# Health, growth and welfare: a theoritical appraisal of the long run impact of medical RD

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# Health, Growth and Welfare: a Theoritical Appraisal of the Long Run Impact of Medical R&D*

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**Abstract**

This paper aims at providing a simple economic framework to address the question of the optimal share of investments in medical R&D in total public spending. In order to capture the long-run impact of tax-financed medical R&D on the growth rate, we develop an endogenous growth model in the spirit of Barro [1990]. The model focuses on the optimal sharing of public resources between consumption and (non-health) investment, medical R&D and other health expenditures. It emphasizes the key role played by the public health-related R&D in enhancing economic growth and welfare in the long run.

**JEL**: H23, H51, I18, O31.

**Keywords**: public health, medical R&D, public spending, endogenous growth.

## 1 Introduction

The issue of the optimal size of specific components of public expenditures – such as education, health or defence – has been extensively addressed in the litterature inspired by the seminal paper of Barro [1990], in the framework of

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endogenous growth models in which government spending plays the role of a productive externality and determines the growth rate of the economy in the long run. Nevertheless, even if many empirical and theoretical articles have focused on the effects of health on economic growth, there is little in the literature on the specific impact of tax-financed medical R&D on economic growth.

In order to study this question and avoid the so-called public-spending dichotomy between *utility-enhancing* and *production-related* expenditures, we focuse on the macroeconomic impact of a tax-financed medical R&D by distinguishing two specific components of health spending: tax-financed medical R&D and other health expenditures. In our model, the R&D externalities play a twofold role as utility-enhancing and production-related public expenditures:

*(i)* on the one hand, health-related R&D and its applications improve the performance of medical equipment, *i.e.* the quality of the health services provided by the public health sector and, eventually, the global welfare.

*(ii)* on the other hand, medical R&D increases the total stock of available scientific and technical knowledge, diffusing sooner or later, to the overall economy and, eventually, promoting a more efficient production process.

The first mechanism directly affects the consumers' utility function, while the second affects the aggregate production function: tax-financed medical R&D, through innovations diffusion, generates spillovers effects from the health sector towards the whole productive system.

Even if our main goal is to provide a fine description of the different components of public spending from a theoretical point of view and disentangling the specific effect of health-related R&D public expenditures, the model developed in the paper not only considers public medical R&D, but also private non-health R&D. Because individual firms, when making R&D investments, do not take into account the positive impact of such investments on other firms and the overall economy, total private R&D spending is far below its socially optimal level ; the role of the government is thus to design the appropriate incentive schemes to encourage private firms to sufficiently invest into R&D, to get closer from the socially optimal level.

Taking into account the two types of R&D – tax-financed medical R&D *vs* private (non-medical) R&D – allows us to analyze how the government manages the optimal allocation of tax resources between public funding of health-related R&D on the one side, and the provision of subsidies to private R&D on the other side.

The paper is organized as follows. After the presentation of the model (section 2) section 3 is devoted to the dynamic general equilibrium analysis. Sections 4 and 5 address the policy issues, while proofs and technicalities are placed in the appendices.

# 2 The model

The purpose of this section is to develop a general equilibrium model to address a policy issue, namely the optimal share of health R&D investment in total public expenditure. An appropriate way to capture the impact of health-related R&D on the growth rate of the economy is to develop an endogenous growth model in the spirit of Barro [1990]. In our model the economy is assumed to be populated by three types of agents: households, firms and the government. Their behavior is characterized in the following subsections.

## 2.1 Households

Households live an infinite number of periods during which they consume a private consumption good $c$, a public consumption good $b$ and health public services denoted by $e$. The overall level of utility reached by the representative household is given by the intertemporal utility function:

$$\sum_{t=0}^{\infty} \beta^t \left[ u\left(c_t\right) + v\left(b_t\right) + w\left(e_t\right) \right] \tag{1}$$

where $0 < \beta \equiv 1/\left(1+i\right) < 1$ denotes the discount factor and $i > 0$ the time-preference rate.

Overall public health expenditures are divided into two components: medical R&D $m$ and other health expenditures $n$. Viewed as accumulable stocks these two components produce the health public good $e_t \equiv e\left(m_t, n_t\right)$ under constant returns to scale: $e\left(\mu m, \mu n\right) = \mu e\left(m, n\right)$. The breakdown of public health spending between medical R&D and other health expenditures enables us to disentangle the specific role played by health R&D investments in producinng social welfare.

Because the public consumption good ($b$) and the public health services ($e$) are supposed to be 100% publicly funded, households expenditures consist of private consumption good $(c)$, private investment in capital $(k)$, private investment in R&D $(p)$ and different kinds of taxes. At each period of time the representative household faces the following budget constraint:

$$c_t + k_{t+1} - \Delta_k k_t + p_{t+1} - \Delta_p p_t \leq \left(1-\tau_k\right) r_{kt} k_t + \left(1-\tau_p\right) r_{pt} p_t + \left(1-\tau_l\right) \omega_t l_t \tag{2}$$

where $\delta_i \equiv 1 - \Delta_i$, with $i = k, p$, denotes different depreciation rates for private capital and private R&D from a period to another.

Consumption and investment net expenditures are on the left side of equation (2) while on the right side figures the disposable income with $r_k$ and $r_p$ the real returns on capital and on private R&D, $\omega$ the real wage and $\tau_k$, $\tau_p$, $\tau_l$ the tax rates on capital, private R&D and labor income, respectively.[1]

For simplicity, labor supply is assumed to be inelastic and normalized to one:

$$l_t = 1 \tag{3}$$

In such a framework the consumer's problem is maximizing the intertemporal utility function (1) subject to the sequence of constraints (2). Deriving the

[1] In this model, the policy maker can take into account the positive externalities associated with private R&D, by reducing the tax rate on the real returns of private R&D, in order to raise private R&D investments.

infinite-horizon Lagrangian function with respect to $k_t$, $p_t$ and $c_t$, eliminating the Lagrange multipliers and rearranging the first-order conditions leads to a No-Arbitrage Condition, which can be interpreted as an equilibrium condition:

$$\Delta_k + (1-\tau_k)\, r_{kt} = \Delta_p + (1-\tau_p)\, r_{pt} \tag{4}$$

to an Euler equation:

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta\left[\Delta_k + (1-\tau_k)\, r_{kt+1}\right] \tag{5}$$

and to the budget constraint (2), now with equality.

Eventually, the optimal solution must satisfy the transversality condition:

$$\lim_{t\to\infty} \lambda_t\,(k_{t+1} + p_{t+1}) = 0 \tag{6}$$

In order to simplify future calculations, and to get tractable equations, we assume that the utility functions $u$, $v$ and $w$ are characterized by constant elasticities of intertemporal substitution in consumption.

**Assumption 1** *CIES preferences:*

$$h(x) \equiv c_h \frac{x^{1-1/\varepsilon_h} - 1}{1 - 1/\varepsilon_h} \text{ if } \varepsilon_h \neq 1;\ h(x) \equiv c_h \ln x \text{ if } \varepsilon_h = 1 \tag{7}$$

*where* $h \equiv u, v, w$ *and, without loss of generality,* $c_u + c_v + c_w = 1$.

## 2.2 Firms

Technology is represented by a production function which processes six inputs: three choice variables for the firm – its stock of capital ($k$), its stock of knowledge resulting from its past and current investments in R&D ($p$), the labor demand ($l$) – and three externalities – the stock of (health-unrelated) public capital ($a$), the stock of knowledge resulting from public investments in medical R&D ($m$) and, eventually, the average stock of knowledge resulting from other firms' private investments in R&D ($\bar{p}$). The public capital $a$, viewed as a productive externality, is simply the result of the accumulation of all past and current public expenditures generating productive externalities. Health R&D expenditures also affect the global productivity through a standard R&D externality.

**Assumption 2** *(i) The production function* $F(k, p, l, a, m, \bar{p})$ *exhibits constant returns to scale in capital, private R&D and labor:*

$$F(\mu k, \mu p, \mu l, a, m, \bar{p}) = \mu F(k, p, l, a, m, \bar{p})$$

*(ii) The intensive production function* $\tilde{f}(\kappa, \pi, a, m, \bar{p}) \equiv F(\kappa, \pi, 1, a, m, \bar{p})$*, where* $\kappa \equiv k/l$ and $\pi \equiv p/l$ *is supposed to be homogeneous of degree one with respect to its arguments:*

$$\tilde{f}(\mu\kappa, \mu\pi, \mu a, \mu m, \mu\bar{p}) = \mu\tilde{f}(\kappa, \pi, a, m, \bar{p})$$

Producers maximizes the profit $F(k_t, p_t, l_t, a_t, m_t, \bar{p}_t) - r_{kt}k_t - r_{pt}p_t - \omega_t l_t$ with respect to the capital stock $k_t$, the R&D $p_t$ and the labor force $l_t$, considering all the externalities – *i.e.* $a$, $m$ and $\bar{p}$ – as constants. The firm equilibrium is thus defined by the equality between the real cost and the productivity of each input *i.e.* in terms of the intensive production function:

$$\begin{aligned} r_{kt} &= \tilde{f}_\kappa(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) \\ r_{pt} &= \tilde{f}_\pi(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) \\ \omega_t &= \tilde{f}(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) - \kappa_t \tilde{f}_\kappa(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) - \pi_t \tilde{f}_\pi(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) \end{aligned} \tag{8}$$

## 2.3 Government

The overall stock of public capital $g$ is the sum of the stocks of *(i)* health-unrelated public capital $a$ (public networks substructures, education *etc.*), *(ii)* public consumption $b$, *(iii)* knowledge $m$ resulting from public investments in health R&D and *(iv)* other non-R&D health spending $n$ (medical equipment, current wages, hospital buildings and so on): $g_t \equiv a_t + b_t + m_t + n_t$. All these stocks result from accumulation of flows and depreciation across time.

The government budget constraint at time $t$ is thus given by:

$$\begin{aligned} & a_{t+1} - \Delta_a a_t + b_{t+1} - \Delta_b b_t + m_{t+1} - \Delta_m m_t + n_{t+1} - \Delta_n n_t \\ \leq\ & \tau_k r_{kt} k_t + \tau_p r_{pt} p_t + \tau_l \omega_t l_t \end{aligned} \tag{9}$$

where $\delta_i \equiv 1 - \Delta_i$ is the depreciation rate of the public capital of type $i$, the right-hand side of (9) representing the total amount of tax receipts.[2]

In such an economy the economic policy of the government is simply described by the tax vector $(\tau_k, \tau_p, \tau_l)$ and the breakdown of the public "capital" $g$ into its four components:

$$(\sigma_a, \sigma_b, \sigma_m, \sigma_n) \equiv (a_t/g_t, b_t/g_t, m_t/g_t, n_t/g_t) \tag{10}$$

with, of course,

$$\sigma_a + \sigma_b + \sigma_m + \sigma_n = 1 \tag{11}$$

Using the sharing (10) and the budget constraint, equation (9) can be rewritten

$$g_{t+1} - \Delta g_t \leq \tau_k r_{kt} k_t + \tau_p r_{pt} p_t + \tau_l \omega_t l_t \tag{12}$$

where the depreciation factor of public capital $g$ can be viewed as a weighted average of specific depreciation factors:

$$\Delta \equiv \sigma_a \Delta_a + \sigma_b \Delta_b + \sigma_m \Delta_m + \sigma_n \Delta_n \tag{13}$$

[2] A lag could be introduced between fiscal revenues and public expenditures, but this would not change the **long-term** analysis and the stationary state of the model.

