Failure risk and quality cost management in the choice of single versus multiple sourcing

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2 October 2011

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KEYWORDS: Supplier selection, latent defects, multiple sourcing, single sourcing, quality cost, total cost of purchasing.

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[†] I thank Stanley Baiman, Eshien Chong, Farok Contractor, Raffi Indjejikian, Francine Lafontaine, Martin Lariviere, Pinghan Liang, Anne Neumann, Eric Noreen, Veronica Santarosa, Richard Saouma, Tim Schmidt-Eisenlohr, Larry Snyder, Steve Tadelis, and Veikko Thiele for their valuable comments on earlier drafts of the paper. I also thank participants of the 2011 Econometric Society European Meeting (ESEM) at Oslo, Norway, 2011 Conference of the European Association for Research in Industrial Economics (EARIE) at Stockholm, Sweden, 2011 World Congress of the International Economic Association (IEA) at Beijing, China, 2011 Accounting Research Workshop (ARW) at the University of Fribourg, Switzerland, 2011 International Industrial Organization Conference (IIOC) at Boston, Massachusetts, 2010 European School on New Institutional Economics (ESNIE) at Cargese Institute of Scientific Studies in France, and 2009 (Third) European Risk Conference on "Risk and Accounting" at Deloitte's training center in London for their useful feedback. All remaining errors are mine.

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Failure Risk and Quality Cost Management in the Choice of Single versus Multiple Sourcing

1. Introduction

This paper examines the supply base composition problem, in particular the choice of single versus multiple sourcing, from a quality cost management perspective. There have been over twenty years of interest in understanding the relative advantages of single versus multiple sourcing, beginning with Deming (1986) and Porter (1985). Many issues have been considered in the literature regarding selecting suppliers to form a supply base. For example, Weber, Current, and Benton (1991) have identified more than twenty criteria for supplier selection decisions. Among the many factors considered, three of them, namely price, delivery, and quality, are often regarded as the core to consider (Lemke et al. 2000). The emphasis of this paper is on the last, but not the least, of these major factors: *quality*.

A strand of the literature on multiple sourcing has focused on the advantage of price reduction due to more suppliers competing with each other (the *competition* advantage).[1] A second, more recent strand examines the advantage arising from the protection against supplier failures (the *protection* advantage).[2] The supplier failures that have been studied are a variety of *unreliable supply* such as late/insufficient/no delivery due to reasons like machine breakdowns, labor strikes, natural disasters, and financial defaults (e.g., Federgruen and Yang 2008, 2009, Babich, Burnetas, and Ritchken 2007, Burke, Carrillo, and Vakharia 2007, Dada, Petruzzi, and Schwarz 2007). Such failures adversely affect the buying company's production and may result in the loss of sale in the end product market. In contrast, the supplier failure studied here is about *undependable* (or even unsafe) *products* sold to the customers of the buying company, as a result of using product parts with latent defects provided by its suppliers.

I emphasize the difference between unreliable supply and undependable products. The former is caused by random yields, which have been studied extensively (see reviews by Yano and Lee 1995 and Grosfeld-Nir and Gerchak 2004). The latter is due to latent defects, which is receiving growing attention (e.g., Chao, Iravani, and Savaskan 2009). *Latent defects* are flaws or weaknesses in product items that

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[1] Studies in this strand often use modeling techniques of the auction literature (e.g., Tunca and Wu 2009). See Mishra and Tadikamalla (2006) for a concise review of major results of the models in this strand. Some most recent study, namely Asker and Cantillon (2010), considers both the quality level and the price factor. Their model, however, focuses on the procurement of an indivisible object. The study is more relevant to settings like defense procurement than the voluminous-quantity setting considered here. Inderst (2008) considers the procurement of a volume of goods, however, with homogeneous quality. He focuses on non-standard assumptions of having two buyers and auctions organized by suppliers, rather then by a buyer.

[2] Examples in this strand are Berger, Gerstenfeld, and Zeng (2004), Berger and Zeng (2006), and Ruiz-Torres and Mahmoodi (2006, 2007). These studies rely mainly on numerical simulation analyses to obtain results, in contrast to the analytical modeling method used in this paper.

could not be discovered by reasonable inspection prior to the sale. They later manifest as field failures in the hands of end customers. Consequences to the buying company (i.e., the manufacturer/retailer to which the supplier has sold the faulty items) are all sorts of quality costs. They include costs due to warranty repairs, product recalls, defect liability claims, reputation damage, loss of sale, and customer confidence restoration efforts (Nagar and Rajan 2001). Such costs originating from latent defects can be huge. For instance, in 2001, Ford announced that it would spend $3 billion to replace millions of Firestone tires causing customer safety concerns (Bradsher 2001).

If latent defects are due to occasional independent mistakes, the impact probably is limited. Latent defects, however, can result from design flaws, systematic manufacturing faults, or the like (Thirumalai and Sinha 2011). In that case, defects can occur in a correlated manner in product items from the same supplier.[3] As a result, voluminous amounts of product items may be affected. Some of these items with latent defects later are identified when field failures occur. Depending on the nature of the product, the consequences to end customers can be substantial or even fatal, e.g., the 34 deaths in accidents allegedly caused by runaway Toyota vehicles (Shepardson 2010). It is thus crucial to understand how the risk of such failures can be diversified and the costs of doing so. I add to the understanding of this issue by analyzing a supply base composition model for the *LUX* (*L*atent defect-*U*ndependable product-e*X*ternal failure) setting described above.

The question asked in this study is similar to Benjaafar, Elahi, and Donohue's (2007). They examine the outsourcing of a fixed demand for a service at a fixed price to a set of potential suppliers. The two competition mechanisms they compare, namely the *supplier-selection* (SS) versus the *supplier-allocation* (SA) approach, are equivalent to single versus multiple sourcing. Their analysis focuses on how the relative advantage of SS versus SA is affected by the presence/magnitude of demand-independent versus demand-dependent service costs. By contrast, the emphasis here is on the convexity of the external failure cost rather than the service/production cost structure.

The buyer in my model can rank suppliers by a composite score,[4] which for simplicity is

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[3] An example of products with correlated latent defects is hard disk drive. It affects many aspects of modern living, from home electronic appliances like digital video recorders to large automation systems controlled by computers that have hard disk drives as massive storage devices. According to Paris and Long (2006), "[b]atch-correlated failures result from the manifestation of a common defect in most, if not all, disk drives belonging to the same production batch. They are much less frequent than random disk failures but can cause catastrophic data losses." Paris and Long advise that redundant copies of the same data should be stored on disks from *different batches* and, possibly, *different manufacturers* to reduce impact of batch-correlated failures.

[4] Firms like Magna Closures require interested companies to provide detailed information about their business operations in order to become certified suppliers of the firms. Only after then would the suppliers receive invitations to submit tenders for the firm's procurement programs. (See, e.g., Magna Closures' *Supplier Quality Manual & Requirements* at http://iweb01ds. magnaclosures.com .)

represented by a supplier's quality level (i.e., a parameter affecting the defect rate). Multiple sourcing cannot be more advantageous than single sourcing unless the expected profit maximizing buyer has a desire for risk diversification. In the model, this is due to a quadratic external failure cost representing the reputation damage suffered by the buyer, as a result of field failures of undependable products. The cost function captures the intuition that as the number of field failures grows, it is increasingly more likely to catch the attention of mass media, and the resulting reputation damage is increasingly more detrimental. Effectively, the buyer is risk averse, as her payoff is concave due to the convex cost.

When ignoring any desire for risk diversification, the buyer would source only from the supplier of the highest quality because this minimizes the quality cost. However, if risk diversification is desirable, the buyer might want to admit another supplier into the supply base and thereby diversify the failure risk somewhat. Whether the buyer would continue to admit more depends on how difficult it is to maintain the overall quality level of the supply base. With more suppliers selected into the supply base, the buyer had to choose from suppliers of further lower qualities. By weighing the incremental quality cost against the incremental benefit from further risk diversification, the optimal supply base is determined.

Under some condition, the optimal supply base has intuitive properties: if a supplier is selected to constitute the supply base, any supplier of higher quality must also be selected; moreover, larger production quotas should be assigned to higher-quality suppliers. The properties hold under a mild condition on the differences among the procurement costs charged by suppliers. This includes equal costs as a special case but also allows unequal costs. As long as the condition is fulfilled, the procurement costs do not need to have any particular order (e.g., higher-quality suppliers may or may not charge higher procurement costs).

I derive a necessary and sufficient condition for determining the size of the optimal supply base, provided the mild condition on the procurement costs holds. If the question is not about determining the exact size but merely about whether single or multiple sourcing is optimal, another necessary and sufficient condition can be derived without requiring any precondition to hold.

By analyzing the advantage of multiple sourcing to protect against the failure risk arising in a LUX setting, this paper contributes to the literature on multiple sourcing, as well as on supplier selection (e.g., Ittner et al. 1999) and on quality management (e.g., Ittner, Nagar, and Rajan 2001). My model highlights the sourcing decision as one about selecting the right *combination* of suppliers to balance between risk diversification and quality cost reduction. In contrast, prior studies on multiple sourcing concern mainly the *number* of suppliers that determines the level of competition, with or without externalities among units or suppliers (e.g., Baiman and Netessine 2004 and Klotz and Chatterjee 1995; see Elmaghraby 2000 for a review of the literature).

The paper adds a new perspective to the debate on single versus multiple sourcing. Deming (1986) argues that a buying company should take quality costs into consideration and choose single sourcing by selecting a high-quality supplier that minimizes *total cost* of sourcing. The result of this paper suggests that in minimizing total cost of sourcing, one might need to use multiple sourcing and include suppliers of lower quality. This can happen when the total cost of sourcing counts in the benefit from supplier failure risk diversification. Recent studies on the buyer-supplier relationship have highlighted the role of trust in the relationship (e.g., Dyer and Chu 2000 and 2011 and Li et al. 2010). However, the breakup of the long-term buyer-supplier relationship between Ford and Firestone suggests that sometimes it might need more than loyalty and trust to deal with the risk of latent defects.

This study illustrates when multiple sourcing can be used to deal with the risk and when single sourcing is preferred despite the risk. It also illustrates how suppliers should be selected into the supply base to best deal with the risk. In this regard, the study also contributes to the strategic management literature on issues related to alliance partner selection (e.g., Shah and Swaminathan 2008).

This paper's closed-form characterization of the optimal supply base with "quality-driven" properties substantially reduces the computational complexity of finding the set of selected suppliers. The tractability of the model provides promising potential for using it as a building block to integrate with another model (e.g., Chao et al. 2009 or Arya and Mittendorf 2007) to study interesting questions. An example is the joint use of multiple sourcing and product recall cost sharing to reduce external failure risks. Another is the interplay between internal transfers and external procurement in controlling quality costs in a LUX setting.

In the next section, I formulate a quality-cost model of supply base composition. The analysis of the baseline model is provided in section 3, where the buyer is supposed to take the per-unit procurement costs charged by the suppliers as given. I consider buyer-initiated procurement contracts in section 4, where the buyer can use full-commitment contracts to elicit the private information on the suppliers' unit variable production costs. Further discussion on the importance of analyzing the LUX setting is given in section 5, where related recent studies are also reviewed. Section 6 contains concluding remarks, with technical proofs and derivations and a brief review of economic studies related to multiple sourcing relegated to the appendix (appendix A). Recent cases of massive recalls are also included in the appendix (appendix B available upon request). Table 1 summarizes the notations used in the paper.

2. A Quality-cost Model of Supply Base Composition

Consider a setting with a single buyer and $n \geq 2$ available suppliers, indexed by $i \in N = \{1, 2, \ldots, n\}$. The buyer, an expected profit maximizer, designs a finished product, owns the brand, and operates as a make-to-order manufacturer. Each unit of the product needs a component part (e.g., the accelerator pedal of a vehicle) to make. Other than this, the rest of the product is manufactured by the buyer. For expositional simplicity, I normalize the buyer's fixed and marginal costs of production to zero. Additionally, I assume that except for the component part in concern, the designs of the rest of the product are foolproof, and the production quality of the buyer is virtually perfect. In other words, if a product failure occurs, it must be due to the failure of the component part. This allows me to focus on the part in concern, without being complicated by less essential details like the buyer's assembly and other parts. The analysis can easily be extended to allow for product failures due to independent random failures of the other parts used by the buyer.

Let $Q > 0$ denote the quantity of the finished products ordered by the end customers of the buyer. The manufacturing of the Q units of the required component part is outsourced to the suppliers. Inventory holding issues are suppressed to focus on other economic forces driving the sourcing decision.

The multiple sourcing arrangement analyzed with the model is of the *split* sense. (See the review of related economic studies provided in the appendix for a clarification of the different senses the term "multiple sourcing" might mean.) Specifically, the buyer's problem is to determine a production quota allocation $\mathbf{Q} = (Q_i)_{i \in N}$ to outsource the part production to the suppliers. The determination of the quota $Q_i \geq 0$ for supplier i takes into account the suppliers' quality levels and the prices they charge. Feasibility requires $\sum_{i=1}^{n} Q_i = Q$.

