IS-LM model for US economy: testing in JMULTI

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Abstract

In this paper IS-LM model, has been introduced as time series model. Standard VAR, VECM test have been applied .Three variables that we estimated were: logarithm of real GDP (q), 3 month interbank interest rate (i), real monetary base (m).VECM mechanism shows that if the system is in disequilibrium alteration in the change of interbank interchange interest rate, log of real US gdp , and monetary base will be downward 5,5%,4,6% and 0,4% respectively.

Keywords: IS-LM, VAR, VECM,JMULTI

Literature review of IS-LM Model

The IS-LM model is macroeconomic model that represents the Keynes's theory. The main idea of the IS-LM model is to show what determined aggregate output in the short run when the prices are fixed. The goal of this model is to analyze the fluctuation of output in the short run through identification of variables that shift aggregate demand. The model gives good base for policymaker in creation adequate macroeconomic policy in short run.

This model is contains form two curves: IS and LM curve. IS curve represents the "investment" and "saving", and the IS curve shows what is going on in the market for goods and services. LM curve represents "liquidity" and "money", and the LM curve shows what is happening to the supply and demand for money.[1]

Interest rate, Investment and the IS Curve

The Keynesian cross is the main path to IS-LM model. The Keynesian cross is useful because it shows how the spending plans of households, firms, and the government determine the output. From macroeconomics, we already know that there is strong relationship between the interest rate and planned investment. The economists explain this causality relationship between interest rate and planned investment in the following way: interest rate is the cost of borrowing to finance investment project, therefore, an increase in the interest rate reduces planned investment. As a result the investment function slopes downward. On the other side, the investment is one of the components of aggregate output[2], and thus, the reduction in planned investment shifts the planned-expenditure function downward. The shift in the planned expenditure function causes the level of output to fall form. As we can see from the final panel of following figure, the IS curve summarize the relationship between the interest rate and the level of output.

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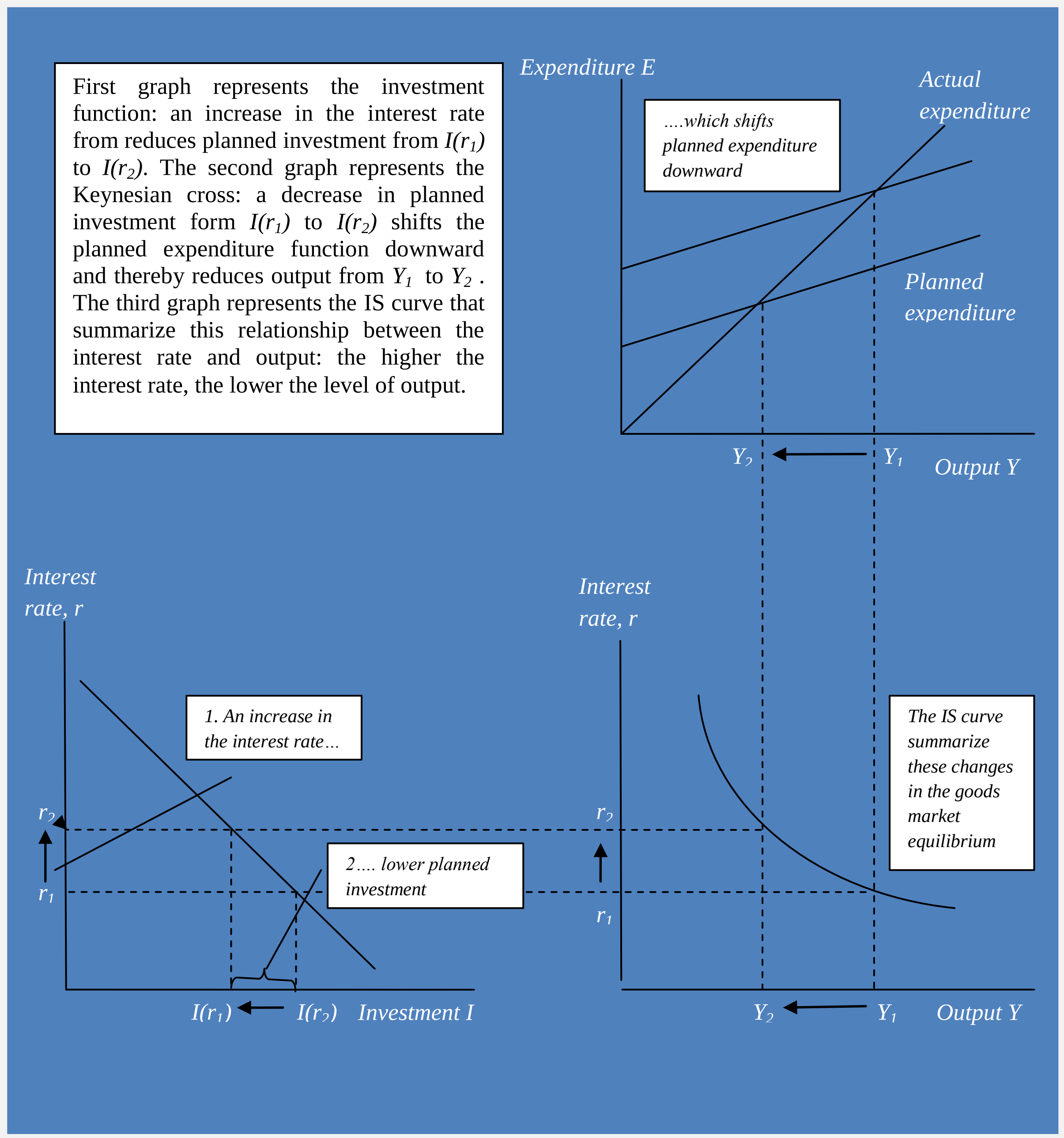
[1]N.,Gregory Mankiw, Macroeconomics, fifth edition, Worth Publishers, 2003. pp.257-178.