Notice that the "usual" breakdown of the total amount of public spending into four flow components – investment (excluding health), consumption, medical R&D and other health expenditures – can be recovered as:[3]

$$(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n) \equiv \left( \frac{a_{t+1} - \Delta_a a_t}{g_{t+1} - \Delta g_t}, \frac{b_{t+1} - \Delta_b b_t}{g_{t+1} - \Delta g_t}, \frac{m_{t+1} - \Delta_m m_t}{g_{t+1} - \Delta g_t}, \frac{n_{t+1} - \Delta_n n_t}{g_{t+1} - \Delta g_t} \right) \tag{14}$$

still with $\tilde{\sigma}_a + \tilde{\sigma}_b + \tilde{\sigma}_m + \tilde{\sigma}_n = 1$.

# 3 Equilibrium dynamics

The equilibrium in the labor market is characterized by an inelastic labor supply (*cf.* (3)): in order to compute the general equilibrium, we focus on the markets of inputs and goods. Since all (competitive) firms are identical, we have in equilibrium: $\bar{p}_t = p_t = \pi_t$ and, since $l_t = 1$,[4]

$$\begin{aligned} r_{kt} k_t + r_{pt} p_t + \omega_t l_t &= r_{kt} \kappa_t + r_{pt} \pi_t + \omega_t \\ &= \tilde{f}(\kappa_t, \pi_t, a_t, m_t, \pi_t) \equiv f(\kappa_t, \pi_t, a_t, m_t) \end{aligned} \tag{15}$$

Let us now define five elasticities of interest:

$$\begin{aligned} s_{kt} &\equiv \frac{f_\kappa(\kappa_t, \pi_t, a_t, m_t)\,\kappa_t}{f(\kappa_t, \pi_t, a_t, m_t)} = \frac{r_{kt}\kappa_t}{f(\kappa_t, \pi_t, a_t, m_t)} \\ s_{pt} &\equiv \frac{\tilde{f}_\pi(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t)\,\pi_t}{\tilde{f}(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t)} = \frac{r_{pt}\pi_t}{f(\kappa_t, \pi_t, a_t, m_t)} \\ s_{\pi t} &\equiv \frac{f_\pi(\kappa_t, \pi_t, a_t, m_t)\,\pi_t}{f(\kappa_t, \pi_t, a_t, m_t)} = 1 - s_{kt} - s_{at} - s_{mt} \\ s_{at} &\equiv \frac{f_a(\kappa_t, \pi_t, a_t, m_t)\,a_t}{f(\kappa_t, \pi_t, a_t, m_t)} \\ s_{mt} &\equiv \frac{f_m(\kappa_t, \pi_t, a_t, m_t)\,m_t}{f(\kappa_t, \pi_t, a_t, m_t)} \end{aligned}$$

The first two elasticities are the shares of capital and private R&D revenues in total income.

In order to simplify the analytical results, but without a substantial loss of generality, we assume a common depreciation rate between private research and capital:

**Assumption 3** $\Delta_p = \Delta_k$.

Moreover, we assume also that the shares of capital income and private R&D in total income are constant as in the case of a Cobb-Douglas technology.

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[3] The link between $\sigma$ and $\tilde{\sigma}$ becomes an explicit function at the steady state (see formula (85) in the Appendix 2).

[4] We notice that $f(\kappa, \pi, a, m)$ is still homogeneous of degree one and $f_\kappa(\kappa, \pi, a, m) = \tilde{f}_\kappa(\kappa, \pi, a, m, \pi)$.

**Assumption 4** *The elasticities vector* $(s_{kt}, s_{pt}, s_{at}, s_{mt}) = (s_k, s_p, s_a, s_m)$ *is constant.*

In addition, we introduce three variables of interest in order to compute the endogenous growth dynamics:

$$\begin{aligned} x_t &\equiv \kappa_t/g_t \\ y_t &\equiv c_t/g_t \quad (16) \\ \gamma_t &\equiv g_{t+1}/g_t \quad (17) \end{aligned}$$

Finally, we define the tax pressure as an average tax rate

$$\tau \equiv s_k\tau_k + s_p\tau_p + (1 - s_k - s_p)\,\tau_l \quad (18)$$

and the intensive production function as follows:

$$\varphi(x_t) \equiv f\left(x_t, \frac{1-\tau_p}{1-\tau_k}\frac{s_{pt}}{s_{kt}}x_t, \sigma_a, \sigma_m\right)$$

The following proposition characterizes the intertemporal equilibrium. The first equation comes from the Euler equation, while the second from the budget constraint. The details of the proof are presented in the **Appendix 1.**

**Proposition 1** *Under the Assumptions 1-4, an intertemporal equilibrium with perfect foresight is a sequence* $(x_t, y_t)_{t=0}^{\infty}$ *that satisfies (i) the initial conditions* $(k_0, g_0)$*, (ii) the transitional dynamics:*

$$[\Delta + \tau\varphi(x_t)]\frac{y_{t+1}}{y_t} = \left(\beta\left[\Delta_k + (1-\tau_k)\frac{s_k}{s_k+s_\pi}\varphi'(x_{t+1})\right]\right)^{\varepsilon_u} \quad (19)$$

$$y_t + \left(1 + \frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)(\Delta x_{t+1} - \Delta_k x_t) = \left[1 - \tau - \left(1 + \frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)\tau x_{t+1}\right]\varphi(x_t) \quad (20)$$

*for* $t = 0, 1, \ldots$*, and (iii) the transversality condition (6).*

**Proof.** See the Appendix 1.

We observe that these equations constitute a two-dimensional dynamic system in $(x_t, y_t)$ where only the variable $x_t$ is predetermined. $y_t$ inherits the status of jump variable from $c_t$.

## 3.1 Stationary state

In order to compute the steady state, we solve the system (19-20) after canceling out the time subscripts:

$$\gamma = \Delta + \tau\varphi(x) = \left(\beta\left[\Delta_k + (1-\tau_k)\frac{s_k}{s_k+s_\pi}\varphi'(x)\right]\right)^{\varepsilon_u} \quad (21)$$

$$y = \left(1 + \frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)(\Delta_k - \Delta)\,x + \left[1 - \tau - \left(1 + \frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)\tau x\right]\varphi(x) \quad (22)$$

Growth is balanced (usual arguments of endogenous growth theory apply): $\gamma \equiv g_{t+1}/g_t = c_{t+1}/c_t = k_{t+1}/k_t = p_{t+1}/p_t$. Noticing that $\lambda_t = \beta^t u'(c_t)$ and using (7), the transversality condition (6) becomes[5] $\beta\gamma^{1-1/\varepsilon_u} < 1$. Thus, we get $\gamma < \Delta_k + \rho$ from (21), where $\rho \equiv (1-\tau_k) r_k = (1-\tau_k) \varphi'(x) s_k/(s_k + s_\pi)$ is the after-tax return on capital.

## 3.2 Local dynamics

Raising the question of saddle-path stability is not a mere theoretical matter. Indeed, as shown by Blanchard and Quah [1989], saddle-path stability implies the uniqueness of equilibrium under rational expectations.

In this section, we show, without introducing additional restrictions on the fundamentals, that the equilibrium is a saddle path and converges to the stationary state. Our proof is sufficiently general to show the uniqueness of equilibrium as a robust feature of Barro-like models.

In the saddle case, the converging path is the unique solution of the dynamic system under rational expectations because the other trajectories either make some variable negative, soon or later, or violate the transversality condition. Since $x_0$ is a predeterminate variable, the control variable $y_0$ jumps to ensure that the starting point $(x_0, y_0)$ belongs to the saddle path.

The following proposition proves the uniqueness of equilibrium transition.

**Proposition 2** *(Saddle-path stability) Under the Assumptions 1-4, the general equilibrium with rational expectations is unique.*

**Proof.** See the Appendix 1.

Proposition 2 recovers the equilibrium determinacy of Barro [1990] where, however, dynamics are poorer due to the lack of short-run transitions. The economy jumps from the very beginning on the steady state because dynamics are driven by a simple equation with one non-predetermined variable and an unstable eigenvalue. In our model, determinacy still prevails, but equilibrium transitions becomes possible.

# 4 Optimal policy

As seen above, a key issue of this model is to find the optimal (welfare-maximizing) breakdown of public capital ($g$) into four components: productive public capital ($a$), public consumption ($b$), stock of knowledge issued from public investments in medical R&D ($m$) and other public expenditures on health ($n$). Until now, economic agents (households and firms) were supposed to solve their programs, taking as given the economic policy, *i.e.* the tax rates and the breakdown of public capital into these four components. Now the government is supposed to compute the optimal policy, that is the vector of optimal shares of public capital and tax rates $\left(\sigma_a^*, \sigma_b^*, \sigma_m^*, \sigma_n^*, \tau_k^*, \tau_p^*, \tau_l^*\right)$, given the private agents' best

[5] Simply, solve the limit: $\lim_{t\to\infty} c_u c_0^{-1/\varepsilon_u} (k_0 + p_0) \gamma \left(\beta\gamma^{1-1/\varepsilon_u}\right)^t = 0$.

responses. As the different shares resulting from the breakdown of public capital add up to unity (*cf.* (11)): $\sigma_a + \sigma_b + \sigma_m + \sigma_n = 1$, the number of policy tools reduces to six endogenous variables and policy making sums up to computing and announcing an optimal vector $\left(\sigma_a^*, \sigma_m^*, \sigma_n^*, \tau_k^*, \tau_p^*, \tau_l^*\right)$.

Under an inelastic labor supply and no restrictions on the tax rates, it is straightforward that the optimal policy would be a corner solution consisting in levying taxes on labor income at a full rate ($\tau_l^* = 1$) and subsidizing ($\tau_k^*, \tau_p^* < 0$) the inputs that generate positive externalities *i.e.* private capital and private R&D. In order to rule out such nonsensical policy, we assume the same tax rate $\tau_q$ on capital and labor income: $\tau_q \equiv \tau_k = \tau_l$. This restriction is far from being unrealistic and is compatible with a balanced growth path.[6] A common tax rate on capital and labor implements an interior solution because capital supply is elastic and the capital is an essential input in the production function (see the Inada conditions).

This restriction on tax rates brings back to five the number of policy variables, while leaving the tax rate $\tau_p$ on private R&D an independent tool; we can freely play with $\tau_p$ in order to evaluate the macroeconomic impact of subsidizing private investments in R&D.

## 4.1 Characterization

The shortcut of a representative agent, makes equivalent for the government to maximize, with respect to the five policy tools $(\sigma_a, \sigma_m, \sigma_n, \tau_p, \tau_q)$, any social welfare function – strictly increasing in the individual utilities – or the representative agent's utility function (1).

