Heterogeneity in stock prices: A STAR model with multivariate transition function

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Abstract

This paper applies a heterogeneous agent asset pricing model, featuring fundamentalists and chartists, to the price-dividend and price-earnings ratios of the S&P500 index. Agents update their beliefs according to macroeconomic information, as an alternative to evolutionary dynamics. For estimation, a STAR model is introduced, with a transition function depending on multiple transition variables. A procedure based on linearity testing is proposed to select the appropriate transition variables, and simultaneously estimate their respective weights. The results show that during periods of favorable economic conditions the fraction of chartists increases, causing stock prices to decouple from fundamentals.

Keywords: Asset pricing, Heterogeneous beliefs, Smooth-transition autoregression

JEL classification: C22, E44, G12

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1 Introduction

Asset pricing models based on the efficient market hypothesis (EMH) have a difficult time explaining the observed dynamics of financial markets. According to these models, asset prices reflect a rational forecast by the market of future cash flows (dividends) generated by the asset and are therefore expected to be smoother than the actual cash flows. However, financial asset prices such as stock prices are historically more volatile than real economic activity including corporate earnings and dividends. Several studies (e.g. LeRoy and Porter, 1981; Shiller, 1981; West, 1988; Campbell and Shiller, 1988, 2001) discuss this excess volatility in financial markets and conclude that stock prices can not be explained by expected dividends alone.

Heterogeneous agent models provide an alternative to the EMH. In these models, the single representative rational agent is replaced by boundedly rational agents who are heterogeneous in beliefs, are not necessarily forecasting future dividends and may switch between trading strategies over time. Hommes (2006) and Manzan (2009) provide surveys of such models in economics and finance. I use data on the S&P500 index to estimate a heterogeneous agent model in which macroeconomic and financial variables simultaneously govern the agents' switching between strategies. This framework can infer the economic conditions under which different types of investor behavior are observed. I find that during periods of economic growth and financial stability, agents loose sight of fundamentals and become more interested in following recent trends in asset prices, which causes asset price bubbles to inflate.

The model in this paper is based on the work by Brock and Hommes (1997, 1998), who introduce a simple analytically tractable heterogeneous agent model with two types of agents: Fundamentalists and chartists. Fundamentalists believe, in accordance with the EMH, that asset prices will adjust toward their fundamental value. Chartists (or trend-followers) speculate on the persistence of deviations from the fundamental value, which generates asset price bubbles if the chartists are dominating the market.

Heterogeneous agent models are typically estimated empirically using regime-switching regression models, with the distinct regimes representing the expected asset pricing processes according to each type of agent. In particular smooth-transition regime-switching models such as the smooth-transition autoregressive (STAR) models (Teräsvirta, 1994) are suitable, as the modeled process is a time-varying weighted average of the distinct regimes. The time-varying weights of the regimes are then interpretable as the fractions of agents belonging to each type.

Recent studies have estimated asset pricing models featuring chartists and fundamentalists for several types of asset prices including exchange rates (Manzan and Westerhoff, 2007; De Jong et al., 2010), option prices (Frijns et al., 2010), oil prices (Reitz and Slopek, 2009) and other commodity prices (Reitz and Westerhoff, 2007). Boswijk et al. (2007) apply the model by Brock and Hommes (1998) to price-dividend (PD) and price-earnings (PE) ratios of the US stock market, finding that the unprecedented stock valuations observed during the 1990s are the result of a prolonged dominant position of the chartist type over the fundamentalist type. Boswijk et al. (2007) follow Brock and Hommes (1998) in assuming that agents switch between strategies based on evolutionary considerations, i.e. based on the realized profits of each type. The transition between regimes can therefore be expressed as a function depending only on lags of the dependent variable, making the model univariate.

The model in this paper is similar to that in Boswijk et al. (2007). The main difference is that the agents' choice of strategy is based on a wider set if information set including macroeconomic variables. In practice, this means I estimate a STAR model, in which the transition function depends on a linear combination of exogenous or predetermined variables. Estimating this multivariate model raises two difficulties compared to the univariate STAR:

Selection of the transition variables to include, and estimation of their weights. Only few earlier applications of STAR models exist that allow for unknown weights of the transition variables. Examples include Medeiros and Veiga (2005) and Becker and Osborn (2011). In both of those papers, however, the models are univariate as the transition function depends on a linear combination of different lagged values of the dependent variable. I propose to apply the linearity test by Luukkonen et al. (1988) to select the transition variables from a large set of information and simultaneously estimate their respective weights in the transition function.

The next section presents the heterogeneous agent model and the STAR specification in more detail. Data descriptions and linearity tests are given in section 3 while section 4 presents estimation results, interpretation and diagnostic checks. Section 5 concludes.