[2] Y=C+I+G, when we assume for close economy.



Income, Money Demand and the LM Curve

The theory of liquidity preference shows how the interest rate is determined in the short run. This theory represents how the interest rate adjusts to balance the supply and demand for the most liquid asset in economy – *money*. To explain the theory of liquidity preference, we start with following equation:

$$(M/P)^s = \overline{M}/\overline{P} \quad (1)$$ [3][4]

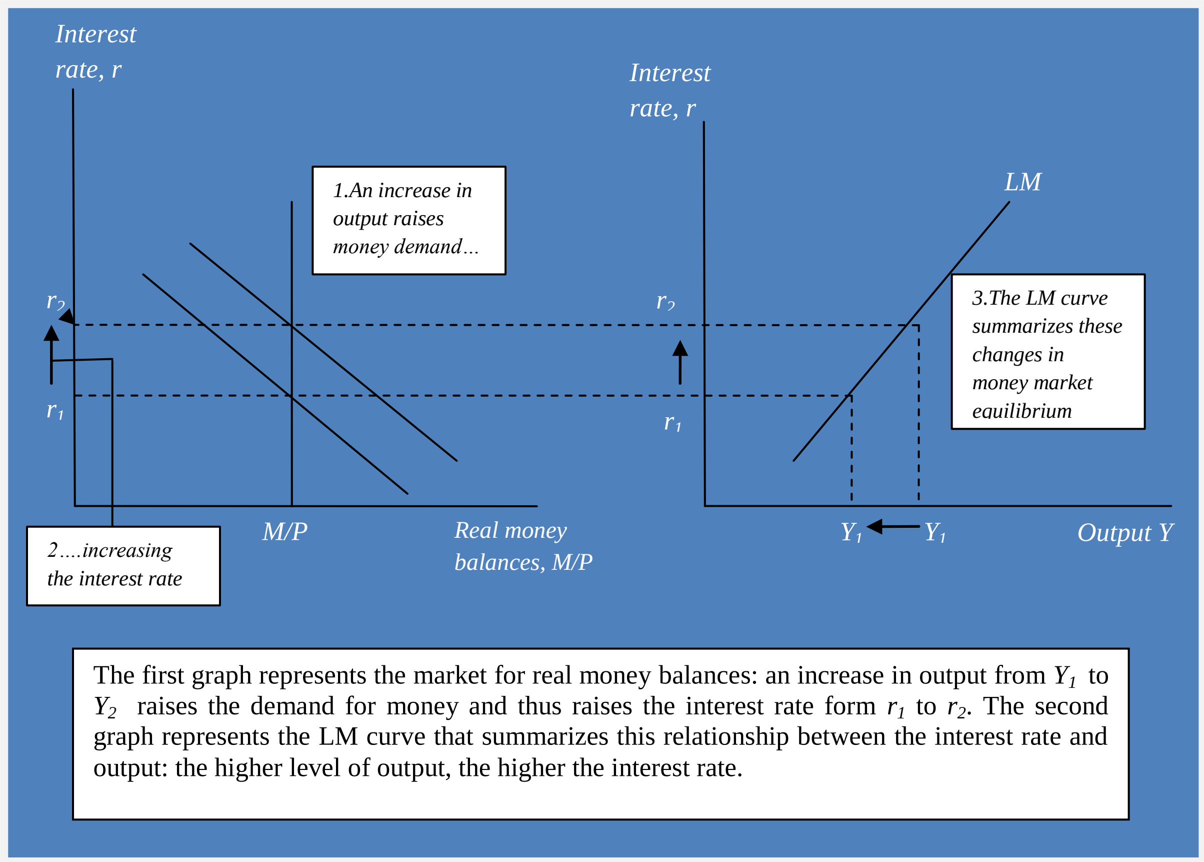
From this equation we can conclude that this theory assumes that supply of real money balances is fixed. This assumption means that the supply of money does not depend of interest rate. The money supply is chosen by a Central bank as exogenous variable. On the other side, the interest rate is important determinant of how much people choose to hold. The reason is that the interest rate is the opportunity cost of holding money. This means, when the interest rate rises, people want to hold lees of their wealth in the form of money. Now, we can write the demand for real money balances:

$$(M/P)^d = L(r,Y) \quad (2)$$

On the other side, the second important factor which determines the demand for money is the level of output. When output is high, expenditure is high, so people engage in more transactions that require the use of money. Thus, greater level of output implies greater money demand. From previous equation, we can conclude that, the quantity of real money balances demanded is negatively related to the interest rate and positively related to output.

Using the theory of liquidity, we can figure out what happens to the equilibrium interest rate when the level of output changes. From first graph in following figure, we can see that an increase in income shifts the money demand curve to the right. The assumption that the supply of real money balances is unchanged, the interest rate must rise from r to r to equilibrate the money market. Therefore, according to the theory of liquidity preference, the higher output leads to higher interest rate. The LM curve plots this relationship between the level of output and the interest rate. The higher the level of output, the higher the demand for real money balances, and the higher the equilibrium interest rate. For this reason, the LM curve slopes upward in the second graph of the figure. [5]

[3] The money supply *M* is an exogenous policy variable chosen by a central bank.
[4] From Keynes's theory, we know that in short run the price level is fixed.
[5] Ibid.



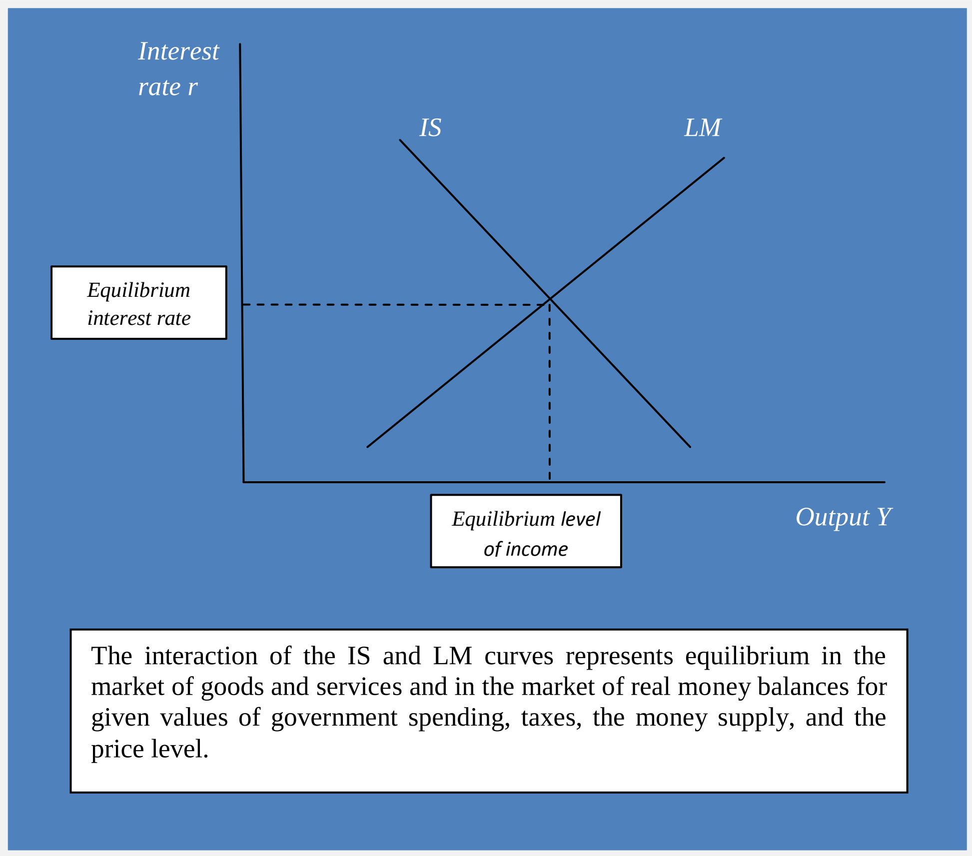
The first graph represents the market for real money balances: an increase in output from Y_1 to Y_2 raises the demand for money and thus raises the interest rate form r_1 to r_2. The second graph represents the LM curve that summarizes this relationship between the interest rate and output: the higher level of output, the higher the interest rate.

The IS-LM model contains two equations that represent the short-run equilibrium in ne close economy:

$$IS \qquad Y = C(Y-T) + I(r) + G \tag{3}$$

$$LM \qquad {}^{M}\!/_{P} = L(r, Y) \tag{4}$$

From first equation, we can conclude that the main determined of output is the interest rate. The fact that the model takes all variables as a give except interest rate, the IS curve provides the combination of r and Y that satisfy the equation representing the goods market. On the other side, the second equation shows the interest rate as a main variable of market for real money balances, and the LM curve provides the combination of r and Y that satisfy the equation representing the money market.



The interaction of the IS and LM curves represents the equilibrium in the market for goods and services and in the market for real money balances for given values of government spending, taxes, the money supply, and the price level. The equilibrium of the economy is the point at which the IS curve and the LM curve cross. This point gives the interest rate r and the level of income *Y* that satisfy conditions for equilibrium in both the goods market and the money market. In this regard, we can conclude that when economy function of equilibrium level, actual expenditure equals planned expenditure and the demand for real money balances equals the supply.

Data description

These are U.S. time series data they contain: logarithm of real GDP (q), 3 month interbank interest rate (i), real monetary base (m).Original time series are from the Federal Reserve Economic Data (FRED) database . The data included in this file are obtained by the following transformations:

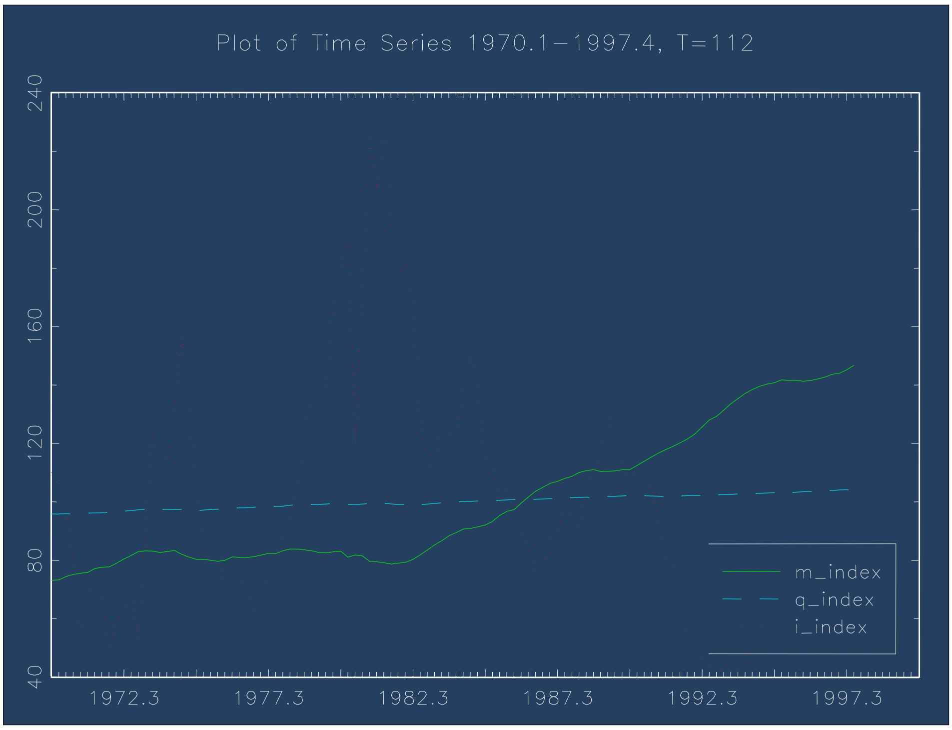
1. Observations for the interest rate and the monetary base are converted to quarterly frequency by averaging the monthly values.

2. q = log("Real Gross Domestic Product")

i = "3-Month Bankers' Acceptance Rate"

m = log("St. Louis Adjusted Monetary Base"/"GDP Implicit Price Deflator")

For a viewers good, we will plot this data on the following graph:



Te data are quarterly US data from the time period from 1970Q1 to 1997Q4. From the above plot we can roughly see that equilibrium, between money market and goods market is achieved in 1985-1986.

Descriptive statistics of the model

sample range:	[1970 Q1, 1997 Q4], T = 112			
DESCRIPTIVE STATISTICS:				
variable	mean	min	max	std. dev.
m	1.00020e+00	7.31711e-01	1.46723e+00	2.30375e-01
q	8.55226e+00	8.19108e+00	8.91000e+00	1.99013e-01
i	7.43699e-02	3.06000e-02	1.68633e-01	2.98795e-02

The above Table rapports the usual statistics of the model, that includes mean minimum, maximum and standard deviation.

The Jarque Bera test of normality and ARCH LM- test of heteroscedasticity with 2 lags

