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Berliant, Marcus and Yu, Chia-Ming

Washington University in St. Louis, National Tsing Hua University

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Marcus Berliant† and Chia-Ming Yu‡

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Abstract: Canonical analysis of the classical general equilibrium model demonstrates the existence of an open and dense subset of standard economies that possess fully-revealing rational expectations equilibria. This paper shows that the analogous result is not true in urban economies under reasonable modifications for this field. An open subset of economies where none of the modified rational expectations equilibria fully reveals private information is found. There are two important pieces. First, there can be information about a location known by a consumer who does not live in that location in equilibrium, and thus the equilibrium rent does not reflect this information. Second, if a consumer's utility depends only on information about their (endogenous) location of residence, perturbations of utility naturally do not incorporate information about other locations conditional on the consumer's location of residence. Existence of equilibrium is proved. Space can prevent housing prices from transmitting information from informed to uninformed households, resulting in an inefficient outcome. (*JEL Classifications*: D51; D82; R13)

Keywords: Urban Economics; General Equilibrium; Private Information; Rational Expectations

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†Department of Economics, Washington University, Campus Box 1208, 1 Brookings Drive, St. Louis, MO 63130-4899. Phone: (1-314) 935-8486, Fax: (1-314) 935-4156, e-mail: berliant@artsci.wustl.edu

‡Department of Economics, National Tsing Hua University, No. 101, Section 2, Kuang-Fu Road, Hsinchu, Taiwan 30013, R.O.C. Phone: +886-3-562-8202, Fax: +886-3-562-9805, e-mail: cmyu.wustl@gmail.com

1 Introduction

1.1 Motivation

People can never fully comprehend the quality and the circumstances of a city until they experience a significant part of their life living in that city. Information on physical amenities of a city (i.e., weather, parks, museums, crime, traffic jams) is easily acquired by both consumers and researchers, so there is institutional and academic work on the quality of life in cities.[1] However, people cannot completely ensure that they choose the right city or location within the city for their family before they start experiencing life there. For example, there could be uncertainty about the quality of schools, congestion of commuting routes contingent on resident and business location, or even major highway closures. Current occupants of the city, or people with friends living in the city, might have information that others don't have. Moreover, even though the current environment of the city can be understood, it is not surprising that the future developments of cities are not known with certainty, but might be known better by current occupants.[2]

On the one hand, information about life in a city is reflected in the demand for housing.[3] Since people are rational in understanding and using the relationship associating a specific state of nature with a specific equilibrium price, depending on what model people have in mind for how equilibrium prices are determined, the price of housing can be a signal for people in choosing a city best suited to their life style. Recall that the concept of rational expectations equilibrium requires agents to use models that are not obviously controverted by their observations of the markets. Therefore, the question of whether the price of housing can play a significant role in transmitting information from

[1] For example, Rosen (1979), Roback (1982), and Blomquist, Berger, and Hoehn (1988) develop a quality of life index for urban areas (QOLI), that measures or implicitly prices the value of local amenities in urban areas.

[2] For example, Cronon (1991) discusses the success of Chicago in surpassing other competitive cities, such as St. Louis, in the early development of the Midwest.

[3] It can also be reflected in wages, but for simplicity we focus on rent.

informed people to uninformed people not only addresses the question of the efficiency of housing markets, but is also related to the issue of the existence of rational expectations equilibrium in urban economics.

Available information is utilized by agents in a rational expectations equilibrium, especially the information conveyed by equilibrium prices. Radner (1979) shows that in a particular asset trading model, if the number of states of initial information is finite then, generically, rational expectations equilibria exist where all traders' private initial information is revealed. In contrast to Radner's model, that fixes state-dependent preferences and then focuses on the information concerning traders' conditional probabilities of various events, Allen (1981) considers a space of economies that is defined by state-dependent preferences and confirms Radner's conclusion in that context. When state space is infinite, Allen (1981) shows that the generic existence of fully-revealing rational expectations equilibria depends on the condition that the price space must have at least as high a dimension as the state space. Jordan (1982) considers the case where the state space has a higher dimension than the price space and demonstrates that generically there does not exist a fully revealing rational expectations equilibrium.

The existence of rational expectations equilibria where prices do not fully reveal the state of nature motivates the development of this paper. As shown in standard general equilibrium models in the literature, fully revealing rational expectations equilibrium demonstrates the efficiency of market prices in information transmission. The cases where the rational expectations equilibrium is not fully revealing are more interesting, for they admit a positive value of private information (that cannot be learned by observing prices) and space for discussing purchases of and strategic behavior using private information. In contrast with standard models, this paper focuses on the existence, with necessary adjustments for urban economics, of non-fully revealing rational expectations equilibrium. In contrast with Allen (1981), who proves the existence of an open and dense subset of economies that possess fully-revealing rational expectations equilibria in the standard general equilibrium model with a finite number of states, this paper shows that the analogous result under reasonable modifications does not hold in urban economies.[4] An open subset of economies is found, where all the modified rational expectations equilibria of these economies do not fully reveal private information.

Though in different settings, the common intuition behind these economies is consistent. First of all, households' bid rents reflect their ex ante valuations for housing, and the expected valuations reflect households' information (and their prior distributions) about the states. However, the equilibrium bid rent reveals only the winner's valuation, instead of being determined by all households' valuations. Therefore, in urban economics, *the equilibrium price of land reflects only the ex ante valuation and the information of the household with the highest willingness-to-pay for a location.* In contrast, the standard general equilibrium model has aggregate excess demand that is dependent on every household's demand. This generates complete information revelation in equilibrium generically, if there are enough prices. The difference between the models is due to the standard assumption in urban economics that each person can only consume housing in one place at one time.

Generically there still exists a fully revealing rational expectations equilibrium in urban economies. Once we suppose that all households have full information, generically each household's marginal rates of substitution for different states are distinct. Thus, bid rents in different states are distinct. However, the self fulfilling property of a standard rational expectations equilibrium is not appropriate in urban economies. The reason is that the winning

[4] Allen (1981, p. 1174) gives an example of rational expectations equilibrium with a spatial flavor: "If a tract of property which might contain natural resources is for sale, persons who own neighboring pieces of property know more, compared to the average trader, about the extent of the mineral wealth contained in the tract which is for sale. The prices that neighbors are willing to pay reflect their superior knowledge about the value of potential resources."

bid does not necessarily reflect this information. *For example, if a household living in one location at equilibrium has information about another location, this information might not be revealed in equilibrium rents.* In this circumstance, it is not reasonable to expect the equilibrium prices to fully reveal households' private information, even if there are many prices and few states.

Consistent with this idea, to allow households to learn states from observing equilibrium prices but avoid the standard logic, instead of using a standard rational expectations equilibrium, in an assignment model we adopt a modified rational expectations equilibrium (MREE) concept admitting that every household can augment his/her private information from observing equilibrium prices in all locations except the one that the household uses as his/her equilibrium location. For that location, they have only their private information. Under this adjustment for urban economies, the MREE is not generically fully revealing.

The other important component, that yields an open set of economies with not all information revealed in equilibrium, concerns perturbations of utility functions. The set of states affecting utility of households living in one location is assumed to be different from the set of such states in another location; in other words, we use a product structure for the state space. Thus, when we consider perturbations of utility functions, we do not allow the utility of households living in one location to depend even a little on states belonging to other locations. This is what we mean when we say perturbations are spatially local.

The model that we present covers both within-city locations and the comparison of different cities, though the latter case is the focus of this paper. This paper is organized as follows: Section 2 gives the basic notation and model. Two explicit examples give the intuition behind the non-existence of fully-revealing MREE in Section 3. For generic results, in Section 4, we find an open subset of economies with no fully-revealing MREE, provided that perturbations are spatially local. In Section 5, the existence of MREE is demonstrated. When some household is insensitive (to be defined precisely in this section), there exists a unique non-fully revealing MREE. When all households are not insensitive, there exists a fully-revealing MREE. When spatially non-local perturbations are considered, the results are the same as the ones in standard general equilibrium models, namely generic existence of fully revealing MREE. In this case, generically households are not insensitive. To extend our results beyond an assignment model where only one person is located in one location, an open-city model is introduced in Section 6. In this context, an open subset of economies with no fully revealing MREE still exists even when many people can live in the same place. In Section 7, it is shown that the introduction of financial markets into our model can restore the existence of a fully-revealing MREE, also restoring efficiency of equilibrium allocations. Whether the introduction of financial markets is reasonable is also examined. Section 8 concludes. An appendix contains proofs.

2 The Framework

Suppose there are n households indexed by $j \in N \equiv \{1, ..., n\}$ and n locations, $k \in K \equiv \{1, ..., n\}$, each endowed with a fixed land supply of $\bar{s}_k$. We consider the case where consumers obtain different utilities from living in different locations. These could represent either areas within a city or in different cities. Each location is endowed with a state variable, $\omega_k \in \Omega_k$, where Ω_k is a finite set, $k \in K$. In our model, ω_k represents state-dependent preference differences, each realized with a probability that is common knowledge. Let $\omega \equiv (\omega_k)_{k\in K}$ and $\Omega = \times_{k\in K} \Omega_k$ denote the state and state space of the economy. Beside locations, in state ω, each household j has to choose the lot size of his/her house and the consumption of composite good in k, denoted by

$s_{jk}(\omega)$, $z_{jk}(\omega)$, respectively. Since it is impossible to consume a house at the same instant in two locations: $s_{jk}(\omega) > 0$ implies $s_{jk'}(\omega) = 0, \forall k' \neq k$. Such a locational indivisibility is one of the unique characteristics of land and houses compared to other commodities. To focus on an exchange economy, standard in both rational expectations general equilibrium and urban economics models, suppose that household j earns a fixed income Y_j of composite good in all states. To placate urban economists, we shall introduce a commuting cost, but all of our arguments hold when commuting cost is set to zero and there is only a utility difference between locations.[5] Consider location 1 as a central business district (CBD) and other locations as suburbs. All job opportunities are located in the CBD. There is only commuting from location k, $k > 1$ to the CBD, where the commuting cost from location k to the CBD is denoted by T_k. It is assumed $0 = T_1 < T_2 < ... < T_n < \min{(Y_j)_{j \in N}}$ to ensure that there is no vacant location.

Each household can consume housing in only one location. Let $k_j : \Omega \rightarrow K$ denote the location where household j lives; that is, $k_j(\omega) = \arg\max_k\{s^*_{jk}(\omega)\}$, $\omega \in \Omega$. Denote household j's consumption plan in k in state ω as $\psi_{jk}(\omega) \equiv (s_{jk}(\omega), z_{jk}(\omega))$ and let $\psi_j(\omega) \equiv (\psi_{jk}(\omega))_{k \in K} \in \bigcup_{k \in K} \mathbb{R}^2_+$ denote j's consumption plan in state ω in all locations. The ex post utility function of household j living in k in state ω, given $\psi_{jk}(\omega)$, is denoted by $u_{jk}(\psi_{jk}(\omega), \omega)$, $\omega \in \Omega$, and the ex post utility of household j choosing to live in their optimal location is

$$u_j(\psi_j(\omega), \omega) \equiv \max_k \{u_{jk}(\psi_{jk}(\omega), \omega)\}_{k \in K}, \ \omega \in \Omega.$$

Let $p_k(\omega)$ denote the rent per unit of housing in location k in state ω, $k \in K$, $\omega \in \Omega$, and normalize the price of freely mobile composite consumption good to be 1. Let $P_k(\omega) \equiv [p_k(\omega) \ \ 1]$ be the price vector for housing and composite good in k in state ω where the composite good is numeraire. Each household

[5] Since the commuting cost is a numeraire cost instead of a time cost, commuting is not a form of production in our model.

j's information structure can be represented by $\mathcal{F}_j$, which is a sigma algebra of measurable subsets of Ω. The general optimization problem for household $j \in N$ with n locations, given his/her information structure $\mathcal{F}_j$, is:[6]

$$\begin{aligned}
&\max_{\psi_j(\omega)} \; Eu_j(\psi_j(\omega)|\mathcal{F}_j) \\
&\text{s.t.} \; \sum_{k\in K} P_k(\omega)\psi_{jk}(\omega) + \sum_{k\in K} \left\lceil \frac{s_{jk}(\omega)}{\sum_{k'\in K} s_{jk'}(\omega)} \right\rceil T_k \leq Y_j, \\
&\quad \psi_{jk}(\omega) \neq 0 \text{ implies that } \psi_{jk'}(\omega) = 0, \; \forall k, k' \in K, k' \neq k \\
&\quad \psi_j(\omega) \in \mathbb{R}^{2n}_+ \text{ is } \mathcal{F}_j\text{-measurable.} \qquad (1)
\end{aligned}$$

Let $P(\omega) \equiv (P_k(\omega))_{k\in K} \in \mathbb{R}^{2n}_+$ and $P_{-k}(\omega) \equiv (P_{k'}(\omega))_{k'\in K\setminus\{k\}}$ denote the prices in all locations in state ω and in all locations except k in state ω, respectively. The rents are collected and consumed by an absentee landlord L who owns all the housing and whose utility is $u_L((s_{Lk})_{k\in K}, z_L) = z_L$ in all states.[7] The landlord is absentee in that he/she has no private information about and does not care about the state (in terms of his/her utility function), but the landlord does care about rent collection. The landlord is endowed with an inelastic supply of housing in all locations.

Since there are the same number of households and locations, from Walras' Law and strict monotonicity of utility, each location is occupied by one and only one household in equilibrium. Since $P^*_k(\omega)$ is determined by bid rents of households living in k and since there is only one household living in k in equilibrium in state ω, $\omega \in \Omega$, $P^*_k(\omega)$ is determined by only the household with $k_j(\omega) = k$. In this case, it is not reasonable to adopt standard rational expectations equilibrium (REE) concept for the reasons discussed in the Introduction.

[6] The ceiling function, denoted by $\ulcorner\theta\urcorner$, is defined by the smallest integer greater than or equal to θ, i.e., $\ulcorner\theta\urcorner \equiv \min\{n \in \mathbb{Z}|\theta \leq n\}$. Notice that $\left\lceil \frac{s_{jk}(\omega)}{\sum_{k'\in K} s_{jk'}(\omega)} \right\rceil$ can be either 1 or 0, depending on whether household j lives in location k or not.

[7] The existence of an absentee landlord is a standard assumption in urban economics that makes our examples simpler. However, it is unnecessary for the existence of a MREE and will be removed in Section 4.

Therefore, a household can augment their private information by and only by using the information conveyed by equilibrium prices except the price in his/her equilibrium location.[8] Denote by P^* an equilibrium price function. Since P^*_k is determined only by the households living in k in equilibrium, and since there is only one household living in each location in equilibrium, it is reasonable in urban economics to require that households maximize their utility given the information $F_j \vee \sigma(P^*_{-k_j})$, where $\sigma(P^*_{-k_j})$ denotes the sigma algebra generated by $P^*_{-k_j}$ (i.e., the sigma algebra of the inverse image of $P^*_{-k_j}$).[9], rather than given the information $F_j \vee \sigma(P^*)$ as assumed in standard rational expectations equilibrium. Denote by $\mathcal{F} \equiv \vee_{j\in N}\mathcal{F}_j$ all households' information. Without loss of generality, $\sigma(\Omega) = \mathcal{F}$. Letting μ denote a (countably) additive probability measure defined on $(\Omega, \mathcal{F})$, then $E[u_j(\psi_j(\omega), \omega)|\mathcal{F}_j \vee \sigma(P^*_{-k_j})] \equiv \sum_{\omega\in\Omega} u_j(\psi_j(\omega), \omega)\ \mu(\omega|\mathcal{F}_j \vee \sigma(P^*_{-k_j}))$ is household j's expected utility of choosing ψ_j, based on j's private information and the information given by $P^*_{-k_j}$. The concept of MREE is formally defined as follows.

