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Abstract

This paper revisits the relationship between money and long-run growth when liquidity demand at the firm level is explicitly modelled. Through a set of sensitivity analyses, I find that this relationship could be positive, negative, or display a hump shape depending on the size of average liquidity demand and the level of financial development. These results explain why existing empirical studies report mixed findings on the relationship.

Keywords: Liquidity Demand, Endogenous Growth, Monetary Supply

JEL Classification: [E51, O16, O42]

1 Introduction

The conventional opinion on the relationship between money and long-run growth, in the empirical literature, is that high inflation hurts the growth of the economy. Correspondingly, in the theoritical literature, many papers have modeled a negative long-run relationship between the rates of growth of money and output. For example, to model this relationship, Paul Gomme (1991) uses the cash in advance approach in an endogenous growth framework; while, Wen-ya Chang (2002) set the real balance as an input into the production of human capital.

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However, recently, there has been evidence supporting a positive relationship between growth and inflation when the inflation rate is low. Ghosh and Phillips (1998) suggests that inflation and growth are positively correlated at very low inflation rates. This can be seen from Figure 1, which reproduces their Figure 2. Interestingly, this chart in fact suggests a hump-shaped relationship between inflation and output growth. Kremer, Bick and Nautz (2009) also reports the significant positive effect of inflation on growth in the industrialized countries, when the inflation rate is below some threshold level. However, there are few theoretical papers modeling this type of positive relationship between inflation and growth, except Wang and Xie (2010) that stressed frictions in labor market and Ploeg and Alogoskoufis (1994) and Paal and Smith (2000) that emphasized the role of "fiat money" in OLG models.

From the point of view of firms' liquidity, this paper provides a microfoundation of the "liquidity channel" at the firm level. Through this channel, the positive relationship between money and output growth is possible, when the rate of the growth of money is low. Higher money supply mitigates the liquidity pressure in the economy, thus enhancing the probability of survival of the production projects. In turn, it is possible to obtain a higher growth rate when the inflation tax is lower.

The methodology used in this paper follows the spirit of Aghion, Angeletos, Banerjee, and Manova (2007) by introducing two types of projects: short-run projects and long-run projects. The long-run projects face liquidity risk, which has to be overcome by borrowing cash from outside. Given the existence of the borrowing constraints, monetary growth is helpful for the survival of the long-run projects. This constitutes the positive effect that money has on growth through the liquidity channel. On the other hand, monetary growth also imposes a traditional "inflation tax", which leads to higher consumption and lower investment.

In a numerical example, our model yields a hump-shaped relationship between money growth and output growth. This is qualitatively consistent with the finding of Ghosh and Phillips (1998) and with the report for the sample of industrialized countries by Kremer, Bick and Nautz (2009). As the subsequent sensitivity analysis shows, the negative relationship between monetary growth and output growth in the high-inflation region is robust; however, the positive effect of monetary growth depends on the average liquidity demand by firms and the borrowing multipler. The former result matches most of the recent evidence, such as Khan and Senhadji (2001), Burdekin, Denzau, Keil, Sitthiyot, and Willett (2004), and Kremer, Bick and Nautz (2009). The latter finding suggests that the differences in the distribution of the demand for liquidity by firms and the degree of development of the financial market might be a potential explanation for the mixed reports on the relationship between inflation rate and growth in the low-inflation region.

The rest of this paper is organized as follows. Section 2 gives the model structure and provides a microfoundation for the "liquidity channel" at the firms level. Section 3 solves this model on the balanced growth path. Section 4 presents the basic result of a numerical experiment. Section 5 conducts the sensitivity analysis. Section 6 concludes.

2 The Model

This section sets up an endogenous growth model with a "liquidity channel" of the monetary transmission mechanism. The monetary economy consists of "firms" and "households". The "households" sector has the standard properties found in the models in the literature on cash in advance (CIA) for consumption. The main difference occurs in the "firms" sector. Firms which own long-run projects, face heterogeneous liquidity risks. This assumption introduces the demand for cash by firms to overcome their liquidity problem. The "liquidity channel" is modeled in this way, with more details to come.

2.1 The Firms

Time is asummed to be discrete in the model. There are two types of firms owned by the households. One type runs the short-run project, which completes in one period and produces goods; the other type runs the long-run project, which needs two periods to complete and to yield an output. The goods produced by these two projects are homogeneous. However, the long-run project has a positive externality that guarantees endogenous improvement of the productivity level (This is described in subsection 2.1.3). This type of project also faces a liquidity risk because of the longer maturity period.

2.1.1 Firms with Short-run Projects

There are an infinite numeber of identical firms of this type, which are continuously and evenly distributed among [0,1]. They rent capitals from the households and use the production technology given by $A_t \mathcal{T}_t^{1-\alpha} K_t^{\alpha}$, to yield goods at the end of each period. Here, A_t is the exogenous technology, $\mathcal{T}_t$ is the endogenous productivity level, and K_t is the capital stock used in the short-run projects.

The objective function of this type of firms can be given by

$$\max \Pi_t^S = P_t(A_t \mathcal{T}_t^{1-\alpha} K_t^{\alpha} - r_t^S K_t),$$

where Π_t^S is the profit of the firm at time t, r_t^S is the real rental rate of the capital in short-run project. From the first order condition, we can show that

$$\alpha A_t \mathcal{T}_t^{1-\alpha} K_t^{\alpha-1} = r_t^S. \tag{1}$$

This means that the marginal productivity of capital used in the short-run project is just equal to the marginal cost, which is the short-run rental rate.

2.1.2 Firms with Long-run Projects

There are also an infinite number of firms with long-run projects. Each of them exists for only two periods. Thus, these firms can be divided into two types, the younger and the older. At any period, both types overlap. When older firms die, new ones come into existence, and younger ones become older. The total number of each type of firms is constant. For convenience, I suppose that each type of these firms are identical and continuously and evenly distributed among [0,1].

Each firm operates only one long-run project. Each long-run project needs one period to build and the other period to produce. Thus, at any period, there are an infinite number of identical long-run projects which are going to yield at the end of the current period; and also, an infinite number of identical long-run projects which are just beginning their installation. A detailed time structure about long-run projects is given in Figure 2.

New firms, owning long-run projects, rent capitial from households at the beginning of the period when they are born. However, the older firms, which have just finished the installation of their long-run capitals, have to face liquidity risks, before their long-run projects can produce any goods. If the liquidity risk of one long-run project is overcome, it will produce goods in next period; if not, it will produce nothing. Irrespective of whether firms with long-run projects produce or not, long-run capital will always depreciate. The rate of depreciation is given by δ^l.

Liquidity Risks The liquidity risk in this paper is modeled following the spirit of Aghion, Angeletos, Banerjee and Manova (2007). Suppose that any long-run project has to incur an additional cost in cash after the installation of its long-run capital. This type of cost can be understood as the demand for cash flow by the long-run projects. These additional costs are heterogenous among the different long-run projects. After the end of the long-run projects, the cash can be collected back by the firms.

Take firm i, which begins its long-run project at time t, as an example. At the beginning of period t, firm i rents Z_t^i units of capital from households and builds its long-run project for the whole period of t. At the end of period t, after finishing the installation, firm i has to pay a random nominal cost, L_t^i. In this model, I suppose that the detrended real value of this additional cost, $l_t^i \equiv \frac{L_t^i}{P_t \mathcal{T}_t}$, follows a lognormal distribution, whose the cumulative distribution function (CDF) is given by

$$F(l) = \frac{1}{2}[1 + \text{erf}(\frac{\ln l - \mu_l}{\sigma_l \sqrt{2}})],$$

and the probability density function (pdf) is

$$f(l) = \frac{1}{\sigma_l l \sqrt{2\pi}} e^{-\frac{(\ln l - \mu_l)^2}{2\sigma_l^2}}.$$

Here, $\text{erf}(\cdot)$ is the error function, μ_l and σ_l are the mean and standard deviation of $\ln l$.

If firm i can borrow enough cash to pay off its nominal additional cost, then it can run its long-run project in next period and will produce goods by the production technology of $A_{t+1}\mathcal{T}_t^{1-\gamma}(Z_t^i)^\gamma$. If this firm cannot pay off its additional cost, its long-run project will fail and it will produce nothing. At the end of firm i's second period, irrespective of whether its long-run project succeeds or fails, the money borrowed from outside is still in this project and can be taken back.

Credit Constraint The credit problem exists in this economy. Each borrower faces a credit constraint and cannot borrow an amount of money that is beyond $\mu(< 1)$ fraction of his evaluated income. Here, the borrowing multiplier, μ, should be interpreted as the loan to value ratio. In the view of lenders, the expected value of the long-run project is equal to its evaluated income, since there is the requirement of paying off the debt first.

Suppose that the lenders are not capable of differentiating between the heterogenous additional costs in the different long-run projects. They have to use the average survival probability $\mathcal{P}_{t+1}$ for the whole industry of long-run projects to evaluate the survival possibility of any single long-run project. Thus, the evaluated income of firm i, which owns a long-run project, is given by $\mathcal{P}_{t+1}P_{t+1}A_{t+1}\mathcal{T}_t^{1-\gamma}(Z_t^i)^\gamma$, where P_{t+1} is the price of goods at time $t+1$. Therefore, the upper limit of money that firm i can borrow, is

$$\bar{D}_{t+1}^i \equiv \mu\mathcal{P}_{t+1}P_{t+1}A_{t+1}\mathcal{T}_t^{1-\gamma}(Z_t^i)^\gamma. \tag{2}$$

Optimal Choice of Long-run Firm In this subsection, I still use the example of firm i, which is born at time t. A detailed description for the optimal choice of this firm is given as below.

