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The Long-run Macroeconomic Impacts of Fuel Subsidies in an Oil-importing Developing Country.

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Abstract

Analytical and numerical results show how the presence of a subsidy on household and firm purchases of oil products distorts long-run macroeconomic aggregates in an oil-importing developing country. Beyond leading to over-consumption of oil products these subsidies also lead to increased labor supply, a distorted emphasis on producing traded goods, and higher real wages. The subsidy also impacts the relative price of non-traded goods, causing it to fall when the non-traded sector is more oil-intensive than the traded sector and vice-versa.

Keywords: oil, fuel-price subsidies, developing countries, fiscal policy
JEL Classifications: Q43, E62, H30, O23
1 Introduction

Subsidies on petroleum products such as diesel and kerosene have been an important issue for many developing countries. One reason is the sheer cost of providing them. For example, one IMF staff report shows that depending upon the year considered, the annual cost of fuel subsidies in Egypt in the latter half of the 2000s ranged from almost four to seven percent of GDP. Another IMF staff report shows that transfers to the state owned petroleum company in Bangladesh were a little less than one and a half percent of GDP in fiscal year 2008. Said and Leigh (2006) show that for a sample of countries in 2005 the average cost of explicit expenditures on fuel subsidies was almost two and a half percent of GDP. These subsidies, therefore, represent a fairly large expenditure for the governments that have them in place.

Another reason fuel subsidies have been an important policy issue has been the difficulty in reducing or removing them, often due to the political turmoil their removal causes. Evidence presented in both Baig, Mati, Coady, and Ntamatungiro (2007) and Coady, Gillingham, Ossowski, Piotrowski, Tareq, and Tyson (2010) suggest this hesitancy appears to have been true even given the large increase in the world price of oil seen in the last half decade. Examples where the pass-through of an increase in world oil prices to domestic prices is less than fifty percent do not appear to be uncommon.

Given their cost and persistence, it is likely that these subsidies have important macroeconomic implications for the countries that choose to enact them. This paper asks the question of how these fuel subsidies affect the macroeconomy of a small, oil-importing developing country in the long-run. The answer to this question, and the main contribution of this paper to the literature, comes in the form of analytical and numerical results that show how these subsidies distort the steady state of a small open economy model that incorporates currency substitution, household and firm demand for oil products, and a subsidy provided by the government on the purchase of those oil products. For a simple model that abstracts from non-traded goods, I present a number of easily interpretable analytical results that provide clear answers about the direction in which a given variable changes when fuel

\footnote{The introduction draws heavily upon a number of IMF working papers and other sources, including but not limited to Bacon and Kojima (2006), Baig, Mati, Coady, and Ntamatungiro (2007), and Coady, Gillingham, Ossowski, Piotrowski, Tareq, and Tyson (2010). More sources are listed in the bibliography.}
subsidies are increased or decreased. In cases where the sign of the change is indeterminate the analytical solutions often show which parameters drive the results. These results then help provide useful intuition for the numerical results produced for a more complicated model that incorporates non-traded goods.

A priori, fuel subsidies would appear to simply promote over-consumption of fuel products. The main point of this paper, however, is that these subsidies have other important effects driven by general equilibrium interactions and the fact that they require government financing. The results show that fuel subsidies drive up hours worked and real wages, lead to an over-emphasis on producing traded goods, and distort the relative price of non-traded goods. In addition, the increased taxation that is necessary to fund the subsidy can lead to a ‘crowding out’ of non-oil consumption under certain conditions. These are important byproducts typically not discussed by policy makers when considering the pros and cons of fuel subsidies.

There is a large literature that focuses on oil and the macroeconomy. This paper joins the subset of that literature that deals specifically with fuel subsidies. Bouakez, Rebei, and Vencatachellum (2008) use a small open economy DSGE model that features nominal rigidities to explore the optimality, in a welfare maximizing sense, of limiting the pass-through of a shock to world oil prices to domestic oil prices. Both Coady, El-Said, Gillingham, Kpodar, Medas, and Newhouse (2006) and del Granado, Coady, and Gillingham (2010) use models based on input-output tables to calculate the real income losses associated with reducing subsidies on fuel products.

Whereas Bouakez, Rebei, and Vencatachellum (2008) focus on optimal monetary policy in the short-run, this work focuses specifically on the long-run impacts that fuel subsidies have on macroeconomic aggregates. As opposed to Coady, El-Said, Gillingham, Kpodar, Medas, and Newhouse (2006) and del Granado, Coady, and Gillingham (2010), the models used in this work are relatively simple, fully specified general equilibrium models. The general equilibrium approach means that the results capture a richer set of interactions between household behavior, firm behavior, and fiscal policy. The simplicity of the models allows the derivation of many analytical results that, in general, would be difficult or impossible to derive in larger scale models. These results enhance our understanding of how subsidies affect the macroeconomy and provide useful intuition for the results from more complicated models.

The rest of the paper proceeds as follows. In the second section, I intro-
duce a simple open economy model that produces traded goods and use this model to derive analytical results about the long-run implications of subsidies on fuel products. The third section adds non-traded goods to the model. The fourth section summarizes and concludes.

2 A Primitive Model

To begin, I consider a small open economy that produces only a composite traded good. Some of the production of this traded good is consumed by households while the rest is used to purchase oil from world oil markets. The economy is small in that it has no effect on the world price of the traded good or the world price of oil.

At this point, I abstract from non-traded goods. Doing so allows me to derive relatively clean analytical results which provide useful intuition for the numerical results presented later for a more complicated model that contains non-traded goods.

The notation used in the exposition is as follows. $dX$ is the differential of the variable $X$, $\dot{X}$ is the time derivative of $X$, $\bar{X}$ is the steady state value of $X$, and $\hat{X}$ is the log-differential of $X$, i.e. $\hat{X} = dX/X$.

2.1 Households

Household activity is controlled by a representative agent who derives disutility from working and utility from consumption of a traded good, consumption of fuel products, and from holdings of real money balances.