To keep things as simple as possible, let us focus directly on the case of regular growth (in the long-run the equilibrium will be sufficiently close to the steady state).[7]

We will use a Cobb-Douglas production function not only to satisfy the homogeneity property (see Assumption 2 and (15)): $f(\mu\kappa, \mu\pi, \mu a, \mu m) = \mu f(\kappa, \pi, a, m)$, but also to simplify numerical simulations. Similarly, we assume a Cobb-Douglas production function for medical care.

**Assumption 5** *The production functions $F$ and $e$ are specified as follows:*

$$\begin{aligned} F(k, p, l, a, m, \bar{p}) &= \theta k^{s_k} p^{s_p} l^{1-s_k-s_p} a^{s_a} m^{s_m} \bar{p}^{1-s_k-s_p-s_a-s_m} \\ e(\sigma_m, \sigma_n) &\equiv B \sigma_m^{\beta_m} \sigma_n^{\beta_n} \end{aligned}$$

*with $\beta_m + \beta_n = 1$.*

[6] Leisure demand is bounded and can not grow as the other arguments in the utility function, namely private and public consumption. The King-Plosser-Rebelo utility function can not be considered because of separability.

[7] In fact, because of the uniqueness of the equilibrium, we could compute utility along a transitional path, whenever the starting point stands off the steady state, and maximize its value with respect to the policy parameters, but it would drive us to hard analytical computations. As the main goal of the paper is to analyze long-run policy effects, we can focus on the steady state or transitional equilibria close enough (by continuity, the optimal rule will change little along an equilibrium path in a neighborhood of the stationary state).

Eventually, we restrict ourselves to the case of logarithmic utility functions, which are easier to handle and widespread in the RBC literature.

**Assumption 6** $u(c) \equiv c_u \ln c$, $v(b) \equiv c_v \ln b$, $w(e) \equiv c_w \ln e$.

A logarithmic utility function corresponds to the case of a unit elasticity of intertemporal substitution. The social welfare function becomes,

$$W = \sum_{t=0}^{\infty} \beta^t c_u \ln c_t + \sum_{t=0}^{\infty} \beta^t c_v \ln b_t + \sum_{t=0}^{\infty} \beta^t c_w \ln e_t \tag{23}$$

where, without loss of generality:

$$c_u + c_v + c_w = 1 \tag{24}$$

The following proposition characterizes the optimal policy in terms of first-order conditions and generalizes the well-known Barro's (1990) result which recommends to implement a fiscal pressure equal to the share of public good externalities in total income (the more productive the public good, the higher the taxation levied to finance it).

**Proposition 3** *Under the Assumption 3, 5, 6, the optimal policy which maximizes the welfare function (23) along the balanced-growth path is a vector*

$$\left(\sigma_a, \sigma_b(\varkappa^*), \sigma_m(\varkappa^*), \sigma_n(\varkappa^*), \tau_k(\varkappa^*), \tau_p(\varkappa^*), \tau_q(\varkappa^*)\right)$$

*where* $\varkappa^* = (\varphi, \sigma_a)^*$ *is solution of a two-dimensional system:*

$$F_1(\varphi, \sigma_a) = 0 \tag{25}$$

$$F_2(\varphi, \sigma_a) = 0 \tag{26}$$

*The explicit form of functions* $\sigma_b$, $\sigma_m$, $\sigma_n$, $\tau_k$, $\tau_p$, $\tau_q$, $F_1$, $F_2$ *is provided in the Appendix 1.*

**Proof.** See the Appendix 1.

## 4.2 Numerical computation

The purpose of this subsection is to compute the optimal economic policy $\left(\sigma_a^*, \sigma_b^*, \sigma_m^*, \sigma_n^*, \tau_p^*, \tau_q^*, \tau^*\right)$ characterized above (see Proposition 3). As the analytical resolution of the system is not possible, we fix plausible values for the structural parameters and we solve the resulting system.

### 4.2.1 Parametrization

The yearly rate of time preference is plausibly set equal to 4%. As our model does not allow us to distinguish between the depreciation rate of private capital $\delta_k$ and the depreciation rate of private R&D $\delta_p$, we assume a common 8% annual depreciation rate for both types of capital, corresponding approximately to a 50% depreciation after 8.5 years. To avoid any bias in favor of public medical R&D and to be consistent with the calibration of the other depreciation rates, we set to 8% the depreciation rate $\delta_m$ of the stock of knowledge issued from public investments in health-related R&D.

The depreciation rate $\delta_a$ of productive public capital is set to 5% to take into account that public and private capital usually depreciate at different rates, reflecting *(i)* the casual observation that some types of governmentally supplied infrastructure (*e.g.* roads, port facilities, nuclear power stations, *etc.*) are typically more durable than those provided by private agents, *(ii)* the fact that a significant part of public investment is devoted to increase human capital which is characterized by a lower depreciation rate, often below 2%, than the depreciation rate of physical capital.

Finally, the depreciation rate $\delta_b$ of public consumption is set at 100% (full yearly depreciation) whereas the depreciation rate of ordinary health expenditures ($\delta_n$) – which is a weighted average of a 100% depreciation rate associated with public health consumption (wage bill of the public health sector, drugs/medical consumption refunded by social security administrations, *etc.*) and the lower depreciation rates associated with medical equipment, hospital buildings, *etc.* – is set at 61%.[8] The share $s_k$ of capital remuneration in GDP is set to 75% according to the empirical estimates by Mankiw, Romer & Weil [1992], Aghion & Howitt [1997] and other empirical estimates. $s_k$ is a measure of both human and physical capital share in total income, while $1 - s_k = s_a + s_m + s_p$ represents the overall weight of the three productive externalities associated with public capital ($a$), private R&D ($p$) and medical R&D ($m$).

In order to provide a conservative evaluation of the macroeconomic impact of R&D expenditures and to avoid any overestimation, we minimize the size of public and private R&D externalities by setting $s_p = s_m = 1\%$.

With the same thought in mind, we chose *(i)* to limit the relative weight of the health public good in the household's utility function – *i.e.* the indirect impact of health-related R&D on social welfare – by considering that households strongly prefer private consumption and public consumption: $c_u = c_v = 46\%$ *i.e.* $c_w = 8\%$,[9] *(ii)* to limit the direct role played by medical R&D in the production of health services: $\beta_m = 10\%$.

This set of conservative and, in a way, pessimistic assumptions, about the R&D mechanisms at work in the economy, should shield us from criticisms about a possible overestimation of their effects on the equilibrium growth rate and on social welfare.

Finally, the productivity parameter $\theta$ is set to 0.5631. More precisely, the

[8] In France, the amount of public health investment (investment in public hospitals, excluding consumption of intermediate goods) in 2006, was around 5.2 billions €, representing approximately 4% of non-R&D public health expenditures. This means that 96% of these expenditures are in fact pure consumption. In such a case, given a 100% depreciation rate for the consumption part and a 6% depreciation rate for the investment part, one gets an average 61% depreciation rate for non-R&D public health expenditures.

[9] Notice that the share of the total amount of public health expenditures in GDP in France is approximately equal to 9%; setting $c_w = 8\%$ is a conservative but reasonable hypothesis.

TFP is revealed by the observed growth rate: we calibrated $\theta$ to generate a growth rate of the economy corresponding to its average yearly value observed in the French economy during the last decade (2%), while setting the policy parameters $(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n, \tau_p, \tau_q, \tau)$ to the values experienced in 2006.[10]

### 4.2.2 Results

In order to compute the optimal policy, we solve numerically the implicit system in Proposition 3. The optimal values we are looking for are:

*(i)* the breakdown of the public capital into the four components: $\sigma_a$, $\sigma_b$, $\sigma_m$ and $\sigma_n$,

*(ii)* the breakdown of the total amount of public spending into the four components: $\tilde{\sigma}_a$, $\tilde{\sigma}_b$, $\tilde{\sigma}_m$ and $\tilde{\sigma}_n$,

*(iii)* the tax pressure (average tax rate) $\tau$ and its breakdown into the tax rate on labor and capital income $\tau_q$ and the specific tax rate on private R&D $\tau_p$,

*(iv)* the growth rate of the economy: $\gamma - 1$,

*(v)* the social welfare: $W$.

Concerning the growth rate and fiscal pressure, results are somewhat usual, in line with the endogenous growth literature *à la* Barro [1990]. The overall tax rate of the economy stands to 15.2% and generates an equilibrium growth rate of the economy equal to 9.3%; these findings are consistent with those generally found in the endogenous growth literature where an optimal tax rate under 20% can sustain a 10% growth rate of the economy.[11] The specific tax rate on labor and capital income stands at 15.5%, *i.e.* above the overall tax rate, allowing the government to save fiscal resources in order to subsidize private R&D through an appropriate transfer characterized by a negative $-13.1\%$ tax rate on private R&D income. The usual breakdown of the total amount of public spending into the four components – investment (46.6%), consumption (44.6%), health R&D (2.6%) and other health expenditures (6.2%) – highlights the central roles played by public medical research and development. Despite a pessimistic set of assumptions concerning the role played by the R&D in the global economy, 2.6% of the total amount of public spending should be devoted to medical R&D, in order for the government to implement an optimal fiscal policy. This result can be usefully compared to the real value observed in France during the year 2006.[12] In that year public health R&D amounted to 2.95 billions € for a total amount of fiscal revenues of 792.49 billions €, which corresponds to a 0.37% share of medical R&D into public spending. According to our numerical simulation, the

[10] An extensive calibration of the model and the evaluation of $(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n, \tau_p, \tau_q, \tau)$ are provided in the next subsection.

[11] In Barro [1990] the second best fiscal pressure has to be equal to the production elasticity w.r.t. the externality of public spending, that is, under a Cobb-Douglas technology, to $1 - \alpha$, where $\alpha$ is the capital share in total income. $1 - \alpha$ can be small under weak externalities (according to empirical estimates), consistently with the assumption (usually retained in the endogenous growth literature) that capital includes human capital (as in Mankiw, Romer and Weil [1992]).

[12] See Fenina & Geffroy [2007] and Appendix 3.

public investment in medical R&D, is 17.6 billions € under its socially optimal level.

With regard to the sensitivity of the optimal policy to the deep parameters, one can clearly distinguish two subsets of parameters:

*(i)* Our main conclusions are relatively insensitive to certain parameters in the households' utility function (weights $c_u$, $c_v$ and $c_w$), to the parameters in the provision of public health services (elasticities $\beta_m$ and $\beta_n$) or to the depreciation rates ($\delta_k$, $\delta_a$, $\delta_b$, $\delta_m$ and $\delta_n$).

*(ii)* Our results are sensitive to the assumptions made on the main production function (parameters $s_k$, $s_a$, $s_m$ and $s_p$). An increase of the size of the externality associated with medical R&D (*resp.* public investment) leads the government to reallocate its fiscal receipts in favor of medical R&D (*resp.* public investment). Symmetrically reducing externalities associated with public spending (medical R&D or public investment) leads the government to reallocate spending in favor of public consumption.