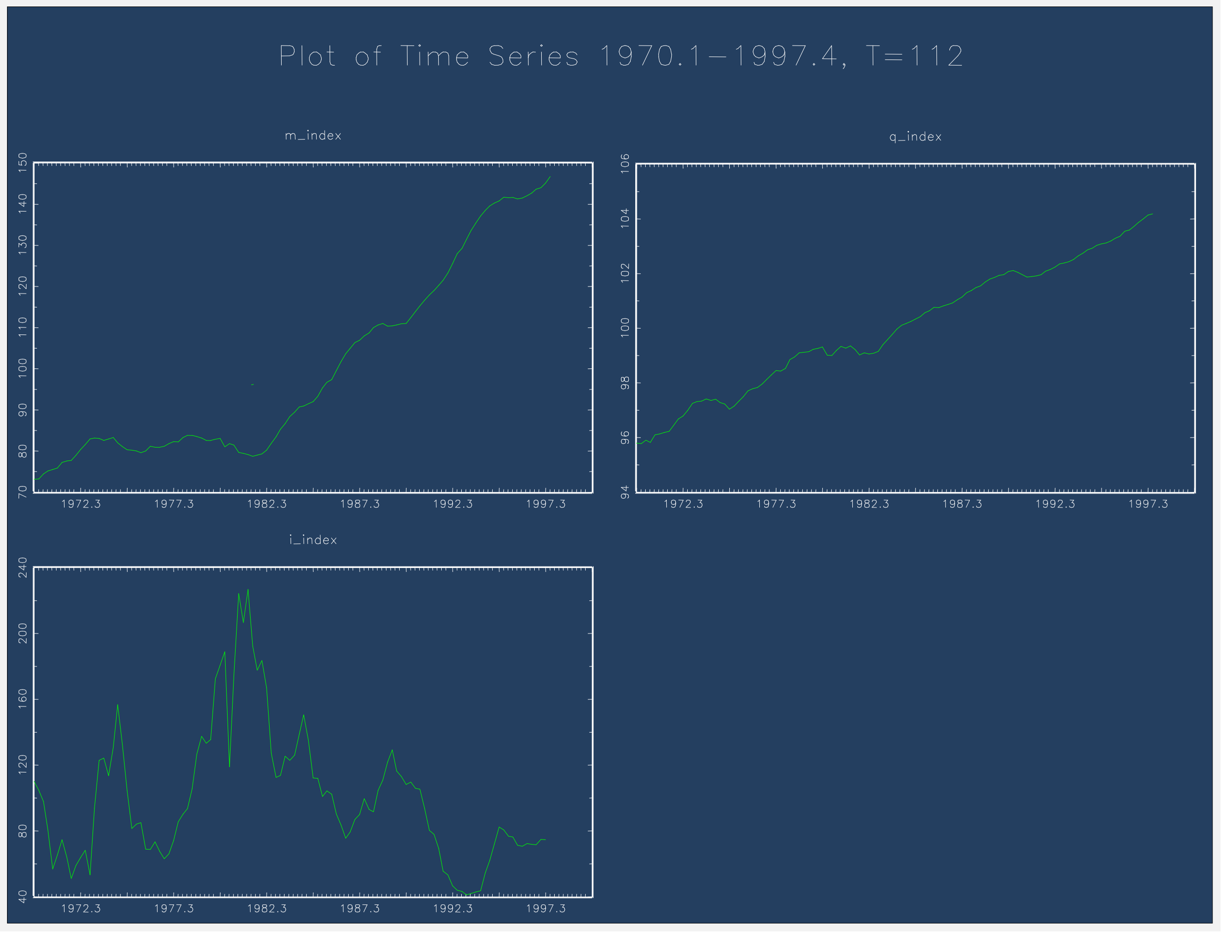
Test of normality and test of heteroscedasticity are being conducted:

JARQUE-BERA TEST				
variable	teststat	p-Value(Chi^2)	skewness	kurtosis
m	13.8522	0.0010	0.7255	2.0711
q	5.7531	0.0563	-0.0623	1.8967
i	22.7546	0.0000	1.0181	3.8545
ARCH-LM TEST with 2 lags				
variable	teststat	p-Value(Chi^2)	F stat	p-Value(F)
m	109.7227	0.0000	21765.2393	0.0000
q	108.9136	0.0000	5514.0497	0.0000
i	67.5512	0.0000	87.5248	0.0000

Normality is not a problem in this model, but heteroscedasticity is present. This is because series have unequal variances. Interest rates are volatile, same as monetary base.

Plot of the series

On the next plot series are being plotted individually.



ADF test

We Augment : $\Delta Y_t = \beta Y_{t-1} + u_t$

1. Constant or "drift" term (α_0)
   - random walk with drift
2. Time trend (T)
   - test H_O: unit root
     - conditional on a deterministic time trend
     - and against H_A: deterministic time trend
3. Lagged values of the dependent variable
   - sufficient for residuals free of autocorrelation

$\Rightarrow$ ADF: $\Delta Y_t = \alpha_0 + \gamma T + \beta Y_{t-1} + \delta_1 \Delta Y_{t-1} + \delta_2 \Delta Y_{t-2} + \ldots + \delta_3 \Delta Y_{t-n} + u_t$

Problems with unit root tests are as follows:

1. Low power in short time series
   - tend to under-reject H_0:unit root
     against H_A: stationarity
   - Endemic problem
2. Critical values for UR tests depend on what the test is conditioned on

- Critical values differ with specification of the testing equation
  - Inclusion/exclusion of
    - drift term
    - deterministic time trend
    - lags of the differenced variable
      - and the number of lags
- Another problem
  - terms to control for structural breaks $\Rightarrow$ also change the critical values

