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Abstract We analyse the stability issue in a Cournot duopoly with heterogeneous players. We show that labour market institutions matter for the stability of the unique interior Cournot-Nash equilibrium. Interestingly, the role played by the existence of firm-specific unions on stability, when the degree of unionism is asymmetric between the two firms, is at all different depending on whether the unionised firm has bounded rational or naive expectations. Indeed, a shift in the union's preference from employment towards wages acts as an economic (de)stabiliser when workers are paid with the (competitive) unionised wage by the bounded rational firm and with the (unionised) competitive wage by the naïve firm.

Keywords Bifurcation; Cournot; Heterogeneous expectations; Monopoly union

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1. Introduction

While a common feature of the standard dynamic oligopoly literature is the assumption of competitive factor markets, in many cases, however, such an assumption does not properly represent real phenomena. For instance, labour markets are often observed to be unionised, especially in Western Europe, and the degree of unionism is higher in concentrated industries. Indeed, a burgeoning literature on unionised oligopoly has recently been developed in a static context (e.g., Horn and Wolinsky, 1988; Dowrick 1989; Bughin, 1995; Naylor 1999; Correa-López and Naylor, 2004; Pal and Saha, 2008; Fanti and Meccheri, 2011). In such models, firms and unions behaviours incorporate two stages of decisions, which, at each stage, are taken by anticipating the equilibrium outcome of subsequent stages. As a consequence, in a static context the analysis of a possible dynamical role of labour market institutions, and, in particular, the relationship between the degree of unionism and product market stability, is neglected, while being the object of the present study.

The existence of firm-specific monopolistic unions in a dynamic Cournot duopoly is assumed by considering that: (1) wages are unilaterally set by the union, which has the object of maximising a specific objective function. Indeed, unions' choices take place simultaneously across firms, *and each union takes the employment reaction function of its own firm as given.* This assumption is intuitive in a dynamic context and differs from the static two-stage game, where each union takes the output market equilibrium employment of its own firm as given; (2) each firm decides on its level of output and optimal factor inputs, given the technology of productions and the wage determined by the union, and plays a non-cooperative oligopolistic game à la Cournot in the output market (as detailed in Section 2).

As regards the firm's expectations formation mechanism with respect to the output choice by the rival, some preliminary comments are indeed in order. The assumption of perfect foresight would eliminate – by definition – any market adjustment dynamics, leaving, however, the fluctuations of output observed in the real world unexplained as an endogenous feature of the market behaviour, with the consequence that they would entirely be due to exogenous random shocks. By contrast, the assumption of bounded rationality (i.e., myopic rather than perfect foresight), which implies a more complicated adjustment mechanism than the Cournot-naïve[1] one, see Dixit (1986), allows for the possibility of complex dynamic events.[2] Given bounded rationality,[3] it is important to understand, from an economic point of view, the dynamic effects of fully micro-founded theoretical advances, such as the trade-unions economics, in the light of the increasing interest for a refined analysis in the nonlinear oligopoly literature (see. e.g., Bischi et al., 2010).

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[1] Cournot (1838) may be considered as the first economist to introduce naive expectations in a duopoly market.

[2] We recall that, as early shown by Theocharis (1960), when both firms have Cournot-naïve expectations the Nash equilibrium is always stable and, hence, only transitional convergent dynamics are possible in such a case.

[3] As known, the different type of expectations (e.g., heterogeneity) leaves unchanged the equilibrium values of output and profit, but it plays a relevant role in the stability conditions of the Nash equilibrium: it is well established that a high level of the speed of adjustment of the bounded rational firm leads the system to instability via flip or Neimark-Sacker bifurcation of the Nash equilibrium (e.g. Agiza and Elsadany, 2003).

Since we are aware that, at the best of our knowledge, no models exist that deal with the effects of trade unions in nonlinear dynamic oligopoly literature, in the present paper we aim to fill this gap by using a duopoly game with quantity competition. For doing this, we have introduced firm-specific union's behaviours in a context of bounded rationality, where each union monopolistically sets the wage, while being either more oriented towards wages or employment, and takes the employment reaction function of its own firm (instead of the equilibrium employment as in the static game) into account, and we address two basic questions: (*i*) does the structure of labour markets (namely, uniformly unionised versus competitive markets, or, alternatively, or, alternatively, a different degree of unionisation between the two firms) matter for stability of the Cournot-Nash equilibrium? (*ii*) May the interaction between the types of firms' expectations formation mechanisms and the structure of labour markets, drastically change the role played by unions on stability outcomes?