# 5 Raising public investment in medical R&D: an evaluation

The purpose of this section is to to calibrate our model using French data and to present the results of additional numerical simulations that use these data. In particular, we analyze the macroeconomic impact on the GDP and the growth rate of the economy of increasing public investment in medical R&D and compare the results with another possible public policy: subsidizing private R&D. The first part of this section is devoted to the calibration of the model using French data which includes the current economic policy. The second part presents the main results of our numerical simulations.

## 5.1 Calibration

The calibration process consists to set the values of two types of parameters needed to implement the numerical simulations:

*(i)* The structural parameters of the model. These deep parameters have been already defined in the previous section; in order to draw a coherent picture, we use in this section the same values that the ones employed to compute the optimal policy.

*(ii)* Other parameters which were endogenous in the optimal policy section (see above), are made exogenous and fixed according to the observed policy practice in the French economy. These parameters are the proportion in the total amount of tax receipts of public investment, public consumption, health R&D and other health expenditures; the tax rate on labor and capital incomes and the tax rate on private R&D income; and the GDP growth rate. The level of GDP, expressed in €, is also used as a convenient basis for providing a monetary evaluation of the impact on the French economy of increasing public investments in medical R&D (rather than providing only the impact on the growth rate).

All theses parameters are taken from French national accounts for the year 2006. The shares, in the total amount of fiscal revenues, of public investment ($\tilde{\sigma}_a$), public consumption ($\tilde{\sigma}_b$), health-related R&D ($\tilde{\sigma}_m$) and other (unproductive) health expenditures ($\tilde{\sigma}_n$), are derived from Table 6 (see Appendix 3). For instance, the amount of health-related public R&D, is 2.95 billions € for the year 2006. This represents 0.37% of the total amount of fiscal receipts; thus we obtain $\tilde{\sigma}_m = 0.37$. The same method was used to compute: $\tilde{\sigma}_a = 7.58\%$, $\tilde{\sigma}_b = 75.88\%$ and $\tilde{\sigma}_n = 16.17\%$.

The ratio of the total amount of taxes (792.49 billions € in 2006) to the French 2006 GDP (1792 billions €) determines the French fiscal pressure in 2006, namely $\tau = 44,22\%$. Since the overall tax rate of the economy $\tau$ is defined in equation (18) as a weighted average of the specific tax rates applied to private capital incomes ($\tau_k$) , private R&D incomes ($\tau_p$), and labor incomes ($\tau_l$), one gets immediately $\tau_q \equiv (\tau - s_p \tau_p) / (1 - s_p)$, where $\tau_q$ denotes the common tax rate on capital and labor income. This formula allow us to compute the tax rate applied to capital and labor incomes as a function *(i)* of the overall tax rate of the economy $\tau$ and *(ii)* of the specific tax rate applied to private R&D incomes.

In order to be consistent with the parametrization of the aggregate production, we assume that private R&D returns represents, approximately, 1% of the GDP, *i.e.* 17.92 billions € for the year 2006. Such incomes would generate, if taxed at the average level $\tau$, a total amount of taxes equal to 44.22% times 17.92 billions €, that is 7.92 billions.

Considering that the so-called *Research Tax Credit (RTC)*, the main tax measure aimed at supporting the development of private R&D, represents an annual cost of about 1.1 billion € for the government budget for year 2006, one can calculate that the total amount of taxes on private R&D incomes is about $7.92 - 1.1 = 6.82$ billions €. Dividing this latter amount of taxes by the corresponding amount of incomes (17.92 billions), one can approximate the value of the specific tax rate on private R&D incomes: $\tau_p = 38,09\%$ and, finally, compute $\tau_q = 44,29\%$.

The yearly real growth rate of the economy has been set equal to 2%, corresponding to the average value observed in the French economy during the last decade, as explained above. In order to have a coherent representation of the economy, we need to calibrate the productivity parameter $\theta$ (TFP) which implements the observed growth rate. Using the observed policy values $(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n, \tau_p, \tau_q, \tau)$ derived above from the national accounts, and the calibration of the structural parameters presented in Table 1, we obtain: $\theta = 0.5631$. The details of this procedure are provided in the Appendix 2.

## 5.2 Results

In this section, we proceed to numerical simulations of the model to address the following questions:

*(i)* What is the macroeconomic impact on the growth rate of the economy and the GDP of an increase of public investment in medical R&D?

*(ii)* Does this impact depend on the way it is financed?

*(iii)* Is it better, from a public policy point of view, to use public money to increase public medical R&D or to subsidize private R&D?

### 5.2.1 Increasing public investment in medical R&D

• First, we assume that the government keeps constant the (*ex-ante*) total amount of fiscal receipts and just switches some fiscal resources (1 billion €) from somewhat "unproductive" public consumption to investments in medical R&D. In this case, the policy change we simulate, is a *permanent transfer of an amount of 1 billion € from public consumption to public medical R&D*. Such a transfer raises the share of public spending in medical R&D from $\tilde{\sigma}_m = 0.372\%$ to about $\tilde{\sigma}'_m = 0.498\%$, while the public consumption share decreases from $\tilde{\sigma}_b = 75.873\%$ to $\tilde{\sigma}'_b = 75.747\%$ ($\tilde{\sigma}_a$ and $\tilde{\sigma}_n$ remaining constants). This change in policy corresponds to an extra-investment in medical R&D of one billion euros under the balanced-budget constraint $\tilde{\sigma}_a + \tilde{\sigma}'_b + \tilde{\sigma}'_m + \tilde{\sigma}_n = 1$.

In the first year the growth rate increases from 2% to about 2.24 %. Over a decade, the GDP discounted total benefit associated with the policy change is more than 60 billions €, corresponding to 3.42% of the 2006 GDP, *i.e.* about 1.7 years of economic growth; after ten years the amount of annual fiscal receipts is 5.157 billions € higher that it would have been without the policy adjustment. In the long run, using public money previously devoted to public consumption, to increase medical R&D public investment, generates an increase of the growth rate of the economy equal to +0.048%.

• Let us now compare the previous results to what is obtained if the government finances the one billion € increase in medical-R&D public investment, by increasing the tax rate on labor and capital incomes (*i.e.* not keeping constant the total amount of fiscal receipts). In this case, the policy change we analize, is a *permanent €1 billion increase of public medical-R&D expenditures, totally funded by a rise of the tax rate on labor and capital incomes*.

First we compute the increase of the tax rate $\tau_q$ on labor and capital income in order to compensate the 1 billion € increase of public medical-R&D expenditures and keep a balanced budget. We know that total fiscal receipts are given by $T = \tau Y = [\tau_p s_p + \tau_q (1 - s_p)] Y$. Then, the variation of $T$ associated with an increase of the tax rate from $\tau_q$ to $\tau'_q$ is given by $\Delta T = (\tau'_q - \tau_q)(1 - s_p) Y$. Setting $\Delta T = 1$ (billion €), we can easily compute the increase of the tax rate $\tau_q$ on labor and capital income which ensures a balanced budget: $\tau'_q = \tau_q + 1/[(1 - s_p) Y] = 44.34\%$, since $Y$, the 2006 GDP, is 1792 billions €. The new proportions, for the four components of the total amount of fiscal revenues (public consumption, public investment, health-related R&D and "other" health expenditures) are given by:

$$(\tilde{\sigma}'_a, \tilde{\sigma}'_b, \tilde{\sigma}'_m, \tilde{\sigma}'_n) = \left( \frac{\tilde{\sigma}_a T}{T+1}, \frac{\tilde{\sigma}_b T}{T+1}, \frac{\tilde{\sigma}_m T + 1}{T+1}, \frac{\tilde{\sigma}_n T}{T+1} \right) = (7.57, 75.78, 0.5, 16.15)\,\%$$

since $T$, the 2006 tax receipt, is 792.49 billions €.

In the first year the growth rate increases from 2% to about 2.21% (*i.e.* +0.21%) , which is less than what we got with the first scenario. Over a decade the GDP discounted total benefit associated with the policy change is close to 55 billions € corresponding approximately to 3% of 2006 GDP *i.e.* 1.5 year of economic growth. In the long run the growth rate of the economy stands to 2.043% (+0.043%).

• Results associated with the two scenarios are in fact close to each others. The macroeconomic impact on the GDP of a 1 billion € increase in public investment in medical-R&D is clearly *positive and strong, whatever the funding process*. However, the impact of increasing publicly funded medical-R&D appears to be higher when the supplementary investment is financed by a transfer from public consumption, than when it is financed by increasing the tax rate on labor and capital incomes. Unsurprisingly, in the latter case, the increase in the overall tax rate of the economy adversely affects, first, labor and investment incentives and then the GDP and fiscal revenues.

#### 5.2.2 Subsidizing private R&D

• We assume here that the government, in order to stimulate private R&D, switches some fiscal resources (1 billion €) from public consumption to the *Research Tax Credit (RTC)* tool. In this case, the policy change we simulate, is a *permanent 1 billion € decrease of taxes on private R&D incomes, totally funded by a decrease of the same amount of public consumption.* Such a change decreases the total amount of taxes on private R&D incomes from 6.82 to 5.82 billions € driving the specific tax rate on private R&D incomes from $\tau_p = 38,09\%$ to $\tau'_p = 32,51\%$. The tax rate $\tau_q$ on labor and capital incomes being constant ($\tau_q = 44,29\%$), we easily compute the new average taxe rate of the economy $\tau' = \tau'_p s_p + \tau_q (1 - s_p) = 44.17\%$ and the new proportions, for the four components of the total amount of fiscal revenues:

$$\left(\tilde{\sigma}'_a, \tilde{\sigma}'_b, \tilde{\sigma}'_m, \tilde{\sigma}'_n\right) = \left(\frac{\tilde{\sigma}_a T}{T-1}, \frac{\tilde{\sigma}_b T - 1}{T-1}, \frac{\tilde{\sigma}_m T}{T-1}, \frac{\tilde{\sigma}_n T}{T-1}\right) = (7.59, 75.85, 0.37, 16.19)\,\%$$

where $T = 792.49$ denotes the initial level of taxes.

In the first year the growth rate of the economy increases from 2% to about 2.02% (*i.e.* +0.02%), far less that what one gets by rising public investment in medical R&D (whatever the funding process). Over a decade the GDP discounted total benefit associated with the policy change, stands under 11 billions € corresponding approximately to 0.61% of 2006 GDP *i.e.* 0.3 year of economic growth; after ten years the amount of annual fiscal receipts is only 0.128 billions € higher that it would have been without the policy adjustment. In the long run, using public money previously devoted to public consumption, to subsidize private R&D, generates an increase of the growth rate of the economy equal to +0.014%.

