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October 2011

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Abstract. In this paper, we use a two-period one-to-one matching model with incomplete information to examine the effect of changes in divorce costs on marital dissolution. Each individual who has a nontransferable expected utility about the quality of each potential marriage decides whether to marry or to remain single at the beginning of the first period. Those who married in the first period learn the qualities of their marriages at the beginning of the second period and then decide whether to stay married or to unilaterally divorce. We show that for any society, there exist matching environments where the probability of the marital dissolution is not decreasing in divorce costs under a gender-optimal matching rule. In such environments an allocation effect of divorce costs with ambiguous sign outweighs an incentive effect which is always negative.

JEL classification: C78, J12.

Key words: One-to-one matching, marriage dissolution, divorce, incomplete information.

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1 Introduction

An unsettled debate in the economics literature is about the role of divorce laws on the probability of marital dissolution, focusing in particular on the big shift in the divorce behavior in the United States (US) in the last five decades. Between 1965 and 1980, the divorce rate in the US more than doubled reaching a rate of around 8 divorces per 1000 adults and then it started to decline steadily together with the marriage rate. As a reason of this profound change in the divorce rate, a number of empirical works cites the adoption, in 1970s, by majority of states of looser divorce laws which allowed spouses to unilaterally file for divorce. While Peters (1986) finds no impact of the newly adopted unilateral divorce laws on divorce rates between 1975 and 1978, Allen (1992) finds, using the same data set, significant and permanent effects when he controls for geographical differences in divorce propensities. Using US state level panel data from 1968 to 1988 so as to control state and year fixed effects as well as state-specific time trends, Friedberg (1998) shows that unilateral divorce laws led to a 6% higher divorce rate and explains 17% of the increase in divorces. Later, Gruber (2004) finds, using the US census data from 1960, 1970, 1980, and 1990, a similar positive effect of the unilateral divorce laws on the stock of divorced women and men. More recently, González and Viitanen (2009) and Kneip and Bauer (2011) show using longitudinal data on divorce rates in large samples of European countries that the results in the literature finding significant positive effects of unilateral divorce laws are not pertinent to US. However, using an extended data set from 1956 to 1998 and adding some variables into the econometric analysis of Friedberg (1998) so as to model the dynamic response of divorce explicitly, Wolfers (2003, 2006) find that the adoption of unilateral divorce laws increased US divorce rates sharply in the first two years following the adoption, but the effects of this legal change decayed within a decade. A theoretical explanation (which we will further address in this section) as to why short-term and long-term effects of a policy switch from a mutual consent regime to a unilateral divorce regime may differ is proposed by Rasul (2006).

In this paper, we use a two-period one-to-one matching model to examine the effect of changes in divorce costs on marital dissolution. Since the exit option in a marriage becomes easier with unilateral divorce, our paper implicitly studies the effects of a shift to a unilateral divorce regime. In our model, individuals who have incomplete information about the nontransferable utility of marriage with each of their potential mates decide whether to marry or to remain single at the beginning of the first period.[2] Those who married in the first period learn the utility of their marriage at the beginning of the second period and then decide whether to stay married or to divorce for the rest of their lives. We show that divorce costs affect not only individuals' decision to divorce in the second period but also their first-period decisions to marry as well as whom to marry. Interestingly, the average probability of marital dissolution in the society is affected by all these three decision channels, which we simply name as 'the divorce channel', 'the marital status channel' and 'the marital composition channel' respectively.

We can separate the effect of divorce costs on marital dissolution through the divorce channel, which we call 'the incentive effect', since the lower the divorce costs, the higher the incentive for married couples to divorce in the second period. In addition, we can also identify in our model a composite effect of divorce costs operating through the other two channels, namely 'the allocation effect'. While it is certain that with lower divorce costs more individuals may decide to change their marital status from single to married in the first period, the extent any cost effects will be transmitted through the described marital status channel to the average probability of marital dissolution entirely depends on the divorce likelihoods of the new couples relative to those of the existing couples. The same uncertainty is true about the operation of the marital composition channel, as well. While a sufficiently big change in divorce costs can change the ordinal preferences of individuals over potential mates and hence the identities of the spouses

[2] In situations where couples can efficiently transfer utilities, the change in divorce costs (or generally divorce laws), as argued by Becker et al. (1977), can only affect the distribution of welfare within marriages, not the marriage or divorce rates, thanks to the Coase theorem.

in an equilibrium matching, the effect of this second type of reallocation on the average probability of marital dissolution is also ambiguous. Hence, the ambiguity of the allocation effect (and consequently the ambiguity of the aggregate effect of divorce costs). So, it is not true that the average probability of marital dissolution can always be reduced by tighter divorce laws that propose higher divorce costs. Conversely, it is not true either that divorce rates will be higher under looser divorce laws. Indeed, as the main result of our paper shows, for any society we can find matching environments where under a gender-optimal matching rule the allocation effect of divorce costs outweighs the incentive effect and thus the average probability of marital dissolution in the society is nondecreasing in divorce costs.

