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Corporate Tax Evasion: the Case for Specialists

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Abstract

We analyze the role of accounting specialists who help corporations evade/avoid taxes in a game of incomplete information played by a tax authority, corporate taxpayers, and an accounting specialist. In addition to a full equilibrium characterization, we establish that (i) marginal changes in enforcement are not effective when evasion/avoidance is pervasive; (ii) fines on firms as opposed to specialists are more effective in such situations; (iii) reducing auditing costs and increasing “creative accounting” costs are effective in curbing evasion when tax compliance is relatively high.

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1 Introduction

The phenomenon of tax noncompliance is pervasive throughout the world¹ and may have unpleasant consequences for the economy. Economic literature usually agrees

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¹See, for example, the survey by Schneider and Enste (2000).

on four main issues: (i) noncompliance may increase the deadweight loss from distortionary taxation (Feldstein 1999); (ii) it may impede the ability of government to collect tax revenue and serve its debt obligations (as in recent Greek experience²); (iii) it calls for wasteful enforcement activities (Slemrod and Yitzhaki 2002); (iv) it may hinder promotion of equity, as the effective tax rate depends on the unobservable characteristic of “tax honesty” and evasion opportunities (Andreoni et al. 1998). Thus, it is important to understand what factors determine equilibrium tax compliance, and this is precisely the research question most of the literature in this field aims at answering.

However, the vast majority of contributions to the tax compliance literature abstract from the specific knowledge necessary for evasion and avoidance³. To show that this is a regrettable omission, in the following, after introducing necessary definitions, we discuss anecdotal evidence suggesting that tax specialists play an important role in determining tax compliance.

Usually, a line between lawful underreporting of tax obligations, also known as *tax avoidance*, and illegal understatement, referred to as *tax evasion*, is drawn. In reality, though, it is virtually impossible to distinguish between the two. In this situation it makes sense to break down underreporting into simple and sophisticated rather than into evasion and avoidance. We define *simple tax evasion* as understatement that does not require special expertise (accounting or financial). Correspondingly, understatement of tax liability that requires such special knowledge will be called *sophisticated tax evasion*.

There are no exact figures about any kind of tax evasion at our disposal. For simple evasion, the shadow-sector estimations presented, e.g., in Schneider et al.(2010) are a good proxy. Sophisticated evasion eludes such attempts, as it is reported and looks legal up to the moment the whole complicated arrangement is uncovered. Thus, what we can observe here are really big cases, the results of firm audits and of changes in the proportion of corporate tax revenues in total tax revenues. The latter, as noted by Slemrod (2004) for the US, have fallen from 6.4 percent of GDP in 1951 to less than 1.5 percent of GDP in recent years. Indirect evidence for growing sophisticated evasion is provided by the fact that “America’s largest and most profitable companies paid less in corporate income taxes in last three years, even as they increased profits,” as Browning (2004) states.

²Online at <http://www.economist.com/node/16792848>.

³For a detailed survey of income tax evasion literature, see Andreoni et al. (1998)

Corporate tax evasion, as opposed to individual, can hardly be simple. For one thing, corporations undergo regular (every three to five years in most countries) audits by the tax authorities. Thus, in order to hide simple evasion, a firm has either to perform a transaction illegally, facing the problems of contract enforcement and depriving itself of the benefits of law, or to officially close down before the corresponding check, rendering it impossible to gain reputation, which is crucial for successful functioning in many markets.

Corporate scandals⁴ become possible with sophisticated evasion, which is hard to detect, as it requires counterchecking of many legal entities, some of which may be in a different tax jurisdiction (another city, state, country) or even liquidated by the time of audit. A broad collection of material about such scandals⁵ has been prepared by Roy Davies and includes Enron as probably the most famous case. An excellent collection about US corporations involved in scandals is due to Citizen Works⁶.

The principal feature of corporate scandals is that fictitious contracts are made to overstate the performance of a company. This is done to boost the benefits of chief executives and stock prices, and tax fraud comes as a by-product of such efforts. Despite being a secondary goal, the tax evasion in these cases is very substantial, and it is increasing, as anecdotal evidence suggests (Johnston 2003a,b).

Apart from the scandals, sophisticated evasion is represented by conventional tax shelters. The following examples of shelters common in the US may seem benign, but taken on a large scale are very detrimental to social welfare⁷: (i) deferring taxes to later years; (ii) obtaining leverage through various financing arrangements; (iii) deducting prepaid interest; (iv) not including prepaid income.

How can the auditors help in performing sophisticated evasion⁸? First, they have to certify the tax reports for public corporations. This also means that the auditors are an essential part of sophisticated evasion schemes. Secondly, they may actually assist smaller corporations, providing tax consulting that may include evasion. That

⁴A recent example is the German scandal related to Liechtenstein as a tax haven; details are available in *The Economist* (Feb 21, 2008), online at http://www.economist.com/world/europe/displaystory.cfm?story_id=10733044

⁵Available at <http://www.exeter.ac.uk/~RDavies/arian/scandals/classic.html#credit>.

⁶Can be found on <http://www.citizenworks.org/enron/corp-scandal.php>

⁷The list and a discussion of tax shelters vs IRS measures to curb them can be found at http://www.usa-federal-state-company-tax.com/tax_shelter_techniques.asp

⁸For a list of cases involving the four big auditors in 2007–09 see http://www.usa-federal-state-company-tax.com/tax_shelters_2007-2009.asp

is why in our paper the key role is played by the accounting specialist, modeled as a local monopolist (alternatively, it can be a number of tacitly colluding specialists) providing sophisticated evasion service for a set of real-sector firms.

Another application of our setup is even more direct: in former Soviet Union (fSU) countries the evasion service is usually provided by an accounting firm associated with a commercial bank. This way every client of a bank has an option to arrange its accounts to minimize its tax obligations. The accounting firm is a local monopolist, as there are substantial costs of changing a bank, and hence of changing the evasion specialist. There is a lot of anecdotal evidence for money laundering through the Russian banks. Sophisticated evasion goes hand in hand with laundering; in most cases it is difficult or impossible to separate the two phenomena⁹.

The study by Levine and Movshovich (2001) shows that sophisticated evasion accounted for about 90% of all corporate tax evasion in fSU countries. Moreover, the famous scandal with Yukos has opened up a bit the mechanism of such evasion to the general public. Briefly, the oil giant managed to reduce its corporate tax liability virtually to zero by shifting its operations on paper to a small republic within Russia and making special arrangements with the regional government. It is common knowledge that other Russian corporations were not very different from Yukos in their tax arrangements, but avoided prosecution.

Despite the evidence summarized above, the modest body of the literature that does take into account tax specialists either (i) assumes away their participation in evasion or (ii) looks at tax specialists inside a firm. Within branch (i), Reinganum and Wilde (1993) focus on the potential of the specialists to lower the costs of filing reports. Using a game of perfect information, they come to the conclusion that the tax authority audits reports prepared by tax specialists more intensively. Franzoni (1999) looks at the auditors as “gatekeepers” who can moderate the relation between tax authority and taxpayers. He finds that multiple perfect Bayesian equilibria become possible due to the substitutability of public and private monitoring. The empirical research is represented by studies by Klepper et al. (1991) and Erard (1993). The principal finding here is that the specialists inhibit evasion on unambiguous items (simple evasion), but stimulate it on ambiguous items (sophisticated evasion).

We are extending this strand of literature by including another type of tax specialists in the analysis. We are not describing the certified lawyers and accountants

⁹An example of a bank connected to an evasion specialist can be found in Zheglov (2006).

that help to fill in tax reports. Rather, we have in mind financial firms that run accounts of the real-sector firms, tax havens that facilitate tax avoidance, or auditors that verify the accounts of public corporations. Therefore, we assume that the firms cannot opt for simple evasion, and that is a special feature of corporate as opposed to personal tax evasion. In this setting, the specialists play a strategic role that is absent in the analysis of Reinganum and Wilde (1993) and Franzoni (1999)¹⁰.

As for branch (ii), it is treated in the corporate tax evasion literature,¹¹ which is complementary to our paper, as it models tax specialists employed directly by firms. Crocker and Slemrod (2005) use a principal-agent framework to study incentives of chief financial officers (CFO) and shareholders to engage in evasion activities. They characterize the optimal contract in the presence of asymmetric information about the magnitude of the legal tax deductions and find that penalties imposed on the tax manager reduce evasion more than do those imposed on the shareholders.

Chen and Chu (2005) look instead at the incentives of chief executive officers when the contracts between them and shareholders are not enforceable. They assume that the rewards can be conditioned on the reported, but not on the actual, profit, and show that the gain from evasion may only come at the expense of a loss in internal control at the firm.

To the best of our knowledge, the only paper so far that takes into account the participation of the external specialists in avoidance/evasion is Damjanovic and Ulph (2010). Assuming an exogenous auditing strategy on the part of the tax authority allows them to analyze equilibrium noncompliance as an intersection of the demand for avoidance schemes and the supply of such schemes by the tax specialists. They find that greater progressivity of the tax schedule may reduce the supply of tax avoidance/evasion schemes and that tax compliance is greater when pretax income is distributed less equally. They also get a result that competition among tax specialists is detrimental for compliance.

To further the analysis of corporate tax evasion and avoidance, we endogenize the response of a tax authority to the reporting behavior of the firms, and we model financial specialists in a novel way. The reason for the former is that exogenous audit probability seems too strong an assumption; for the latter, that we want to target

¹⁰In their work, the presence of the specialist only changes the tax agency's auditing behavior and serves as an additional source of revenue. I am grateful to an anonymous referee for this remark.

¹¹Kopczuk and Slemrod (2006) provide a general macroeconomic framework for analyzing tax evasion by firms, showing that standard equivalences of different taxes break down.

the auditor firms or external accounting specialists (or even tax havens) rather than internal financial services.

Our research question is what the equilibrium relation between tax collection parameters and the amount of evasion in the presence of tax specialists is. We do not aim at explaining how the evasion industry comes into existence, or whether the evasion specialists play some useful role in society. We take this sector as given and look at how tax rates, fines, and industry structure can affect it. The interaction between tax authority, firms, and evasion specialists is modeled as a static game of incomplete information in the spirit of Reinganum and Wilde (1986). The perfect Bayesian equilibrium is used as a solution concept, and simple intuition is used for equilibrium selection.

Our main result and contribution is the characterization of equilibria in this non-trivial three-player game.

First, we find equilibria of the evasion game with an exogenous price for specialist service. In addition to the separating equilibrium (every type submits a distinct report) analyzed by Reinganum and Wilde (1986), we discuss a pooling equilibrium, in which everybody evades everything, and a hybrid equilibrium, in which low-profit firms are pooling their reports and high-profit firms are separating. We only select equilibria with pooling at zero report, which are natural focal points for the firms. If the costs of tax-authority audits are high, only pooling is possible in equilibrium; with medium auditing costs, a hybrid equilibrium exists; and with low costs, a separating equilibrium exists, given that the price of the specialist is not too low.

Secondly, we provide a full description of the selected equilibrium of the whole game for both monopolistic and competitive structure of the industry of tax specialists. We show that the monopolistic specialist chooses either full cheating, or the separating (hybrid) equilibrium with an evasion level that is constant across incomes.

Fines on firms are more effective in driving the economy away from complete evasion than are fines on the specialist. This is in contrast to the result in Crocker and Slemrod (2005), stating that fines on managers (agents) are preferable to fines on firms (principals). Intuitively, in their framework higher CFO fines lead to the restructuring of the evasion-favoring contract. In our setting, the firms play a role of agents (the specialist being a principal), and it is better to fine firms, as they imperfectly adjust to the fines, taking the specialist price as given. The specialists, on the contrary, do not have this restriction – they can adjust perfectly to the fines

by choosing an appropriate price.

For markets with low auditing costs (separating equilibrium, low evasion) our model points out the importance of compliance-enhancing factors other than conventional enforcement. Indeed, in the separating equilibrium it is infinitely costly to ensure extinction of specialists by raising fines or simple auditing intensity. Moreover, with higher compliance it is increasingly costly to reduce sophisticated evasion by tougher punishment – a more promising way to fight it is through an increase in costs of muddling accounts. This result is driven by the lack of commitment on the side of the tax authority: The specialists can implement a low enough evasion level to ensure that revenue-maximizing auditing intensity is arbitrarily small.

