Productivity shocks and housing market inflations in new Keynesian models

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Abstract

Econometric evidence suggests the existence of two dynamics in the postwar U.S. housing market: (i) housing rental and purchase prices co-move positively in response to productivity shocks, and (ii) the purchase price exhibits much larger volatile movements than the rental price in response to the shocks. A standard New Keynesian model with nominal rigidity in the production sector is inconsistent with these facts. We incorporate a rental market into an otherwise standard New Keynesian model with durables and show that nominal rigidity in the rental market contributes to our empirical findings.

1 Introduction

New Keynesian (NK) models focus on characterizing monetary policies in the environment with nominal rigidities and imperfect competition. These sticky-price models are often abstracted from the rental market, and they focus solely on the production sector. Empirically, we document two main features related to the evolution of housing rental prices and purchase prices. First, these two prices co-move positively in response to productivity shocks. Second, the purchase price is considerably more sensitive than the rental price to exogenous shocks. We perform vector autoregression (VAR) analysis of the U.S. quarterly total factor productivity (TFP), gross domestic output, rental and housing purchase prices, and interest rate. We find that the purchase price exhibited a positive response after the increase of about 0.6 percent one year after the shock and an increase of about 0.8 percent two years after the shock. On the other hand, the corresponding response sizes of the rental price were about 0.1 percent and 0.2 percent, respectively.

A standard NK model with housings is generally inconsistent with the abovementioned findings. This is because the price-rent ratio relationship plays a crucial role in the case of a long-lived durable good. In a flexible-price economy with long-lived housing capitals, the price-rent ratio increases in response to the persistently positive productivity shock. This is because the currently purchased housing plays the role of an asset for the future. However, when the housing purchase price is sticky, its adjustment process is triggered, and hence, the rental price should be lowered instead. When the rental market does not witness any friction, the rental price implicitly serves as a frictionless shadow price

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of the corresponding durable service flows. Hence, when purchase prices are sticky and rental prices are flexible, a persistently expansionary productivity shock slightly increases purchase prices and significantly lowers the rental prices so that the price-rent ratio moves efficiently. This result is at odds with the fact that the two prices co-move in response to a productivity shock.

In this paper, we show that the presence of nominal rigidity in the rental market can reconcile an otherwise standard NK model with the empirical evidence of productivity shocks on housing rental and purchase prices. The fundamental contribution of this paper is as follows. First, the empirical evidence on rental price rigidity for housing services is considerably more relevant than the evidence for sticky housing purchase prices. When the rental price is sticky, the price adjustment cost induces a smaller user cost response to exogenous shocks. The housing purchase price, therefore, should respond actively so that the user cost equation holds in the equilibrium. This implies that the nominal rigidity in the rental market can be one reason why the purchase price is volatile. Second, the co-movement problem can be solved by the introduction of the nominal rigidity of the rental price and shock persistence. In the existence of durables, two effects should be considered. The first effect is the intertemporal elasticity of substitution for the purchases of durable goods, as discussed in Barsky, House, and Kimball (2007). The second effect is the consumption smoothing effect of durable services in response to a persistent shock. When a shock is temporary, a household's demand in the impact period is at the highest level because of the first effect. Hence, the purchase price and the price-rent ratio are inclined to increase in the short run. Because firms in the rental market can produce durable services at a higher input cost, they want to increase their rental prices. With the passage of time, the demand for the durable service decreases, and hence the purchase and rental prices also decrease. On the other hand, when the shock is persistent, the level of the household's demand in the impact period is not as high as that in the temporary shock case because of the second effect. Because of the accumulation effect of total durable stocks and the persistence of the exogenous shocks, it is expected that an economic boom will occur in the near future. Therefore, the purchase price and rental price show an increasing pattern in response to the increasing demand.

The rest of the paper is organized as follows. Section 2 presents our empirical evidence. Section 3 presents the theoretical model. Section 4 defines the equilibrium. Section 5 presents a descriptive analysis in the flexible case. Section 6 discusses a distortion economy. Section 7 concludes the paper.

2 Productivity shocks and durable inflation: the evidence

In this section, we document two features that characterize the dynamic evolution of the rental and purchase prices of housing. First, both the housing rental and purchase prices increase in response to positive productivity shocks. Second, the volatility of housing prices to productivity shocks is significantly larger than that of rental prices. In the previous literature that employed VAR, measured TFP or measured labor productivity is used as an explanatory variable to identify productivity shocks. For example, Beaudry and Lucke (2009) assume that only TFP shocks may have contemporaneous effects on measured TFP. Galí (1999) assumes that only productivity shocks can influence labor productivity in the long run. We follow these approaches to investigate the effect of productivity shocks on housing rental and purchase prices.

To assess the impact of exogenous shocks, we estimate a VAR model for the U.S. economy, specified as follows:

$$X_t = \sum_{j=1}^{L} A_j X_{t-j} + D + B u_t, \tag{1}$$

where u_t is a vector of contemporaneous disturbances. The vector X_t comprises five variables in the following order: (i) measured TFP, (ii) (imputed) rental price of housing, (iii) gross domestic output, (iv) federal funds rate, and (v) purchase price of housing. Except for the funds rate, all variables are in logs. Output and price variables are normalized by the size of the working age population. The sample size is 1960Q1 to 2007Q2. A constant is included as a determinstic term, D, and six lags are selected that were recommended by Akaike's information criterion.

To identify a productivity shock, we use a standard recursive identification scheme. Our identified productivity shock is the only shock that can influence measured TFP in the impact period. Therefore, the exogeneity of productivity can distinguish a productivity shock from other shocks. Fig. 1 displays the estimated responses of explanatory variables to the estimated productivity shock. The blue lines represent the point estimates of each explanatory variable. The shaded areas represent two-standard error bands. In response to the productivity shock, we can observe the boom of the economy with the increase of output. Furthermore, in the second row, the rental and purchase prices of housing show an increase. Point estimates of the housing purchase prices exhibit an abrupt adjustment while the rental price exhibits a slow and steady adjustment process. The response size of the purchase price is about 0.6 percent one year after the shock and 0.8 percent two years after the shock. On the other hand, the corresponding response sizes of the rental price are about 0.1 percent and 0.2 percent, respectively. The response of the interest rate is slightly negative in the impact period, and then, its sign becomes positive.

As a robustness check, we also estimate a vector-error correction model. Two identification schemes are used: short-run restrictions and a combination of short-run and long-run restrictions. The red dashed lines represent the responses with the latter identification scheme, and we have a robust result.

Our evidence, positive responses of housing rental and purchase prices to productivity shocks, should not be viewed as evidence against the sticky price models. In a standard NK model, prices decrease in response to productivity shocks. We re-estimate the six-variable VAR model including the GDP deflator, and find that the GDP deflator decreases in response to productivity shocks. In the following sections, we show that the prices can increase in a sticky-price economy with long-lived durables.

3 The model

In this section, we develop an infinite-horizon economy with long-lived durable goods and services and with corresponding prices. The key feature of our model is that durable goods can be traded in two ways: as an ownership of a good or a lease contract of its service flow in each period. To investigate the role of durable services and the corresponding prices, we segment the durable market into product and service markets.