The agent’s problem is to maximize

$$\int_{0}^{\infty} \left\{ \frac{(C^{\frac{1}{\sigma_c}} + a_1 O_k^h \frac{1}{\sigma_c} - 1)^{1-\frac{1}{\sigma_c}}}{1 - \frac{1}{\tau}} - \kappa_1 \frac{L^{1+\frac{1}{\rho}}}{1 + \frac{1}{\rho}} + \kappa_2 \frac{(m^{\frac{1}{\sigma_m}} + b_1 F^{\frac{1}{\sigma_m}})^{1-\frac{1}{\sigma_m}}}{(P^{\cdot CPI})^{1-\frac{1}{\sigma_c}}(1 - \frac{1}{\tau})} \right\} e^{-\rho s} ds,$$

subject to a real wealth constraint,

$$A = m + b + F,$$  

and the flow constraint

$$\dot{A} = W^T L^T + (i - \chi) b - C^T - P^s O^h - T - \chi m.$$  

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$C_T$ is consumption of the traded good. PPP holds so the domestic price of $C_T$, denoted as $\bar{P}$, is given by

$$\bar{P} = eP^T,$$

where $e$ is the nominal exchange rate and $P^T$ is the world price of the traded good in dollars. The nominal exchange rate measures the number of units of domestic currency per one dollar and the rate of depreciation is given by

$$\chi = \frac{\dot{e}}{e}.$$

Core inflation, i.e. the inflation rate of $\bar{P}$, is denoted as $\pi$ and is equal to

$$\pi = \chi + \pi^T.$$ 

Since the economy in question does not affect the world price of the traded good, it is convenient to assume that $P^T$ is constant and equal to one which implies that $\pi^T$ is equal to 0 at all times. I assume for the rest of the paper that the traded good is the numeraire and deflate all nominal variables by $e$.

The agent also derives utility from the use of fuel products, $O^h$. The price of these products on world markets is given by $P^o$. Households do not pay the world price, however, but instead face a subsidized price, $P^s$. As with traded goods, the economy is small and does not affect the world price of oil.

The variable $m$ is real money balances of the domestic currency and $F$ the stock of foreign currency. The term $P_{CPI}$ is the theoretic consumer price index (CPI) deflated by the nominal exchange rate. The exact equation for $P_{CPI}$ is determined by the assumptions made on the utility function and, in this case, is given by

$$P_{CPI} = \left(1 + a_i^o e P^{s1-\sigma_e} \right)^{\frac{1}{1-\sigma_e}}. \quad (4)$$

Besides the two currencies, the agent also has access to a domestically traded nominal bond. This asset pays a nominal interest rate of $i$, and its real value is given by $b$. In equilibrium the bond is in net-zero supply.

The parameters $\tau$ and $\mu$ are the intertemporal elasticity of substitution and the wage elasticity of labor supply, respectively. The elasticity of substitution between consumption of traded goods and oil products is $\sigma_e$, and the elasticity of substitution between domestic and foreign currency is $\sigma_m$. 

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Income from labor is given by $WTLT$, where $WT$ is the real wage in the traded sector and $LT = L$ is labor supplied to the traded sector. The agent also pays taxes in two forms. The total value of lump sum taxes are denoted by $T$. Income lost due to the inflation tax is given by the term $\chi m$.

Define $\lambda_1$ as the multiplier on the flow constraint. The first order conditions for the agent’s problem are

$$
\left(CT^{\frac{\sigma_c-1}{\sigma_c}} + a_1O^{\frac{\sigma_c-1}{\sigma_c}}\right)\left(\frac{\sigma_c}{\sigma_c-1}\right)(1-\frac{1}{\sigma_c})^{-1} C^{\frac{1}{\sigma_c}} T = \lambda_1,
$$

Equations (5) and (6) set the marginal cost of the two consumption goods equal to their marginal utilities. Equation (7) sets the marginal benefit of working more equal to the marginal dis-utility of doing so. Equation (9) sets the benefit of holding an extra unit of domestic currency equal to the opportunity cost of doing so, which is the nominal interest rate $i$. Equation (8) does likewise for the domestic bond, with the benefit being equal to $i - \chi$ and the opportunity cost equal to the foregone utility that would have been derived by holding more of the foreign currency.

$$
\kappa_2 b_1 F^{-\frac{1}{\sigma_m}} \left( m^{\frac{\sigma_m-1}{\sigma_m}} + b_1 F^{\frac{\sigma_m-1}{\sigma_m}} \right) \left(\frac{\sigma_m}{\sigma_m-1}\right)(1-\frac{1}{\sigma_m})^{-1} \lambda_1 \left( P^{\frac{1}{\sigma_m}} \right)^{\frac{1}{\sigma_m}} = i,
$$

2.2 Production

Production of the traded good is done by a representative firm operating under perfect competition using the CES technology

$$
Q^T = \left[ \left(A^T L^T \right)^{\frac{\sigma_T-1}{\sigma_T}} + c_1 \left(O^T \right)^{\frac{\sigma_T-1}{\sigma_T}} \right]^{\frac{\sigma_T}{\sigma_T-1}},
$$

(11)
where $A^T$ is a scaling factor, $c_1$ a distribution parameter, $O^T$ is oil demanded by the firm, and $\sigma_T$ is the elasticity of substitution between labor and oil demanded.

Assuming the firm pays the subsidized price for oil, the first order conditions for the firm’s profit maximization problem are

\[ Q^T \frac{1}{\sigma_T} \left( A^T L^T \right)^{-\frac{1}{\sigma_T}} A^T = W^T, \tag{12} \]
\[ Q^T \frac{1}{\sigma_T} c_1 \left( O^T \right)^{-\frac{1}{\sigma_T}} = P^s. \tag{13} \]

These set the marginal products of the inputs equal to their marginal costs. Note that if $P^s < P^o$, it is in the firms interest to overuse oil products and, because of the complementarity between oil and labor, to also overuse labor.