For the markets with high auditing costs (pooling equilibrium, high evasion), marginal increases in enforcement parameters are generally not sufficient to bring about a reduction in tax evasion. For that reason, drastic reforms may be needed to ensure an improvement in compliance. The intuition here is that the specialists as first movers make sure that in response to higher fines, the real-sector firms reduce their rents from evasion before reducing the evasion volume itself

The model setup is presented in section 2; the description of equilibria for an exogenous price follows in section 3. Price setting by the specialist is considered in section 4, followed by the government problem in section 5. Section 6 contains a discussion of alternative specifications of the model. In conclusion, the results are summarized and policy implications are suggested.

2 The Model

Imagine a world in which there is a continuum of firms with measure one, each characterized by some profit π . The magnitude of this profit is a realization of a random variable $\boldsymbol{\pi}$ distributed over the interval $[\pi_{\min}, \pi_{\max}]$ according to a cdf F that has a finite mean and strictly positive density everywhere on its domain. We require that π_{\min} be nonnegative.

There is a profit tax with a flat rate t , together with surcharge rates *per unit of evaded tax*, s_1 on the firms and s on the specialist, set by the government. After observing its profit, each firm has to decide how much tax it wants to evade. To do so, the firm has to ask the tax specialist for assistance, e.g., to forge some bills issued by fictitious firms.

There is a tax authority that visits firms costlessly with a basic frequency r_1 . Conditioned upon a visit, the probability of detecting sophisticated evasion is r . The tax authority can alter this probability, but it is costly to do so. The simple auditing probability r_1 is exogenous to the decision of the tax authority, as we think of it as reflecting resources that the government has decided to invest in tax compliance monitoring, e.g., a law determines how often the authority should visit the firms. As long as the tax authority has a limited budget, we have to assume $r_1 < 1$, though we shall also discuss a degenerate case in which $r_1 = 1$.

2.1 Sequence of moves and information

The evasion specialist moves first, quoting the price p per unit of unreported income at which it is ready to forge documents. The second move is made by nature, which assigns a type embodied in the profit level π for each of the firms. The firms move third, deciding on how much profit to report, π_r . The tax authority moves last, deciding on the auditing probability r after observing the firms' reports. After this, payoffs to the tax specialist, firms, and tax authority are realized. The tax rate t , surcharge rates s_1 and s , and basic auditing frequency r_1 are exogenous parameters characterizing the institutional arrangement of the game.

All these parameters are common knowledge. The realization of its own profit π is known to the firm and to the tax specialist; the distribution F of the random variable π is common knowledge.

2.2 Players and strategies

1. *Specialist.* Consider a local monopolist that sets a price $p \in (0, 1)$ to maximize its expected profit, taking into account the response of the real sector, $\pi_r(p, \pi)$ (below we drop the arguments for brevity) and a punishment for soliciting evasion, s . The realized profit of the specialist is

$$\Pi = pE(\pi - \pi_r) - c_s(E(\pi - \pi_r)) - str_1Er(\pi - \pi_r), \quad (1)$$

where $E(\pi - \pi_r)$ is the total evaded income, and $c_s(\cdot)$ is the cost function of the specialist. Note that the realized and the expected profit are equal, as the specialist serves an infinite population of the real-sector firms.

2. *Taxpayers.* Each of the firms characterized by profit $\pi \in [\pi_{\min}, \pi_{\max}]$ maximizes the expected after-tax profit I by choosing the tax report $\pi_r(p, \pi) \in [0, \pi]$:

$$I(\pi, \pi_r) = \pi - t\pi_r - p(\pi - \pi_r) - t(1 + s_1)r_1r(\pi - \pi_r). \quad (2)$$

3. *Tax authority.* It chooses $r(\pi_r, p)$, given a belief about π , observed report π_r , and known profit distribution F , to maximize expected revenue

$$R(\pi_r, r; \nu) = t\pi_r + rr_1t(1 + s + s_1)(E_\nu\{\boldsymbol{\pi}|\pi_r\} - \pi_r) - c(r), \quad (3)$$

where $\nu(\pi|\pi_r)$ is a belief about the distribution of true profits given the reported profits:

$$E_\nu\{\boldsymbol{\pi}|\pi_r\} = \int_{\pi_{\min}}^{\pi_{\max}} \pi d\nu(\pi|\pi_r).$$

In the case of separating equilibrium there are point beliefs

$$E_\nu\{\boldsymbol{\pi}|\pi_r\} = \hat{\pi}(\pi_r) : [0, +\infty) \rightarrow [\pi_{\min}, \pi_{\max}].$$

Notice that despite the complete revelation in a separating equilibrium, the tax authority has to incur the auditing costs in order to prove that the sophisticated evasion has actually taken place. Sending letters to the taxpayers with claims about their hidden income would not be credible.

3 Exogenous price

We first consider the exogenous specialist price, i.e., the subgame that excludes the specialist from the list of players. We call a perfect Bayesian Nash equilibrium of this subgame *complete pooling* if everybody submits zero reports; we call it *complete separating* if each firm submits a different report; we call it *hybrid* if some reports are distinct and some are pooled. In any perfect Bayesian Nash equilibrium, (i) the reporting strategy of each firm is maximizing its expected after-tax profit given the verification policy of the authority; (ii) the verification strategy of the authority is maximizing its tax revenue given the beliefs about the reporting strategy; (iii) the beliefs about the reporting strategy are consistent with the actual reports by the firms.

Before characterizing the equilibrium of the game formally, let us discuss the game intuitively and see what we may expect the equilibrium to look like. Firstly,

since the tax authority has no commitment ability, the existence of the equilibrium is not granted *per se*. In principle, it could be that we have a classical predator–prey relation in which tax authority chooses low audit in response to low evasion. But low auditing makes firms evade more, which in turn makes the tax authority raise auditing intensity. Such cycling of best responses would mean that an equilibrium in pure strategies does not exist.

Fortunately, Reinganum and Wilde (1986) establish the existence of separating equilibrium in our setting without a specialist price. The way out of cycling is that low reports are audited relatively heavily, and high reports relatively lightly. As a result, the firms that enjoy low auditing do not have an incentive to evade more (lower reports would trigger higher auditing intensity); the firms that evade a lot do not have an incentive to make a higher report, as the decrease in auditing probability does not compensate the loss from paying additional tax.

But if the tax authority is not efficient enough, it will find it too costly to audit the lowest reports sufficiently intensively for this incentive-compatible arrangement to work. In such a situation, the lower end of the distribution might want not to report any profit. In the case of extreme inefficiency, when the expected revenue from auditing a randomly drawn report is lower than the cost of such auditing, no firm will report anything.

This intuition is confirmed by the propositions below, with a qualification that a low enough price of the specialist service induces complete pooling (full evasion) for any positive costs of auditing, if fines are not too high ($(1 + s_1)r_1 < 1$). We discuss this assumption after presenting the results formally.

We take the audit cost function from an example in Reinganum and Wilde (1986) with $c(r) = -c \ln(1 - r)$. We first characterize a complete separating equilibrium, in which we greatly borrow from Reinganum and Wilde (1986). We call the separating equilibrium *strict* if each firm strictly prefers to make its equilibrium report; we call it *weak* if all the firms are indifferent between making the equilibrium report and some other report.

Denote the equilibrium values of the report by π_r^* , which is a function of the profit π , and the probability of deep auditing by r^* ; the equilibrium point belief about the true income is $\hat{\pi}^*(\pi_r)$. Before characterizing the equilibria of our subgame, we state a lemma that specifies how the tax authority responds to a profit report (denote $\mu := (1 + s + s_1)tr_1$):

Lemma 1 Consider a subgame defined by given specialist price p . The best response of the tax authority with a belief ν to a tax report π_r is

$$r(\pi_r; \nu) = 1 - \frac{c}{\mu(E_\nu\{\boldsymbol{\pi}|\pi_r\} - \pi_r)}. \quad (4)$$

The proof of the lemma and propositions 1–3 can be found in the appendix.

In the following proposition we establish the existence conditions for the separating equilibrium in our subgame (denote $B := \mu - (t - p)(1 + s + s_1)/(1 + s_1)$):

Proposition 1 (separating equilibrium) Consider a subgame defined by a given specialist price p . Assume the auditing is cheap: $c/\mu < \pi_{\min}$.

(i) If $t(1 - r_1(1 + s_1)) < p \leq \min\{t(1 - r_1(1 + s_1)(1 - c(\mu\pi_{\min})^{-1})), t\}$, there exists a complete separating (strict) equilibrium characterized by the triple $\{\pi_r^*, r^*; \hat{\pi}^*\}$ with π_r^* defined by

$$\pi - \pi_r^*(\pi) = \left(\frac{c}{\mu} - \frac{c}{B}\right) e^{\frac{B}{c}(\pi_r^*(\pi) - \pi_r^*(\pi_{\max}))} + \frac{c}{B}, \quad (5)$$

$$r^* = 1 - \frac{c}{\mu(\hat{\pi}^* - \pi_r^*)}, \quad (6)$$

$$\hat{\pi}^*(\pi_r) = \begin{cases} \pi_{\min}, & \pi_r < \pi_r^*(\pi_{\min}), \\ \pi_r^{*-1}(\pi), & \pi_r \in [\pi_r^*(\pi_{\min}), \pi_r^*(\pi_{\max})], \\ \pi_{\max}, & \pi_r > \pi_r^*(\pi_{\max}). \end{cases}$$

(ii) If $t(1 - r_1(1 + s_1)(1 - c(\mu\pi_{\min})^{-1})) < p \leq t$, there exists a complete separating (weak) equilibrium characterized by the triple $\{\pi_r^*, r^*; \hat{\pi}^*\}$ with π_r^* defined by

$$\pi - \pi_r^*(\pi) = \frac{c}{\mu} \left(1 - \frac{t - p}{(1 + s_1)tr_1}\right)^{-1} \quad (7)$$

and the expressions (6).

(iii) If $p > t$, there exists a complete separating (honesty) equilibrium characterized by

$$\begin{aligned} \pi_r^* &= \pi, \\ r^* &\equiv 0, \\ \hat{\pi}^*(\pi_r) &= \begin{cases} \pi_r^{*-1}(\pi), & \pi_r \in [\pi_r^*(\pi_{\min}), \pi_r^*(\pi_{\max})], \\ \pi_{\max}, & \pi_r \notin [\pi_r^*(\pi_{\min}), \pi_r^*(\pi_{\max})]. \end{cases} \end{aligned} \quad (8)$$

The analysis of the pooling equilibria in our game is complicated by the fact that there is a continuum of them¹². However, we choose the pooling at zero report as an obvious focal point. We also do not consider all kinds of hybrid equilibria that could potentially arise in the subgame, but only the pooling at zero report for the types below a certain profit, and the separating equilibrium for the types above it. The reason for this is that we want to distinguish clearly the high-evasion regime (complete pooling) and the low-evasion regime (complete separation or hybrid).

Proposition 2 (hybrid equilibrium) *Consider a subgame defined by the given specialist price p . Assume the auditing is not very expensive: $\pi_{\min} < c/\mu < E(\boldsymbol{\pi}|\pi \leq \pi^0)$, where π^0 simultaneously solves (5) and the indifference condition $I(\pi^0, \pi_r^*(\pi^0)) = I(\pi^0, 0)$.*

(i) *If $t(1 - r_1(1 + s_1)) < p \leq \min \left\{ t \left(1 - r_1(1 + s_1) \left(1 - c(\mu E(\boldsymbol{\pi}|\pi \leq \pi^0))^{-1} \right) \right), t \right\}$, there exists a hybrid (strict) equilibrium characterized by the triple $\{\pi_r^*, r^*; v^*\}$ with π_r^* defined by (5), r^* defined by (6) for any $\pi \geq \pi^0$, and*

$$\begin{aligned} \pi_r^*(\pi) &= 0, \\ r^* &= 1 - \frac{c}{\mu E(\boldsymbol{\pi}|\pi \leq \pi^0)} \end{aligned} \tag{9}$$

for any $\pi < \pi^0$. The beliefs are

$$\nu^*(\pi | \pi_r) = \begin{cases} F(\pi | \pi \leq \pi^0) & \text{if } \pi_r = 0, \\ D & \text{if } 0 < \pi_r < \pi_r^*(\pi^0), \\ \pi_r^{*-1}(\pi), & \pi_r \in [\pi_r^*(\pi^0), \pi_r^*(\pi_{\max})], \\ \pi_{\max}, & \pi_r > \pi_r^*(\pi_{\max}) \end{cases} \tag{10}$$

$$\forall D | E_D \{\boldsymbol{\pi} | \pi_r\} \geq E(\boldsymbol{\pi} | \pi \leq \pi^0) + \pi_r.$$

(ii) *If $t \left(1 - r_1(1 + s_1) \left(1 - c(\mu E(\boldsymbol{\pi}|\pi \leq \pi^0))^{-1} \right) \right) < p \leq t$, there exists a hybrid (weak) equilibrium characterized by the triple $\{\pi_r^*, r^*; v^*\}$ defined by (6)–(7) for any $\pi \geq \pi^0$ and (9) for any $\pi < \pi^0$. The beliefs are determined according to (10).*

(iii) *If $p > t$, there exists a complete separating (honesty) equilibrium characterized by (8).*

The following proposition establishes existence conditions for the complete pooling at zero equilibrium:

¹²Note that intuitive or divinity criteria are not applicable in our subgame, as it has a continuum of types.