For analytical convenience, I assume Q_i is a real variable, although for ease of interpretation I sometimes refer to it as though it were an integer. In practice, the production target Q is likely to be quite large and the number of selected suppliers tends to be small. The indivisibility of Q_i should be negligible.

Related economic studies suggest that scale economies in production inherently bias against multiple sourcing, whereas scale diseconomies bias towards it. To eliminate such inherent biases, I assume the suppliers have constant marginal costs of production and no fixed costs.[5] Consequently, it is reasonable to suppose that the price quotations provided by the suppliers are piece rates.

Let $c_i > 0$ denote the procurement cost of the part for each unit ordered from supplier i. My analysis starts from the price quotations, c_i's, provided to the buyer and focuses on the quota allocation given the price quotations. Nesting the model into an extended setting to analyze the strategic pricing of the suppliers is left for future research. In section 4, I explicitly model an asymmetric information setting where a buyer-initiated procurement contract can be used to elicit the private information on the suppliers'

[5] In accounting, constant marginal cost is arguably a more widely accepted assumption on cost behavior than increasing or decreasing marginal cost. For example, in product costing, the variable cost per unit is typically assumed to be constant within the relevant range of production. Zero fixed cost, though uncommon, is a justifiable assumption if accepting the procurement order is an incremental production decision falling within the relevant range of production of a supplier.

unit variable production costs. With endogenously determined procurement costs in this setting, my results continue to hold if the roles of c_i's are substituted by the *virtual* unit variable production costs (to be defined in section 4).

The design of the component part is believed to be very robust under a wide range of operating conditions. Still, it is not infallible. Whether a field failure of the component part will occur depends on whether it has a latent defect. By definition, such a defect cannot be discovered by inspection prior to sale. For simplicity, I assume that if a latent defect exists in a component part, it will surely reveal itself through a field failure. The chance of having a defective part depends on the buyer's design quality level, as well as the production quality of the supplier that manufactures the part.

Specifically, let D_i denote the amount of defective parts in the Q_i units of parts produced by supplier i. Defects occur in a manner following stochastically proportional yield loss, i.e., $D_i = R_i\delta_i Q_i$. The random variables R_i's are independently and identically distributed with mean $E(R_i) = \mu$, where $0 < \mu \leq \bar{\mu} << 1$, and variance $\text{var}(R_i) = \sigma^2 > 0$. Intuitively, $R_i\delta_i$ may be referred to as the *random yield loss*, although for convenience I sometimes use this to refer to R_i alone.

Besides the variable μ that characterizes the buyer's design quality level, the random yield loss also depends on the parameter δ_i, where $0 < \delta_i \leq 1$. I refer to $1-\delta_i$ as the *quality-based scoring index* of supplier i, or simply its *quality* level. For analytical convenience, assume δ_i's are distinct. Without loss of generality, I suppose $\delta_1 < \delta_2 < \ldots < \delta_n$ so that supplier 1 has the highest quality level, and other suppliers are ordered accordingly. Note that although self-reported information collected from the suppliers (see, e.g., footnote 4) may be used to determine the parameters δ_i's, potentially biased information can be crossed-checked with independent observations from other sources, e.g., direct visits at suppliers' production facilities. Therefore, the δ_i's should be interpreted as the ultimate figures used after adjusting for strategic concerns not explicitly modeled here, rather than self-reported figures taken at face value.

Suppose the latent defect described above is the only sort of defects that could occur in the buyer's or a supplier's production process. Thus, the total quantity ordered by the buyer is always the same as the total quantity delivered by the suppliers. The buyer sells all Q units of finished product to end customers. Denote by $D = \sum_{i=1}^{n} D_i$ the total amount of defective products sold. Each customer with a defective product will experience a field failure and take the product back for warranty repair. Knowing that the product has a defective part, the buyer finds it in its best interest to apply a costly remedy to address the problem. For simplicity, I assume the fix is effective: after the warranty repair, the product will not fail again.

Suppose that all warranty related costs are proportional to the amount of products returned for repair, i.e., ωD, where $\omega > 0$. Moreover, the defective products will lead to customer dissatisfaction and eventually some additional external failure costs borne by the buyer.[6] In particular, I suppose that such costs are primarily due to reputation damage. I argue that it is increasingly more likely to catch the attention of mass media as D grows, and the resulting reputation damage is increasingly more detrimental. This feature is captured by the analytically tractable assumption of a quadratic *other external failure cost* function: $C_E(D) = c_E D^2$, where $c_E > 0$.

Summing up this and the warranty related costs gives the total external failure cost $\omega D + c_E D^2$. Because the increasing marginal cost of C_E is with respect to D (i.e., the sum of the D_i's), not its individual constituents, the quadratic functional form does not by itself favor multiple sourcing.[7]

To see how the allocation of the production quotas to the suppliers may affect the external failure costs to the buyer, one can examine the expected value of $\omega D + c_E D^2$ below (see the appendix for the derivation):

$$E[\omega D + c_E D^2] = \omega\mu[\textstyle\sum_{i=1}^{n} \delta_i Q_i] + c_E\mu^2[\textstyle\sum_{i=1}^{n} \delta_i Q_i]^2 + c_E\sigma^2[\textstyle\sum_{i=1}^{n} \delta_i^2 Q_i^2].$$

Interestingly, although the expected external failure cost is a second-order polynomial of Q_i's, sharing the production target Q among the suppliers cannot reduce the cost unless $\sigma^2 > 0$. If instead the "random yield losses" R_i's were deterministic, the third term in $E[\omega D + c_E D^2]$ would vanish. In that case, assigning all the Q to the supplier with the lowest δ_i would minimize the expected external failure costs.

Summarized below is the event sequence of the model:

1. The suppliers provide price quotations on the per-unit procurement costs c_i's they will charge to the buyer.
2. The buyer outsources the production of the component part to the suppliers, with a production quota Q_i allocated to supplier i.
3. Suppliers manufacture the parts according to the allocated quotas. The ordered amounts, Q_i's, are delivered to the buyer. Out of the amount Q_i, $D_i = R_i\delta_i Q_i$ are defective parts. (The values of D_i's are unobservable to the buyer until later.)

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[6] Though not critical in this study (but can be important to nesting the model into an extended setting to examine related issues), the suppliers are assumed to have no external failure cost. This is in line with the belief that "[a]utomakers generally don't like to focus on their suppliers, or even identify them" (Healey 2010). When identifying the involved suppliers is unavoidable, e.g., due to the US National Highway Traffic Safety Administration (NHTSA) requirements, it is believed that the suppliers "almost always avoid attention, even when their name is in the public record" (Szczesny 2010). A legal professional also commented that "[s]uppliers usually aren't part of legal actions unless the automaker can't be sued. … It's harder to prove fault at suppliers, which often build parts to specifications and aren't involved in testing them[.]" (Green and Ramsey 2010).

[7] To see this, imagine the hypothetical case where the proportions of D_i's in D could be directly controlled by the buyer. Then it is clear that as long as the total, D, remained constant, there could not be any gain or loss from shifting among the D_i's. Even though D_i's actually are random variables, multiple sourcing cannot create any benefit unless the randomness of D_i's resulting from the supplier failure risk is diversifiable. This last point will become clear shortly below.

4. The buyer uses the parts to manufacture the finished products and sells them to end customers.

5. Customers who purchased the $D = \sum_{i=1}^{n} D_i$ units of defective products eventually experience field failures and return the products for warranty repair. The values of D_i's become known to the buyer. The buyer incurs warranty related expenses of ωD and suffers reputation damage equivalent to a dollar cost of $C_E(D) = c_E D^2$.

3. Optimal Quota Allocation: Single versus Multiple Sourcing

The buyer's problem is to choose a production quota allocation $\mathbf{Q} = (Q_i)_{i\in N}$ to minimize its expected total cost of sourcing. The allocation must specify quotas that sum to the required procurement quantity Q. The optimization program below formally summarizes the buyer's sourcing problem:

[SB] $$\min_{Q_i} \sum_{i=1}^{n} c_i Q_i + \omega\mu[\sum_{i=1}^{n} \delta_i Q_i] + c_E\mu^2[\sum_{i=1}^{n} \delta_i Q_i]^2 + c_E\sigma^2[\sum_{i=1}^{n} \delta_i^2 Q_i^2]$$

subject to

QC: $$\sum_{i=1}^{n} Q_i = Q$$

with non-negative Q_i's. The choice of the production quotas Q_i's indirectly determines the *supply base* $B = \{ i\in N \mid Q_i > 0 \}$, i.e., the set of suppliers selected by the buyer. Let b denote the *size* of the supply base, i.e., the number of selected suppliers. The type of a supply base, B, is said to be *single sourcing* if $b = 1$ and *multiple sourcing* if $b \geq 2$. In particular, if $b = 2$, B is also said to be of the *dual sourcing* type.

To simplify the expression of the buyer's expected total cost of sourcing, I define the following notations:

$$\eta \equiv (\mu/\sigma)^2$$

$$s_i \equiv (c_i+\omega\mu\delta_i)/c_E\sigma^2$$

The parameter η is the *squared standardized mean* of the "random yield loss" R_i, which means η^{-1} is the *squared coefficient of variation*. The parameter s_i is a ratio representing the relative unimportance of the quadratic other external failure cost, characterized by c_E, in constituting the buyer's expected total cost. With these notations, the expected total cost of sourcing can be expressed as

$$C(\mathbf{Q}; \mathbf{s}) \equiv c_E\sigma^2[\sum_{i=1}^{n} s_i Q_i + \eta(\sum_{i=1}^{n} \delta_i Q_i)^2 + \sum_{i=1}^{n} \delta_i^2 Q_i^2],$$

where $\mathbf{s} = (s_i)_{i\in N}$. Clearly, $C(\mathbf{Q}; \mathbf{s})$ is strictly convex in $\mathbf{Q}$ and linear in $\mathbf{s}$. Another, even more intuitive, way to express $C(\mathbf{Q}; \mathbf{s})$ is

$$\sum_{i=1}^{n} c_i Q_i + C_X(\mathbf{Q}),$$

where the first term is the outlay procurement cost and the second term $C_X(\mathbf{Q}) \equiv C(\mathbf{Q};(\omega\mu/c_E\sigma^2)\delta)$, with $\delta = (\delta_i)_{i\in N}$, is the expected external failure cost. The functions $C(\mathbf{Q}; \mathbf{s})$ and $C_X(\mathbf{Q})$ will appear again in section 4 where I model the suppliers' private information on their production costs explicitly.

Whether multiple sourcing has advantages over single sourcing, or the other way around, depends on the optimal choice of Q_i's and the resulting size of B. The following result tells us when the protection advantage of multiple sourcing may fail to exist.

PROPOSITION 1 (CONDITIONS FOR NON-EXISTENCE OF PROTECTION ADVANTAGE OF MULTIPLE SOURCING): *Suppose the ascending ranking of the suppliers based on s_i also has supplier 1 ranked highest. Or, alternatively, suppose that for any distinct j and k with $(\delta_k - \delta_j)(s_k - s_j) \leq 0$,*

$$(s_j - s_k)/(\delta_k - \delta_j) < 2\eta\delta_1 Q.$$

Then multiple sourcing has no advantage over single sourcing if one of the following holds:

(a) *The variance of the "random yield loss" is negligible, i.e., $\sigma^2 \to 0$;*

(b) *The marginal other external failure cost is negligible, i.e., $c_E \to 0$.*

The result of this proposition needs one of two preconditions: either that both the s_i-based and δ_i-based rankings have supplier 1 ranked highest, or that whenever they differ in ranking suppliers j and k, the cardinal difference $s_j - s_k$, relative to $\delta_k - \delta_j$, is not too large. When one of these preconditions holds, the reason for multiple sourcing to be advantageous comes solely from the protection against supplier failure risk due to latent defects. Multiple sourcing becomes unattractive if there is little risk to protect against, or the benefit (i.e., external failure cost saved) from the protection is tiny.

Two model elements contributing to the protection advantage of multiple sourcing are highlighted by the proposition. The first is about the nature of the supplier failure risk. If the "risk" is not due to variation in the yield but merely about not knowing which unit is defective (e.g., the fully predictable yields assumed in Burke et al. 2007), there might not be room for risk diversification by multiple sourcing.

The second element is related to the nature of the external failure costs considered here. With genuine random yields, the quadratic other external failure cost induces a desire for spreading the supplier failure risk by multiple sourcing. If the marginal other external failure cost is negligible, the buyer will remain "risk-neutral" to the risk. The protection advantage of multiple sourcing therefore cannot exist.

Below I will characterize the unique optimal production quota allocation for the buyer's sourcing problem, identify some intuitive features of the allocation, and derive necessary and sufficient conditions for determining the exact size of the optimal supply base and whether single or multiple sourcing is optimal. These results are stated in terms of the *quality-adjusted cost-based scoring index* defined as follows:

$$S_i\,(W^*) \equiv \left[\frac{c_i + (\omega + 2c_E\mu\, W^*)\mu\delta_i}{c_E\sigma^2}\right].$$

It will be clear shortly that the μW^* in the index is simply the expected number of external failures given the optimally allocated production quotas.