The closest works that study the effects of divorce costs on marital dissolution are Bougheas and Georgellis (1999) and Rasul (2006). In the multi-period setup of both works, each individual is randomly matched with a potential partner and then decide to marry or to remain single in the first period depending on an imperfect signal about the quality of the potential marriage. Those who marry learn the true quality of the marriage at the beginning of the next period and decide to remain married forever or to divorce and remain divorced forever. While Bougheas and Georgellis (1999) allow the individuals unilaterally divorce, Rasul (2006) examines the mutual consent regime as well to identify the effect of a policy switch to a unilateral divorce regime. In both studies, the divorce channel and the associated incentive effect (which is called 'pipeline effect' in Rasul, 2006) work in the same unambiguous way as in our model: the lower the divorce costs, the higher the divorce rate of existing couples in the short-run.

The marital status channel operates in Rasul (2006) by the effect of divorce costs on the reservation (marriage market) signal of individuals, for any signal level below which they would choose to remain single and draw a new potential match in the next period from the pool of available mates. When divorce costs decrease, the reservation signal of individuals increases; implying an increased selection into marriage and consequently a fall in the divorce rate. Rasul (2006) shows that this so called (by himself) 'selection effect' the newly formed marriages have on the divorce rate may dominate in the long-run the pipeline effect of lower divorce costs leading to higher divorce rates among existing married couples.[3] As remarked by Rasul (2006), the tension between the pipeline and the selection effect explains the earlier empirical result of Wolfers (2003) that looser divorce laws in the US led, in 1970s, to immediate positive effects on divorce rates that died out in the long-term. On the other hand, the marital composition channel is missing in the search theoretic matching models of Bougheas and Georgellis (1999) and Rasul (2006) since all individuals of the same gender are *ex ante* identical and the potential partner of each individual is randomly matched in the pre-marriage stage; thus all possible couples are *ex ante* subject to the same likelihood of divorce.

The problem of marriage formation and marital dissolution was recently studied by Mumcu and Saglam (2008) in a one-to-one matching framework with 'transferable' utilities under complete information. The solution procedure they borrowed from Crawford and Rochford (1986) had been proposed for a general problem of matching under cooperative bargaining and is known to be inefficient. Thus, this procedure allows Mumcu and Saglam (2008) to search for allocational effects of divorce costs in a non-Coasian environment. Mumcu and Saglam (2008) show that even small changes in divorce costs that yield insignificant effects on the post-divorce distribution of welfare may lead to wide fluctuations in the marital status of individuals with heterogenous endowments. Our paper differs from Mumcu and Saglam (2008) in that utilities in our case are not transferable among individuals. In fact, our work is, to the best of our knowledge, the first attempt to deal with the problem of marital dissolution in a one-to-one matching framework with 'nontransferable' utilities. The non-Coasian framework we use allows us to

[3] A similar effect potentially exists, yet not explicitly elaborated, also in Bougheas and Georgellis (1999), where divorce costs negatively affect the number of marriages formed in the society, with the effect being more pronounced when the signals received in the pre-marriage stage are less informative. In such situations, many individuals marry when divorce costs are low; however, the high noise in the signal implies that the probability of divorce is also high.

study the effect of divorce costs on marital decisions in a society with heterogenous individuals, while solely this heterogeneity activates the part of the allocation effect of divorce costs operating through the marital composition channel, which the existing search-theoretic matching literature is currently missing.

The organization of the paper is as follows: Section 2 introduces the model. Section 3 presents our results, and Section 4 concludes.

2 The Model

There are two nonempty, finite and disjoint sets of individuals: a set of men, M and a set of women, W. We assume $\min\{|M|, |W|\} \geq 2$. The society is denoted by $N = M \cup W$.

Each individual has two periods to live. The beginning of the first period involves a 'matching' stage, where individuals enter a marriage market where all matchings between men and women take place according to their preferences which we will define in Section 2.2. The individuals decide whether to marry or to remain single at the matching stage under a given matching rule that selects a stable matching with respect to their expected utilities calculated under their beliefs about the qualities of their potential marriages.