The main findings of the present paper can be resumed as follows. In the standard Cournot duopoly, when the labour markets become uniformly unionised and the relative importance of wages in the union's objective increases, the parametric stability region is always enlarged irrespective of whether firms' expectations are bounded rational or naïve.[4] Moreover, we show that when firms' expectations are heterogeneous and labour market institutions are different (i.e., unionised and competitive), the role played by the relative degree unionism on stability is different depending on the interaction between the types of expectations.

In particular, we show that the process of unionisation of workers (as represented by an increase in the relative importance of wages in the union's objective) only in the bounded rational firm tends to destabilise the Nash-Cournot equilibrium.

The rest of the paper is organised as follows. Section 2 presents a dynamic Cournot duopoly with heterogeneous (bounded rational and naïve) expectations. Section 3 further develops the model by introducing micro-foundations of trade-unions' wage choice. Section 4 studies the steady-state and dynamics of the model under asymmetric unionism, ands shows the parametric conditions of the existence, local stability and bifurcation of the Cournot-Nash equilibrium. Section 5 concludes.

2. The Cournot duopoly model with heterogeneous firms

We consider a duopolistic Cournot market for a single homogenous product with inverse demand determined by:

$$p = a - Q, \tag{1}$$

where $a > 0$ is an index that captures the market size, p denotes the price and Q is the sum of outputs q_1 and q_2 produced by firm 1 and firm 2, respectively.

We assume that firm $i = \{1,2\}$ produces goods and services of variety i through the following production function with constant (marginal) returns to labour: $q_i = L_i$, where L_i represents the labour force employed by firm i. The ith firm faces (constant) average and marginal cost $w_i \geq 0$ for every unit of output produced, where w_i is the wage per unit of labour. Therefore, the firm i's cost function is linear and described by:

$$C_i(q_i) = w_i L_i = w_i q_i. \tag{2}$$

[4] This is shown in an unpublished extended version of the present paper.

Let $q_{i,t}$ be the firm i's quantity produced at time $t = 0,1,2,....$ Then, $q_{i,t+1}$ is obtained through the following optimisation programmes:

$$q_{1,t+1} = \arg\max_{q_{1,t}} \Pi_{1,t}\left(q_{1,t}, q^e{}_{2,t+1}\right), \tag{3.1}$$

$$q_{2,t+1} = \arg\max_{q_{2,t}} \Pi_{2,t}\left(q^e{}_{1,t+1}, q_{2,t}\right). \tag{3.2}$$

where $q^e{}_{i,t+1}$ represents the quantity that the rival (i.e., firm j) today (time t) expects will be produced by firm i in the future (time $t+1$) to maximise profits, $\Pi_{i,t}$.

Following a burgeoning strand of literature (see, amongst many others, Agiza and Elsadany, 2003; Zhang et al., 2007; Tramontana, 2010), in this paper we assume that firms have heterogeneous (i.e., bounded rational or naïve) expectations about the quantity to be produced in the future. Therefore, the bounded rational player 1 uses information on its current profits ($\Pi_{1,t}$) to increase or decrease the quantity produced at time $t+1$ depending on whether marginal profits (i.e., $\partial\Pi_{1,t}/\partial q_{1,t}$) are either positive or negative. Following Dixit (1986), the adjustment mechanism of quantities over time of the bounded rational player 1 is described by:

$$q_{1,t+1} = q_{1,t} + \alpha q_{1,t} \frac{\partial \Pi_{1,t}}{\partial q_{1,t}}, \tag{4}$$

where $\alpha > 0$ is a coefficient that captures the speed of adjustment of firm 1's quantity with respect to a marginal change in profit when q_1 varies at time t. In contrast, we assume that firm 2 has Cournot-naïve expectations, so that $q_{2,t+1} = q_{2,t}$.[5]

3. The union's wage setting

Following the well-established static unionised oligopoly literature mentioned above, we assume that the cost of production of the ith firm (i.e., the wage per unit of labour, w_i) is no longer exogenous while being the outcome of a strategic decision of its upstream supplier (labour union), as described below.