2 The model

In a simple present-value asset pricing model, consistent with the efficient market hypothesis, the price of a financial asset (P_t) equals the discounted sum of the expected asset price next period and any expected cash flows (dividends, D_{t+1}) paid out on the asset in the coming period (Gordon, 1959). Iterating forward, the price can be expressed as a infinite sum of discounted expected dividends:

$$P_t = E_t\left[\frac{P_{t+1}+D_{t+1}}{1+r_{t+1}}\right] = \sum_{i=1}^{\infty} E_t\left[\left(\prod_{j=1}^{i}\frac{1}{1+r_{t+j}}\right)D_{t+i}\right], \tag{1}$$

in which the discount factor is given by $(1+r_t)^{-1}$. By introducing the dividend growth rate g_t, such that $D_t = (1+g_t)D_{t-1}$, this equation can be rewritten as:

$$\frac{P_t}{D_t} = \sum_{i=1}^{\infty} E_t\left[\left(\prod_{j=1}^{i}\frac{1+g_{t+j}}{1+r_{t+j}}\right)\right]. \tag{2}$$

According to equation (2), any movements of the PD ratio ($\frac{P_t}{D_t}$) can be caused only by changed expectations on future dividend growth rates or discount factors. Under the assumption of a constant discount factor, an increase in the PD ratio should predict an increase in future dividends and vice versa. However, Campbell and Shiller (2001) argue that neither the PD nor the PE ratio are good predictors for future dividend growth rates. Instead, both valuation ratios work well as a predictor for future stock returns. High valuation ratios predict decreasing stock prices, while low ratios predict increasing prices (Campbell and Shiller, 2001).

The assumption of a constant discount factor is very restrictive. Instead, modern asset pricing models often incorporate a stochastic discount factor (SDF), representing the time-varying risk aversion of the representative agent (Cochrane, 2011). Nevertheless, Campbell and Shiller (1988) show that the finding of excess volatility is robust to several time-varying discount factors, including discount factors based on consumption, output, interest rates and return volatility.

Brock and Hommes (1998) provide an alternative to the present-value relationship (1), by allowing asset prices to depend on the expectations of H different types of boundedly rational agents:

$$P_t = \frac{1}{1+r}\sum_{h=1}^{H} G_{h,t} E_t^h \left[P_{t+1} + D_{t+1}\right], \tag{3}$$

with $E_t^h[\cdot]$ representing the beliefs of agent type h. The fraction of agents following trading strategy h at time t is denoted by $G_{h,t}$. For analytical tractability, Brock and Hommes (1998) assume a constant discount factor. This model nests the standard present-value model; when all types have rational beliefs (i.e. $E_t^h[\cdot] = E_t[\cdot]\ \forall h$), model (3) reduces to (1). Boswijk et al. (2007) show that if dividends are specified as a geometric random walk process, model (3) can be reformulated as follows:

$$y_t = \frac{1}{1+r}\sum_{h=1}^{H} G_{h,t} E_t^h \left[y_{t+1}\right], \tag{4}$$

in which y_t is defined as the PD ratio in deviation from its fundamental value. The results of Campbell and Shiller (2001) suggest to estimate mispricings in the market as the PD ratio in deviation from its long-run average:

$$y_t \ = \frac{P_t}{D_t} - \mu, \tag{5}$$

in which $\mu = \frac{1}{T}\sum\limits_{t=1}^{T}\frac{P_t}{D_t}$ represents an estimate of the fundamental value of the PD ratio. y_t gives the size of the bubble in the market, which can be negative as well as positive. The asset is over-valued if $y_t > 0$ and under-valued if $y_t < 0$. The price of the asset P_t can be decomposed in an estimated fundamental value μD_t and bubble $y_t D_t$:

$$P_t \ = \mu D_t + y_t D_t \tag{6}$$

A widely cited example of model (3) distinguishes two types of agents, fundamentalists and chartists, who are both aware of the fundamental value, but disagree about the persistence of the deviation from this fundamental value. The fundamentalists' strategy is to buy stocks when the market is undervalued and sell when the market is overvalued. They believe in mean reversion; mispricings in the market should disappear over time: $E_t^F\left[y_{t+1}\right] = \eta_F y_{t-1}$, with $\eta_F < 1+r$. Chartists (or trend-followers), on the other hand, speculate that the stock market will continue to diverge from its fundamental valuation: $E_t^C\left[y_{t+1}\right] \ = \ \eta_C y_{t-1}$, with $\eta_C \geqslant 1+r$.

By allowing the fraction of chartists and fundamentalists to change over time, the asset pricing process can be described by a smooth-transition autoregressive (STAR) process:

$$y_t = \alpha_F y_{t-1}(1 - G_t) + \alpha_C y_{t-1} G_t + \varepsilon_t, \tag{7}$$

with $\alpha_F = \eta_F/(1+r) < 1$ and $\alpha_C = \eta_C/(1+r) \geqslant 1$. The transition function G_t defines the fraction of chartist in the market. The fraction of fundamentalists is in this two-type model is given by $1 - G_t$. Although both types use a linear prediction rule, the time-varying fractions of each agent type makes the process nonlinear and, under certain parametrizations, chaotic (Brock and Hommes, 1998).