Since each firm of long-run project exists for only two periods, its optimal question can be solved backwards. At its second period, firm i makes decisions on whether to borrow money against its liquidity risk, and how much money is needed. Given its optimal strategy in the second period, firm i makes the decision on renting capital at its first period.

To Borrow or Not Borrow Money At the last period of firm i (time $t+1$), its nominal additional cost L_t^i is already known. The problem faced by this firm, is whether to borrow money and how much to borrow.

At first, I need to know how much money this firm volunteers to borrow, when no credit constraint exists.

If firm i does not borrow against its liquidity risk, the value of its long-run project is given by

$$V_{t+1}^N = -P_{t+1}r_t^L Z_t^i,$$

where r_t^L is the real long-run rental at time t. When this firm borrows enough money, the value of its long project can be written as

$$V_{t+1}^B = P_{t+1}A_{t+1}T_t^{1-\gamma}(Z_t^i)^\gamma - R_{t+1}L_t^i - P_{t+1}r_t^L Z_t^i,$$

where R_{t+1} is the nominal interest rate at time $t+1$. Since borrowers have to pay off interest, they would not like to borrow more (or less) money than the amount actually needed. For firm i to have an incentive to borrow from outside, it is needed that

$$V_{t+1}^B \geq V_{t+1}^N.$$

From above inequation, we can find that the maximum amount of money that firm i volunteers to borrow is

$$\bar{L}_t^i = \frac{P_{t+1} A_{t+1} \mathcal{T}_t^{1-\gamma} (Z_t^i)^\gamma}{R_{t+1}}. \tag{3}$$

In this economy, the borrowing multiplier, μ, is less than 1 and the average surviving probability, $\mathcal{P}_{t+1}$, is also not larger than 1. At the same time, generally, an appropriate nominal interest rate R_{t+1} must be less than 1. So, the upper limit of money that firm i can borrow, should be much less than the maximum amount of money that firm i volunteers to borrow, i.e.,

$$\bar{D}_{t+1}^i << \bar{L}_t^i.$$

Therefore, at this stage (time $t+1$), the optimal strategy for firm i, which faces an additional cost, L_t^i, is: to borrow zero unit of money, when this additional cost is higher than the upper limit that it can borrow, $\bar{D}_{t+1}^i$; to borrow L_t^i units of money, when L_t^i is not larger than $\bar{D}_{t+1}^i$.

To Rent Capital At its first period (time t), firm i makes the choice on how much capital it should rent. Given its optimal strategy at time $t+1$, firm i maximizes its expected profit, $E_t \Pi_{t+1}^L(i)$, that is,

$$\begin{aligned} &E_t P_{t+1} \int_0^{\frac{\bar{D}_{t+1}^i}{P_t \mathcal{T}_t}} A_{t+1} \mathcal{T}_t^{1-\gamma} (Z_t^i)^\gamma f(l) dl \\ &- E_t R_{t+1} \int_0^{\frac{\bar{D}_{t+1}^i}{P_t \mathcal{T}_t}} P_t \mathcal{T}_t l \cdot f(l) dl - E_t P_{t+1} r_t^L Z_t^i. \end{aligned}$$

Here, additional cost, L_t^i, can be taken as a state, and the expected survival probability of long-run project, $E_t \mathcal{P}_{t+1}^i$, depends on the realization of L_t^i. As Figure 3 illustrates, the probability that additional cost, L_t^i, is not larger than the upper limit of borrowing, $\bar{D}_{t+1}^i$, is equal to the survival probability of this long-run project.

Above objective function can be rewritten as

$$
\begin{aligned}
&E_t\{\mathcal{P}^i_{t+1}P_{t+1}A_{t+1}\mathcal{T}_t^{1-\gamma}(Z^i_t)^\gamma \\
&-R_{t+1}P_t\mathcal{T}_t\int_0^{\frac{\bar{D}^i_{t+1}}{P_t\mathcal{T}_t}} lf(l)dl - P_{t+1}r^L_tZ^i_t\},
\end{aligned}
$$

where

$$
\mathcal{P}^i_{t+1} \equiv \frac{1}{2}+\frac{1}{2}\,\mathrm{erf}(\frac{\ln\frac{\bar{D}^i_{t+1}}{P_t\mathcal{T}_t}-\mu_l}{\sigma_l\sqrt{2}}). \tag{4}
$$

Given the complexity of this profit function, the parameters restrictions needed to garantee its concavity, are not easily obtained. However, I argue that in this modeled economy, the expected nominal interest rate is controlled to be much lower than 1. In such a monetary environment, a sufficient condition for strict concavity of this objective function can be given by

$$
\mu_l < \ln\mu A(1+g^M) - \frac{\sigma_l^2}{(1-\mu\Lambda)}
$$

(see more details in Appendix 6.1).

Thus, the first order condition of above optimality problem is given as below.

$$
\begin{aligned}
Z^i_t \quad : \quad & \gamma E_t[\frac{1}{2}+\frac{1}{2}\,\mathrm{erf}(\frac{\ln\frac{\mu\mathcal{P}_{t+1}P_{t+1}A_{t+1}\mathcal{T}_t^{1-\gamma}(Z^i_t)^\gamma}{P_t\mathcal{T}_t}-\mu_l}{\sigma_l\sqrt{2}})]A_{t+1}\mathcal{T}_t^{1-\gamma}(Z^i_t)^{\gamma-1} \\
& +\frac{\gamma}{\sigma_l\sqrt{2\pi}}E_te^{-[\frac{\ln\frac{\mu\mathcal{P}_{t+1}P_{t+1}A_{t+1}\mathcal{T}_t^{1-\gamma}(Z^i_t)^\gamma}{P_t\mathcal{T}_t}-\mu_l}{\sigma_l\sqrt{2}}]^2}A_{t+1}\mathcal{T}_t^{1-\gamma}(Z^i_t)^{\gamma-1} \\
& -\frac{\gamma}{\sigma_l\sqrt{2\pi}}E_t\mu R_{t+1}\mathcal{P}_{t+1}e^{-\frac{[\ln\frac{\mu\mathcal{P}_{t+1}P_{t+1}A_{t+1}\mathcal{T}_t^{1-\gamma}(Z^i_t)^\gamma}{P_t\mathcal{T}_t}-\mu_l]^2}{2\sigma_l^2}}A_{t+1}\mathcal{T}_t^{1-\gamma}(Z^i_t)^{\gamma-1} \\
= \quad & r^L_t
\end{aligned} \tag{5}
$$

It is necessary to mention that the marginal effect of long-run capital is composed of the marginal increments of both the expected sales amount and the expected cost of interest. This is because an increment of long-run capital increases not only the survival probability of the long-run project and input factors, but also the expected debt.

2.1.3 Endogenous Productivity

Long-run projects not only produce goods, but also have the positive externality of improving the endogenous productivity level. Following the traditional method in the literature on endogenous growth, I also assume that the evolvement of endogenous productivity follows the rule given as below.

$$\mathcal{T}_{t+1} = \int_0^1 n_{t+1}^i \mathcal{T}_t^{1-\gamma} (Z_t^i)^\gamma di,$$

where n_{t+1}^i is an indicator variable, such that $n_{t+1}^i = 1$, when firm i's long-run project survives; and $n_{t+1}^i = 0$, otherwise.

Since each type of firms with long-run projects are identical and independent, based on the law of large numbers, there are $\mathcal{P}_{t+1} = \frac{1}{2} + \frac{1}{2}\operatorname{erf}(\frac{\ln \frac{\bar{D}_{t+1}}{P_t \mathcal{T}_t} - \mu_l}{\sigma_l \sqrt{2}})$ fraction of long-term projects that can survive at time $t+1$. Therefore, the process of endogenous productivity can be rewritten as

$$\mathcal{T}_{t+1} = \mathcal{P}_{t+1} \mathcal{T}_t^{1-\gamma} (Z_t)^\gamma. \tag{6}$$

2.2 Households

The "households" sector is treated in the standard way. There are an infinite number of independent and identical households. The representive household faces the cash in advance constraint on consumption (8) besides the budget constraint (7). Thus, its optimality question can be given by

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

s.t.:

$$\begin{aligned} & P_t C_t + P_t[K_{t+1} - (1-\delta)K_t + Z_{t+1} - (1-\delta^L)Z_{t-1}] + D_{t+1} + M_{t+1} \quad (7) \\ = \; & P_t(r_t^S K_t + r_{t-1}^L Z_{t-1}) + (1+R_t)D_t + M_t + X_t + \Pi_t^S + \int_0^1 \Pi_t^L(j)dj, \end{aligned}$$

$$P_t C_t \leq M_t + X_t, \tag{8}$$

where C_t denotes the consumption at time t, D_{t+1} is the amount of money that is lent to firms with long-run projects, M_{t+1} is the amount of money that the household holds at the end of time t, and X_t is the money ejection at the beginning of time t.