A decentralised union, distinctly oriented towards wages (employment), unilaterally chooses the wage. As is known, union objectives are not necessarily dominated by wages. In order to derive analytical tractable results for the wage, we assume – following, amongst many others, Pencavel (1984, 1985), Dowrick and Spencer (1994), and Petrakis and Vlassis (2000) –, that the union determines the wage by maximising the following Stone-Geary objective function:

$$V = \left(w - w^{\circ}\right)^{\theta} L, \tag{5}$$

where w_i is the union's wage, w° is the reservation or competitive wage, L is the labour employed by the firm and $\theta > 0$ (see, e.g., Pencavel, 1984; Dowrick and Spencer, 1994). A value of $\theta = 1$ gives the rent-maximising case (i.e., the union seeks to maximise the total rent); values of θ smaller (higher) than 1 imply that the union is less (more) concerned about wages and more (less) concerned about jobs (see, e.g., Mezzetti and Dinopoulos, 1991; Fanti and Gori, 2011). Moreover, the unions aims to maximise the wage bill when $w^{\circ} = 0$.

[5] As regards the Cournot-naïve case, we follow, e.g., Theocharis (1960), and specify in the simplest way the adjustment process, which depends on firms' expectations formation mechanism: i.e., firm i expects that the rival's output in period $t+1$ will be unchanged with respect to the quantity produced in period t.

Let firm i have average and marginal cost of production given by w_i, as stated in Section 2. Profits of firm i can then be written as:

$$\Pi_i = (p - w_i)q_i \,. \tag{6}$$

Using Eq. (1) to substitute out for p into Eq. (6) and maximising profit with respect to q_i, gives the following firm i's best-reply function:

$$q_i(q_j) = \frac{a - w_i - q_j}{2} \,. \tag{7}$$

This is the firm's output for any given level of wages, which are chosen by the monopolistic decentralised union.

We now turn to the union's choice. By recalling that $q_i = L_i$, the firm-specific (decentralised) union i's objective Eq. (5) can then be written as follows:

$$\max_{\{w_i\}} V_i = (w_i - w^\circ)^\theta q_i \,, \tag{5.1}$$

subject to Eq. (7), since each firm-specific labour union considers the employment (output) reaction function of its own firm. Therefore, the constrained maximisation of Eq. (5.1) with respect to w_i gives the following union i's wage as a function of the quantity produced by the firm i's rival, that is:

$$w_i(q_j) = \frac{\theta(a - q_j) + w^\circ}{1+\theta} \,. \tag{8}$$

Eq. (8) which defines the best-reply function in wages of union-firm pair i under the assumption of a non-cooperative Cournot-Nash equilibrium in the product market.

Now, substituting out Eq. (7) into Eq. (8) for w_i, we definitely obtain firm i's best-reply function as follows:

$$q_i(q_j) = \frac{a - w^\circ - q_j}{2(1+\theta)} \,. \tag{9}$$

4. Equilibrium and local stability analysis under asymmetric unionism

4.1. Case $\boxed{BU/NC}$

In this section we study the local stability properties of the Cournot-Nash equilibrium when the bounded rational firm 1 pays the non-competitive wage determined at the firm-specific union level w_1 (BU), that is $\theta > 0$, and the naïve firm 2 pays the competitive or reservation wage w° (NC), that is $\theta = 0$, where $w_1 > w^\circ$.