\subsection{2.3 The Government}

The government provides a subsidy on fuel products and earns revenue from levying lump sum taxes and from the inflation tax. I assume that the government purchases oil at the world price of $P^o$ and then sells it at the subsidized price $P^s$, with $P^s \leq P^o$.

In the developing world, it is often the case that fuel subsidies fall most heavily on kerosene and diesel. The former is typically used as a heating oil while the latter is used in transportation and electricity generation. This suggests that a good starting point would be to assume that both households and firms benefit from the subsidies. In this case, the government budget constraint is

\[ \dot{m} = (P^o - P^s) \left( O^h + O^T \right) - T - \chi m. \tag{14} \]

In the steady state this equation reads

\[ \bar{T} + \bar{\chi} \bar{m} = \left( \bar{P}^o - \bar{P}^s \right) \left( \bar{O}^h + \bar{O}^T \right), \]

where the left hand side is the total revenue available to the government, while the right hand side is the total expenditures made by the government. This equation makes clear that lowering $P^s$ requires the government to increase revenues by either increasing lump sum taxes (raising $\bar{T}$), or by increasing seigniorage revenue through an increase in the steady state rate of inflation (raising $\bar{\chi}$).

\footnote{In the steady state the domestic inflation rate of the traded good, $\pi$, is exactly equal to $\chi$ so raising $\chi$ is analogous to raising $\pi$.}
2.4 Government Taxation and Household Decisions

The government’s revenue comes from lump sum taxation, $T$, and from seigniorage revenue, $\chi_m$. Any increase in spending on subsidies must be financed by increasing the revenue derived from one or both of those sources. Increased taxes can change household behavior by reducing the disposable income available to households and, potentially, by distorting the first-order conditions that hold when they make their optimal decisions.

Lump sum taxation, by its very nature, does not distort the first-order conditions of the agent. It does, however, reduce the agent’s disposable income and hence produce income effects. This can be seen by evaluating the agent’s budget constraint, equation (3), at a steady state, which produces

$$\tilde{C}^T + P_s \tilde{O}^h = W^T L^T - T - \tilde{\chi} \tilde{m}.$$  

Holding all else constant a rise in $T$ reduces the income available to be spent on consumption goods. This causes changes in how much the agent consumes and, through variations in the marginal utility of consumption, the agent’s decision about how much to work.

Financing the subsidy through increased seigniorage also reduces the income available to households. In addition to that, it distorts the agent’s holdings of $m$ and $F$ by increasing the opportunity cost of holding domestic currency. To see this, note that equation (10) evaluated at a steady state reads

$$\tilde{i} = \rho + \tilde{\chi}.$$  

Using the inflation tax means raising $\tilde{\chi}$, which drives up the steady state nominal interest rate. As shown in equation (9), the agent’s first order condition for $m$, the nominal interest rate is the opportunity cost of holding domestic currency so when it goes up the agent will choose to re-allocate his holdings of $m$ and $F$.

2.5 The Current Account

The equation linking the current account to the accumulation of foreign currency can be derived by combining the agent’s flow constraint with the government budget constraint and then substituting out $W^t L^T$ using the zero-profit condition of the firm. Doing so gives

$$\dot{F} = Q^T - C^T - P^o (O^h + O^T).$$  \hspace{1cm} (15)
This states that the economy accumulates foreign assets whenever the economy produces more of the traded good than it consumes of the traded good and oil products. Evaluating equation (15) at the steady state gives

$$Q_T^* = C_T^* + P_o^* \left( O_h^* + O_T^* \right).$$

In the long-run, trade must balance so any spending on the traded consumption good and oil products, which are also traded commodities, must be met by increased production of the traded good.

### 2.6 Steady State Implications of the Subsidy

It is possible to derive some useful analytical results for how small changes in $P^s$ change the economy’s steady state. Equations (4), (5), (6), (7), (11), (12), (13), and (15) provide the solutions for $P^CPI$, $C_T$, $O_h$, $L$, $Q_T$, $W$, $O_T$, and $\lambda_1$. These variables can be solved separately from the other ones due to the separability of money in the utility function and the fact that taxation in this model is non-distortionary with respect to the non-monetary variables. With these solutions, it is then possible to solve for how $m$, $F$, $i$, and $T$ or $\chi$ vary when $P^s$ changes. This can be done using equations (8), (9), (10), and (14).

The solutions are a combination of the effects brought about because the agent faces a different relative price for $O_h$ and because the firm faces a different relative cost for $O_T$. It pays dividends, though, to consider how each of these channels affects the solutions in isolation from the other. To do this, I first solve for a case where the household pays the subsidized price but the firm pays the world price of oil. I then consider what happens if the household pays the world price of oil but the firm pays the subsidized price.

#### 2.6.1 Subsidy Benefits Only Households

Begin by differentiating the two first-order conditions for the firm, which always pays $P^o$ for oil products in this case. Since the economy is small and does not affect world oil prices, one immediately finds that

$$\hat{W}^T = 0.$$  

(16)

Since there is no change in the marginal product of oil the real wage remains constant. It is also easy to show that in this case

$$\hat{O}_T^* = \hat{L}_T^*.$$
a result driven by the fact that the relative price of oil to labor remains unchanged in this case.