The first order conditions can be obtained as follows,

$$C_t : C_t^{-\sigma} = \lambda_{1,t}P_t + \lambda_{2,t}P_t, \tag{9}$$

$$D_{t+1} : \lambda_{1,t} = \beta E_t \lambda_{1,t+1}(1 + R_{t+1}), \tag{10}$$

$$M_{t+1} : \lambda_{1,t} = \beta E_t(\lambda_{1,t+1} + \lambda_{2,t+1}), \tag{11}$$

$$K_{t+1} : P_t\lambda_{1,t} = \beta E_t P_{t+1}\lambda_{1,t+1}[r_{t+1}^S + (1 - \delta)], \tag{12}$$

$$Z_{t+1} : P_t\lambda_{1,t} = \beta^2 E_t P_{t+2}\lambda_{1,t+2}[(1 - \delta^l) + r_{t+1}^L]. \tag{13}$$

Here, $\lambda_{1,t}$ and $\lambda_{2,t}$ are the Lagrangian multipliers associated with the budget constraint and the cash in advance constraint, respectively.

2.3 Market Clearing

The goods market clearing condition can be easily written as

$$\begin{aligned} & C_t + K_{t+1} - (1 - \delta)K_t + Z_{t+1} - (1 - \delta^l)Z_{t-1} \\ = \quad & A_t\mathcal{T}_t^{1-\alpha}K_t^\alpha + \mathcal{P}_t A_t \mathcal{T}_{t-1}^{1-\gamma}Z_{t-1}^\gamma \equiv Y_t, \end{aligned} \tag{14}$$

where Y_t is the gross output at time t. Given the timing of money ejection, the money market clearing condition is given as

$$D_{t+1} + M_{t+1} = D_t + M_t + X_t, \tag{15}$$

where M_t is the money held by households and D_t is the money left in long-run projects. Thus, the money growth rate can be written as

$$g_{t+1}^M = \frac{D_{t+1} + M_{t+1}}{D_t + M_t} - 1.$$

2.4 Balanced Growth Path

Suppose the exogenous technology level is always the constant, A. On the balanced growth path, $c_t \equiv \frac{C_t}{\mathcal{T}_t}, k_t \equiv \frac{K_t}{\mathcal{T}_t}, z_t \equiv \frac{Z_t}{\mathcal{T}_t}, 1 + \bar{\pi}_{t+1} \equiv \frac{P_{t+1}}{P_t}, 1 + g_{t+1}^{\mathcal{T}} \equiv \frac{\mathcal{T}_{t+1}}{\mathcal{T}_t}$, $\mathcal{P}_t$, R_t and g_{t+1}^M are all constants.

The equation system on the balanced growth path is summarized as follows. The derivation details can be obtained in Appendix 6.2.

From the binding cash in advance constraint on consumption, we can obtain that

$$(1+g^M) = (1+\bar{\pi})(1+g^{\mathcal{T}}).$$

This long-run relationship still holds even if this CIA constraint does not exists at all. This result implies that the "CIA only on consumption" channel in the monetary transmission mechanism has no real effect in the long-run. This is consistent with the usual conclusion in the relevant literature.

By the intertemporal substitution condition of households, we know that

$$[\frac{1+\bar{\pi}}{\beta(1+R)}]^{-\frac{1}{\sigma}} = 1+g^{\mathcal{T}}.$$

Together with the definitions of real short-run rental and real long-run rental, we obtain following two equations respectively.

$$(1+g^{\mathcal{T}})^{\sigma} = \beta[\alpha A k^{\alpha-1} + (1-\delta)],$$

$$(1+g^{\mathcal{T}})^{2\sigma} = \beta^2\{(1-\delta^L) + r^L\},$$

where

$$\begin{aligned} r^L \quad \equiv \quad & \{\gamma[\frac{1}{2}+\frac{1}{2}\operatorname{erf}(\frac{\ln(\mu A(1+g^M))-\mu_l}{\sigma_l\sqrt{2}})] \\ & + \frac{\gamma}{\sigma_l\sqrt{2\pi}} e^{-\frac{[\ln(\mu A(1+g^M))-\mu_l]^2}{2\sigma_l^2}} (1-\mu R\mathcal{P})\} A z^{\gamma-1}. \end{aligned}$$

From the evolvement process of endogenous technology, we obtain that

$$1+g^{\mathcal{T}} = \mathcal{P} z^{\gamma}, \tag{16}$$

where the survival probability is given as

$$\mathcal{P} = \frac{1}{2}+\frac{1}{2}\operatorname{erf}(\frac{\ln(\mu A(1+g^M))-\mu_l}{\sigma_l\sqrt{2}}). \tag{17}$$

3 Model Simulations

This section presents simulated long-run relationships of above monetary economy by a numerical experiment on the balance growth path.

3.1 Model Parameterization

I set the technology level A as 1. The elasticity of intertemporal substitution of households is chosen to be 1/2. This means that the value of parameter σ is 2. The discount factor β is set as 0.96. The capital shares in the short-run production function and in the long-run production function, α and γ, are both set as 0.3. The depreciation rate of short-run capital, δ, is 0.1. These parameter values are often adopted in business cycle literature. The depreciation rate of long-run capital, δ^L, is set to be equal to $1-(1-\delta)^2$. As in the relevent literature, the borrowing multiplier μ is given by 0.77.

In the baseline numerical experiment, the parameters in the distribution function of additional cost are chosen in following way. As we know, the variance of logarithm of additional cost σ_l^2, must not be negative, and $\mu\Lambda \equiv \mu R_{t+1}\mathcal{P}_{t+1}$ should be less than 1 when there is no hyperinflation. Given the special parameters restriction of

$$\mu_l < \ln \mu A(1+g^M) - \frac{\sigma_l^2}{(1-\mu\Lambda)},$$

μ_l should at least be less than $\ln \mu A(1+g^M)$. In order to analyze the implications of money growth, the minimum value of the rate of money growth, g^M, should be 0. Under above parameterization, we find that $\ln \mu A = \ln 0.77 = -0.261\,36$. Thus, I set the mean of logarithm of additional cost, μ_l, as -0.3. From above parameter restriction, we also obtain that

$$\sigma_l^2 < (1-\mu\Lambda)[\ln \mu A(1+g^M) - \mu_l].$$

In order not to violate this restriction, I set the standard deviation of the logarithm of additional cost, σ_l, as $\frac{1}{7.8}$.

3.2 Effects of the Growth of Money

The long-run effects of the rate of money growth are presented in Figure 4.

As the top left diagram, whose longitudinal coordinate is labelled by "Growth

Rate" shows, with the increase in the rate of money growth, the long-run growth rate first increases from 1%, and approaches its peak at 4%, when the rate of money growth is at 10%. It then begins to decrease. Obviously, 10% is the "threshold" level of the rate of money growth. Above this threshold level, the pace of decline in the growth rate also changes very fast. This pace of decline is much larger in the region of money growth rate between 10% and 40% than in the area where the money growth rate is above 40%. This hump-shaped relationship between output growth and money growth illustrates that modest growth in money supply is good for economic growth; but, excessive money supply hurts growth.

Similarly, a hump-shaped relationship between the rate of real return and money growth can also be generated by this model. It is presented by the first diagram in the second line, whose longitudinal coordinate is labelled by "Real Return of D". The peak of this real return is also approached, when the monetary growth rate is 10%.

As the right-most diagram in the second line shows, there is an almost one-to-one relationship between the inflation rate and the rate of money growth. So, it is appropriate to say that the long-run relationship between the growth and the inflation rate is very similar to that between output growth and money growth, which is shown in the top left diagram. This hump-shaped relationship is qualitatively consistent with the general conclusion in recent empirical reports, such as Ghosh and Phillips (1998) and the studies on industrialized countries by Kremer, Bick and Nautz (2009). In addition, the long-run relationship between investment and inflation should also be similar to what is presented in the top right diagram, whose longitudinal coordinate is labelled by "Investment". This negative correlation is consistent with the finding by Barro (1995).

The reasons behind these results can be given from the point of view of the financial market equilibrium.

It is clear that the upper limit of borrowing, $\bar{D}$, is generally less than the upper bar of floating debt that firms volunteer to borrow, $\bar{L}$. Given the distribution of additional cost in long-run projects, we can easily see that there is always a shortage of liquidity in firm sector. However, the supply of loan for cash, D, is determined by the inflation rate. When the rate of money growth is small, inflation rate is also low. Households have not much incentive to provide cash for loan. However, by equation (17), the increment of the money growth rate would increase the average survival probability of the long-run projects. In turn, this stimulates output and raises the upper limit of borrowing, $\bar{D}$, as shown by equation of (2). Thus, there is extra demand in the loan market. This promotes the nominal interest rate much higher than the inflation rate. Therefore, the real return on nominal debt, D, has to go up. Contrary to Tobin's effect, households would decrease their investment and increase nominal debt supply. Moreover, when the "inflation tax" is low, the positive effect of money supply on output growth, which is through mitigating liquidity pressure, dominates the negative effect from inflation. Therefore, we see a positive relationship between money and growth in the low-inflation region.

However, when money growth rate is higher than the threshold level of 10%, the inflation rate becomes high enough to stimulate the supply of cash for loan. Given the high level of nominal interest rate, the cost of borrowing is much higher. However, the survival probability is already high enough. The marginal benefit to increase survival rate of long-run projects further is low. Thus, the incentive to borrow cash decreases, leading to oversupply in the loan market. The real return of loan would be depressed. Correspondingly, the short-run investment goes up. But, due to the lower increment of the survival rate and the higher loan cost, the long-run investment still goes down and overwhelms the increase in short-run investment. Thus, the long-run capital level always decreases with the increasing rate of money growth, as shown by the diagram, whose longitudinal coordinate is labelled by "Long-run Capital" in Figure 4. So does investment. Even if money growth is still helpful for the survival rate of long-run projects (as shown by the diagram whose longitudinal coordinate is labelled by "Surviving Probability"), its positive effect on output is already much weaker in the high-inflation region. However, the negative impact of "inflation tax" is much stronger, leading to a negative relationship between output growth and money growth when the inflation rate is high.