If suppliers' qualities are nearly identical (i.e., δ_i's are almost the same), the ranking by $S_i(W^*)$ is not much different from that by c_i. Alternatively, if production costs are equal, $S_i(W^*)$ and δ_i give the same ranking. Even when the costs are unequal, the ranking by $S_i(W^*)$ and by δ_i can still be the same, provided c_i's are "not too unequal." Specifically, this means

$$\max_{i\in\{2,\ldots,n\}}\left(\frac{c_{i-1}-c_i}{\delta_i-\delta_{i-1}}\right)\leq(\omega+2c_{\mathrm{E}}\mu\delta_1 Q)\mu.$$

In words, the condition requires that the cost saving from using a lower-quality supplier is not too attractive given the loss in quality.

The first major result below characterizes the optimal quota allocation without imposing the condition of "not too unequal" costs. Subsequently, it is added to put more structure on the optimal quota allocation.

PROPOSITION 2 (OPTIMAL QUOTA ALLOCATION): *A quantity vector* $\mathbf{Q}^* = (Q_i^*)_{i\in N}$ *is the unique optimal quota allocation for the buyer's sourcing problem if and only if for some* $\theta^* > 0$*, the following marginal conditions are satisfied:*

$$Q_i^* \geq \left[\frac{\theta^* - S_i(W^*)}{2\delta_i^2}\right] \qquad \text{for all } i \in N$$

with the equality holding for all i's in the supply base $B^* \equiv \{\, i\in N \mid Q_i^* > 0 \,\}$, *where*

$$S_i\,(W^*) \equiv \left[\frac{c_i + (\omega + 2c_{\mathrm{E}}\mu\ W^*)\mu\delta_i}{c_{\mathrm{E}}\sigma^2}\right]$$

is a quality-adjusted cost-based scoring index and θ^* *and* W^* *are given by the following formulas*:

$$\theta^* \;=\; \frac{(\eta^{-1}+b^*)[2Q+\sum_{B^*}(s_l/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)][\sum_{B^*}(s_l/\delta_l)]}{(\eta^{-1}+b^*)[\sum_{B^*}(1/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)]^2} \tag{1}$$

$$W^* \;=\; \frac{[\sum_{B^*}(1/\delta_l)][2Q+\sum_{B^*}(s_l/\delta_l^2)] - [\sum_{B^*}(1/\delta_l^2)][\sum_{B^*}(s_l/\delta_l)]}{2\eta[\,(\eta^{-1}+b^*)[\sum_{B^*}(1/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)]^2\,]}\;, \tag{2}$$

with $b^* \equiv |B^*|$ *denoting the size of the supply base,* $\eta \equiv (\mu/\sigma)^2$ *denoting the squared standardized mean of the "random yield loss"* R_i*, and* $s_i \equiv (c_i+\omega\mu\delta_i)/c_{\mathrm{E}}\sigma^2$*. Moreover,* $W^* = \sum_{B^*}\delta_l Q_l^* = \sum_{i=1}^{n}\delta_i Q_i^*$.

This proposition provides a closed-form characterization of the unique optimal quota allocation $\mathbf{Q}^*$. Once the supply base B^* is determined, the optimal quota for a selected supplier i can be computed with the simple formula

$$Q_i^* = \left[\frac{\theta^* - S_i(W^*)}{2\delta_i^2} \right]$$

whose key constituents, θ^* *and* W^*, are given by another two formulas specified in the proposition. Although the procedure is straightforward, determining the optimal combination of suppliers to constitute the supply base can be prohibitively complex. This is especially so when the number of available suppliers is large. In a different but related setting, Federgruen and Wang (2008) show that a similar combinatorial optimization problem of supplier selection is NP-complete.

However, suppose that the ranking of the suppliers by the *quality-adjusted cost-based scoring index* $S_i(W^*) = [c_i + (\omega+2c_E\mu W^*)\mu\delta_i]/c_E\sigma^2$ is the same as that by δ_i. This would be the case if the differences among the costs c_i's are sufficiently small and the marginal other external failure cost c_E, or the production target Q and hence $W^* = \sum_{i=1}^{n} \delta_i Q_i^*$, is sufficiently large. Under such circumstances, the weight attached to the second component of $S_i(W^*)$ will be large enough to let δ_i dominate this scoring index. Consequently, the unique optimal quota allocation $\mathbf{Q}^*$ will have some simple, intuitive properties.

The first property is a positive association between the quality of a selected supplier and the quota assigned to it. That is to say, the higher the quality of a supplier (i.e., with a lower δ_i), the (weakly) larger the quota assigned to it. As a result, if a supplier is selected into the supply base, any suppliers of higher quality will also be selected. These intuitive properties are defined below formally.

DEFINITION 1: The supply base B^* of the optimal quota allocation $\mathbf{Q}^*$ is (*weakly*) *quality-driven* if the selection of a supplier into the supply base implies also the selection of any higher-quality suppliers, i.e., $j \in B^* \Rightarrow j-1 \in B^*$ for all $j \in B^* \backslash \{1\}$, or equivalently $B^* = \{1, 2, \ldots, b^*\}$.

DEFINITION 2: The supply base B^* of the optimal quota allocation $\mathbf{Q}^*$ is *strongly quality-driven* if the quota allocated to a supplier is at least as high as those allocated to any lower-quality suppliers, i.e., $Q_j^* \leq Q_{j-1}^*$ for all $j \in B^* \backslash \{1\}$.

Obviously, a strongly quality-driven B^* is also (weakly) quality-driven but not necessarily the other way around.

PROPOSITION 3 (QUALITY-DRIVEN SUPPLY BASE): *Suppose* $\max_{i \in \{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \leq (\omega + 2c_E\mu\delta_1 Q)\mu$. *Then the supply base* B^* *of the optimal quota allocation* $\mathbf{Q}^*$ *is strongly quality-driven, i.e.,* $Q_j^* \leq Q_{j-1}^*$ *for all* $j \in B^* \backslash \{1\}$. *Consequently, it is also (weakly) quality-driven, i.e.,* $B^* = \{1, 2, \ldots, b^*\}$.

Despite no fixed costs for selecting more suppliers, expanding the supply base can be costly because it means using suppliers of lower quality than the incumbent ones. The increase in this cost as lower-quality suppliers are included into the supply base eventually may limit its size. An interesting question to ask is when it will stop at the size of one (i.e., single sourcing is optimal) and when it will stop at a size larger than one (i.e., multiple sourcing is optimal). The following results shed some light on this question.

PROPOSITION 4 (SUFFICIENT AND NECESSARY CONDITION FOR DETERMINING THE SIZE OF THE OPTIMAL SUPPLY BASE): *Suppose* $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \le (\omega + 2c_E\mu\delta_1 Q)\mu$. *Then the size of the (smallest) optimal supply base is j (i.e.,* $b^* = j$*), where* $j \in N\backslash\{n\}$*, if and only if there exist positive* θ_j *and* W_j *defined by formulas (1) and (2) with* B^* *substituted by* $B_j \equiv \{1, \ldots, j\}$ *such that*

$$[\theta_j - S_j(W_j)]/2\delta_j^2 > 0 \ge [\theta_j - S_{j+1}(W_j)]/2\delta_{j+1}^2,$$

where $S_i(W) \equiv [c_i + (\omega+2c_E\mu W)\mu\delta_i]/c_E\sigma^2$. *If the condition above is not satisfied by any* $j < n$, $b^* = n$.

This proposition provides a characterization of b^*, the size of the optimal supply base. Determining b^* can be rather complex owing to the combinatorial nature of the supplier selection problem. However, if the precondition of the proposition holds, i.e., the differences between consecutive c_i's are "not too large," then the problem of determining the size of the optimal supply base can be reduced to simply comparing the n quality-driven supply bases, i.e., $B_j \equiv \{1, \ldots, j\}$ for $j \in N$. This comparison only requires solving for each j a linear equation system with two unknowns, i.e., θ_j *and* W_j, and then search for the j with the lowest positive value of $[\theta_j - S_j(W_j)]/2\delta_j^2$. Such a problem takes much less time to solve than the original problem.

Suppose one only needs to determine whether single sourcing or multiple sourcing is optimal, without actually identifying the size of the optimal supply base for the latter case. Then the problem can be simplified even without a precondition. Why? Proposition 2 already provides the necessary and sufficient condition for checking whether any given supply base B is optimal. For the case of single sourcing, there are only n such candidate supply bases to check. If none of them is optimal, the optimal arrangement must be multiple sourcing. To check whether a singleton supply base is optimal, it does not require the remaining unselected suppliers to be lined up in certain orders. One only needs to ensure that the buyer cannot be better off by shifting some quota away from a candidate single-sourcing supplier. This result is the next proposition.

PROPOSITION 5 (SUFFICIENT AND NECESSARY CONDITION FOR SINGLE SOURCING TO BE OPTIMAL): *Let* $\theta^h \equiv s_h + 2(1+\eta)Q\delta_h^2$ *and* $W^h \equiv \delta_h Q$ *for all* $h \in N$. *Then single sourcing is optimal if and only if there exists* $h \in N$ *such that*

$$\max_{i\in N\backslash\{h\}} [(\theta^h - S_i(W^h))/2\delta_i^2] \le 0,$$

where $S_i(W) \equiv [c_i + (\omega+2c_E\mu W)\mu\delta_i]/c_E\sigma^2 = s_i + 2\eta W\delta_i$ and $s_i \equiv (c_i+\omega\mu\delta_i)/c_E\sigma^2$. *The h satisfying the condition above is the only selected supplier of the single-sourcing supply base.*

In the necessary and sufficient condition of this proposition, there is no counterpart of the $[\theta_j - S_j(W_j)]/2\delta_j^2 > 0$ required in Proposition 4. The reason is that such a requirement is automatically satisfied for the case of single sourcing. Using the definitions of θ^h and W^h, it is easy to verify that $[\theta^h -$

$S_h(W^h)]/2\delta_h^2 = Q > 0$, regardless of the $h \in N$ in concern. The requirement of $0 \geq [\theta_j - S_{j+1}(W_j)]/2\delta_{j+1}^2$ in Proposition 4 as well as its precondition $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \leq \omega\mu + 2\delta_1 Q c_E \mu^2$ are substituted by the condition that $\max_{i\in N\setminus\{h\}} [(\theta^h - S_i(W^h))/2\delta_i^2] \leq 0$ for some h. While this looks more complicated, it does not impose any ordering on the c_i's or restrictions on their differences. In terms of computational complexity, checking the condition requires calculating $n-1$ values of $[(\theta^h - S_i(W^h))/2\delta_i^2]$ for each $h \in N$. This amounts to a total of $n(n-1)$ calculations, a more complicated task but still manageable within a reasonable time.

I end this section with the following comparative statics result on the optimal quota allocation characterized in Proposition 2. This is a general result without requiring "not too unequal" costs. It confirms the intuition of reducing the quota allocated to a supplier if it charges a higher cost. Beyond this, the result also paves the road for proving a main result in the next section for the asymmetric information setting with buyer-initiated procurement contracts.

PROPOSITION 6 (NON-INCREASING RESPONSE OF A SUPPLIER'S QUOTA TO THE COST IT CHARGES): *For any given* $\mathbf{c}_{-i} = (c_j^*)_{j\in N\setminus i}$ *charged by other suppliers, the optimal production quota* Q_i^* *allocated to supplier i is non-increasing in the cost* c_i *charged by the supplier*.

In proving this proposition, it would be nice if some function involved is differentiable. However, this is not obvious. To get around the problem, I apply the technique of supermodularity. This makes the proof simple even without differentiability.

4. Buyer-initiated Procurement Contracts with Privately Known Production Costs

So far the buyer is supposed to take the per-unit procurement costs charged by the suppliers as given and allocate production quotas accordingly. In this section, I continue to assume the quality parameters are known to the buyer through a prior supplier certification process (not modeled here). However, owing to the changing environments specific to the individual suppliers, they have private information on their unit variable costs of production. I consider a setting whether the buyer can use full-commitment, take-it-or-leave-it contracts to elicit the private information from the suppliers. The basic setup remains the same as that in the previous section, except that the roles of the procurement costs previously taken as given are now substituted by the contractual payments specified in the optimal procurement contracts.

Let v_i denote the unit variable cost of production privately known to supplier i. This is the type of the supplier. For analytical simplicity, independent types are assumed, which means v_i's are independently drawn from the probability distribution functions $F_i(v_i)$'s. For expositional simplicity, I assume $F_i(v_i) = F(v_i)$ with the probability density function $f(v_i) > 0$ for all $v_i \in [\underline{v}, \overline{v}]$, where $0 < \underline{v} < \overline{v} < \infty$, and $f(v_i) = 0$ otherwise. I also assume that $f(v_i)$ is continuous on $[\underline{v}, \overline{v}]$. Despite the symmetric assumption on $F(v_i)$'s, the model is still asymmetric because of the unequal quality parameters assumed. The analysis below can easily be extended to the more general setting with asymmetric $F_i(v_i)$'s.