The solutions for $C^T$, $O^h$, $\lambda_1$, and $L^T$ must be derived jointly. The current account equation gives

$$\hat{L}^T = \frac{\theta_{ct}}{1 - \theta_{ot}} \hat{C}^T + \frac{\theta_{oh}}{1 - \theta_{ot}} \hat{O}^h,$$

where

$$\theta_{ct} = \frac{\bar{C}^T}{\bar{Q}^T},$$

$$\theta_{ot} = \frac{\bar{P}^o \bar{O}^T}{\bar{Q}^T},$$

$$\theta_{oh} = \frac{\bar{P}^o \bar{O}^h}{\bar{Q}^T}.$$

A solution for $\lambda_1$ in terms of $O^h$ and $C^T$ can be produced using the household’s first order condition for $L^T$ and the equation just derived from the current account. The first order conditions for $C^T$ and $O^h$ can then be used to show that their solutions are

$$\hat{O}^h = -\frac{\sigma_c}{\tau + \frac{1}{\mu}} \left[ \frac{\tau \gamma_{oh} + \sigma_c \gamma_{ct}}{\sigma_c \tau} + \frac{\theta_{ct}}{\mu(1 - \theta_{ot})} \right] \hat{P}^s, \quad (17)$$

$$\hat{C}^T = -\frac{\sigma_c}{\tau + \frac{1}{\mu}} \left[ \frac{(\tau - \sigma_c) \gamma_{oh}}{\sigma_c \tau} - \frac{\theta_{oh}}{\mu(1 - \theta_{ot})} \right] \hat{P}^s, \quad (18)$$

where

$$\gamma_{ct} = \frac{\bar{C}^T}{\bar{C}^T + \bar{P}^s \bar{O}^h};$$

$$\gamma_{oh} = \frac{\bar{P}^s \bar{O}^h}{\bar{C}^T + \bar{P}^s \bar{O}^h};$$

are the expenditure shares for the two consumption goods in the original steady state.

As one would expect, for $O^h$ the coefficient in front of $\hat{P}^s$ is negative so that lowering $P^s$ leads to increased consumption of fuel products. The solution for $C^T$, however, is more complicated as the coefficient can be positive or
negative depending upon the calibration of the model. As shown in the technical appendix, $C^T$ and $O^h$ are Edgeworth substitutes, independent goods, or Edgeworth complements as $\tau \leq \sigma_c$. When $\tau \leq \sigma_c$, i.e. the two goods are substitutes or independent, lowering $P^s$ unambiguously lowers consumption of the traded good. On the other hand, when $\tau > \sigma_c$ consumption rises iff

$$\mu > \frac{\theta_{oh} \sigma_c \tau}{(1 - \theta_{ot}) \gamma_{oh} (\tau - \sigma_c)}.$$

At first glance, this result may seem odd since the two goods are complements when $\tau > \sigma_c$. A partial equilibrium approach would say that in this case $C^T$ should rise since $P^s$ has been lowered. But, this is a general equilibrium model and the solution for $C^T$ captures both substitution effects driven by the change in $P^s$ and income effects driven by increased taxation. Increased subsidies (lower $P^s$) require increased taxation which, in the end, the agent pays for by reducing consumption and working more. When the labor supply is very inelastic, i.e. $\mu$ is very small, the optimal choice is to reduce consumption of the traded good, even if the two goods are complements.

Besides distorting decisions regarding $O^h$ and $C^T$, a change in $P^s$ also affects the demand for inputs by the firm. To see this, substitute the solutions for $O^h$ and $C^T$ into the current account equation. This produces

$$\hat{L}^T = \frac{\mu [-\tau \gamma_{oh} (1 - \theta_{ot}) + \sigma_c (\theta_{ct} \gamma_{oh} - \gamma_{ct} \theta_{oh})]}{(1 - \tau)(1 - \theta_{ot})} \hat{P}^s.$$  \hspace{1cm} (19)

By substituting out the $\gamma$ and $\theta$ terms one can show that the coefficient on this term is always negative so lowering $P^s$ always brings about greater hours worked. As $\hat{O}^T = \hat{L}^T$, we get the automatic result that a decrease in $P^s$ also drives up the demand for oil by the firm, even though the firm does not directly benefit from the subsidy. Given the fact that both $O^T$ and $L^T$ increase we also know that $\hat{Q}^T$ will be positive.

Increased output in the economy could be seen as a positive result of the subsidy by some. But, the reason the economy produces more is because the subsidy has led to over-consumption of fuel products. These products must be traded for and, therefore, lead to the higher levels of output seen. The only case where this would not occur would be if labor was inelastically supplied. But, in that special case we would get the result that $C^T$ would be completely crowded out to pay for the extra consumption of $O^h$. This result
highlights the fact that the subsidy is not a free lunch at the aggregate level. The benefits households derive from the extra consumption of fuel products comes at the expense of working more and, possibly, consuming less of other goods.

2.6.2 Subsidy Benefits Only Firms

It is possible to use the exact same procedure to solve for the steady state changes when the subsidy only benefits firms. The change in wages in this case is given by

$$\hat{W}^T = -\frac{\alpha_o^T}{\alpha_l^T} \hat{P}^s,$$  \hspace{1cm} (20)

where

$$\alpha_o^T = \frac{\bar{P}^s \bar{O}^T}{\bar{W}^T \bar{L}^T + \bar{P}^s \bar{O}^T},$$

$$\alpha_l^T = \frac{\bar{W}^T \bar{L}^T}{\bar{W}^T \bar{L}^T + \bar{P}^s \bar{O}^T},$$

are the cost shares of oil and labor in the traded sector, respectively. Lowering \(P^s\) unambiguously drives up wages in the economy as it increases demand for oil products which, due to the complementarity between oil and labor, drives up the marginal product of labor.

In this case the marginal product of oil is now less than the true cost of said oil as \(P^s < P^o\). This leads to a slight change in the equation one derives from the current account equation,

$$\hat{L}^T = \frac{\theta_{ct}}{1 - \theta_{ot}} \hat{C}^T + \frac{\theta_{oh}}{1 - \theta_{ot}} \hat{O}^h + \frac{\sigma_T}{\alpha_l^T (1 - \theta_{ot})} \hat{P}^s.$$

A \(\hat{P}^s\) term directly appears this time and since \(\alpha_l^T < \theta_{ot}\), it has a negative sign. This reflects the gap between the cost of the oil to society and the extra production that the oil generates, a gap that must eventually be paid for somehow.