4 Sensitivity Analysis

As reported above, the results of the baseline experiment depend on the distribution of additional cost in the long-run projects and the credit constraint. In this section, I will analyze the sensitivity of the relationship between output growth and money growth to the mean of logarithm of additional costs, μ_l, and the borrowing multiplier μ in the credit constraint. The effects of the cash in advance constraint on investment are also checked.

4.1 Mean of Logarithm of Additional Costs

4.1.1 Exogenous μ_l

Suppose that the mean of the logarithm of additional costs is set exogenously. To an fixed standard deviation σ_l, a smaller μ_l[1] means less expected additional cost. Thus, the extra demand of loan is less and the survival probability is higher for any fixed rate of money growth. Since 1 is the natual upper limit of the survival probability, the marginal positive effect of money growth to survival probability is lower. Therefore, as presented by Figure 5, the hump-shaped relationship between growth rate and money growth disappears with the decrease of μ_l.

4.1.2 Endogenous μ_l

Compared to the exogenous mean of the logarithm of additional costs, an endogenous mean is more meaningful. It is also more acceptable if the endogenous mean is a strictly increasing function of the detrand long-run capital level, $\frac{Z_t}{\mathcal{T}_t}$.

Suppose the form of function $\mu_l(\frac{Z_t}{\mathcal{T}_t})$, is given by

$$\ln(e^{\eta}(\frac{Z_t}{\mathcal{T}_t})^{\theta}),$$

where η is a constant calibrated by -0.3, the value of parameter θ is non-negative. The equation systems on the balanced growth path becomes to following.

$$1 + g^{\mathcal{T}} = \mathcal{P} z^{\gamma}$$

$$[\frac{1+\bar{\pi}}{\beta(1+R)}]^{-\frac{1}{\sigma}} = 1 + g^{\mathcal{T}}$$

$$(1+g^{\mathcal{T}})^{\sigma} = \beta[\alpha A k^{\alpha-1} + (1-\delta)]$$

$$(1+g^{\mathcal{T}})^{2\sigma} = \beta^2[r^l + (1-\delta^l)]$$

$$\mathcal{P} = \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}(\frac{\ln \mu \mathcal{P} A(1+\bar{\pi}) + (\gamma-\theta)\ln z - \eta}{\sigma_l\sqrt{2}})$$

[1] Given the parameterization in this model, too low a value of μ_l will volates the parameter constraint on concavity of the expected profit of long-run firms.

$$
\begin{aligned}
[\qquad & \gamma\mathcal{P} + \frac{\gamma-\theta-\gamma\mu R\mathcal{P}}{\sigma_l\sqrt{2\pi}} e^{-\frac{[\ln\mu\mathcal{P}A(1+\bar{\pi})+(\gamma-\theta)\ln z-\eta]^2}{2\sigma_l^2}}]Az^{\gamma-1} \\
& -\frac{\theta R}{\sqrt{2\pi}\sigma_l^3(1+\bar{\pi})z}\int_0^{\mu\mathcal{P}(1+\bar{\pi})Az^{\gamma}} e^{-\frac{[\ln l-\ln(e^{\eta}z^{\theta})]^2}{2\sigma_l^2}} \cdot [\ln l - \ln(e^{\eta}z^{\theta})]dl \\
= \quad & r^l
\end{aligned}
$$

When the value of θ is set as 0, this equation systems reverts to the original form and η corresponds to the exogenous μ_l. When θ is positive and small enough to satisfy the parameter restriction on concavity, the hump-shaped relationship between growth and money can still be obtained. The upper diagram with the title of "eta=-0.3" in Figure 6 gives this basic result. Now, let us focus on the region where the money growth rate is below the threshold level. With the increasing value of θ, the positive marginal effect of money growth to output growth becomes stronger. However, when I allow the value of η to be changed, this marginal effect disappears with the decrease of η. The reason is similar to that in the case of exogenous μ_l. Take the case of θ calibrated by 0.01 as an example. Its qualitative property is illustrated by the lower diagram whose title is "theta=0.01" in Figure 6.

4.2 Credit Constraint

If the value of borrowing multiplier μ becomes higher[2], which indicates a more developed financial market, then the maximum amount of loan borrowed by firms with long-run projects will increase, given a fixed long-run capital level. This implies that the development of the financial market can facilitate the mitigation of the liquidity problem. Similar to the case of exogenous μ_l, the marginal effect of money growth to survival probability also decreases with the increase of μ. As Figure 7 illustrates, the marginal effects of money growth rate to output growth, in the area where money growth is below the threshold level, also disappear with the increase of μ.

4.3 CIA on Investment

Suppose that the requirement of cash in advance is not only for consumption but also for ψ fraction of investment. Thus, the new cash in advance constraint

[2] Given the parameterization in this model, a sensitivity analysis on decreasing μ is impossilbe. Because lower value of μ will violates the parameter restriction on concavity of expected profit function of firms who own long-run projects.

can be written as follows,

$$P_t\{C_t + \psi[K_{t+1} - (1-\delta)K_t + Z_{t+1} - (1-\delta^l)Z_{t-1}]\} \leq M_t + X_t.$$

When $\psi = 0$, this constraint is back to original form; while, when $\psi = 1$, it becomes CIA on both consumption and investment. As Stockman (1981) argued, CIA on investment will introduce the effect of "inflation tax" on investment.

Figure 8 presents the basic implications of money growth when $\psi = 1$. The result is very similar to that in the case of CIA on consumption except that the current investment is lower. This is because the constraint of CIA on investment enhances the "inflation tax". With the increasing inflation rate, more money is lent to long-run projects but not held for investment in the next period. However, the stable hump-shaped relationship between growth and money verifies that this basic result is robust whether or not the cash in advance constraint on investment exists.

5 Conclusion

From above numerical experiments, we find that the relationship between output growth and money growth depends on the distribution of additional costs in long-run projects and the value of the borrowing multiplier, which indicates the degree of development of the financial market. This relationship is still robust when the cash in advance constraint on investment is introduced. However, with the decrease of additional cost for liquidity in firms, e^{μ_l}, or, the increase of the borrowing multipler, μ, the positive marginal effect of money growth (or, inflation) to output growth disappears. But, the negative relationship between money growth and output growth, in the region of high-inflation, is very robust.

Given the controversial empirical reports on this relationship in the region of low-inflation, this paper suggests that the differences in liquidity demand and the development of the financial market among individual samples might be a potential explanation. Thus, empirical research incorporating the average requirment for liquidity at the firm level and the borrowing multipler as independent variables, should be the direction of future research.

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6 Appendix

6.1 Concavity of the Expected Profit of Firm i

In this appendix, I check the conditions for the strict concavity of the objective function of firm i who runs a long-run project. Given the objective function as follows,

$$
\begin{aligned}
E_t \Pi^L_{t+1}(i) \quad = \quad & E_t[\frac{1}{2} + \frac{1}{2}\,\mathrm{erf}(\frac{\ln \frac{\mu \mathcal{P}_{t+1} P_{t+1} A_{t+1} \mathcal{T}_t^{1-\gamma} (Z_t^i)^\gamma}{P_t \mathcal{T}_t} - \mu_l}{\sigma_l \sqrt{2}})] P_{t+1} A_{t+1} \mathcal{T}_t^{1-\gamma} (Z_t^i)^\gamma \\
& - E_t R_{t+1} P_t \mathcal{T}_t \int_0^{\frac{\mu \mathcal{P}_{t+1} P_{t+1} A_{t+1} \mathcal{T}_t^{1-\gamma} (Z_t^i)^\gamma}{P_t \mathcal{T}_t}} x f(x) dx - E_t P_{t+1} r_t^l Z_t^i,
\end{aligned}
$$

we can easily obtain the first derivative to Z_t^i :

$$
\begin{aligned}
& \frac{\partial E_t \Pi^L_{t+1}(i)}{\partial Z_t^i} \\
= \quad & \{E_t[\frac{1}{2} + \frac{1}{2}\,\mathrm{erf}(\frac{\ln \mu \Theta (Z_t^i)^\gamma - \mu_l}{\sigma_l \sqrt{2}})] \\
& + \frac{1}{\sigma_l \sqrt{2\pi}} E_t e^{-[\frac{\ln \mu \Theta (Z_t^i)^\gamma - \mu_l}{\sigma_l \sqrt{2}}]^2} (1 - E_t \mu \Lambda)\} \gamma \varkappa (Z_t^i)^{\gamma - 1} \\
& - P_{t+1} r_t^l,
\end{aligned}
$$

where $\Theta \equiv \frac{\mathcal{P}_{t+1}P_{t+1}A_{t+1}\mathcal{T}_t^{1-\gamma}}{P_t\mathcal{T}_t} > 0, \varkappa \equiv P_{t+1}A_{t+1}\mathcal{T}_t^{1-\gamma} > 0, \Lambda \equiv R_{t+1}\mathcal{P}_{t+1}$, and the second derivative to Z_t^i :