3.1 Household

The utility function of the representative household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big[U(S_{D,t}) - V(N_t) \Big], \tag{2}$$

where $\beta \in (0,1)$ is a subjective discount factor. $S_{D,t}$ denotes the total durable service flows to be consumed in period t, and N_t denotes the amount of labor supplied in the production sector. The period utility is assumed to be continuous and twice differentiable, with $U_{S_D,t} \equiv \frac{\partial U(S_{D,t})}{\partial S_{D,t}} > 0$, $U_{S_D S_D,t} \equiv \frac{\partial^2 U(S_{D,t})}{\partial S_{D,t}^2} \leq 0$, $V_{N,t} \equiv \frac{\partial V(N_t)}{\partial N_t} \leq 0$, and $V_{NN,t} \equiv \frac{\partial^2 V(N_t)}{\partial N_t^2} \leq 0$.

The purchase of durable service is financed by the labor income, the ownership of the intermediate firms in the service and production sectors, government transfers, and assets. The nominal budget constraint for all t is given by

$$r_{D,t} S_{D,t} + R_{t-1} B_{t-1} = B_t + W_t N_t + \Gamma_t + T_t,$$

where $r_{D,t}$ is a service price, B_t denotes a nominal bond, R_t denotes a nominal return of the nominal bond, Γ_t denotes dividends from the ownership of firms in all sectors, and T_t denotes the lump-sum transfer from the government.

Dividing this by the service price, $r_{D,t}$, we obtain the real budget constraint:

$$S_{D,t} + R_{t-1} \frac{b_{t-1}}{\pi_{r_{D,t}}} = b_t + \frac{W_t}{r_{D,t}} N_t + \frac{\Gamma_t + T_t}{r_{D,t}}. \tag{3}$$

We also assume that no Ponzi scheme holds:

$$\lim_{T \to \infty} E_t B_T \leq 0$$

for all t.

3.1.1 Optimal allocation and implication

Given the initial value, b_{-1}, the household chooses the labor, consumption, and asset profile $\{N_t, S_{D,t}, b_t,\}_{t=0}^{\infty}$ to maximize (2) subject to (3). The first-order necessary conditions become

$$-\frac{V_{N,t}}{U_{S_D,t}} = \frac{W_t}{r_{D,t}}, \tag{4}$$

$$1 = \beta E_t \left[\frac{U_{S_D,t+1}}{U_{S_D,t}} \frac{R_t}{\pi_{r_D,t+1}} \right]. \tag{5}$$

Equation (4) is the intra-temporal decision condition between the labor supply and the consumption of durable service flows in period t. Equation (5) is a standard Euler condition with respect to the inter-temporal consumption decision of durable service flows for all t.

3.2 Rental market

In the service market, a perfectly competitive final firm purchases $D_t(i)$ units from the intermediate firm i. The final firm operates the production function

$$D_t \equiv \left(\int_0^1 D_t(i)^{\frac{\varepsilon_{r_D}-1}{\varepsilon_{r_D}}} di \right)^{\frac{\varepsilon_{r_D}}{\varepsilon_{r_D}-1}},$$

where $D_t(i)$ is the demanded quantity of the intermediate service i by the final firm, and ε_{r_D} is the elasticity of substitution among differentiated varieties. The maximization of profits yields the demand function for the intermediate service i for all $t \geq 0$:

$$D_t(i) = \left(\frac{r_{D,t}(i)}{r_{D,t}} \right)^{-\varepsilon_{r_D}} D_t, \tag{6}$$

where $r_{D,t} \equiv \left(\int_0^1 r_{D,t}(i)^{1-\varepsilon_{r_D}} di \right)^{\frac{1}{1-\varepsilon_{r_D}}}$ is the service price index.

There is a continuum of firms producing differentiated services indexed in the interval $[0, 1]$. Each firm i is a monopolistic competitor in the rental market, but, following Rotemberg (1982), the firm is assumed to face a quadratic cost proportional to the total durable services in changing its price equal to $\frac{\theta_{r_D}}{2}(\frac{r_{D,t}(i)}{r_{D,t-1}(i)} - 1)^2 D_t$, measured by the finished service. θ_{r_D} governs the magnitude of the price adjustment cost, measuring the degree of sectoral nominal price rigidity.

If the intermediate service firm $i \in [0, 1]$ can set a new price and input in period t, it chooses $r_{D,t}(i)$ and $I_{D,t}$ to maximize

$$E_t \sum_{k=t}^{\infty} \Lambda_{t,k} \left[\Gamma_{r_{D,t}}(i) - \frac{\theta_{r_D}}{2} \left(\frac{r_{D,t}(i)}{r_{D,t-1}(i)} - 1 \right)^2 r_{D,t} D_t \right]$$
$$\text{s.t. (6) and } D_t(i) = (1 - \delta) D_{t-1}(i) + I_{D,t}(i),$$

where E is an expectation operator, and

$$\Lambda_{t,k} = \beta^{k-t} \frac{U_{S_D,k}}{U_{S_D,t}} \frac{r_{D,t}}{r_{D,k}}$$

is a stochastic discount factor, where $U_{S_D,t}$ measures the marginal utility value to the household of an additional unit of real profits during period t. Profits, $\Gamma_{r_{D,t}}(i)$, are the difference between rental revenue and the expenses of new purchases from the production sector, and they are paid out as dividends to shareholders:

$$\Gamma_{r_{D,t}}(i) \equiv r_{D,t}(i) D_t(i) - P_{D,t}(i) I_{D,t}(i).$$

δ is a depreciation rate of housings and $I_{D,t}(i)$ is the newly purchased output flows from the production sector in period t. The "used" or second hand $(1 - \delta) D_{t-1}(i)$ stocks returned to the intermediate firm at the end of the previous period can be sold at $P_{D,t}(i)$ in the current period. Thus, the intermediate firms newly demand the durable goods as much as $D_t(i) - (1 - \delta) D_{t-1}(i)$ from the final-good firm in the production sector.

In the symmetric equilibrium where $r_{D,t}(i) = r_{D,t}$ for all i, the first order condition is

$$(\pi_{r_D,t} - 1)\pi_{r_D,t} = E_t\left[\Lambda_{t,t+1}\frac{r_{D,t+1}}{r_{D,t}}\frac{D_{t+1}}{D_t}(\pi_{r_D,t+1} - 1)\pi_{r_D,t+1}\right] + \frac{\varepsilon_{r_D}}{\theta_{r_D}}\left\{\Xi_{r_D,t} - \frac{\varepsilon_{r_D} - 1}{\varepsilon_{r_D}} - (1-\delta)E_t\left[\Lambda_{t,t+1}\frac{r_{D,t+1}}{r_{D,t}}\Xi_{r_D,t+1}\right]\right\}. \quad (7)$$

This equation implies that the current gross rental inflation rate, $\pi_{r_D,t}(\equiv \frac{r_{D,t}}{r_{D,t-1}})$, is a function of the expected inflation in the next period, $\pi_{r_D,t+1}$, and the real marginal cost in period t in the rental sector, $\Xi_{r_D,t}$. Furthermore, one distinctive feature is that an additional term influences the current inflation: the expectation of the real marginal cost in the next period. This feature insulates the most important mechanism in our model.

Even in the case of flexible prices with $\theta_{r_D} = 0$, the real marginal cost is not constant:

$$\Xi_{r_D,t} = \frac{1}{\mu_{r_D}} + (1-\delta)E_t\left[\Lambda_{t,t+1}\frac{r_{D,t+1}}{r_{D,t}}\Xi_{r_D,t+1}\right], \quad (8)$$

where $\mu_{r_D} \equiv \frac{\varepsilon_{r_D}}{\varepsilon_{r_D}-1}$ is a mark-up in the rental sector. Note that the current real marginal cost depends on the future real marginal cost. For example, the real marginal cost increases as far as the future real marginal cost is positive.