Proposition 3 (pooling equilibrium) *If $p \leq t(1 - r_1(1 + s_1))$ or $c/\mu > E\pi$, there exists a complete pooling equilibrium characterized by the triple $\{\pi_r^*, r^*; \nu^*\}$ with*

$$\begin{aligned} \pi_r^* &\equiv 0, \\ r^* &= \max \left\{ 0, 1 - \frac{c}{\mu E\pi} \right\}, \\ \nu^*(\pi | \pi_r) &= \begin{cases} F(\pi) & \text{if } \pi_r = 0, \\ D & \text{if } \pi_r > 0, \end{cases} \\ D &: E_D \{\pi | \pi_r\} = E\pi. \end{aligned} \tag{11}$$

A separating equilibrium does not exist.

If $p > t(1 - r_1(1 + s_1))$, the pooling equilibrium described above does not exist¹³.

The table below summarizes the results in Propositions 1–3:

		c/μ		
		Low	Medium	High
p	Low	Full cheating	Full cheating	Full cheating
	Medium	Separation	Hybrid	Full cheating
	High ($p > t$)	Full honesty	Full honesty	Full honesty

We see that there are two factors that determine what kind of equilibrium exists: (i) the relative auditing costs c/μ , (ii) the specialist price p . When the auditing costs are very small ($c/\mu < \pi_{\min}$), there may be complete separating or complete pooling equilibrium, depending on the specialist price. When the costs are higher ($\pi_{\min} < c/\mu < E(\pi | \pi \leq \pi^0)$), complete separating equilibrium does not exist; the specialist price determines whether hybrid or pooling equilibrium is played. For substantial auditing costs ($c/\mu \geq E(\pi | \pi \leq \pi^0)$), the equilibrium is either pooling or in mixed strategies, so in our subsequent analysis we assume $c/\mu < E(\pi | \pi \leq \pi^0)$.

For a small specialist price there is complete pooling equilibrium, i.e., no firm submits a truthful report, and the tax authority picks the firms for auditing with uniform probability. For a higher price there is separating or hybrid equilibrium, in which each firm with high enough profit conceals a part of it, and the tax authority can deduce the profit of such firms from their reports. When the price is prohibitively

¹³This amounts to the uniqueness of the equilibria of Propositions 1–3 in the space of separating, hybrid, and all complete pooling equilibria. I am grateful to an anonymous referee for this observation.

high, all the firms submit truthful reports and the tax authority does not perform deep auditing, so we have full honesty equilibrium.

Intuitively, in the pooling equilibrium any firm should prefer submitting a zero report to submitting a positive one. Thus, the gain from decreased auditing probability at higher report should not outweigh the loss of the profit from forgone evasion. If the authority believes that any positive report implies higher than average profit, it will not reduce its optimal auditing effort enough to make a nonzero report attractive. Then for such out-of-equilibrium belief of the authority, pooling at zero is the equilibrium, with the correct equilibrium belief being unconditional distribution of profits.

In any equilibrium, a firm plays the best response to both the verification strategy of the tax authority and the reporting behavior of other firms. In a separating equilibrium with $\pi_r(\pi)$ strictly increasing, every firm writes its report in such a way that it reveals its profit level. This happens if $p > t(1 - r_1(1 + s_1))$. When the price of auditing is high enough ($t(1 - r_1(1 + s_1)(1 - c(\mu\pi_{\min})^{-1})) < p \leq t$), the firms are indifferent between evading and reporting, so we have weak separating equilibrium.

The same logic applies to the hybrid equilibrium. The difference is that all firms with the profit below π^0 submit zero reports. This allows for more intensive auditing of the lowest reports than in the complete separation case. Hence, hybrid equilibrium exists when auditing is costly enough for complete separation to fail. Hybrid equilibrium is not distribution-independent: the threshold level of profit, π^0 , at which a firm is indifferent between submitting zero and positive report is determined by the shape of the profit distribution.

For $p > t$, evasion would not make sense, so the firms report full profit. The lower threshold for price, $t(1 - r_1(1 + s_1))$, determines whether there is pooling or separating equilibrium (recall that we select pooling at zero whenever there is neither separating nor hybrid equilibrium). This threshold is determined by the tax rate and the fine faced by a firm in case of detection: the higher the fine, the smaller is the range where the separating (or hybrid) equilibrium does not exist.

At the extreme, when all the firms are visited ($r_1 = 1$), the threshold becomes $-ts_1$, that is, separation exists for any specialist price, given that the auditing is not too expensive. The threshold is very intuitive, as t can be thought of as the marginal benefit from evasion, whereas $p + tr_1(1 + s_1)$ can be thought of as the marginal cost for an audited firm ($r = 1$). Thus, when the benefits are higher even for a detected

evader, no separation can exist (everybody evades everything). However, in reality the tax authority never has the resources to inspect all the firms, so $r < 1$. In fact, the evidence summarized in Andreoni et al. (1998) implies that in reality $r_1(1 + s_1) < 1$, that is, the auditing probability is on average below its Nash equilibrium counterpart¹⁴.

This completes the description of the game between tax authority and taxpayers. It is valid for any industry structure of the specialist service, ranging from perfect competition to monopoly. In any case, for a low specialist price an equilibrium with firms evading everything is played; for a higher price a separating equilibrium with firms evading some part of their profit is played; for a price higher than the tax rate all firms report honestly.

Before looking more closely at the price-setting behavior of the specialist, the interested reader may find in appendix E two results that characterize the evasion behavior in the separating or hybrid equilibrium considered.

4 Price setting

In the previous section we considered evasion subgame equilibria for any fixed specialist price p . The interesting question, though, is what price a specialist would want to charge if it were a monopolist¹⁵. For simplicity we assume here a linear cost function of the specialist, $c_s(x) = c_s x$. Recall that we have also assumed $c/\mu < E(\pi|\pi \leq \pi^0)$, as otherwise only pooling equilibrium exists and the problem of choosing the specialist price is trivial.

Clearly, if $c_s \geq t$, honesty prevails, so there is no room for the specialist. Otherwise, if $c_s < t$, the specialist can choose any price $p \in (t(1 - r_1(1 + s_1)), t)$ to get a separating or hybrid equilibrium, or any price $p \leq t(1 - r_1(1 + s_1))$ to get pooling, or else close down, ensuring zero profit and complete honesty. Formally, the specialist maximizes its profit by choosing p :

$$\int_{\pi_{\min}}^{\pi_{\max}} (p - c_s - str_1 r^*(\pi_r^*(p, \pi))) (\pi - \pi_r^*(p, \pi)) dF(\pi). \quad (12)$$

¹⁴We do not endogenize the basic auditing probability r_1 , as we believe this decision has substantially longer horizon than the auditing intensity r . Moreover, it is likely to be a government, not a tax-authority, decision. Hence, it is may be chosen to maximize welfare and not the tax revenue.

¹⁵Monopolistic structure seems most realistic for the evasion industry. For the details see the introduction.

The following condition turns out to be important for the specialist's decision:

$$t < c_s + (1 + s + s_1) tr_1. \quad (13)$$

Intuitively, this condition states that the marginal benefit from evasion, t , is lower than the sum of expected marginal costs of evasion for the specialist, $c_s + str_1$, and for the real-sector firms, $(1 + s_1) tr_1$. Accordingly, there is no scope for full evasion in this configuration. Note that $r = 1$ is implicit in this condition: the deep auditing always discovers sophisticated evasion. Without perfect detection, we still expect no complete evasion, as the deep auditing should be high when evasion is high (recall that the tax authority always plays the best response).

When the condition (13) is not satisfied, that is, if the marginal benefit from evasion is higher than the sum of marginal costs even for perfect auditing, we expect that pooling may be possible. This should happen if the specialist finds it attractive to share the evasion gain, i.e., when the (implicit) price elasticity of demand for evasion is high enough. This intuition is formalized in the following proposition¹⁶, which characterizes the equilibrium play in our game:

Proposition 4 *The specialist anticipates the subgame play given by Propositions 1–3. Its profit is maximized at:*

- (i) *A separating equilibrium with the price t , if the condition (13) is satisfied.*
- (ii) *A pooling equilibrium with the price $p^p = t(1 - r_1(1 + s_1))$, if the condition (13) is not satisfied and the following relation holds¹⁷:*

$$(p^* - c_s - str_1) \int_{\pi_{\min}}^{\pi_{\max}} e^*(p^*, \pi) dF(\pi) < (t - c_s - \mu) E\pi + \frac{cs}{1 + s + s_1}. \quad (14)$$

Here p^* is determined by the following relation:

$$\int_{\pi_{\min}}^{\pi_{\max}} e^*(p^*, \pi) dF(\pi) + (p^* - c_s - str_1) \int_{\pi_{\min}}^{\pi_{\max}} e_p^*(p^*, \pi) dF(\pi) = 0.$$

- (iii) *A separating equilibrium with the price $p^* \in (p^p, t]$, if the conditions (13) and (14) are not satisfied.*

¹⁶There is no straightforward relation between the parameters of the model and the elasticity of demand for evasion; (14) states a precise condition under which the elasticity is “high enough.”

¹⁷Where $e^*(\pi) := \pi - \pi_r^*(\pi)$.

The formal proof of (i) is left to appendix F. If the condition (13) is satisfied, the profits from separating or hybrid equilibrium are maximized at the highest possible separating price, t . In (ii) and (iii) the condition (13) is not satisfied, so the specialist's profit from separating equilibrium may be maximized at an interior. That is why in (14) we implicitly compare profits from separation and pooling. Note that positive profit in pooling equilibrium is assured by the violation of the condition (13), and this implies that whenever the condition (14) is satisfied, there is a positive profit in separation as well.

Roughly, if the fines are too low ($t > c_s + (1 + s + s_1)tr_1$), then either pooling or separation can be played, depending on the structure of the fines. Otherwise the system ends up in a separating or hybrid equilibrium with $p = t$.

5 Government

Now, consider the government, which cares about reduction of evasion. Similarly to Crocker and Slemrod (2005), we look at the effectiveness of different fines in deterring evasion and get a nonequivalence result. In separating equilibrium both fines are equally effective, as the evasion $e^*(t) = c/\mu$ is minimal and constant across income levels (apart from the pooled types in a hybrid equilibrium). In pooling equilibrium, they are *marginally* equally ineffective, as the evasion volume is locally insensitive to the enforcement parameters. An interesting case is a jump from pooling equilibrium (ii) to separating or hybrid equilibrium (iii), and here fines on the real-sector firms do a better job, as the following proposition states (the proof is presented in appendix H).

Proposition 5 *If the condition (13) is not satisfied and there is a pooling equilibrium, fines on the real firms are more effective in pushing the system to a separating equilibrium than fines on the specialist are.*

Thus, our model predicts that the whole burden of punishment should lie on the firms actually evading tax. One might think that this result stems from the monopoly power of the specialist, who extracts all the rents in the separating equilibrium. Indeed, the firms are favoring full cheating more, because they get a part of the pie in the pooling equilibrium. However, in section 6 we show that the result on the effectiveness of the fines persists in the case of a competitive evasion industry. The

essential thing is that the specialist is a first mover: It is her decision what price to charge and thereby which equilibrium to induce in the subgame.