At the beginning of the second period, each married individual enters a 'perfect learning' stage where the information about the quality of the marriage is acquired and the individual has to decide, as an optimal response to this new information, whether to stay married or to divorce. We assume that the society has a unilateral divorce regime in which either spouse in a couple has the right to unilaterally end the marriage. For simplicity, we assume that a marriage market does not open in the second period; therefore individuals that choose to become single in the matching stage will remain single in both periods, whereas married individuals that decide to divorce in the perfect learning stage will become single in the second period. Here, we let $\beta > 0$ to denote the common discount factor of all individuals for the second period.[4] Before deriving the equilibrium in this two-period matching model, we will describe a matching environment.

2.1 Matching Environment

A matching is a one-to-one function, $\mu : N \to N$ such that for each $(m, w) \in M \times W$, $\mu(m) = w$ if and only if $\mu(w) = m$; $\mu(m) \notin W$ implies $\mu(m) = m$ and similarly $\mu(w) \notin M$ implies $\mu(w) = w$. If $\mu(m) = w$, then m and w are matched to one another. For any $i \in M \cup W$, i is single if $\mu(i) = i$. Let $\mathcal{M}_N$ denote the set of all matchings for the society N.

For any $i \in N$, we denote by $A_N(i)$ the set of admissible mates in N; i.e., the set of individuals of the opposite gender. Then, $A_N(m) = W$ for all $m \in M$ and $A_N(w) = M$ for all $w \in W$.

We assume that no individual completely knows in the first period the quality (hence the utility) of any potential marriage. Let θ^i_j denote a random variable representing the perception of individual $i \in N$ of the quality of a marriage with individual $j \in A_N(i)$. For all $i \in N$ and $j \in A_N(i)$, we restrict the support of θ^i_j to the interval $[0, 1]$ and we assume that all individuals know these supports. We let $f^i_j(\theta^i_j)$ denote the probability density function representing the beliefs of $i \in N$ about the quality parameter θ^i_j corresponding to a marriage with $j \in A_N(i)$. We assume that the densities $(f^i_j)_{j \in A_N(i),\, i \in N}$ are all independent. For each f^i_j, we denote by F^i_j the corresponding distribution function. We assume that for each $i \in N$ and $j \in A_N(i)$, the beliefs $\{f^i_j, f^j_i\}$ are mutually known to both i and j.

For each individual, a function U maps the type space $[0, 1]$ to reals, with $U(\theta)$ representing the instantaneous utility the individual derives from a marriage with the perceived quality θ. We set the lowest utility individuals may get from

[4]Although we do not drop the conventional assumption that individuals discount future periods in their intertemporal choices, we allow β to exceed unity so as to simply model in our paper also situations in which the first period is shorter than the second period, whereby the remaining singlehood period of a divorced individual would be longer than the period in which he or she was married.

marriage to zero, i.e., $U(0) = 0$. Moreover, we assume that $U(.)$ is strictly increasing (hence the inverse function $U^{-1}(.)$ is defined and strictly increasing, too).

For each individual, we denote by U^s the instantaneous utility of being single and by c the instantaneous divorce costs. For convenience, we assume $U^s - c < U(1)$; i.e., the instantaneous net utility any individual obtains after getting divorced is below his or her instantaneous utility from a marriage with the highest quality. Here, we also assume that each individual knows the list $(\beta, U(.), U^s, c)$ and that this list is common for all individuals.

For a given society N, a utility-belief structure is described by the list $\Gamma_N = (\beta, U, U^s, (f^i_j)_{j \in A_N(i)})_{i \in N}$, and a matching environment by the list $\Phi = (\Gamma_N, c)$. Define also the notation $\Phi_- = \Gamma_N$ and $\Phi = (\Phi_-, c)$.

2.2 Equilibrium

Below, we will describe the equilibrium in our two-period matching problem, going backwards from the perfect learning stage in the second period to the matching stage in the first period.

Perfect Learning Stage: Individual $i \in N$ who was matched to $j \in A_N(i)$ in the first period learns the private quality parameter θ^i_j associated with his or her marriage and decides whether to stay married or to divorce. The net utility in the second period becomes $U(\theta^i_j)$ if individual i stays married whereas $U^s - c$ if individual i divorces. So, individual i decides to stay married to individual j if and only if θ^i_j is not below a calculated threshold $\bar{\theta}(c)$, where

$$\bar{\theta}(c) = \begin{cases} 1 & \text{if } \; U^s - c \geq U(1), \\ U^{-1}(U^s - c) & \text{if } \; U(0) \leq U^s - c \leq U(1), \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

We can immediately note that the quality threshold, $\bar{\theta}(c)$, is nonincreasing in divorce costs, c, and nondecreasing in the instantaneous utility of being single, U^s.