As before, the first order conditions for \(C^T\) and \(O^h\) provide the solutions for those two variables,

$$\hat{O}^h = \frac{1}{\sigma_T} \left( \frac{\sigma_T}{\mu \alpha_l^T (1 - \theta_{ot})} \hat{P}^s + \hat{W}^T \right),$$  \hspace{1cm} (21)

$$\hat{C}^T = \hat{O}^h,$$  \hspace{1cm} (22)
When the subsidy only benefits firms, the consumption variables are pulled in opposite directions by two different forces. The $\hat{W}^T$ term represents the fact that lowering $P^s$ drives wages up, which pushes for increased consumption of both goods. On the other hand, the $\hat{P}^s$ term, which has a positive coefficient on it, captures the increased taxation required to finance the subsidy and the gap that appears in the current account equation. These forces push for decreased consumption.

Unfortunately, after substituting out the $\hat{W}^T$ term, it is not possible to sign the combined coefficient on the term. It is possible, however, to show that the coefficient will be negative iff

$$\mu > \frac{\sigma_T (\theta_{ot} - \alpha^T_0)}{(1 - \theta_{ot}) \alpha^T_0}.$$  

In other words, when labor is sufficiently elastic the agent consume more of both goods. Intuitively, this is similar to what happens when only households benefit from the subsidy. Inelastic labor supply leads to sufficiently small responses in $L^T$ which then makes it optimal for the household to pay for the tax by reducing consumption of both goods.

While the results for $C^T$ and $O^h$ require some assumptions about how elastically labor is supplied, it is possible to derive unambiguous results for the labor supply itself. The solution for that variable is

$$\hat{L}^T = \frac{\mu \sigma_T (\alpha^T_0 - \theta_{ot})}{\alpha^T_l (\tau + \mu)} \hat{P}^s + \frac{1}{\tau + \frac{1}{\mu}} \hat{W}^T.$$  

After substituting out $\hat{W}^T$, the coefficient on this solution is always negative so that once again, lowering $P^s$ leads to more labor being supplied. The first order conditions for the firm can also be manipulated to give

$$\hat{O}^T = \hat{L}^T - \frac{\sigma_l}{\alpha^T_l} \hat{P}^s,$$

which leads to the immediate result that $\hat{O}^T$ is positive and, therefore, that $Q^T$ increases in the long-run when $P^s$ is lowered.

### 2.6.3 Subsidy Benefits Households and Firms

The results just derived apply to cases where the subsidy benefits either households or firms. This helps highlight the effects at play that determine
how a specific variable adjusts when $P^s$ is changed. The empirical evidence suggests that usually both households and firms benefit from the subsidy. In that case the long run impacts are a combination of the effects that occur in the two special cases just discussed.

Table one summarizes what we know so far by showing how $C^T$, $O^h$, $L^T$, $O^T$, $Q^T$, and $W^T$ vary when $P^s$ is changed. The box contains a - sign if the variable decreases, a + if it increases, and a ? if the change is ambiguous. The three cases where the subsidy falls on the household, the firm, or both households and firms are considered.

Definite answers can be given for the variables related to production. In all three cases, lowering $P^s$ leads to a long-run increase in labor supplied, oil demanded by firms, and output in the traded sector. Wages remain unchanged or rise, with the most realistic case calling for a rise in wages.

For the consumption variables, the changes depend upon the calibration of the model. More specifically, the directions for $O^h$ and $C^T$ depend upon the elasticity of the labor supply and whether or not the two goods are Edgeworth complements. The analytical results suggest that crowding out of the non-oil consumption good may occur when the goods are substitutes consumption or if $\mu$ is small enough.

2.6.4 Monetary and Fiscal Variables

As of now, nothing has been said of the variables $m$, $F$, $i$, $T$ or $\chi$. With the solutions for the real variables in hand, though, the second block of equations can be used to solve for these. In the case where lump sum taxes are used to finance the spending, we know that $T$ must rise to pay for the increased spending on the subsidy. Given that $\chi$ is fixed, we know that the nominal interest rate does not change across steady states. In the polar case where the inflation tax is used to finance spending, we know that $T$ is fixed, and that $\chi$ and $i$ rise across steady states. Unfortunately, easily signed solutions for $m$ and $F$ were impossible to derive. Therefore a discussion of how the subsidy impacts money demand is delayed until the next section when numerical results are presented.
3 An Economy with Non-traded Goods

The primitive model is useful as a starting point because the analytical results derived from it are relatively clean cut. This helps provide useful intuition about the various ways in which the subsidy affects certain macroeconomic variables. An important caveat, though, is that most developing countries are not nearly as open as the model assumes in that a significant portion of economic activity is in non-traded goods. This section introduces non-traded goods into the primitive model and then re-considers the long-run effects of the subsidy. As I will show shortly, this is an important extension as fuel subsidies distort relative prices and the extent to which the economy produces traded or non-traded goods.

3.1 The Model

Technically, adding a non-traded good is relatively simple. First, the utility from consumption is changed to

$$\frac{\left( C^T \sigma_c^{-1} + a_1 O^h \sigma_c^{-1} + a_2 C^n \sigma_c^{-1} \right)^{(1-\frac{1}{\tau})}}{1-\frac{1}{\tau}},$$

where $C^n$ is consumption of the non-traded good. The equation for $P^{CPI}$ now reads

$$P^{CPI} = \left(1 + a_1 \sigma_c P^{s1-\sigma_c} + a_2 \sigma_c P^{n1-\sigma_c} \right)^{-\frac{1}{1-\sigma_c}},$$

where $P^n$ is the relative price of the non-traded good to the traded good. Aggregate labor is now defined as $L = L^n + L^T$ and the disutility from working is now modeled as

$$-\kappa_1 \frac{(L^n + L^T)^{1+\frac{1}{\mu}}}{1+\frac{1}{\mu}}.$$

The traded good remains the numeraire so the wealth constraint is the same but the flow constraint for the agent now reads

$$\dot{A} = W^n L^n + W^T L^T + (i - \chi) b - C^T - P^n C^n - P^s O^h - T - \chi m,$$

where $W^n$ is the real wage paid in the non-traded sector, denominated in terms of the traded good.
The first order conditions for labor supply in the traded and non-traded sector merit some discussion. These equations are given by

\[ \kappa_3 L^T = \lambda_1 W, \]
\[ \kappa_3 L^n = \lambda_1 W_T. \]