The profit in the separating equilibrium, $\Pi^s = (t - c_s) c/\mu$, stays the same regardless of whether the specialist or the firms are fined. Thus, for our result about the effectiveness of fines to hold, charging the specialist must affect her profits in pooling equilibrium more weakly than charging the firms. Essentially, proposition 5 proves just that. The intuition is fairly general: the specialist (or the evasion industry in the competition case), as a first mover, has but an imperfect instrument to adjust the reaction of the real sector to the fine; at the same time, she can perfectly adjust her own choice in response to the fine on the specialist.

Having looked at what it takes to go from full cheating to separation, it is natural to also ask what it takes to go from separation to full honesty. In other words, if the government wants to destroy specialists, as many newspapers recommend, how costly would it be? As the evasion in separating equilibrium is fixed at c/μ , it is obviously infinitely costly to get rid of it completely by raising fines. A much more effective way to make specialists inactive is to raise their costs all the way up to the tax rate. This could be achieved through employing better accounts monitoring and cross-checking systems, or by increasing the costs of running accounts.

The intuition here is straightforward: however large the fines are, it is always possible to make underreporting so small that the auditing is rare and the expected value from evasion is nonnegative. And even small evasion, if done by many firms, may generate substantial revenue for the specialist. At the same time, if the operating costs in the evasion industry are high enough, it simply does not pay to provide the service. This result crucially depends on the auditing rule assumed in this paper: the authority performs revenue-maximizing random auditing. If the tax authority were committed to a fixed auditing frequency, for example, high enough fines could obviously eliminate evasion completely.

Finally, let us emphasize how the optimal tools for fighting evasion depend on the extent to which the evasion is spread in an *audit class*. When the sophisticated evasion is pervasive (pooling¹⁸), a marginal increase in enforcement is not likely to affect compliance behavior. In terms of our model, the system exhibits inertia with respect to compliance, when in the pooling equilibrium. It is the specialist who adjusts the

¹⁸The evasion volume in the pooling equilibrium that we select is maximal. Under a weak condition presented in appendix D, the evasion volume in *any* complete pooling equilibrium is larger than that in the separating equilibrium.

price and the tax authority that adjusts auditing – the firms keep evading everything. In such circumstances, the system should be pushed to the separating equilibrium in order to achieve any reduction in evasion. This can be done by the means of conventional enforcement (fines and auditing intensity), but the corresponding change is bound to be nontrivial.

When sophisticated evasion is rare, marginal changes in enforcement are effective in reducing it, but at increasingly lower rate. In our model, the separating equilibrium exhibits full honesty only at the limit of infinitely high fines. This happens because no matter what the fines are, the specialist can effectively insure herself against them and provide as little evasion service as is profitable for given fines. On the other hand, if the specialist faces high costs of producing the evasion service, she will decide to close down anyway. Therefore, with small evasion rates the government may be better off making falsification of accounts more costly.

In addition to these findings, we summarize the comparative statics for the two types of equilibria we have¹⁹. The auditing probability in the pooling equilibrium, $1 - c/(\mu E\pi)$, is increasing in the fines, the tax rate, and the superficial auditing frequency. It is decreasing in the costs of auditing. The intuition is straightforward: the former factors increase the direct benefits of a deep audit; the latter one increases its costs. The evasion volume is obviously not sensitive to parameter changes in the pooling equilibrium.

In separation with $p = t$, there is no auditing, so the auditing probability is insensitive to the parameter shifts. The cheating volume c/μ is a standard result also obtainable from conventional models without specialists. Of course, we should keep in mind that sufficiently cheap auditing is crucial for all our results, as otherwise full cheating is an unchallengeable outcome.

6 Discussion: alternative specifications

6.1 Industry structure

One may wonder what happens in an industry plagued with sophisticated evasion if the specialists are not local monopolists as we have assumed above, but rather compete in prices. In the first stage of the game, then each specialist quotes a price,

¹⁹Payoffs and comparative-statics results associated with the equilibrium play are derived in appendix G.

and the subgame with the lowest price as a fixed price for specialist service follows. As in a simplest Bertrand setting, the competition will drive the prices down to the marginal costs. A complication in our model is that the marginal costs are increasing, and the demand is kinked. Namely, the specialist's marginal cost is $c_s + str_1 r(\pi_r, p)$, and the demand for the service is the subgame equilibrium evasion whenever $p \leq t$.

One equilibrium candidate is charging the minimal marginal cost c_s . This is an equilibrium only in the very special case of $t = c_s$. If $t > c_s$, the specialist charging $p = c_s$ suffers a loss of $str_1 r E(\pi - \pi_r^*(\pi))$, and has a profitable deviation consisting in not providing the service at all. The following proposition characterizes the equilibrium in this case:

Proposition 6 *If the specialists compete in prices à la Bertrand in anticipation of the subgame play given by Propositions 1–3 and $t > c_s$, the equilibrium is characterized by the following specialist price:*

(i)

$$p^{wc} = \frac{c_s(1 + s_1) + st}{1 + s + s_1} \quad (15)$$

if $c_s(1 + s_1) + st > (\mu - ce^{-1}(\pi_{\min}))(r_1^{-1} - (1 + s_1))$. The weak separating equilibrium of the subgame is played.

(ii)

$$p^{sc} = c_s + str_1 \left(1 - \frac{c}{\mu e(p^{sc})} \right), \quad (16)$$

$$e(p^{sc}) = \int_{\pi_{\min}}^{\pi_{\max}} (\pi - \pi_r^*(p^{sc}, \pi)) dF(\pi) \quad (17)$$

if $t(1 - r_1(1 + s_1)) < p^{sc} \leq t(1 - r_1(1 + s_1)(1 - c(\mu e(\pi_{\min}))^{-1}))$. The strict separating equilibrium is played.

(iii)

$$p^{pc} = c_s + str_1 \left(1 - \frac{c}{\mu E\pi} \right) \quad (18)$$

if $c_s - str_1 c(\mu E\pi)^{-1} < t - \mu$. The pooling equilibrium is played in the subgame.

(iv) No pure strategy Bayesian SPNE exists if none of conditions (i)–(iii) is satisfied.

Sketch of the proof. Any competitive equilibrium in our game must be characterized by a zero-profit condition for the specialists. Depending on the subgame equilibrium, this is (15), (16), or (18). Indeed, the cost of serving the firms is $\int_{\pi_{\min}}^{\pi_{\max}} (c_s + str_1 r^*(\pi_r^*(p, \pi))) (\pi - \pi_r^*(p, \pi)) dF(\pi)$, whereas the revenue is

$p \int_{\pi_{\min}}^{\pi_{\max}} (\pi - \pi_r^*(p, \pi)) dF(\pi)$. After equating and rearranging we get the expressions above.

Consider possible deviations of any of the specialists (there can be any finite number of them larger than one) charging the price specified by the proposition. The deviations to any higher price bring about the same payoff, i.e., zero. The deviations to any lower price bring about losses. Note that rationing customers at a lower price does not help to reduce costs, as the tax authority observes the price, not the quantity, and audits according to the best response specified in the subgame. Thus, we indeed have an equilibrium strategy for all the specialists, if the specified price corresponds to the subgame equilibrium.

Finally, we provide the conditions under which each of the subgame equilibria is played. When the specialist costs are sufficiently high, the separating equilibrium results in zero profit. When the cost are low, a separating equilibrium brings about positive profits, so the firms charge a price low enough to trigger a pooling equilibrium. When any separating equilibrium brings about positive profit, but the pooling equilibrium results in a negative profit, the specialists randomize, and no pure-strategy equilibrium obtains. ■

Competition is effective in our setting: it drives the profits of the specialists to zero independently of parameter values. Contrary to the monopolistic case, the real-sector firms are left with an expected surplus from evasion. In pooling equilibrium this is of magnitude $(t(1 - r_1(1 + s_1)) - p^{pc})\pi$.

We want to compare the two industry structures according to the volume of evasion they generate. First, the condition for the pooling is $c(t - c_s - str_1) < (t - c_s - \mu)\mu E\pi$ in the monopoly case versus $-str_1c < (t - c_s - \mu)\mu E\pi$ in the competition case. It can be seen that the former condition is more stringent. Thus, competition results in a larger evasion volume than a monopoly does, whenever $-str_1 < (t - c_s - \mu)\mu E\pi/c < t - c_s - str_1$. Second, in a separating equilibrium the evasion volume is larger with competition whenever $p^c < t$, as the monopolist chooses a minimum of c/μ . This leads us to the following corollary.

Corollary 1 If the condition (13) is satisfied, the evasion volume with competition among the specialists is at least as high as with a monopoly.

The usual intuition from industrial organization theory goes through in our model: monopoly leads to a reduction of production relative to the competitive benchmark.

In our setup, it is the sophisticated evasion service that is being produced. Hence, in so far as we do not like monopoly in the production of goods, we should like it in the production of “bads,” supposing it is too costly to eliminate such production altogether.

As we can see, our previous result about the relative effectiveness of the fines still holds in the absence of monopoly power. Namely, the same term $-str_1$ makes fining specialists better for the pooling. In the weak separating equilibrium the fines are equally ineffective: their influence is completely offset by the adjustment of price, and the evasion level stays at a constant $c(1 - (t - c_s))^{-1}$. Remarkably, the evasion level here is increasing in tax rate, contrary to the monopolistic case.

6.2 Specialist cost function

One of the assumptions that restrict applicability of our analysis is the linear cost function of the specialists. Indeed, one may believe that it becomes increasingly costly to muddle through the accounts as the evasion volume increases, both on the firm and on the industry level. While the cost rise on the firm level is partially reflected in the detection probability increase, the industry-level cost rise has been left out of our model so far.

At the same time, there can be the opposite spillover effect: it is costly to develop a complicated evasion scheme, but, once developed, it can be applied to many firms relatively inexpensively. This technology effect may actually provide an additional justification for the monopolistic structure of the industry of sophisticated evasion.

We shall think of the spillover effect as some fixed costs needed to start the business. The opposite effect can be captured by a convex cost function. The specialist’s objective function (12) then becomes

$$\int_{\pi_{\min}}^{\pi_{\max}} (p - str_1 r^*(p, \pi)) e(p, \pi) dF(\pi) - c_s \left(\int_{\pi_{\min}}^{\pi_{\max}} e(p, \pi) dF(\pi) \right) - C. \quad (19)$$

Consider the fixed costs C first. As they do not distort the pricing decision of the monopolist, all our results go through, with the qualification that for high enough fixed costs full honesty equilibrium results.

The convexity does introduce additional complications, but the condition for profits to be maximized at $p = t$ can be reduced to an expression like (13), where instead of c_s we use $c'_s(e)$. A more precise condition can be written as $e(t) + (t - str_1 - c'_s(e)) e'(t) \geq 0$, which is of course a familiar inverse elasticity rule of

the monopolist adapted to our setting. Intuitively, depending on whether equilibrium evasion is larger or smaller than that in the linear-cost case, the condition will be correspondingly less or more restrictive. The separating equilibrium will be preferred for a larger set of parameter values, as full cheating becomes relatively more costly. The rest of the story is virtually unaltered.

6.3 The authority's cost function

The cost function of the authority used in our model may seem very specific. However, Reinganum and Wilde (1986) have shown that the separating equilibrium of the type that we discuss exists for a large class of auditing functions. Namely, they assume $c(0) = 0$, and twice continuous differentiability with $0 < c'(r) < \infty$, $0 < c''(r) < \infty$, $\lim_{r \rightarrow 1} c'(r) = \infty$, and $c'(r)/c''(r) + r > 1/(1+s)$. The best response of the tax authority is then

$$r(\pi_r; v) = c'^{-1}(\mu(E_\nu \{\boldsymbol{\pi} | \pi_r\} - \pi_r)), \quad (20)$$

which is a generalized form of the expression (4).

Obviously, our strict separating equilibrium of the subgame is valid under the same restrictions, as it completely mimics the equilibrium of Reinganum and Wilde. The weak separating equilibrium also exists under these conditions, with

$$e = \frac{1}{\mu} c' \left(\frac{t-p}{(1+s_1)tr_1} \right). \quad (21)$$

Finally, the pooling at zero equilibrium also survives the generalization, with the existence conditions modified appropriately.