$$
\begin{aligned}
& \frac{\partial^2 E_t \Pi_{t+1}^L(i)}{\partial (Z_t^i)^2} \\
= \; & E_t[\frac{1}{2} + \frac{1}{2}\operatorname{erf}(\frac{\ln \mu\Theta(Z_t^i)^\gamma - \mu_l}{\sigma_l\sqrt{2}})]\gamma(\gamma-1)\varkappa(Z_t^i)^{\gamma-2} \\
& + \frac{1}{\sigma_l\sqrt{2\pi}}E_t e^{-[\frac{\ln \mu\Theta(Z_t^i)^\gamma - \mu_l}{\sigma_l\sqrt{2}}]^2}(1 - E_t\mu\Lambda)\gamma(\gamma-1)\varkappa(Z_t^i)^{\gamma-2} \\
& + \gamma\varkappa(Z_t^i)^{\gamma-1}[1 - \frac{(1-E_t\mu\Lambda)(\ln \mu\Theta(Z_t^i)^\gamma - \mu_l)}{\sigma_l^2}]\frac{1}{\sqrt{\pi}}E_t e^{-[\frac{\ln \mu\Theta(Z_t^i)^\gamma - \mu_l}{\sigma_l\sqrt{2}}]^2}\frac{\gamma}{\sqrt{2}\sigma_l Z_t^i} \\
= \; & \gamma\varkappa(Z_t^i)^{\gamma-2}\{E_t(\gamma-1)[\frac{1}{2} + \frac{1}{2}\operatorname{erf}(\frac{\ln \mu\Theta(Z_t^i)^\gamma - \mu_l}{\sigma_l\sqrt{2}})] \\
& + \frac{1}{\sigma_l\sqrt{2\pi}}E_t e^{-[\frac{\ln \mu\Theta(Z_t^i)^\gamma - \mu_l}{\sigma_l\sqrt{2}}]^2}(1 - E_t\mu\Lambda)(\gamma-1) \\
& + \frac{\gamma}{\sigma_l\sqrt{2\pi}}E_t e^{-[\frac{\ln \mu\Theta(Z_t^i)^\gamma - \mu_l}{\sigma_l\sqrt{2}}]^2}(1 - \frac{(1-E_t\mu\Lambda)(\ln \mu\Theta(Z_t^i)^\gamma - \mu_l)}{\sigma_l^2})\}
\end{aligned}
$$

Thus, the sufficient and necessary condition for strict concavity of $E_t\Pi_{t+1}^l(i)$ is

$$
\begin{aligned}
& E_t(\gamma-1)[\frac{1}{2} + \frac{1}{2}\operatorname{erf}(\frac{\ln \mu\Theta(Z_t^i)^\gamma - \mu_l}{\sigma_l\sqrt{2}})] \\
& + \frac{1}{\sigma_l\sqrt{2\pi}}E_t e^{-[\frac{\ln \mu\Theta(Z_t^i)^\gamma - \mu_l}{\sigma_l\sqrt{2}}]^2}(1 - E_t\mu\Lambda)(\gamma-1) \\
& + \frac{1}{\sigma_l\sqrt{2\pi}}E_t e^{-[\frac{\ln \mu\Theta(Z_t^i)^\gamma - \mu_l}{\sigma_l\sqrt{2}}]^2}\gamma(1 - \frac{(1-E_t\mu\Lambda)(\ln \mu\Theta(Z_t^i)^\gamma - \mu_l)}{\sigma_l^2}) \\
< \; & 0
\end{aligned}
$$

In the usual monetary situation, $E_tR_{t+1} < 1$, thus $E_t\mu\Lambda < 1$. Given $\gamma < 1$, we can easily find that the first two terms are both negative. Since the economy in this model is supposed to be around the balanced growth path, together with the fact that on the balanced growth path

$$\mu\Theta(Z_t^i)^\gamma = \mu A(1 + g^M),$$

the condition

$$1 < \frac{(1-E_t\mu\Lambda)(\ln \mu A(1+g^M) - \mu_l)}{\sigma_l^2}$$

can guarantee the third term to be negative.

Therefore, a sufficient condition for concavity of $E_t \Pi_{t+1}^l(i)$ is

$$\mu_l < \ln \mu A(1+g^M) - \frac{\sigma_l^2}{(1-\mu\Lambda)}. \tag{18}$$

This gives a constraint for the choice of parameters of the liquidity cost distribution.

6.2 Derivation of Equations on the BGP

From equation (10) and equation (11), we find that

$$E_t(\lambda_{1,t+1} R_{t+1}) = E_t(\lambda_{2,t+1}) \tag{19}$$

with the rewritten form of equation (10)

$$\frac{E_t \lambda_{1,t+1}}{\lambda_{1,t}} = \frac{1}{\beta(1+R_{t+1})} \neq 0 \tag{20}$$

we know that the value of λ_1 will never be zero. Similarly for λ_2. Thus, at the balanced growth path, the CIA constraint is always binding:

$$P_t C_t = M_t + X_t$$

together with the market clearing condition, we find that

$$P_t C_t + D_t = D_{t+1} + M_{t+1}$$

Thus,

$$1 + g_{t+1}^M \equiv \frac{D_{t+1} + M_{t+1}}{D_t + M_t} = \frac{P_t C_t + D_t}{P_{t-1} C_{t-1} + D_{t-1}}$$

Since at balanced growth path,

$$\frac{C_t}{C_{t-1}} = 1 + g_t^Y \equiv \frac{Y_t}{Y_{t-1}} = \frac{A_t(\frac{\mathcal{T}_t}{\mathcal{T}_{t-1}} k_t^\alpha + \mathcal{P}_t z_{t-1}^\gamma)}{A_{t-1}(k_{t-1}^\alpha + \frac{\mathcal{P}_{t-1} z_{t-2}^\gamma}{\mathcal{T}_{t-1}/\mathcal{T}_{t-2}})}$$

Thus,

$$g^Y = g^{\mathcal{T}}$$

Given

$$D_{t+1} = P_t \mathcal{T}_t \int_0^{\frac{\mu \mathcal{P}_{t+1} P_{t+1} A_{t+1} \mathcal{T}_t^{1-\gamma} (Z_t)^\gamma}{P_t \mathcal{T}_t}} l f(l) dl$$

we can find that at BGP,

$$1 + g_{t+1}^D \equiv \frac{D_{t+1}}{D_t} = \frac{P_t \mathcal{T}_t \int_0^{\frac{\mu \mathcal{P}_{t+1} P_{t+1} A_{t+1} \mathcal{T}_t^{1-\gamma} (Z_t)^\gamma}{P_t \mathcal{T}_t}} l f(l) dl}{P_{t-1} \mathcal{T}_{t-1} \int_0^{\frac{\mu \mathcal{P}_t P_t A_t \mathcal{T}_{t-1}^{1-\gamma} (Z_{t-1})^\gamma}{P_{t-1} \mathcal{T}_{t-1}}} l f(l) dl}.$$

This implies that

$$1 + g^D = (1 + \bar{\pi})(1 + g^{\mathcal{T}}).$$

Therefore,

$$1 + g^M = (1 + \bar{\pi})(1 + g^{\mathcal{T}}). \tag{21}$$

When there is no cash in advance constraint on consumption, the money market clearing condition implies that the growth rate of debt for cash g^D is equal to the growth rate of money supply g^M. Thus, the equation (21) still holds in the case of no CIA.

From equation (9), we find that

$$E_t \frac{C_{t+2}^{-\sigma}}{C_{t+1}^{-\sigma}} = E_t \frac{(\lambda_{1,t+2} + \lambda_{2,t+2}) P_{t+2}}{(\lambda_{1,t+1} + \lambda_{2,t+1}) P_{t+1}}$$

Together with equation (11) and (20), we obtain that

$$E_t \frac{C_{t+2}^{-\sigma}}{C_{t+1}^{-\sigma}} = E_t \frac{1 + \pi_{t+2}}{\beta(1 + R_{t+1})}.$$

Thus,

$$1 + g^C \equiv \frac{C_{t+1}}{C_t} = [\frac{1 + \bar{\pi}}{\beta(1 + R)}]^{-\frac{1}{\sigma}} = 1 + g^{\mathcal{T}} \tag{22}$$

From equation (9), we also find that

$$E_t C_{t+1}^{-\sigma} = E_t (\lambda_{1,t+1} + \lambda_{2,t+1}) P_{t+1}$$

together with equation (19), we obtain that

$$E_t \lambda_{1,t+1} P_{t+1} = E_t \frac{C_{t+1}^{-\sigma}}{1 + R_{t+1}}.$$

At the balanced growth path, together with equation (1), (5) and equation (6), equation (12) and (13) can be rewritten as

$$(1+g^{\mathcal{T}})^{\sigma} = \beta[\alpha A k^{\alpha-1} + (1-\delta)] \tag{23}$$

$$(1+g^{\mathcal{T}})^{2\sigma} = \beta^2\{(1-\delta^L) + r^L\} \tag{24}$$

where $r^L \equiv Az^{\gamma-1}\{\gamma[\frac{1}{2}+\frac{1}{2}\operatorname{erf}(\frac{\ln[\mu A(1+\bar{\pi})(1+g^{\mathcal{T}})]-\mu_l}{\sigma_l\sqrt{2}})]+\frac{\gamma}{\sigma_l\sqrt{2\pi}}e^{-\frac{(\ln[\mu A(1+\bar{\pi})(1+g^{\mathcal{T}})]-\mu_l)^2}{2\sigma_l^2}}(1-\mu R\mathcal{P})\}$. And by the equation (6), we obtain that

$$1+g^{\mathcal{T}}_{t+1} \equiv \frac{\mathcal{T}_{t+1}}{\mathcal{T}_t} = \mathcal{P}_{t+1}{z_t}^{\gamma}.$$