To proceed with the analysis further, we build on the two-dimensional system that characterises the dynamics of the BU/NC duopoly game, that is:

$$\begin{cases} q_{1,t+1} = q_{1,t} + \alpha q_{1,t} \dfrac{\partial \Pi_{1,t}}{\partial q_{1,t}} \,, \\ q_{2,t+1} = q_{2,t} \end{cases} \tag{10}$$

where marginal profits are $\dfrac{\partial \Pi_{1,t}}{\partial q_{1,t}} = \dfrac{a - 2(1+\theta)q_{1,t} - q_{2,t} - w^\circ}{1+\theta}$. Therefore, Eq. (10) can alternatively be written as follows:

$$\begin{cases} q_{1,t+1} = q_{1,t} + \dfrac{\alpha q_{1,t}}{1+\theta}\left[a - 2(1+\theta)q_{1,t} - q_{2,t} - w^{\circ}\right] \\ q_{2,t+1} = \dfrac{a - w^{\circ} - q_{1,t}}{2} \end{cases}. \tag{11}$$

Equilibrium implies that $q_{1,t+1} = q_{1,t} = q_1$ and $q_{2,t+1} = q_{2,t} = q_2$. Therefore, the dynamic system defined by (11) can be reduced in equilibrium to:

$$\begin{cases} \dfrac{\alpha q_1}{1+\theta}\left[a - 2(1+\theta)q_1 - q_2 - w^{\circ}\right] = 0 \\ \dfrac{a - w^{\circ} - q_1}{2} - q_2 = 0 \end{cases}. \tag{12}$$

The unique non-negative fixed point $E_{BU/NC} = (q^*_1, q^*_2)$ of the dynamic system (11) is determined by the following non-negative solution of Eq. (12), that is:

$$E_{BU/NC} = (q^*_1, q^*_2) = \left(\frac{a - w^{\circ}}{3+4\theta}, \frac{(1+2\theta)(a - w^{\circ})}{3+4\theta}\right), \tag{13}$$

where $q^*_2 > q^*_1$ since production costs (wages) faced by the bounded rational firms 1 are higher than those faced by the naïve firm 2.

In order to investigate the local stability properties of the Cournot-Nash equilibrium (13) of the two-dimensional system (11), we build on the Jacobian matrix J evaluated at the equilibrium point $E_{BU/NC}$, that is:

$$J_{BU/NC} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} \dfrac{3+4\theta-2\alpha(a-w^{\circ})}{3+4\theta} & \dfrac{-\alpha(a-w^{\circ})}{(1+\theta)(3+4\theta)} \\ \dfrac{-1}{2} & 0 \end{pmatrix}. \tag{14}$$

whose trace and determinant are respectively given by:

$$T := Tr(J_{BU/NC}) = J_{11} + J_{22} = \frac{3+4\theta-2\alpha(a-w^{\circ})}{3+4\theta}, \tag{15}$$

$$D := Det(J_{BU/NC}) = J_{11}J_{22} - J_{12}J_{21} = \frac{-\alpha(a-w^{\circ})}{2(1+\theta)(3+4\theta)}. \tag{16}$$

Therefore, the characteristic polynomial of (14) can be written as follows:

$$G(\lambda) = \lambda^2 - T\lambda + D, \tag{17}$$

with its discriminant being determined by $Q := T^2 - 4D$.

The local stability properties of the Cournot-Nash equilibrium $E_{BU/NC}$ are given by the well-known Jury's condition that follows (see, e.g., Gandolfo, 2010):

$$\begin{cases} (i) \quad F := 1 + T + D = \dfrac{4(1+\theta)(3+4\theta) - \alpha(5+4\theta)(a-w^{\circ})}{2(1+\theta)(3+4\theta)} > 0 \\ (ii) \quad TC := 1 - T + D = \dfrac{\alpha(a-w^{\circ})}{2(1+\theta)} > 0 \\ (iii) \quad H := 1 - D = \dfrac{8\theta^2 + 14\theta + 6 + \alpha(a-w^{\circ})}{2(1+\theta)(3+4\theta)} > 0 \end{cases}. \tag{18}$$

The violation of any single inequality in (18), with the other two being simultaneously fulfilled leads to: (i) a flip or period-doubling bifurcation (a real eigenvalue that passes through -1) when $F = 0$; (ii) a fold or transcritical bifurcation (a real eigenvalue that passes through $+1$) when $TC = 0$; (iii) a Neimark-Sacker bifurcation (i.e., the modulus of a complex eigenvalue pair that passes through 1) when $H=0$, namely $Det(J)=1$ and $|Tr(J)|<2$. While from (18) it is clear that conditions (*ii*) and (*iii*) are always fulfilled, condition (*i*) can be violated.