We have chosen a specific function for a clear characterization of the subgame equilibrium, but it can be seen that our equilibrium structure admits generalization to the class of auditing functions in Reinganum and Wilde (1986).

7 Conclusion

The game between tax authority, taxpayers, and a tax specialist featuring stylized reality of corporate scandals and sophisticated evasion is analyzed in this paper. We consider illiterate firms, i.e., firms that do not know how to evade taxes. We identify three types of equilibria for a given specialist price: (i) complete pooling at zero report; (ii) complete separation with true profit revelation; (iii) hybrid equilibrium with low

types submitting zero reports and high types revealing their profit. Furthermore, complete separating and hybrid equilibria can be of two different types, strict or weak. As suggested by the terms, in strict equilibrium the firms strictly prefer to submit equilibrium reports; in weak equilibrium the firms are indifferent between cheating and reporting honestly. Finally, there is a special case of the separating equilibrium in which all firms report truthfully.

Introducing the specialist who can choose the price for her services reduces the number of equilibrium types for a large set of parameter values: the specialist chooses (i) complete pooling or (ii) separating or hybrid equilibrium at the highest possible price. In separating equilibrium the specialist gets all the evasion rent, whereas in pooling she has to share it with the firms. For the high-evasion regimes (pooling at zero equilibrium), fines on the evading firms are more effective in driving the system out of full evasion than fines on the specialist preparing documents for this evasion. For the low-evasion regimes (separating or hybrid equilibrium), increasing the costs of complicating accounts is more effective than conventional enforcement measures.

Our results are robust to a number of changes in the model specification. Competition between the specialists expands the set of possible equilibria and increases tax evasion, but it does not change the equilibrium structure or the effectiveness of the enforcement instruments. A convex cost function of the specialist does not alter the analysis. Finally, the equilibrium structure is preserved for a rather broad class of monotonically increasing convex auditing functions²⁰.

While interpreting our results, one should keep in mind that the derived equilibrium describes a situation in a given audit class. The differences in audit classes are summarized by the auditing cost function of the tax authority. In the presence of a monopolistic tax specialist, our model predicts polarization of evasion/avoidance levels in different audit classes, viz., very high levels for classes with expensive auditing, and low levels for classes with cheap auditing. This prediction is potentially testable empirically, in that with competitive tax specialists or simple (noncorporate) evasion, the distribution of evasion volume across different audit classes should be more uniform.

Aggregation issues aside, our results may be in line with some economy-wide experiences. For example, the success of Russian tax law enforcement policy in the

²⁰The equilibrium structure is also robust to introducing (not too high) risk aversion – the equilibrium prices are just shifted down from t and $t(1 - (1 + s_1)r_1)$.

flat-tax reform, which was accompanied with both a large increase in punishment for evasion for the firms and no change in the responsibility of accounting specialists, is consonant with our inequivalence result. The Sorbannes–Oxley act in the US, on the other hand, may be justified in light of the result that the costs of falsifying accounts are effective in reducing evasion in high-compliance separating equilibrium.

This might suggest that developing countries with low compliance could find themselves better off by following the reform path, whereas developed countries with high compliance could benefit from introducing stricter regulation of financial intermediaries. The interpretation of developing countries experiencing high levels of evasion and developed countries enjoying high compliance is a bit simplistic, as compliance behavior is endogenous in our model and differences in compliance may come from various sources, e.g., difficulty of law enforcement, differences in detection technology, tax morale, etc.²¹ However, all these factors may be summarized by inclusion in the costs of auditing (where we may understand costs broadly, including any social and psychological factors) and the enforcement parameters in our model. Given these parameters, our results allow us to describe theoretically what kind of equilibrium is played. Thus, though the developing–developed interpretation should be applied with caution, it may deserve some attention in policy debates.

Recently, a rapidly growing literature on tax havens (see Slemrod and Wilson (2009) and references therein) has started analyzing tax avoidance and its effect on tax competition between governments. Tax havens may be looked upon as tax specialists, because they are essential in the provision of the avoidance and sophisticated-evasion service. Clearly, tax havens cannot be punished with fines, as they are separate jurisdictions. Our model suggests that this does not represent a constraint for a government interested in reduction of sophisticated tax evasion, as fines on domestic firms are at least as good a deterrent as fines on havens would be. All our main results are new to the tax-haven literature, as it assumes away strategic interaction between tax authority, firms, and tax havens.

The analysis presented is by no means limited to the tax avoidance/evasion phenomenon. A very similar problem arises, for example, in the interaction of a competition authority and firms that are colluding. Most often collusion agreements are bound to be detected if not executed through intermediaries, which are specialist firms. The same story applies: not punishing these intermediaries does not represent

²¹I am grateful to an anonymous referee for pointing this out.

a problem per se.

This work could be extended in a number of ways. The government could be added as an active first mover to set institutional parameters in a way to maximize social welfare or some other objective. Further, the model could be extended to general equilibrium in order to study welfare aspects of enforcement policies. Finally, it would be interesting to estimate the effect of changing enforcement and cost parameters on sophisticated evasion and avoidance.

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Appendices

A - Proof of Lemma 1 - Tax authority best response

As we have seen, the tax authority maximizes

$$R(\pi_r, r; \mu) = t\pi_r + r\mu(E_\nu \{\pi | \pi_r\} - \pi_r) - c(r),$$

the first order condition is hence

$$\mu(E_\nu \{\boldsymbol{\pi} | \pi_r\} - \pi_r) - c'(r) = 0, \quad (22)$$

and the second order condition is simply $c''(r) > 0$.

For the assumed cost function $c(r) = -c \ln(1 - r)$ the FOC can be rewritten as

$$r(\pi_r; \nu) = 1 - \frac{c}{\mu(E_\nu \{\boldsymbol{\pi} | \pi_r\} - \pi_r)}, \quad (23)$$

which is the statement of the Lemma. In a separating equilibrium $E_\nu \{\boldsymbol{\pi} | \pi_r\} = \hat{\boldsymbol{\pi}}$; in case of pooling at zero or any other report below π_{\min} , we have $E_\nu \{\boldsymbol{\pi} | \pi_r\} =$

$E\pi$, as the only consistent belief of the authority is the true underlying distribution $\nu(\pi|\pi_r) = F(\pi)$. When the pooling is at a report above π_{\min} , the belief is updated according to Bayes formula $\nu(\pi|\pi_r) = (F(\pi) - F(\pi_r)) / (1 - F(\pi_r))$ for $\pi \geq \pi_r$ and $\nu(\pi|\pi_r) = 0$ for $\pi < \pi_r$ as the tax authority knows that the taxpayers are rational. But in this paper we mostly restrict our attention to the pooling at zero equilibrium.

B - Proof of Proposition 1 - Separating equilibrium

The separating equilibrium has to satisfy the following conditions: 1) absence of deviation incentives for the taxpayer when the tax authority audits each report with equilibrium probability; 2) absence of deviation incentives for the tax authority when every taxpayer submits equilibrium report; 3) consistent beliefs of the tax authority about the true income of the taxpayers who submit equilibrium reports; 4) arbitrary beliefs of the tax authority about the true income of the taxpayer who submits out-of-equilibrium report.

Concerning 2), the best response of the authority given its belief $\hat{\pi}(\pi_r)$ is given by (4). This is straightforward from the authority maximization problem (the payoff is concave in r). The restriction on r is obviously $0 \leq r \leq 1$, and it is satisfied whenever

$$(\hat{\pi}(\pi_r) - \pi_r)\mu \geq c \quad (24)$$

In words, each report should bring more revenue than costs. If this condition does not hold, the authority's best response is not to audit the report.

For 1) and 3) we have to consider two cases, strict and weak equilibrium.

strict equilibrium

In the strict equilibrium (the firms strictly prefer the equilibrium reporting strategy) the firms maximize their after-tax expected profit

$$\pi - t\pi_r - (p + t(1 + s_1)r_1r)(\pi - \pi_r).$$

The first order condition to this problem is

$$-t - t(1 + s_1)r_1r'(\pi_r)(\pi - \pi_r) + p + t(1 + s_1)r_1r = 0,$$

and the second order condition is

$$-r''(\pi_r)(\pi - \pi_r) + 2r'(\pi_r) \leq 0.$$

One can check that it is satisfied in equilibrium for our auditing function $c(r) = -c \ln(1 - r)$.

Plugging in the tax authority best response (assume $\hat{\pi} - \pi_r > \frac{c}{\mu}$), we can rewrite the first order condition as

$$1 - \frac{c}{\mu(\hat{\pi} - \pi_r)} - (\pi - \pi_r) \frac{c}{\mu(\hat{\pi} - \pi_r)^2} (\hat{\pi}'(\pi_r) - 1) = \frac{t - p}{t(1 + s_1)r_1}.$$

Using the consistent beliefs in the candidate equilibrium $\hat{\pi} = \pi$, we get

$$-\frac{c}{\mu(\hat{\pi} - \pi_r)} \hat{\pi}'(\pi_r) = \frac{t - p}{t(1 + s_1)r_1} - 1,$$

For convenience denoting evasion associated with a given report as $e(\pi_r) \equiv \pi(\pi_r) - \pi_r$, $e'(\pi_r) \equiv \pi'(\pi_r) - 1$, we have

$$c(e' + 1) = \left(1 - \frac{t - p}{t(1 + s_1)r_1}\right) \mu e.$$

Using B defined as in the text ($B := \mu - (t - p)(1 + s + s_1) / (1 + s_1)$) we have

$$ce' - Be + c = 0. \tag{25}$$

This is a first order ordinary differential equation (DE). Its solution is a sum of general solution to the corresponding homogenous DE and a particular solution to the non-homogenous DE. Homogenous equation is $ce'(\pi_r) - Be(\pi_r) = 0$. Its solution is

$$e(\pi_r) = A \exp\{\rho\pi_r\}; \rho = \frac{B}{c}.$$

To find a particular solution, put $e'(\pi_r) = 0$ to get $e(\pi_r) = \frac{c}{B}$. Thus, the general solution of our equation will be $e(\pi_r) = A \exp\left\{\frac{B}{c}\pi_r\right\} + \frac{c}{B}$. To pin down the constant A we need an initial condition, $r(\pi_{r \max}) = 0$ reflecting the fact that the maximal report should not be audited (we assume that it is not profitable, which, strictly speaking, does not have to be true unless $\max \int R(\pi_r(\pi), e(\pi_r(\pi))) dF(\pi)$ is achieved at $r(\pi_{r \max}) = 0$). Thus,

$$\begin{aligned} 1 - \frac{c}{\mu(\pi_{\max} - \pi_{r \max})} &= 0 \\ \pi_{r \max} &= \pi_{\max} - \frac{c}{\mu}. \end{aligned}$$

After rearrangement, we have

$$A = \left(\frac{c}{\mu} - \frac{c}{B}\right) \exp\left\{-\frac{B}{c}\pi_{r \max}\right\}.$$

This provides us with the expression (5).

For 3) we need the consistency of beliefs, that is the authority's belief about $\pi(\pi_r)$ must coincide with actual reporting strategy $\pi_r(\pi)$. This is only possible, if $\pi(\pi_r)$ is increasing, and in this case consistency is actually ensured by the best responses in our formulation. Then, the following condition should be satisfied:

$$\pi'(\pi_r) = 1 + \left(\frac{c}{\mu} - \frac{c}{B} \right) \frac{B}{c} e^{\frac{B}{c}(\pi_r - \pi_{\max} + \frac{c}{\mu})} > 0 \forall \pi_r \in [\pi_{r \min}, \pi_{r \max}]$$

It turns out to be useful to look at the coefficient of the exponent, $B/\mu - 1$. It is negative for $p < t$, and hence the coefficient is negative. Using this to simplify the condition for positive derivative, we get $\mu < \mu - B$ in case of $B > 0$, which is obviously true. In case of $B < 0$ the condition is $\pi_{\max} < c/\mu$, but then the strict separating equilibrium does not exist for $p < t$, as $r^* \equiv 0$ and all firms prefer evading.