3.3 Production sector

In the production sector, a perfectly competitive final-good producer purchases $Y_t(i)$ units of intermediate good j. The final-good producer operates the production function

$$Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\varepsilon_{P_D}-1}{\varepsilon_{P_D}}} di\right)^{\frac{\varepsilon_{P_D}}{\varepsilon_{P_D}-1}},$$

where $Y_t(j)$ is the quantity of the intermediate good j demanded by the final-good producer, and ε_{P_D} is the elasticity of substitution between differentiated varieties. The maximization of profits yields the demand function for the intermediate good j for all t:

$$Y_t(j) = \left(\frac{P_{D,t}(j)}{P_{D,t}}\right)^{-\varepsilon_{P_D}} Y_t, \quad (9)$$

where the price index is $P_{D,t} \equiv \left(\int_0^1 P_{D,t}(j)^{1-\varepsilon_{P_D}} dj\right)^{\frac{1}{1-\varepsilon_{P_D}}}$.

There is a continuum of firms producing differentiated products indexed in the interval $[0, 1]$. A typical firm j hires $N_t(j)$ units of labor from households to produce $Y_t(j)$ units of intermediate good j, using a linear production technology:

$$Y_t(j) = A_t N_t(j), \quad j \in [0, 1], \quad (10)$$

where A_t is a productivity shock. a_t, which is a logarithm of the t-period productivity shock in the production sector, follows

$$a_{t+1} = \rho_a a_t + u^a_{t+1}, \quad \rho_a \in [0, 1), \quad (11)$$

where $E_t u^a_{t+1} = 0$ and $E_t u^a_t {u^a}'_t = \sigma^{a2}$.

Each firm j is a monopolistic competitor in the product markets, but, following Rotemberg (1982), the firm is assumed to face a quadratic cost proportional to output in changing its price equal to $\frac{\theta_{P_D}}{2}(\frac{P_{D,t}(j)}{P_{D,t-1}(j)} - 1)^2 Y_t$, measured by the finished good. θ_{P_D} governs the magnitude of the price adjustment cost, measuring the degree of sectoral nominal price rigidity.

Subject to (9) and (10), the intermediate firm $j \in [0,1]$ in the good sector who can adjust its price in period t chooses its price and quantity to maximize

$$E_k \sum_{t=k}^{\infty} \Lambda_{t,k} \left[\Gamma_{P_{D,t}}(j) - \frac{\theta_{P_D}}{2} \left(\frac{P_{D,t}(j)}{P_{D,t-1}(j)} - 1 \right)^2 P_{D,t} Y_t \right].$$

Profits are the difference between the revenue and the expenses of paying for workers, and they are immediately paid out as dividends, $\Gamma_{P_{D,t}}(j)$, to shareholders:

$$\Gamma_{P_{D,t}}(j) \equiv P_{D,t}(j) Y_t(j) - W_t N_t(j),$$

where W_t denotes a nominal wage rate.

In the symmetric equilibrium where $P_{D,t}(j) = P_{D,t}$ for all j, the first order condition becomes

$$\begin{aligned} (\pi_{P_D,t} - 1)\pi_{P_D,t} =& E_t \left[\Lambda_{t,t+1} \frac{P_{D,t+1}}{P_{D,t}} \frac{Y_{t+1}}{Y_t} (\pi_{P_D,t+1} - 1)\pi_{P_D,t+1} \right] \\ &+ \frac{\varepsilon_{P_D}}{\theta_{P_D}} \left[\Xi_{P_D,t} - \frac{\varepsilon_{P_D} - 1}{\varepsilon_{P_D}} \right], \end{aligned} \tag{12}$$

where $\pi_{P_D,t} \equiv \frac{P_{D,t}}{P_{D,t-1}}$ is the gross producer inflation rate, and $\Xi_{P_D,t}$ is the real marginal cost in the production sector.

On solving cost minimization problems, we get the real marginal costs common to all firms within the sector. In the case of flexible prices when $\theta_{P_D} = 0$, the real marginal cost becomes

$$\Xi_{P_D,t} = \frac{W_t}{P_{D,t} A_t} = \frac{1}{\mu_{P_D}},$$

where $\frac{1}{\mu_{P_D}} \equiv \frac{\varepsilon_{P_D} - 1}{\varepsilon_{P_D}}$ is a mark-up in the production sector.

3.4 Monetary policy

We assume that the monetary authority obeys the Taylor-type rule. We consider the following instrument:

$$\frac{R_t}{R} = \left(\frac{\pi_{r_D,t}}{\pi_{r_D}} \right)^{\rho_{\pi r_D}} \left(\frac{\pi_{P_D,t}}{\pi_{P_D}} \right)^{\rho_{\pi P_D}} exp(\eta^R_t),$$

where variables with no subscript denote the steady state levels of corresponding variables.

The discretionary monetary shock, η^R_t, is assumed to follow:

$$\eta^R_{t+1} = \rho_\pi \eta^R_t + u^R_{t+1}, \tag{13}$$

where $E_t u^R_{t+1} = 0$ and $E_t u^R_t u'^R_t = \sigma^{R2}$.

3.5 Market clearing condition

The market clearing conditions in the service and good markets are

$$D_t = S_{D,t} + \frac{\theta_{r_D}}{2}(\pi_{r_D,t} - 1)^2 D_t, \tag{14}$$

$$Y_t = I_{D,t} + \frac{\theta_{P_D}}{2}(\pi_{D,t} - 1)^2 Y_t, \tag{15}$$

where some proportions of the final service and good are allocated to resource costs originating from price adjustment. Labor and bond markets also clear in the equilibrium.

4 Equilibrium

Equilibrium consists of the allocations $S_{D,t}$, b_t, N_t, for households; the allocations $D_t(i)$ and $I_{D,t}(i)$, and price $r_{D,t}(i)$ for the rental service firm $i \in [0, 1]$; the allocations $Y_t(i)$ and $N_t(i)$, and price $P_{D,t}(i)$ for the housing producer $i \in [0, 1]$. Along with wages W_t, that satisfies: taking prices and the wage given, the household's allocations solve its utility maximizing problem; taking the wage and all prices but its own as given, the allocations and the price of each rental service firm solve its profit maximizing problem; taking all prices but its own as given, the allocations and the price of each housing producer solve its profit maximizing problem; taking the wage and all prices but its own as given, the market for bonds and labor clear.

In the following sections, we investigate the differece between the allocations of the efficient and distorted economies.

5 Efficient allocation

5.1 Standard user cost in a frictionless economy

To find the details of the price relationship between rent and purchase, we derive a traditional user cost equation in a frictionless economy. In the case that the rental price is flexible, the intermediate service firm's first-order-condition equation (8) with respect to the rental price becomes

$$r_{D,t} = \mu_{r_D}\left\{P_{D,t} - (1-\delta)E_t\left[\frac{P_{D,t+1}}{R_t}\right] + \iota_t\right\}, \tag{16}$$

$$\text{where } \iota_t \equiv -(1-\delta)\mathrm{COV}_t\Big(P_{D,t+1}, \Lambda_{t,t+1}\Big) \tag{17}$$

and $\frac{1}{R_t} = E_t \Lambda_{t,t+1}$ is a risk-free bond rate from the durable service Euler equation. The current user cost is determined by four components. The rental-sector mark-up, μ_{r_D}, inefficiently raises the current user cost, $r_{D,t}$. The current nominal marginal cost, $P_{D,t}$, also directly raises the user cost. Most interestingly, the third and fourth terms are the expected value for the leftover stocks after depreciation. In a long-lived durable economy, the third term mitigates the increase of the user cost. When it is expected that the good price in the next period will increase, after the interest rate is discounted, the burden of the current purchase will be eased. This is because the remaining housing stocks become the future asset with a higher price. Thus, the user cost decreases as the expectation of the future purchase price increases. In the last term, the covariance of the expected resale price with the households' marginal utility of consumption in $t+1$ reflects the excess return from durable investment. Unlike the risk-free bond, the current purchase of the housings is influenced by the future risk. In general, this covariance term is negative, and hence, the risk premium increases the user cost.