• We now compare the previous results with what we get if the governement – instead of decreasing public consumption in order to keep a balanced budget – decides to finance the 1 billion € increase of the RTC, by increasing the tax rate on labor and capital incomes. The policy change we analize, is thus a *permanent €1 billion decrease of the amount of taxes on private R&D incomes,* *totally offset by a increase of the amount of taxes on labor and capital incomes.* Like before, the total amount of taxes on private R&D incomes shifts from 6.82 to 5.82 billions € driving the specific tax rate on private R&D incomes to $\tau'_p = 32,51\%$, but now the total amount of taxes ($T = 792.49$ billions €) remains constant and it is the same for the average taxe rate of the economy ($\tau = 44,22\%$); we thus easily compute the new tax rate on labor and capital incomes $\tau'_q \equiv \left(\tau - s_p \tau'_p\right) / (1 - s_p) = 44.34\%$. Futhermore it is straightforward that the vector $(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n)$ remains the same.

The impact of this policy shock on the growth rate of the economy is about +0.03% in the short run (one year), but is close to zero in the long run (+0.002%); once again far less that what one gets by promoting medical R&D public investment. Over a decade the GDP discounted total benefit associated with the policy change is under 5 billions € corresponding approximately to 0.3% of 2006 GDP *i.e.* less than 2 months of economic growth.

• Like in the previous subsection, results associated with the two scenarios are close to each others and, once again, the funding process seems to play a second order role : the macroeconomic impact on the GDP of a 1 billion € decrease of of taxes on private R&D incomes is *positive but weak, whatever the funding process.*

# 6 Conclusion

The general equilibrium endogenous growth model presented in this paper emphasizes the key role played by public health R&D investments in determining the long-run rate of economic growth and welfare.

From a theoretical point of view we found four main results: *(i)* the equilibrium path is unique, *(ii)* market imperfections – externalities and taxes – make the equilibrium inefficient under arbitrary policies, *(iii)* an appropriate fiscal policy (tax rate and public spending shares) can achieve the second best, *(iv)* health R&D matters more than other health expenditures in achieving the social welfare target in the long run. This last point is crucial as it stresses how the health-related research and development, as a productive externality, is a powerful engine for growth compared to alternative policies such as, for example, public consumption.

The numerical simulations provided in the paper can be seen as an illustration of possible benefits associated with a permanent increase of medical R&D public investment. More precisely, we found that such an increase has a strong impact on the growh rate of the economy, whatever the funding process (reallocating public money from "unproductive" public consumption to medical R&D or rising the tax rate on labor and capital incomes) ; moreover, for the same amount of public money, the long run benefit of increasing public investment in medical R&D, is always higher that the one associated with subsidizing private R&D.

# 7 Appendix

## 7.1 Appendix 1: Proofs of propositions

**Proof of Proposition 1** Since $a_t = \sigma_a g_t$ and $m_t = \sigma_m g_t$, the representative agent budget constraint (2) rewrites as an aggregate resources constraint:

$$\begin{aligned} & c_t + \kappa_{t+1} - \Delta_k \kappa_t + \pi_{t+1} - \Delta_p \pi_t \\ \leq \quad & \left[ (1-\tau_k) s_{kt} + (1-\tau_p) s_{pt} + (1-\tau_l)(1 - s_{kt} - s_{pt}) \right] f(\kappa_t, \pi_t, \sigma_a g_t, \sigma_m g_t) \end{aligned} \tag{27}$$

On its side, the government budget constraint (12) becomes:

$$g_{t+1} - \Delta g_t = \left[ \tau_k s_{kt} + \tau_p s_{pt} + \tau_l (1 - s_{kt} - s_{pt}) \right] f(\kappa_t, \pi_t, \sigma_a g_t, \sigma_m g_t) \tag{28}$$

Substituting (8) in the Euler equation (5), one gets:

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta \left[ \Delta_k + (1-\tau_k) f_\kappa (\kappa_{t+1}, \pi_{t+1}, a_{t+1}, m_{t+1}) \right] \tag{29}$$

Observing that the homogeneity property of the intensive production function implies that its derivatives are homogeneous of degree zero: $f_\kappa(\mu\kappa, \mu\pi, \mu a, \mu m) = f_\kappa(\kappa, \pi, a, m)$, it follows immediately $f_\kappa(\kappa_t, \pi_t, \sigma_a g_t, \sigma_m g_t) = f_\kappa(\kappa_t/g_t, \pi_t/g_t, \sigma_a, \sigma_m)$. NAC (4) becomes:

$$\Delta_k + (1-\tau_k) f_\kappa(\kappa_t, \pi_t, a_t, m_t) = \Delta_p + (1-\tau_p) \frac{\kappa_t}{\pi_t} \frac{s_{pt}}{s_{kt}} f_\kappa(\kappa_t, \pi_t, a_t, m_t)$$

since $r_{pt} = (\kappa_t/\pi_t)(s_{pt}/s_{kt}) f_\kappa(\kappa_t, \pi_t, a_t, m_t)$.

Under this Assumption 3, we find**:**

$$\pi_t = \frac{1-\tau_p}{1-\tau_k} \frac{s_{pt}}{s_{kt}} \kappa_t \tag{30}$$

Then

$$r_{kt} = f_\kappa(\kappa_t, \pi_t, a_t, m_t) = f_\kappa\left( \frac{\kappa_t}{g_t}, \frac{\pi_t}{g_t}, \sigma_a, \sigma_m \right) = f_\kappa\left( \frac{\kappa_t}{g_t}, \frac{1-\tau_p}{1-\tau_k} \frac{s_{pt}}{s_{kt}} \frac{\kappa_t}{g_t}, \sigma_a, \sigma_m \right)$$

Under Assumption 4, we obtain:

$$\varphi'(x_t) = f_\kappa + \frac{1-\tau_p}{1-\tau_k} \frac{s_p}{s_k} f_\pi = \frac{s_k + s_\pi}{s_k} f_\kappa \left( x_t, \frac{1-\tau_p}{1-\tau_k} \frac{s_p}{s_k} x_t, \sigma_a, \sigma_m \right)$$

since (30) holds and

$$f_\pi = \frac{s_\pi}{s_k} \frac{\kappa_t}{\pi_t} f_\kappa = \frac{1-\tau_k}{1-\tau_p} \frac{s_\pi}{s_p} f_\kappa$$

where $s_\pi = 1 - s_k - s_a - s_m$. Therefore,

$$r_{kt} = f_\kappa(\kappa_t, \pi_t, a_t, m_t) = \frac{s_k}{s_k + s_\pi} \varphi'(x_t)$$

Using the average tax pressure (18), under Assumptions 3 and 4, equations (27), (28) and (29) write:

$$\begin{aligned} c_t+\left(1+\frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)(\kappa_{t+1}-\Delta_k\kappa_t) &\leq (1-\tau)\,g_t\varphi(x_t) & (31)\\ g_{t+1}-\Delta g_t &= \tau g_t\varphi(x_t) & (32)\\ \frac{u'(c_t)}{u'(c_{t+1})} &= \beta\left[\Delta_k+(1-\tau_k)\frac{s_k}{s_k+s_\pi}\varphi'(x_{t+1})\right] \end{aligned}$$

since

$$f(\kappa_t,\pi_t,\sigma_a g_t,\sigma_m g_t)=g_t f\left(\frac{\kappa_t}{g_t},\frac{\pi_t}{g_t},\sigma_a,\sigma_m\right)=g_t\varphi(x_t)$$

Under definitions (16) and (17), dividing both sides of (31) and (32) by $g_t$, one eventually gets:

$$\begin{aligned} y_t+\left(1+\frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)(\gamma_t x_{t+1}-\Delta_k x_t) &\leq (1-\tau)\,\varphi(x_t) & (33)\\ \gamma_t &= \Delta+\tau\varphi(x_t) & (34) \end{aligned}$$

On the other hand, the Euler equation can be revisited under Assumption 1:

$$\frac{c_{t+1}}{c_t}=\left(\beta\left[\Delta_k+(1-\tau_k)\frac{s_k}{s_k+s_\pi}\varphi'(x_{t+1})\right]\right)^{\varepsilon_u}$$

So, we have:

$$\frac{y_{t+1}}{y_t}\gamma_t=\left(\beta\left[\Delta_k+(1-\tau_k)\frac{s_k}{s_k+s_\pi}\varphi'(x_{t+1})\right]\right)^{\varepsilon_u} \tag{35}$$

Substituting (34) into (35) and (33) gives the dynamic system (19)-(20). ■

**Proof of Proposition 2** In the following, (1) we linearize the dynamic system (19-20) around the steady state and we compute the Jacobian matrix, then (2) we prove the saddle-path stability entailing the equilibrium uniqueness under rational expectations.

(1) Differentiating (19) with respect to dynamic variables $(x_{t+1},y_{t+1},x_t,y_t)$ and using (21-22), one gets,

$$\gamma\varepsilon_u\frac{\rho}{\rho+\Delta_k}\frac{x\varphi''}{\varphi'}\frac{dx_{t+1}}{x}-\gamma\frac{dy_{t+1}}{y}=\tau\varphi' x\frac{dx_t}{x}-\gamma\frac{dy_t}{y} \tag{36}$$

where the differentials are relative to the stationary state.

Linearizing now equation (20) around the steady state, one has:

$$\eta\gamma\frac{dx_{t+1}}{x}=\left[\eta(\Delta_k-\tau x\varphi')+(1-\tau)\varphi'\right]\frac{dx_t}{x}-\frac{y}{x}\frac{dy_t}{y} \tag{37}$$

where,

$$\eta\equiv 1+\frac{s_p}{s_k}\frac{1-\tau_p}{1-\tau_k} \tag{38}$$

We observe that (38) implies:

$$y = \eta (\Delta_k - \Delta) x + (1 - \tau - \eta\tau x) \varphi (x) \tag{39}$$

Let $\varepsilon_2 \equiv x\varphi''/\varphi' < 0$ denote the elasticity of the interest rate with respect to the ratio $\kappa/g$ (capital per head over public spending). The linear system (36-37) rewrites equivalently:

$$\begin{bmatrix} \frac{dx_{t+1}}{x} \\ \frac{dy_{t+1}}{y} \end{bmatrix} = \begin{bmatrix} \gamma\varepsilon_u\varepsilon_2 \frac{\rho}{\rho+\Delta_k} & -\gamma \\ \gamma\eta & 0 \end{bmatrix}^{-1} \begin{bmatrix} \tau x \varphi' & -\gamma \\ \Delta_k\eta + (1 - \tau - \tau x\eta) \varphi' & -\frac{y}{x} \end{bmatrix} \begin{bmatrix} \frac{dx_t}{x} \\ \frac{dy_t}{y} \end{bmatrix}$$

The trace (sum of eigenvalues) and the determinant (product of eigenvalues) of the Jacobian matrix evaluated at the steady state are given by

$$D = \frac{1}{\gamma}\left[\Delta_k + \frac{\varphi'}{\eta}\left(1 - \tau - \tau x\eta - \tau\frac{y}{\gamma}\right)\right] \tag{40}$$

$$T = 1 + D + \frac{1}{\gamma}\frac{y}{x\eta}\left(\frac{\varphi'}{\gamma}\tau x - \frac{\rho}{\rho+\Delta_k}\varepsilon_2\varepsilon_u\right) \tag{41}$$

(2) We want to prove that the steady state is a saddle point. Since the system is two-dimensional with one predetermined variable, saddle-path stability entails equilibrium uniqueness under rational expectations (with or without transition).