$\Rightarrow$

$$1+g^{\mathcal{T}} = \mathcal{P}z^{\gamma} \tag{25}$$

where

$$\mathcal{P} = \frac{1}{2}+\frac{1}{2}\operatorname{erf}(\frac{\ln[\mu A(1+\bar{\pi})(1+g^{\mathcal{T}})]-\mu_l}{\sigma_l\sqrt{2}}) \tag{26}$$

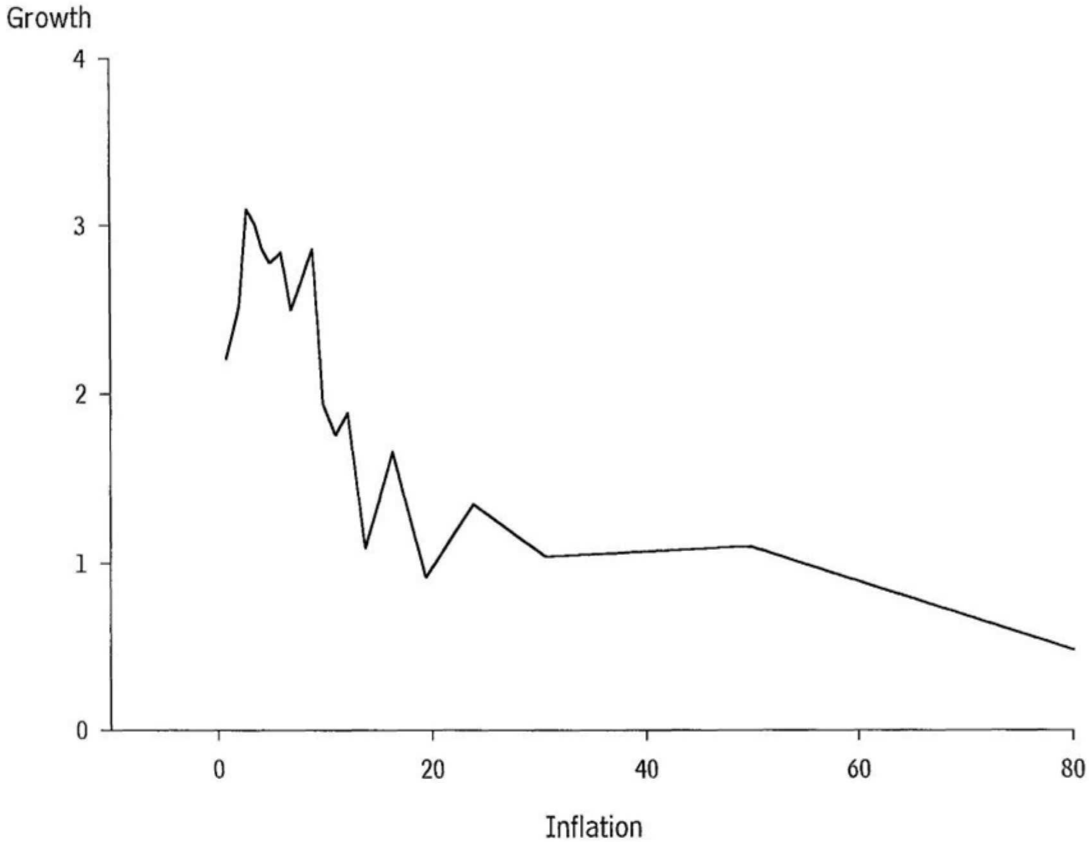


Figure 1 Relationship between Inflation and Growth in Ghosh and Phillips (1998)

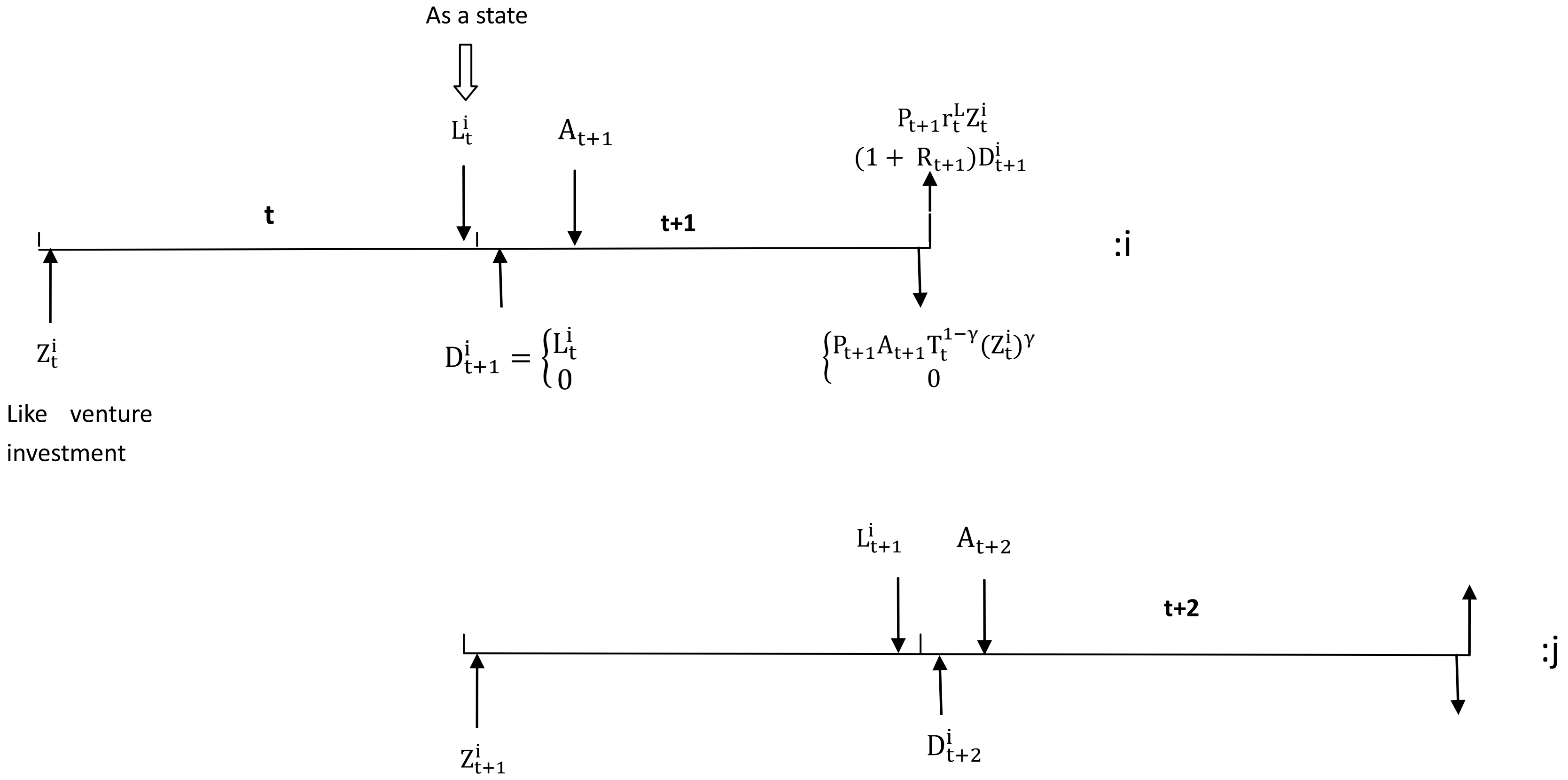
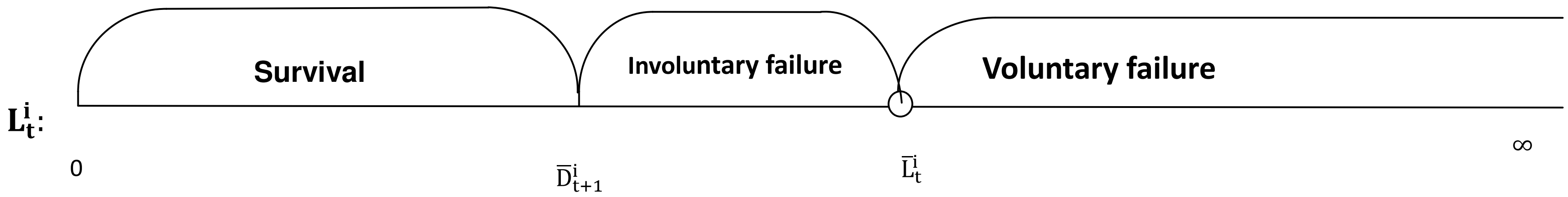


Figure 2: The Timing of Long-Run Projects

For long-run project i:

$\mathcal{P}^{\mathbf{i}}_{\mathbf{t+1}}$ **depends on the realization of** $\mathbf{L^{i}_{t}}$



$\mathbf{D^{i}_{t+1}}$: **the money borrowed from outside at the beginning of time t+1**

