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Short communication

Inaccurate Approximation in the Modelling of Hyperinflations

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Abstract. In time series macroeconometric models, the first difference in the logarithm of a variable is routinely used to represent the rate of change of that variable. It is often overlooked that the assumed approximation is accurate only if the rates of change are small. Models of hyper-inflation are a case in point, since in these models, by definition, changes in price are large. In this letter, Cagan's model is applied to Hungarian hyper-inflation data. It is then demonstrated that use of the approximation in the formation of the price inflation variable is causing an upward bias in the model's key parameter, and therefore an exaggeration of the effect postulated by Cagan.

Key words: hyperinflation, model specification, difference in logarithms.

1. Introduction

Models of hyper-inflation are considered very useful by macroeconomists because during periods of very high inflation, the effects of expected inflation on key variables such as money demand tend to drown out all other influences, allowing the econometrician to focus exclusively on the effect of inflationary expectations, thereby estimating this effect with maximal precision (see, for example, Sargent and Wallace, 1973).

When the rate of price inflation appears as a variable in a macroeconometric model, the variable routinely used to represent it in estimation is:

$$\Delta p_t = \Delta \ln(P_t) = \ln(P_t) - \ln(P_{t-1}), \quad (1)$$

that is, the first difference in the natural logarithm of the price level P_t.

This routine is usually adopted in models of hyper-inflation (e.g. Sargent and Wallace, 1973; Salemi, 1979; Taylor, 1991). In this letter, it is argued

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that to measure inflation in this way is illogical, since it is only a valid measure when it takes values close to zero, and in hyper-inflations, by definition, changes in price are high.

2. Measuring Inflation

Let i_t be the actual rate of inflation in period t, that is:

$$i_t = \frac{P_t - P_{t-1}}{P_{t-1}}. \tag{2}$$

Δp_t defined in (1) provides an accurate approximation to i_t when i_t takes values close to zero, because:

$$\Delta p_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(1 + i_t) = i_t - \frac{i_t^2}{2} + O\left(i_t^3\right). \tag{3}$$

It is also obvious from (3) that whenever i_t is not close to zero, the approximation will not be accurate, and we would normally expect it to be biased downwards. As i_t approaches unity, the approximation breaks down completely. The use of Δp_t in this situation may be loosely perceived as a case of measurement error.

The obvious solution to these problems is to use i_t itself as the inflation variable.

3. Models of Hyper-inflation

Cagan's (1956) theory of hyper-inflation postulates that the demand for real cash balances (M/P) is inversely related to the expected rate of inflation (McCallum, 1989: 136). We use the natural logarithm of real cash balances as the dependent variable. This is:

$$\log\left(\frac{M_t}{P_t}\right) = \log(M_t) - \log(P_t) = m_t - p_t \equiv (m-p)_t\,. \tag{4}$$

Cagan's model is therefore:

$$(m-p)_t = \alpha + \beta(E_t i_{t+1}) + u_t. \tag{5}$$

Notice that the log of real money balances in the current month is assumed to depend on the expectation, formed in the current month, of the inflation rate in the following month.

According to Cagan's theory, $\beta < 0$ in (5).

3.1. ADAPTIVE EXPECTATIONS (AE)

If expectations are formed adaptively, then:

$$E_t\,(i_{t+1}) = E_{t-1}\,(i_t) + \gamma\,[i_t - E_{t-1}\,(i_t)] \tag{6}$$

Combining (5) and (6), we construct the estimable model:

$$(m-p)_t = \alpha\gamma + \beta\gamma\, i_t + (1-\gamma)\,(m-p)_{t-1} + v_t \tag{7}$$

(7) may be estimated by OLS, and the three parameters deduced from the OLS estimates. Standard errors may be obtained using the delta method.

3.2. RATIONAL EXPECTATIONS (RE)

The reasons usually cited for the *a priori* rejection of the Adaptive Expectations model in favour of Rational Expectations (e.g. Attfield et al., 1991: Ch. 1) are especially relevant to the modelling of hyper-inflations (see Salemi, 1979). This is because, during hyper-inflations, agents are likely to show greater interest in inflation, and to increase their efforts in the formation of expectations thereof. Also, the penalties from the persistent forecasting errors necessarily made under Adaptive Expectations are likely to be more severe during hyper-inflations.

For these reasons, we turn our attention to the assumption of rational expectations. That is, we assume that actual inflation is expected inflation plus a random, mean zero, unserially correlated "forecast error":

$$i_{t+1} = E_t i_{t+1} + \varepsilon_{t+1} \tag{8}$$

Combining (8) with (5), we then obtain:

$$(m-p)_t = \alpha + \beta\;(i_{t+1} - \varepsilon_{t+1}) + u_t. \tag{9}$$

Due to the presence of the forecast error term, $-\varepsilon_{t+1}$, in (9), least squares regression of the dependent variable on i_{t+1} would yield inconsistent estimates of the model's parameters. Instrumental variables (IV) estimation can be used to obtain consistent estimates, with $(m-p)_{t-1}$ and i_t being suitable candidates for instruments.

4. Empirical Example

The dependent variable in Cagan's model is the logarithm of real cash balances, defined as notes in circulation plus demand deposits, divided by the price index. Both this variable, and the inflation rate, for Hungary, are available on a monthly basis between July 1921 and March 1925. One source of this data set is Maddala (1988: table 10.1).