Now, define

$$\alpha^F{}_{BU/NC} = \frac{4(1+\theta)(3+4\theta)}{(5+4\theta)(a-w^\circ)}, \tag{19}$$

as the flip bifurcation value of α in the case BU/NC. Then, from (18) and (19) we have the following propositions.

Proposition 1. *(1) The Cournot-Nash equilibrium $E_{BU/NC}$ of the two-dimensional system (11) is locally asymptotically stable for every $0<\alpha<\alpha^F{}_{BU/NC}$. (2) If $\alpha=\alpha^F{}_{BU/NC}$ a flip or period-doubling bifurcation emerges. (3) The Cournot-Nash equilibrium $E_{BU/NC}$ is locally unstable for every $\alpha>\alpha^F{}_{BU/NC}$.*

Proof. Since $F>0$ for any $0<\alpha<\alpha^F{}_{BU/NC}$, then point (1) holds. Since $F=0$ when $\alpha=\alpha^F{}_{BU/NC}$, then point (2) holds. Since $F<0$ for any $\alpha>\alpha^F{}_{BU/NC}$, then point (3) holds. **Q.E.D.**

Proposition 2. *An increase in the relative importance of wages in the union's objective, θ, increases the flip bifurcation value $\alpha^F{}_{BU/NC}$, and then it acts as an economic stabiliser.*

Proof. Since $\dfrac{\partial \alpha^F{}_{BU/NC}}{\partial \theta} = \dfrac{4(16\theta^2+40\theta+23)}{(5+4\theta)^2(a-w^\circ)} > 0$, then Proposition 2 follows. **Q.E.D.**

4.2. Case $\boxed{BC/NU}$

In this section we assume that the bounded rational firm 1 pays the reservation wage w° (BC), that is $\theta=0$, and the naïve firm 2 pays the unionised wage determined at the firm-specific level w_2 (NU), that is $\theta>0$, where $w_2>w^\circ$.

We now build on the two-dimensional system that characterises the dynamics of the BC/NU duopoly game à la Cournot, that is:

$$\begin{cases} q_{1,t+1} = q_{1,t} + \alpha q_{1,t} \dfrac{\partial \Pi_{1,t}}{\partial q_{1,t}} \\ q_{2,t+1} = q_{2,t} \end{cases}, \tag{20}$$

where marginal profits are $\dfrac{\partial \Pi_{1,t}}{\partial q_{1,t}} = a - 2q_{1,t} - q_{2,t} - w^\circ$. Therefore, Eq. (20) can alternatively be written as follows:

$$\begin{cases} q_{1,t+1} = q_{1,t} + \alpha q_{1,t}(a - 2q_{1,t} - q_{2,t} - w^\circ) \\ q_{2,t+1} = \dfrac{a - w^\circ - q_{1,t}}{2(1+\theta)} \end{cases}. \tag{21}$$

Equilibrium implies that $q_{1,t+1} = q_{1,t} = q_1$ and $q_{2,t+1} = q_{2,t} = q_2$. Therefore, the dynamic system defined by (21) can be reduced to:

$$\begin{cases} \alpha q_1 (a - 2q_1 - q_2 - w^\circ) = 0 \\ \dfrac{a - w^\circ - q_1}{2(1+\theta)} - q_2 = 0 \end{cases} . \tag{22}$$

The unique non-negative fixed point $E_{BC/NU} = (q^*{}_1, q^*{}_2)$ of the dynamic system defined by Eq. (21) is determined by the following non-negative solution of Eq. (22), that is:

$$E_{BC/NU} = (q^*{}_1, q^*{}_2) = \left(\frac{(1+2\theta)(a - w^\circ)}{3+4\theta}, \frac{a - w^\circ}{3+4\theta} \right), \tag{23}$$

where $q^*{}_1 > q^*{}_2$ since production costs faced by the bounded rational firms 1 are lower than those faced by the naïve firm 2.