Matching Stage: Each individual $i \in N$ calculates the expected utility, $E[U^i_j(c)]$, derived from a potential match with individual $j \in A_N(i)$. (Note that $E[.]$ is the expectation operator.) By backward induction, we can calculate $E[U^i_j(c)]$ in a given matching environment (Φ_-, c) as

$$\begin{aligned} E[U^i_j(c)] &= \int_0^1 d\theta^i_j\, f^i_j(\theta^i_j) U(\theta^i_j) + \\ &\quad \beta \left[(U^s - c) D^{i,j}_{\Phi_-}(c) + [1 - F^j_i(\bar{\theta}(c))] \int_{\bar{\theta}(c)}^1 d\theta^i_j\, f^i_j(\theta^i_j) U(\theta^i_j) \right]. \end{aligned} \tag{2}$$

Above, the first term is the expected utility from the first-period of marriage and the second term is the discounted second-period expected utility, where

$$D^{i,j}_{\Phi_-}(c) = F^i_j(\bar{\theta}(c)) + F^j_i(\bar{\theta}(c)) - F^i_j(\bar{\theta}(c)) F^j_i(\bar{\theta}(c)) \tag{3}$$

denotes the probability that individuals i and j will divorce in the second period.[5]

For notational convenience, we also define $E[U^i_i(c)] = (1+\beta)U^s$ for all $i \in N$. Thus, we have defined the expected utility $E[U^i_j(c)]$ of each individual i derived from a marriage with $j \in A_N(i) \cup \{i\}$. Apparently, the preferences represented by these expected utilities are complete and transitive. For any matching environment (Φ_-, c), we define the list $E[U(c)] = (E[U^i_j(c)])_{j \in A_N(i) \cup \{i\},\, i \in N}$ as the preference (or expected utility) profile of N and we denote the associated marriage market by the triple $(M, W, E[U(c)])$.

We say that a mate $j \in A_N(i) \cup \{i\}$ is acceptable for $i \in N$ at the preference profile $E[U(c)]$ if $E[U^i_j(c)] \geq E[U^i_i(c)]$. (Obviously, individual i is acceptable for i.) Given a marriage market $(M, W, E[U(c)])$, a matching μ is *individually rational* if for all $i \in N$, the mate $\mu(i)$ is acceptable for individual i. For a given matching μ, (m, w) is a *blocking pair* if they are not matched to one another but prefer one another to their matches at μ; i.e., $\mu(m) \neq w$ and

[5] Equation (3) follows from the fact that under a unilateral divorce law, any pair of individuals i and j married in the first period will not divorce in the second period with probability $[1 - F^i_j(\bar{\theta}(c)][1 - F^j_i(\bar{\theta}(c))]$, since f^i_j and f^j_i are independent.

$E[U^m_w(c)] > E[U^m_{\mu(m)}(c)]$ and $E[U^w_m(c)] > E[U^w_{\mu(w)}(c)]$. A matching is *stable* if it is individually rational and if there are no blocking pairs.

A well-known theorem by Gale and Shapley (1962) shows the existence of a stable matching for every marriage market. We denote by $\mathcal{M}^S_{\Phi_-}(c)$ the set of stable matchings for the marriage market associated with the matching environment (Φ_-, c). We require that all matchings realized in the first stage of the first period are in $\mathcal{M}^S_{\Phi_-}(c)$. This completes the description of the matching stage.

2.3 Gender-Optimal Equilibrium Matchings

As stable matchings in a matching environment are not necessarily unique, we will restrict ourselves to a special selection of equilibrium, namely gender-optimal matchings, which were first introduced and studied by Gale and Shapley (1962).

We say that men strictly prefer μ to μ' if all men like μ at least as well as μ', with at least one man strictly preferring μ to μ'; i.e., $E[U^i_{\mu(i)}(c)] \geq E[U^i_{\mu'(i)}(c)]$ with the inequality being strict for some i. We also say that men weakly prefer μ to μ' if either all men strictly prefer μ to μ' or that all men are indifferent between μ and μ', i.e. $E[U^i_{\mu(i)}(c)] = E[U^i_{\mu'(i)}(c)]$ for all $i \in N$. Similarly, we define strict and weak preference relations for women.

For a given marriage market $(M, W, E[U(c)])$, a stable matching μ is said to be M*-optimal* if men weakly prefer μ to any other stable matching. Similarly, a stable matching ν is said to be W*-optimal* if women weakly prefer ν to any other stable matching. We denote M-optimal and W-optimal matchings for the matching environment Φ by μ^M_Φ and μ^W_Φ, respectively. We also say that a stable matching is gender-optimal if it is M-optimal or W-optimal.