Figure 3: The Distribution of Liquidity Cost

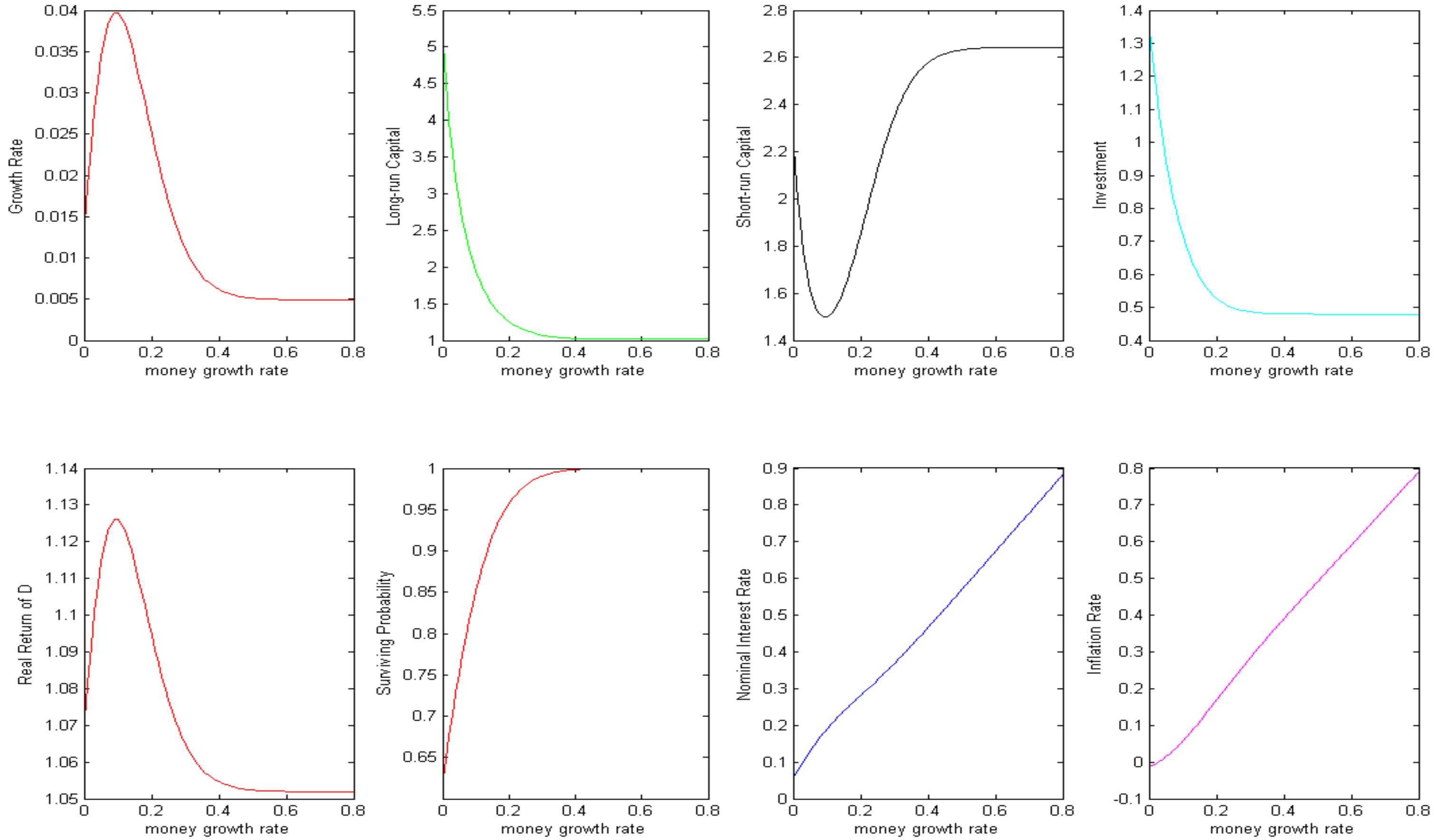


Figure 4 Long-Run Effects of Money Growth

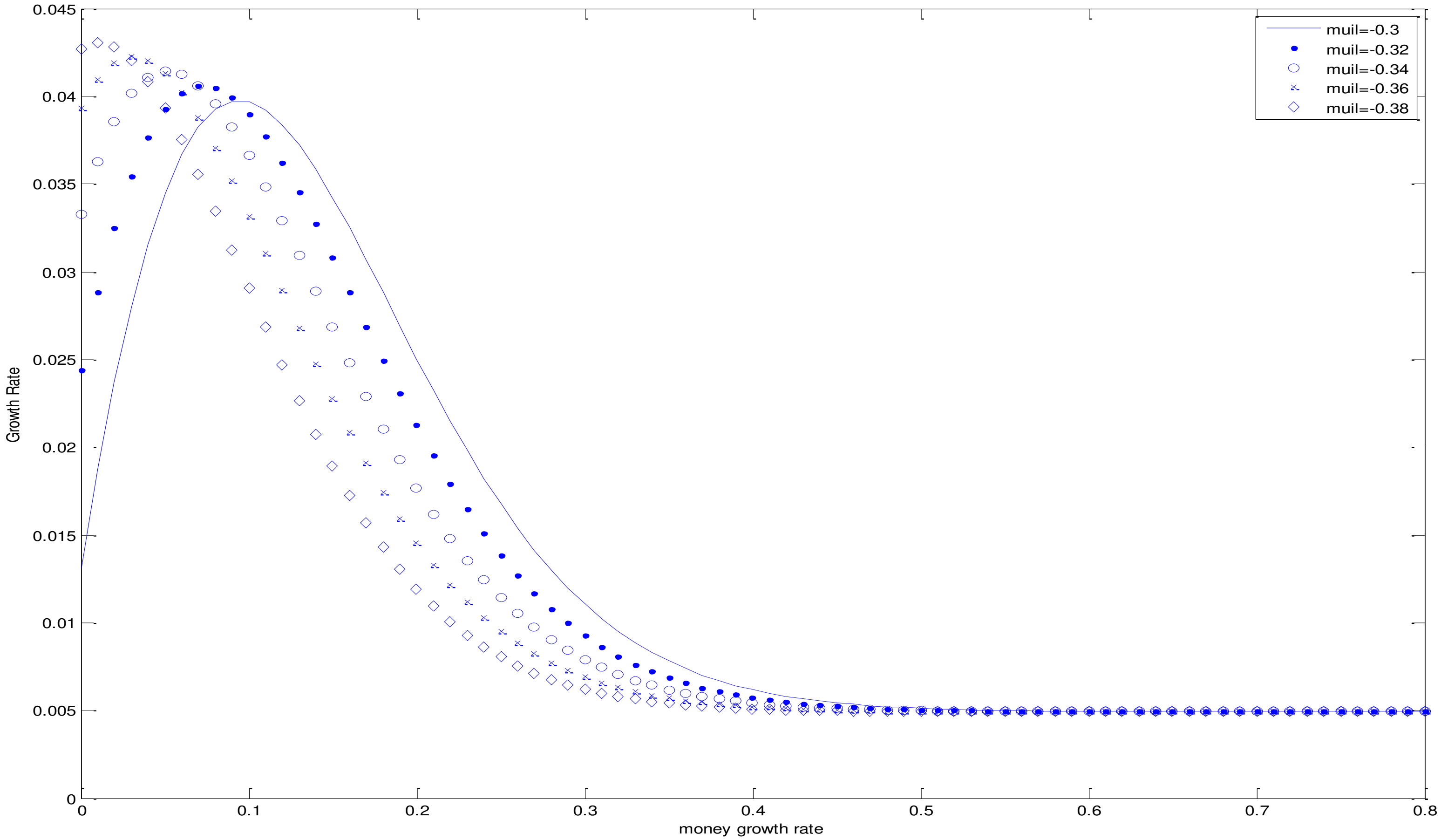


Figure 5 Sensitivity Analysis of Exogenous μ_1

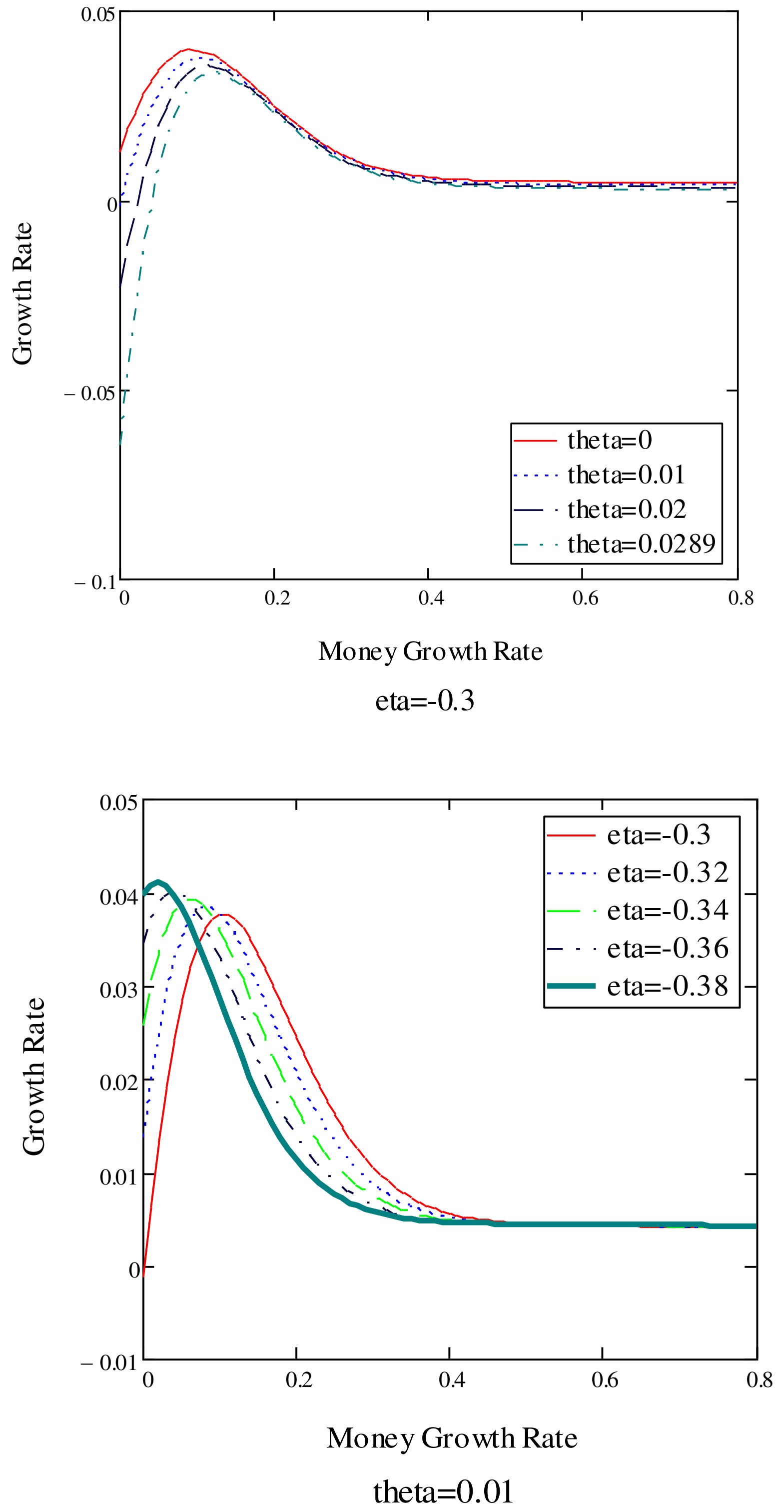