We now build on the Jacobian matrix J evaluated at the equilibrium point $E_{BC/NU}$, that is:

$$J_{BC/NU} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} \dfrac{3+4\theta - 2\alpha(a - w^\circ)(1+2\theta)}{3+4\theta} & \dfrac{-\alpha(a - w^\circ)(1+2\theta)}{3+4\theta} \\ \dfrac{-1}{2(1+\theta)} & 0 \end{pmatrix}. \tag{24}$$

whose trace and determinant are respectively given by:

$$T := Tr(J_{BC/NU}) = J_{11} + J_{22} = \frac{3+4\theta - 2\alpha(a - w^\circ)(1+2\theta)}{3+4\theta}, \tag{25}$$

$$D := Det(J_{BC/NU}) = J_{11}J_{22} - J_{12}J_{21} = \frac{-\alpha(a - w^\circ)(1+2\theta)}{2(1+\theta)(3+4\theta)}. \tag{26}$$

Therefore, the characteristic polynomial of (24) can be written as follows:

$$Z(\lambda) = \lambda^2 - T\lambda + D, \tag{27}$$

with its discriminant being determined by $P := T^2 - 4D$.

The local stability properties of the Cournot-Nash equilibrium $E_{BC/NU}$ are now given by:

$$\begin{cases} (i) \quad F := 1 + T + D = \dfrac{4(1+\theta)(3+4\theta) - \alpha(1+2\theta)(5+4\theta)(a - w^\circ)}{2(1+\theta)(3+4\theta)} > 0 \\ (ii) \quad TC := 1 - T + D = \dfrac{\alpha(a - w^\circ)(1+2\theta)}{2(1+\theta)} > 0 \\ (iii) \quad H := 1 - D = \dfrac{8\theta^2 + 2[7 + \alpha(a - w^\circ)]\theta + 6 + \alpha(a - w^\circ)}{2(1+\theta)(3+4\theta)} > 0 \end{cases} . \tag{28}$$

While from (28) it is clear that conditions (*ii*) and (*iii*) are always fulfilled, condition (*i*) can be violated.

Now, define

$$\alpha^F{}_{BC/NU} = \frac{4(1+\theta)(3+4\theta)}{(1+2\theta)(5+4\theta)(a - w^\circ)} = \frac{\alpha^F{}_{BU/NC}}{1+2\theta}, \tag{29}$$

as the flip bifurcation value of α in the case BC/NU. Then, from (28) and (29) we have the following propositions.

Proposition 3. *(1) The Cournot-Nash equilibrium* $E_{BC/NU}$ *of the two-dimensional system (11) is locally asymptotically stable for every* $0 < \alpha < \alpha^F{}_{BC/NU}$. *(2) If* $\alpha = \alpha^F{}_{BC/NU}$ *a flip or period-doubling bifurcation emerges. (3) The Cournot-Nash equilibrium* $E_{BC/NU}$ *is locally unstable for every* $\alpha > \alpha^F{}_{BC/NU}$.

Proof. Since $F > 0$ for any $0 < \alpha < \alpha^F{}_{BC/NU}$, then point (1) holds. Since $F = 0$ when $\alpha = \alpha^F{}_{BC/NU}$, then point (2) holds. Since $F < 0$ for any $\alpha > \alpha^F{}_{BC/NU}$, then point (3) holds. **Q.E.D.**

Proposition 4. *An increase in the relative importance of wages in the union's objective, θ, reduces the flip bifurcation value $\alpha^F{}_{BC/NU}$, and then it acts as an economic de-stabiliser.*

Proof. Since $\frac{\partial \alpha^F{}_{BC/NU}}{\partial \theta} = \frac{-4(7+8\theta)}{(1+2\theta)^2(5+4\theta)^2(a-w^\circ)} < 0$, then Proposition 4 follows. **Q.E.D.**

From Propositions 2 and 4 we may conclude that the process of unionisation of only one firm[6] may stabilise or de-stabilise the market equilibrium depending on whether the unionised firm has naïve or bounded rational expectations. This shed some light on the complicated dynamic events unions can contribute to explain and the importance of the type of firms' expectations under unionism.

Furthermore, a comparison of the duopoly games BU/NC and BC/NU gives the following corollary.