Definition 1 *A modified rational expectations equilibrium (MREE) is defined as an equivalence class of $\mathcal{F}$-measurable price functions $P^* : \Omega \to \mathbb{R}^{2n}_+$, and for each $j \in N$, an equivalence class of $\mathcal{F}_j \vee \sigma(P^*_{-k_j})$-measurable allocation functions $\psi^*_j : \Omega \to \bigcup_{k\in K} \mathbb{R}^2_+$ such that*
(i) $P^*_k(\omega) \cdot \psi^*_{jk}(\omega) \leq Y_j - T_k$ for μ-almost every $\omega \in \Omega$;
(ii) If $\psi'_j : \Omega \to \bigcup_{k\in K} \mathbb{R}^2_+$ satisfies the informational constraint that ψ'_j is $\mathcal{F}_j \vee \sigma(P^*_{-k_j})$-measurable and the budget constraint $P^*_k(\omega) \cdot \psi'_{jk}(\omega) \leq Y_j - T_k$, $\forall k \in K$, for μ-almost every $\omega \in \Omega$, then $\forall j \in N$,

$$E[u_j(\psi'_j(\omega), \omega)|\mathcal{F}_j \vee \sigma(P^*_{-k_j})] \leq E[u_j(\psi^*_j(\omega), \omega)|\mathcal{F}_j \vee \sigma(P^*_{-k_j})];$$

(iii) $\sum_{k\in K}\sum_{j\in N} z^*_{jk}(\omega) + z^*_L(\omega) + \sum_{k\in K}\sum_{j\in N} \ulcorner \frac{s^*_{jk}(\omega)}{\sum_{k'\in K} s^*_{jk'}(\omega)} \urcorner T_k = \sum_{j\in N} Y_j$,

[8] When households condition their expectations on additional market variables, the equilibrium concept is defined as a generalized rational expectations equilibrium; see Allen (1998).

[9] Following Aumann (1976), the join $F_j \vee \sigma(P^*_{-k_j})$ denotes the coarsest common refinement of F_j and $\sigma(P^*_{-k_j})$.

$\sum_{j\in N} s^*_{jk}(\omega) = \bar{s}_k$, *and for each* j, $\psi^*_{jk}(\omega) \neq 0$ *implies that* $\psi^*_{jk'}(\omega) = 0$, $\forall k' \in K, k' \neq k$ *for* μ*-almost every* $\omega \in \Omega$.

Condition (i) says that the budget constraint holds for every state that can happen with a positive probability. Condition (ii) represents maximization of expected utility subject to the budget. Condition (iii) represents material balance and restricts each consumer to own housing in one and only one location. This is the minimal perturbation of the standard general equilibrium model necessary to make it compatible with urban economics, i.e., it is the standard general equilibrium model with the standard assumption in urban economics that restricts each consumer to consume housing in one and only one location, and can learn from the prices in all but his/her equilibrium location. In what follows, we will introduce and solve for a bid rent equilibrium with uncertainty, which is equivalent to a MREE (see Lemma 1 below). This device is common in urban economics, and is used "almost everywhere."

Given a vector of households' utility levels in state ω, $u(\omega) \equiv (u_j(\omega))_{j\in N}$, bid rent $\Psi_{jk}(u_j(\omega), \omega)$ is the maximum rent per unit of housing that the household j is willing to pay for residing in k in state ω while enjoying a given utility level $u_j(\omega)$, $j \in N$, $k \in K$, $\omega \in \Omega$. For a given $u^*(\omega) \equiv (u^*_j(\omega))_{j\in N}$, denoting $\Psi^*(\omega) \equiv (\Psi^*_k(\omega))_{k\in K}$ and $\Psi^*_{-k}(\omega) \equiv (\Psi^*_{k'}(\omega))_{k'\in K\backslash\{k\}}$, where $\Psi^*_k(\omega) \equiv \Psi_k(u^*(\omega), \omega) = \max_j\{\Psi_{jk}(u^*_j(\omega), \omega)\}$, then households form expected utilities based on private information and the information revealed by Ψ^*_{-k}; however, $\Psi_{jk}(u^*_j(\omega), \omega)$ is determined by households' optimization. Given $u^* : \Omega \to \mathbb{R}^n_+$ and $\Psi^* : \Omega \to \mathbb{R}^n_+$, mappings from the state space to the utility and the bid rent space, respectively, and denoting $\Psi_{jk}(u^*_j(\omega), \omega) \equiv \max_{\psi_{jk}(\omega)}\{\frac{Y_j - T_k - z_{jk}(\omega)}{s_{jk}(\omega)} | E[u_j(\psi_j(\omega), \omega)|\mathcal{F}_j \vee \sigma(\Psi^*_{-k})] = u^*_j(\omega)\}$, a bid rent equilibrium is constituted when the given mappings u^*, Ψ^* and the corresponding $\Psi_{jk}(u^*_j(\omega), \omega)$ are consistent in that $\Psi^*_k(\omega) = \max_j\{\Psi_{jk}(u^*_j(\omega), \omega)\}$, $\forall k \in K$, $\omega \in \Omega$. When we solve the maximization problem defining $\Psi_{jk}(u^*_j(\omega), \omega)$, we obtain the optimal lot size $S_{jk}(u_j^*(\omega),\omega)$. Comparing this with $\psi_{jk}(\omega) \equiv (s_{jk}(\omega), z_{jk}(\omega))$ in a standard utility-maximization problem, here we denote $\varphi_{jk}(u_j(\omega),\omega) \equiv (S_{jk}(u_j(\omega),\omega),\ Z_{jk}(u_j(\omega),\omega))$ to be the optimal consumptions (arg max) in a bid-maximization problem. It can be checked that $S_{jk}(u_j^*(\omega),\omega) = s_{jk}(\omega)$ when $\Psi_{jk}(u_j^*(\omega),\omega) = p_k(\omega)$ and $u_j^*(\omega) = u_{jk}(\psi_{jk}(\omega))$. Furthermore, recall again that in Lemma 1, we will show that the equilibrium solutions of these two maximization problems are exactly the same. Given u^*, for notational convenience, also denote $S_{jk}^*(\omega) \equiv S_{jk}(u_j^*(\omega),\omega)$, $Z_{jk}^*(\omega) \equiv Z_{jk}(u_j^*(\omega),\omega))$, $\varphi_{jk}^*(\omega) \equiv (S_{jk}^*(\omega), Z_{jk}^*(\omega))$, and $\varphi_j^*(\omega) \equiv (\varphi_{jk}^*(\omega))_{k\in K}$.

Definition 2 *A bid rent equilibrium is defined by an equivalence class of $\mathcal{F}$-measurable house price functions $\Psi^* : \Omega \to \mathbb{R}_+^n$, and for each $j \in N$, an equivalence class of $\mathcal{F}_j \vee \sigma(\Psi_{-k_j}^*)$-measurable utility functions $u_j^* : \Omega \to \mathbb{R}_+$ such that for each location $k \in K$, for μ-almost every $\omega \in \Omega$:*

$$\Psi_k^*(\omega) \equiv \Psi_k(u^*(\omega),\omega) = \max_j\{\Psi_{jk}(u_j^*(\omega),\omega)\}; \tag{2}$$

$$\varphi_{jk}^*(\omega) \equiv \varphi_{jk}(u_j^*(\omega),\omega)$$
$$= \begin{cases} \arg\max_{\psi_{jk}(\omega)}\{\frac{Y_j - T_k - z_{jk}(\omega)}{s_{jk}(\omega)} \big| E[u_j(\psi_j(\omega),\omega)|\mathcal{F}_j \vee \sigma(\Psi_{-k_j}^*)] = u_j^*(\omega)\} \\ \qquad\qquad\qquad\qquad \textit{if } \ j \in \arg\max_j\{\Psi_{jk}(u_j^*(\omega),\omega)\}, \\ (0,0) \quad \textit{if } \ j \notin \arg\max_j\{\Psi_{jk}(u_j^*(\omega),\omega)\}; \end{cases} \tag{3}$$

$$\sum_{j\in N} S_{jk}^*(\omega) = \bar{s}_k,$$
$$\sum_{k\in K}\sum_{j\in N} Z_{jk}^*(\omega) + z_L^*(\omega) + \sum_{k\in K}\sum_{j\in N} \ulcorner \frac{S_{jk}^*(\omega)}{\sum_{k'\in K} S_{jk'}^*(\omega)} \urcorner\, T_k = \sum_{j\in N} Y_j,$$
$$\textit{and } \varphi_{jk}^*(\omega) \neq 0 \textit{ implies that } \varphi_{jk'}^*(\omega) = 0,\ \forall k' \in K, k' \neq k, \forall j \in N. \tag{4}$$

Here, condition (2) says that the equilibrium housing price in every location is determined by the highest bid rent among households for the housing there. Condition (3) says that the equilibrium consumption of the household who lives in k maximizes that household's bid rent in k, given his private information and the information revealed by equilibrium prices. Again, condition (4) represents material balance and the standard urban economics assumption that each consumer lives in one and only one location.

Since each household can consume housing in at most one location, the consumption set is $\bigcup_{k\in K}\mathbb{R}^2_+$, and the ex post state-dependent preferences of living in k, $k \in K$, can be specified by utilities $u_{jk} : \Omega_k \to \kappa_{jk}$, where κ_{jk} is a compact subset of $C^r(\mathbb{R}^2_+, \mathbb{R})$, $r \geq 2$, endowed with the weak C^r compact-open topology. Assume that for μ-almost every $\omega \in \Omega$, $u_{jk}(\varphi, \omega) \in \kappa_{jk}$ satisfies for each $\varphi \in \mathbb{R}^2_+$:

(a) strict (differentiable) monotonicity: $D_\varphi u_{jk}(\varphi, \omega) \in \mathbb{R}_{++}$,

(b) strict (differentiable) concavity: $D_{\varphi\varphi} u_{jk}(\varphi, \omega)$ is negative definite, and

(c) smooth boundary condition: the closure in $\mathbb{R}^2$ of the upper contour set $\{\varphi' \in \mathbb{R}^2_{++} | u_{jk}(\varphi', \omega) \geq u_{jk}(\varphi, \omega)\}$ is contained in $\mathbb{R}^2_{++}$.

These conditions ensure that every household's state-dependent preferences are smooth in the sense of Debreu (1972) so that, conditional on any state with a positive probability, demands are well defined C^{r-1} functions. Our examples satisfy these assumptions.

Although it is well-known that bid-rent and competitive equilibria are closely connected (see for example Fujita, 1989), results in the literature cover only the context of no uncertainty. If the MREE were known to be fully revealing, this result could be applied state by state. We require an equivalence result in the context of uncertainty, especially when the MREE might not be fully revealing. The proof uses classical duality.

Lemma 1 *Given that all households' preferences are representable by a utility function satisfying conditions (a), (b), and (c),* $(\Psi^*(\omega), u^*(\omega))$ *constitutes a bid rent equilibrium if and only if the corresponding* $(\Psi^*(\omega), (\varphi^*_j(\omega))_{j\in N})$ *constitutes a modified rational expectations equilibrium in a competitive economy.*

Proof. See Appendix A.

3 The Examples

Before stating formally and proving the results, to illustrate the non-existence of fully revealing MREE, let us examine a few examples. In the first example, one of the households is fully informed, whereas the other has no information. In the second example, both households have partial information about the states of nature in different locations. In both examples, the equilibrium prices are the same in different states, and hence illustrate an economy where the MREE do not fully reveal the private information of households. Examples similar to these appear in the literature on rational expectations in the standard general equilibrium model, i.e., Allen (1981), though in that literature they belong to the complement of a generic set, and have a very different flavor. The idea behind Example 1 comes from the examples in Kreps (1977), Jordan and Radner (1979), and Allen (1984).

3.1 Example 1

Suppose that there are two households ($j \in \{1, 2\}$) with the same income ($Y_1 = Y_2 = Y$), and two locations ($k \in \{x, y\}$) with land endowments $\bar{x}$ and $\bar{y}$, respectively. Household 1's utility is state-dependent but the utility function of household 2 is independent of states. In each location k, there are two states (Low and High) denoted by $\omega_k \in \Omega_k \equiv \{L, H\}$, $k \in \{x, y\}$, which are equally likely to occur and the states in different locations are not correlated. What each agent can observe are events that are subsets of $\Omega \equiv \Omega_x \times \Omega_y$. Denote $\omega \equiv \omega_x \times \omega_y$ as an element of Ω. Furthermore, household 1 has no information, and household 2 knows what the state will be. That is, households' information is represented respectively by $\mathcal{F}_1 = \{\phi, \Omega_x\} \times \{\phi, \Omega_y\}$, $\mathcal{F}_2 = \{\phi, \{H\}, \{L\}, \Omega_x\} \times \{\phi, \{H\}, \{L\}, \Omega_y\}$ which are sub-σ-fields of $\mathcal{F}$, where $\mathcal{F} \equiv \mathcal{F}_1 \vee \mathcal{F}_2 = \mathcal{F}_2$. Everything except the true state is common knowledge, so households are assumed to know the relationship between states and prices.

Given information structure $\mathcal{F}_1$, the state variable in household 1's allocation can be ignored for simplicity until he/she learns something. Utilities will be Cobb-Douglas. The optimization problem for household 1 is to maximize expected utility subject to the budget constraint:

$$\begin{aligned}
\max_{s_{1x}, s_{1y}, z_{1x}, z_{1y}} \quad & E u_1(s_{1x}, s_{1y}, z_{1x}, z_{1y} | \mathcal{F}_1) \\
& = \max\{E[\alpha_1^\omega \ln(s_{1x}) + \ln(z_{1x}) | \mathcal{F}_1],\ E[\beta_1^\omega \ln(s_{1y}) + \ln(z_{1y}) | \mathcal{F}_1]\} \\
\text{s.t.} \quad & p_x(\omega) s_{1x} + p_y(\omega) s_{1y} + z_{1x} + z_{1y} + \ulcorner \frac{s_{1y}}{s_{1x} + s_{1y}} \urcorner\, t \le Y, \\
& s_{1k}\, s_{1l} = 0,\ s_{1k}\, z_{1l} = 0,\ z_{1k}\, z_{1l} = 0, \\
& s_{1k}, z_{1k} \ge 0, \quad \forall k, l = x, y,\ k \ne l,
\end{aligned}$$

where $\alpha_1^\omega, \beta_1^\omega \in \mathbb{R}_{++}$. In contrast, since household 2's utility is state-independent, his/her optimization problem is, for $\omega \in \Omega$,

$$\begin{aligned}
\max_{s_{2x}(\omega), s_{2y}(\omega), z_{2x}(\omega), z_{2y}(\omega)} \quad & u_2(s_{2x}(\omega), s_{2y}(\omega), z_{2x}(\omega), z_{2y}(\omega), \omega) \\
& = \max\{\alpha_2 \ln(s_{2x}(\omega)) + \ln(z_{2x}(\omega)),\ \beta_2 \ln(s_{2y}(\omega)) + \ln(z_{2y}(\omega))\} \\
\text{s.t.} \quad & p_x(\omega) s_{2x}(\omega) + p_y(\omega) s_{2y}(\omega) + z_{2x}(\omega) + z_{2y}(\omega) \\
& + \ulcorner \frac{s_{2y}(\omega)}{s_{2x}(\omega) + s_{2y}(\omega)} \urcorner\, t \le Y, \\
& s_{2k}(\omega)\, s_{2l}(\omega) = 0,\ s_{2k}(\omega)\, z_{2l}(\omega) = 0,\ z_{2k}(\omega)\, z_{2l}(\omega) = 0, \\
& s_{2k}(\omega), z_{2k}(\omega) \ge 0, \quad \forall k, l = x, y,\ k \ne l,
\end{aligned}$$

where $\alpha_2, \beta_2 \in \mathbb{R}_{++}$. Suppose that on average household 1 likes the housing in the CBD (x) more than household 2, and household 2 prefers y more than household 1, i.e., $E[\alpha_1^\omega] > \alpha_2$ and $E[\beta_1^\omega] < \beta_2$.