Boswijk et al. (2007) estimate a variant of this model for both the PD and PE ratio of the S&P 500 index, in deviation from their mean, for the period 1871 to 2003. They follow Brock and Hommes (1998) by letting agents update their beliefs based on the realized profits of each type in the previous period. Under these evolutionary dynamics, agents switch from the less profitable strategy to the more profitable strategy. The transition function therefore depends on lagged values of the dependent variable:

$$G_t = \left(1 + exp[-\gamma(\eta_C - \eta_F) y_{t-3}(y_{t-1} - (1+r)y_{t-2})]\right)^{-1}, \tag{8}$$

in which γ represents the intensity of choice of the agents. If $\gamma \to \infty$ all agents choose the strategy that was most profitable in the previous period. On the other hand, if $\gamma = 0$, the fraction of both types is exactly 50% in all periods, independent of the realized profits.

Instead of these evolutionary dynamics, I let the agents base their choice of strategy on macroeconomic and financial information, which can be interpreted as an extension of the agents' information set. Of interest is to find which economic conditions can be associated with each type of agent.

The transition function G_t is a logistic function, as in the logistic STAR model (Teräsvirta, 1994):

$$G_t = (1 + exp[-\gamma(x_{t-1} - c)])^{-1}. \tag{9}$$

I consider the logistic STAR only, since a logistic transition function follows directly from the logit switching rule in the model by Brock and Hommes (1998). In principle, however, the transition may also be expressed by other specifications such as an exponential function (Teräsvirta, 1994). The transition variable x_{t-1} is usually a lagged value or lagged difference of the dependent variable, but can be any predetermined or exogenous variable. The transition function may also depend on a linear combination of variables:

$$G_t = \left(1 + exp[-\gamma(X'_{t-1}\beta - c)]\right)^{-1}, \tag{10}$$

with $X_{t-1} = \left[x_{1,t-1} \dots x_{p,t-1}\right]'$ and p is the number of included transition variables. For this model; γ, c and β can not be all identified. This problem can be solved by placing a restriction on β. In this paper, the elements of β are restricted to sum to one, so that $X'_{t-1}\beta$ is a weighted average of multiple transition variables.

3 Data and linearity tests

Figure 1 shows quarterly data of the PD (left) and PE (right) ratios of the S&P500 index since 1881[1]. These valuation ratios show the level of the S&P500 index relative to the cash flows that the indexed stocks are generating. In particular the path of the PE ratio (right) seems stable or mean-reverting in the long run. Even after reaching record levels around the start of this century, the PE ratio recently dropped again below its average value during the credit crisis in 2008. This latest peak is comparable in size to earlier episodes, most notably the 1920s. For the PD ratio, this pattern is less clear. Due to relatively low dividend payouts by

[1]Source: Robert Shiller, http://www.irrationalexuberance.com/index.htm

listed firms in recent decades (Fama and French, 2001), the PD ratio climbs during the 1990s to much higher levels than during any earlier peaks in the market. Although the model in section 2 is expressed in terms of the PD ratio, I estimate the STAR model with both these valuation ratios as the dependent variable. I smooth earnings over a period of ten years, creating the so-called cyclically adjusted PE ratio. Both valuation ratios are taken in logs and in deviation from their average value.

I follow the specification, estimation and evaluation cycle for STAR models proposed by Teräsvirta (1994). The specification stage includes the selection of the appropriate lag structure and justification of STAR modeling by testing for linearity. To find the optimal lag length, I estimate linear AR(q) models including up to six lags for both the PD and PE ratio. Table 1 shows the Akaike (1974) Information Criteria (AIC) for all specifications. For both valuation ratios, the AR(1) model is selected as the appropriate specification. The STAR model is therefore estimated with an autoregressive structure of one lag, as in equation (7). At the end of this paper, I verify the sufficiency of this lag structure by submitting the residuals from the final STAR model to a test of serial independence.