Corollary 1. (1) *With asymmetric unionism, the Cournot-Nash equilibrium under BU/NC is more likely to be destabilised than under BC/NU for any $\theta > 0$. (2) Moreover, the stabilising effect of θ under BU/NC is stronger than de-stabilising effect of θ under BC/NU.*

Proof. Since $\alpha^F{}_{BU/NC} = \alpha^F{}_{BC/NU}$ when $\theta = 0$, $\alpha^F{}_{BU/NC} > \alpha^F{}_{BC/NU}$ for any $\theta > 0$ and since Propositions 2 and 4 hold, then Point (1) holds. Since (*i*) $\alpha^F{}_{BU/NC}$ is a monotonic increasing function of θ and $\lim_{\theta \to +\infty} \alpha^F{}_{BU/NC} = +\infty$, and (*ii*) $\alpha^F{}_{BC/NU}$ is a monotonic decreasing function of θ and $\lim_{\theta \to +\infty} \alpha^F{}_{BC/NU} = \frac{2}{a - w^\circ}$. **Q.E.D.**

[6] This result of course also holds when the degree of unionisation of workers in one firm is higher than in the other firm. By passing, we note this result is robust to the existence of a technology with decreasing returns to scale (i.e. quadratic wage costs) (the proof is here omitted here for economy of space and is available upon request).

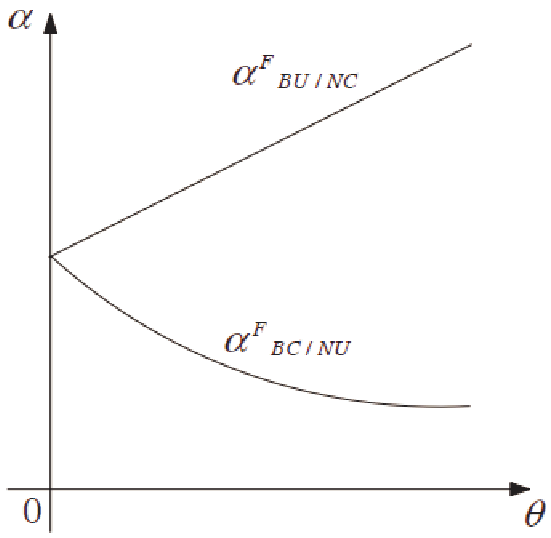

Figure 1. Stability-instability regions in (α, θ) plane.

Figure 1 clearly shows that when θ increases, the parametric stability region (i.e., the regions below both curves $\alpha^F{}_{BU/NC}$ and $\alpha^F{}_{BC/NU}$) under BU/NC increases more rapidly than the extent of decrease in it under BC/NU.

5. Conclusions

We analysed the dynamics of a Cournot duopoly where (1) firms are different in the extent of the decision mechanism they adopt in a context of bounded rationality, and (2) firm-specific unions set the wage, while acting asymmetrically between the two firms. Following the recent static unionised oligopoly literature, we assumed that each union unilaterally chooses the wage, according to an objective function that embodies both wages and employment as arguments, while taking the employment reaction function (instead of the equilibrium employment, as in the static case), of its own firm into account.

The main result of the present study concerns the role played by the degree of unionism on stability of the unique positive Cournot-Nash equilibrium: in contrast with the case in which both labour markets become uniformly unionised – where the uniform degree of unionisation always acts as an economic stabiliser – we show that when the firms' expectations formation mechanisms are heterogeneous and labour markets institutions are different (i.e., workers in one firm are unionised while being paid according to the rules of competition in the other firm), the stability effects of the degree of unionism are different depending on the interaction between the types of firms' expectations and the existence (absence) of unionism.

In particular, we showed that the higher the degree of unionisation (i.e., the higher the relative importance of wages in the union's objective) of workers within the bounded rational firm, the smaller the parametric stability region of Cournot-Nash equilibrium.

This present analysis has been by no means exhaustive as regards the dynamic role played by unions in a nonlinear Cournot duopoly, and researchers have several further challenges in front with: for instance, the analysis of a duopoly game with price or quantity competition under horizontal product differentiation where firms are homogeneous as regards the type of expectations formation mechanisms but the degree of unionism of workers within each firm is different can be the object of future research.

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