Following Alonso (1964), Fujita (1989), and our Lemma 1, people live where their bid rents are maximal in equilibrium, and these bid rents constitute equilibrium rents. The bid rent functions of the two households for the housing in x and y are

$$\Psi_{1x}(Eu_1,\omega) = \max_{s_{1x}} \frac{Y - e^{Eu_1}(s_{1x})^{-E[\alpha_1^\omega]}}{s_{1x}}, \tag{5}$$

$$\Psi_{1y}(Eu_1,\omega) = \max_{s_{1y}} \frac{Y - t - e^{Eu_1}(s_{1y})^{-E[\beta_1^\omega]}}{s_{1y}}, \tag{6}$$

$$\Psi_{2x}(u_2(\omega),\omega) = \max_{s_{2x}(\omega)} \frac{Y - e^{u_2(\omega)}(s_{2x}(\omega))^{-\alpha_2}}{s_{2x}(\omega)}, \tag{7}$$

$$\Psi_{2y}(u_2(\omega),\omega) = \max_{s_{2y}(\omega)} \frac{Y - t - e^{u_2(\omega)}(s_{2y}(\omega))^{-\beta_2}}{s_{2y}(\omega)}, \tag{8}$$

where $\omega \in \Omega$. From first and second-order conditions, the optimal land lot sizes for households are

$$S_{1x}^*(Eu_1,\omega) = [\frac{e^{Eu_1}(1+E[\alpha_1^\omega])}{Y}]^{\frac{1}{E[\alpha_1^\omega]}}, \tag{9}$$

$$S_{1y}^*(Eu_1,\omega) = [\frac{e^{Eu_1}(1+E[\beta_1^\omega])}{Y-t}]^{\frac{1}{E[\beta_1^\omega]}}, \tag{10}$$

$$S_{2x}^*(u_2(\omega),\omega) = [\frac{e^{u_2(\omega)}(1+\alpha_2)}{Y}]^{\frac{1}{\alpha_2}}, \tag{11}$$

$$S_{2y}^*(u_2(\omega),\omega) = [\frac{e^{u_2(\omega)}(1+\beta_2)}{Y-t}]^{\frac{1}{\beta_2}}. \tag{12}$$

Since households 1 and 2 prefer to live at x and y, respectively, from market clearing conditions, $S_{1x}^*(\omega) = \bar{x}$ and $S_{2y}^*(\omega) = \bar{y}$, $\forall \omega \in \Omega$, we have

$$Eu_1^* = \ln[Y] + E[\alpha_1^\omega]\ln[\bar{x}] - \ln[1+E[\alpha_1^\omega]], \tag{13}$$

$$u_2^*(\omega) = \ln[Y-t] + \beta_2 \ln[\bar{y}] - \ln[1+\beta_2], \tag{14}$$

for $\omega \in \Omega$. So the equilibrium bid rents of agents in the two locations in the two states are

$$\Psi_{1x}^*(\omega) = \frac{E[\alpha_1^\omega]}{1+E[\alpha_1^\omega]}\frac{Y}{\bar{x}}, \tag{15}$$

$$\Psi_{1y}^*(\omega) = (1+E[\alpha_1^\omega])^{\frac{1}{E[\beta_1^\omega]}} E[\beta_1^\omega](\bar{x}^{E[\alpha_1^\omega]}Y)^{\frac{-1}{E[\beta_1^\omega]}} \left(\frac{Y-t}{1+E[\beta_1^\omega]}\right)^{1+\frac{1}{E[\beta_1^\omega]}}, \tag{16}$$

$$\Psi_{2x}^*(\omega) = (1+\beta_2)^{\frac{1}{\alpha_2}}\alpha_2[\bar{y}^{\beta_2}(Y-t)]^{\frac{-1}{\alpha_2}} \left(\frac{Y}{1+\alpha_2}\right)^{1+\frac{1}{\alpha_2}}, \tag{17}$$

$$\Psi_{2y}^*(\omega) = \frac{\beta_2}{1+\beta_2}\frac{Y-t}{\bar{y}}, \tag{18}$$

for $\omega \in \Omega$. The equilibrium bid rents are presented in Figure 1, where the horizontal axis represents the location whereas the vertical axis represents the individuals' bid rents.[10]

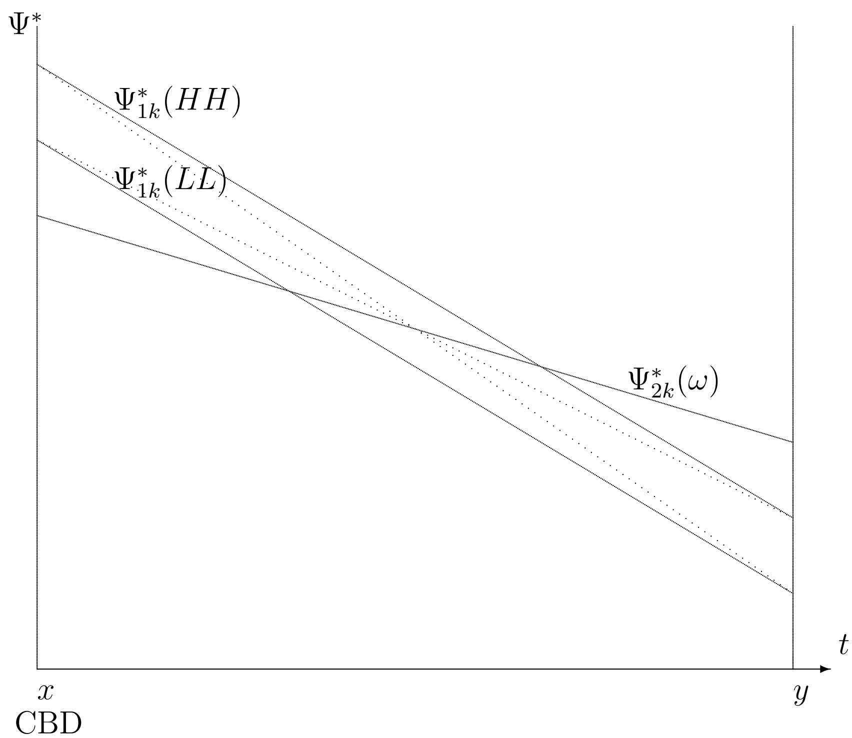

Figure 1: The bid rent functions in Example 1, where the dotted lines represent $\Psi^*_{1k}(HL)$ and $\Psi^*_{1k}(LH)$, respectively.

Choosing parameters satisfying $\Psi^*_{1x}(\omega) > \Psi^*_{2x}(\omega)$ and $\Psi^*_{2y}(\omega) > \Psi^*_{1y}(\omega)$, $\forall\omega \in \Omega$, the bid rent of household 1 for the housing in x (y) is higher (*lower*) than that of household 2 for the housing in x (y) in all states.[11] That is, the equilibrium location pattern where household 1 lives at x and household 2 lives in y is verified for some parameters.

Notice that there is no equilibrium that fully reveals information. Suppose there is a fully revealing MREE, that is, $\Psi^*_x(HH) = \Psi^*_x(HL) \neq \Psi^*_x(LH) =$

[10]If we used Definition 1 rather than Definition 2 for the examples, although they are equivalent by Lemma 1, we could not draw the figures.

[11]For example, for $Y = 20$, $t = 1$, $\alpha_2 = 1$, $E[\beta_1^\omega] = 1$, $\beta_2 = 2$, $\bar{x} = 10$, and $\bar{y} = 10$, it can be checked that $\forall E[\alpha_1^\omega] > 1$, $\Psi^*_{1x}(\omega) > \Psi^*_{2x}(\omega)$ and $\Psi^*_{2y}(\omega) > \Psi^*_{1y}(\omega)$, $\forall\omega \in \Omega$.

$\Psi_x^*(LL)$ in equilibrium. Since $\Psi_y^*(\omega)$ is a constant for all $\omega \in \Omega$, different valuations of household 1 for the housing in x in different states conflict with the assumption that household 1 has no information about the state in x. Notice also that $\Psi_x^*(\omega)$ and $\Psi_y^*(\omega)$ depend only on the mean of α_1, β_2, and the values of Y, t, $\bar{x}$, and $\bar{y}$. Therefore, the equilibrium rents in the two locations are independent of the realized state, and thus, there exists no fully-revealing MREE. Even though household 2 knows the state, since household 2 doesn't care about the state, equilibrium prices don't reveal household 2's private information.

3.2 Example 2

Follow the same setting as in the previous example, but suppose that household 1 knows the state in location y and has no information about location x. On the other hand, household 2 knows only the state in x, but not the state in y. Let $\Omega \equiv \Omega_x \times \Omega_y$, where $\Omega_x = \Omega_y \equiv \{H, L\}$ represent the state spaces in locations x and y. $\mathcal{F}_1 = \{\phi, \Omega_x\} \times \{\phi, \Omega_y, \{H\}, \{L\}\}$, $\mathcal{F}_2 = \{\phi, \Omega_x, \{H\}, \{L\}\} \times \{\phi, \Omega_y\} \subseteq \mathcal{F}$ are sub-σ-fields representing households' private information. Again, the relationship between states and prices is common knowledge.

Each household lives in one and only one location. Moreover, households make their decisions simultaneously. Given an event $\omega \in \Omega$, both households' utilities are state-dependent, so their optimization problems are

$$\begin{aligned}
\max_{s_{1x}, s_{1y}(\omega), z_{1x}, z_{1y}(\omega)} \quad & Eu_1(s_{1x}, s_{1y}(\omega), z_{1x}, z_{1y}(\omega)|\mathcal{F}_1) \\
&= \max\{E[\alpha_1^\omega \ln(s_{1x}) + \ln(z_{1x})|\mathcal{F}_1],\ \beta_1^\omega \ln(s_{1y}(\omega)) + \ln(z_{1y}(\omega))\} \\
\text{s.t.} \quad & p_x(\omega)s_{1x} + p_y(\omega)s_{1y}(\omega) + z_{1x} + z_{1y}(\omega) + \ulcorner \frac{s_{1y}(\omega)}{s_{1x}(\omega) + s_{1y}(\omega)} \urcorner t \leq Y, \\
& s_{1x}\, s_{1y}(\omega) = 0,\ s_{1x}\, z_{1y}(\omega) = 0,\ z_{1x}\, s_{1y}(\omega) = 0,\ z_{1x}\, z_{1y}(\omega) = 0, \\
& s_{1x}, s_{1y}(\omega), z_{1x}, z_{1y}(\omega) \geq 0;
\end{aligned}$$

$$\max_{s_{2x}(\omega),s_{2y},z_{2x}(\omega),z_{2y}} Eu_2(s_{2x}(\omega),s_{2y},z_{2x}(\omega),z_{2y}|\mathcal{F}_2)$$
$$= \max\{\alpha_2^\omega \ln(s_{2x}(\omega)) + \ln(z_{2x}(\omega)),\ E[\beta_2^\omega \ln(s_{2y}) + \ln(z_{2y})|\mathcal{F}_2]\}$$
$$\text{s.t.}\quad p_x(\omega)s_{2x}(\omega) + p_y(\omega)s_{2y} + z_{2x}(\omega) + z_{2y} + \ulcorner \frac{s_{2y}(\omega)}{s_{2x}(\omega)+s_{2y}(\omega)} \urcorner\, t \le Y,$$
$$s_{2x}(\omega)\, s_{2y} = 0,\ s_{2x}(\omega)\, z_{2y} = 0,\ z_{2x}(\omega)\, s_{2y} = 0,\ z_{2x}(\omega)\, z_{2y} = 0,$$
$$s_{2x}(\omega), s_{2y}, z_{2x}(\omega), z_{2y} \ge 0;$$

Note that, in fact, the optimized utility of household 1 is state-dependent at y, denoted by $u_{1y}^*(\omega)$, and state-independent at x, denoted by Eu_{1x}^*; $u_{2x}^*(\omega)$ and Eu_{2y}^* are similarly defined. To present a MREE without revelation of private information, suppose that $E[\alpha_1^\omega] > \alpha_2^\omega$ and $E[\beta_2^\omega] > \beta_1^\omega$, for all $\omega \in \Omega$.