To sum up the argument, $B > 0 \Rightarrow y' > 0, B < 0 \Rightarrow \nexists y$, so the separating equilibrium may only exist for $B > 0$ or

$$p > t(1 - r_1(1 + s_1)). \quad (26)$$

What happens if the reporting is decreasing in the profit? Then the initial condition is

$$1 - \frac{c}{\mu(\pi_{\min} - \pi_{r \max})} = 0$$

$$\pi_{r \max} = \pi_{\min} - \frac{c}{\mu}$$

And so A is the same. The difference is that there is no problem for $\pi'(\pi_r) < 0$, so that

$$\pi'(\pi_r) = 1 + \left(\frac{c}{\mu} - \frac{c}{B} \right) \frac{B}{c} e^{\frac{B}{c}(\pi_r - \pi_{\max} + \frac{c}{\mu})} < 0$$

$$\left(\frac{B}{\mu} - 1 \right) e^{\frac{B}{c}(\pi_r - \pi_{r \max})} < -1$$

with $B < 0$ satisfied for sure. So the really binding in this case is of course $\pi_r(\pi_{\max}) \geq 0$

$$\left(\frac{c}{\mu} - \frac{c}{B} \right) \exp \left\{ \frac{B}{c} \pi_{r \max} \right\} + \frac{c}{B} \leq \pi_{\max}$$

We have to also respect the individual rationality constraint, that is the above described reporting strategy should be preferred to honest reporting:

$$\pi - t\pi_r^* - (p + t(1 + s_1)r_1r)(\pi - \pi_r^*) > (1 - t)\pi,$$

which can be rearranged to obtain

$$p < t(1 - r_1(1 + s_1)r^*).$$

Since $\max r^* = 1 - \frac{c}{\mu(\pi_{\min} - \pi_{r\min})}$, we have

$$p < t \left(1 - r_1(1 + s_1) \left(1 - \frac{c}{\mu(\pi_{\min} - \pi_{r\min})} \right) \right). \quad (27)$$

The last check is for the equilibrium reports to be positive, as the negative reports are not allowed (or do not make sense, since no negative tax is paid). The corresponding restriction can be formulated as

$$\left(\frac{c}{\mu} - \frac{c}{B} \right) \exp \left\{ \frac{B}{c} \left(\frac{c}{\mu} - \pi_{\max} \right) \right\} \leq \pi_{\min} - \frac{c}{B}.$$

Since $0 < B < \mu$ (the positivity required by consistency of beliefs discussed later), the condition is satisfied for $c/B < \pi_{\min}$, that is when $p \in \left(t - \left(1 - \frac{c}{\mu\pi_{\min}} \right) t(1 + s_1)r_1, t \right)$ - exactly when the rationality constraint is not satisfied.

When $c/B > \pi_{\min} > c/\mu$, the condition is actually satisfied for the interval $p \in \left(t(1 - r_1(1 + s_1)), t - \left(1 - \frac{c}{\mu\pi_{\min}} \right) t(1 + s_1)r_1 \right)$, as can be directly checked at the borders ($B = 0$ and $B = \frac{c}{\pi_{\min}}$) and by monotonicity and continuity applies to the whole interval.

Collecting the restrictions, we have $c/\mu < \pi_{\min}$, $p \in \left(t(1 - r_1(1 + s_1)), t - \left(1 - \frac{c}{\mu\pi_{\min}} \right) t(1 + s_1)r_1 \right)$ as necessary and sufficient conditions for complete separating (strict) equilibrium existence.

weak equilibrium

In the weak equilibrium the firms are indifferent between submitting reports truthfully and engaging into evasion. Formally,

$$\pi - t\pi_r - (p + t(1 + s_1)r_1r(\pi_r))(\pi - \pi_r) = \pi - t\pi. \forall \pi$$

After rearranging this condition using the tax authority best response (4), which with the constant evasion takes the form $1 - \frac{c}{\mu\hat{e}}$, we arrive at

$$\hat{e} = \frac{c}{\mu} \left(1 - \frac{t - p}{t(1 + s_1)r_1} \right)^{-1}.$$

Imposing the consistency of beliefs $\hat{e} = e$ we get the expression in the proposition.

Note that restriction on evasion volume in this case is

$$\frac{c}{\mu} < e < \pi_{\min},$$

which in terms of the price looks like

$$t \left(1 - (1 + s_1) r_1 \left(1 - \frac{c}{\mu \pi_{\min}} \right) \right) < p < t.$$

In this equilibrium the expected punishment is exactly equal to the expected gain from the evasion regardless of the evasion level: $t(1 + s_1)r_1r(\pi_r) = t - p$. Profitable deviations are impossible, as any report brings about the same payoff.

To complete the characterization of subgame equilibrium (requirement 4)), out-of-equilibrium beliefs of the tax authority (for both strict and weak equilibrium) are specified as $\pi(\pi_r) = \begin{cases} \pi_{\min}, & \pi_r < \pi_{r \min} \\ \pi_{\max}, & \pi_r > \pi_{r \max} \end{cases}$. Note that we do not have to specify beliefs for any possible deviation to reports in $[\pi_{r \min}, \pi_{r \max}]$, as the tax authority has no chance of observing such a deviation.

C - Proof of Proposition 2 - Hybrid equilibrium

We have shown that for $c/\mu > \pi_{\min}$, the complete separation equilibrium does not exist. Here we are interested whether a hybrid equilibrium with the following properties exists: (i) all the taxpayers with $\pi \in [\pi_{\min}, \pi^0]$ submit zero reports; (ii) all the taxpayers with $\pi \in (\pi^0, \pi_{\max}]$ submit different reports $(\pi_r^0, \pi_{r \max}]$; (iii) $r(\pi_r > 0) < r(\pi_r = 0)$; (iv) $I(\pi^0, 0) = I(\pi^0, \pi_r^0)$. The first two are defining properties, the second is the decreasing in report auditing, the third follows from continuity of the distribution. let us look at (iii) more closely.

Since we know the auditing has to be a best response in equilibrium, from (22) we get for our auditing cost function $r(\pi_r > 0) = 1 - \frac{c}{\mu(\hat{\pi} - \pi_r)} < 1 - \frac{c}{\mu E(\pi | \pi \leq \pi^0)} = r(\pi_r = 0)$ or, in particular,

$$E(\pi | \pi \leq \pi^0) > \hat{\pi}^0 - \pi_r^0. \quad (28)$$

This is a consistency requirement on the side of the tax authority.

For the firm's consistency, we need

$$\begin{aligned} I(\pi^0, 0) &= \pi^0 - (p + t(1 + s_1)r_1r(0))\pi^0 \\ &= \pi^0 - t\pi_r^0 - (p + t(1 + s_1)r_1r(\pi_r^0))(\pi^0 - \pi_r^0) = I(\pi^0, \pi_r^0) \end{aligned}$$

we need the equality, because otherwise by continuity there is incentive to deviate. Working out this condition, we arrive at

$$\frac{\pi^0 - \pi_r^0}{(\hat{\pi}^0 - \pi_r^0)} - \frac{\pi^0}{E(\boldsymbol{\pi}|\pi \leq \pi^0)} = -\frac{B}{c}\pi_r^0$$

using consistency of beliefs, we get

$$\frac{B}{c}\pi_r^0 = \frac{\pi^0 - E(\boldsymbol{\pi}|\pi \leq \pi^0)}{E(\boldsymbol{\pi}|\pi \leq \pi^0)} \quad (29)$$

This actually defines the level of report π_r^0 as a function of p and π^0 in the hybrid equilibrium. This also immediately imposes $B > 0$ on the parameters. The problem is that nothing pins down π^0 . However, if profit is distributed uniformly over $[0, \pi_{\max}]$, this becomes $\pi_r^0 = \frac{c}{B}$. Here the profit does not enter, because with uniform distribution the ratio $\frac{\pi^0 - E(\boldsymbol{\pi}|\pi \leq \pi^0)}{E(\boldsymbol{\pi}|\pi \leq \pi^0)}$ is constant, which is not true for a general distribution. We have to make sure that after-tax income is increasing in pre-tax income plus check incentive and participation constraints as in case of complete separation.

strict equilibrium

Since the separation part of the problem is identical to the previous one (complete separation), we get the same result up to π^0 , only cut at the point defined by (29). So solving two equations simultaneously, we get π^0 and π_r^0 . For the uniform, we have

$$\pi^0 = \left(\frac{c}{\mu} - \frac{c}{B}\right) \exp\left\{\frac{B}{c}\left(\frac{c}{B} - \pi_{\max} + \frac{c}{\mu}\right)\right\} + 2\frac{c}{B}.$$

Combining the consistency conditions of the tax authority (28) and the firms (29), we get

$$\frac{c}{B} > E(\boldsymbol{\pi}|\pi \leq \pi^0) > \pi^0 - \pi_r^0.$$

For the uniform distribution, for example, it takes the form $\pi^0 < c/B$, which is satisfied.

The report is positive by construction, and evasion must be also positive. And it is, since it is decreasing in report and at the maximal profit is positive c/μ . We have to modify rationality constraint (27) to the analogous expression for the polled types in hybrid equilibrium:

$$p \leq t(1 - r_1(1 + s_1)r(0, \pi_r^0)).$$

To check for absence of deviation from the pooling part, we have

$$\begin{aligned} I(\pi^-, 0) &= \pi^- - (p + t(1 + s_1)r_1r(0))\pi^- \\ &> \pi^- - t\pi_r^0 - (p + t(1 + s_1)r_1r(\pi_r^0))(\pi^- - \pi_r^0) = I(\pi^-, \pi_r^0) \end{aligned}$$

or

$$\left(\frac{t-p}{t(1+s_1)r_1} - r(\pi_r^0) \right) \pi_r^0 > (r(0) - r(\pi_r^0))\pi^-$$

Since $r(0) - r(\pi_r^0) > 0$, the rhs increases in π^- . At π^0 it reaches maximum with equality, so the condition is indeed satisfied.

From the separating part,

$$\begin{aligned} I(\pi^+, 0) &= \pi^+ - (p + t(1 + s_1)r_1r(0))\pi^+ \\ &< \pi^+ - t\pi_r^+ - (p + t(1 + s_1)r_1r(\pi_r^+))(\pi^+ - \pi_r^+) = I(\pi^+, \pi_r^+) \end{aligned}$$

or

$$\left(\frac{t-p}{t(1+s_1)r_1} - r(\pi_r^+) \right) \pi_r^+ < (r(0) - r(\pi_r^+))\pi^+$$

by the same logic works for π_r^0 . Thus also true for π_r^+ , as $I(\pi^+, \pi_r^+) > I(\pi^+, \pi_r^0)$ by the separating part condition.

To complete the characterization of the hybrid equilibrium, out-of-equilibrium beliefs of the tax authority should be specified. It is sufficient that $\forall \pi_r \in (0, \pi_r^0)$ $r(0) \leq r(\pi_r) \implies E_D(\boldsymbol{\pi}|\pi_r) \geq \pi_r + E(\boldsymbol{\pi}|\pi \leq \pi^0)$. Actually, for the pooled types it suffices to have $E_D(\boldsymbol{\pi}|\pi_r) \geq E(\boldsymbol{\pi}|\pi \leq \pi^0)$ by the same logic as considered below for complete pooling. For the separated types, a weaker sufficient condition is a mess, so we do not state it here.

weak equilibrium

In the weak hybrid equilibrium the types below π^0 evade everything; the types above are indifferent between honesty and cheating. Using the same logic, we arrive at (7) for the separating part. The restriction is slightly different,

$$p > t - t(1 + s_1)r_1r(0, \pi^0),$$

and complements the rationality constraint for the strict hybrid equilibrium.

D - Proof of Proposition 3 - Pooling equilibrium

A pooling equilibrium has to satisfy the following conditions: 1) absence of deviation incentives for the taxpayer when the tax authority audits any report with equilibrium probability; 2) absence of deviation incentives for the tax authority when every taxpayer submits zero report; 3) arbitrary beliefs of the tax authority about the true income of the taxpayer who submits out-of-equilibrium report.

As mentioned in the text, we only consider the pooling at zero equilibrium, as we consider zero report a natural focal point for underreporting.

As far as 1) is concerned, the payoff from zero report should be preferred to any deviation for any profit level. As for 2), tax authority chooses the auditing probability (11) that maximizes its revenues given zero report. Finally, for 3) we need an analogous expression for a deviator, and that should depend on the out-of-equilibrium beliefs of the tax authority, that is it can be any belief. We consider an arbitrary belief D , but notice that the most adverse for the deviator belief (and hence most favorable for the equilibrium) is that the deviator has maximum profit π_{\max} .