As labor is fully mobile between the traded and non-traded sector, wages across sectors must be equal. This can be seen by combining the two equations together. I, therefore, simplify the exposition by defining the equilibrium real wage as \( W \). The remaining first order condition then reads

\[ \kappa_3 L^T = \lambda_1 W. \]

I assume that the non-traded good is produced by a representative firm operating under perfect competition using oil and labor as inputs. Technology in this sector is

\[ Q^n = \left[ \left( \frac{A^n L^n}{\sigma_n - 1} \right)^{\frac{\sigma_{n-1}}{\sigma_n}} + \frac{d_1}{\sigma_n} \left( \frac{O^n}{\sigma_{n-1}} \right)^{\frac{\sigma_{n-1}}{\sigma_n}} \right]^{\frac{\sigma_n}{\sigma_n - 1}}, \]

where \( A^n \) is a scaling factor, \( d_1 \) is a distribution parameter, \( O^n \) is oil demanded by the firm, and \( \sigma_n \) is the elasticity of substitution between labor and oil demanded. The profit maximization problem for the firm, denominated in dollars, is

\[ \max_{L^n, O^n} Q^n - \frac{W^n}{P_n} L^n - \frac{P_s}{P_n} O^n. \]

The first order conditions for this problem are

\[ Q^n \frac{1}{\sigma_n} (A^n L^n)^{-\frac{1}{\sigma_n}} A^n = \frac{W^n}{P_n}, \]
\[ Q^n \frac{1}{\sigma_n} d_1 (O^n)^{-\frac{1}{\sigma_n}} = \frac{P_s}{P_n}. \]

These equate the marginal products of each input with its respective marginal cost, denominated in dollars. The relative price term appears in the first order conditions due to the choice of the numeraire. The market clearing condition for the non-traded sector is given by

\[ C^n = Q^n, \]

and holds at all times.
Since the firm in the non-traded sector also uses oil products, the government budget constraint and the current account equation are slightly modified. They are, respectively,

\[ \dot{m} = (P^o - P^s) \left( O^h + O^T + O^n \right) - T - \chi m. \]  
(27)

\[ \dot{F} = Q^T - C^T - P^o O^h - P^o O^T - P^o O^n. \]  
(28)

### 3.2 Analytical Results

The procedure used earlier to derive results about how the subsidy distorts the steady state can be used with this model as well. While most of the analytical results end up being too complicated to easily interpret, there are some important exceptions. Results for the change in wages and the relative price of the non-traded good are particularly easy to derive. It is also possible, in the case where the subsidy only benefits households, to derive solutions for \( C^T \) and \( O^h \).

#### 3.2.1 Subsidy Benefits Only Firms

The four first-order conditions for the firms can be used by themselves to solve for the changes in \( W \) and \( P^n \) across steady states. In the case that firms benefit from the subsidy but households do not, the results state that

\[ \hat{W} = -\frac{\alpha^T}{\alpha^T_i} \hat{P}^s, \]  
(29)

\[ \hat{P}^n = \frac{\alpha^n_o - \alpha^T_o}{\alpha^T_i} \hat{P}^s, \]  
(30)

where \( \alpha^n_o \) is the cost share of oil in the non-traded sector. The solution for \( \hat{W} \) is exactly the same as before and shows that lowering \( P^s \) unambiguously increases wages in the economy.

The result for \( \hat{P}^n \) depends upon whether the cost share of oil is higher in the non-traded sector or the traded sector. If it is higher in the traded sector then \( \hat{P}^n \) falls and vice-versa. The key to understanding why this holds is to realize that the change in \( P^s \) may have asymmetric effects on costs in the two sectors. Wages are guaranteed to rise when \( P^s \) falls, but the exact amount by which \( W \) rises is determined by how oil intensive the traded sector is. The rise in wages drives up costs in the non-traded sector but this is, potentially,
offset by lower costs for fuel products. If the non-traded sector is relatively
more oil-intensive than the traded sector, the decline in the domestic price
of oil is sufficient to bring about lower costs for the non-traded firm. This
reflects itself in a lower relative price in the long-run. In the opposite case
the reduction in the price of oil is not enough to override the rise in wages
and costs rise in the non-traded sector, which drives up \( P^n \).

Regardless of whether \( P^n \) rises or falls, the fact that it does change means
the subsidy has an additional unintended consequence on the economy. The
change in \( P^n \) creates an additional substitution effect that will distort house-
hold decisions about the relative mix of traded to non-traded consumption
goods. If \( P^n \) rises, for example, this pushes for households to substitute away
from non-traded goods. Given the market clearing condition for non-traded
goods, this indirectly has implications for the amount of labor supplied to
the non-traded sector, as well.

### 3.2.2 Subsidy Benefits Only Households

When the subsidy only falls on households the results are simple,

\[
\hat{W} = 0, \quad \hat{P}^n = 0. \tag{31}
\]

With the world price of oil constant wages, and therefore costs in both sectors,
remain unchanged across steady states. As costs do not change for either
sector the relative price of the non-traded good is also fixed in this case.