Given these conditions, suppose that households 1 and 2 choose to live in locations x and y, respectively. Their bid rent functions are, $\forall\omega \in \Omega$,

$$\Psi_{1x}(Eu_1,\omega) = \max_{s_{1x}} \frac{Y - e^{Eu_1} s_{1x}^{-E[\alpha_1^\omega]}}{s_{1x}}, \tag{19}$$
$$\Psi_{1y}(u_1(\omega),\omega) = \max_{s_{1y}} \frac{Y - t - e^{u_1(\omega)} s_{1y}^{-\beta_1^\omega}}{s_{1y}}, \tag{20}$$
$$\Psi_{2x}(u_2(\omega),\omega) = \max_{s_{2x}} \frac{Y - e^{u_2(\omega)} s_{2x}^{-\alpha_2^\omega}}{s_{2x}}, \tag{21}$$
$$\Psi_{2y}(Eu_2,\omega) = \max_{s_{2y}} \frac{Y - t - e^{Eu_2} s_{2y}^{-E[\beta_2^\omega]}}{s_{2y}}. \tag{22}$$

Thus, the optimal lot sizes for household 1 and 2 are, $\forall\omega \in \Omega$,

$$S_{1x}^*(Eu_1,\omega) = [\frac{e^{Eu_1}(1+E[\alpha_1^\omega])}{Y}]^{\frac{1}{E[\alpha_1^\omega]}}, \tag{23}$$
$$S_{1y}^*(u_1(\omega),\omega) = [\frac{e^{u_1(\omega)}(1+\beta_1^\omega)}{Y-t}]^{\frac{1}{\beta_1^\omega}}, \tag{24}$$
$$S_{2x}^*(u_2(\omega),\omega) = [\frac{e^{u_2(\omega)}(1+\alpha_2^\omega)}{Y}]^{\frac{1}{\alpha_2^\omega}}, \tag{25}$$
$$S_{2y}^*(Eu_2,\omega) = [\frac{e^{Eu_2}(1+E[\beta_2^\omega])}{Y-t}]^{\frac{1}{E[\beta_2^\omega]}}. \tag{26}$$

From $S_{1x}^*(\omega) = \bar{x}$ and $S_{2y}^*(\omega) = \bar{y}$, $\forall\omega \in \Omega$, we have

$$Eu_1^*(\cdot|\mathcal{F}_1) = \ln[Y] + E[\alpha_1^\omega]\ln[\bar{x}] - \ln[1+E[\alpha_1^\omega]], \tag{27}$$
$$Eu_2^*(\cdot|\mathcal{F}_2) = \ln[Y-t] + E[\beta_2^\omega]\ln[\bar{y}] - \ln[1+E[\beta_2^\omega]]. \tag{28}$$

Again, households' equilibrium bid rents are

$$\Psi^*_{1x}(\omega) = \frac{E[\alpha^\omega_1]}{1+E[\alpha^\omega_1]}\frac{Y}{\bar{x}}, \tag{29}$$

$$\Psi^*_{1y}(\omega) = (1+E[\alpha^\omega_1])^{\frac{1}{\beta^\omega_1}}\beta^\omega_1(\bar{x}^{E[\alpha^\omega_1]}Y)^{\frac{-1}{\beta^\omega_1}}\left(\frac{Y-t}{1+\beta^\omega_1}\right)^{1+\frac{1}{\beta^\omega_1}}, \tag{30}$$

$$\Psi^*_{2x}(\omega) = (1+E[\beta^\omega_2])^{\frac{1}{\alpha^\omega_2}}\alpha^\omega_2[\bar{y}^{E[\beta^\omega_2]}(Y-t)]^{\frac{-1}{\alpha^\omega_2}}\left(\frac{Y}{1+\alpha^\omega_2}\right)^{1+\frac{1}{\alpha^\omega_2}}, \tag{31}$$

$$\Psi^*_{2y}(\omega) = \frac{E[\beta^\omega_2]}{1+E[\beta^\omega_2]}\frac{Y-t}{\bar{y}}, \tag{32}$$

where $\omega \in \Omega$. The equilibrium bid rents are drawn in Figure 2, where the horizontal axis represents the location, whereas the individual bid rents are represented by the vertical axis.

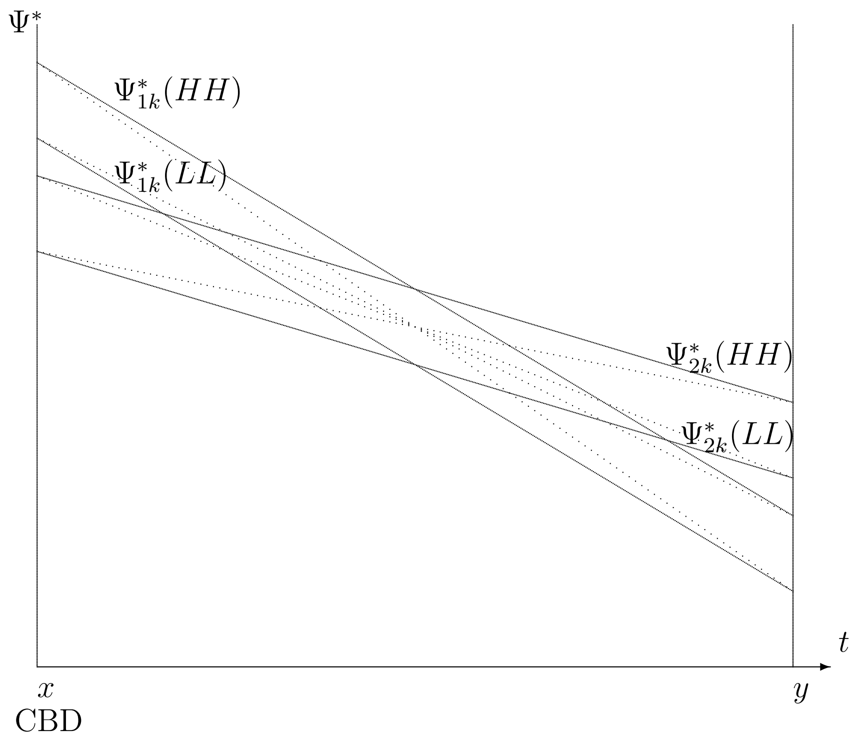

Figure 2: The bid rent functions in Example 2, where the dotted lines represent $\Psi^*_{1k}(HL)$, $\Psi^*_{1k}(LH)$, $\Psi^*_{2k}(HL)$, and $\Psi^*_{2k}(LH)$, respectively.

Given parameters satisfying $\Psi^*_{1x}(\omega) > \Psi^*_{2x}(\omega)$ and $\Psi^*_{2y}(\omega) > \Psi^*_{1y}(\omega)$, $\forall\omega \in \Omega$, the bid rent of household 1 (household 2) for the housing in x (y) is always higher than that of household 2 (household 1). So the equilibrium location pattern where household 1 lives at x and household 2 lives at y is verified.[12]

Again, there is no fully revealing equilibrium in this example. Since Ψ_x^* and Ψ_y^* depend only on Y, t, the mean of the preference parameters and the endowments of land in each location, the equilibrium bid rents are the same in all the realized states. That is, the mapping from prices to preferences is not injective, so fully-revealing MREE does not exist.[13]

These examples illustrate different causes for the equilibrium not fully revealing private information: The first example arises because the informed household doesn't care about different states. The second one arises due to the mismatch between informed households and their equilibrium locations. In the next section, we show that these unfortunate circumstances can persist under small perturbations.

4 An Open Subset of Economies without Fully Revealing Equilibria

The examples represent two points in the space of economies with no fully revealing MREE. In this section, we generalize the examples and show that, in economies under uncertainty where there is no market for contingent claims or financial contracts, fully revealing MREE is not present for an open set of economies. But for all parameters satisfying a condition, there exists a MREE (that might not be fully revealing). This will be proved in the next

[12] For example, for $Y = 20$, $t = 1$, $\alpha_2^{HH} = \alpha_2^{HL} = 1.1$, $\alpha_2^{LH} = \alpha_2^{LL} = 1$, $\beta_1^{HH} = \beta_1^{LH} = 1.1$, $\beta_1^{HL} = \beta_1^{LL} = 1$, $E[\beta_2^{\omega}] = 2$, $\bar{x} = 10$, and $\bar{y} = 10$, it can be checked that $\forall E[\alpha_1^{\omega}] > 1$, $\Psi_{1x}^*(\omega) > \Psi_{2x}^*(\omega)$ and $\Psi_{2y}^*(\omega) > \Psi_{1y}^*(\omega)$, $\forall \omega \in \Omega$.

[13] In these two examples, each household has either full information or no information about the state of a location. We can consider another example where each household has partial information about the state of a location, i.e., $\Omega_x = \Omega_y = \{H, M, L\}$, $\mathcal{F}_1 = \{\phi, \{H, M\}, \{L\}, \Omega_x\} \times \{\phi, \{H\}, \{M, L\}, \Omega_y\}$, and $\mathcal{F}_2 = \{\phi, \{H\}, \{M, L\}, \Omega_x\} \times \{\phi, \{H, M\}, \{L\}, \Omega_y\}$. Then if household 1 (2) lives in x (y) in equilibrium, except in state LL, states of two locations are not fully revealed by equilibrium bid rents. So there does not exist a fully revealing MREE.

section.

Suppose there are two households ($j \in N \equiv \{1, 2\}$), and two locations ($k \in K \equiv \{x, y\}$). Let $\Omega \equiv \Omega_x \times \Omega_y = \{H, L\} \times \{H, L\}$ be the finite payoff-relevant state space of the economy. For each state ω, the economy $(Y, u_j(\psi_j(\omega), \omega)_{j \in N})$ is a smooth economy as defined by Debreu (1972). It is important to notice that u_{jk} is payoff-relevant to only Ω_k; that is, we assume that people living in location k care only about the state in k. Later, we consider the perturbations that maintain this property.

Before we prove the results, some characteristics of equilibrium must be defined. In a MREE, the information can be fully revealing, which means that all households can learn the state of nature by observing the equilibrium prices in all locations except their equilibrium locations and using their private information. Alternatively, the information can be non-fully revealing in a MREE, where at least one household cannot know the state of nature from the equilibrium price and their private information. Their formal definitions are as follows.

Definition 3 *A fully-revealing modified rational expectations equilibrium is a modified rational expectations equilibrium such that*

$$\mathcal{F}_j \vee \sigma(\Psi^*_{-k_j}) = \mathcal{F}, \quad \forall j \in N. \tag{33}$$

When there is at least one j such that the above equality does not hold, we say it is a non-fully-revealing modified rational expectations equilibrium.

In other words, conditioning on a fully revealing equilibrium price function is equivalent to knowing the pooled information of all households in the economy. Though Allen (1981) proves the existence of an open and dense subset of economies with fully-revealing rational expectations equilibrium in the classical framework, when perturbations location-by-location are considered, Theorem 1 will show that the same statement does not hold for MREE in urban economics. Utility functions defined location-by-location are formally called local utilities.[14] We have been using them in this paper up to this point.

Definition 4 (Local Utilities)
Households' preferences are called local when their preferences satisfy $\forall j \in N, k \in K$, $u_{jk} : \Omega_k \to \kappa_{jk}$. *If for some* j, k, *there exists* k', $k' \neq k$, $k' \in K$, *such that* $u_{jk} : \Omega_k \times \Omega_{k'} \to \kappa_{jk}$ *is not constant for some* $\omega_{k'}, \omega'_{k'} \in \Omega_{k'}$, *then it is called non-local.*

That is, saying that utilities are local requires that each household's utility at location k is measurable with respect to only Ω_k when they live in location k. We shall require that when utility functions are perturbed, if they start local, they remain local. We call this a "spatially local perturbation." *Spatially local perturbation* means that if people living in a location care only about the state in the location where they live, then when their utility function is perturbed, it continues to have this property. Spatially local perturbations are more realistic than non-local perturbations in urban economics, since it is not persuasive to say that the perturbed preferences conditional on residence in location k depend on the state in another location. For example, when preference perturbations are considered, in most cases, the state of commuting congestion or crime (or the quality of schools) in Chicago is irrelevant to that in New York. Therefore, in urban economics, it doesn't make sense to consider spatially non-local perturbations as used in standard models. Throughout this paper, to highlight the distinct essence of urban economics, we focus on spatially local perturbations.

It is possible to add other kinds of perturbations to the model, for example national or regional uncertainty, but this would only complicate notation.

Theorem 1 *Given the discrete state space* Ω, *consider local perturbations of households' preferences. There exists an open subset of economies that possess no fully-revealing modified rational expectations equilibrium.*

[14] Throughout this paper, only preference perturbations are considered since endowment perturbations give households more information if they are state-dependent, and perturbations of ex ante information are not smooth.

Proof. See Appendix B.

Thus, if one household has information about a specific location, if it doesn't live there in equilibrium, the housing price in that location will not reveal his information. If a household lives in the location about which he is informed, there is an information gain (in that he can maximize ex post utility instead of ex ante utility), but also a information spillover to all other households in that they can learn private information about that location by observing the equilibrium housing price. When local utility and spatially local perturbations are considered, the information spillover plays no role for the households living in other locations. However, when spatially non-local perturbations are considered, a small perturbation makes the states of all locations relevant to the utility of living in k. So, as shown in Allen (1981), there exists an open and dense set of economies possessing fully revealing rational expectations equilibrium.

Finally, we make a remark here: If there is no fully revealing MREE, an equilibrium allocation can fail to be Pareto optimal. Consider a variation of Example 1 shown in Figure 3. When probability is quite evenly distributed over states in Ω_k, $k = 1, 2$, household 1's bid rent for the CBD is larger than that of household 2, and household 2's bid rent for location 2 is larger than that of household 1. So in equilibrium, household j lives in location j, $j = 1, 2$ in both states. However, in a Pareto optimum, household j lives in $3 - j$, $j = 1, 2$ when $\omega = LH$. Therefore, we have an example with an equilibrium allocation that is ex ante but not ex post efficient.

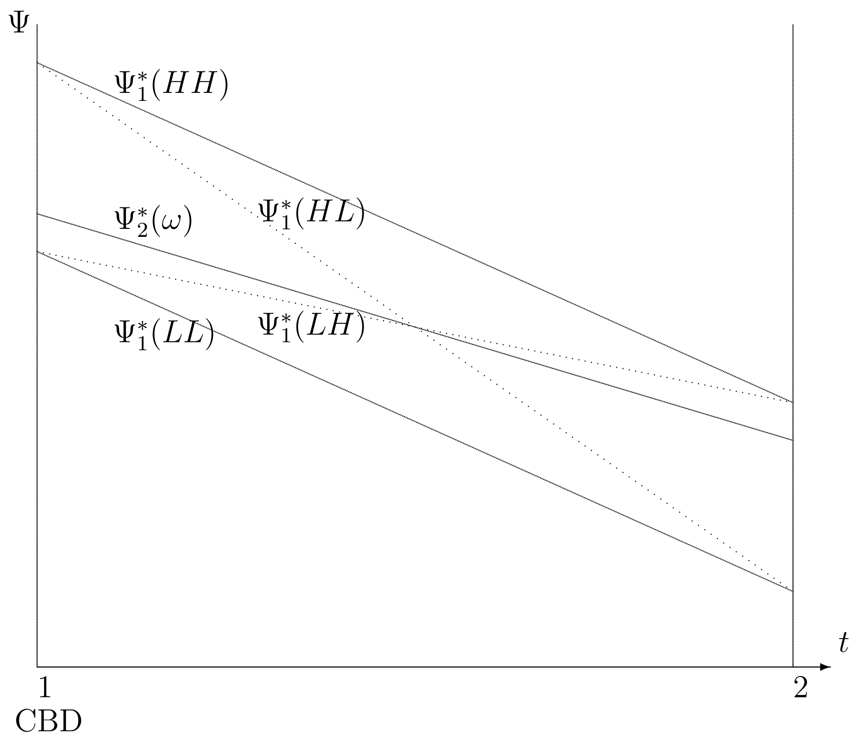

Figure 3: The non-fully revealing MREE allocation can fail to be ex post Pareto optimal.

5 The Existence of Modified Rational Expectations Equilibrium

After presenting an open subset of economies that possess non-fully-revealing MREE, it is natural to ask: Can a MREE fail to exist in urban economies? This can undermine the minimal requirement for further analysis in urban economics with uncertainty. In this section, the existence of (not necessarily fully-revealing) MREE is examined, given the assumption of ordered relative steepness of bid-rents, to be defined shortly. Since an absentee landlord is not needed here, we can assume that private ownership of land is distributed among all households. Denoting e_{jk} as household j's land endowment in location k, the results with no landlord are the same as the situation with an absentee landlord, except that $Y_j = \sum_{k \in K} \Psi_k^*(\omega)\, e_{jk}$, where $\sum_{j \in N} e_{jk} = \bar{s}_k$, $\forall j \in N$, $k \in K$. First we describe how the existence of equilibrium depends on the number of locations relative to the number of households.