5.1.1 Frictionless user cost

Consider a perfectly competitive market case where ε_{r_D} converges to infinity. In this case, the model can be interpreted as a variant of Lucas' (1978) asset pricing model.[1] Iterating the user cost equation forward, the nominal marginal cost of the investor becomes

$$P_{D,t} = \sum_{k=t}^{\infty}(1-\delta)^{k-t} E_t\left[\frac{r_{D,k} - \iota_k}{\Pi_{l=t}^{k} R_{l-1}}\right], \tag{18}$$

where the left-hand side represents the nominal marginal cost of housing durables in period t, which is the purchase price from the production sector, while the right-hand side is composed of the corresponding rental gains and risk premium from period t on. The implication is straightforward: the purchase price of housing should be equated to the present value of earnings from the corresponding services and risk premium.

5.1.2 Frictionless price-rent ratio

We can also derive a price-rent ratio in line with the Lucas-tree model. We can interpret the real marginal cost, $\Xi_{r_D,t}$, as a price-rent ratio of the service firm. It can be written as a function of the depreciation rate and the marginal utility gap:

$$\Xi_{r_D,t} = E_t\left\{\frac{\sum_{k=t}^{\infty}[(1-\delta)\beta]^{k-t} U_{S_D,k}}{U_{S_D,t}}\right\}. \tag{19}$$

This equation has at least two important implications. First, the price-rent ratio is an increasing function of durability, $(1-\delta)$. High durability implies that the purchased good survives for a long period, and hence, the value of the good also increases. Second, the price-rent ratio is a function of the marginal rate of substitution of consumption between today and the future. We call these terms *marginal utility gaps*. Because the households are the owners of service firms, the firms' current and future profits can be valued in terms of the gaps of marginal utility of consumption, which are $U_{S_D,t}$ and $U_{S_D,k+1}$ for all $k \geq t$. If the marginal utility of the consumption in the future is relatively higher than the current marginal utility, the price-rent ratio increases. When the households lend greater value to future consumption, the price-rent ratio should increase.

5.1.3 Descriptive analysis

To learn the dynamics of two prices, we analyze how our model economy reacts to exogenous shocks. Fig. 2 shows the evolution of the impulse responses under a different

[1]The major difference is that his model is an endowment economy with no depreciation rate, while in our model, durable goods are depreciated and there are always new production in each period. Therefore, our model always has transactions of durable goods and services even in the steady state.

durability to a persistent technology shock: quarters after the shock, from the impact to the 32nd periods, are plotted on the x-axis; the degree of durability is plotted on the y-axis, from 0 to 1; and the response scale is plotted on the z-axis. The responses of service flow in the rental market and new output flow in the production market are respectively displayed in the first row. When the depreciation rate is 100 percent ($\delta = 1$), both the service and output flows feature the same responses because this situation is the case with a only nondurable goods' economy.[2] The existing total stock against the new production ratio, the so-called *stock-flow ratio*, in the steady state is a function of durability: a lower depreciation rate increases the volume of the total stocks compared to new production flows. Therefore, as the durability increases, the response of the durable stocks and the corresponding service flows are smoothed while the output flows show a more active response. It is well known that durable goods respond more sensitively than nondurable goods to macro shocks such as productivity and monetary shocks. For example, Basu and Fernald (1997) find that durable industries are more cyclical, employing a large share of the marginal inputs in a boom. In our model, this is the case because the service firms' capital gain is an increasing function of durability.[3] Therefore, the response scale of output flows becomes bigger. Furthermore, the consumption of service flows exhibits a hump-shaped response because newly produced output flows get accumulated in the short run.

The price-rent ratio shows positive signs in all regions. When the depreciation rate is 100 percent, the price-rent ratio does not deviate from one. This is because the price of nondurable goods equals the corresponding service price. The price-rent ratio in the steady state is an increasing function of durability because high durability means sustained high earnings. Its response to the productivity innovation depends on the sequence of the marginal utility gap:

$$\hat{\Xi}_{r_D,t} = [(1-\delta)\beta]E_t(\hat{U}_{S_D,t+1} - \hat{U}_{S_D,t}) + [(1-\delta)\beta]^2 E_t(\hat{U}_{S_D,t+2} - \hat{U}_{S_D,t}) + \cdots ,$$

which is a log-linearized version of equation (19).[4] When durability is considerably low or the shock is temporary, the path of marginal utility gaps ($\hat{U}_{S_D,k+1} - \hat{U}_{S_D,t}$ for all $k \geq t$) are positive in general because the future consumption is lower. On the other hand, when durability is high and the expansionary shock is persistent, a consumption boom is expected in the near future. Expecting future demands for the currently purchased durables, service firms demand more durables and the prices increase.[5] Thus, the price-rent ratio gap exhibits little deviation in the short run under the negative marginal utility gap effect. Overall, the price-rent ratio shows a hump-shaped response.

The nominal interest rate exhibits a positive sign in a long-lived durable economy. No arbitrage condition between the purchases of bonds and durable goods should hold:

$$R_t = E_t(Z_t), \tag{20}$$

[2] On the other hand, when the depreciation rate reaches zero percent, the economy converges to the Lucas-tree economy.

[3] Another explanation is that the elasticity of substitution for the purchase of durable goods is much higher. For example, see Barsky, House, and Kimball (2007).

[4] The hatted value denotes the log-deviation of the corresponding value from the steady state level.

[5] In other words, the response of durable consumption shows a hump-shaped response because of the high accumulation effect of durable goods. Therefore, the marginal utility gaps are negative until the future durable consumption returns to the level in the impact period.

from equation (16). The right-hand side represents a one-period holding return, $Z_t \equiv \frac{(1-\delta)P_{D,t+1}}{P_{D,t}-\mu_{r_D,t}^{-1}r_{D,t}+\iota_t}$, from buying the durable goods in period t and selling them on period $t+1$. The term in the numerator is the capital gain. It is divided by the net purchase price in the denominator to provide a rate of return. Therefore, the returns of riskless bonds and the expected return in investing durable goods are the same.[6] In the economy where the expected return of durable investment is high, the return of riskless bonds also should increase.

6 Co-movement and volatility

In this section, we analyze the cases where the distortions exist in each sector.