The next step is to test for linearity and simultaneously select the transition variables. I consider a set of stock market indicators, business cycle indicators, interest rates and monetary aggregates as potential transition variables[2]. The stock market indicators include both dependent variables (PD and PE) and the volatility of the S&P500 index, defined as the variance of daily returns in each quarter (VOL). The business cycle indicators are real GDP (GDP), industrial production (IP), consumer sentiment (CS) and the Aruoba-Diebold-Scotti Business Conditions Index (ADS), which is a measure of business conditions in the US, based on a number of real-time macroeconomic variables (Aruoba et al., 2009). The inter-

[2] Source: FRED® (Federal Reserve Economic Data)

est rates include short- and long-term rates as well as sovereign and commercial rates: The yield on 3-month US treasury bills (INT_{3m}), on ten-year US treasury notes (INT_{10y}) and on Aaa-rated commercial bonds (INT_{Aaa}). From these interest rates I also derive the term spread ($SPR_{10y-3m} = INT_{10y} - INT_{3m}$) and the yield spread of commercial bonds over sovereign bonds ($SPR_{Aaa-US} = INT_{Aaa} - INT_{10y}$). The monetary aggregates are given by M2 and the stock of consumer credit outstanding ($M2$ and CC). For GDP, IP, $M2$ and CC I take quarter-on-quarter growth rates. For the interest rates I take both levels and first differences (denoted by $\triangle$). These data are not available for the full period of S&P500 data, so the model is estimated for 1961Q1-2009Q3. All variables are standardized (demeaned and divided by their standard deviation), to accommodate numerical estimation of the nonlinear model. All explanatory variables are lagged one period with respect to the dependent variable, so are predetermined.

To determine which of these variables are valid transition variables in the STAR model, they are submitted to a linearity test based on a Taylor approximation of the STAR model following Luukkonen et al. (1988). First, I consider the univariate transition function (9). A third-order Taylor approximation of (7) with univariate transition function (9) around $\gamma = 0$ gives:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \sum_{i=1}^{3} \phi_{1+i} y_{t-1} x_{t-1}^{i} + e_t. \tag{11}$$

Linearity can now be tested by estimating this Taylor approximation by OLS and testing the null hypothesis $H_o : \phi_2 = \phi_3 = \phi_4 = 0$. Rejection of linearity implies that x is a valid transition variable.

Results of the linearity tests are given in Table 2, which shows the test statistics and corresponding p-values. The test statistic is asymptotically $F(n, T-k-n-1)$ distributed under the null, with $T = 199$ (observations), $k = 2$ (unrestricted parameters) and $n = 3$ (restricted parameters). An asymptotically equivalent χ^2-test may be applied here as well, but the F-test has preferable properties in small samples (Teräsvirta et al., 2010). Table 2 shows that with both valuation ratios, linearity can not be rejected when the transition variable is a lag of the dependent variable. This result does not imply that *PD* and *PE* are not regime-switching processes, but rather that it is not the level of their lagged values that triggers a regime switch. In fact, the results in Table 2 show that several variables are valid transition variables, with linearity rejected at the 1% level. Teräsvirta (1994) proposes to estimate the STAR model with the transition variable for which rejection of linearity is the strongest. However, the fact that linearity is rejected for different transition variables suggests to incorporate more than one variable in the transition function.

Allowing for a multivariate transition function, I now propose a similar procedure based on linearity tests to select the appropriate transition variables $X = [x_1 \dots x_p]$ and simultaneously estimate their respective weights β. A first-order Taylor approximation of (7), with a multivariate transition function (10) around $\gamma = 0$:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-1}(X'_{t-1}\beta) + e_t, \tag{12}$$

or:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \sum_{i=1}^{p} \theta_i y_{t-1} x_{i,t-1} + e_t, \tag{13}$$

such that $\theta_i = \phi_2 \beta_i$. The restriction $\phi_2 = 0$ is identical to the restriction $\theta_1 = \dots = \theta_p = 0$. I estimate (13) for several sets of explanatory variables, test for linearity ($H_o : \theta_1 = \dots = \theta_p = 0$) and choose the set of variables that leads to the strongest rejection of the null-hypothesis. This exercise includes all possible sets of one to five variables, which never include more than one variable out of each of the following five groups: Stock market indicators, business cycle indicators, interest rates, interest rate spreads and monetary aggregates. This approach limits the number of sets under consideration and avoids multicollinearity.

The test statistic is asymptotically $F(n, T-k-n-1)$ distributed under the null, with $n = p$. This test is based on a first-order Taylor approximation rather the than third-order approximation in the univariate case. Besides computational efficiency, this first-order test has the advantage that the test results reveal the estimated weights β of the selected transition variables, as I show below. After selecting the set of transition variables, I use the OLS estimates $\widehat{\theta}$ and the initial condition $\sum_{i=1}^{p} \beta_i = 1$ to derive estimates of β:

$$\begin{aligned}
\theta_i &= \phi_2 \beta_i \\
\sum_{i=1}^{p} \beta_i &= \phi_2^{-1} \sum_{i=1}^{p} \theta_i = 1 \\
\phi_2 &= \sum_{i=1}^{p} \theta_i \\
\widetilde{\beta}_j &= \left(\sum_{i=1}^{p} \widehat{\theta}_i \right)^{-1} \widehat{\theta}_j.
\end{aligned} \tag{14}$$

With $y = PD$, the set of transition variables found by this procedure consists of three variables: IP, $\triangle INT_{10y}$ and SPR_{10y-3m}. In the test based on equation (13) these variables result in an F-statistic of 6.58 with p-value 2.97×10^{-4}. The vector of weights of the transition variables, derived with equation (14), is $\widetilde{\beta} = (0.58, 0.72, -0.30)'$. As a robustness check, I substitute $x_t = X_t' \widetilde{\beta}$ into the third order Taylor approximation (11) and run the original linearity test, which gives an F-statistic of 9.59 with p-value 6.22×10^{-6}; a stronger rejection of linearity than for any variable in Table 2.

