the sample path is viewed. Indeed for series with few turning points one needs to inspect the whole of the sample path. This is simply too complex a task to code into an algorithm.

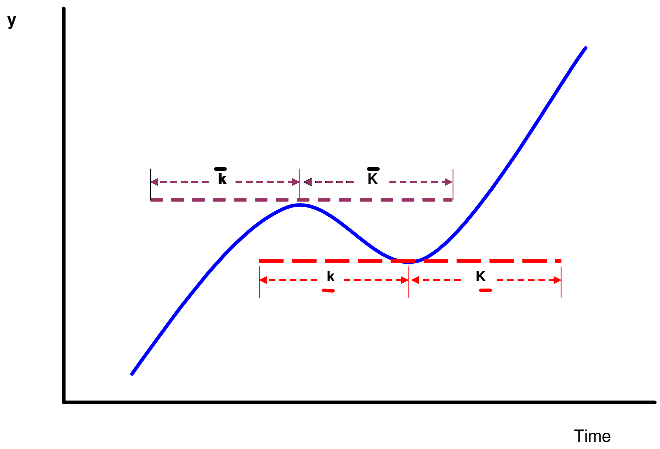
The second approach, which is implemented below, recognizes that the bulk of the action from NBER censoring procedures occur within a short window about each turning point. In implementing this approach I have assumed that the window goes back k periods and forward K periods where (k, K) might be allowed to be different for peaks and troughs. This simplifies the research question as now one only needs to establish how $\left(\overline{k}, \overline{K}, \underline{k}, \underline{K}\right)$ vary with the properties of the data so as to generate business cycle chronologies that are good approximations to BBQ and Bry Boschan.

$$
\begin{aligned}
\wedge_t^{\overline{k},\overline{K}} &= \mathbf{1}\{(y_{t-\overline{k}}, \cdots, y_{t-1}) < y_t > (y_{t+1}, \cdots, y_{t+\overline{K}})\} \\
\vee_t^{\underline{k},\underline{K}} &= \mathbf{1}\{(y_{t-k}, \cdots, y_{t-1}) > y_t < (y_{t+1}, \cdots, y_{t+K})\}.
\end{aligned}
\tag{8}
$$

Intuition regarding the choice of the parameters $\left(\overline{k}, \overline{K}, \underline{k}, \underline{K}\right)$ in (8) is given in Figures 1, 2 and 3 and the accompanying discussion.

For strongly trended series such as that shown in the stylized representation of Figure 1 the two main parameters are $\underline{k}$ and $\overline{K}$. The strong positive trend in the series means that the parameters $\overline{k}$ and $\underline{K}$ will only determine a small proportion of turning points. Thus for positively trended series the local peak is effectively determined by $\overline{K}$. So if for example $\overline{K} = 4$ then we are determining local peaks in the series by asking whether y exceeds its current level for four quarters. Similarly if $\underline{k} = 4$ then we are determining local troughs in the series by asking whether y is currently lower than it was four quarters ago.

Figure 1: Choice of window width for series with a strong positive trend



As shown in Figure 2 the parameters $\left(\overline{k}, \overline{K}, \underline{k}, \underline{K}\right)$ are equally important for determining turning points in untrended series. This argues for setting $\overline{k} = \overline{K} = \underline{k} = \underline{K} = k$ when dating turning points in such series.

$\underline{K}$ and $\overline{k}$ are the two key parameters for determining turning points in strongly negatively trended series. As is shown in in Figure 3 $\overline{k}$ influences the location of peaks in strongly negatively trended series while $\underline{K}$ determines the location of troughs in these series.