6.1 Sticky purchase price

When the purchase price is sticky, the relationship of the purchase price with the nominal wage rate becomes

$$P_{D,t} = \frac{\Psi_{P_D,t} W_{D,t}}{A_t}, \tag{21}$$

where $\Psi_{P_D,t} \equiv \left[\frac{1}{\frac{1}{\mu_{P_D}}+\psi_{P_D,t}}\right]$.[7] $\Psi_{P_D,t}$ is an efficiency parameter in the production sector: the first term in the denominator represents the mark-up friction, and the second term, $\psi_{P_D,t}$, is a distortion from the price stickiness in the good sector. Note that when the production sector is perfectly competitive ($\mu_{P_D} = 1$) and the purchase price is flexible ($\theta_{P_D} = 0$ and $\psi_{P_D,t} = 1$), all distortions disappear ($\Psi_{P_D,t} = 1$). In this case, the movement of the real wage replicates the productivity innovation. However, $\Psi_{P_D,t}$ deviates from its efficient level when the purchase price is sticky. Thus, the firms in the production sector set the current prices inefficiently at a lower level than in the flexible case and vice versa. For example, in a case where the firms intend to increase their prices today, a positive gap may occur between current inflation and the expectation of future inflation in the production sector. In this case, the price adjustment cost depresses the firms' decision.

6.2 Sticky rental price

In a case where the rental price is sticky, the user cost equation becomes

$$r_{D,t} = \Psi_{r_D,t}\left\{P_{D,t} - (1-\delta)E_t\left[\frac{P_{D,t+1}}{R_t}\right] + \iota_t\right\}, \tag{22}$$

where $\Psi_{r_D,t} \equiv \left[\frac{1}{\frac{1}{\mu_{r_D}}+\psi_{r_D,t}}\right]$.[8] $\Psi_{r_D,t}$ is an efficiency parameter that consolidates all frictions in the service sector. $1/\mu_{r_D}$, the first term in the denominator, represents the friction

[6] Calza, Monacelli, and Stracca (2007) analyze the difference between flexible and fixed rates in the housing market. We do not analyze those effects in this paper.

[7] $\psi_{P_D,t} \equiv \frac{\theta_{P_D}}{\varepsilon_{P_D}}\left\{(\pi_{D,t}-1)\pi_{D,t} - \beta E_t\left[\frac{U_{S_D,t+1}}{U_{S_D,t}}\frac{\Xi_{r_D,t+1}}{\Xi_{r_D,t}}\frac{Y_{t+1}}{Y_t}(\pi_{D,t+1}-1)\pi_{D,t+1}\right]\right\}$.

[8] $\psi_{r_D,t} \equiv \frac{\theta_{r_D}}{\varepsilon_{r_D}}\left\{(\pi_{r_D,t}-1)\pi_{r_D,t} - \beta E_t\left[\frac{U_{S_D,t+1}}{U_{S_D,t}}\frac{D_{t+1}}{D_t}(\pi_{r_D,t+1}-1)\pi_{r_D,t+1}\right]\right\}$.

originating from the monopolistic competition, and $\psi_{r_D,t}$ represents distortion from the price stickiness. Note that this equation holds even when the purchase price is sticky.

6.3 The relationship between nominal rigidity and co-movement

Fig. 3 displays the effect of a productivity shock on the selected variables. Two cases are exhibited: (i) sticky purchase prices (and flexible rental prices) and (ii) sticky rental prices (and flexible purchase prices). The key parameters are the physical depreciation rate, δ, and the price adjustment costs, θ_{P_D} and θ_{r_D}. For calibration, we consider a housing market where the degree of price stickiness in the service market is high while the purchase price is flexible. In the preceding literature, the annual depreciation rate of housing is shown to be between 1 percent to 5 percent in the U.S. economy, so we calibrate δ on a quarterly basis as $\delta = \frac{0.05}{4}$. Following Genesove (2003), we set the degree of nominal rigidity in the rental market, so that the annual probability of no rental price adjustment is 0.29. In the counterfactual simulation, we assume the same degree of nominal rigidity in the production sector. For a central bank's rule, we assume a simple Taylor rule that responds to the sticky inflation rate, ρ_i (for $i \in \{\pi_{P_D}, \pi_{r_D}\}$): the coefficient parameter is set to be two and the others to be zero.

We start with the benchmark case of a standard NK model with sticky price in the production sector. The blue lines represent the sticky purchase prices and flexible rental prices case. In this case, the rental price falls substantially in response to the shock. This is caused by the nominal rigidity of the purchase price in the production sector. As we saw in the flexible price case, the purchase price and the price-rent ratio are likely to increase. However, when the purchase price is sticky, the adjustment cost depresses the production firms to increase their goods' prices enough. Instead, the rental price in the frictionless rental market efficiently replicates the price-rent ratio. Note that the price-rent ratio is efficient when the rental market is frictionless.

The red dashed lines in the middle panels represent the price dynamics when the rental price is sticky and the purchase price is flexible. These graphs present at least two interesting features. The nominal rigidity of the rental price becomes one reason to make the purchase price volatile. Since the rental price, $r_{D,t}$, is stable or motionless, the housing price, $P_{D,t}$, should respond to the shocks. A considerably high degree of rental price stickiness induces a large swing of the purchase prices to equate the user cost equation. More interestingly, two prices show an increasing pattern after the introduction of the nominal rigidity of the rental price and the shock persistence.

6.4 The relationship between shock persistence and co-movement

Fig. 4 displays the impulse responses to the different persistences of productivity shocks. The panels in the first column display the sticky purchase price case, and the co-movement behavior is not observed in all cases. The panels in the second column are the responses under the sticky rental price economy. We observe that the co-movement behavior disappears when the shock persistence is low.

In the existence of durables, two effects should be considered. The first effect is the intertemporal elasticity of substitution for the purchases of durable goods, as discussed in Barsky, House, and Kimball (2007). The second effect is the consumption smoothing effect of durable services in response to the persistent shock.

When the shock is temporary, the demand for the rental services of households is at the highest level in the impact period because of the first effect.[9] Hence, the purchase price and the price-rent ratio are inclined to increase in the short run. Because firms in the rental market can produce rental services at a higher input cost, the rental service firms want to increase their rental prices. With the passage of time, the demand for the durable service decreases, and hence, the purchase and rental prices also decrease. On the other hand, when the shock is persistent, the households' rental demand level in the impact period is not as high as the temporary shock case because of the second effect. Because of the accumulation effect of the total durable stocks and the persistence of the exogenous shocks, it is expected that the future demand for the rental services will be higher and an economic boom will occur in the near future. Therefore, the purchase price and rental price show an increasing pattern in response to the increasing demand.

6.5 Does the rental price stickiness amplify the purchase price volatility?

Asset price volatility has been a recurring concern in the preceding asset market literature.[10] In the previous studies that aim to identify the relationship between stock prices and dividends, the stability of dividends were considered as one of reasons for the volatile movements of stock prices. For example, Shiller (1981) investigates whether the excessively volatile movements of stock prices are justified by subsequent changes in dividends. Cochrane (1992) reports the characteristics of price-dividend ratio variances that are reflected by the dividend growth as well as discount rates. Furthermore, in the new-open-economy-macro literature, a high volatility for a nominal exchange rate and the terms of trade can be explained by the nominal price rigidity in the domestic market. For example, Gali and Monacelli (2005) show that a policy of strict domestic inflation targeting implies a substantially great volatility in the nominal exchange rate and terms of trade. Therefore, we analyze whether the volatility of the purchase price can be explained by the nominal rigidity of the rental price.