Invoking the revelation principle (Myerson 1981), the buyer only needs to focus on direct-revelation contracts in searching for an optimal procurement arrangement on the quota allocation and payments. With only adverse selection and risk neutral parties, there is no gain to make the contracts contingent on the numbers of defective items D_i's. Moreover, because of the strictly convex expected external failure cost $C_X(\mathbf{Q})$ and the risk neutrality in payments, the buyer can confine to deterministic contracts without loss of generality.

Let $m = (m_i)_{i\in N} \in [\underline{v}, \overline{v}]^n$ be the profile of reports submitted by the suppliers. For any report m, let $Q_i(m)$ and $P_i(m)$ denote the allocation and payment to supplier i, respectively, if m is received. I assume that the allocation and payment rules $Q_i(\bullet)$ and $P_i(\bullet)$ are continuous and non-negative functions. A feasible procurement contract is a profile of allocation and payment rules $\langle \mathbf{Q}(\bullet), \mathbf{P}(\bullet)\rangle$, where $\mathbf{Q}(\bullet) = (Q_i(\bullet))_{i\in N}$ and $\mathbf{P}(\bullet) = (P_i(\bullet))_{i\in N}$, such that they satisfy the incentive compatibility and individual rationality constraints, in addition to the quota constraint:

IC(v_i, m_i):	$\mathrm{E}\pi_i(v_i) \;\geq\; \mathrm{E}\pi_i(m_i, v_i)$		for all $i \in N$, $v_i \in [\underline{v}, \overline{v}]$, and $m_i \in [\underline{v}, \overline{v}]$
IR(v_i):	$\mathrm{E}\pi_i(v_i) \;\geq\; 0$		for all $i \in N$ and $v_i \in [\underline{v}, \overline{v}]$
QC(v):	$\sum_{i=1}^{n} Q_i(v) \;=\; Q$		for all $v = (v_i)_{i\in N} \in [\underline{v}, \overline{v}]^n$,

where $\mathrm{E}\pi_i(v_i) = \mathrm{E}\pi_i(v_i, v_i)$, $\mathrm{E}\pi_i(m_i, v_i) = \mathrm{E}_{-i}[\pi_i(m_i, v_i)]$, with the expectation $\mathrm{E}_{-i}[\bullet]$ taken over $v_{-i} = (v_j)_{j\in N\setminus i}$, and $\pi_i(m_i, v_i) = P_i(m_i, v_{-i}) - v_iQ_i(m_i, v_{-i})$.

The following result completely characterizes the set of feasible procurement contracts.

LEMMA 1 (FEASIBLE PROCUREMENT CONTRACTS): *A procurement contract* $\langle \mathbf{Q}(\bullet), \mathbf{P}(\bullet)\rangle$ *is feasible if and only if the following conditions hold: For any* $i \in N$, $v_i \in [\underline{v}, \overline{v}]$, *and* $m_i \in [\underline{v}, \overline{v}]$,

(i) $m_i \geq v_i \;\Rightarrow\; \mathrm{E}_{-i}[Q_i(v_i, v_{-i})] \geq \mathrm{E}_{-i}[Q_i(m_i, v_{-i})]$, *i.e., exaggerating v_i results in less quota allocation on average;*

(ii) $\mathrm{E}\pi_i(v_i) = \mathrm{E}\pi_i(\overline{v}) + \int_{v_i}^{\overline{v}} \mathrm{E}_{-i}[Q_i(t_i, v_{-i})]dt_i$, *i.e., a lower type's expected payoff is the highest type's plus an information rent;*

(iii) $\mathrm{E}\pi_i(\overline{v}) \geq 0$, *i.e., it suffices to ensure only the highest type's individual rationality constraint;*

(iv) $\sum_{i=1}^{n} Q_i(v) = Q$, *which is the quota constraint.*

It turns out that what determines the minimized total expected cost of sourcing is only the quota allocation rules. Given the optimal allocation rules determined, payment rules can always be constructed to fulfill all the feasibility conditions in Lemma 1 that ensure incentive compatibility and individual rationality. It will also be clear that the buyer's expected total cost of sourcing under this asymmetric information setting is basically the same as that in the previous section with the per-unit procurement costs c_i's taken as given. The key difference is the replacement of c_i by the *virtual* unit variable cost of production defined as follows:

$$p(v_i) \equiv v_i + F(v_i)/f(v_i).$$

Below is the lemma summarizing these findings.

LEMMA 2 (SUFFICIENT CONDITION FOR OPTIMAL PROCUREMENT CONTRACTS): *Let* $\boldsymbol{\gamma}(v) = ((p(v_i)+\omega\mu\delta_i)/c_{\mathrm{E}}\sigma^2)_{i\in N}$ *with* $p(v_i) = v_i + F(v_i)/f(v_i)$. *Suppose that given* $\boldsymbol{\gamma}(v)$, *the allocation rule profile* $\mathbf{Q}^*(\bullet)$ *minimizes the expected total cost of sourcing* $\mathrm{E}[C(\mathbf{Q}(v); \boldsymbol{\gamma}(v))]$ *subject to the quota constraint* $\sum_{i=1}^{n} Q_i(v) = Q$ *for all* $v \in [\underline{v}, \overline{v}]^n$. *Then* $\langle \mathbf{Q}^*(\bullet), \mathbf{P}^*(\bullet)\rangle$ *is an optimal procurement contract if for all* $i \in N$,

(a) the allocation rule $Q_i^*(v_i, v_{-i})$ *is non-increasing in* v_i *given* v_{-i}, *and*

(b) the payment rule is $P_i^*(v) = v_iQ_i^*(v) + \int_{v_i}^{\overline{v}} Q_i^*(t_i, v_{-i})dt_i$.

The last main result below shows that all the findings in the previous section continue to hold under asymmetric information on the unit variable production costs v_i's, provided that the role of the per-unit procurement cost previously taken as given is substituted by the virtual unit variable production cost $p(v_i)$ with $p(v_i)$ being non-decreasing in v_i. This last condition is satisfied if the hazard rate $F(v_i)/f(v_i)$ is a non-decreasing function.

PROPOSITION 7 (OPTIMAL PROCUREMENT CONTRACT): *Suppose for any realization* $v \in [\underline{v}, \overline{v}]^n$, $\mathbf{Q}^*(v)$ *is a quota allocation satisfying the conditions in Proposition 2 with* $c_i = p(v_i) \equiv v_i + F(v_i)/f(v_i)$. *If* $p(v_i)$ *is non-decreasing in* v_i, *then* $\langle \mathbf{Q}^*(\bullet), \mathbf{P}^*(\bullet)\rangle$, *with* $P_i^*(v) = v_iQ_i^*(v) + \int_{v_i}^{\overline{v}} Q_i^*(t_i, v_{-i})dt_i$, *is an optimal procurement contract. Furthermore, if the variation in* v *is sufficiently small, e.g.,* $p(\overline{v}) - p(\underline{v}) \le (\omega + 2c_{\mathrm{E}}\mu\delta_1 Q)\mu[\min_{i\in\{2,\ldots,n\}} (\delta_i - \delta_{i-1})]$, *then the optimal quota allocation* $\mathbf{Q}^*(v)$ *is strongly quality-driven for any realized* v.

5. Discussions: Importance of Quality Costs and Related Recent Studies

Quality, in particular product safety, is an important factor to consider in supplier selection decisions. Supporting this view, 78% of the senior executives participating in a survey answered that product safety is among the greatest risks in relation to the integrity of their supply chains. In addition, 68% of them cited quality as the main risk of global sourcing (PricewaterhouseCoopers 2008).

Recently, product safety concerns related to global sourcing have hit the headlines repeatedly. The most noticeable cases are recalls of unsafe products manufactured by suppliers in China (more details given in the appendix). For instance, Mattel recalled 436,000 Chinese-made toy cars covered with lead paint in 2007 (Story and Barboza 2007). Other recent recalls for unsafe products include a drug, namely heparin, and dairy products. In both cases, the products were tainted by contaminants structurally so similar to the real ingredients that they could not have been distinguished by routine tests. Such latent defects, like the use of lead paint, are flaws affecting voluminous amounts of products and yet unforeseeable or undiscoverable at the time of buying from the suppliers. By the time the flaws are noticed, the defective products have reached the hands of end customers, or may even have caused irreversible damages. Quality costs to the buying companies of such defective products are no doubt an essential factor to consider in forming the supply base.

Although recent cases of massive recalls often involve suppliers from low-cost countries like China, product quality problems are a universal business issue rather than exclusive to sourcing from developing countries. For example, alongside with the recall for lead-paint toys in 2007, Mattel also recalled 18.2 million magnetized toys made in China following Mattel's design specification (Story and Barboza 2007). In other words, any high-quality suppliers, western or Asian, would have made these unsafe toys designed by Mattel, the world's largest toy company.

Other well-known companies from developed countries have also recalled defective products for safety reasons. For instance, in 2000 Bridgestone/Firestone recalled 6.5 million tires that seemed to have an unusually high risk of tread failures (Bridgestone/Firestone 2000). Tires with this defect were linked to crashes killing 271 people and causing over 800 injuries (NHTSA 2001). Many of the tires were installed on Ford Motor's vehicles because Firestone is a long-term major supplier of Ford. To restore customer confidence, Ford later announced the replacement of about 13 million Firestone tires installed on Ford vehicles by non-Firestone brands (Bradsher 2001). This move ended the nearly century-long buyer-supplier relationship between Ford and Firestone (Hakim 2004).[8]

Regardless of low-cost suppliers from China or well-known suppliers from the western world, latent defects may exist in their products. The quality costs to the buying companies are enormous. Mattel estimated that the cost of recalling 1.5 million toys coated with toxic levels of lead paint could amount $30 million (Barboza 2007). Ford expected to spend $3 billion in a nine-month program to replace millions of Firestone tires causing customer safety concerns (Bradsher 2001, Hakim 2004). Besides the loss of sale and other quality costs that can be estimated, damage claims in defect liability lawsuits are difficult to

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[8] Besides Bridgestone, other first-tier suppliers in the automotive sector also are involved in recalls of unsafe products. For example, "Chrysler said the cause of [a steering-related] defect was a change by its axle supplier Dana Corp. [in the manufacturing process]" (Shepardson 2011). Another example is that "a defective [module] was found at a Chrysler instrument panel supplier[, TRW Automotive Electronics Group,] and "[i]n the event of a crash, the driver's airbag will not deploy" (Rudnitsky Law Firm 2009). A third example is Federal-Mogul's recall of replacement control arm assemblies for Chrysler vehicles (Seetharaman 2010). Dana, TRW, and Federal-Mogul are Tier 1 suppliers in the automotive sector, according to Ernst & Young (2008, p. 3).

assess.

Despite the importance of the product safety issue arising from latent defects, models analyzing the protection advantage of multiple sourcing have concentrated on supplier failures due to unreliably supply. In this paper, I use a latent-defect model with non-linear external failure costs to capture the sort of supplier failure risk relevant to the product safety issues discussed above. To highlight the benefit from risk diversification as a reason for multiple sourcing, the model assumes away other reasons such as capacity constraints. Moreover, certainty in lead time and delivery is assumed to avoid overlapping with prior studies' emphases.[9] To focus on the choice of single versus multiple sourcing, the model also abstracts away from other aspects of supply chain management already extensively studied in the literature, such as coordination for information sharing. (See Kouvelis, Chambers, and Wang 2006 for a review of the supply chain management literature and Cachon 2003 for a review specifically on supply chain coordination with incentive contracts.)

I take the perspective of viewing the choice of single versus multiple sourcing as a supply base composition problem. The question asked is about what combination of suppliers can diversify the risk of latent defects most efficiently, in terms of the incremental quality cost to pay as a sacrifice. Instead of formulating a very general model that requires combinatorial mathematical techniques to solve, I structure the model in a tractable stylized fashion, yet rich enough to capture the fundamental economic tradeoff.[10]

Recently, Federgruen and Yang (2008) has examined a general setting of the supplier selection problem that also emphasizes the optimal combination of suppliers. They show that this is an NP-complete combinatorial optimization problem. Their focus is to develop an accurate approximation method to overcome the computational complexity of the problem. Dada et al. (2007) and Federgruen and Yang (2009) have also considered similar general settings, with a focus on characterizing and solving the supplier selection problem with random yields and uncertain demand.

In contrast, I consider a simple setting with no random yields nor uncertain demand but only latent defects. Complementing prior studies, this paper emphasizes the linkage between supplier selection and

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[9] Reasons for favoring single or multiple sourcing that have been studied include supplier capacity constraints, saving in outgoing order/incoming inspection/transportation costs, saving in inventory holding costs by shortening the delivery lead time, quantity discounts due to production scale economies, saving in purchasing costs by supplier competition (with or without information asymmetry), and encouraging investment by suppliers to reduce production costs or to improve product quality. These reasons have been examined extensively in economic and operations research/management science (OR/MS) studies. Concise reviews of related OR/MS studies can be found in Berger and Zeng (2006) and Mishra and Tadikamalla (2006). A review of related studies in economics is provided in the appendix.