In the $(T, D)$-plane, the saddle points match with the two cones:

$$\begin{aligned} -T - 1 &< D < T - 1 \\ T - 1 &< D < -T - 1 \end{aligned}$$

As $\varepsilon_2 < 0$, (41) implies:

$$D = T - 1 - \frac{1}{\gamma}\frac{y}{x\eta}\left(\frac{\varphi'}{\gamma}\tau x - \frac{\rho}{\rho+\Delta_k}\varepsilon_2\varepsilon_u\right) < T - 1 \tag{42}$$

To show that the stationary state is a saddle point, one needs only to prove that $D > -T - 1$.

Substituting formulas (40) and (41) into $D > -T - 1$, one gets the following condition:

$$D > \frac{1}{2}\frac{1}{\gamma}\frac{y}{x\eta}\left(\frac{\rho}{\rho+\Delta_k}\varepsilon_2\varepsilon_u - \frac{\varphi'}{\gamma}\tau x\right) - 1$$

or, equivalently,

$$\gamma + \Delta_k + \frac{\varphi'}{\eta}\left(1 - \tau - \tau x\eta - \frac{\tau}{2}\frac{y}{\gamma}\right) > \frac{1}{2}\frac{y}{x\eta}\frac{\rho}{\rho+\Delta_k}\varepsilon_2\varepsilon_u$$

Since $\varepsilon_2 < 0$, it is sufficient to prove that

$$\gamma + \Delta_k + \frac{\varphi'}{\eta}\left(1 - \tau - \tau x\eta - \frac{\tau}{2}\frac{y}{\gamma}\right) > 0 \tag{43}$$

From equation (22) and definition (38), using $\gamma = \Delta + \tau\varphi$ we observe that

$$y = \varphi (1 - \tau) - \eta x (\gamma - \Delta_k) \tag{44}$$

Replacing (44) in (43), we get:

$$\begin{aligned} & \gamma + \Delta_k + \frac{\varphi'}{\eta}\left(1 - \tau - \tau x\eta - \frac{\tau}{2}\frac{y}{\gamma}\right) \\ = \;& \Delta + (1-\tau)\frac{\varphi'}{\eta}\left(1 - \frac{1}{2}\frac{\tau\varphi}{\Delta + \tau\varphi}\right) + \left(\frac{1}{2}\frac{\Delta + \tau\varphi - \Delta_k}{\Delta + \tau\varphi} - 1\right)\frac{\varphi' x}{\varphi}\tau\varphi + \tau\varphi + \Delta_k \\ \geq \;& \Delta + (1-\tau)\frac{\varphi'}{\eta}\left(1 - \frac{1}{2}\frac{\tau\varphi}{\Delta + \tau\varphi}\right) + \left(\frac{1}{2}\frac{\Delta + \tau\varphi - \Delta_k}{\Delta + \tau\varphi} - 1\right)\varepsilon_1\tau\varphi + \varepsilon_1(\tau\varphi + \Delta_k) \\ = \;& \Delta + (1-\tau)\frac{\varphi'}{\eta}\left(1 - \frac{1}{2}\frac{\tau\varphi}{\Delta + \tau\varphi}\right) + \left(\frac{1}{2}\tau\varphi + \left(1 - \frac{1}{2}\frac{\tau\varphi}{\Delta + \tau\varphi}\right)\Delta_k\right)\varepsilon_1 > 0 \end{aligned}$$

where

$$\varepsilon_1 \equiv \frac{\varphi' x}{\varphi} \in (0, 1)$$

■

**Proof of Proposition 3** Before maximizing the welfare function (23), we need to transform it in an appropriate form. We compute the welfare function (utility function) along the balanced growth path: $(c_t, b_t, m_t, n_t) = (c_0, b_0, m_0, n_0)\gamma^t$**,** where $\gamma$ is the common (regular) growth factor: $e_t \equiv e(m_t, n_t) = e(m_0\gamma^t, n_0\gamma^t) = e(m_0, n_0)\gamma^t = e_0\gamma^t$ **(**notice that the health good production function is supposed to be homogeneous of degree one). Denoting $e_0 \equiv e(m_0, n_0)$, one gets under restriction (24):

$$\begin{aligned} W \;&=\; c_u \sum_{t=0}^{\infty} \beta^t \ln\left(c_0\gamma^t\right) + c_v \sum_{t=0}^{\infty} \beta^t \ln\left(b_0\gamma^t\right) + c_w \sum_{t=0}^{\infty} \beta^t \ln\left(e_0\gamma^t\right) \\ &=\; (c_u \ln c_0 + c_v \ln b_0 + c_w \ln e_0)\sum_{t=0}^{\infty}\beta^t + (c_u + c_v + c_w)\ln\gamma \sum_{t=0}^{\infty}\beta^t t \\ &=\; \frac{1}{1-\beta}\left(c_u \ln c_0 + c_v \ln b_0 + c_w \ln e_0 + \frac{\beta}{1-\beta}\ln\gamma\right) \end{aligned}$$

Equilibrium uniqueness under rational expectations (Proposition 2) requires $c_0$, $b_0$, $e_0$ to be compatible with the regular growth factor $\gamma$. Definition (10) details the economic policy and implies at the beginning: $(a_0, b_0, m_0, n_0) = (\sigma_a, \sigma_b, \sigma_m, \sigma_n) g_0$ and $e_0 = e(m_0, n_0) = e(\sigma_m g_0, \sigma_n g_0) = e(\sigma_m, \sigma_n) g_0$. From definition (16) one gets $c_0 = y g_0$. The endogenous growth of steady state implies a regular growth path; under restriction (24), we obtain:

$$\begin{aligned} W \;&=\; \frac{1}{1-\beta}\left(c_u \ln(y g_0) + c_v \ln(\sigma_b g_0) + c_w \ln\left[e(\sigma_m, \sigma_n) g_0\right] + \frac{\beta}{1-\beta}\ln\gamma\right) \\ &=\; \frac{1}{1-\beta}\left[c_u \ln y + c_v \ln\sigma_b + c_w \ln e(\sigma_m, \sigma_n) + \ln g_0 + \frac{\beta}{1-\beta}\ln\gamma\right] \end{aligned}$$

where $g_0 \equiv a_0 + b_0 + m_0 + n_0$ is an initial condition.

As $\beta$ and $g_0$ are not choice variables, the problem of maximizing $W$ turns out to be equivalent to the following:

$$\max \left[ c_u \ln y + c_v \ln \sigma_b + c_w \ln e(\sigma_m, \sigma_n) + \frac{\beta}{1-\beta} \ln \gamma \right] \tag{45}$$

Under Assumption 4, the policy of public spending (10) entails:

$$\begin{aligned} \tilde{f}(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) &= \theta \kappa_t^{s_k} \pi_t^{s_p} a_t^{s_a} m_t^{s_m} \bar{p}_t^{1-s_k-s_p-s_a-s_m} \\ f(\kappa_t, \pi_t, a_t, m_t) &= \theta \kappa_t^{s_k} \pi_t^{1-s_k-s_a-s_m} a_t^{s_a} m_t^{s_m} \\ \varphi(x) &= \frac{f(\kappa_t, \pi_t, a_t, m_t)}{g_t} = f(x, (\eta-1)x, \sigma_a, \sigma_m) \\ &= \theta \sigma_a^{s_a} \sigma_m^{s_m} (\eta-1)^{1-s_k-s_a-s_m} x^{1-s_a-s_m} \qquad (46) \\ \varphi'(x) &= (1-s_a-s_m) \theta \sigma_a^{s_a} \sigma_m^{s_m} (\eta-1)^{1-s_k-s_a-s_m} x^{-s_a-s_m} \end{aligned}$$

where now $\eta \equiv 1 + (s_p/s_k)[(1-\tau_p)/(1-\tau_q)]$. Still under Assumption 4 we have: $\varepsilon_1 \equiv x\varphi'/\varphi = 1 - s_a - s_m$.

Since $\varepsilon_u = 1$, one gets from (21) an implicit equation defining the stationary state $x$:

$$\begin{aligned} &\Delta + \tau \theta \sigma_a^{s_a} \sigma_m^{s_m} (\eta-1)^{1-s_k-s_a-s_m} x^{1-s_a-s_m} \\ = \; &\beta \left[ \Delta_k + (1-\tau_q) s_k \theta \sigma_a^{s_a} \sigma_m^{s_m} (\eta-1)^{1-s_k-s_a-s_m} x^{-s_a-s_m} \right] \end{aligned} \tag{47}$$

where, according to (18):

$$\tau = s_p \tau_p + (1-s_p) \tau_q \tag{48}$$

Taking into account that $\tau\varphi = \gamma - \Delta$, equation (21) becomes

$$\gamma = \beta \left[ \Delta_k + \frac{s_k}{s_k + s_\pi} \frac{1-\tau_q}{\tau} \frac{1}{x} \frac{x\varphi'}{\varphi} (\gamma - \Delta) \right] \tag{49}$$

Substituting $\varepsilon_1$ into equation (49), noticing that $s_\pi \equiv 1 - s_k - s_a - s_m$ and solving for $\gamma$, the growth factor is now explicitly computed:

$$\gamma = \beta \frac{\Delta s_k (1-\tau_q) - \Delta_k \tau x}{\beta s_k (1-\tau_q) - \tau x} \tag{50}$$

Under Assumption 4, the implicit equation (47) becomes,

$$\theta \sigma_a^{s_a} \sigma_m^{s_m} (\eta-1)^{1-s_k-s_a-s_m} = \frac{(\Delta - \beta\Delta_k) x^{s_a+s_m}}{\beta s_k (1-\tau_q) - \tau x} \tag{51}$$

Instead of maximizing the welfare with respect to policy tools $(\sigma_a, \sigma_m, \sigma_n, \tau_p, \tau_q)$, one maximizes it indirectly with respect to an alternative vector $(\sigma_a, \sigma_m, \sigma_n, \eta, h)$, where $h$ is given by

$$h \equiv \gamma - \Delta_k \tag{52}$$

and finally compute $(\tau_p, \tau_q)^*$ using $(\sigma_a, \sigma_m, \sigma_n, \eta, h)^*$.

$\sigma_b$ is given by (11) and program (45) becomes

$$\max\left[c_u \ln y + c_v \ln(1-\sigma_a-\sigma_m-\sigma_n) + c_w \ln e(\sigma_m, \sigma_n) + \frac{\beta}{1-\beta}\ln\gamma\right] \quad (53)$$

Let us express $y$ in terms of $(\sigma_a, \sigma_m, \sigma_n, \eta, h)$.