With $y = PE$, the set of transition variables consists of IP and SPR_{Aaa-US}. These variables give an F-statistic of 9.68 with p-value 9.90×10^{-5}. The vector of weights is $\widetilde{\beta} = (-1.43, 2.43)'$. Substituting $x_t = X_t' \widetilde{\beta}$ into (11) gives an F-statistic of 7.56 with p-value 8.31×10^{-5}.

In the next section, I use these transition variables and estimates of β to estimate the STAR model. The section starts by estimating the model with a univariate transition function, using the results in Table 2.

4 Results

Since the test results in Table 2 are very close for some potential transition variables, I estimate the STAR model (7) with univariate transition function (9) with two different transition variables for each dependent variable: With $y = PD$ the two variables giving the strongest rejection of linearity are ADS and IP, while with $y = PE$ these are ADS and SPR_{Aaa-US}.

Table 3 shows the parameter estimates for all four STAR models, estimated by nonlinear least squares. Starting values for the parameters are found with a two-dimensional grid search for γ and c. The autoregressive parameters of each regime are denoted by α_1 and α_2, rather than α_C and α_F, because the latter notation implies restrictions on these parameters that I do not impose during estimation. Table 3 also shows the R^2 and AIC for both the STAR model and for an alternative linear model $y_t = \omega_1 y_{t-1} + \omega_2 x_{t-1}$, which includes the same explanatory variable. According to the AIC, all four nonlinear models fit the data better than their linear alternatives. They also provide a fit better than the univariate AR(q) models in Table 1.

Although the models outperform their linear alternatives, these results are not entirely consistent with the spirit of the heterogeneous agent model by Brock and Hommes (1998). None of the four estimated specifications has an autoregressive parameter significantly larger than one. If both autoregressive parameters are smaller than one, it seems incorrect to speak of a model with chartists and fundamentalists. Instead, there are two different types of fundamentalists. They disagree about the pace of adjustment, but agree that the price must converge towards its fundamental value. Moreover, the intensity of choice parameter γ is so high that the fraction of of each type is either zero or one. The entire population of agents makes the same switch simultaneously, which is inconsistent with the concept of heterogeneous beliefs.

To solve these issues, I proceed by estimating the model with a multivariate transition function, as in equation (10).

Using the set of transition variables found in section section 2, I estimate γ, c and α with NLS in the same manner as for the univariate case, while keeping $\widetilde{\beta}$ fixed at the levels derived with equation (14). As a final stage, I estimate all parameters once more, with the previous estimates as starting values, fixing none of the parameters (except $\sum_{i=1}^{p} \beta_i = 1$). I find in practice that the final parameter estimates stay close to the starting values, suggesting that the Taylor approximation (13) produces reasonable estimates of β. Table 4 presents all parameter estimates and goodness-of-fit measures. The R^2 and AIC show that both these models outperform their linear alternatives as well as the STAR models with univariate transition functions in Table 3.

In both estimated models, two distinct regimes are identified. Each specification has one autoregressive parameter significantly smaller than one (representing the fundamentalist type), while the other autoregressive parameter is significantly greater than one (representing the chartist type).

For both models, all elements of β are significant at the 1% level. Interpreting β reveals that chartists become more dominant during periods of high (expected) growth, while the fraction of fundamentalists increases during (expected) economic downturns. For $y = PD$ Industrial production growth (IP) has a positive coefficient, implying in this case it supports the chartist type: An increase in industrial production causes an increase in the fraction of chartists in the economy.

The differenced yield on ten-year treasury notes ($\triangle INT_{10y}$) also has a positive coefficient, while the coefficient for term spread (SPR_{10y-3m}) is negative. Decreasing rates on treasury notes and high term spreads signal an increased preference of relatively safe assets to riskier assets. The role of interest rates and spreads in predicting economic downturns is discussed by several authors including Bernanke (1990); Estrella and Mishkin (1998); Estrella (2005). These signals increase the fraction of fundamentalists.

With $y = PE$, the model does not include the exact same set of transition variables, but the results tell a similar story: Chartists are dominant with high industrial production growth and low spreads of Aaa-rated commercial bonds over treasury notes. Also these yield spreads signal financial stress, as widening spreads are a sign of investors preferring treasury notes to riskier commercial bonds.