Figure 3 also helps to illustrate an important feature of the NBER censoring procedures. In Figure 3 there is a clear peak at A and a clear trough at D but it is uncertain whether B should be a trough and C should be a peak. The NBER procedures codified in BBQ and Bry Boschan eliminate the trough at B because the subsequent trough at D is lower than at B. These procedures would also eliminate the peak at C because it is lower than the previous peak at A. The censoring procedure just described means that one needs to select peaks and troughs that one is certain about and then proceed to eliminate intermediate peaks if they are lower than the previous peak and troughs if they are higher than the subsequent trough. This means that to

Figure 2: Choice of window width for untrended series

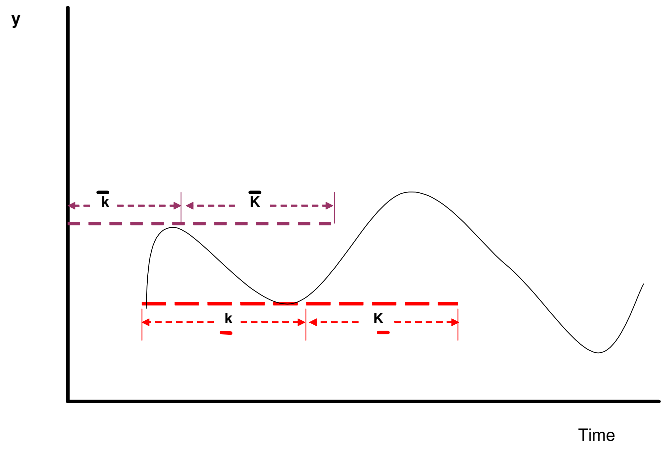
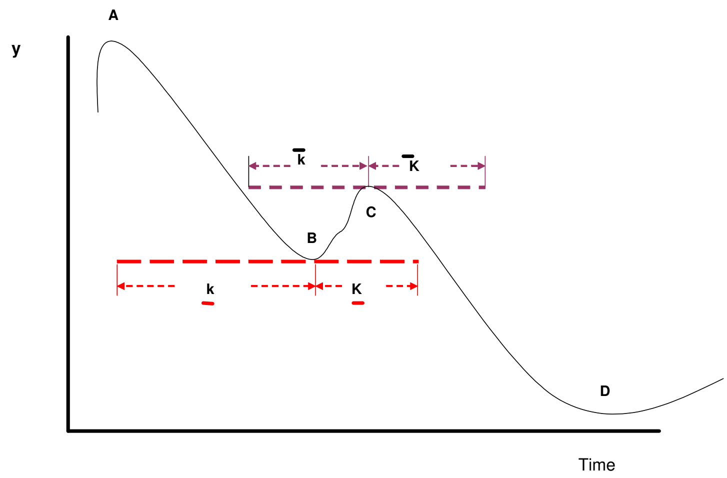


Figure 3: Choice of window width for strongly negatively trended series



fully implement the NBER procedures in a way that can be described mathematically one either needs to use the whole sample or select a window with width that is determined by the data. Both of these are things that make it very difficult to write down a likelihood function that is conditioned on the past values of y_t and the business cycle states.

The feature just described is the main reason why the NBER dating committee has to wait so long before it determines a turning point. For example, in determining a trough at B in Figure 3 the committee needs to wait until one of two events occur viz

- y exceeds its value at A which then confirms that the next peak will be higher than A and thus valid thereby making the trough at B valid;

- y is less than the value at B thereby confirming that B is not a valid turning point.

The feature just described means that even when using simulated data and BBQ the forecasting of turning points is extremely difficult because how far one needs to forecast into the future to ensure that all of the censoring is completed depends on the data generating process for y_t. The feature also means that NBER turning points can have little effective role in guiding policy decisions because turning points cannot be established until well after they have occurred. Clearly, this is an unsatisfactory situation from both a forecasting and public policy role. Of course the problems just described relate primarily to the use of NBER turning points in forecasting and public policy they do are of less important where the BBQ or Bry Boschan algorithms are used to generate statistics to distinguish between econometric models in terms of their capacity to match summary statistics measuring business cycle features.

Thus, in the approximation to the BBQ algorithm I propose to omit the censoring requirement just described.

The requirement that completed cycles have a duration of at least five quarters causes a similar problem that one needs to look a long way ahead to determine whether a current potential turning point is valid. This feature also makes it difficult to obtain the likelihood for the states and for this reason I have omitted this form of censoring from the approximation to the BBQ algorithm.