It is well known that housing productions and the corresponding prices are considerably more volatile than nondurable goods and prices in the business cycle movements. For example, Iacoviello and Neri (2010) show that the cyclical standard deviation of housing productions is almost five times as volatile as that of the nondurable consumption in the U.S. economy: 9.97 versus 1.22. This also holds for the prices: 1.87 versus 0.40. In our model, these high volatilities can be explained by the stickiness in the rental market. When the user cost is sticky and the purchase price is flexible, the purchase price becomes volatile in response to the exogenous shocks. From equation (22), the user cost on the left-hand side does not move much when Ψ_{r_D} in the right-hand side is low (ϕ_{r_D} is high). Therefore, when an exogenous shock, such as a productivity innovation, occurs, the purchase price on the right-hand side should be adjusted.

Fig. 5 depicts the standard deviations of the simulated variables.[11] As the rental price rigidity increases, the standard deviation of the user cost inflation decreases. On the other hand, the rental price stickiness let the purchase price become more volatile.

[9] The same effect was analysed by Barsky, House, and Kimball (2007) under the temporary monetary shock.

[10] For example, see Lucas (1989).

[11] We perform the numerical simulation using the methods described by Uhlig (1999).

The price-rent ratio also becomes very volatile as the nominal rigidity of the user cost increases. Cochrane (2011) discusses that the movements of the price-rent ratio are largely explained not by the dividend growth rate but by the sequence of the stochastic discount factors. However, Fig. 5 reveals that the purchase price inflation and the price-ratio volatility correspond to the degree of the nominal rigidity of user cost inflation.

7 Concluding remarks

How do the prices in the housing market respond to productivity shocks? Empirical evidence reveals that both the rental and purchase prices in the housing sector increase in response to productivity shocks.

In this paper, we incorporate the rental market into the otherwise conventional sticky price model with long-linved durables. We show that the behavior of rental prices and purchase prices depends heavily on where the nominal rigidity originates. If the purchase price in the production sector is sticky, the co-movement phenomenon observed in the data cannot be generated. When all prices are flexible, the purchase price and the price-rent ratio are inclined to increase in response to productivity shocks. However, when the purchase price is sticky, it does not increase enough compared to the flexible price economy. Therefore, the rental-service firm, a price taker of the purchase price, can purchase the new output at a low input cost, and hence, can offer rental services at a lower price. This is at odds with the empirical findings.

The purchase price volatility and co-movement of two prices can be resolved by the introduction of the nominal rigidity of rental prices. When the rental price is sticky, it becomes motionless. Instead, the purchase price positively reacts to the shocks. The rental-service firm should consider the future rental demand for the current purchase. Furthermore, when the shock is persistent, the demand for the rental services shows a hump-shaped pattern, and hence, the rental price and the purchase price also increase.

Several unresolved issues need to be pursued in future works. First, in this paper, we do not consider a case where goods with different durabilities coexist. Many different characteristics introduce many goods. Second, we need to introduce other features into the model in order to capture more implications of exogenous shocks to durable goods. The lumpiness of durables or news shocks can be good examples. Third, we only consider the case of a rational response to the shocks. The introduction of irrational behavior may be necessary to explain the recent housing market boom and bust in the U.S. economy.

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Appendix A: deterministic steady state

We consider a steady-state in which all the shocks are zero and in which monetary policymakers set their respective CPI inflation rates to zero:

$$\pi_{r_D} = 1; \pi_{P_D} = 1.$$

Equation (5) implies the nominal interest rate and the price of claims become:

$$R = \frac{1}{\beta}.$$

The real marginal costs are

$$\Xi_{r_D} = \frac{\mu_{r_D}^{-1}}{1-(1-\delta)\beta}; \quad \Xi_{P_D} = \mu_{P_D}^{-1}.$$

Compared to the production sector, the real marginal cost in the rental sector is higher.

The real wage becomes:

$$w = \Xi_{P_D}.$$

We set the steady state labor level as one third. ($N = 1/3$) Then combining production functions, the good market conditions, and the law of motion of durable goods, we get

$$N = Y = I_D = \delta D$$
$$\therefore D = \frac{1}{3\delta}.$$

From the rental market clearing condition

$$S_D = D.$$

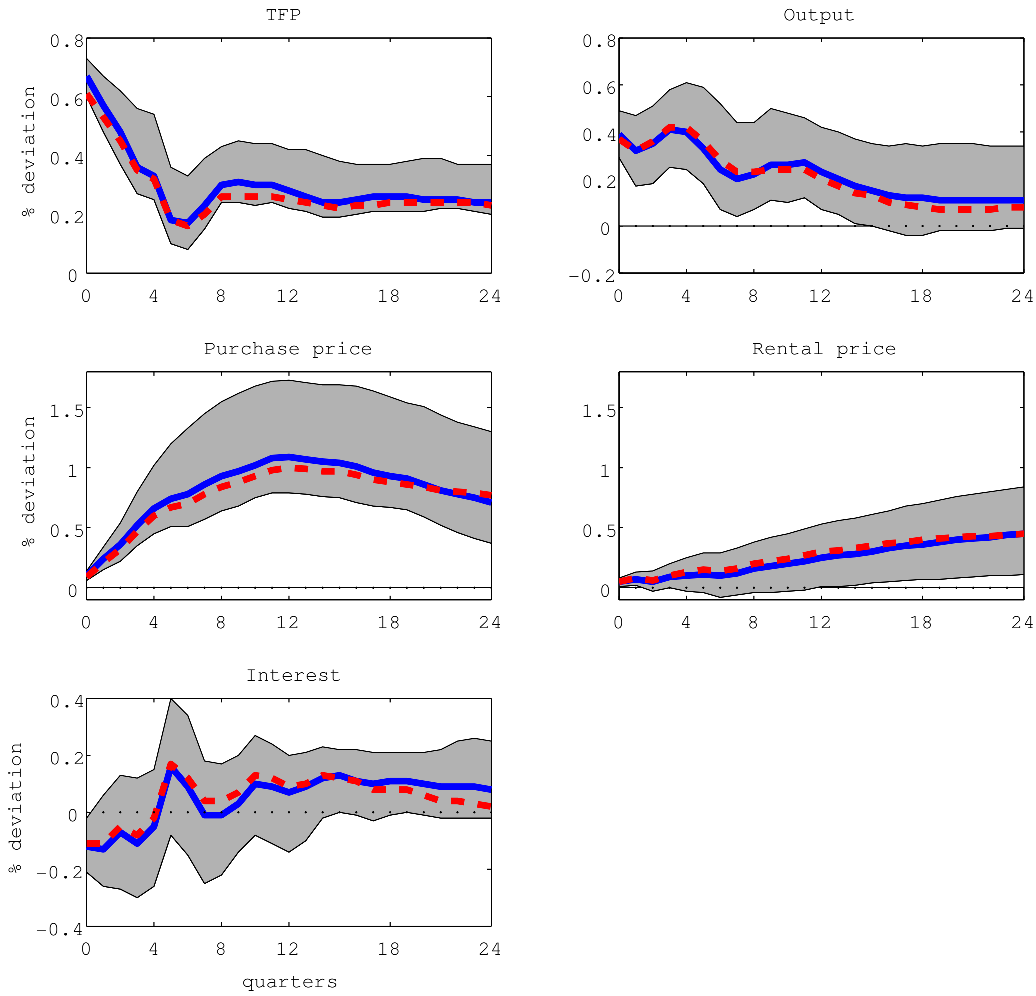
Appendix B: Calibration and numerical simulation results

Time is in quarters and we set the quarterly discount factor as $\beta = 0.99$. It implies that the annual real interest rate is pinned down by the household's patience rate and is equal to 4 %. The annual depreciation rate for houses is 5 % ($\delta = 0.05/4$). Following Monacelli (2009), the elasticity of substitution between varieties in the non-durable and the durable sectors ε_{P_D} and ε_{P_C} are set equal to 6, which yields a steady state mark-up of 20 %.