When the number of locations is greater than the number of households, since each household can consume housing in at most one location, there must exist at least one location where no household lives. In these abandoned locations, by Walras' Law, the price of housing is zero. Therefore, given $\forall \bar{z} > 0$, $\lim_{s_{jk} \to \infty} u_{jk}((s_{jk}, \bar{z}), \omega) = \infty$, unless the commuting cost is very high and these locations are far away from the CBD, households have an incentive to move into these locations to enjoy a higher utility. In this case, there is no equilibrium.

Note that the ordering of households is independent of state. When the number of locations is the same as the number of households, the assumption of ordered relative steepness of bid rents ensures that every location is occupied by exactly one household in equilibrium. Therefore, we can settle households one-by-one from the core to periphery in the order of the slopes of their bid rents, constituting an equilibrium allocation.[15] Thus, we know ex ante what information will be revealed by equilibrium prices, so we can add this information to the consumer's optimization problem. The case when the number of households is larger than the number of locations is left to Section 5. This would be the case, for example, if there were a continuum of consumers.

Suppose there are n households and n locations. Before proving a theorem on the existence of equilibrium, we need to make following assumptions on households' bid rents. These assumptions are standard in urban economies; see for example Fujita (1985, 1989).[16] To avoid abuse of notation, let $\tilde{s}_j(t, \omega)$ and $\tilde{z}_j(t, \omega)$ denote the consumptions of lot size and composite good at a

[15]Without the assumption of ordered relative steepness of bid rents, we must find a fixed point in the information structure, which is hard.

[16]In fact, in standard urban economies, the assumption of ordered relative steepness relates only to the uniqueness of equilibrium and makes the proof of existence of equilibrium easier, but existence of equilibrium in urban economies can be proved without this assumption when there is no uncertainty; see Fujita and Smith (1987).

distance t from the CBD in state ω. Given a specific state ω and a utility level u, denote $\tilde{\Psi}_j(t, u, \omega) \equiv \max_{\tilde{s}_j(t,\omega), \tilde{z}_j(t,\omega)} \{\frac{Y_j - t - \tilde{z}_j(t,\omega)}{\tilde{s}_j(t,\omega)} | u_j(t, \omega) = u\}$ as household j's bid rent for housing at distance t from the CBD.[17]

Assumption 1 (Ordered Relative Steepness of Bid Rent)
Households' bid rent functions are ordered by their relative steepnesses. That is, given $j < j' \leq n$, $\tilde{\Psi}_j(t, u_j, \omega)$ is steeper than $\tilde{\Psi}_{j'}(t, u_{j'}, \omega)$: Given $\omega \in \Omega$, whenever $\tilde{\Psi}_j(\bar{t}, u_j, \omega) = \tilde{\Psi}_{j'}(\bar{t}, u_{j'}, \omega) > 0$ for some $\bar{t}$, u_j and $u_{j'}$, then

$$\tilde{\Psi}_j(t, u_j, \omega) > \tilde{\Psi}_{j'}(t, u_{j'}, \omega), \quad \forall\, 0 \leq t < \bar{t}, \tag{34}$$

$$\tilde{\Psi}_j(t, u_j, \omega) < \tilde{\Psi}_{j'}(t, u_{j'}, \omega), \quad \forall\, t > \bar{t} \textit{ wherever } \tilde{\Psi}_j(t, u_j, \omega) > 0. \tag{35}$$

Notice that the ordering of households is independent of state. When households have the same utility function but different incomes, and when housing is a normal good, ordered relative steepness of bid rents is naturally satisfied.[18] However, when households have different utilities, ordered relative steepness of bid rent is not implied. The assumption of ordered relative steepness of bid rents ensures that given arbitrary levels of utilities for two agents, for each state, their bid rents must cross at (no more than) one point as shown in Figure 4, where the bid rent curves shift down as the utility levels increase. For example, the Cobb-Douglas utilities in Examples 1 and 2 satisfy the assumption of ordered relative steepness of bid rents, and so do quasi-linear utilities. In what follows, we prove the existence of MREE, given the assumption of ordered relative steepness of bid rents.

5.1 When households are insensitive

To begin, given ordered steepness of bid rents and the same number of consumers and locations, use Assumption 1 to order consumers so that consumer

[17]Notice that though locations are discrete points on the line representing distance to the CBD, households' bid rents are in fact continuous functions of the distance from the CBD.

[18]See Fujita (1989), pages 28-29.

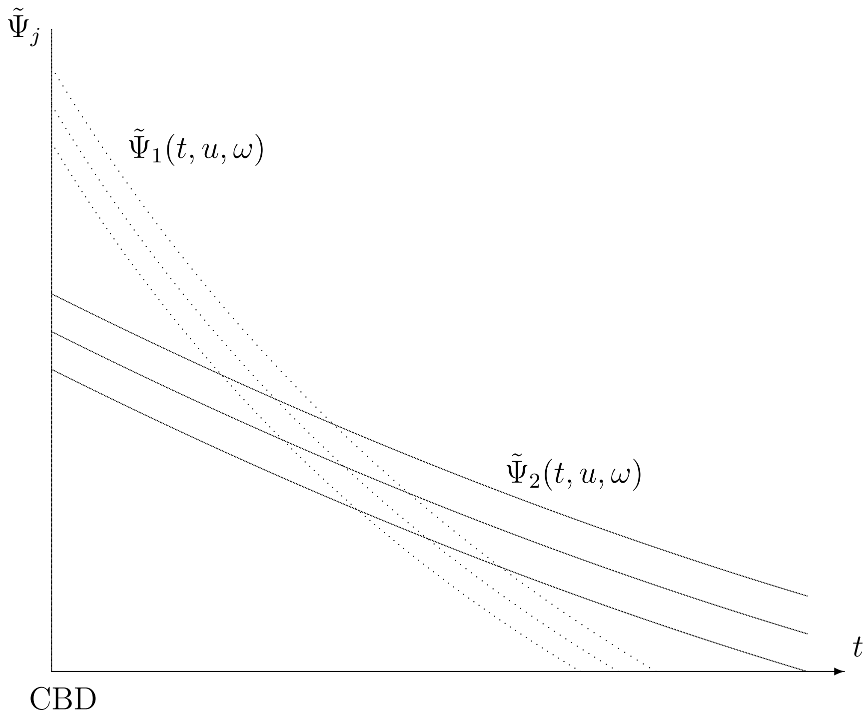

Figure 4: Example where households' bid rents satisfy ordered relative steepness of bid rents.

1 has the steepest bid rent, consumer 2 the next steepest, and so forth. Since the examples in Section 2 highlight the condition required for the existence of non-fully revealing MREE, in what follows we focus on the case where households present insensitivity. Recall that the utility of household j in state ω from living in location j is denoted by $u_{jj}(\psi_{jj}(\omega),\omega)$.

Definition 5 (Insensitivity)
There exist states $(\omega,\omega')\in\Omega\times\Omega$ that for each household $j\in N$ such that ω and ω' are in different partition elements of $\mathcal{F}_j$,[19]

$$\frac{D_{s_{jj}(\omega)}u_{jj}(\psi_{jj}(\omega),\omega)}{D_{z_{jj}(\omega)}u_{jj}(\psi_{jj}(\omega),\omega)}\Bigg|_{\varphi^*_{jj}(\omega)}=\frac{D_{s_{jj}(\omega')}u_{jj}(\psi_{jj}(\omega'),\omega')}{D_{z_{jj}(\omega')}u_{jj}(\psi_{jj}(\omega'),\omega')}\Bigg|_{\varphi^*_{jj}(\omega')}, \tag{36}$$

but there exists $j'\in N$ for whom ω and ω' are in the same element of $\mathcal{F}_{j'}$ (with a positive probability), $u^*_{j'j'}(\psi_{jj}(\omega),\omega)\neq u^*_{j'j'}(\psi_{jj}(\omega'),\omega')$.

[19] States that nobody can distinguish and that do not matter to anyone can be combined.

Given that housing is a normal good, we will show that equilibrium always exists and that insensitivity is a necessary and sufficient condition for the existence of a non-fully revealing MREE.

The intuition for the first part of the definition of insensitivity is that for any household who has information in distinguishing two states, his/her marginal rate of substitution in k is independent of these realized states. However, to ensure that the household's information is not trivial, we need the second part of the definition which implies that his/her information about location k does matter for another household. Insensitivity can result from one or more of several sources: utility could be quasi-linear, or information about conditions in one location can be irrelevant to the consumer living there, or some information is irrelevant to all consumers.

Let $\mathcal{P}(\Omega)$ be the power set of Ω. Now, consider a public partitional information function $\mathcal{I} : \Omega \rightarrow \mathcal{P}(\Omega)\backslash\{\phi\}$ such that for every $\omega \in \Omega$, a nonempty subset $\mathcal{I}(\omega)$ of Ω is assigned, where: (1) for every $\omega \in \Omega$, $\omega \in \mathcal{I}(\omega)$; (2) $\omega' \in \mathcal{I}(\omega)$ implies $\mathcal{I}(\omega') = \mathcal{I}(\omega)$. Moreover, for every (ω, ω') satisfying insensitivity, $\mathcal{I}(\omega') = \mathcal{I}(\omega)$. This condition implies that when ω and ω' are insensitive, and ω' and ω'' are insensitive, then $\mathcal{I}(\omega) = \mathcal{I}(\omega') = \mathcal{I}(\omega'')$. So it can be checked that

$$\mathcal{I}(\omega) = \{\omega' \big| \mathcal{I}(\omega') = \mathcal{I}(\omega)\} \tag{37}$$

In other words, $\mathcal{I}(\omega)$ is a partition element collecting states that are directly or transitively insensitive with ω. Intuitively, for all states in $\mathcal{I}(\omega)$, either households have no information to distinguish them, or the informed household cannot reflect its information by differences in its marginal rate of substitution. The non-fully revealing MREE is supported by the σ-algebra generated by the public partitional information function.

Theorem 2 *Given Assumption 1 and that housing consumption is a normal good, under insensitivity, for $j \in N$, there is an equivalence class of $\sigma(\mathcal{I})$-measurable bid rent functions $\Psi^* : \Omega \rightarrow \mathbb{R}^{2n}_+$ and $\mathcal{F}_j \vee \sigma(\Psi^*_{-j})$-measurable* *consumption functions* $\varphi_j^*: \Omega \to \cup_{k\in K}\mathbb{R}_+^2$ *that constitute a unique non-fully revealing MREE such that, for* $k \in K$*,*

$$\begin{aligned}\Psi_k^*(\omega) &\equiv \Psi_{kk}(u_k^*(\omega),\omega)\\ &= \max_{s_{kk}(\omega),z_{kk}(\omega)}\{\frac{Y_j - T_k - z_{kk}(\omega)}{s_{kk}(\omega)}\Big| E[u_{kk}(\psi_{kk}(\omega),\omega)|\mathcal{F}_k \vee \sigma(\Psi_{-k}^*)] = u_k^*(\omega)\};\end{aligned} \tag{38}$$

$$\varphi_{jk}^*(\omega) \equiv \varphi_{jk}(u_k^*(\omega),\omega) = \begin{cases}(\bar{s}_k,\ Y_j - T_k - \Psi_k^*(\omega)\,\bar{s}_k), & \textit{if } \ j = k,\\ (0,0), & \textit{if } \ j \neq k;\end{cases} \tag{39}$$

and the unique equilibrium utility level $u_k^*(\omega)$*,* $k \in K$*, satisfies*

$$\Psi_{kk}(u_k^*(\omega),\omega) = \frac{D_{s_{kk}(\omega)}E[u_{kk}(\psi_{kk}(\omega),\omega)|\mathcal{F}_k \vee \sigma(\Psi_{-k}^*)]}{D_{z_{kk}(\omega)}E[u_{kk}(\psi_{kk}(\omega),\omega)|\mathcal{F}_k \vee \sigma(\Psi_{-k}^*)]}\Bigg|_{\varphi_{kk}^*(\omega)}. \tag{40}$$

Proof. See Appendix C.

5.2 When households are not insensitive

Insensitivity is necessary and sufficient for the existence of a non-fully revealing MREE. Since, with insensitivity, there is some useful information that is not transmitted from informed to uninformed households, the MREE is non-fully revealing. Let $\tilde{\sigma}_k \equiv \sigma(\Omega_k) \times (\times_{k'\neq k}\{\phi, \Omega_{k'}\})$, which is the σ-algebra indicating that only the state in k is known, whereas all states in other locations are completely unknown. Without insensitivity, the equilibrium can only be fully-revealing.

Theorem 3 *Given Assumption 1 and housing consumption is a normal good, under no insensitivity, there exists a unique modified rational expectations equilibrium, and it is fully revealing.*

Proof. See Appendix D.

In the literature, an open and dense subset of standard economies with fully revealing rational expectations equilibrium is found. However, under the natural assumption of spatially local perturbations of utility functions, as shown in the previous section, an open subset of economies with only a non-fully revealing equilibrium is found. Recall that, consistent with what is shown in standard general equilibrium models, there is also an open subset of urban economies with only fully revealing equilibria: The easiest way to present this is to exchange the information given to households 1 and 2 in our examples and use spatially local perturbations of utility functions. Then within these perturbations, the MREE can only be fully revealing (since there is no mismatch between the information known by households and their locations). Therefore, neither the set of fully revealing nor the set of non-fully revealing economies can be dense under the structure of urban economics. Non-fully revealing equilibrium is more interesting in highlighting the potential positive value and the strategic use of information. When non-local perturbations are considered, though they are not so reasonable in urban economics, the results are the same as the ones in standard general equilibrium models. That is, there is an open and dense subset of economies that possess a fully revealing MREE.

As shown in the comparison in Table 1, the inefficiency in information transmission in a housing/land market rests on two key assumptions: spatially local utility perturbations and the standard setting in urban economics that every household can consume housing in only one place. When either of them is violated, the result in standard models is restored. That is, in economic circumstances where there is no location structure or no spatially local property of utility, generically, the efficiency of prices in information transmission is attained even in a (modified) rational expectations equilibrium. We conclude that geographic structure, together with spatially local utility properties, can play a role in distorting the efficiency of the market in transmitting information from informed to uninformed households.

	Households can consume housing in only one place	Ordinary consumption set
Spatially local utility perturbations	Open subsets of economies with fully revealing and non-fully revealing equilibria (Urban economics)	An open and dense subset of economies with fully revealing equilibria
Spatially non-local utility perturbations	An open and dense subset of economies with fully revealing equilibria	An open and dense subset of economies with fully revealing equilibria (Standard model)

Table 1: A comparison of the results in this paper with the results in the literature.

If households can be redistributed so that location is coincident with information, then we can create a fully-revealing MREE. However, this idea seems impractical since in most cases, unless the households are very risk averse, households' subjective preferences for location do not necessarily depend on the information that they have.