Thus in summary the approximate Bry Boschan algorithm is a follows

- For quarterly data equations 4 and 8 with values of k and K chosen between 2 and 5 quarters;

- For monthly data equations 5 and 8 with values of k and K chosen between 5 and 9 months; and

- Censored peaks and troughs for forecasting defined by equations 6 and 7 respectively.

3.2 Evaluation of the approximate BBQ rule

To test the usefulness of the approximate BBQ rule I compared the correlation between the business cycle states obtained via the MBBQ rule S_t^{MBBQ} and the approximate BBQ rule S_t^A for a range of values of k and K . The results are in Tables 5, 6, 7, 8 and suggest that by adjusting k and K one can find an approximate BBQ rule that produces a chronology that closely matches that obtained via BBQ for the classical and acceleration cycles. But it is not possible to find an approximate BBQ rule that works well for the growth cycle. The reasons for this are not yet known.

Table 5: Correlation between the MBBQ chronology and the chronology with k=2 and K=2 and turning points censored so they allternate and have minimum phase length, various generating processes

Series	Data		Simulated models		
	US GDP	UK GDP	RW	AR2 L	AR1 GR
y_t	0.94	0.80	0.65	0.53	0.54
$y_t - d_t$	0.33	0.33	0.27	0.26	0.25
$y_t - y_{t-4}$	0.95	0.70	0.78	0.80	0.79
N	246	201	$4,000,000$	$4,000,000$	$4,000,000$

Table 6: Correlation between the MBBQ chronology and the chronology with k=2 and K=4 and turning points censored so they allternate and have minimum phase length, various generating processes

Series	Data		Simulated models		
	US GDP	UK GDP	RW	AR2 L	AR1 GR
y_t	0.94	0.85	0.88	0.66	0.67
$y_t - d_t$	0.37	0.39	0.30	0.30	0.27
$y_t - y_{t-4}$	0.98	0.80	0.88	0.86	0.85
N	246	201	$4,000,000$	$4,000,000$	$4,000,000$

Table 7: Correlation between the MBBQ chronology and the chronology with k=4 and K=4 and turning points censored so they allternate and have minimum phase length, various generating processes

Series	Data		Simulated models		
	US GDP	UK GDP	RW	AR2 L	AR1 GR
y_t	0.94	0.85	0.84	0.64	0.65
$y_t - d_t$	0.37	0.39	0.30	0.26	0.26
$y_t - y_{t-4}$	0.93	0.75	0.87	0.85	0.84
N	246	201	$4,000,000$	$4,000,000$	$4,000,000$

Table 8: Correlation between the MBBQ chronology and the chronology with k=3 and K=3 and turning points censored so they allternate and have minimum phase length, various generating processes

Series	Data		Simulated models		
	US GDP	UK GDP	RW	AR2 L	AR1 GR
y_t	0.94	0.85	0.81	0.62	0.63
$y_t - d_t$	0.38	0.36	0.30	0.27	0.26
$y_t - y_{t-4}$	0.93	0.72	0.85	0.84	0.83
N	246	201	$4,000,000$	$4,000,000$	$4,000,000$

4 Forecasting business cycle states and turning points

Forecasting of business cycle states involves forming the conditional expectation $E\left(S_{t+j}|F_t\right)$ where F_t is an information set. Forecasting turning points involves the conditional expectations $E\left(\wedge_{t+j}^c|F_t\right)$ and $E\left(\vee_{t+j}^c|F_t\right)$ where $\left(\wedge_t^c, \vee_t^c\right)$ are defined by equations (6) and (7) respectively. Thus forecasting the probability of a peak at $t+j$ involves

$$\Pr\left(Peak\,at\,t+j|F_t\right) = E\left(S_{t+j}\left(1-S_{t+j+1}\right)|F_t\right) \tag{9}$$

and the probability of a trough at $t+j$ is

$$\Pr\left(trough\,at\,t+j|F_t\right) = E\left(\left(1-S_{t+j}\right)S_{t+j+1}|F_t\right) \tag{10}$$