In the benchmark case, we set the degree of nominal rigidity in service and good prices to generate a frequency of price adjustment of about four quarters. Let κ be the probability of not resetting prices in the standard Calvo-Yun model. Log linearized Phillips curve in this model is $\frac{\varepsilon}{\theta}$, while it is $\frac{(1-\kappa)(1-\beta\kappa)}{\kappa}$ in the Calvo-Yun model. A price rigidity of four quarters is a standard in the recent literature so we take it as a benchmark parameter ($\kappa = 0.75$).

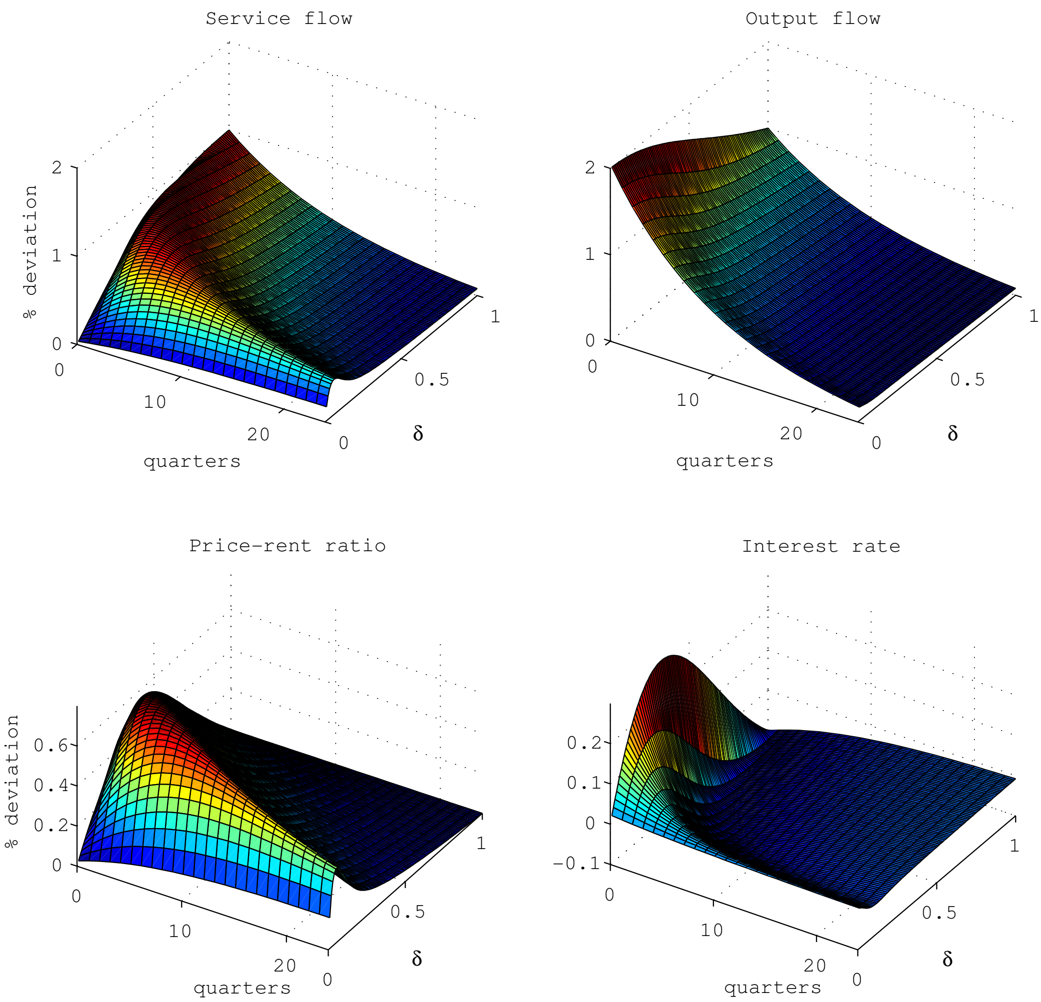
The period utility function is assumed to be: $\frac{S_{D,t}^{\sigma-1}}{\sigma-1} - \nu\frac{N_t^{1+\phi}}{1+\phi}$. Following the existing literature on durable goods, we set $\sigma = 1$ and $\phi = 1$. In the analysis of optimal monetary policy, we change the value of σ and search for the implication of welfare. The elasticity parameter, ϕ, is set to one in all cases. Therefore, the scale parameter, ν, is adjusted for the intra-temporal condition to hold in equality with the change of durability.

Fig. 1: Impulse responses to the productivity shock



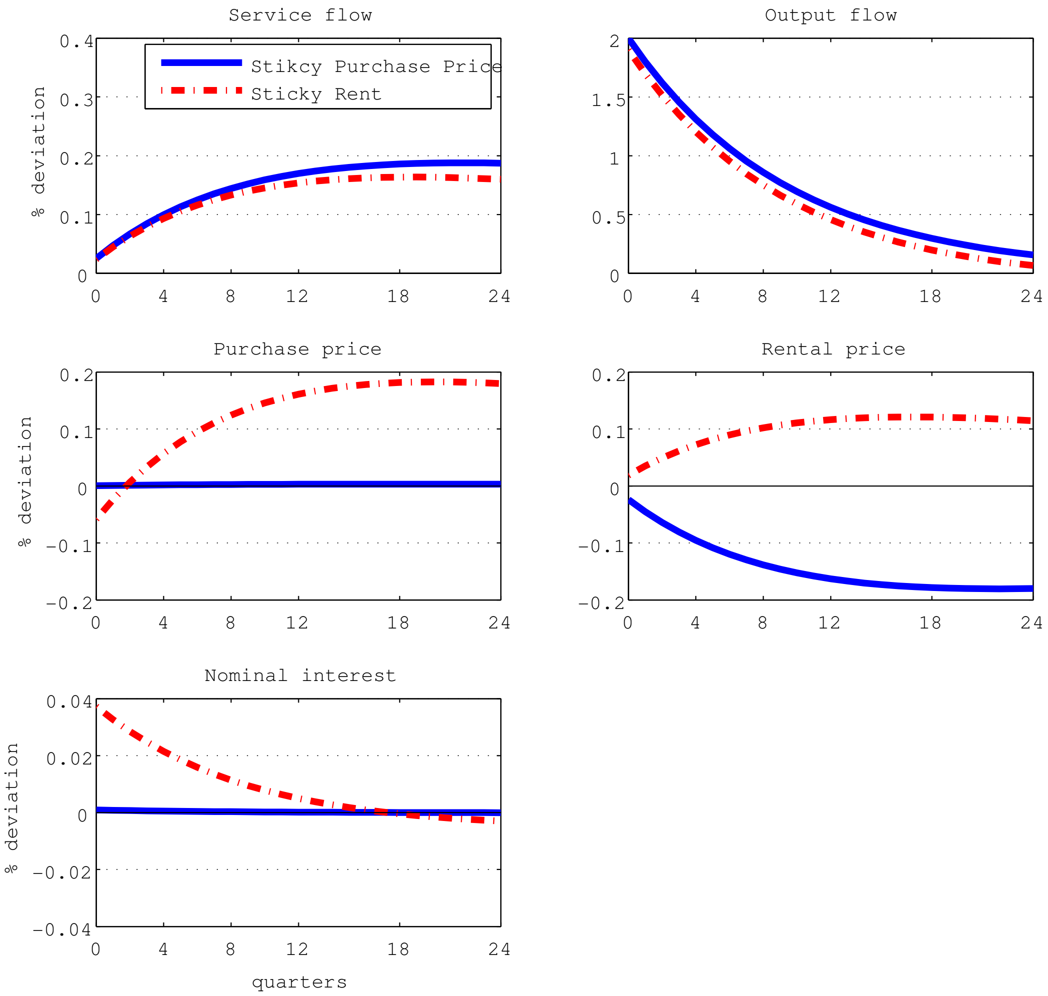
Note: The responses to the one percent increase in TFP innovation. The shaded areas represent the two standard deviation bands.