As \( P^n \) is constant it is possible to solve for the changes in the three
consumption variables,

\[
\hat{O}^h = -\frac{\sigma_c}{\tau + \frac{1}{\mu}} \left[ \frac{\tau \gamma_{oh} + \sigma_c(1 - \gamma_{oh})}{\sigma_c \tau} + \frac{\eta(\theta_{ct} + \theta_{cn})}{(1 - \theta_{ot})\mu} + \frac{1 - \eta}{\mu} \right] \hat{P}^s, \tag{33}
\]

\[
\hat{C}^T = -\frac{\sigma_c}{\tau + \frac{1}{\mu}} \left[ \frac{(\tau - \sigma_c)\gamma_{oh}}{\tau \sigma_c} - \frac{\eta \theta_{oh}}{\mu(1 - \theta_{ot})} \right] \hat{P}^s, \tag{34}
\]

\[
\hat{C}^n = \hat{C}^T, \tag{35}
\]

where \( \eta = \frac{LT}{L} \) is the share of labor used in the traded sector out of total
labor supplied. When \( \eta = 1 \) the solutions collapse to those in the previous
section, as \( \theta_{cn} = 0 \).
Despite the addition of a few extra terms, the qualitative results are exactly the same as in the case with only traded goods. Lowering $P^s$ always increases consumption of fuel products but may or may not lead to crowding out of the other consumption goods available to the household. More specifically, as before if $\tau \leq \sigma_c$ the three goods are Edgeworth substitutes and the consumer reduces consumption of $C^m$ and $C^T$. If $\tau > \sigma_c$ then consumption rises iff

$$\mu > \frac{\eta \theta_{oh} \tau \sigma_c}{(1 - \theta_{ol})(\tau - \sigma_c)\gamma_{oh}},$$

with the result again being driven by income effects associated with increased taxation.

### 3.2.3 Subsidy Benefits both Firms and Households

Unfortunately, the analytical results are fairly sparse for the model with non-traded goods. In a realistic case where households and firms benefit from the subsidy we can rest assured that lowering $P^s$ causes wages to rise in the economy. Given cost-shares for oil in the two sectors we can also make accurate predictions about what happens to the relative price of the non-traded good. Beyond this it is impossible to know for sure which direction the variables will move.

### 3.3 Numerical Results

Given the lack of easily interpretable analytical solutions, I turn to numerical solutions to help fill in the gap. This requires calibrating the model, specifying a specific value for $\hat{P}^s$, and choosing how fiscal policy finances the subsidy.

I calibrate the model’s initial steady state to match features of a typical oil-importing developing country that has had experience with fuel subsidies, such as Cambodia, Bangladesh or Honduras. Real GDP is calibrated to unity so that GDP ratios can be used to aide in calibrating many of the model’s variables. I consider a baseline calibration and several alternative calibrations for $\mu$, $\sigma_c$, and the relative usage of oil in the traded and non-traded sectors. Table two at the end of the paper records the baseline and alternative values used for the parameters and many of the starting values for the model’s variables.
Some evidence presented in Coady, El-Said, Gillingham, Kpodar, Medas, and Newhouse (2006) show that the consumption shares of fuel products for several developing countries were somewhere in the range of 3 to 6.5 percent. The numbers for developed countries are typically in that range as well, so I choose to calibrate the consumption share of fuel products at 5 percent of GDP.

Unfortunately, little data is available that would allow one to pin down spending on oil products by the traded and non-traded sectors. Given the lack of data I use the following strategy. I set total spending on oil products by both sectors equal to 5 percent of GDP. This is an educated guess made with reference to what this number was for developed countries when they were more energy intensive. For the baseline calibration, I assume that spending on oil products is equal to 2.5 percent of GDP in each sector. Under one alternative calibration I set spending in the traded sector equal to 1 percent of GDP while spending in the non-traded sector is equal to 4 percent. Another alternative sets spending in the traded sector to 4 percent of GDP and spending in the non-traded sector at 1 percent. These calibrations allow me to explore the importance of the relative oil intensities of the two sectors.

The analytical results earlier highlighted the importance of $\mu$. A baseline case sets $\mu$ to 1 while an alternative calibration considers a low-elasticity of $1/8$. The parameter $\sigma_c$ controls the price-elasticity of demand for fuel products and, in conjunction with $\tau$, determines the complementarity of the consumption goods. I set $\tau$ equal to .50 and consider settings of .25 and .75 for $\sigma_c$. The former sets the goods as Edgeworth complements and provides a low price elasticity of demand while the latter sets the goods as Edgeworth substitutes and gives a price elasticity of demand for fuel products on the high range of what empirical studies typically find.

In specifying a value for $\hat{P}^s$, I assume that initially $P^s = P^o$ and then ask what happens to the steady state when $P^s$ is lowered by 10 percent. For the calibrations used, this usually results in a subsidy that costs roughly 1 percent of GDP in the new steady state.

Finally, fiscal policy must be specified. I consider two polar cases: one where $\chi$ is held fixed and $T$ adjusts to clear the budget constraint and one where $T$ is held fixed while $\chi$ is adjusted to clear the budget constraint.

Table 3 shows the results for the case where lump sum taxation adjusts to clear the government budget constraint and $\sigma_c$ is set to .25 so that the consumption goods are Edgeworth complements. There are seven columns in this table with each row in the first column identifying what variable is
being discussed. The next three columns set \( \mu \) to 1 and show the results for the three calibrations of oil usage. In column 2 we have higher spending on oil in the traded sector, column 3 equal spending, and column 4 higher spending in the non-traded sector. Columns 5 - 7 is for the alternative, lower calibration of \( \mu \).

Several results presented in table 3 perfectly mirror the analytical results presented earlier for the model with non-traded goods. Reading across the row for \( W \), we see that the change in wages is driven by the relative usage of oil by the two sectors, with larger changes associated with greater usage by the traded sector. Likewise, the response of \( P^m \) is in line with the analytical result derived earlier, rising when oil is more heavily used by the traded sector and falling when oil is used more heavily in the non-traded sector.

The changes in \( C^T \) and \( C^n \) are small in magnitude, sometimes positive and sometimes negative. Substitution effects brought about by changes to \( P^s \) and \( P^n \), increased income from higher wages, and increased taxation seem to essentially wash out for the two non-oil consumption goods. As predicted by the analytical results, reducing the size of \( \mu \) brings about smaller increases in total labor supplied and makes it more attractive for the agent to reduce consumption to pay for increased taxes. This can be seen by comparing the results for the consumption variables in columns 2 and 5, 3 and 6, and 4 and 7. In each case the change is about .25 percent lower for the low setting of \( \mu \). For certain cases it is possible to see reductions in one or both of the non-oil consumption goods, although quantitatively the amounts are small.