Using (38) and (48), we find that

$$\begin{aligned} 1-\tau_p &= \frac{1-\tau}{1-s_p+(\eta-1)s_k}\frac{s_k}{s_p}(\eta-1) \\ 1-\tau_q &= \frac{1-\tau}{1-s_p+(\eta-1)s_k} \end{aligned} \quad (54)$$

From (46) and (51), we know that

$$\varphi(x) = \frac{(\Delta-\beta\Delta_k)x}{\beta s_k(1-\tau_q)-\tau x} \quad (55)$$

From (21), we know also that

$$\tau = \frac{\gamma-\Delta}{\varphi(x)} \quad (56)$$

or, equivalently,

$$\varphi(x) = \frac{\gamma-\Delta}{\tau} \quad (57)$$

Replacing (54) in (55) and (55) so modified in (56) and solving for $\tau$, we get

$$\tau = \left[1 + x\frac{\gamma-\beta\Delta_k}{\gamma-\Delta}\frac{1-s_p+(\eta-1)s_k}{\beta s_k}\right]^{-1} \quad (58)$$

Substituting (57) and (58) in (39), we get

$$y = x\left[\frac{1-s_p+(\eta-1)s_k}{\beta s_k}(\gamma-\beta\Delta_k) - \eta(\gamma-\Delta_k)\right] \quad (59)$$

Replacing (59) in (53) and using (52), we obtain

$$\begin{aligned} & \tilde{W}(\sigma_a, \sigma_m, \sigma_n, \eta, h) \\ \equiv\ & c_u \ln x + c_u \ln\left[\frac{1-s_p+(\eta-1)s_k}{\beta s_k}(h+(1-\beta)\Delta_k) - \eta h\right] \\ & + c_v \ln(1-\sigma_a-\sigma_m-\sigma_n) + c_w \ln e(\sigma_m, \sigma_n) + \frac{\beta}{1-\beta}\ln(h+\Delta_k) \end{aligned}$$

We observe that $x$ is determined by (51), where we have substituted (52), (54) and (58):

$$\theta\sigma_a^{s_a}\sigma_m^{s_m}(\eta-1)^{1-s_k-s_a-s_m}x^{1-s_a-s_m} = h+\Delta_k-\Delta+x(h+(1-\beta)\Delta_k)\frac{1-s_p+(\eta-1)s_k}{\beta s_k} \quad (60)$$

where

$$\Delta = \Delta_b + \sigma_a \left(\Delta_a - \Delta_b\right) + \sigma_m \left(\Delta_m - \Delta_b\right) + \sigma_n \left(\Delta_n - \Delta_b\right) \tag{61}$$

From (60), according to the Implicit Function Theorem, we locally define $x = x\left(\sigma_a, \sigma_m, \sigma_n, \eta, h\right)$ with partial derivatives:

$$\left(x_a, x_m, x_n, x_\eta, x_h\right) = \left(\frac{\partial x}{\partial \sigma_a}, \frac{\partial x}{\partial \sigma_m}, \frac{\partial x}{\partial \sigma_n}, \frac{\partial x}{\partial \eta}, \frac{\partial x}{\partial h}\right)$$

These partial derivatives can be computed by totally differentiating (60).

$$\frac{x_a}{x} = \frac{\Delta_a - \Delta_b + \varphi \frac{s_a}{\sigma_a}}{\left(s_a + s_m\right)\varphi - z} \tag{62}$$

$$\frac{x_m}{x} = \frac{\Delta_m - \Delta_b + \varphi \frac{s_m}{\sigma_m}}{\varphi\left(s_a + s_m\right) - z} \tag{63}$$

$$\frac{x_n}{x} = \frac{\Delta_n - \Delta_b}{\varphi\left(s_a + s_m\right) - z} \tag{64}$$

$$\frac{x_\eta}{x} = \frac{\varphi \frac{1 - s_k - s_a - s_m}{\eta - 1} - \left(z + \Delta - \beta \Delta_k\right)\frac{x}{\beta}}{\varphi\left(s_a + s_m\right) - z} \tag{65}$$

$$\frac{x_h}{x} = -\frac{1 + \frac{1 - s_p + (\eta - 1)s_k}{s_k}\frac{x}{\beta}}{\varphi\left(s_a + s_m\right) - z} \tag{66}$$

where $z \equiv \gamma - \Delta = h + \Delta_k - \Delta$.

The optimal policy implements a vector $\left(\sigma_a, \sigma_m, \sigma_n, \eta, z\right)^*$ satisfying the following system:

$$\left(\frac{\partial \tilde{W}}{\partial \sigma_a}, \frac{\partial \tilde{W}}{\partial \sigma_m}, \frac{\partial \tilde{W}}{\partial \sigma_n}, \frac{\partial \tilde{W}}{\partial \eta}, \frac{\partial \tilde{W}}{\partial h}\right) = 0$$

Noticing that, under Assumption 2,

$$\frac{\partial e}{\partial \sigma_m}\frac{1}{e} = \frac{\beta_m}{\sigma_m}, \quad \frac{\partial e}{\partial \sigma_n}\frac{1}{e} = \frac{\beta_n}{\sigma_n}$$

we obtain

$$\frac{\partial \tilde{W}}{\partial \sigma_a} = c_u \frac{x_a}{x} - c_v \frac{1}{1 - \sigma_a - \sigma_m - \sigma_n} = 0$$

$$\frac{\partial \tilde{W}}{\partial \sigma_m} = c_u \frac{x_m}{x} - c_v \frac{1}{1 - \sigma_a - \sigma_m - \sigma_n} + c_w \frac{\beta_m}{\sigma_m} = 0$$

$$\frac{\partial \tilde{W}}{\partial \sigma_n} = c_u \frac{x_n}{x} - c_v \frac{1}{1 - \sigma_a - \sigma_m - \sigma_n} + c_w \frac{\beta_n}{\sigma_n} = 0$$

$$\frac{\partial \tilde{W}}{\partial \eta} = c_u \frac{x_\eta}{x} + c_u \frac{\frac{1}{\beta}\left(z + \Delta - \beta \Delta_k\right) - \left(z + \Delta - \Delta_k\right)}{\frac{1 - s_p + (\eta - 1)s_k}{\beta s_k}\left(z + \Delta - \beta \Delta_k\right) - \eta\left(z + \Delta - \Delta_k\right)} = 0$$

$$\frac{\partial \tilde{W}}{\partial h} = c_u \frac{x_h}{x} + c_u \frac{\frac{1 - s_p + (\eta - 1)s_k}{\beta s_k} - \eta}{\frac{1 - s_p + (\eta - 1)s_k}{\beta s_k}\left(z + \Delta - \beta \Delta_k\right) - \eta\left(z + \Delta - \Delta_k\right)} + \frac{\beta}{1 - \beta}\frac{1}{z + \Delta} = 0$$

Substituting equations (62-66) and taking into account equation (60), we get the following system. The optimal policy is a vector $(\sigma_a, \sigma_m, \sigma_n, \eta, z)^*$ such that $(\sigma_a, \sigma_m, \sigma_n, \eta, z, \varphi)^*$ is solution of:

$$\begin{aligned}
\theta \sigma_a^{s_a} \sigma_m^{s_m} (\eta - 1)^{1-s_k-s_a-s_m} x^{1-s_a-s_m} &= \varphi &(67)\\
c_u \frac{\Delta_a - \Delta_b + \frac{s_a}{\sigma_a}\varphi}{(s_a + s_m)\varphi - z} - c_v \frac{1}{1-\sigma_a-\sigma_m-\sigma_n} &= 0 &(68)\\
c_u \frac{\Delta_m - \Delta_b + \frac{s_m}{\sigma_m}\varphi}{(s_a + s_m)\varphi - z} - c_v \frac{1}{1-\sigma_a-\sigma_m-\sigma_n} + c_w \frac{\beta_m}{\sigma_m} &= 0 &(69)\\
c_u \frac{\Delta_n - \Delta_b}{(s_a + s_m)\varphi - z} - c_v \frac{1}{1-\sigma_a-\sigma_m-\sigma_n} + c_w \frac{\beta_n}{\sigma_n} &= 0 &(70)
\end{aligned}$$

$$\frac{\frac{1-s_k-s_a-s_m}{\eta-1}\varphi - x\left(z + \Delta - \beta\Delta_k\right)\frac{1}{\beta}}{(s_a + s_m)\varphi - z} + \frac{(1-\beta)(z+\Delta)}{\frac{1-s_p-s_k}{s_k}\left(z + \Delta - \beta\Delta_k\right) + \eta(1-\beta)(z+\Delta)} = 0 \quad (71)$$

$$-c_u \frac{1 + x\frac{1-s_p+(\eta-1)s_k}{\beta s_k}}{(s_a + s_m)\varphi - z} + c_u \frac{\frac{1-s_p+(\eta-1)s_k}{\beta s_k} - \eta}{\frac{1-s_p+(\eta-1)s_k}{\beta s_k}\left(z + \Delta - \beta\Delta_k\right) - \eta\left(z + \Delta - \Delta_k\right)} + \frac{\beta}{1-\beta}\frac{1}{z+\Delta} = 0 \quad (72)$$

where

$$\varphi = h + \Delta_k - \Delta + x\left(h + (1-\beta)\Delta_k\right)\frac{1 - s_p + (\eta-1)s_k}{\beta s_k} \quad (73)$$

Replacing (67) in (68) and (69), we obtain

$$\begin{aligned}
\sigma_m &= \frac{\beta_m \frac{c_w}{c_u}\left((s_a + s_m)\varphi - z\right) + s_m\varphi}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}} &(74)\\
\sigma_n &= \frac{\beta_n \frac{c_w}{c_u}\left((s_a + s_m)\varphi - z\right)}{\Delta_a - \Delta_n + \varphi\frac{s_a}{\sigma_a}} &(75)
\end{aligned}$$

Substituting (74) and (75) in (67) and solving for $z$, we find

$$z(\varphi, \sigma_a) \equiv \varphi(s_a + s_m) - \frac{c_u\left(\Delta_a - \Delta_b + \varphi\frac{s_a}{\sigma_a}\right)\left(1 - \sigma_a - \frac{\varphi s_m}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}}\right)}{c_v + c_w\left(\Delta_a - \Delta_b + \varphi\frac{s_a}{\sigma_a}\right)\left(\frac{\beta_m}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}} + \frac{\beta_n}{\Delta_a - \Delta_n + \varphi\frac{s_a}{\sigma_a}}\right)} \quad (76)$$

Replacing (76) in (74) and (75), we obtain

$$\sigma_n(\varphi, \sigma_a) = \frac{\beta_n}{\Delta_a - \Delta_n + \varphi\frac{s_a}{\sigma_a}} \frac{c_w\left(\Delta_a - \Delta_b + \varphi\frac{s_a}{\sigma_a}\right)\left(1 - \sigma_a - \frac{\varphi s_m}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}}\right)}{c_v + c_w\left(\Delta_a - \Delta_b + \varphi\frac{s_a}{\sigma_a}\right)\left(\frac{\beta_m}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}} + \frac{\beta_n}{\Delta_a - \Delta_n + \varphi\frac{s_a}{\sigma_a}}\right)} \quad (77)$$

$$\sigma_m(\varphi, \sigma_a) \equiv \frac{\varphi s_m}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}} + \sigma_n(\varphi, \sigma_a)\frac{\beta_m}{\beta_n}\frac{\Delta_a - \Delta_n + \varphi\frac{s_a}{\sigma_a}}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}} \quad (78)$$