We know that in any matching environment Φ where men and women have strict preferences, the 'Deferred Acceptance Algorithm' by Gale and Shapley (1962) produces μ^M_Φ if men propose to women and μ^M_Φ if women propose to men. The algorithm with men proposing is simply as follows: In the initial step, each man proposes to the most preferred one among all acceptable women. Each women rejects the proposal of any man who is not acceptable to her and gets engaged to the most preferred man among those whose proposals she has not rejected. At any step $k \geq 2$, any man who was not engaged in the previous step deletes the woman who rejected him in step $k-1$ from his list of acceptable woman and proposes to his favourite woman, if any, in the updated list. Each woman receiving proposals gets engaged to the most preferred acceptable man among the group consisting of the man to whom she may have engaged in step $k-1$ and men who have just proposed in step k, and rejects all other members of this group. The algorithm stops after any step in which no man is rejected. By changing the role of men and women in the above procedure, we can similarly define the Deferred Acceptance Algorithm with women proposing.

Now, for any society we define a matching rule as a function from the set of all matching environments to the set of all matchings. For any matching rule $\mu(.)$ and any matching environment Φ, we denote by $\mu(\Phi)$ the matching selected at the matching environment Φ. We say that a matching rule $\mu(.)$ is the M-optimal matching rule if it selects the M-optimal matching at all matching environments; i.e., $\mu(\Phi) = \mu^M_\Phi$ for all Φ. We also define the W-optimal matching rule analogously. We denote by $\mu^M(.)$ and $\mu^W(.)$ the M-optimal and W-optimal matching rule respectively.

3 Results

We first define, using (3), the average probability of marital dissolution in the society N for the matching environment (Φ_-, c) under the matching rule $\mu(.)$ as

$$D^N_{\Phi_-}(c, \mu) = \frac{1}{|W^\mu_{\Phi_-}(c)|} \sum_{w \in W^\mu_{\Phi_-}(c)} D^{m,\mu(\Phi)(m)}_{\Phi_-}(c), \tag{4}$$

where $W^\mu_{\Phi_-}(c) = \{w \in W \,|\, (w, w) \notin \mu(\Phi)\}$ is the set of women who were not single in the first period.

Below, we observe that the probability of marital dissolution under any given matching is nonincreasing in divorce costs.

Remark 1. *For any society N, any matching environment (Φ_-, c), and any matching $\mu \in \mathcal{M}_N$, we have $D^N_{\Phi_-}(\hat{c}, \mu) \leq D^N_{\Phi_-}(c, \mu)$ if $\hat{c} > c$.*

The above remark follows from (1), (3) and (4) together with the facts that $\partial\bar{\theta}(c)/\partial c \leq 0$ and that $dF^i_j(\theta)/d\theta \geq 0$ for all $i \in N$ and $j \in A_N(i)$. We call this effect, which also works in search theoretic models, to be the 'incentive effect' of divorce costs; since the higher the divorce costs, the higher the incentive for the marriages that were formed in the first period to continue in the second period as well. Indeed, divorcing becomes totally unattractive for the whole society when divorce costs are sufficiently high; i.e., for all N and $\mu \in \mathcal{M}_N$ we have $D^N_{\Phi_-}(c, \mu) = 0$ if $c \geq U^s$.

The incentive effect is only a partial specification of the relation between divorce costs and marital dissolution, since given any non-constant matching rule $\mu(.)$ and any matching environment (Φ_-, c), a change in divorce costs c can also change equilibrium matchings $\mu(\Phi)$, which we have fixed in the above remark.[6] This dependence is evident from equations (1) and (2), since a sufficiently large change in divorce costs can change the 'ordinal' preferences of the individuals over their potential mates because of the heterogeneity of the individuals' beliefs and utilities. Naturally, impacts of divorce costs on the equilibrium matchings will also be transmitted to the average probability of divorce now due to the heterogeneity of the individuals' beliefs. We call this second effect of divorce costs on the marital dissolution as the 'allocation effect'. Below we formally define these two effects.

Given any society N and any matching environment (Φ_-, c), the change in the average probability of marital dissolution under a matching rule $\mu(.)$ when the cost value is changed from c to $\hat{c}$ is given by

$$D^N_{\Phi_-}(\hat{c}, \mu(\Phi_-, \hat{c})) - D^N_{\Phi_-}(c, \mu(\Phi_-, c)) = \Delta^R_\mu(\hat{c}, c, \Phi_-) + \Delta^I_\mu(\hat{c}, c, \Phi_-), \tag{5}$$

[6] A matching rule is constant if it selects the same constant matching at all matching environments.