Figure 6 Sensitivity Analysis of Endogenous μ_1

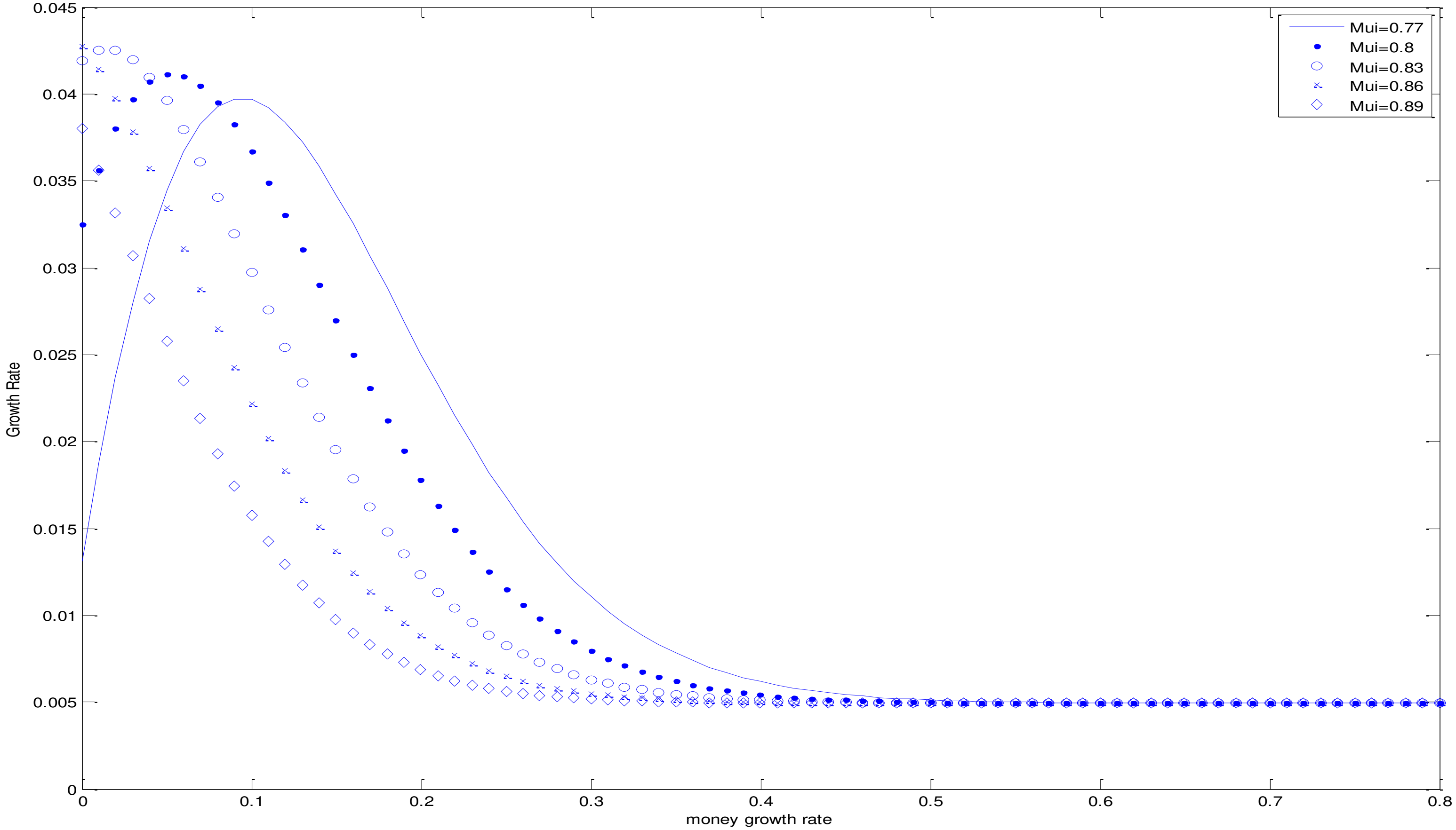


Figure 7 Sensitivity Analysis of Credit Constraint

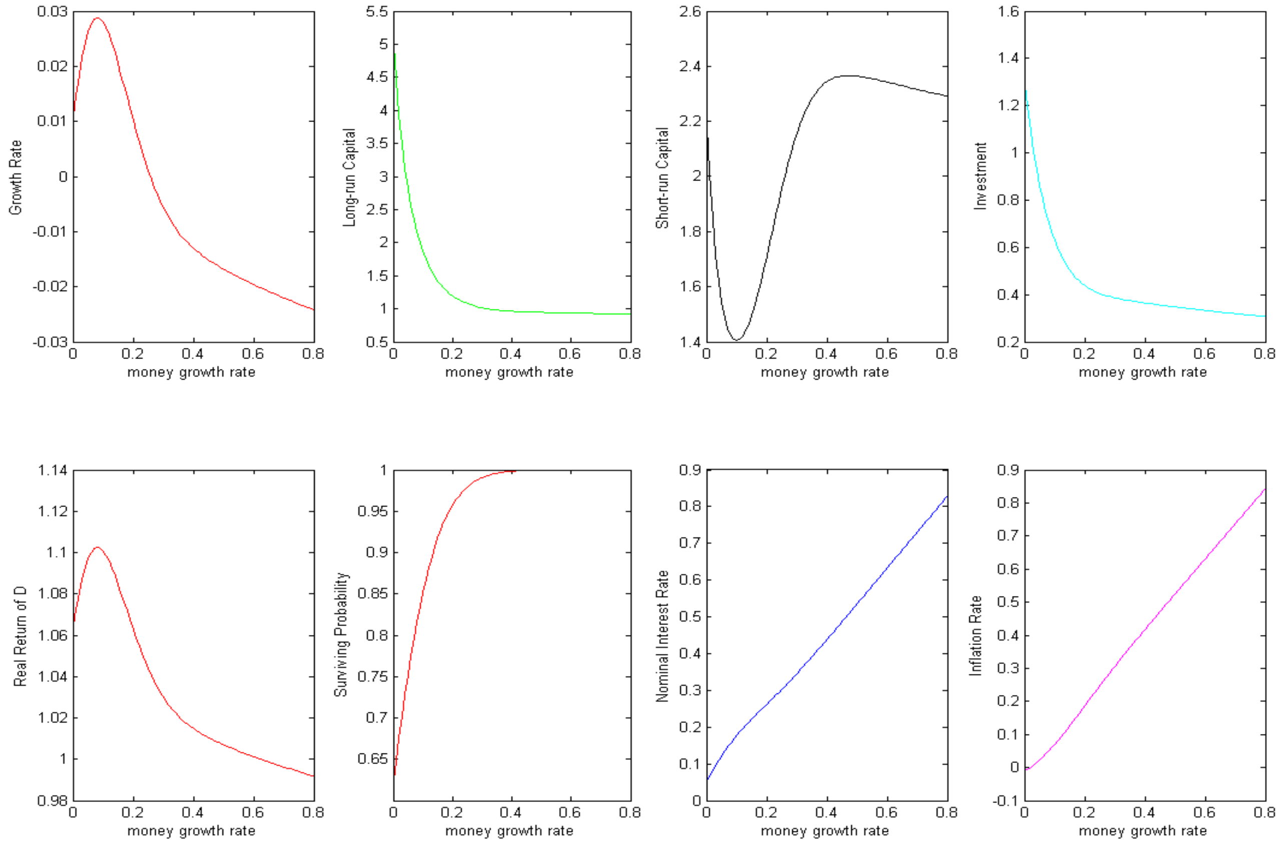


Figure 8 Effects of Money Growth with CIA Constraint on Investment