Fig. 2: Response to persistent productivity shock (flexible case)



Note: The responses to the one percent increase in productivity. This is a persistent case with $\rho^a = 0.9$.

Fig. 3: Sticky purchase price versus sticky rent price



Note: The responses to the one percent increase in productivity. This is a persistent case with $\rho^a = 0.9$.

Fig. 4: Different degree of shock persistency

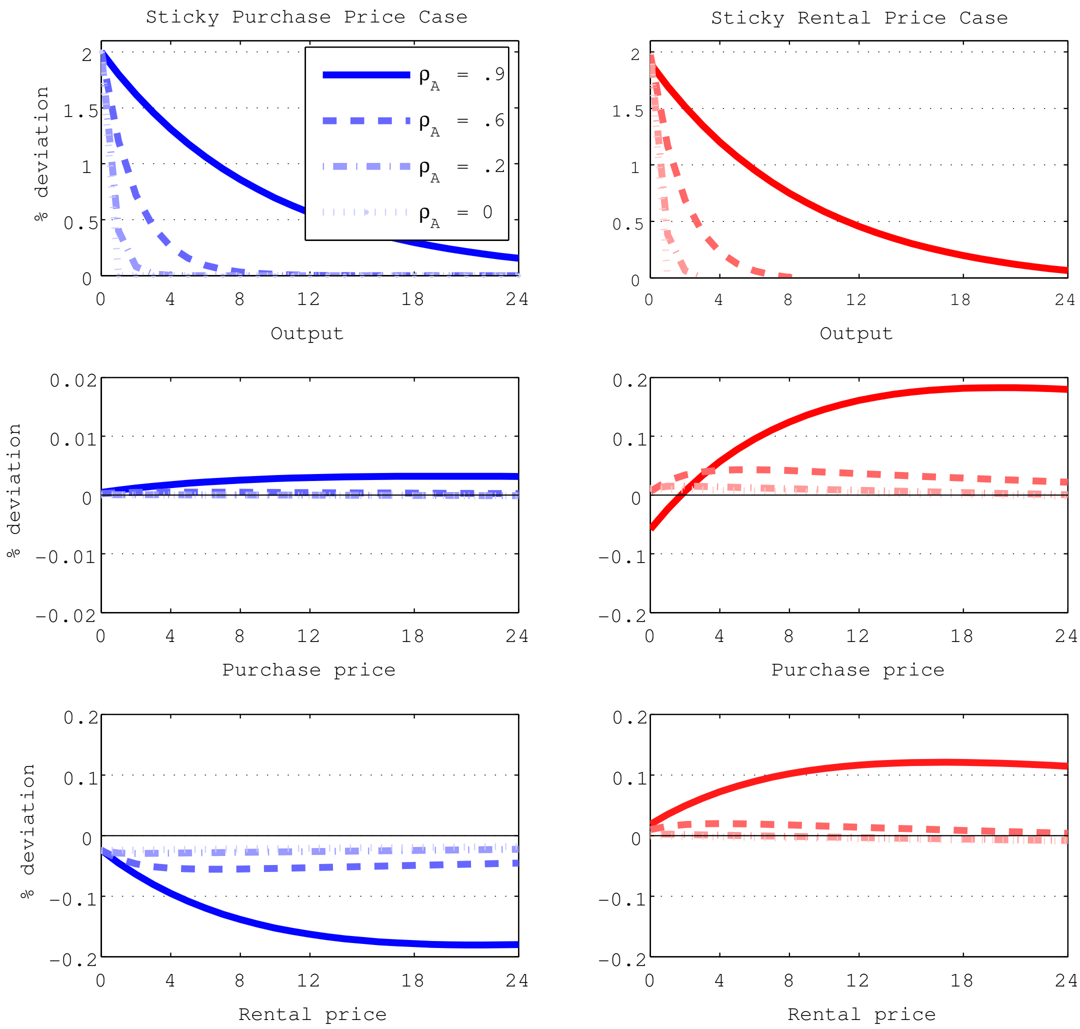
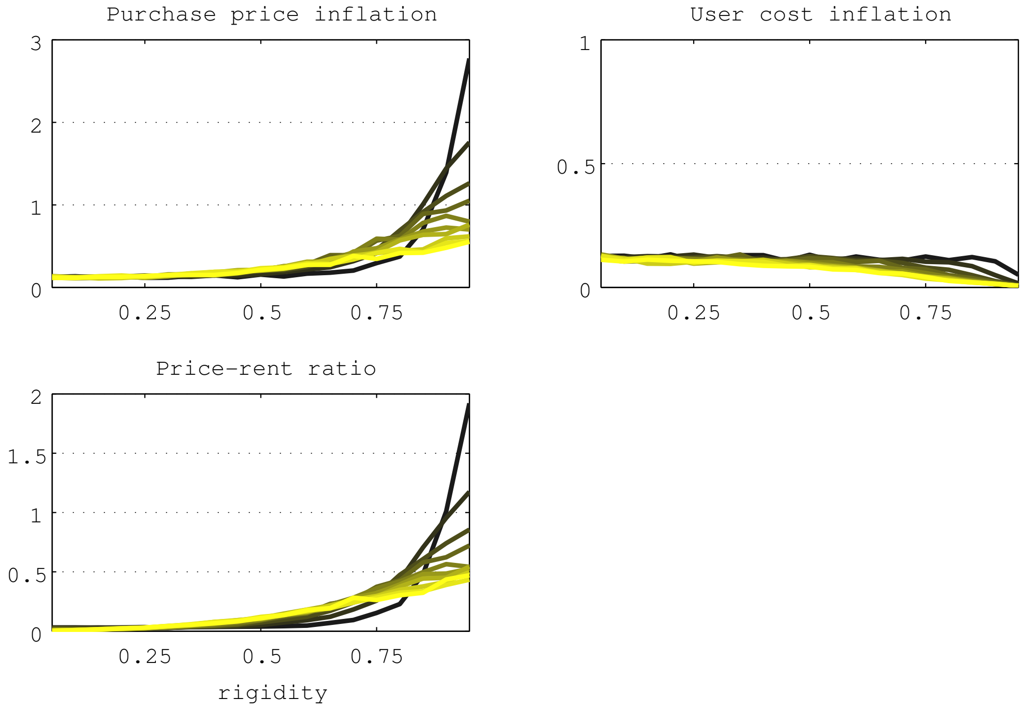


Fig. 5: Standard deviations with different rigidity



Note: The values in the x-axis correspond to the Calvo parameters. For example, 0.75 implies the firms can change their prices once a year. The brightest yellow line is the case when $\delta = 0.1$, and the darkest line is the case when $\delta = 1$.