[10] Other studies in the quality management literature have focused on incentive contracting related to a variety of quality improvement arrangements like vender certification, incoming inspection, and product recall/warranty cost sharing (e.g., Balachandran and Radhakrishnan 2005, Hwang et al. 2006, Chao et al. 2009, Baiman et al. 2000, 2001). These models have only one supplier without considering multiple sourcing and supplier failure risk diversification.

external failure costs. My model allows a closed-form characterization of the optimal quota allocation through which the fundamental economic driving forces can be clearly seen.

6. Concluding Remarks

Challenged by the success of Japanese manufacturers, western firms have adopted Japanese management methods, most notably just-in-time (JIT) operations and total quality management (TQM).[11] These methods are inter-related. For example, excellent supplier relations are important to both JIT operations and TQM.[12] Successful implementation of these methods is impossible unless suppliers can commit to timely deliveries of defect-free items. Given the substantial rework required, timely deliveries of defective items are not obviously better than late deliveries of defect-free items. Thus, quality is an important factor in supplier selection even when delivery is taken into consideration.

In selecting suppliers, a key issue is the choice of single versus multiple sourcing. Relying on only a single supplier, or just a few, in sourcing an item is a distinct characteristic of buyer-supplier relationships in Japan. This is also believed to be crucial to JIT implementation and the success of Japanese companies (see discussions in Asanuma 1989, Cusumano and Takeishi 1991, and Nishiguchi and Anderson 1995 for further details). Richardson (1993) notes that although U.S. quality experts (e.g., Deming 1986) recommend single sourcing, the traditional practice in the western world and the strategic management literature then (e.g., Porter 1985) generally favor multiple sourcing. Likewise, Morris and Imire (1992) point out that multiple sourcing has traditionally been used by British manufacturers as “a means of ensuring competitive tendering, by playing the suppliers off against each other.”

The successful experience of Japanese industries has led western firms to reconsider their purchasing strategies (Deming 1986, Ansari and Modarress 1990). Although there have been studies rigorously examining the competition advantage of multiple sourcing, its advantage to protect against supplier failures has received little attention until recent years. While there are more studies examining the protection against supply disruption risks, research focusing on the LUX setting discussed in this paper remains sparse. Difficult situations like the one Toyota recently faced suggest more research in this strand would better inform managers of important considerations in the sourcing and supplier selection decisions. My study contributes to the growing literature addressing this need.

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[11] Elements of TQM were introduced to Japan by American advisors such as Drs. Deming and Juran after WWII. The ideas were then refined and further developed in Japan before they drew the attention of western industries in 1970’s as a consequence of the challenge by successful Japanese companies such as Sony and Toyota. An overview of this history can be found in Steeples (1992) and Cali (1993). Cases on the attempts of North American manufacturers to implement JIT production and total quality control (TQC) can be found in Schonberger (1987).

[12] See Flynn, Sakakibara, and Schroeder (1995) for discussions on the relationship between JIT and TQM and Hutchins (1992) for discussions on the importance of supplier relations to these management methods.

Quality cost concepts have been introduced into managerial accounting textbooks for some years (e.g., Horngren, Foster, and Datar 1994, Garrison, Noreen, and Brewer 2008). Although attempts have been made to investigate TQM and JIT operations (e.g., Wruck and Jensen 1994, Alles, Datar, and Lambert 1995, Ittner and Larcker 1995, Cremer 1995, and Barron and Gjerde 1996), there remain a lot to explore in relation to quality costs. This paper represents another step towards this direction with regard to quality cost management in the supply base composition decision.

There are several interesting directions for extending the analysis of this paper. A possible extension is to incorporate the buyer's quality improvement effort to raise the design quality level μ and the suppliers' efforts to reduce the defect rate parameter δ_i's. Another is to nest the model into an extended setting to analyze the strategic pricing and competition among the suppliers. A third direction is to identify circumstances where even without the condition of "not too unequal" costs, the optimal supply base is still quality-driven. Given the limited space here, these interesting extensions are left for future research.

Appendix A

DERIVATION OF THE EXPECTED TOTAL EXTERNAL FAILURE COST: To derive $E[\omega D + c_E D^2]$, first note that $E(D^2) = \text{var}(D) + E(D)^2$. Hence,

$$
\begin{aligned}
&E[\omega D + c_E D^2] \\
&\quad = \omega E[D] + c_E E[D^2] \\
&\quad = \omega E[D] + c_E E[D]^2 + c_E \text{var}[D] \\
&\quad = \omega E[\textstyle\sum_{i=1}^n D_i] + c_E(E[\sum_{i=1}^n D_i])^2 + c_E \text{var}[\sum_{i=1}^n D_i] \\
&\quad = \omega \textstyle\sum_{i=1}^n E[D_i] + c_E(\sum_{i=1}^n E[D_i])^2 + c_E \sum_{i=1}^n \text{var}[D_i] \\
&\quad = \omega \textstyle\sum_{i=1}^n \mu\delta_i Q_i + c_E[\sum_{i=1}^n \mu\delta_i Q_i]^2 + c_E \sum_{i=1}^n \sigma^2(\delta_i Q_i)^2 \\
&\quad = \omega\mu[\textstyle\sum_{i=1}^n \delta_i Q_i] + c_E\mu^2[\sum_{i=1}^n \delta_i Q_i]^2 + c_E\sigma^2[\sum_{i=1}^n \delta_i^2 Q_i^2]. \qquad \textit{Q.E.D.}
\end{aligned}
$$

PROOF OF PROPOSITION 1 (CONDITIONS FOR NON-EXISTENCE OF PROTECTION ADVANTAGE OF MULTIPLE SOURCING): When one of the two conditions holds, i.e., either $\sigma^2 \to 0$ or $c_E \to 0$, the buyer's expected total cost becomes $c_E\mu^2[\sum_{i=1}^n \Delta_i Q_i + (\sum_{i=1}^n \delta_i Q_i)^2]$ or simply $\sum_{i=1}^n (c_i+\omega\mu\delta_i)Q_i$, where $\Delta_i \equiv (c_i+\omega\mu\delta_i)/c_E\mu^2$. Suppose the ascending ranking of the suppliers based on Δ_i's also has supplier 1 ranked highest. Then obviously setting $Q_1 = Q$ minimizes $\sum_{i=1}^n \delta_i Q_i$ as well as $\sum_{i=1}^n \Delta_i Q_i$ individually. Consequently, the expected total cost must also be minimized when $Q_1 = Q$. Thus, multiple sourcing cannot be better than single sourcing.

Alternatively, suppose that for any distinct j and k with $(\delta_k - \delta_j)(s_k - s_j) \le 0$, $(s_j - s_k)/(\delta_k - \delta_j) < 2\eta\delta_1 Q$. Then if multiple sourcing is better than single sourcing, the supply base must not contain any j and

n with the property above. Otherwise, assuming without loss of generality that $\delta_j < \delta_k$, I can rearrange the allocation by shifting some amount of Q_k to Q_j and thereby reducing the sum $\sum_{i=1}^{n} \Delta_i Q_i + (\sum_{i=1}^{n} \delta_i Q_i)^2$ in the expected total cost.

To see this, simply differentiate the sum with respect to Q_i to get the derivative $\Delta_i + 2\delta_i(\sum_{i=1}^{n} \delta_i Q_i)$. Note that $s_i \equiv (c_i+\omega\mu\delta_i)/c_{\mathrm{E}}\sigma^2 = \eta\Delta_i$. So for $\delta_j < \delta_k$, $(s_j-s_k)/(\delta_k-\delta_j) < 2\eta\delta_1 Q$ implies $(\Delta_j-\Delta_k) < 2(\delta_k-\delta_j)\delta_1 Q < 2(\delta_k-\delta_j)(\sum_{i=1}^{n} \delta_i Q_i)$. Hence,

$$\Delta_j + 2\delta_j(\sum_{i=1}^{n} \delta_i Q_i) < \Delta_k + 2\delta_k(\sum_{i=1}^{n} \delta_i Q_i),$$

implying that shifting some amount of Q_k to Q_j will reduce the expected total cost further. This leads to the conclusion that any multiple-sourcing supply base must include only suppliers with δ_i's and s_i's showing exactly the same ranking.

However, with such a ranking of the selected suppliers, the expected total cost can be minimized with Q assigned solely to the supplier ranked highest in the supply base, i.e., the one with the lowest baseline defect rate among the suppliers selected. This contradicts the initial supposition that multiple sourcing can be better than single sourcing if $(s_j - s_k)/(\delta_k - \delta_j) < 2\delta_1 Q$ for any distinct j and k with $(\delta_k - \delta_j)(s_k - s_j) \leq 0$. *Q.E.D.*

PROOF OF PROPOSITION 2 (OPTIMAL QUOTA ALLOCATION): The existence of an optimal quota allocation is guaranteed because any feasible allocation must be from the closed and bounded domain $[0,Q]^n$ and the objective function and constraints of the optimization problem are concave and linear, respectively. The following is the Lagrangian of program SB (with constraint QC decomposed into two inequality constraints):

$$\boldsymbol{L} = -c_{\mathrm{E}}\sigma^2[\sum_{i=1}^{n} s_i Q_i + \eta(\sum_{i=1}^{n} \delta_i Q_i)^2 + \sum_{i=1}^{n} \delta_i^2 Q_i^2] + \overline{\theta}(\sum_{i=1}^{n} Q_i - Q) + \underline{\theta}(Q - \sum_{i=1}^{n} Q_i)$$

with $\eta \equiv (\mu/\sigma)^2$ and $s_i \equiv (c_i+\omega\mu\delta_i)/c_{\mathrm{E}}\sigma^2$. Since $\boldsymbol{L}$ is strictly concave in $\mathbf{Q} = (Q_i)_{i\in N}$, a $\mathbf{Q}^*$ is the unique optimal quota allocation for the program if and only if the first-order conditions of the program are satisfied (see Takayama 1985, Chapter 1, Section D).

Differentiating the Lagrangian with respect to Q_i yields the first-order partial derivative below:

$$\boldsymbol{L}_i = \overline{\theta} - \underline{\theta} - c_{\mathrm{E}}\sigma^2[s_i + 2\eta\delta_i(\sum_{h=1}^{n} \delta_h Q_h) + 2\delta_i^2 Q_i].$$

The first-order conditions require that if $\mathbf{Q}^*$ has some $Q_i^* > 0$, then $\mathbf{Q}^*$ has to satisfy the equation $\boldsymbol{L}_i = 0$ for some $\overline{\theta} \geq 0$ and $\underline{\theta} \geq 0$. These $\overline{\theta}$ and $\underline{\theta}$ must be the same for all such i's with $Q_i^* > 0$. In addition, if $\mathbf{Q}^*$ has some $Q_j^* = 0$, then $\mathbf{Q}^*$ has to satisfy the inequality $\boldsymbol{L}_j \leq 0$ for any such j's for the same $\overline{\theta}$ and $\underline{\theta}$. Moreover, $\mathbf{Q}^*$ must satisfy constraint QC.

As some Q_i^* has to be positive, so must the difference $\overline{\theta} - \underline{\theta}$. Define $\theta^* = (\overline{\theta} - \underline{\theta})/c_{\mathrm{E}}\sigma^2$. The first-order conditions are equivalent to the following ones:

$$\sum_{i=1}^{n} Q_i^* = Q$$

and some $\theta^* > 0$ exists such that

[MC$_i$]: $$\theta^* \le s_i + 2\eta\delta_i(\sum_{h=1}^{n} \delta_h Q_h^*) + 2\delta_i^2 Q_i^* \qquad \forall\ i \in N$$

with the equality holding for all $i \in B^* \equiv \{\ i{\in}N \mid Q_i^* > 0\ \}$. Another expression of the marginal condition MC$_i$ is as follows:

$$Q_i^* \ge \left[\frac{\theta^* - S_i(W^*)}{2\delta_i^{\ 2}}\right]$$

with $S_i(W^*) = [c_i + (\omega + 2c_{\mathrm{E}}\mu W^*)\mu\delta_i]/c_{\mathrm{E}}\sigma^2$ and $W^* \equiv \sum_{i=1}^{n} \delta_i Q_i^* = \sum_{B^*} \delta_l Q_l^*$.