From (73), we get

$$x = \frac{\varphi - z}{z + \Delta - \beta\Delta_k} \frac{\beta s_k}{1 - s_p + (\eta - 1) s_k} \tag{79}$$

Replacing in (72) and solving for $\eta$, we have

$$\eta(\varphi, \sigma_a) \equiv \frac{1}{1-\beta} \frac{1 - s_p - s_k}{s_k} \frac{1 - \frac{\varphi + \Delta(\varphi,\sigma_a) - \beta\Delta_k}{\varphi(s_a + s_m) - z(\varphi,\sigma_a)} + \frac{1}{c_u} \frac{\beta}{1-\beta} \frac{z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a) - \beta\Delta_k}{z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a)}}{\frac{\varphi + \Delta(\varphi,\sigma_a) - \beta\Delta_k}{\varphi(s_a + s_m) - z(\varphi,\sigma_a)} \frac{z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a)}{z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a) - \beta\Delta_k} - \frac{1}{c_u}\frac{\beta}{1-\beta} - 1} \tag{80}$$

where

$$\Delta(\varphi, \sigma_a) \equiv \Delta_b + \sigma_a(\Delta_a - \Delta_b) + \sigma_m(\varphi, \sigma_a)(\Delta_m - \Delta_b) + \sigma_n(\varphi, \sigma_a)(\Delta_n - \Delta_b)$$

Substituting (79) in (70), we find $F_1(\varphi, \sigma_a) = 0$, where

$$\begin{aligned} F_1(\varphi, \sigma_a) &\equiv \varphi - \theta\sigma_a^{s_a}\sigma_m(\varphi,\sigma_a)^{s_m}[\eta(\varphi,\sigma_a) - 1]^{1 - s_k - s_a - s_m} \\ &\times \left(\frac{\varphi - z(\varphi,\sigma_a)}{z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a) - \beta\Delta_k} \frac{\beta s_k}{1 - s_p + [\eta(\varphi,\sigma_a) - 1]s_k}\right)^{1 - s_a - s_m} \end{aligned} \tag{81}$$

Eventually, replacing (79) in (71), we obtain $F_2(\varphi, \sigma_a) = 0$, where

$$\begin{aligned} F_2(\varphi, \sigma_a) &\equiv \frac{\varphi\frac{1 - s_k - s_a - s_m}{\eta(\varphi,\sigma_a) - 1} - [\varphi - z(\varphi,\sigma_a)]\frac{s_k}{1 - s_p + [\eta(\varphi,\sigma_a) - 1]s_k}}{\varphi(s_a + s_m) - z(\varphi,\sigma_a)} \\ &+ \frac{(1-\beta)[z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a)]}{\frac{1 - s_p - s_k}{s_k}[z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a) - \beta\Delta_k] + \eta(\varphi,\sigma_a)(1-\beta)[z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a)]} \end{aligned} \tag{82}$$

The optimal policy $(\varphi, \sigma_a)^*$ is solution of the two-dimensional system (25-26), where the functions $z(\varphi, \sigma_a)$, $\sigma_m(\varphi, \sigma_a)$, $\sigma_n(\varphi, \sigma_a)$, $\eta(\varphi, \sigma_a)$ are given by (76), (77), (78) and (80), respectively.

After this system has been solved, we get $\sigma_a, \sigma_m, \sigma_n, \eta, z, \varphi$ and $\Delta$; then we compute $\sigma_b = 1 - \sigma_a - \sigma_m - \sigma_n$, the growth factor $\gamma \equiv z + \Delta$ and, using (79), $x$. Finally, $\tau$ is computed using (58), while $\tau_p$ and $\tau_q$ are obtained as follows:

$$\begin{aligned} \tau_p &= 1 - \frac{1 - \tau}{1 - s_p + (\eta - 1) s_k} \frac{s_k}{s_p}(\eta - 1) \\ \tau_q &= 1 - \frac{1 - \tau}{1 - s_p + (\eta - 1) s_k} \end{aligned}$$

■

## 7.2 Appendix 2: Calibration procedures

**Calibrating $\theta$.** Using (21) and (46), we get

$$\theta = \frac{\gamma - \Delta}{\tau\sigma_a^{s_a}\sigma_m^{s_m}(\eta - 1)^{1 - s_k - s_a - s_m} x^{1 - s_a - s_m}} \tag{83}$$

Replacing in (47) and solving for $x$, we obtain

$$x = \beta s_k \frac{1-\tau_q}{\tau} \frac{\gamma - \Delta}{\gamma - \beta \Delta_k}$$

Replacing in (83), we find

$$\theta = \frac{\gamma - \Delta}{\tau \sigma_a^{s_a} \sigma_m^{s_m} (\eta - 1)^{1-s_k-s_a-s_m} \left( \beta s_k \frac{1-\tau_q}{\tau} \frac{\gamma-\Delta}{\gamma-\beta\Delta_k} \right)^{1-s_a-s_m}} \tag{84}$$

that is the right way of calibrating the unobserved $\theta$, given the observed $\gamma$.

From (14), the breakdown of the total amount of *public spending* into its four components (investment without health, consumption, health R&D and other health expenditures) takes the form:

$$(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n) = \left( \frac{a_{t+1}/a_t - \Delta_a}{g_{t+1}/g_t - \Delta} \sigma_a, \frac{b_{t+1}/b_t - \Delta_b}{g_{t+1}/g_t - \Delta} \sigma_b, \frac{m_{t+1}/m_t - \Delta_m}{g_{t+1}/g_t - \Delta} \sigma_m, \frac{n_{t+1}/n_t - \Delta_n}{g_{t+1}/g_t - \Delta} \sigma_n \right)$$

At the steady state (regular growth), we obtain

$$(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n) = \left( \frac{\gamma - \Delta_a}{\gamma - \Delta} \sigma_a, \frac{\gamma - \Delta_b}{\gamma - \Delta} \sigma_b, \frac{\gamma - \Delta_m}{\gamma - \Delta} \sigma_m, \frac{\gamma - \Delta_n}{\gamma - \Delta} \sigma_n \right) \tag{85}$$

Using (13) and solving for $(\sigma_a, \sigma_b, \sigma_m, \sigma_n)$, we obtain $(\sigma_a, \sigma_b, \sigma_m, \sigma_n)^T = M (\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n)^T$, where

$$M \equiv \gamma \begin{bmatrix} \gamma - (1-\tilde{\sigma}_a)\Delta_a & \tilde{\sigma}_a \Delta_b & \tilde{\sigma}_a \Delta_m & \tilde{\sigma}_a \Delta_n \\ \tilde{\sigma}_b \Delta_a & \gamma - (1-\tilde{\sigma}_b)\Delta_b & \tilde{\sigma}_b \Delta_m & \tilde{\sigma}_b \Delta_n \\ \tilde{\sigma}_m \Delta_a & \tilde{\sigma}_m \Delta_b & \gamma - (1-\tilde{\sigma}_m)\Delta_m & \tilde{\sigma}_m \Delta_n \\ \tilde{\sigma}_n \Delta_a & \tilde{\sigma}_n \Delta_b & \tilde{\sigma}_n \Delta_m & \gamma - (1-\tilde{\sigma}_n)\Delta_n \end{bmatrix}^{-1}$$

Numerically, we set $\gamma = 1.02$, $\tilde{\sigma}_a = 0.0758432283057199$, $\tilde{\sigma}_b = 0.758\,728\,816\,8$, $\tilde{\sigma}_m = 0.00371613521937185$, $\tilde{\sigma}_n = 0.161711819707504$, $\Delta_a = 1 - \delta_a$, $\Delta_b = 1 - \delta_b$, $\Delta_m = 1 - \delta_m$, $\Delta_n = 1 - \delta_n$, $\Delta_k = 1 - \delta_k$, $\delta_a = 0.05$, $\delta_b = 1$, $\delta_m = 0.08$, $\delta_n = 0.61$, $\delta_k = 0.08$, in order to find $\sigma_a = 0.510\,790\,285\,2$, $\sigma_b = 0.350\,679\,408\,5$, $\sigma_m = 0.017\,519\,244\,2$, $\sigma_n = 0.121\,011\,062\,1$. Notice that $\sigma_a + \sigma_b + \sigma_m + \sigma_n = 1$.

Using (13) and setting also $\tau = 0.442\,237\,723\,3$, $\tau_q = 0.442857763$, $\tau_p = 0.38085379464$, $s_k = 0.75$, $s_a = 0.23$, $s_m = 0.01$, $s_p = 0.01$, $\beta = 0.961\,538\,461\,5$ and using (38) with $\tau_k = \tau_q$, eventually, we get from (84): $\theta = 0.563096\,6639$.

### 7.3 Appendix 3: Public spending in France

Public spending and percentages in 2006.

| YEAR 2006 | | Billions € | % of Overall Taxes | Type of Public Expenditures |
|---|---|---|---|---|
| **National Budget** (3) | | **280,75** | **35,43%** | |
| *After transfers to Local Public Administrations* | Health expenditures (2) | 3,70 | 0,47% | |
| | *including Medical R&D expenditures* (1) | *2,95* | *0,37%* | Medical R&D |
| | *Other health expenditures* | *0,76* | *0,10%* | Health expenditures (excl. R&D) |
| | Other expenditures | 277,05 | 34,96% | |
| | *including Investment* (4) | *10,27* | *1,30%* | Public investment |
| | *Consumption* | *266,79* | *33,66%* | Public consumption |
| **Local Public Administrations** (3) | | **101,32** | **12,79%** | |
| | Health expenditures (2) | 1,50 | 0,19% | Health expenditures (excl. R&D) |
| | Other expenditures | 99,82 | 12,60% | |
| | *including Investment* (4) | *43,51* | *5,49%* | Public investment |
| | *Consumption* | *56,31* | *7,11%* | Public consumption |
| **Social Security Administrations** (3) | | **405,75** | **51,20%** | |
| | Health expenditures (2) | 125,90 | 15,89% | Health expenditures (excl. R&D) |
| | Other expenditures | 279,85 | 35,31% | |
| | *including Investment* (4) | *6,33* | *0,80%* | Public investment |
| | *Consumption* | *273,52* | *34,51%* | Public consumption |
| **European Union (U.E.)** (3) | | **4,67** | **0,59%** | Public consumption |
| **Overall Taxes** (3) | | **792,49** | **100,00%** | |

Table 6. Breakdown of taxes paid by French citizens by type of expenditures (2006)

Sources:

(1) Ministry of National Education, Advanced Instruction, and Research, quoted in Fenina & Geffroy [2007], p. 43.

(2) National Institute for Statistics and Economic Studies, INSEE, National accounts (base 2000), in http://www.insee.fr/fr/themes/comptes-nationaux/.

(3) Report on 2006 National Accounts, quoted in *Ministère de l'intérieur, de l'outre-mer et des collectivités territoriales* [2008], p. 34.

(4) 2006 National Accounts, quoted in *Ministère de l'intérieur, de l'outre-mer et des collectivités territoriales* [2008], p. 40.

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