Figure 1 shows the time paths of the two measures of monthly inflation, i_t and Δp_t. This graph confirms (cf. (3)) that the latter is a serious under-representation of the former at times of high inflation. Figure 2 shows the log of real cash balances $(m - p)_t$. Some evidence in favour of Cagan's hypothesis is seen in a comparison of Figures 1 and 2, because the peaks in the former appear to correspond closely to the troughs in the latter.

Both the Adaptive Expectations model (7) and the Rational Expectations model (9) are estimated using the Hungarian data just described. Each model is estimated twice, once using the log-difference of price, and once using actual inflation. Results are presented in Table I.

Note that the estimate of β is negative and significant in every case, providing strong support for Cagan's hypothesis. However, note that under both AE and RE, the estimate of β is smaller in magnitude when i_t is

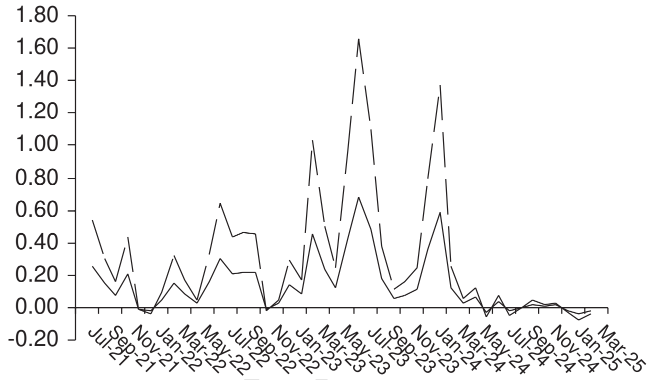

Figure 1. The two measures of monthly Inflation over time (Hungary: July 1921–March 1925), —,Log-difference; ---,Actual rate.

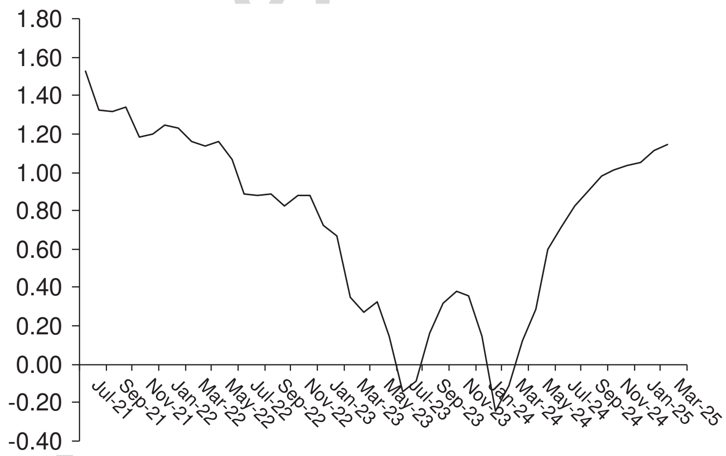

Figure 2. Log of real money balances (Hungary: July 1921–March 1925).

Table I. Results of models (7) and (9) for Hungarian data (monthly, July 1921–March 1925), using two different measures of inflation

	AE (with Δp_t)	AE (with i_t)	RE (with Δp_t)	RE (with i_t)
α	0.22(0.11)	0.21(0.10)	1.12(0.15)	1.11(0.15)
β	−4.62(0.63)	−3.42(0.46)	−2.91(0.85)	−2.41(0.71)
γ	0.17(0.02)	0.17(0.02)		
Sample size	42	42	42	42

Standard errors in parentheses. Standard errors for each parameter in the adaptive expectations model are obtained using the delta method. Instruments used in the RE models are lagged real money balances and current inflation.

used in place of Δp_t. This implies, as expected, that the use of the latter is exaggerating the negative relationship between expected inflation and real money balances.

Further note that β is estimated with greater precision when i_t is used. A consequence of this is that statistical evidence in favour of Cagan's theory is still strong (in one case stronger), despite the smaller coefficients.

5. Conclusion

The main point made in this letter is that the convention of taking the first difference of the logarithm to represent a rate of change should not be adopted automatically; it should first be checked that the rates of change appearing in the data are sufficiently small for the approximation to be accurate. It has been demonstrated that use of the approximation in the estimation of a hyper-inflation model leads to an exaggeration of the key parameter. In the illustration, we actually found strong support for Cagan's hypothesis whichever measure of inflation has been used, so it may appear that the choice between the two measures is unimportant. However, it is likely that other studies exist for which erroneous use of the approximation has led to misleading conclusions. In any case, the focus of the letter is on the logical rather than empirical problems associated with using the approximation.

When the first difference in the logarithm is used, the econometric problem which results may be seen as a manifestation of measurement error, although, as made clear by (3) the sign of the measurement error will usually be negative, with value highly correlated with the actual inflation. This means that, if this problem were to be analysed in a measurement error context, the distributional assumptions usually made in such a framework would need to be relaxed somewhat.

The data used for illustrative purposes in this letter is from Hungary in the early 1920s. This is just one of many hyper-inflations that occurred during the 20th Century and have been subjected to empirical analysis. A comprehensive collation of these hyper-inflations has been provided by Blanchard (2003: table 23-1). Perhaps the most famous was the German hyper-inflation of 1921–1924. A feature of that data set is that for part of the sample inflation reached astronomically high values: prices rose by a factor of 295 in Germany in the month of October 1923. A consequence of this is that the German inflation data has a strong positive skew. In this situation it is very tempting to use the first difference of the logarithm as an approximation, since this brings extreme values down to a reasonable size. However, (3) should remind us that this is exactly the situation in which the approximation is most misleading, and should be avoided.

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