To determine the values of θ^* and W^*, I divide MC$_i$ by $2\delta_i$ and then sum over the equality marginal conditions, i.e., MC$_i$'s $\forall i{\in}B^*$. This yields the following equation of θ^* and W^*:

$$\theta^* [(1/2)\sum_{B^*} (1/\delta_l)] = [(1/2)\sum_{B^*} (s_l/\delta_l)] + W^* [1 + \eta b^*],$$

where $b^* \equiv |B^*|$ is the size of the supply base. Similarly, divide MC$_i$ by $2\delta_i^2$ and sum over the equality marginal conditions. Then incorporate the quota constraint, QC. This gives a second equation of θ^* and W^*:

$$\theta^* [(1/2)\sum_{B^*} (1/\delta_l^2)] = [Q + (1/2)\sum_{B^*} (s_l/\delta_l^2)] + W^* [\eta \sum_{B^*} (1/\delta_l)].$$

The solution of the two equations is as follows:

$$\theta^* = \frac{(\eta^{-1}+b^*)[2Q+\sum_{B^*} (s_l/\delta_l^2)] - [\sum_{B^*} (1/\delta_l)][\sum_{B^*} (s_l/\delta_l)]}{(\eta^{-1}+b^*)[\sum_{B^*} (1/\delta_l^2)] - [\sum_{B^*} (1/\delta_l)]^2}$$

$$W^* = \frac{[\sum_{B^*} (1/\delta_l)][2Q+\sum_{B^*} (s_l/\delta_l^2)] - [\sum_{B^*} (1/\delta_l^2)][\sum_{B^*} (s_l/\delta_l)]}{2\eta[\ (\eta^{-1}+b^*)[\sum_{B^*} (1/\delta_l^2)] - [\sum_{B^*} (1/\delta_l)]^2\ ]}\ .$$

In summary, the first-order conditions imply the conditions specified in this proposition. The reverse also holds with $\overline{\theta}$ set to $c_{\mathrm{E}}\sigma^2\theta^*$ and $\underline{\theta}$ set to zero. *Q.E.D.*

PROOF OF PROPOSITION 3 (QUALITY-DRIVEN SUPPLY BASE): For any multiple-sourcing supply base B^*, let supplier j be a selected supplier other than the highest-quality supplier in the supply base. Suppose

$$\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \le (\omega + 2c_{\mathrm{E}}\mu\delta_1 Q)\mu.$$

Because $W^* = \sum_{B^*} \delta_l Q_l^* \ge \delta_1 Q$,

$$(c_{j-1} - c_j)/(\delta_j - \delta_{j-1}) \leq \omega\mu + 2\delta_1 Q c_E \mu^2 \leq \omega\mu + 2W^* c_E \mu^2.$$

Hence, $c_j + (\omega\mu + 2W^* c_E\mu^2)\delta_j \geq c_{j-1} + (\omega\mu + 2W^* c_E\mu^2)\delta_{j-1}$, or equivalently,

$$S_j(W^*) \geq S_{j-1}(W^*),$$

where $S_i(W^*) = [c_i + (\omega + 2c_E\mu W^*)\mu\delta_i]/c_E\sigma^2$. By Proposition 2,

$$Q_j^* = [\theta^* - S_j(W^*)]/2\delta_j^2 > 0 \text{ and}$$
$$Q_{j-1}^* \geq [\theta^* - S_{j-1}(W^*)]/2\delta_{j-1}^2.$$

Thus, $Q_{j-1}^* \geq [\theta^* - S_{j-1}(W^*)]/2\delta_j^2 \geq [\theta^* - S_j(W^*)]/2\delta_j^2 = Q_j^* > 0$. Since $b^* \equiv |B^*|$, it has to be that $B^* = \{1, 2, \ldots, b^*\}$. *Q.E.D.*

PROOF OF PROPOSITION 4 (SUFFICIENT AND NECESSARY CONDITION FOR DETERMINING THE SIZE OF THE OPTIMAL SUPPLY BASE): Suppose $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \leq (\omega + 2c_E\mu\delta_1 Q)\mu$. If the size of the optimal supply base is $j \in N\backslash\{n\}$, Proposition 3 implies $B^* = \{1, \ldots, j\}$. Consequently, Proposition 2 implies the existence of positive θ^* and W^*, as defined by formulas (1) and (2), such that

$$Q_j^* = [\theta^* - S_j(W^*)]/2\delta_j^2 > 0 \text{ and}$$
$$0 = Q_{j+1}^* \geq [\theta^* - S_{j+1}(W^*)]/2\delta_{j+1}^2.$$

Define $\theta_j = \theta^*$ and $W_j = W^*$. The condition of this proposition is thus satisfied.

For the "if" part, suppose there exist positive θ_j and W_j defined by formulas (1) and (2) with B^* substituted by $B_j \equiv \{1, \ldots, j\}$ such that $[\theta_j - S_j(W_j)]/2\delta_j^2 > 0 \geq [\theta_j - S_{j+1}(W_j)]/2\delta_{j+1}^2$. Define $\theta^* = \theta_j$ and $W^* = W_j$. Then θ^* and W^* by construction satisfy formulas (1) and (2) for $B^* = B_j$. Moreover, define a quota allocation $\mathbf{Q}^*$ with $Q_i^* = [\theta^* - S_i(W^*)]/2\delta_i^2$ for all $i \leq j$ and $Q_i^* = 0$ for all $i > j$. Because $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \leq \omega\mu + 2\delta_1 Q c_E \mu^2$, a procedure similar to the proof of Proposition 3 will show that $S_{i+1}(W^*) \geq S_i(W^*)$ for all $i \in N\backslash\{n\}$. Consequently, $[\theta^* - S_j(W^*)]/2\delta_j^2 > 0$ implies $Q_i^* = [\theta^* - S_i(W^*)]/2\delta_i^2 > 0$ for all $i \leq j$. Similarly, $0 \geq [\theta^* - S_{j+1}(W^*)]/2\delta_{j+1}^2$ implies $Q_i^* = 0 \geq [\theta^* - S_i(W^*)]/2\delta_i^2$ for all $i \geq j+1$. Thus, the marginal conditions in Proposition 2 are satisfied by the $\mathbf{Q}^*$ defined above. This means it is the unique optimal allocation for the buyer's sourcing problem, and B_j is the optimal supply base. Hence, $b^* = j$. *Q.E.D.*

PROOF OF PROPOSITION 5 (SUFFICIENT AND NECESSARY CONDITION FOR SINGLE SOURCING TO BE OPTIMAL): The "only if" part follows directly from Proposition 2. For the "if" part, it is straightforward to verify that for each $h \in N$, the θ^h and W^h defined in this proposition satisfy the formulas (1) and (2) of Proposition 2 for the single-sourcing supply base $B^h \equiv \{h\}$. Define for each h a quota allocation $\mathbf{Q}^h$ with $Q^h{}_h \equiv [\theta^h - S_h(W^h)]/2\delta_h^2$ and $Q^h{}_i \equiv 0$ for all $i \in N\backslash\{h\}$. Because $\theta^h \equiv s_h + 2(1+\eta)Q\delta_h^2$ and $W^h \equiv \delta_h Q$, clearly $Q^h{}_h = Q > 0$.

To see if one or none of the B^h's is the optimal supply base, it suffices to check whether one or none of them meets the remaining marginal conditions of Proposition 2, i.e., for any given h,

$Q^h{}_i \geq [(\theta^h - S_i(W^h))/2\delta_i^2$ for all $i \in N\backslash\{h\}$.

Some h will meet this last requirement if the condition of the proposition is fulfilled. When this is the case, the $\{h\}$ is the optimal supply base and single sourcing is optimal. *Q.E.D.*

PROOF OF PROPOSITION 6 (NON-INCREASING RESPONSE OF A SUPPLIER'S QUOTA TO THE COST IT CHARGES): Because of the quota constraint, the feasible choice set defined on $\mathbf{Q}$ is not a sublattice of $\Re^n$. This prevents applying some supermodularity result directly. To get around the problem, I consider an equivalent formulation of the buyer's sourcing problem as follows.

First, the buyer takes Q_i as given and choose $\mathbf{Q}_{-i} = (Q_j^*)_{j\in N\backslash i}$ to minimize the part of the expected total cost that depends on $\mathbf{Q}_{-i}$, i.e.,

$$C_{-i}(\mathbf{Q}_{-i}; Q_i) = \sum_{j\in N\backslash i} c_jQ_j + \omega\mu[\sum_{i=1}^{n} \delta_iQ_i] + c_{\mathrm{E}}\mu^2[\sum_{i=1}^{n} \delta_iQ_i]^2 + c_{\mathrm{E}}\sigma^2[\sum_{i=1}^{n} \delta_i^2Q_i^2],$$

subject to the constraint that $\sum_{j\in N\backslash i} Q_j \geq Q - Q_i$. This constraint is equivalent to the quota constraint because $C_{-i}(\mathbf{Q}_{-i}; Q_i)$ being increasing in $\mathbf{Q}_{-i}$ ensures that the inequality constraint will bind at optimum. This first-stage minimization problem is analogous to the original problem. Following the same proof as for Proposition 2, I can show that a unique minimizer $\mathbf{Q}_{-i}^*(Q_i)$ exists. By the maximum theorem (Sundaram 1996, p. 235), the minimized value $C_{-i}^*(Q_i) = C_{-i}(\mathbf{Q}_{-i}^*(Q_i); Q_i)$ is continuous in Q_i. The non-negativity of $Q_j^*(Q_i)$'s and Q_i ensures that $C_{-i}^*(Q_i)$ is non-negative and hence bounded from below. Moving on to the second stage, the buyer chooses $Q_i \in [0, Q]$ to minimize $c_iQ_i + C_{-i}^*(Q_i)$. A minimizer Q_i^* clearly exists.

The two-stage formulation above is equivalent to the original problem, which has a unique solution characterized in Proposition 2. The quota allocation $(Q_i^*, \mathbf{Q}_{-i}^*(Q_i^*))$ obtained from the two-stage formulation must be the same unique solution. In order to apply below a theorem directly, it is convenient to define the following: $\theta \equiv -c_i$, $x \equiv Q_i$, and $f(x, \theta) \equiv \theta x - C_{-i}^*(x)$. Note that for any (x, θ) and (x', θ') with $x \geq x'$ and $\theta \geq \theta'$,

$$\begin{aligned} & f(x, \theta) - f(x', \theta) \\ &\quad = [\theta x - C_{-i}^*(x)] - [\theta x' - C_{-i}^*(x')] \\ &\quad = (\theta x - \theta x') - (\theta' x - \theta' x') + [\theta' x - C_{-i}^*(x)] - [\theta' x' - C_{-i}^*(x')] \\ &\quad = (\theta - \theta')(x - x') + f(x, \theta') - f(x', \theta') \\ &\quad \geq f(x, \theta') - f(x', \theta'), \end{aligned}$$

with the inequality holding strictly whenever $x > x'$ and $\theta > \theta'$. So f satisfies strictly increasing differences in (x, θ). By Theorem 10.6 of Sundaram (1996, p. 258), the x^* maximizing f over the feasible set $[0, Q] \subset \Re$ is non-decreasing in θ. In other words, Q_i^* is non-increasing in c_i given $\mathbf{c}_{-i} = (c_j^*)_{j\in N\backslash i}$. *Q.E.D.*

PROOF OF LEMMA 1 (FEASIBLE PROCUREMENT CONTRACTS): The incentive compatibility constraint implies that for any m_i and v_i,

$$\begin{aligned}
\mathrm{E}\pi_i(v_i) &\geq \mathrm{E}\pi_i(m_i, v_i) \\
&= \mathrm{E}_{-i}[P_i(m_i, v_{-i}) - v_iQ_i(m_i, v_{-i})] \\
&= \mathrm{E}\pi_i(m_i) + (m_i - v_i)\mathrm{E}_{-i}[Q_i(m_i, v_{-i})].
\end{aligned}$$

Similarly, $\mathrm{E}\pi_i(m_i) \geq \mathrm{E}\pi_i(v_i) + (v_i - m_i)\mathrm{E}_{-i}[Q_i(v_i, v_{-i})]$. Thus, for $m_i \geq v_i$,

$$(m_i - v_i)\mathrm{E}_{-i}[Q_i(v_i, v_{-i})] \geq \mathrm{E}\pi_i(v_i) - \mathrm{E}\pi_i(m_i) \geq (m_i - v_i)\mathrm{E}_{-i}[Q_i(m_i, v_{-i})] \geq 0.$$

This implies $\mathrm{E}_{-i}[Q_i(v_i, v_{-i})] \geq \mathrm{E}_{-i}[Q_i(m_i, v_{-i})]$ for $m_i \geq v_i$. Moreover, since the inequalities above hold for any arbitrarily close m_i and v_i, $\mathrm{E}\pi_i'(v_i) = d\mathrm{E}\pi_i(v_i)/dv_i = -\mathrm{E}_{-i}[Q_i(v_i, v_{-i})] \leq 0$. Hence

$$\mathrm{E}\pi_i(v_i) = \mathrm{E}\pi_i(\bar{v}) + \int_{v_i}^{\bar{v}} \mathrm{E}_{-i}[Q_i(t_i, v_{-i})]dt_i.$$

That $\mathrm{E}\pi_i(\bar{v}) \geq 0$ follows directly from the individual rationality constraint. Condition (iv) is simply the quota constraint of the original feasibility requirement. Therefore, the "only if" part of the proof is completed.

For the "if" part, first note that $Q_i(v_i, v_{-i}) \geq 0$ and conditions (ii) and (iii) imply the individual rationality constraint.