where

$$\Delta^R_\mu(\hat{c}, c, \Phi_-) = D^N_{\Phi_-}(\hat{c}, \mu(\Phi_-, \hat{c})) - D^N_{\Phi_-}(\hat{c}, \mu(\Phi_-, c)) \tag{6}$$

is the change in the average probability of marital dissolution for the matching environment $(\Phi_-, \hat{c})$ due to a change in the matching from $\mu(\Phi_-, c)$ to $\mu(\Phi_-, \hat{c})$, and

$$\Delta^I_\mu(\hat{c}, c, \Phi_-) = D^N_{\Phi_-}(\hat{c}, \mu(\Phi_-, c)) - D^N_{\Phi_-}(c, \mu(\Phi_-, c)) \tag{7}$$

is the change in the average probability of marital dissolution under the matching $\mu(\Phi_-, c)$ due to a change in the matching environment from (Φ_-, c) to $(\Phi_-, \hat{c})$. We call the terms $\Delta^R_\mu(\hat{c}, c, \Phi_-)$ and $\Delta^I_\mu(\hat{c}, c, \Phi_-)$ respectively the allocation and the incentive effect due to the change in divorce costs from c to $\hat{c}$.

By Remark 1, we know that the incentive effect is always negative, i.e., $\Delta^I_\mu(\hat{c}, c, \Phi_-) < 0$ if $\hat{c} > c$. But, unlike the incentive effect, the allocation effect does not have a determinate sign, as will be evident from Examples 1 and 2. Below, we first consider a matching environment in which the allocation and incentive effect may work in opposite directions.

Example 1. Consider a society N involving $M = \{m_1, m_2\}$ and $W = \{w_1, w_2\}$ with Φ_- given by $\beta = 0.99$, $U(\theta) = \sqrt{0.07\,\theta}$, $U^s = 0.15036$, and

$$f^{m_1}_{w_1} = f^{w_1}_{m_1} = f^{w_1}_{m_2} = f^{w_2}_{m_1} = f^{m_2}_{w_2} = f^{w_2}_{m_2} = f^a,$$

$$f^{m_1}_{w_2} = f^{m_2}_{w_1} = f^b,$$

where

$$f^a(\theta) = \begin{cases} 0.1380 & \text{if } \theta \in [0.200, 0.269] \\ 1.0639 & \text{otherwise,} \end{cases}$$

and

$$f^b(\theta) = \begin{cases} 0.1429 & \text{if } \theta \in [0.200, 0.270] \\ 1.0645 & \text{otherwise.} \end{cases}$$

Let $c = 0.01286$. We compute $\bar{\theta}(c) = 0.2701$. Then, the expected utility profile $E[U(c)]$ is calculated as follows:

$E[U^i_j(c)]$ (i: rows; j: columns)

	m_1	m_2	w_1	w_2
m_1	0.299216	—	0.299313	0.299314
m_2	—	0.299216	0.299314	0.299313
w_1	0.299313	0.299314	0.299216	—
w_2	0.299314	0.299313	—	0.299216

Consider the M-optimal matching rule $\mu^M(.)$. From the above table, it follows that $\mu^M(\Phi) = \mu^W(\Phi) = \mu^a$, where

$$\mu^a = \begin{pmatrix} m_1 & m_2 \\ w_2 & w_1 \end{pmatrix},$$

i.e., m_1 and m_2 are matched to w_2 and w_1, respectively. We then compute $D^N_{\Phi_-}(c, \mu^M) = 0.396624$.

Now, consider the matching environment $(\Phi_-, \hat{c})$ with $\hat{c} = 0.01291$. We compute $\bar{\theta}(\hat{c}) = 0.2699$. Then, the expected utility profile $E[U(\hat{c})]$ is calculated as follows:

$E[U^i_j(\hat{c})]$ (i: rows; j: columns)

	m_1	m_2	w_1	w_2
m_1	0.299216	—	0.299296	0.299293
m_2	—	0.299216	0.299293	0.299296
w_1	0.299296	0.299293	0.299216	—
w_2	0.299293	0.299296	—	0.299216

It follows that $\mu^M((\Phi_-, \hat{c})) = \mu^W((\Phi_-, \hat{c})) = \mu^b$, where

$$\mu^b = \begin{pmatrix} m_1 & m_2 \\ w_1 & w_2 \end{pmatrix}.$$

We calculate $D^N_{\Phi_-}(\hat{c}, \mu^M) = 0.396656$, which is higher than $D^N_{\Phi_-}(c, \mu^M)$. Finally, we compute $\Delta^R_\mu(\hat{c}, c, \Phi_-) = 0.000279$ and $\Delta^I_\mu(\hat{c}, c, \Phi_-) = -0.000248$. The allocation effect for this example is positive and it dominates the incentive effect, hence we get the increase in the average probability of divorce. $\square$

Our proposition below generalizes Example 1 to state that for any society there exists a matching environment where some increases in divorce costs raise the average probability of marital dissolution under the M-optimal matching rule. (Obviously, a similar result for the W-optimal matching rule is immediate by interchanging the names of men and women in the proof of Proposition 1.)