So, the net expected profit of the firm with gross profit π and a report π_r can be written as

$$I(\pi_r, \pi) = \pi - t\pi_r - p(\pi - \pi_r) - t(1 + s_1)(\pi - \pi_r)r_1r(\pi_r),$$

where the tax authority auditing is given by (22):

$$r(\pi_r, D) = \begin{cases} 1 - \frac{c}{\mu E\boldsymbol{\pi}}, & \pi_r = 0; \\ 1 - \frac{c}{\mu(E_D\{\boldsymbol{\pi}|\pi_r\} - \pi_r)}, & \pi_r \in \left(0, E_D\{\boldsymbol{\pi}|\pi_r\} - \frac{c}{\mu}\right); \\ 0, & \pi_r \geq E_D\{\boldsymbol{\pi}|\pi_r\} - \frac{c}{\mu}. \end{cases}$$

We have to then consider three cases:

$$I(0, \pi) = \pi - p\pi - t(1 + s_1)\pi r_1 \left(1 - \frac{c}{\mu E\boldsymbol{\pi}}\right),$$

$$I(\pi_r, \pi) = \pi - t\pi_r - p(\pi - \pi_r) - t(1 + s_1)(\pi - \pi_r)r_1 \left(1 - \frac{c}{\mu(E_D\{\boldsymbol{\pi}|\pi_r\} - \pi_r)}\right),$$

$$I(\pi_r \geq, \pi) = \pi - t\pi_r - p(\pi - \pi_r).$$

First, we show that $I(0, \pi) \geq I(\pi_r \geq, \pi) \forall \pi, \pi_r \geq E_D\{\boldsymbol{\pi}|\pi_r\} - \frac{c}{\mu}$. As $I(\pi_r \geq, \pi)$ is decreasing in π_r , it is enough to show that the inequality holds for $\pi_r = E_D\{\boldsymbol{\pi}|\pi_r\} - \frac{c}{\mu}$:

$$\pi - p\pi - t(1 + s_1)\pi r_1 \left(1 - \frac{c}{\mu E\boldsymbol{\pi}}\right) \geq (1 - p)\pi - (t - p) \left(E_D\{\boldsymbol{\pi}|\pi_r\} - \frac{c}{\mu}\right)$$

Thus, a sufficient condition for no deviation with a belief D such that $E_D \{\boldsymbol{\pi} | \pi_r\} > \pi_{\max} - \left(\frac{\pi_{\max}}{E\boldsymbol{\pi}} - 1\right) \frac{c}{\mu}$ is

$$t(1 + s_1)r_1 \leq t - p,$$

which as we have seen is complementary to the separating equilibrium existence condition. A necessary one for a belief D is

$$\frac{t(1 + s_1)r_1}{t - p} \pi_{\max} \left(1 - \frac{c}{\mu E\boldsymbol{\pi}}\right) + \frac{c}{\mu} \leq E_D \{\boldsymbol{\pi} | \pi_r\}$$

Secondly, we show that $I(0, \pi) \geq I(\pi_r, \pi) \forall \pi, \pi_r \in \left(0, E_D \{\boldsymbol{\pi} | \pi_r\} - \frac{c}{\mu}\right]$. Note first that $I(\pi_r, \pi)$ is decreasing in π_r , if the condition $t(1 + s_1)r_1 \leq t - p$ is satisfied and if $\partial E_D \{\boldsymbol{\pi} | \pi_r\} / \partial \pi_r \leq 1$. Then it is enough to consider a marginal deviation:

$$\pi - p\pi - t(1 + s_1)\pi r_1 \left(1 - \frac{c}{\mu E\boldsymbol{\pi}}\right) \geq \pi - p\pi - t(1 + s_1)\pi r_1 \left(1 - \frac{c}{\mu E_D \{\boldsymbol{\pi} | \pi_r\}}\right).$$

This is clearly satisfied for any out-of-equilibrium belief D such that $E_D \{\boldsymbol{\pi} | \pi_r\} \geq E\boldsymbol{\pi}$. So with $p \leq t(1 - (1 + s_1)r_1)$ the pooling at zero equilibrium exists.

In the rest of the paper we assume that the pooling at zero equilibrium is played whenever the separating equilibrium (or considered hybrid equilibrium) does not exist. As we have seen, this is only true for certain out-of-equilibrium beliefs. However, (i) there is no complete pooling equilibrium that exists for any out-of-equilibrium beliefs, (ii) considering all possible hybrid equilibria complicates the analysis substantially; moreover, they are not likely to be more robust or realistic than the equilibria considered, (iii) having two extreme cases of separation (or hybrid) and pooling provides a clear benchmark for the analysis of factors that affect evasion volume in our setting.

When $t(1 + s_1)r_1 > t - p$ (separating or hybrid equilibrium exists), the first (now necessary) condition is

$$\frac{t(1 + s_1)r_1}{t - p} \left(1 - \frac{c}{\mu E\boldsymbol{\pi}}\right) \pi_{\max} + \frac{c}{\mu} \leq E_D \{\boldsymbol{\pi} | \pi_r\},$$

which breaks down if

$$\pi_{\max} \left(\frac{t(1 + s_1)r_1}{t - p} - 1\right) \geq \frac{c}{(1 + s + s_1)tr_1} \left(\frac{E_D \{\boldsymbol{\pi} | \pi_r\}}{E\boldsymbol{\pi}} - 1\right).$$

Clearly, $\exists D : E_D \{\boldsymbol{\pi} | \pi_r\} \geq E\boldsymbol{\pi}$ for which this is satisfied, meaning that pooling equilibrium does not exist.

On pooling at a positive level $\pi^\circ \leq \pi_{\min}$

The three cases now are

$$\begin{aligned} I(\pi^\circ, \pi) &= \pi - t\pi^\circ - p(\pi - \pi^\circ) - t(1 + s_1)(\pi - \pi^\circ)r_1 \left(1 - \frac{c}{\mu(E\boldsymbol{\pi} - \pi^\circ)}\right), \\ I(\pi_r, \pi) &= \pi - t\pi_r - p(\pi - \pi_r) - t(1 + s_1)(\pi - \pi_r)r_1 \left(1 - \frac{c}{\mu(E_D\{\boldsymbol{\pi}|\pi_r\} - \pi_r)}\right), \\ I(\pi_r \geq, \pi) &= \pi - t\pi_r - p(\pi - \pi_r). \end{aligned}$$

First, we show that $I(\pi^\circ, \pi) \geq I(\pi_r \geq, \pi) \forall \pi, \pi_r \geq E_D\{\boldsymbol{\pi}|\pi_r\} - \frac{c}{\mu}$. As $I(\pi_r \geq, \pi)$ is decreasing in π_r , it is enough to show that the inequality holds for $\pi_r = E_D\{\boldsymbol{\pi}|\pi_r\} - \frac{c}{\mu}$:

$$t(1 + s_1)r_1(\pi - \pi^\circ) \left(1 - \frac{c}{\mu(E\boldsymbol{\pi} - \pi^\circ)}\right) \leq (t - p) \left(E_D\{\boldsymbol{\pi}|\pi_r\} - \frac{c}{\mu} - \pi^\circ\right).$$

A necessary condition for a belief D is

$$\frac{t(1 + s_1)r_1}{t - p}(\pi_{\max} - \pi^\circ) \left(1 - \frac{c}{\mu(E\boldsymbol{\pi} - \pi^\circ)}\right) + \frac{c}{\mu} + \pi^\circ \leq E_D\{\boldsymbol{\pi}|\pi_r\},$$

which is more stringent than the condition for pooling at zero whenever $t(1 + s_1)r_1 < t - p$. In fact, $E_D\{\boldsymbol{\pi}|\pi_r\} > E\boldsymbol{\pi}$ has to be satisfied, so no pooling at positive report is possible, if $E_D\{\boldsymbol{\pi}|\pi_r\} = E\boldsymbol{\pi}$.

Since only $E_D\{\boldsymbol{\pi}|\pi_r\} \leq \pi_{\max}$ is consistent with rationality, we have

$$\frac{t(1 + s_1)r_1}{t - p} \left(1 - \frac{c}{\mu(E\boldsymbol{\pi} - \pi^\circ)}\right) \leq 1 - \frac{c}{\mu(\pi_{\max} - \pi^\circ)}.$$

Secondly, we show that $I(\pi^\circ, \pi) \geq I(\pi_r, \pi) \forall \pi, \pi_r \in \left(0, E_D\{\boldsymbol{\pi}|\pi_r\} - \frac{c}{\mu}\right]$. Note first that $I(\pi_r, \pi)$ is decreasing in π_r , if

$$\begin{aligned} & -t + p + t(1 + s_1)r_1 + \frac{ct(1 + s_1)r_1}{\mu} \times \\ & \times \frac{-(E_D\{\boldsymbol{\pi}|\pi_r\} - \pi_r) - (\pi - \pi_r)(\partial E_D\{\boldsymbol{\pi}|\pi_r\}/\partial \pi_r - 1)}{(E_D\{\boldsymbol{\pi}|\pi_r\} - \pi_r)^2} \leq 0. \end{aligned}$$

We can obviously always find an appropriate belief $E_D\{\boldsymbol{\pi}|\pi_r\}$ such that the condition $t(1 + s_1)r_1 \leq t - p$ is sufficient.

With a strictly positive equilibrium report, though, there is a third type of deviation to consider, the one to a lower report. This kind of deviation provides a direct

benefit of higher income, but also a danger of more intensive auditing. By the same logic as above, the most profitable deviation is to the zero report. Then we should have

$$\frac{(t-p)\pi^\circ}{t(1+s_1)\pi r_1} + \left(1 - \frac{\pi^\circ}{\pi}\right) \left(1 - \frac{c}{\mu(E\pi - \pi^\circ)}\right) \leq 1 - \frac{c}{\mu E_D\{\pi|0\}}.$$

Clearly, for $\pi^\circ = \pi_{\min}$, it is impossible to find a belief D that satisfies the inequality above. Indeed, the left-hand side then turns into $\frac{t-p}{t(1+s_1)r_1}$, which is greater or equal to 1. Intuitively, the lowest type should prefer honest reporting to zero reporting, that is

$$(1-t)\pi \geq \pi - p\pi - t(1+s_1)\pi r_1 \left(1 - \frac{c}{\mu E_D\{\pi|0\}}\right),$$

which clearly does not happen. By continuity, for π° close to π_{\min} , a pooling equilibrium cannot exist. On the other hand, for π° close enough to zero, this should not be a problem. Thus, there exists π^1 such that

$$\frac{(t-p)\pi^1}{t(1+s_1)\pi_{\min}r_1} + \left(1 - \frac{\pi^1}{\pi_{\min}}\right) \left(1 - \frac{c}{\mu(E\pi - \pi^1)}\right) = 1 - \frac{c}{\mu\pi_{\max}}.$$

At the extreme of $t(1+s_1)r_1 = t-p$

$$\pi^1 = \pi_{\min} \frac{\pi_{\max} - E\pi}{\pi_{\max} - \pi_{\min}},$$

and no belief can rationalize an equilibrium with reports pooling above π^1 .

Compliance in pooling and separating equilibria

Finally, we consider the possibility to have more compliance in a pooling equilibrium than in the separating equilibrium. As we have shown above, the lowest evasion level in a pooling equilibrium is

$$\int_{\pi_{\min}}^{\pi_{\max}} \pi dF(\pi) - \pi^1 = E\pi - \pi_{\min} \frac{\pi_{\max} - E\pi}{\pi_{\max} - \pi_{\min}} = \frac{\pi_{\max}(E\pi - \pi_{\min})}{\pi_{\max} - \pi_{\min}} < E\pi$$

In the weak separating equilibrium with $p = t$, the evasion volume is $c/\mu < \pi_{\min}$ from the existence condition. A sufficient condition on distribution is then

$$\pi_{\max}(E\pi - 2\pi_{\min}) + \pi_{\min}^2 > 0.$$

For symmetric distributions, this condition simplifies to

$$\pi_{\max} > 2\pi_{\min},$$

which seems to be a very weak condition (if the minimal profit is 100 dollars, the maximal should be at least 200 dollars). The condition holds *a fortiori* for the distributions with higher mass on higher profits.

E - Properties of complete separating equilibrium

Define equilibrium evasion volume $e^*(\pi) := \pi - \pi_r^*(\pi)$.