The rationale behind the investigation in the preceding sections is that as is demonstrated in Harding and Pagan (2007) the two conditional expectations (9) and (10) are complex statistical entities when the S_t are obtained using the Bry Boschan, BBQ and MBBQ algorithms. The discussion above shows that it is feasible to obtain reasonable approximations to S_t^{BBQ} that might permit

- the writing down of explicit statistical expressions for (9) and (10); and/or
- simulation procedures that can obtain the simulated expectations (9) and (10) without having too look (data determined) long way into the future as is the case when using the BBQ and MBBQ algorithms for forecasting.

There is much to do here in further extensions of the paper. But I want to make one simple point that shows why even with the approximate BBQ algorithm it will still be difficult to obtain explicit forms of the likelihood.

Essentially, the ease or difficulty of writing down the likelihood for S_t depends on whether or not the information set F_t contains S_{t-1} and S_{t-2}. To understand this issue let $\widetilde{S}_{t-1} = \{S_{t-1}, S_{t-2}, ...\}$ be the past history of the business cycle states and let $\widetilde{F}_t = \{y_t, y_{t-1}, y_{t-2}, ..x_t, x_{t-1}, ...\}$ be the past history of the $y's$ and $x's$. Then in the most favourable circumstance we can expect to be in is where the information set is $F_t = \left\{ \widetilde{S}_{t-1}, \widetilde{F}_t \right\}$ in this case

$$
\begin{aligned}
E\left(S_t | \widetilde{S}_{t-1}, \widetilde{F}_t\right) &= S_{t-1}\left(1 - S_{t-2}\right) + S_{t-1}S_{t-2} - S_{t-1}S_{t-2}E\left(\wedge_{t-1} | \widetilde{S}_{t-1}, \widetilde{F}_t\right) \\
&\quad + \left(1 - S_{t-1}\right)\left(1 - S_{t-2}\right) E\left(\vee_{t-1} | \widetilde{S}_{t-1}, \widetilde{F}_t\right)
\end{aligned}
$$

Which is a relatively simple entity to calculate.

Unfortunately in most cases BBQ and the approximation to the BBQ algorithm means that the information set $F_t = \left\{ \widetilde{S}_{t-K}, \widetilde{F}_t \right\}$ where K is the parameter governing how far forward the approximate BBQ algorithm looks when locating a turning point. Even when $K = 2$ we can see that

$$
\begin{aligned}
E\left(S_t | F_t\right) &= E\left(S_t | \widetilde{S}_{t-2}, \widetilde{F}_t\right) \\
&= E\left(S_{t-1} | \widetilde{S}_{t-2}, \widetilde{F}_t\right)\left(1 - S_{t-2}\right) + E\left(S_{t-1} | \widetilde{S}_{t-2}, \widetilde{F}_t\right) S_{t-2} \\
&\quad - S_{t-2}E\left(S_{t-1} \wedge_{t-1} | \widetilde{S}_{t-2}, \widetilde{F}_t\right) \\
&\quad + \left(1 - S_{t-2}\right) E\left(\left(1 - S_{t-1}\right) \vee_{t-1} | \widetilde{S}_{t-2}, \widetilde{F}_t\right)
\end{aligned}
$$

And this involves more complex conditional expectations to calculate than the previous case. But it looks feasible to obtain expressions for these expectations and thus to obtain the likelihood for the approximate BBQ business cycle states either explicitly or via simulation.

5 Conclusions

The paper has three significant findings. First, the business cycle states obtained by the BBQ algorithm are complex statistical processes and it is not possible to write down an exact likelihood function for them. Second, for the classical and acceleration cycles it is possible to obtain a reasonably simple approximation to the BBQ algorithm that may permits one to write down a likelihood function. Third, when evaluating these algorithms there is a large difference between the results using US GDP as compared to UK GDP or simulated data from models fit to US GDP. Specifically, turning points are much easier to detect in US GDP than in other series. One needs to take this into account when using US based research on detecting and forecasting business cycle turning points.

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