Proposition 1. *For any society N, there exists a matching environment $\Phi = (\Phi_-, c)$ and divorce costs $\hat{c} > c$ such that we have $D^N_{\Phi_-}(\hat{c}, \mu^M) > D^N_{\Phi_-}(c, \mu^M)$.*

Proof. Consider the society N with the list Φ_- in Example 1. Let $N' = M' \cup W'$ be any larger society such that $M' \supseteq M$ and $W' \supseteq W$ with at least one of the inclusions being strict. Consider the two cost values $c = 0.01286$ and $\hat{c} = 0.01291$. Consider the list $\Phi'_- = (\beta, U, U^s, (f'^i_j)_{j \in A_{N'}(i)})_{i \in N'}$ such that $\beta = 0.99$, $U(\theta) = \sqrt{0.07\,\theta}$, and $U^s = 0.15036$. (Thus, we have extended all utility specifications for N in Example 1 to N'.) By this construction, we have $\bar{\theta}(c) = 0.2701$ and $\bar{\theta}(\hat{c}) = 0.2699$. Now, let $f'^i_j = f^i_j$ if $i \in N$ and $j \in A_N(i)$. Moreover, for all $(i,j) \in M' \times W'$ such that $(i,j) \notin M \times W$, let $f'^i_j = f^c$, where

$$f^c(\theta) = \begin{cases} 0 & \text{if } \theta \in [0.200, 0.260] \\ 1.0638 & \text{otherwise.} \end{cases}$$

Then, we have $E[U^i_j(c)] = 0.299209$ and $E[U^i_j(\hat{c})] = 0.299191$ for all $(i,j) \in M' \times W'$ such that $(i,j) \notin M \times W$. Since we also have $E[U^i_i(c)] = E[U^i_i(c)(\hat{c})] = 0.299216$ for all $i \in N' \backslash N$, it follows that under both c and $\hat{c}$, all individuals in $N' \backslash N$ will decide to remain single in both periods. Thus, $\mathcal{M}^S_{\Phi'_-}(c) = \{\mu^{M'}(\Phi'_-, c)\}$ and $\mathcal{M}^S_{\Phi'_-}(\hat{c}) = \{\mu^{M'}(\Phi'_-, \hat{c})\}$, where $\mu^{M'}(\Phi'_-, c)(i) = \mu^M(\Phi_-, c)(i)$ and $\mu^{M'}(\Phi'_-, \hat{c})(i) = \mu^M(\Phi_-, \hat{c})(i)$ for all $i \in N$ (with $\mu^M(\Phi_-, c)$ and $\mu^M(\Phi_-, \hat{c})$

defined in Example 1); and $\mu^{M'}(\Phi'_-, c)(i) = \mu^{M'}(\Phi'_-, \hat{c})(i) = \{i\}$ for all $i \in N' \setminus N$. It then follows that $D^{N'}_{\Phi'_-}(c, \mu^{M'}) = D^{N}_{\Phi_-}(c, \mu^{M})$ and $D^{N'}_{\Phi'_-}(\hat{c}, \mu^{M'}) = D^{N}_{\Phi_-}(\hat{c}, \mu^{M})$, which completes the proof. $\square$

The following example shows that there are also matching environments where the allocation effect can be negative under gender-optimal matching rules.

Example 2. Consider a society N involving $M = \{m_1, m_2\}$ and $W = \{w_1, w_2\}$ with Φ_- given by $\beta = 0.99$, $U(\theta) = \sqrt{0.07\,\theta}$, $U^s = 0.15036$, and

$$f^{m_1}_{w_1} = f^{w_1}_{m_1} = f^{m_2}_{w_2} = f^{w_2}_{m_1} = f^a,$$

$$f^{m_1}_{w_2} = f^{m_2}_{w_1} = f^{w_1}_{m_2} = f^{w_2}_{m_2} = f^b,$$

where

$$f^a(\theta) = \begin{cases} 10 & \text{if } \theta \in [0.20, 0.22] \\ 0.8163 & \text{otherwise,} \end{cases}$$

and

$$f^b(\theta) = \begin{cases} 6.25 & \text{if } \theta \in [0.20, 0.24] \\ 0.7813 & \text{otherwise.} \end{cases}$$

Let $c = 0.01286$. We compute $\bar{\theta}(c) = 0.2701$. Then, the expected utility profile $E[U(c)]$ is calculated as follows:

$E[U^i_j(c)]$ (i: rows; j: columns)

	m_1	m_2	w_1	w_2
m_1	0.299216	—	0.307935	0.307887
m_2	—	0.299216	0.308197	0.308259
w_1	0.307935	0.308197	0.299216	—
w_2	0.308259	0.307887	—	0.299216

Consider the M-optimal matching rule $\mu^M(.)$. From the above table, it follows that $\mu^M(\Phi) = \mu^W(\Phi) = \mu^a$, where

$$\mu^a = \begin{pmatrix} m_1 & m_2 \\ w_1 & w_2 \end{pmatrix}.$$

We then compute $D^N_{\Phi_-}(c, \mu^M) = 0.652596$.