6 The Open-City Model

The model developed in the previous sections is, in practice, an assignment model; that is, only one household is allowed to reside in each location. An open-city model allows us to depart from this restriction. Although each household must reside in only one location, the location is no longer indivisible in that many households can reside in the same location. In order to allow more consumers than the number of locations, a continuum of consumers of a finite number (m) of types is considered in the open-city model, $m \geq 2$, where each type of household is indexed by $j \in M \equiv \{1, ..., m\}$. Households of the same type have the same gross income Y_j, $j \in M$, the same information, and the same preference for housing and composite goods. Moreover, bid rent functions of different types of households are ordered by their relative steepnesses such that type 1 (m) households have the steepest (flattest) bid rent curves, as defined in Assumption 1.

Assume that there are a large number of outside cities and, following Henderson (1974), Henderson and Abdel-Rahman (1991), Abdel-Rahman (1996), Anas and Xiong (2003), and Anas (2004), all cities are ex ante identical. Moreover, there is no aggregate uncertainty among outside cities. Thus, the utility for type j households from living in an outside city is a constant $\bar{u}_j > 0$. Locations are indexed by $k \in K \equiv \{1, ..., n\}$, $n \geq 2$, in the representative city that we are analyzing. For the generic existence of equilibrium, it is assumed that $m \leq n$. Each location k is endowed with an inelastic land supply $\bar{s}_k$ and a random variable ω_k, $k \in K$. Let $\omega \equiv (\omega_k)_{k \in K} \in \Omega$ represent the states of all locations. Assume that households are costlessly mobile. That is, each type of household can costlessly migrate into and out of any location in the representative and outside cities. Following the literature on open-city models, the utility levels of households are exogenously given in the range so that equilibrium always exists and the city populations are endogenous.[20] Denoting $N^*_{jk}(\omega)$ as the equilibrium population of type j households living in k, $N^*_{jk}(\omega)$ is endogenously determined by the equilibrium bid rent and the land supply in k, $k \in K$, $j \in M$, $\omega \in \Omega$.

Given these assumptions, one location is occupied by more than one

[20] There is in practice a system of cities behind our open-city model. In the systems-of-cities literature, total (national) population is exogenous and both the number of cities and city populations are endogenous. Following Henderson (1974), given that there is only one type of city, in equilibrium cities are identical except for the realized states. The cities can be set up by developers, as in Abdel-Rahman and Fujita (1993) and Anas (2004), or they can be self-organized, as in Henderson (1974), Henderson and Abdel-Rahman (1991), Anas (1992), Henderson and Becker (2000), and Pines (2000).

Following the literature of representative cities, we focus on symmetric equilibria where households of the same type get the same *ex ante* utility level in all cities. Utility levels for all types are endogenous shadow prices corresponding to the number of cities and city populations. To get city population for each type and the number of cities to match the national population in a symmetric equilibrium, we can adjust utility levels and find a fixed point $(\bar{u}_j)_{j \in M}$ where all the population fits and land markets clear.

household, and can be occupied by more than one type. The definition of a bid rent equilibrium in the open-city model is the same as Definition 2 except that $u_j^*(\omega) = \bar{u}_j$, $\forall \omega \in \Omega$, and the information that households can augment from observing prices, specified as follows.

Though many real-world economic phenomena appear to hinge critically on the process of asymmetric information revelation in equilibrium,[21] as emphasized in Ausubel (1990) and Allen and Jordan (1998), it is very difficult to obtain a partially revealing rational expectations equilibrium. The reason is the informational discontinuity in households' demand invalidates the use of a fixed-point argument when the existence of equilibrium is demonstrated. In the literature, the existence of partially revealing equilibria generally depends on some form of approximation, either to exact market clearing or to complete rationality in agents' use of information. Here, on the other hand, we use no approximation but a generalized version of MREE according to the characteristics of urban economies. The MREE concept given in Definition 1 is fine in the open city model as long as, for each location, the same type lives there in all states. However, if the type who lives in a location changes with the state, it is not clear what information households can condition on for that location. To solve this problem, we encapsulate a multi-stage learning process in "test prices" when the MREE concept is defined for the open city model. This learning process postulates a selection of equilibrium which is reasonable in urban economics.

Since utility levels are fixed in all states, we can proceed to analyze each location independently. Fixing any location, for each state, first we take every person's information to be their private information and find the "test" equilibrium price or maximal bid rent based on it. (Due to a lack of full information, some test prices will be the same in different states.) Next, after adding the information conveyed by these test prices to each person's infor-

[21] For example, Grossman and Stiglitz (1976) show that if information can be freely conveyed by prices, no trader would have an incentive to collect costly information.

mation, we compute again new test prices. Repeating this process, at each step every household's information is getting weakly better in that his/her sigma algebra does not get smaller. Eventually, the process stops when the renewed test price function becomes measurable in all households' sigma algebra obtained in the previous step, which means no household can further augment his/her information from observing test prices. Since any price function is measurable in full information (the finest sigma algebra), given a finite state space, this process must terminate in a finite number of steps. The information augmentation process and the MREE are formally defined as follows.

Definition 6 (Modified Rational Expectations Equilibrium in the Open City Model)
For each $k \in K$, consider a finite sequence of test bid rent functions, $(\Psi_k^{(i)})_{i=0,1,...,I}$, satisfying the following conditions:

$$\Psi_k^{(0)}(\omega) = \max_{j \in M} \Psi_{jk}^{(0)}(\bar{u}_j, \omega), \ \textit{where}$$
$$\Psi_{jk}^{(0)}(\bar{u}_j, \omega) = \max_{s_{jk}(\omega), z_{jk}(\omega)} \left\{ \frac{Y_j - T_k - z_{jk}(\omega)}{s_{jk}(\omega)} \middle| E[u_j(\psi_j(\omega), \omega) \middle| \mathcal{F}_j] = \bar{u}_j \right\}; \tag{41}$$

For $i > 0$, $\Psi_k^{(i)}(\omega) = \max_{j \in M} \Psi_{jk}^{(i)}(\bar{u}_j, \omega)$, $\forall \omega \in \Omega$, where

$$\Psi_{jk}^{(i)}(\bar{u}_j, \omega) = \max_{s_{jk}(\omega), z_{jk}(\omega)} \left\{ \frac{Y_j - T_k - z_{jk}(\omega)}{s_{jk}(\omega)} \middle| E[u_j(\psi_j(\omega), \omega) \middle| \left(\bigvee_{\tau=0,1,...,i-1} \sigma(\Psi_k^{(\tau)}) \right) \vee \mathcal{F}_j] = \bar{u}_j \right\}; \tag{42}$$

$$\Psi_k^{(I)}(\omega) \ \ \textit{is} \ \left(\bigvee_{\tau=0,1,...,I} \sigma(\Psi_k^{(\tau)}) \right) \vee \mathcal{F}_j - \textit{measurable}, \ \forall j \in M, \ \ \textit{and}$$
$$\Psi_k^{(i)}(\omega) \ \ \textit{is not} \ \left(\bigvee_{\tau=0,1,...,i} \sigma(\Psi_k^{(\tau)}) \right) \vee \mathcal{F}_j - \textit{measurable for at least one} \ \ j \in M,$$
$$\forall i = 1, ..., I-1. \tag{43}$$

Notice that the MREE concept in the open city model reduces to the previous definition in the assignment model where the locations of consumers are state-independent, as in the assignment model under Assumption 1. In this case, $I = 2$, because .because the private information of consumers living at k is reflected in step $i = 1$. When $I > 1$, $\bigvee_{\tau=0,1,...,i-1}\sigma(\Psi_k^{(\tau)})$ represents type j households' extra information augmented from observing test prices before step i, $i = 1, ..., I$.[22]

For each location $k \in K$, letting $I(k)$ be the smallest positive number of steps such that $\Psi_k^{(I(k))}(\omega)$ is measurable in $\left(\bigvee_{\tau=0,1,...,I(k)-1}\sigma(\Psi_k^{(\tau)})\right) \vee \mathcal{F}_j$, $\forall j \in M$, then $I = \max\{I(k)\}_{k\in K}$. With the help of an open city model framework, Theorem 4 shows that even if the location is not indivisible, there is an open subset of economies that possess no fully revealing MREE.

Theorem 4 *In the open city model, given Assumption 1, the modified rational expectations equilibrium always exists and is unique. Furthermore, there exists an open subset of economies where the unique modified rational expectations equilibrium is not fully revealing.*

Proof. See Appendix E.

Theorem 4 shows that when the location is no longer indivisible in that multiple households (that are of the same type or different types) can reside in the same location, there still exists an open subset of urban economies that possess no fully-revealing MREE. Though it is for a selection of reasonable equilibrium in urban economics, the key feature of the designed information augmentation process is perfect recall of all historical test prices. The following example highlights the importance of this feature in urban economies.

For a given location $k \in K$, suppose that there are two states, $\Omega_k \equiv \{H, L\}$, and two types of households, $M \equiv \{1, 2\}$. Suppose that each type

[22] Notice that outside the case $I = 2$, the equilibrium information structure might not be $\mathcal{F}_j \vee \sigma(\Psi^*_{-k})$. Moreover, the equilibria of the assignment model and the open city model can be different because the setups of the two models are different.

1 household's bid rent for the housing in k is 5 in H and 2 in L, and each type 2 household's bid rent for the housing in k is 3 in H and 5 in L. Type 1 households have full information while type 2 households do not. Type 2 households have a prior belief that both states are equally likely to happen, so each type 2 household's bid rent for the housing in k is 4 when they are given only their private information. It is interesting to notice that, in this example, there is no standard rational expectations equilibrium, fully revealing or not. However, due to perfect recall of historical test prices, there exists a MREE where households have full information in equilibrium but equilibrium prices are the same in different states.[23] Though type 2 households augment their information from the test prices in step 1 instead of the equilibrium prices in this example, this phenomenon is not generic.

Theorem 5 *Generically, the set of MREE is a subset of REE.*

Proof. See Appendix F.

Under perfect recall, since the final prices generically reveal all the information carried by the test prices, the process by which we obtain the final prices using test prices is just a way of proving the existence of MREE. That is, the designed information augmentation process is helpful for solving equilibrium but not essential for the results. On the other hand, since the result based on an embedded perfect-recall learning process seems more reasonable than the self-fulfilling logic used behind the standard rational expectations equilibrium concept, the adoption of MREE is justified in urban economies.

A classical way to induce households to reveal their private information, as shown in Debreu (1959) Chapter 7 and Arrow (1964), is to consider contingent claim or financial markets. This idea is discussed in Section 6.

[23]From the examples in Section 3, it is evident that the set of REE is not a subset of MREE. From this example, it is evident that the set of MREE is not a subset of REE. MREE is a subset of REE only when $I = 1$.

7 Adding Financial Markets

When contingent claims or financial markets are included, do our examples with no fully-revealing MREE survive? This interesting question is examined here.

Similar to Hirshleifer's (1971) conclusion in cases with technological uncertainty, speculative profits from price revaluation give individuals incentives to disseminate their information. We show that when there is market uncertainty, the same incentives exist and thus all households' information is revealed in equilibrium.

Following the setting of our Example 1 and Magill and Quinzii (1996), consider that before consuming composite good and housing, the two households can buy and sell state-contingent financial securities in financial markets. That is, consider a one-period, two-stage model as follows. At the beginning of the first stage, households are endowed with e_j^0 units of numeraire (composite consumer good), $j = 1, 2$. Household 2 has complete information about the states in the two locations, whereas household 1 has no information. The financial markets are opened in stage 1, where the two households can buy and sell securities. Assume that the financial markets are complete in that the number of securities is the same as the number of states, so we can use the same index for securities and states. Specifically, the security ω, $\omega \in \Omega$, is a contract promising to deliver one unit of numeraire (income) in state ω, and 0 in other states, in the second stage. All securities are perfectly monitored and perfectly enforced. After closing the financial markets and the end of the first stage, the state is realized and all security returns are paid at the beginning of the second stage. Then an absentee landlord trades with households in spot housing markets. The game is complete when the housing markets are closed. We want to know whether or not there is a fully-revealing MREE under the new setting.

Let e_j^ω be household j's endowment in state ω in the second stage, and let the row vector $\nu_j \equiv (\nu_j(\omega))_{\omega\in\Omega} \in \mathbb{R}^4$ be household j's portfolio. Let $q \equiv (q(\omega))_{\omega\in\Omega} \in \mathbb{R}^4$ and $V \equiv (V(\omega))_{\omega\in\Omega} \in \mathbb{R}^{16}$ where $q(\omega) \in \mathbb{R}$ and $V(\omega) \in \mathbb{R}^4$ represent the price vector of security ω and the payoff matrix of securities in state ω, respectively. That is, $V(\omega)$ is a row vector of zeros except that the element representing state ω is 1, and $V(\omega) \neq V(\omega')$, for all $\omega \neq \omega'$. The fully revealing MREE under the new setting can be solved by backward induction as follows.

Suppose there exists a fully revealing MREE. From Section 2.2, given $Y_j(\omega)$, households' indirect utility functions with optimization in stage 2 are

$$U_1(\omega) = \alpha_1^\omega \, \ln \bar{x} - \ln(1+\alpha_1^\omega) + \ln Y_1(\omega),$$
$$U_2(\omega) = \beta_2^\omega \, \ln \bar{y} - \ln(1+\beta_2^\omega) + \ln Y_2(\omega).$$

Through monotonic transformations of these indirect utility functions, household j's optimization problem in stage 1 can be written as

$$\begin{aligned} \max_{\nu_j} \;\; & \tilde{U}_j(\omega) \equiv \ln Y_j(\omega) \\ \text{s.t.} \;\; & q \cdot \nu_j^T = e_j^0, \\ & Y_j(\omega) - e_j^\omega = V(\omega)\, \nu_j^T, \quad \omega \in \Omega, \end{aligned}$$

where ν_j^T denotes the transpose of j's portfolio vector. Denoting the true state as $\hat{\omega}$, since households learn the true state by observing prices in a fully revealing MREE, it is obvious that the equilibrium security prices must satisfy $q(\hat{\omega}) = 1$ and $q(\omega) = 0$, $\forall \omega \neq \hat{\omega}$. Since for arbitrary different $\hat{\omega}$, $\hat{\omega}'$, the corresponding equilibrium price vectors are not the same, each q^* reveals a unique $\hat{\omega}$. Therefore, it follows that q^* supports a fully-revealing MREE.

Though we show that adding financial markets helps to reveal the informed household's private information, there are some issues with this idea. Grossman and Stiglitz (1980) argue that the informed household can use their private information to take advantage of uninformed households. Thus, if the financial markets and the corresponding fully-revealing equilibrium prices make private information publicly available to every household, the informed household could not earn an information rent (coming from asymmetric information) and has an incentive to hide his/her private information (by pretending to be uninformed). Therefore, though adding financial markets can restore the existence of a fully-revealing MREE, there are reasons why these financial markets might not function. Of course, if financial asset markets are incomplete for whatever reason, the problems we have discussed return.

8 Conclusions

Radner (1979), Allen (1981), and Jordan (1982) prove the existence of an open and dense subset of standard economies that possess fully-revealing rational expectations equilibria. Since in urban economies there is an open subset of economies without fully-revealing MREE, Allen's theorem about the existence of a dense subset of economies possessing fully-revealing MREE does not extend to urban economies when spatially local perturbations of utilities are considered. These perturbations retain the property that the utility of living at a location depends only on the consumption bundle at that location and the resolution of uncertainty about local variables only. Furthermore, since an open subset of economies with fully revealing MREE can easily be constructed, we cannot challenge the existence of an open subset of economies that possess fully-revealing MREE in the context of urban economies. Therefore, neither the set of fully revealing nor the set of non-fully revealing economies can be dense under the structure of urban economics.