Next, note that condition (ii) implies that for $m_i \geq v_i$,

$$\begin{aligned}
\mathrm{E}\pi_i(v_i) &= \mathrm{E}\pi_i(m_i) + \int_{v_i}^{m_i} \mathrm{E}_{-i}[Q_i(t_i, v_{-i})]dt_i \\
&\geq \mathrm{E}\pi_i(m_i) + \int_{v_i}^{m_i} \mathrm{E}_{-i}[Q_i(m_i, v_{-i})]dt_i \qquad \text{by condition (i)} \\
&= \mathrm{E}\pi_i(m_i) + (m_i - v_i)\mathrm{E}_{-i}[Q_i(m_i, v_{-i})] \\
&= \mathrm{E}\pi_i(m_i, v_i).
\end{aligned}$$

Similarly, $\mathrm{E}\pi_i(m_i) \geq \mathrm{E}\pi_i(v_i) + (v_i - m_i)\mathrm{E}_{-i}[Q_i(v_i, v_{-i})] = \mathrm{E}\pi_i(v_i, m_i)$ for $m_i \geq v_i$. Swapping the roles of m_i and v_i, I get $\mathrm{E}\pi_i(v_i) \geq \mathrm{E}\pi_i(m_i, v_i)$ for $m_i \leq v_i$. Hence, for any m_i and v_i, the incentive compatibility constraint is implied. This completes the "if" part of the proof. *Q.E.D.*

PROOF OF LEMMA 2 (SUFFICIENT CONDITION FOR OPTIMAL PROCUREMENT CONTRACTS): The buyer's sourcing problem is to find an optimal procurement contract to minimize the expected total cost of sourcing below, subject to the feasibility conditions (i) to (iv) in Lemma 1:

$$\mathrm{E}[\textstyle\sum_{i=1}^{n} P_i(v) + C_X(\mathbf{Q}(v))],$$

where the expectation $\mathrm{E}[\bullet]$ is taken over $v = (v_i)_{i\in N}$ and $C_X(\mathbf{Q}) \equiv C(\mathbf{Q};(\omega\mu/c_E\sigma^2)\delta)$, with $C(\mathbf{Q};\ \mathbf{s}) \equiv c_E\sigma^2[\sum_{i=1}^{n} s_iQ_i + \eta(\sum_{i=1}^{n} \delta_iQ_i)^2 + \sum_{i=1}^{n} \delta_i^2Q_i^2]$. By definition, $P_i(v) = v_iQ_i(v) + \pi_i(v_i, v_i)$. Therefore,

$$\begin{aligned}
\mathrm{E}_{-i}[P_i(v)] &= \mathrm{E}_{-i}[v_iQ_i(v)] + \mathrm{E}\pi_i(v_i) \\
&= v_i\mathrm{E}_{-i}[Q_i(v)] + \int_{v_i}^{\bar{v}} \mathrm{E}_{-i}[Q_i(t_i, v_{-i})]dt_i + \mathrm{E}\pi_i(\bar{v}).
\end{aligned}$$

Note that

$$\mathrm{E}[\int_{v_i}^{\bar{v}} \mathrm{E}_{-i}[Q_i(t_i, v_{-i})]dt_i] = \int_{\underline{v}}^{\bar{v}}\int_{v_i}^{\bar{v}} \mathrm{E}_{-i}[Q_i(t_i, v_{-i})]f(v_i)dt_i dv_i$$

$$= \int_{\underline{v}}^{\bar{v}}\int_{\underline{v}}^{t_i} \mathrm{E}_{-i}[Q_i(t_i, v_{-i})]f(v_i)dv_i dt_i$$

$$= \int_{\underline{v}}^{\bar{v}} \mathrm{E}_{-i}[Q_i(t_i, v_{-i})]F(t_i)dt_i$$

$$= \int_{\underline{v}}^{\bar{v}} \{\mathrm{E}_{-i}[Q_i(v_i, v_{-i})]F(v_i)/f(v_i)\}f(v_i)dv_i$$

$$= \mathrm{E}[\mathrm{E}_{-i}[Q_i(v_i, v_{-i})]F(v_i)/f(v_i)]$$

$$= \mathrm{E}[Q_i(v)F(v_i)/f(v_i)].$$

Hence,

$$\mathrm{E}[P_i(v)] = \mathrm{E}[\mathrm{E}_{-i}[P_i(v)]]$$

$$= \mathrm{E}[v_i\mathrm{E}_{-i}[Q_i(v)]] + \mathrm{E}[\int_{v_i}^{\bar{v}} \mathrm{E}_{-i}[Q_i(t_i, v_{-i})]dt_i] + \mathrm{E}\pi_i(\bar{v})$$

$$= \mathrm{E}[(v_i + F(v_i)/f(v_i))Q_i(v)] + \mathrm{E}\pi_i(\bar{v})$$

and

$$\mathrm{E}[\textstyle\sum_{i=1}^{n} P_i(v)] = \textstyle\sum_{i=1}^{n} \mathrm{E}[p(v_i)Q_i(v)] + \textstyle\sum_{i=1}^{n} \mathrm{E}\pi_i(\bar{v}),$$

where $p(v_i) = v_i + F(v_i)/f(v_i)$. The buyer's objective function can thus be expressed as

$$\mathrm{E}[\textstyle\sum_{i=1}^{n} p(v_i)Q_i(v) + C_{\mathrm{X}}(\mathbf{Q}(v))] + \textstyle\sum_{i=1}^{n} \mathrm{E}\pi_i(\bar{v}),$$

where $C_{\mathrm{X}}(\mathbf{Q}(v)) = c_{\mathrm{E}}\sigma^2[\sum_{i=1}^{n} (\omega\mu/c_{\mathrm{E}}\sigma^2)\delta_i Q_i(v) + \eta(\sum_{i=1}^{n} \delta_i Q_i(v))^2 + \sum_{i=1}^{n} \delta_i^2 Q_i(v)^2]$. Define

$$\gamma_i(v_i) \equiv (p(v_i)+\omega\mu\delta_i)/c_{\mathrm{E}}\sigma^2$$

The expression of the objective function can be further simplified as

$$\mathrm{E}[C(\mathbf{Q}(v); \boldsymbol{\gamma}(v))] + \textstyle\sum_{i=1}^{n} \mathrm{E}\pi_i(\bar{v}),$$

where $\boldsymbol{\gamma}(v) = (\gamma_i(v_i))_{i\in N}$.

Note that the payment rules appear only in the second term of the objective function. Suppose that given any profile of allocation rules fulfilling the quota constraint, the payment rules can be chosen to satisfy the feasibility conditions (ii) and (iii) in Lemma 1 and also minimize $\sum_{i=1}^{n} \mathrm{E}\pi_i(\bar{v})$ to zero (i.e., its lowest possible value). Then a profile of optimal allocation rules can be determined by simply minimizing the first term of the objective function subject to the remaining feasibility conditions. One can also find a candidate profile first by ignoring the feasibility condition (i) and later show that the profile also satisfies the constraint although it was not explicitly included in the minimization process initially.

In the following, I will show that if $\mathbf{Q}^*(\bullet)$ minimizes $\mathrm{E}[C(\mathbf{Q}(v); \boldsymbol{\gamma}(v))]$ subject to the quota constraint, with $Q_i^*(v_i, v_{-i})$ being non-increasing in v_i given v_{-i}, and $\mathbf{P}^*(\bullet)$ is set such that $P_i^*(v) = v_iQ_i^*(v) +$

$\int_{v_i}^{\bar{v}} Q_i^*(t_i, v_{-i})dt_i$, then $\langle \mathbf{Q}^*(\bullet), \mathbf{P}^*(\bullet)\rangle$ is a feasible contract minimizing $\sum_{i=1}^{n} \mathrm{E}\pi_i(\bar{v})$ to zero. Consequently, the contract must also be optimal.

To begin, note that because $Q_i^*(v_i, v_{-i})$ is non-increasing in v_i,

$m_i \geq v_i \quad \Rightarrow \quad Q_i(v_i, v_{-i}) \geq Q_i(m_i, v_{-i})$ for any given v_{-i}.

Consequently, $\mathrm{E}_{-i}[Q_i(v_i, v_{-i})] \geq \mathrm{E}_{-i}[Q_i(m_i, v_{-i})]$. So the feasibility condition (i) in Lemma 1 is satisfied. Next, the payment rule specified in the proposition implies that $P_i^*(v) - v_iQ_i^*(v) = \int_{v_i}^{\bar{v}} Q_i^*(t_i, v_{-i})dt_i$. Therefore, $\mathrm{E}\pi_i(v_i) = \mathrm{E}\pi_i(v_i, v_i) = \mathrm{E}_{-i}[\pi_i(v_i, v_i)] = \mathrm{E}_{-i}[P_i^*(v) - v_iQ_i^*(v)] = \int_{v_i}^{\bar{v}} \mathrm{E}_{-i}[Q_i^*(t_i, v_{-i})]dt_i$. Clearly, $\mathrm{E}\pi_i(\bar{v}) = 0$ for all i. Hence the feasibility condition (iii) is also satisfied and $\sum_{i=1}^{n} \mathrm{E}\pi_i(\bar{v}) = 0$. In addition, $\mathrm{E}\pi_i(v_i) = \mathrm{E}\pi_i(\bar{v}) + \int_{v_i}^{\bar{v}} \mathrm{E}_{-i}[Q_i^*(t_i, v_{-i})]dt_i$ for all i. The feasibility condition (ii) is met as well. By the supposition of the proposition, $\mathbf{Q}^*(\bullet)$ minimizes $\mathrm{E}[C(\mathbf{Q}(v); \gamma(v))]$ subject to the quota constraint. Thus, the feasibility condition (iv) is also met. Since $\langle \mathbf{Q}^*(\bullet), \mathbf{P}^*(\bullet)\rangle$ is a feasible contract minimizing the two terms of the buyer's objective function individually, it must also be an optimal procurement contract minimizing the objective function. *Q.E.D.*

PROOF OF PROPOSITION 7 (OPTIMAL PROCUREMENT CONTRACT): Given $\gamma(v) = ((p(v_i)+\omega\mu\delta_i)/c_{\mathrm{E}}\sigma^2)_{i\in N}$, the pointwise minimization of $C(\mathbf{Q}(v); \gamma(v))$ by an allocation $\mathbf{Q}(v)$ subject to the constraint of $\sum_{i=1}^{n} Q_i(v) = Q$ will give an allocation rule $\mathbf{Q}(\bullet)$ that minimizes $\mathrm{E}[C(\mathbf{Q}(v); \gamma(v))]$ subject to the quota constraint for all $v \in [\underline{v}, \bar{v}]^n$. Note that $C(\mathbf{Q}(v); \gamma(v))$ is simply the buyer's expected total cost of sourcing when the per-unit procurement costs taken as given are $c_i = p(v_i) \equiv v_i + F(v_i)/f(v_i)$ for all i. So for any v, the unique $\mathbf{Q}^*(v)$ that satisfies the conditions in Proposition 2 for such c_i's will constitute an allocation rule minimizing $\mathrm{E}[C(\mathbf{Q}(v); \gamma(v))]$ subject to the quota constraint regardless of v. By Proposition 6, $Q_i^*(v_i, v_{-i})$ is non-increasing in $p(v_i)$. If $p(v_i)$ is non-decreasing in v_i, $Q_i^*(v_i, v_{-i})$ is non-increasing v_i. By Lemma 2, $\langle \mathbf{Q}^*(\bullet), \mathbf{P}^*(\bullet)\rangle$, with $P_i^*(v) = v_iQ_i^*(v) + \int_{v_i}^{\bar{v}} Q_i^*(t_i, v_{-i})dt_i$, is an optimal procurement contract. Finally, suppose further that $p(\bar{v}) - p(\underline{v}) \leq (\omega + 2c_{\mathrm{E}}\mu\delta_1 Q)\mu[\min_{i\in\{2,\ldots,n\}} (\delta_i - \delta_{i-1})]$. By Proposition 3, the optimal quota allocation $\mathbf{Q}^*(v)$ is strongly quality-driven for any realized v. *Q.E.D.*

REVIEW OF RELATED STUDIES IN ECONOMICS: Before giving a brief account of related economic studies, let me clarify some confusion on the usage of the term "multiple sourcing." In management studies, multiple sourcing generally means *split* procurement arrangement, i.e., relying on two or more suppliers in procuring an item. The sourcing literature in economics, however, also uses multiple sourcing to mean *sharable* procurement arrangement, i.e., having two or more suppliers compete for a share in supplying an item without precluding the one-supplier-take-all outcome. The key difference is that multiple sourcing of the sharable sense (hereafter, *sharable* multiple sourcing) refers to an *ex ante* arrangement concerning the *degree* of supplier competition, i.e., at least two *competitors*, whereas multiple sourcing of the split sense (hereafter, *split* multiple sourcing) refers to an *ex post* arrangement concerning the *outcome* of supplier competition, i.e., at least two *winners*. In other words, *sharable* multiple sourcing can end up having only one selected supplier, as long as multiple suppliers have competed for being the one selected. Some studies (e.g., Laffont and Tirole 1988, Riordan and Sappington 1989) claim to analyze multiple sourcing but are better described as studying *source switching*, i.e., about examining the effects of introducing additional suppliers to compete with an incumbent in *one-supplier-take-all* settings.[13]

The sourcing literature in economics is largely based on auction models (e.g., Myerson 1981, Laffont and Tirole 1987). It typically concludes that *split* multiple sourcing is undesirable unless suppliers have increasing marginal costs with sufficiently low fixed costs (e.g., the third case studied by Dasgupta and Spulber 1989/90 and the second case analyzed by Auriol and Laffont 1992). As noted by McMillan (1990), "the disadvantage of multiple sourcing is that economies of scale may be forgone." So for multiple sourcing to be better than single sourcing, scale diseconomies have to be sufficiently great.