An intuitive interpretation of the results is found by giving (7) the alternative formulation of an AR(1) process with a time-varying parameter:

$$y_t = \delta_t y_{t-1} + \varepsilon_t, \tag{15}$$

in which $\delta_t = \alpha_1(1-G_t) + \alpha_2 G_t$, which can be interpreted as an indicator of market sentiment. When $\delta_t \geqslant 1$ the valuation ratio is diverging from its mean, implying that the chartist regime is dominant, while the valuation ratio is adjusting towards its mean when $\delta_t < 1$. Figure 2 offers a graphical evaluation of both estimated models by showing plots of δ_t over time as well as scatter plots of G_t against $X'_{t-1}\beta$. Because of the relatively low value of the intensity of choice parameter γ, both scatter plots on the right side of Figure 2 clearly show a logistic curve. At almost every point in time, both chartists and fundamentalists are represented in the economy: A considerable proportion of the population speculates on further deviation from the fundamental value, with δ_t close to one. On several occasions, however, the market turns almost completely to the fundamentalist type, which causes bubbles to deflate. The end of the "dot-com" era in 2001 and the credit-crisis in 2008 are recent examples of such episodes.

Figure 2 shows furthermore that when the model is applied to the PD ratio, the distribution of agents is more skewed towards the chartist regime. An explanation for this result is given in Figure 1. The bubble during the 1990s is much larger in terms of the PD ratio than in terms of the PE ratio, as a result of disappearing dividends (Fama and French, 2001).

Finally, the estimated models in Table 4 are evaluated with diagnostic checks. Table 5 presents results on tests of serial independence, parameter constancy and no remaining non-linearity. Eitrheim and Teräsvirta (1996) provide technical details on all three tests.

The test of serial independence test the null hypothesis of no q^{th} order autocorrelation in the residuals. This Lagrange Multiplier test is a generalization of the test for serial independence in linear models by Godfrey (1988). For a q^{th} order test, the resulting test statistic is asymptotically $F(q, T-q-4)$ distributed under the null, with $T = 199$ (sample size). I execute this test for first- up to fourth-order autocorrelation. For both models, the test results give no reason the reject the null hypothesis. This finding confirms the sufficiency of an autoregressive structure of only one lag.

Under the null hypothesis of no time-variation of the parameters in (7) and (10), the parameter constancy test statistic is asymptotically $F(6, T-10)$ distributed. Also this test gives no reason to reject the specification.

The test of no remaining nonlinearity checks whether any variable has a significant non-linear effect on the residuals. This could be the case when a transition variable is omitted, or when these variables have an effect on y_t through some other nonlinear channel. The test statistic is asymptotically $F(3, T-6)$ distributed under the null. I repeat this test for all potential transition variables considered in this paper. Table 5 shows the test statistics for the same variables as listed in Table 2. For the majority of the variables, the null hypothesis of no remaining non-linearity can not be rejected. When the model is estimated with the PE ratio, the null is rejected for VOL and $M2$, but including these variables in the transition function does not improve the fit of the model. Given that the test is repeated for many variables, it is possible that the two rejections are simply Type I errors. Overall, the results of these diagnostic checks are positive and provide support to the specification of the model.

5 Conclusion

In this paper, I identify two types of agents: fundamentalists and chartists. The presence of chartists, who are predicting trends rather than fundamentals, explains the existence of bubbles in asset prices. To estimate the effects of macroeconomic conditions on the behavior of agents, I propose a STAR model with a multivariate transition function. This STAR model outperforms STAR models with a single transition variable as well as linear alternatives in terms of goodness-of-fit.

Agents are more willing to believe in the persistence of bubbles during times of positive macroeconomic news. Chartists gain influence during periods of favorable economic conditions, measured by industrial production growth and tranquility on the bond market. The fraction of fundamentalists increases during economic downturns and expectations thereof (signalled by a flight to safe assets), which encourage agents to focus on fundamentals.

I apply the model to US stock prices in deviation from an estimated fundamental value based on dividends or earnings. This framework is, however, suitable to find the conditions under which any asset price deviates from some measure of fundamental value. Other applications may include the deviation of exchange rates from purchasing power parity (see e.g. Rogoff, 1996), or the term structure of interest rates in deviation from the expectations hypothesis (see e.g. Mankiw and Miron, 1986).