Now, consider the matching environment $(\Phi_-, \hat{c})$ such that $\hat{c} = 0.02286$. We compute $\bar{\theta}(\hat{c}) = 0.2322$. Then, the expected utility profile $E[U(\hat{c})]$ is calculated as follows:

$E[U^i_j(\hat{c})]$ (i: rows; j: columns)

	m_1	m_2	w_1	w_2
m_1	0.299216	—	0.302274	0.303467
m_2	—	0.299216	0.303424	0.302192
w_1	0.302274	0.303424	0.299216	—
w_2	0.302192	0.303467	—	0.299216

It follows that $\mu^M((\Phi_-, \hat{c})) = \mu^W((\Phi_-, \hat{c})) = \mu^b$, where

$$\mu^b = \begin{pmatrix} m_1 & m_2 \\ w_2 & w_1 \end{pmatrix}.$$

We calculate $D^N_{\Phi_-}(\hat{c}, \mu^M) = 0.592446$. Finally, we compute $\Delta^R_\mu(\hat{c}, c, \ \Phi_-) = -0.009867$ and $\Delta^I_\mu(\hat{c}, c, \Phi_-) = -0.050283$. $\square$

We should finally remark that the ambiguous sign of the allocation effect should not be surprising, since in our one-to-one matching framework the equilibrium (stability) notion that determines the final matching allocations involves individual comparisons of expected utilities under alternative matchings, but not comparisons of the corresponding expected divorce rates.

4 Conclusions

In this paper, we have studied the effect of changes in divorce costs on marital dissolution in a two-period one-to-one matching model with nontransferable utilities under incomplete information. We show that divorce costs affect not only individuals' decision to divorce and to marry but also their decision whom to marry. Consequently, the average probability of marital dissolution in the society is determined by these three decision channels, which we call 'the divorce channel', 'the marital status channel' and 'the marital composition channel', respectively. Divorce costs always operate through the divorce channel in an unambiguous way and yields a negative incentive effect on the average probability of marital dissolution in the society. The same effect has also been found in search-theoretic models of matching. In this study, we also identify the allocation effect of divorce costs through the marital status and the marital composition channel. The effect through the marital status channel has already been studied in the search-theoretic matching literature: Rasul (2006) identifies it and calls it the selection effect always working in the negative direction of the incentive effect (or the pipeline effect as he calls it) . However, the effect through the marital composition channel is novel to this study. We find that the allocation effect of divorce costs through the marital status and the marital composition channel is ambiguous. For any gender-optimal matching rule that always selects a stable matching which is optimal for men or women, we can find matching environments in which the allocation effect has a positive sign and dominates the incentive effect as well as environments in which the allocation effect has a negative sign and reinforces the incentive effect.

Following a similar classification of Rasul (2006), we call the incentive effect and the allocation effect of a change in divorce costs to be the short-term effect and the long-run effect, respectively. While a decrease in divorce costs always increases divorce rates by the short-term effect, there are environments where the long-term effect may outweigh the short-term effect and leads to a fall in the divorce rates eventually. This theoretical result may help to explain the empirical observation in Wolfers (2003, 2006) that a sharp immediate rise in the US divorce rate after the divorce reform in 1970s is reversed in the next decade to such an extent that the divorce rate is lower 15 years after the reform.

We believe that another contribution of this study is to show that the one-to-one matching theory with nontransferable utilities which has been heavily used in studying stable matchings for given economic environments can also be successfully employed to study the issue of separation once an algorithm or rule that produces stable matchings is chosen. While search theoretic framework has its clear advantage in terms of formulating the equilibrium in a simple way in large matching environments where the information about the potential partners is more limited, the one-to-one matching framework may be powerful in small environments that allow for assortative mating.

Finally, we believe that future research may benefit from the search theoretical works of Bougheas and Georgellis (1999) and Rasul (2006) to analyse in a one-to-open matching framework also the effects of imperfect learning about the potential partners on the marital dissolution, along with several potential strategic issues in acquiring knowledge.

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