This paper highlights the importance of "local conditions" for the existence of MREE in urban economies. The existence of a unique MREE is proved with the assumption of ordered relative steepness of bid rents. Whether the MREE is fully revealing or non-fully revealing depends on the insensitivity condition: When insensitivity is satisfied, the unique MREE is non-fully revealing; otherwise, the equilibrium is fully revealing. When an open-city model is considered where multiple people can reside in the same location, it is demonstrated that an open subset of economies with no fully revealing equilibrium can still exist even when the location is not indivisible. Though introducing financial markets can restore the existence of fully-revealing MREE, many provisos also accompany it. In summary, geography can play a role in undermining the efficiency of market prices in transmitting information from informed to uninformed households.

Other topics for future research are to extend the intuition behind our results to other models. For example, in an overlapping generations model, time may play a role similar to the spatial structure in preventing information transmission. Moreover, when search/matching models are considered, stable equilibrium may also pick only the best of all potential matches. In either of these cases, we conjecture that there exists an open subset of economies with no fully-revealing MREE, since agents with information about states in other lifetimes (in the overlapping generations framework) or in other equilibrium matches (in the search framework) might not have their information reflected in equilibrium prices.

Appendix A. Proof of Lemma 1

Comparing Definition 1 and Definition 2, since condition (iii) is the same as equations (4), for μ-almost every $\omega \in \Omega$, we only need to prove that $((\psi_j^*(\omega))_{j\in N}, P^*(\omega))$ satisfies (i) and (ii) if and only if $((\varphi_j^*(\omega))_{j\in N}, \Psi^*(\omega))$ satisfies (2) and (3), given $\varphi_j^*(\omega) = \psi_j^*(\omega)$, $\Psi_k^*(\omega) = p_k^*(\omega)$, and $u_j^*(\omega) = u_{jk}(\psi_{jk}^*(\omega), \omega)$, $\forall j \in N$, $k \in K$.

First, to prove this, given that (2) and (3) are satisfied but either (i) or (ii) is not true, we want to show contradictions. If (i) is not true, there exists $\Omega_0 \subseteq \Omega$ with $\mu(\Omega_0) > 0$ such that $P_k^*(\omega) \cdot \psi_{jk}^*(\omega) > Y_k - T_k$, $\forall \omega \in \Omega_0$. Then for $\omega \in \Omega_0$, we have $p_k^*(\omega)s_{jk}^*(\omega) + z_{jk}^*(\omega) > Y_k - T_k$, which together with $\Psi_k^*(\omega) = p_k^*(\omega)$ implies

$$\Psi_k^*(\omega) > \frac{Y_k - T_k - z_{jk}^*(\omega)}{s_{jk}^*(\omega)}, \quad \forall \omega \in \Omega_0,$$

a contradiction with (2) and (3), given that the utility level is the same as the optimal level in Definition 1, i.e., $u_j^*(\omega) = u_{jk}(\psi_{jk}^*(\omega), \omega)$.

On the other hand, if (ii) is not true, then $\exists j \in N$ and $\psi_j'(\omega)$ within the budget constraint such that

$$E[u_j(\psi_j'(\omega), \omega)\big|\mathcal{F}_j \vee \sigma(\Psi_k^*)] > E[u_j(\psi_j^*(\omega), \omega)\big|\mathcal{F}_j \vee \sigma(\Psi_k^*)]. \tag{44}$$

For this household j and for location k where she lives in equilibrium, we can choose $u_j^*(\omega) = E[u_j(\psi_j^*(\omega), \omega)\big|\mathcal{F}_j \vee \sigma(\Psi_k^*)]$, and then by strict concavity and strict monotonicity, there exists $\epsilon > 0$ and $\psi_j''(\omega) \equiv \frac{\psi_j'(\omega)+\psi_j^*(\omega)}{2} - \epsilon$ such that $E[u_j(\psi_j''(\omega), \omega)|\mathcal{F}_j \vee \sigma(\Psi_k^*)] = u_j^*(\omega)$. Since $[\Psi_k^*(\omega) \;\; 1] \cdot \psi_{jk}''(\omega) < Y_k - T_k$ implies $\Psi_k^*(\omega) < \frac{Y_k - T_k - z_{jk}''(\omega)}{s_{jk}''(\omega)}$,[24] letting $p_k''(\omega) \equiv \frac{Y_k - T_k - z_{jk}''(\omega)}{s_{jk}''(\omega)}$, we have $p_k''(\omega) > \Psi_k^*(\omega)$, though $\psi_{jk}''(\omega)$ and $\psi_{jk}^*(\omega)$ yield the same expected utility level $u_j^*(\omega)$. That is, given $u_j^*(\omega)$, $\psi_{jk}''(\omega)$ supports a higher $p_k''(\omega)$ than $\Psi_k^*(\omega)$. Therefore, $\varphi_{jk}^*(\omega) = \psi_{jk}^*(\omega)$ does not maximize $\Psi_{jk}(u_j^*(\omega), \omega)$, a contradiction with equation (3).

[24] Recall that $\Psi_k^*(\omega) \equiv \Psi_k(u^*(\omega), \omega) = \max_j\{\Psi_{jk}^*(\omega)\}$.

Secondly, supposing that (i) and (ii) hold, but either (2) or (3) is not satisfied, we want to prove that there is a contradiction. If (2) does not hold, there exists $k \in K$, $j \in N$, and $\Omega_0 \subseteq \Omega$ with $\mu(\Omega_0) > 0$ such that $\forall \omega \in \Omega_0$, $\Psi^*_{jk}(\omega) > \Psi^*_k(\omega)$ but j does not live in location k. Suppose that j lives in location $k' \neq k$. Then for this household j, since she can pay less for the housing in k than the price that makes her indifferent between the housing in k and k', household j has an incentive to move from k' into location k to increase his/her utility for all $\omega \in \Omega_0$, a contradiction with condition (ii) that φ^*_j maximizes j's conditional expected utility.

If (3) does not hold, the budget line with price $\Psi^*_{jk}(\omega)$ is not tangent to the indifference curve for a given u for some states $\omega \in \Omega_0$, where $\mu(\Omega_0) > 0$. By strict concavity, there exists $\psi^{'}_{jk}(\omega) \neq \varphi^*_{jk}(\omega)$ such that $u_{jk}(\psi^{'}_{jk}(\omega), \omega) = u_{jk}(\varphi^*_{jk}(\omega), \omega)$, and thus $E[u_j(\psi'_j(\omega), \omega)|\mathcal{F}_j \vee \sigma(\Psi^*_k)] = Eu^*_j$, where Eu^*_j is the optimal utility level solved from Definition 1. Choosing $\psi^{''}_{jk}(\omega) \equiv \frac{\varphi^*_{jk}(\omega)+\psi^{'}_{jk}(\omega)}{2}$, then by strict concavity, $\psi^{''}_{jk}(\omega)$ is available for household j in achieving a higher utility level, i.e., $E[u_j(\psi''_j(\omega), \omega)|\mathcal{F}_j \vee \sigma(\Psi^*_k)] > Eu^*_j$, a contradiction with (ii) that φ^*_j maximizes household j's expected utility conditional on the private information and the information revealed by equilibrium prices.

Appendix B. Proof of Theorem 1

Consider example 1 first. Notice that in equilibrium, household 1's marginal rate of substitution for housing in terms of composite commodity in location x is $\frac{E[\alpha^\omega_1]}{1+E[\alpha^\omega_1]}\frac{Y}{\bar{x}}$. On the other hand, household 2's marginal rate of substitution for housing in x is $\frac{\alpha^\omega_2}{1+\alpha^\omega_2}\frac{Y}{\bar{x}}$. Let $\alpha^{HH}_1 = \alpha^{HL}_1 > \alpha^{LH}_1 = \alpha^{LL}_1$ and $\beta^{HH}_1 = \beta^{LH}_1 > \beta^{HL}_1 = \beta^{LL}_1$.

Since in the example $E[\alpha^\omega_1] > \alpha_2$ and $E[\beta^\omega_1] < \beta_2$, we can choose $\epsilon^\alpha = \frac{E[\alpha^\omega_1]-\alpha_2}{(E[\alpha^\omega_1]+\alpha_2)Y+(2+E[\alpha^\omega_1]+\alpha_2)\bar{x}} > 0$, $\epsilon^\beta = \frac{\beta_2-E[\beta^\omega_1]}{(E[\beta^\omega_1]+\beta_2)Y+(2+E[\beta^\omega_1]+\beta_2)\bar{y}} > 0$, and $\epsilon =$

$\min\{\epsilon^\alpha, \epsilon^\beta\}$. Recall that the equilibrium marginal utilities in example 1 are

$$v^* \equiv (D_{s_{1x}} Eu_1^*, D_{s_{1y}} Eu_1^*, D_{z_{1x}} Eu_1^*, D_{z_{1y}} Eu_1^*, D_{s_{2x}} u_2^*(\omega), D_{s_{2y}} u_2^*(\omega), D_{z_{2x}} u_2^*(\omega), D_{z_{2y}} u_2^*(\omega)). \tag{45}$$

Centered at v^*, consider all spatially local perturbations of utility functions within an open set in the weak C^r topology such that

$$D_{s_{1k}} Eu_1 \in (D_{s_{1k}} Eu_1^* - \epsilon, D_{s_{1k}} Eu_1^* + \epsilon), \tag{46}$$

$$D_{z_{1k}} Eu_1 \in (D_{z_{1k}} Eu_1^* - \epsilon, D_{z_{1k}} Eu_1^* + \epsilon), \tag{47}$$

$$D_{s_{2k}} u_2(\omega) \in (D_{s_{2k}} u_2^*(\omega) - \epsilon, D_{s_{2k}} u_2^*(\omega) + \epsilon), \tag{48}$$

$$D_{z_{2k}} u_2(\omega) \in (D_{z_{2k}} u_2^*(\omega) - \epsilon, D_{z_{2k}} u_2^*(\omega) + \epsilon), \qquad k \in K. \tag{49}$$

These perturbations are evaluated in k, $k \in K$, individually, and are thus spatially local perturbations. Then it can be checked that all utilities within this neighborhood generate bid rents that are within ϵ of the equilibrium bid rents in example 1. Furthermore, household 1's realized marginal rate of substitution for housing in terms of composite good in location x is always higher than the marginal rate of substitution of household 2; household 2's marginal rate of substitution for housing in location y is always higher than that of household 1.[25]

Now we can prove the non-existence of fully revealing MREE for all economies in this neighborhood. Suppose for any set of preferences within these local perturbations, there exists a fully revealing MREE $(\varphi_1^*, \varphi_2^*, \Psi^*)$. Then the uninformed household (household 1) can infer the state of nature by observing $\Psi^*_{-k_j}$. However, within the perturbations, the equilibrium bid rents are the same across states, contradicting that Ψ^* is a fully-revealing MREE price.

[25]In location x, for example, since the lowest MRS for household 1 is $\frac{E[\alpha_1^\omega]/\bar{x}-\epsilon}{(1+E[\alpha_1^\omega])/Y+\epsilon}$, and the highest MRS for household 2 is $\frac{\alpha_2/\bar{x}+\epsilon}{(1+\alpha_2)/Y-\epsilon}$, household 1's MRS is greater than household 2's MRS if and only if $\epsilon < \epsilon^\alpha = \frac{E[\alpha_1^\omega]-\alpha_2}{(E[\alpha_1^\omega]+\alpha_2)Y+(2+E[\alpha_1^\omega]+\alpha_2)\bar{x}}$. Similarly, household 2's MRS in location y is greater than that of household 1 if and only if $\epsilon < \epsilon^\beta = \frac{\beta_2 - E[\beta_1^\omega]}{(E[\beta_1^\omega]+\beta_2)Y+(2+E[\beta_1^\omega]+\beta_2)\bar{y}}$.

Obviously, a similar argument works for the cases with more than 2 states and example 2.

Appendix C. Proof of Theorem 2

Before beginning the proof, a general remark about existence of rational expectations equilibrium is in order. For equilibrium that is not fully revealing, one generally requires a fixed point in prices and the public information structure, since prices determine what consumers learn from them, and the information structure determines which prices must be the same in different states due to measurability. In general, the information structure does not have the mathematical properties required for application of a fixed point theorem. So we sidestep this issue.

First, we use the implication from Lemma 1 that bid rent equilibrium is a MREE. Next, a bid rent equilibrium will be constructed, and the existence and uniqueness of the equilibrium will be proved. Finally, it will be shown that the unique bid rent (modified rational expectations) equilibrium is non-fully revealing.

Following a standard argument in urban economics, given Assumption 1, every location is occupied by exactly one household. Since household 1 has the steepest bid rent, from equation (2) in Definition 2, she must occupy the housing in location 1 in equilibrium. After settling household 1, we can consider the problem as the one with $n-1$ households ($j \in \{2, ..., n\}$) and $n-1$ locations ($k \in \{2, ..., n\}$). Then, household 2 has a steeper bid rents than remaining households, so his/her equilibrium bid rent for the housing in location 2 is higher than that of other households. Therefore, in equilibrium, household 2 occupies the housing in location 2. Following the same logic, in equilibrium all households are arranged so that household j lives in location j, $j \in N$, or say that location k is occupied by household k, $k \in K$.

Given that household k is located in location k, as shown in Figure 5, the intercept of budget line $Y_j - T_k$ and the housing supply $\bar{s}_k$ are determined by parameters. Now, given arbitrary u, the slope of budget line $\Psi_k(u,\omega)$ and the corresponding $\varphi_{kk}(u,\omega)$ are uniquely determined (by the cross point of the budget line and the vertical line $\bar{s}_k$). Furthermore, given consumption point $\varphi_{kk}(u,\omega)$, since households' preferences are smooth, the slope of the indifference curve passing through $\varphi^*_{kk}(u,\omega)$ is uniquely determined. Formally, letting $\Phi_{kk}(u,\omega) \equiv \frac{D_{s_{kk}(\omega)}E[u_{kk}(\psi_{kk}(\omega),\omega)|\mathcal{F}_k \vee \sigma(\Psi^*)]}{D_{z_{kk}(\omega)}E[u_{kk}(\psi_{kk}(\omega),\omega)|\mathcal{F}_k \vee \sigma(\Psi^*)]}\Big|_{\varphi_{kk}(u,\omega)}$, the equilibrium utility level (and the equilibrium housing price in location k) is given by solving $\Psi_k(u,\omega) = \Phi_{kk}(u,\omega)$, $\omega \in \Omega$, as shown in Figure 5. Letting $f_{kk}(u,\omega) \equiv \Psi_k(u,\omega) - \Phi_{kk}(u,\omega)$, since Ψ_k and Φ_{kk} are continuous in u, f_{kk} is continuous in u. At $\bar{E}$, $f_{kk}(u,\omega) < 0$ since $\Psi_k(u,\omega) = 0$ and $\Phi_{kk}(u,\omega) > 0$ at $\bar{E}$ by monotonicity. Given $\bar{s}_k > 0$, $\Psi_k(u,\omega)$ is increasing as $z_{kk}(\omega)$ decreases and, by the smooth boundary condition, $\Phi_{kk}(u,\omega) \to 0$ as $z_{kk}(\omega) \to 0$, which implies that $\exists \underline{u}$ such that $f_{kk}(u,\omega) > 0, \forall u \leq \underline{u}$. Therefore, by the intermediate value theorem, there exists a $u^*_k(\omega)$ solving $f_{kk}(u,\omega) = 0$, $\omega \in \Omega$, and thus, there exists a MREE. The uniqueness of equilibrium can be guaranteed by the condition that $\Phi_{kk}(u,\omega)$ is increasing in u, which is true when the consumption of housing is a normal good as shown in Berliant and Fujita (1992).