When suppliers have constant marginal costs with positive fixed costs (e.g., the first case analyzed by Auriol and Laffont 1992 and the settings studied by Demski, Sappington, and Spiller 1987 and Riordan 1996), split multiple sourcing usually is undesirable because of duplication of fixed costs.[14] However, it is always good to have more suppliers competing for a procurement contract as this increases the chance of finding a lower-cost supplier. This *sampling effect* makes sharable multiple sourcing desirable, regardless of the suppliers' cost structure.

Some studies have examined the effects of incomplete information about suppliers' types on the dis/advantages of multiple sourcing and the sourcing decision. Auriol and Laffont (1992) find that the sampling effect is higher under incomplete information than under complete information, provided the consumer demand is sufficiently price-inelastic. If the consumer demand is sufficiently price-elastic, the

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[13] More precisely, Laffont and Tirole (1988) uses the term "second sourcing," which is a special case of multiple sourcing when there are only two suppliers. Second sourcing also has the connotation of finding a second supplier in addition to an incumbent, or to replace the incumbent when used in the source switching sense. Such usages are particularly suitable for describing the sourcing decision in a multiple-period or multiple-stage setting, as in Demski, Sappington, and Spiller (1987), Laffont and Tirole (1988), and Riordan and Sappington (1989). In a single-period setting, dual sourcing seems to be a better term for describing the two-supplier case of multiple sourcing.

[14] Dana and Spier (1994) and McGuire and Riordan (1995) have studied settings with constant marginal costs and positive fixed costs yet still found a split-award outcome (i.e., the duopoly market structure in their models) sometimes desirable. Dana and Spier's result is driven by smaller efficiency loss resulting from Cournot competition by the suppliers in the case of dual sourcing, as compared to unregulated monopoly in the case of single sourcing. McGuire and Riordan's result is due to the spatial competition model embedded in their model: a product differentiation benefit, playing the same role as increasing marginal costs, arises when aggregating the social value of the treatment to clients uniformly distributed on a line of unit length.

contrary instead holds. Riordan (1996) studies a setting with an exogenous procurement quantity requirement, which resembles a price-inelastic demand. He finds that incomplete information biases the choice of the market structure in his model towards sharable multiple sourcing, a result consistent with Auriol and Laffont's.

By contrast, Dana and Spier (1994) draw a different conclusion about split multiple sourcing. Because the split-award outcome is a weaker penalty to a lying supplier than the no-award outcome (i.e., single sourcing from a rival supplier), split multiple sourcing is less powerful in discouraging suppliers from lying about their types. Thus, it should be used less often under incomplete information than under complete information. McGuire and Riordan (1995) obtain a similar result for some parameter values of their model, and the contrary for some other values.

The research discussed so far follows a normative approach to study the optimal sourcing decision. Some other research by contrast uses a positive approach to investigate when multiple sourcing can arise in equilibrium. Anton and Yao (1989) consider a setting with two suppliers playing a sharable procurement auction. Interestingly, the buyer is indifferent among all equilibria, which include single-sourcing from the lower-cost supplier. So although dual sourcing can arise in equilibrium, it brings no benefit to the buyer. Extending their model to include incomplete information, Anton and Yao (1992) show that dual sourcing can arise in equilibrium if a technical condition is fulfilled. This condition ensures that dual-source production is less expensive than sole-source production.

Seshadri, Chatterjee, and Lilien (1991) study a procurement auction model with endogenous choice of participation. They point out that specifying in advance a greater number of winners to split the procurement contract may increase the chance of winning. This encourages more suppliers to send in bids and stimulates supplier competition. The downside of expanding the supply base is the higher production costs of the marginal winner, which are borne by the buyer.[15]

From this review, it is clear that the sourcing literature in economics has focused only on the competition advantage of multiple sourcing, with no attention given to the protection advantage of multiple sourcing. Neither has this advantage received sufficient attention in OR/MS studies, as explained in the introduction.

[15] Seshadri, Chatterjee, and Lilien's approach has some normative favor. Although the winner selection and award splitting rules are exogenously specified, they discuss the implications of expanding the supply base as if it were a design instrument of the buyer.

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The appendix in the following pages is provided to the reviewers to facilitate their evaluation. It is understood that the material will not be printed in the journal or as an electronic companion. The contents of the material are not critical for the proper evaluation of the manuscript.

Table 1. Notations

N	=	$\{1, 2, \ldots, n\}$ is the index set of the n suppliers ($n \geq 2$).
Q_i	=	the production quota allocated to supplier i ($Q_i \geq 0$).
Q	=	$\sum_{i=1}^{n} Q_i$ is the total procurement quantity ($Q > 0$).
$\mathbf{Q}$	=	$(Q_i)_{i\in N}$ is the production quota allocation.
c_i	=	per-unit procurement cost charged by supplier i.
D_i	=	$R_i\delta_i Q_i$ is the amount of defective parts manufactured by supplier i.
R_i	≥	0, or more precisely $R_i\delta_i$, is referred to as the *random yield loss* of supplier i. The random variables R_i's are independently and identically distributed with mean $E(R_i) = \mu$, where $0 < \mu \leq \overline{\mu} << 1$, and variance $\mathrm{var}(R_i) = \sigma^2 > 0$.
η	=	$(\mu/\sigma)^2$ is the *squared standardized mean* of the "random yield loss" R_i, or equivalently, η^{-1} is referred to as the *squared coefficient of variation*.
δ_i	>	0 is a parameter affecting the random yield loss of supplier i ($\delta_i \leq 1$). The value $1-\delta_i$ is referred to as the *quality-based scoring index* of the supplier, or simply its *quality* level. It is assumed that $\delta_1 < \delta_2 < \ldots < \delta_n$.
D	=	$\sum_{i=1}^{n} D_i$ is the total amount of defective products sold to end customers by the buyer. It becomes observable after the customers have experienced field failures of the products and take them back for warranty repair.
ω	>	0 is the constant marginal cost of warranty repair.
$C_{\mathrm{E}}(D)$	=	$c_{\mathrm{E}}D^2$ is the *other external failure cost* (e.g., reputation damage) borne by the buyer, in addition to the warranty-related cost, as a result of the defective products sold to customers (where $c_{\mathrm{E}} > 0$).
s_i	=	$(c_i+\omega\mu\delta_i)/c_{\mathrm{E}}\sigma^2$ is a ratio representing the relative unimportance of the quadratic other external failure cost, characterized by c_{E}, in constituting the buyer's expected total cost.
$S_i(W^*)$	=	$[c_i + (\omega+2c_{\mathrm{E}}\mu W^*)\mu\delta_i]/c_{\mathrm{E}}\sigma^2 = = s_i + 2\eta W^*\delta_i$ is referred to as the *quality-adjusted cost-based scoring index* for supplier i, evaluated at $W^* = \sum_{i=1}^{n} \delta_i Q_i^*$ based on the optimal quota allocation $\mathbf{Q}^* = (Q_i^*)_{i\in N}$.
B	=	$\{ i\in N \mid Q_i > 0 \}$ is the set of selected suppliers constituting the supply base.
b	=	$\lvert B\rvert$ is the size of the supply base B.
$C(\mathbf{Q}; \mathbf{s})$	=	$c_{\mathrm{E}}\sigma^2[\sum_{i=1}^{n} s_iQ_i + \eta(\sum_{i=1}^{n} \delta_iQ_i)^2 + \sum_{i=1}^{n} \delta_i^2Q_i^2]$, where $\mathbf{s} = (s_i)_{i\in N}$, is the buyer's expected total cost of sourcing.
$C_{\mathrm{X}}(\mathbf{Q})$	=	$C(\mathbf{Q};(\omega\mu/c_{\mathrm{E}}\sigma^2)\delta)$, where $\delta = (\delta_i)_{i\in N}$, is the buyer's expected external failure cost.
$\langle\mathbf{Q}(\bullet), \mathbf{P}(\bullet)\rangle$	=	a procurement contract constituted of a profile of allocations $\mathbf{Q}(\bullet) = (Q_i(\bullet))_{i\in N}$ and payments $\mathbf{P}(\bullet) = (P_i(\bullet))_{i\in N}$ for any profile of the suppliers' unit costs of production $v = (v_i)_{i\in N} \in [\underline{v}, \overline{v}]$.
$\mathrm{E}\pi_i(v_i)$	=	$\mathrm{E}\pi_i(v_i, v_i)$, where $\mathrm{E}\pi_i(m_i, v_i) = \mathrm{E}_{-i}[\pi_i(m_i, v_i)]$, with the expectation $\mathrm{E}_{-i}[\bullet]$ taken over $v_{-i} = (v_j)_{j\in N\setminus i}$, and $\pi_i(m_i, v_i) = P_i(m_i, v_{-i}) - v_iQ_i(m_i, v_{-i})$ for any $v_i \in [\underline{v}, \overline{v}]$ and $m_i \in [\underline{v}, \overline{v}]$.
$p(v_i)$	=	$v_i + F(v_i)/f(v_i)$ is the *virtual* unit variable production cost of supplier i; $f(v_i) > 0$ for all $v_i \in [\underline{v}, \overline{v}]$ is the density function of the probability distribution function $F(v_i)$.

Appendix B

RECENT CASES OF MASSIVE PRODUCT RECALLS: The most noticeable cases are recalls of unsafe products manufactured by suppliers in China. These include

(i) *Toys*. Mattel recalled 436,000 Chinese-made toy cars covered with lead paint in 2007 (Story and Barboza 2007). This incident raised concerns about the insufficient enforcement of existing laws banning lead paint. Such concerns forced other U.S. manufacturers to recall over a million toy ovens, trains, dolls, and other popular toys.

According to Egan, Campbell, and Vogel (2009), "[t]he purported culprit was a Chinese supplier that had subcontracted its work to another Chinese company that had coated the toy cars with lead paint without the knowledge of the U.S. manufacturer. … The foreseeability of these events by any U.S. manufacturer is doubtful. In China, as in the United States, lead paint is illegal. Nevertheless, it appears that a number of Chinese companies began using lead paint because it is more resistant to corrosion and dries faster, thereby, in part, decreasing production time.

Until the 2007 toy recalls, the Chinese government was seemingly unaware that lead paint was widely used in its manufacturing sector, and therefore, did not sufficiently enforce lead paint prohibitions. Similarly the U.S. Consumer Product Safety Commission was also unable to enforce existing laws banning lead paint because it had only around 100 field investigators who were responsible for inspecting $22 billion in toys." For further details of this incident, see also Barboza (2007) and Lipton and Barboza (2007).

(ii) *Drugs*. Heparin is a blood thinner widely used in surgery and dialysis. In early 2008, heparin sold by Baxter International was linked to at least 19 deaths and hundreds of allergic reactions in the U.S.. After recalling nine lots of the drug, problems continued. So the company suspended the manufacturing of the drug associated with the problems.

Investigations later discovered that the heparin at issue, with its raw components bought from a Chinese plant, contained a contaminant mimicking heparin. By that time, Baxter had expanded the recall to cover almost all its heparin products. Because the company supplies about half the U.S.'s heparin, the production suspension and the widespread recall caused worries about shortage problems in the short and long runs. The impact was so huge that some even warned that "many more patients would be likely to experience significant blood loss during dialysis." For further details of the incident, see Harris (2008a,b), Bogdanich (2008a,b), and Barboza (2008a).

(iii) *Dairy products*. In July 2008, infant milk formula produced by Sanlu Group, the largest milk power maker in China for 15 years in a row, were found to contain a toxic industrial chemical called melamine. Follow-up investigations discovered the same problem in the products of 21 other companies.

Melamine was added to milk to raise the protein count artificially and fool safety tests for protein content. The contamination caused at least six infants dying from kidney stones and other complications and sickened over 50,000 children.

Exports of food ingredients from China have been growing in recent years. Because milk powder is an ingredient to many dairy products, the milk scandal scared the international community. Tests showed that many products of international brands were also tainted by melamine, leading to worldwide recalls of contaminated products. Affected brands include Nabisco, Kraft Foods, Heinz, Mars, Cadbury, Lipton, and Nestles.

The scope of the contamination later spread to eggs traced back to the use of melamine-tainted animal feed, even though the chemical had been banned as an animal feed additive since July 2007. In reacting to the scandal, over 25 countries banned imports of dairy and other affected food products from China. For further details of the scandal, see Barboza (2008b,c) and Fuller (2008).