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Tables and charts

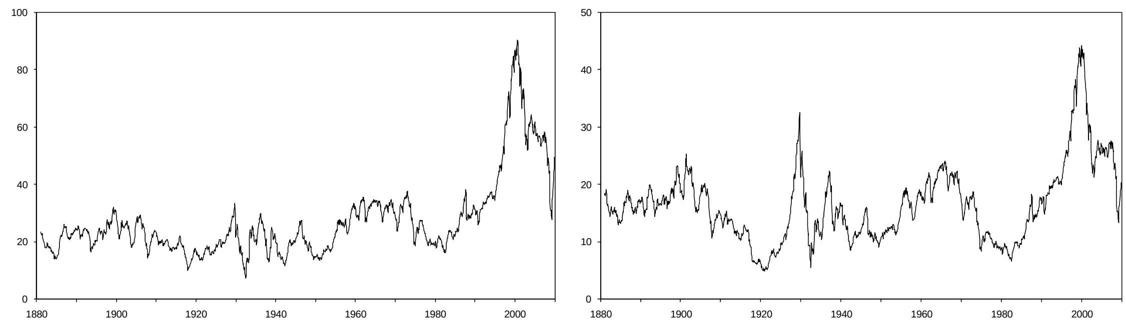

Figure 1: S&P 500 index 1881-2009: price-dividend ratio (left) and price-earnings ratio (right).

TABLE 1. AIC: Univariate linear autoregressions

lags	1	2	3	4	5	6
$y_t = PD$	-634.6	-633.4	-629.2	-624.9	-618.7	-613.4
$y_t = PE$	-555.8	-555.0	-550.3	-546.1	-540.5	-535.2

Notes: Akaike Information Criteria for AR(q) models. Sample size (for $y_t = PD$ and $y_t = PE$) is 199 observations: 1960Q2-2009Q4.

TABLE 2. Linearity tests

	$y_t = PD$		$y_t = PE$	
x_{t-1}	F-statistic	p-value	F-statistic	p-value
PD	0.364	0.779	2.090	0.103
PE	1.399	0.244	1.289	0.279
VOL	0.510	0.676	2.341	0.075
GDP	2.367	0.072	1.020	0.385
IP	4.174	0.007	4.038	0.008
CS	1.746	0.159	0.795	0.498
ADS	3.922	0.010	4.239	0.006
INT_{3m}	1.004	0.392	1.011	0.389
$\triangle INT_{3m}$	0.295	0.829	0.141	0.935
INT_{10y}	0.233	0.873	0.388	0.762
$\triangle INT_{10y}$	0.721	0.540	1.690	0.170
INT_{Aaa}	0.242	0.867	0.786	0.503
$\triangle INT_{Aaa}$	0.224	0.880	1.604	0.190
SPR_{10y-3m}	1.958	0.122	3.619	0.014
SPR_{Aaa-US}	2.172	0.093	4.283	0.006
$M2$	0.678	0.567	0.454	0.715
CC	1.564	0.200	0.993	0.397

Notes: Test statistics and corresponding p-values for $H_o: \phi_2 = \phi_3 = \phi_4 = 0$ in equation (11).

TABLE 3. Parameter estimates: univariate transition function

y_t	x_{t-1}	α_1	α_2	γ	c	R^2	AIC
PD	IP	0.814	1.002	70.78	-1.226	0.964	-645.8
		(0.012)	(0.004)	(30.49)	(0.012)	{0.961}	{-631.5}
PD	ADS	0.829	1.000	15.15	-1.376	0.963	-641.4
		(0.0145)	(0.004)	(5.086)	(0.048)	{0.961}	{-631.0}
PE	ADS	0.819	0.997	404.9	-1.278	0.945	-563.3
		(0.009)	(0.004)	(337.2)	(3.734)	{0.941}	{-551.9}
PE	SPR_{Aaa-US}	0.994	0.750	25.94	0.807	0.946	-566.2
		(0.004)	(0.013)	(8.487)	(0.0157)	{0.942}	{-554.4}

Notes: NLS parameter estimates for model (7) with univariate transition function (9). Standard errors in parenthesis. Sum of Squared Residuals and Akaike Information Criterion are reported for the STAR model and the linear model $y_t = \omega_1 y_{t-1} + \omega_2 x_{t-1} + \varepsilon_t$. The latter are reported in accolades. All estimated models include a constant, which are not significantly different from zero and are therefore not reported.

TABLE 4. Parameter estimates: multivariate transition function

y_t	X_{t-1}	α_1	α_2	γ	c	β_1	β_2	β_3	R^2	AIC
PD	$(IP, \triangle INT_{10y}, SPR_{10y-3m})'$	0.722	1.034	1.781	-1.496	1.04	0.629	-0.666	0.965	-649.1
		(0.024)	(0.009)	(0.249)	(0.238)	(0.084)	(0.049)	(0.123)	{0.961}	{-630.8}
PE	$(IP, SPR_{Aaa-US})'$	1.08	0.789	0.532	1.024	-1.742	2.742	.	0.947	-568.5
		(0.016)	(0.016)	(0.154)	(0.378)	(0.615)	(0.615)	.	{0.942}	{-552.6}

Notes: NLS parameter estimates for model (7) with multivariate transition function (10). Standard errors in parenthesis. Sum of Squared Residuals and Akaike Information Criterion are reported for the STAR model and the linear model $y_t = \omega_1 y_{t-1} + \omega_2' X_{t-1} + \varepsilon_t$. The latter are reported in accolades. All estimated models include a constant, which are not significantly different from zero and are therefore not reported.