Rows 4 - 6 show the changes in the total labor supply and the breakdown in labor supplied to the two sectors. Total labor supply rises in both cases considered, with the traded sector seeing a proportionally greater increase than the non-traded sector. When \( \mu \) is low it is even possible to see labor flow out of the non-traded sector. The results for \( Q^T \) and \( Q^n \) mirror the implications for \( L^T \) and \( L^n \) as there is always a larger response in output from the traded sector. The emphasis on producing traded goods is caused by the same reason output increased in the model without non-traded goods. Lowering \( P^s \) causes increased use of fuel products in the long-run. The numerical results show that this is paid for mainly through increased production in the traded sector.

Table 4 presents the results for the case where \( T \) once again adjusts, but \( \sigma_c \) is equal to .75 instead of .25. This increases the price elasticity of demand for fuel products and makes the different consumption goods Edgeworth substitutes, as \( \sigma_c > \tau \) now. The main changes in the results are driven
by the fact that the 10 percent drop in $P^s$ now creates a larger substitution effect towards $O^h$ away from $C^T$ and $C^n$. This leads to reductions in one or both of the non-oil consumption goods in all cases considered. Given the significantly larger response in $O^h$, it also brings about greater inflows of labor to the traded sector versus the non-traded sector and, in fact, leads to reductions in labor supplied to the non-traded sector in all cases considered.

Tables 5 and 6 repeat the experiments of table 3 and 4, respectively, but instead of lump sum taxation adjusting to clear the government budget constraint the inflation tax is used to finance the spending. Since the two financing methods are non-distortionary with respect to the real variables in the model, the results for variables such as $C^T$ and so on are equivalent for the two financing choices. The results for rows 1 - 12 ($C^T$ to $P^n$) are exactly the same in table 3 and 5, and in 4 and 6.

Different results hold for the monetary variables $m$ and $F$, which are respectively shown in rows 13 and 14. This is true because, as explained in section 2.4, adjusting $\chi$ to clear the government budget constraint distorts the first-order conditions for holdings of $m$ and $F$. In tables 3 and 4, the results for $m$ and $F$ are driven entirely by changes in aggregate consumption spending, $C^T + P^n C^n + P^s O^h$, since the nominal interest rate does not change across steady states. In that case, the results are fairly boring. The results are more dramatic, though, when $\chi$ is used to finance the new spending. Given the cost of the subsidy, the steady state inflation rate rises dramatically in the long-run, around 16 percent. This causes significant capital flight and a large increase in the nominal interest rate. Of course, the results are driven by the fact that this is the worst case scenario where the inflation tax is used to finance 100 percent of the spending increase. For some combination of increased lump sum taxation and reliance on the inflation tax, the increase in $\chi$ will be less and the corresponding amount of capital flight reduced in the long-run.

4 Summary and Conclusion

This paper has examined the long-run implications of fuel subsidies in an oil-importing developing country. The main contribution to the literature is a set of analytical and numerical results that show how the economy’s steady state is distorted when the domestic price of oil is permanently reduced below that of the world price of oil. These results show that fuel subsidies
have important effects on a number of macroeconomic variables above and beyond just promoting over-consumption of fuel products.

For an economy that produces only traded goods, analytical results showed that the subsidy drives up wages in the economy, leads to inefficiently high labor supply, and increases production of the traded good to pay for the over-consumed oil. There is also a distinct possibility that non-oil consumption could be crowded out depending upon how elastic labor is supplied.

A number of similar results hold for an economy that also produces non-traded goods. As before, the subsidy drives up wages and leads to an over supply of labor. In addition, it distorts the allocations between the traded and non-traded sectors, generally leading to an over-emphasis on producing traded goods to pay for the increased consumption of fuel products. This occurs regardless of whether or not the relative price of the non-traded good rises or falls in the economy. When the inflation tax is used to finance the subsidy, there are also significant impacts on monetary variables as well.

All of these results highlight impacts of these subsidies that are usually ignored when considering the pros and cons of fuel subsidies in these countries. While households certainly benefit from extra consumption of fuel products, this comes at the indirect cost of working more, potentially reduced consumption of other goods, and inefficient allocation of resources towards the traded sector.

This paper has focused on long-run implications that fuel subsidies have for macroeconomic aggregates. There appear to be several promising avenues for future research that go beyond this focus. First, one could use the model in this paper to explore the welfare implications of permanently removing a subsidy already in place or the opposite policy of imposing the subsidy where there was none before. This would require tracking the transition paths of the model’s variables from the initial steady state to the new steady state brought about by the policy change and deriving an accurate measure of the welfare gains/losses brought about by the policy. Second, a shortcoming of this model is that it is essentially silent on distributional issues. An interesting extension would be to design a model where agents are heterogenous in their income. This would lead to differences in spending on fuel products and, consequently, differences in who benefits from the subsidies. These are left for future research.
Table 1: Qualitative Changes in Select Macroeconomic Variables

<table>
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<th>Variable</th>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$C^T$</td>
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<td>?</td>
<td>?</td>
</tr>
<tr>
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<td>?</td>
<td>?</td>
</tr>
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<td>+</td>
<td>+</td>
</tr>
<tr>
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<td>+</td>
<td>+</td>
</tr>
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Table 2: Calibrated Parameters and Steady State Values

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<th>Alternative Value</th>
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Table 4: Percent Change Across Steady States when $T$ Adjusts, $\sigma_c = .75$

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</tr>
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</tr>
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</tr>
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Table 5: Percent Change Across Steady States when $\chi$ Adjusts, $\sigma_c = .25$

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<td>0.03</td>
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<td>2.70</td>
<td>2.63</td>
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<td>2.55</td>
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<td>0.71</td>
<td>0.62</td>
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<td>0.47</td>
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<td>-0.03</td>
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<tr>
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<tr>
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<td>19.30</td>
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Table 6: Percent Change Across Steady States when $\chi$ Adjusts, $\sigma_c = .75$

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<tbody>
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<td>0.18</td>
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<td>7.87</td>
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<tr>
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27
References