Proposition A1 In the complete separating equilibrium defined by the reporting function (5), evasion volume is a decreasing and concave function of profit, $\frac{de^*(\pi)}{d\pi} < 0$, $\frac{d^2e^*(\pi)}{d\pi^2} < 0$.

Taking the reporting strategy (5), we differentiate evasion volume with respect to profit

$$\frac{de(\pi)}{d\pi} = 1 - \frac{d\pi_r^*}{d\pi} = \left(\frac{B}{\mu} - 1\right) e^{\frac{B}{c}(\pi_r^*(\pi) - \pi_r^*(\pi_{\max}))} \frac{d\pi_r^*}{d\pi}.$$

We know that for $p < t$, $B/\mu - 1 < 0$. As $\frac{d\pi_r^*}{d\pi} > 0$ and the exponent is positive, $\frac{de(\pi)}{d\pi} < 0$.

Differentiating the evasion volume second time, we obtain

$$\frac{de^2(\pi)}{d\pi^2} = -\frac{d^2\pi_r^*}{d\pi^2} = \left(\frac{B}{\mu} - 1\right) e^{\frac{B}{c}(\pi_r^*(\pi) - \pi_r^*(\pi_{\max}))} \left(\left(\frac{d\pi_r^*}{d\pi}\right)^2 + \frac{d^2\pi_r^*}{d\pi^2} \right).$$

Rearranging, we get

$$\frac{de^2(\pi)}{d\pi^2} = \left(\frac{B}{\mu} - 1\right) e^{\frac{B}{c}(\pi_r^*(\pi) - \pi_r^*(\pi_{\max}))} \left(\frac{d\pi_r^*}{d\pi}\right)^2 / \left(\left(\frac{B}{\mu} - 1\right) e^{\frac{B}{c}(\pi_r^*(\pi) - \pi_r^*(\pi_{\max}))} + 1 \right).$$

As the nominator is negative, the second derivative can only be positive, if the denominator is negative. Since the minimum of the first term of denominator is achieved at $\pi = \pi_{\max}$, we must have $\frac{B}{\mu} - 1 + 1 < 0$, which is impossible. Thus, the second derivative is negative, $\frac{de^2(\pi)}{d\pi^2} < 0$.

The decreasing evasion is a counter-intuitive result, as we would expect the rich to evade more. After all, their reports are audited less. In our setting though, if they evade more, they get audited disproportionately more, thus preferring to stay at their separating equilibrium report. This does not contradict a common sense that evasion makes the tax system more regressive. Indeed, without evasion the linear tax rate implies a neutral tax system. In separating equilibrium though the tax system

becomes regressive, as after tax expected income is increasing faster than before-tax income. Formally, this leads us to the following corollary:

Corollary A1 In the complete separating equilibrium defined by the reporting function (5), the linear tax is regressive, $\frac{d^2 I}{d\pi^2} = (t - p - t(1 + s_1)r_1) \frac{\partial^2 e^*(\pi)}{\partial \pi^2} > 0$.

This result can be obtained by direct differentiation of the expected after-tax profit (2).

F - Proof of proposition 4

From the equilibrium structure in propositions 1-3 we can see that the specialist's profit maximization (12) can be split into three subproblems: pooling, strict separation and (possibly) weak separation. The maximization in the pooling equilibrium is trivial: since the evasion volume is fixed, the local profit maximizing price is $p = t(1 - r_1(1 + s_1))$. For the weak separating equilibrium the profit function can be written as $(p - c_s - str_1r)e$, the first derivative is

$$e + (p - c_s - str_1) e'.$$

Using the explicit expression for e from (7) in proposition 1 this can be rewritten as $\mu - t + c_s$, which is positive iff condition (13) is satisfied. Thus, in this case the local profit maximizing price is $p = t$. Correspondingly, the local maximizing price is the minimal price that supports weak separating or hybrid equilibrium, iff condition (13) is not satisfied.

The least tractable case is the strong separating or hybrid equilibrium. However, we are able to show that under condition (13) it is strictly dominated by an equilibrium with $p = t$. The condition for that is

$$(p - c_s - str_1) \int_{\pi_{\min}}^{\pi_{\max}} (\pi - \pi_r(p, \pi)) dF(\pi) < (t - c_s - str_1) \frac{c}{\mu} \quad (30)$$

$$\text{for } t(1 - r_1(1 + s_1)) < p < t(1 - r_1(1 + s_1)r).$$

After rearranging and using the fact that $\int_{\pi_{\min}}^{\pi_{\max}} dF(\pi) = 1$ and $p - c_s - str_1 \geq 0$ (otherwise the separating equilibrium brings about less than sure minimum of $\frac{cs}{1+s+s_1}$ to the specialist) we get

$$\int_{\pi_{\min}}^{\pi_{\max}} (\pi - \pi_r(p, \pi)) dF(\pi) < \frac{c(t - c_s - str_1)}{\mu(p - c_s - str_1)}. \quad (31)$$

From (5) we know that

$$\pi - \pi_r(p, \pi) < \frac{c}{B} \forall \pi,$$

(for a hybrid equilibrium this is only true for the separating part, and for the pooling part there is a weaker condition $E(\pi | \pi \leq \pi^0) < c/B$, which is sufficient for us, as $\int_{\pi_{\min}}^{\pi_{\max}} (\pi - \pi_r(p, \pi)) dF(\pi) = E(\pi | \pi \leq \pi^0) + \int_{\pi^0}^{\pi_{\max}} (\pi - \pi_r(p, \pi)) dF(\pi)$ so that

$$\int_{\pi_{\min}}^{\pi_{\max}} (\pi - \pi_r(p, \pi)) dF(\pi) < \int_{\pi_{\min}}^{\pi_{\max}} \frac{c}{B} dF(\pi) = \frac{c}{B}. \quad (32)$$

Now, if also

$$\frac{c}{B} < \frac{c(t - c_s - str_1)}{\mu(p - c_s - str_1)}, \quad (33)$$

then the profit in any strict separating equilibrium is dominated by the profit at t . Simply rearranging (33) we get

$$0 < \mu + c_s - t$$

This is the condition (13), so it is only left to show that under this condition the full cheating equilibrium is never preferred:

$$\frac{c}{\mu} (t - c_s - str_1) > (t - c_s - \mu) E\pi. \quad (34)$$

Since $c/\mu < E\pi$ and $str_1 < \mu$, $t - c_s - str_1 > t - c_s - \mu$. We have just established that $t - c_s - \mu < 0$. If $t - c_s - str_1 \geq 0$, then (34) is clearly satisfied. If $t - c_s - str_1 < 0$, then notice that $0 < c/\mu < E\pi$ and hence (34) is satisfied again.

G - Payoffs and comparative statics

The proposition 4 characterizes the equilibrium of the game with a specialist. In the following we discuss the factors that (i) affect cheating and auditing in the separating equilibrium; (ii) influence the behavior of agents in the pooling equilibrium; (iii) drive the system into separation or pooling.

Separation at $p = t$

The specialist profits under separation or hybrid equilibrium can be obtained by substituting $p = t$ into (12):

$$\Pi^s = \frac{c}{\mu} (t - c_s). \quad (35)$$

The profit is increasing in the tax rate, auditing costs and decreasing in enforcement parameters and specialist's costs. Notice that the specialist extracts all the rent from the tax evasion, leaving the firms indifferent between cheating and being honest.

In this equilibrium the auditing never happens, and evasion $e^*(t) = c/\mu$ is minimal and constant across income levels (apart from the pooled types in a hybrid equilibrium). The comparative statics is conventional here: evasion is increasing in auditing costs and decreasing in enforcement parameters; fines on firms and the specialist have an equivalent impact.

Pooling

Substituting $p = t(1 - r_1(1 + s_1))$ into (12), we get specialist profits in pooling

$$\Pi^p = (t - c_s - \mu) E\pi + \frac{cs}{1 + s + s_1}. \quad (36)$$

The profit is trivially decreasing in the specialist's costs and increasing in auditing costs. It is increasing in the tax rate if the enforcement is not sufficiently strong $(1 + s + s_1)r_1 < 1$ and decreasing otherwise. The fines on the firms unambiguously decreases specialist's profitability²². In the pooling equilibrium a part of the rent is left with the firms to make sure they do not prefer partial evasion of separating equilibrium. Everybody evades everything in this case: $\pi_r \equiv 0$.

The deep auditing probability is given by $r = 1 - c/(\mu E\pi)$, and it approaches unity as the auditing costs approach zero. The probability has conventional properties: it is decreasing in auditing costs and decreasing in enforcement parameters, tax rate and firms' profit.

²²Notice that the fine on the specialist would increase her profits for $\frac{c}{\mu} > E\pi \frac{1+s+s_1}{1+s_1}$, but this is ruled out by the assumption that $\frac{c}{\mu} < E\pi$.

Separation versus pooling

We have seen that the low auditing costs and high fines are good for separation. The higher specialist costs give more chances for separation as well. The impact of the enforcement mix is ambiguous: stricter enforcement in terms of s_1 decreases "marginal attractiveness" of separation at $p = t$ by $(t - c_s) c / \mu^2$, but also decreases that of pooling by $E\pi + cs / (1 + s + s_1)^2$. Thus, the condition for a stricter enforcement to work into the direction of separation is $E\pi (1 + s + s_1)^2 / c + s > (t - c_s) / (tr_1)^2$. This is only not satisfied for small values of r_1 .

H - Proof of proposition 5

Define a function P that takes negative values if and only if the equilibrium of the game is pooling,

$$P := (p^* - c_s - str_1) \int_{\pi_{\min}}^{\pi_{\max}} e(p^*, \pi) dF(\pi) - (t - c_s - \mu) E\pi - \frac{cs}{1 + s + s_1}.$$

The effectiveness of a fine in the sense of the present proposition is then just a derivative of the function P with respect to the corresponding fine. It is left to show that $\frac{dP}{ds} < \frac{dP}{ds_1}$. We have then

$$\begin{aligned} \frac{dP}{ds} &= \left(\frac{dp^*}{ds} - tr_1 \right) \int_{\pi_{\min}}^{\pi_{\max}} e(p^*, \pi) dF(\pi) \\ &\quad + (p^* - c_s - str_1) \int_{\pi_{\min}}^{\pi_{\max}} \frac{e(p^*, \pi)}{dp^*} \frac{dp^*}{ds} dF(\pi) + tr_1 E\pi - \frac{c(1 + s + s_1) - cs}{(1 + s + s_1)^2}, \\ \frac{dP}{ds_1} &= \frac{dp^*}{ds_1} \int_{\pi_{\min}}^{\pi_{\max}} e(p^*, \pi) dF(\pi) \\ &\quad + (p^* - c_s - str_1) \int_{\pi_{\min}}^{\pi_{\max}} \frac{e(p^*, \pi)}{dp^*} \frac{dp^*}{ds_1} dF(\pi) + tr_1 E\pi + \frac{cs}{(1 + s + s_1)^2}. \end{aligned}$$

The difference is

$$\begin{aligned} &\frac{dP}{ds_1} - \frac{dP}{ds} = \\ &\quad \left(\frac{dp^*}{ds_1} - \frac{dp^*}{ds} + tr_1 \right) \int_{\pi_{\min}}^{\pi_{\max}} e(p^*, \pi) dF(\pi) \\ &\quad + (p^* - c_s - str_1) \left(\int_{\pi_{\min}}^{\pi_{\max}} e_p(p^*, \pi) \frac{dp^*}{ds_1} dF(\pi) - \int_{\pi_{\min}}^{\pi_{\max}} e_p(p^*, \pi) \frac{dp^*}{ds} dF(\pi) \right) + \frac{c}{1 + s + s_1}. \end{aligned}$$

Recall that from the definition of p^* we have

$$\int_{\pi_{\min}}^{\pi_{\max}} e(p^*, \pi) dF(\pi) + (p^* - c_s - str_1) \int_{\pi_{\min}}^{\pi_{\max}} e_p(p^*, \pi) dF(\pi) = 0,$$

so that the expression above becomes

$$\frac{dP}{ds_1} - \frac{dP}{ds} = tr_1 \int_{\pi_{\min}}^{\pi_{\max}} e(p^*, \pi) dF(\pi) + \frac{c}{1 + s + s_1} > 0.$$