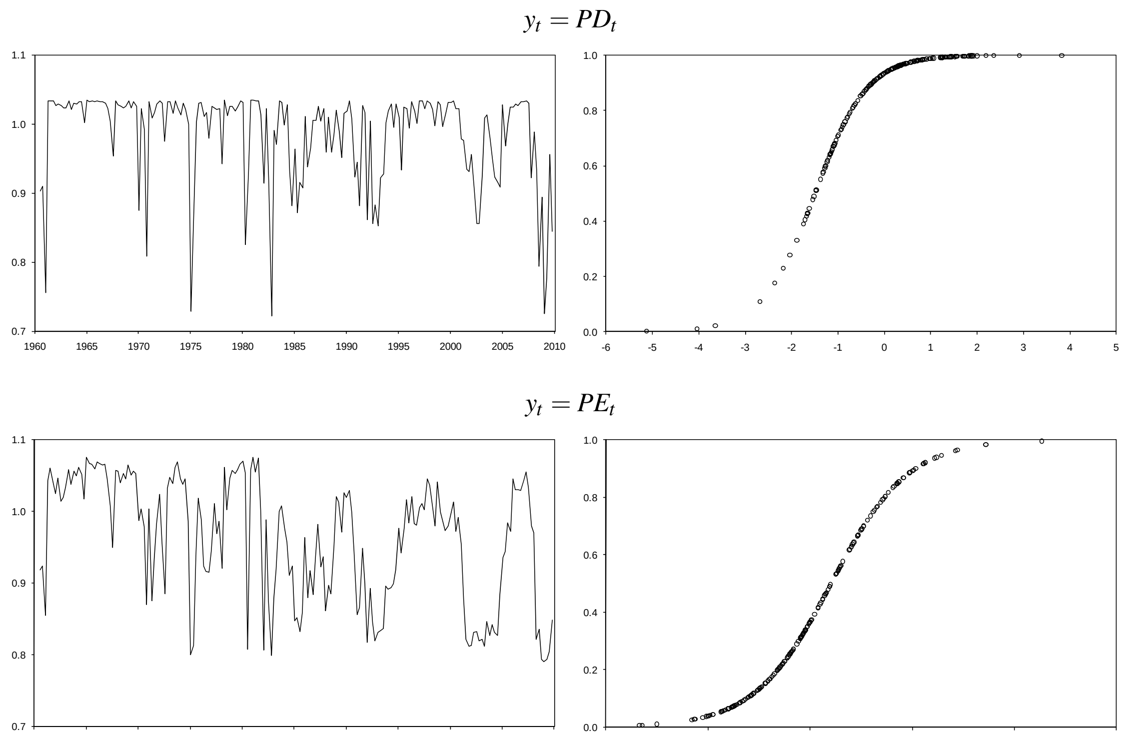

Figure 2 :Regression results: Plot (left) of $\delta_t = \alpha_1(1-G_t) + \alpha_2 G_t$ over time and scatterplot (right) of G_t against $X_{t-1}'\beta$, evaluated at parameter estimates in Table 4.

TABLE 5. Diagnostic tests

		$y_t = PD$		$y_t = PE$	
		F-statistic	p-value	F-statistic	p-value
Serial independence:	1^{st}	0.563	0.454	1.154	0.284
	2^{nd}	0.620	0.539	0.738	0.480
	3^{rd}	0.416	0.742	0.521	0.669
	4^{th}	0.365	0.833	0.398	0.810
Parameter constancy:		0.574	0.751	0.564	0.759
No remaining nonlinearity:	PD	0.893	0.446	2.051	0.108
	PE	0.229	0.876	0.657	0.580
	VOL	0.190	0.903	2.639	0.051
	GDP	1.444	0.231	1.262	0.289
	ADS	0.567	0.638	1.366	0.255
	IP	0.333	0.801	0.718	0.542
	CS	0.121	0.948	0.637	0.592
	INT_{3m}	0.108	0.955	0.551	0.648
	$\triangle INT_{3m}$	2.040	0.110	1.170	0.322
	INT_{10y}	0.511	0.675	0.242	0.867
	$\triangle INT_{10y}$	1.341	0.262	1.664	0.176
	INT_{Aaa}	0.823	0.483	0.285	0.836
	$\triangle INT_{Aaa}$	1.249	0.293	2.335	0.075
	SPR_{10y-3m}	0.468	0.705	0.440	0.724
	SPR_{Aaa-US}	0.632	0.595	0.189	0.904
	$M2$	0.989	0.399	3.162	0.026
	CC	0.133	0.940	0.392	0.759

Notes: Test statistics and corresponding p-values for first- to fourth-order serial independence, parameter constancy and no remaining non-linearity (Eitrheim and Teräsvirta, 1996)