Under insensitivity, we want to prove that the unique MREE is non-fully revealing. Suppose on the contrary that the equilibrium is fully-revealing, then choosing arbitrary k, we have

$$\Psi^*_k(\omega) = \Psi_{kk}(u^*(\omega),\omega) \neq \Psi_{kk}(u^*(\omega),\omega') = \Psi^*_k(\omega'), \tag{50}$$

$\forall \omega, \omega' \in \Omega, \omega \neq \omega'$. First, for household k (living in location k in equilibrium), any such pair (ω,ω') must be in different partition elements. That is, $\mathcal{F}_k \vee \sigma(\Psi^*_{-k}) = \mathcal{F}$. Second, from (38) and (40), $\Psi_k(u,\omega) \neq \Psi_k(u,\omega')$ implies,

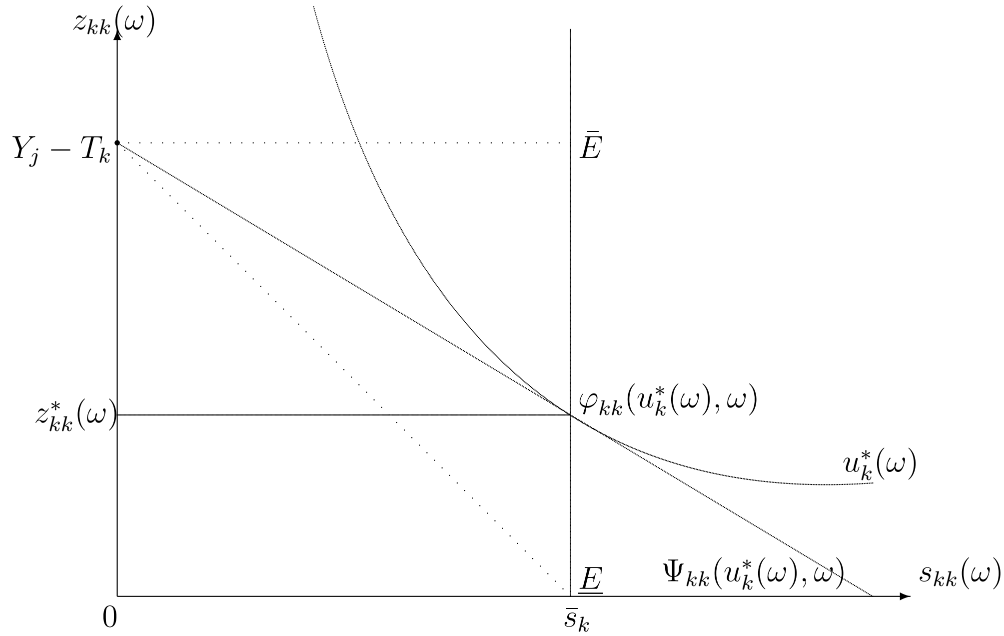

Figure 5: The determination of equilibrium housing price and equilibrium utility for household k in location k in state ω, $k \in K$.

$\forall \omega, \omega' \in \Omega$,

$$\frac{D_{s_{kk}(\omega)} u_{kk}(\psi_{kk}(\omega), \omega)}{D_{z_{kk}(\omega)} u_{kk}(\psi_{kk}(\omega), \omega)} \Big|_{\varphi^*_{kk}(\omega)} \neq \frac{D_{s_{kk}(\omega')} u_{kk}(\psi_{kk}(\omega'), \omega')}{D_{z_{kk}(\omega')} u_{kk}(\psi_{kk}(\omega'), \omega')} \Big|_{\varphi^*_{kk}(\omega')}. \tag{51}$$

However, from insensitivity, there exist ω, $\omega' \in \Omega$ such that

$$\frac{D_{s_{kk}(\omega)} u_{kk}(\psi_{kk}(\omega), \omega)}{D_{z_{kk}(\omega)} u_{kk}(\psi_{kk}(\omega), \omega)} \Big|_{\varphi^*_{kk}(\omega)} = \frac{D_{s_{kk}(\omega')} u_{kk}(\psi_{kk}(\omega'), \omega')}{D_{z_{kk}(\omega')} u_{kk}(\psi_{kk}(\omega'), \omega')} \Big|_{\varphi^*_{kk}(\omega')}, \tag{52}$$

a contradiction with (51).

Appendix D. Proof of Theorem 3

From Assumption 1 and Lemma 1, as in the proof of Theorem 2, there exists a MREE which corresponds to the bid rent equilibrium. When the insensitivity condition is violated, the realized marginal rates of substitution are different $\forall \omega_k \in \Omega_k$, for the consumer living in location k. Furthermore, with no insensitivity, $\tilde{\sigma}_k \subseteq \mathcal{F}_k$ for the household k living in k; otherwise, there exist ω_k, $\omega'_k \in \Omega_k$ that can be distinguished by $j' \neq k$ who does not live in k, a contradiction with no insensitivity. Since this is true for all $k \in K$, and the equilibrium bid rent in k is equal to the marginal rate of substitution of household living in k, the equilibrium bid rents are different in each state, implying that the MREE is fully revealing.

Appendix E. Proof of Theorem 4

First, we want to show the equilibrium always exists and is unique in the open city model. Secondly, an open-city economy with no fully revealing equilibrium is found. Finally, centered at this open-city economy, a spatially local perturbation in utilities is introduced so that all corresponding open-city economies possess no fully revealing equilibrium.

When condition (43) is satisfied with $I = 1$, a unique MREE is constituted immediately. When $I > 1$, condition (43) implies that in steps 1 to $I - 1$, there exists at least one household who can learn something new from observing test prices, and in step I no one can further augment her accumulated information. Therefore, the test prices until step I and the corresponding augmented information constitute a MREE. Recall that the state space is finite and every household's sigma algebra is weakly getting bigger during the information augmentation process. During the process, if there exists one step such that no household's sigma algebra is getting strictly bigger, then nothing new is revealed by test prices and a MREE is attained. Otherwise, at least one household's sigma algebra is getting strictly bigger in each step. Since there are at most $2^{||\Omega||}$ elements in $\sigma(\Omega)$ and all households' bid rent functions must be measurable in $\sigma(\Omega)$,[26] there exists a finite integer $I \leq 2^{||\Omega||}$ such that conditions (42) and (43) are both satisfied, constituting an equilibrium which is fully revealing. Given households' initial private information, since the information augmentation process is unique, there always exists a unique MREE in the open city model.

[26] $||\Omega||$ denotes the number of elements in Ω.

Consider an open city model analogous to Example 2 in Section 2. That is, there are two types of households ($M \equiv \{1, 2\}$) with Cobb-Douglas utility functions specified in Example 2. There are two available locations, $K \equiv \{x, y\}$, each of which is endowed with two states, i.e., $\Omega \equiv \{HH, HL, LH, LL\}$. Recall that $\mathcal{F}_1 = \{\phi, \Omega_x\} \times \{\phi, \Omega_y, \{H\}, \{L\}\}$ and $\mathcal{F}_2 = \{\phi, \Omega_x, \{H\}, \{L\}\} \times \{\phi, \Omega_y\}$ is every type 1 and 2 household's private information, respectively. Given $\bar{u} \equiv (\bar{u}_1, \bar{u}_2)$ and $\mathcal{F}_j$, $j \in M$, such that $x \in k_1(\omega)$ and $y \in k_2(\omega)$, $\forall \omega \in \Omega$, from (42),

$$\Psi_x^{(0)}(\omega) = E[\alpha_1^\omega](1 + E[\alpha_1^\omega])^{-(1+\frac{1}{E[\alpha_1^\omega]})} e^{-\frac{\bar{u}_1}{E[\alpha_1^\omega]}} (Y_1 - T_x)^{1+\frac{1}{E[\alpha_1^\omega]}}, \tag{53}$$

$$\Psi_y^{(0)}(\omega) = E[\beta_2^\omega](1 + E[\beta_2^\omega])^{-(1+\frac{1}{E[\beta_2^\omega]})} e^{-\frac{\bar{u}_2}{E[\beta_2^\omega]}} (Y_2 - T_y)^{1+\frac{1}{E[\beta_2^\omega]}}, \ \forall \omega \in \Omega. \tag{54}$$

Since $\mathcal{F}_j \vee \sigma(\Psi_k^{(0)}) = \mathcal{F}_j$, $\forall j \in M$, $k \in K$, it can be checked that $\Psi_k^{(1)}(\omega) = \Psi_k^{(0)}(\omega)$, $\forall \omega \in \Omega$. Therefore, $\Psi_k^{(1)}$ is measurable in $F_j \vee \sigma(\Psi_k^{(0)})$, $\forall j \in M$, $k \in K$. In the MREE, $\Psi_k^*(\omega) = \Psi_k^{(1)}(\omega) = \Psi_k^{(0)}(\omega)$, $\forall k \in K$, $\omega \in \Omega$. Moreover, the equilibrium population of type j households living in k is

$$N_{jk}^*(\omega) = \begin{cases} \frac{\bar{s}_k}{s_{jk}^*(\omega)}, & \text{for } \ k \in k_j(\omega), \\ 0, & \text{if } \ k \notin k_j(\omega). \end{cases} \tag{55}$$

where $s_{1x}^*(\omega) = (\frac{(1+E[\alpha_1^\omega])e^{\bar{u}_1}}{Y_1 - T_x})^{\frac{1}{E[\alpha_1^\omega]}}$ and $s_{2y}^*(\omega) = (\frac{(1+E[\beta_2^\omega])e^{\bar{u}_2}}{Y_2 - T_y})^{\frac{1}{E[\beta_2^\omega]}}$, $\forall \omega \in \Omega$. Since (53), (54), and $\Psi_k^*(\omega) = \Psi_k^{(1)}(\omega)$ imply that $\Psi_k^*(\omega)$ is a constant for all states, $k \in K$, the equilibrium is not fully revealing. Furthermore, since the MREE is always uniquely, there is no fully revealing MREE in the illustrated open-city economy.

Centered at $\bar{u}$, consider arbitrary perturbations in utility levels $(\Delta \bar{u}_j)_{j \in M}$ such that no household changes his/her equilibrium location. That is, letting $\alpha_2^{HH} = \alpha_2^{HL} > \alpha_2^{LH} = \alpha_2^{LH}$ and $\beta_1^{HH} = \beta_1^{LH} > \beta_1^{HL} = \beta^{LL}$, we choose $\epsilon_1 \equiv \frac{1}{E[\alpha_1^\omega] + \beta_2^{HH}}(E[\alpha_1^\omega]\bar{u}_2 - \beta_2^{HH}\bar{u}_1 + E[\alpha_1^\omega]\beta_2^{HH}(\ln[E[\alpha_1^\omega]] - \ln[\beta_2^{HH}]) + (1 + E[\alpha_1^\omega])\beta_2^{HH}(\ln[Y_1 - T_x] - \ln[1 + E[\alpha_1^\omega]]) - (1 + \beta_2^{HH})E[\alpha_1^\omega](\ln[Y_2 - T_x] - \ln[1 + \beta_2^{HH}]))$, $\epsilon_2 \equiv \frac{1}{\alpha_1^{HH} + E[\beta_2^\omega]}(E[\beta_2^\omega]\bar{u}_1 - \alpha_1^{HH}\bar{u}_2 + \alpha_1^{HH}E[\beta_2^\omega](\ln[E[\beta_2^\omega]] - \ln[\alpha_1^{HH}]) + (1 + E[\beta_2^\omega])\alpha_1^{HH}(\ln[Y_2 - T_y] - \ln[1 + E[\beta_2^\omega]]) - (1 + \alpha_1^{HH})E[\beta_2^\omega](\ln[Y_1 - T_y] - \ln[1 +

$\alpha_1^{HH}]))$, and $\epsilon = \min\{\epsilon_1, \epsilon_2\}$, then it can be checked that $\Psi_{1x}^{(1)}(\omega) > \Psi_{2x}^{(1)}(\omega)$ and $\Psi_{2y}^{(1)}(\omega) > \Psi_{1y}^{(1)}(\omega)$, $\forall \omega \in \Omega$. Since bid rent functions are continuous in $\bar{u}_j$, for all perturbations $(\Delta\bar{u}_j)_{j\in M}$ such that $|\Delta\bar{u}_j| < \epsilon$, $\forall j \in M$, there exists a unique non-fully revealing MREE in each perturbed economy. Therefore, it is proved that there exists an open subset of economies that possess no fully-revealing MREE.

Appendix F. Proof of Theorem 5

The statement of Theorem 5 is equivalent to saying that under perfect recall final prices $\Psi_k^{(I)}$ generically reveal the information carried by test prices $(\Psi_k^{(i)})_{i=0,1,\ldots,I-1}$.

For the openness part, when $I = 1$, the proof is trivial. When $I > 1$, $(\Psi_k^{(i)})_{i=0,1,\ldots,I-1}$ implies a process of changes in households' locations and in test prices. Condition (43) implies that for each $0 \leq i \leq I-1$, $\exists k \in K, \omega, \omega' \in \Omega$, $\omega \neq \omega'$, such that $\Psi_k^{(i)}(\omega) \neq \Psi_k^{(i)}(\omega')$. Denoting $d_k^{(i)} \equiv |\Psi_k^{(i)}(\omega) - \Psi_k^{(i)}(\omega')|$ and $\bar{d} \equiv \min\{(d_k^{(i)})_{i=0,1,\ldots,I-1;j\in M}\}$, similar to the proof of Theorem 1, we can consider perturbations of utility functions such that the same process of households' locations is implied. That is, $0 < |\Psi_k^{(i)}(\omega) - \Psi_k^{(i)}(\omega')| < \bar{d}$ for the same $k \in K$ and the same $\omega, \omega' \in \Omega$ after perturbations. Then, the same information augmentation process is followed, and it is proved the subset of economies where final prices reveal the information carried by test prices is always open.

For the denseness part, as illustrated in the example in the text, when final prices cannot reveal the information implied by test prices, there must exist $k \in K$, $j, j' \in M$, $j \neq j'$, and $\omega, \omega' \in \Omega$, $\omega \neq \omega'$, such that $\Psi_{jk}^{(I)}(\omega) = \Psi_{j'k}^{(I)}(\omega')$. Since this equality is necessary for final prices failing in revealing the information of test prices, however, we can always find $\Psi_{jk}^{(I)}(\omega) \neq \Psi_{j'k}^{(I)}(\omega')$ for arbitrary small preference perturbations, so the subset of economies where final prices reveal the information carried by test prices is dense.

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