Here is a sample of time series modeling but with time break

$$y_t = \hat{\mu} + \hat{\theta} DU_t + \hat{\beta}_t + \hat{\gamma} DT_t + \hat{d} D(TB)_t$$
$$+ \hat{\alpha} y_{t-1} + \sum_{i=1}^{k} \hat{c}_i \Delta y_{t-i} + \hat{e}_t$$

- Same as in any ADF test

μ_t : constant or estimated "drift" term

β_t : (deterministic) time trend

y_{t-1}: 1st lag

Δy_{t-i}: lagged differences

- To implement empirically
  - subtract y_{t-1} from both sides

$\Rightarrow \beta_1$ = ([ά-hat] – 1)

We use JMULTI software that adds seasonal dummy variables in the models and adds Trend break dummies.

Definition: T_B Time of the break is a period in which a one-time break in structure occurs i.e., a change in the parameters of the trend function .How to identify T_B? (Perron, 1990, p.161) Usually "visual inspection is sufficient", Relate T_B to "major" events (Great Stock or Oil crash)

Terms added to the ADF test

$D(TB)_t$ Models a one-time change in the intercept, i.e., in the level of the series a "crash" , = 1 if $t = T_B+1$; otherwise 0, DV=1 for the single period immediately ***after*** the break .

ADF test for m- log("St. Louis Adjusted Monetary Base"/"GDP Implicit Price Deflator")

```
ADF Test for series:        m
sample range:               [1970 Q4, 1997 Q4], T = 109
lagged differences:         2
intercept, time trend, seasonal dummies
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%          5%          10%
-3.96       -3.41       -3.13
value of test statistic: -1.3650
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)          -0.0099       -1.3650
dx(-1)           0.5203        5.4222
dx(-2)           0.1756        1.8228
constant         0.0113        1.5284
trend            0.0001        1.9546
sdummy(2)       -0.0009       -0.5206
sdummy(3)        0.0027        1.5531
sdummy(4)        0.0011        0.6533
RSS              0.0040
```

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA	
sample range: [1972 Q4, 1997 Q4], T = 101	
	optimal number of lags (searched up to 10 lags of 1. differences)
Akaike Info Criterion	3
Hannan-Quinn Criterion	3
Final Prediction Error	3
Schwarz Criterion	1

From the above tables about the monetary base, this variable is unit root with a drift variable. Coefficient on the trend variable is small 0.0001 but significant above 1.96 t-stats. From the optimal endogenous lags info criteria optimal number of lags for this variable I three.

ADF Test for series: i

```
sample range:               [1970 Q4, 1997 Q4], T = 109
lagged differences:         2
intercept, time trend, seasonal dummies
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%          5%          10%
-3.96       -3.41       -3.13
value of test statistic: -1.9914
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)         -0.0803       -1.9914
dx(-1)          0.1314        1.3427
dx(-2)         -0.1243       -1.2623
constant        0.0058        1.5563
trend          -0.0000       -0.7589
sdummy(2)      -0.0025       -0.7985
sdummy(3)       0.0020        0.6330
sdummy(4)       0.0006        0.1733
RSS             0.0141
```

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:	[1972 Q4, 1997 Q4], T = 101
	optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion	5
Final Prediction Error	5
Schwarz Criterion	0
Hannan-Quinn Criterion	0

This variable interest rates in US economy has unit root and optimal number of endogenous lags by the info criteria is up to 5 lags.

ADF Test for series: q

```
sample range:              [1970 Q4, 1997 Q4], T = 109
lagged differences:        2
intercept, time trend, seasonal dummies
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%          5%          10%
-3.96       -3.41       -3.13
value of test statistic: -3.3346
regression results:
---------------------------------------
variable       coefficient    t-statistic
---------------------------------------
 x(-1)         -0.1182        -3.3346
dx(-1)          0.2972         3.1691
dx(-2)          0.2157         2.2363
constant        1.0142         3.3470
trend           0.0007         3.3096
sdummy(2)       0.0011         0.5058
sdummy(3)       0.0004         0.2040
sdummy(4)      -0.0008        -0.3685
RSS             0.0060
```

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

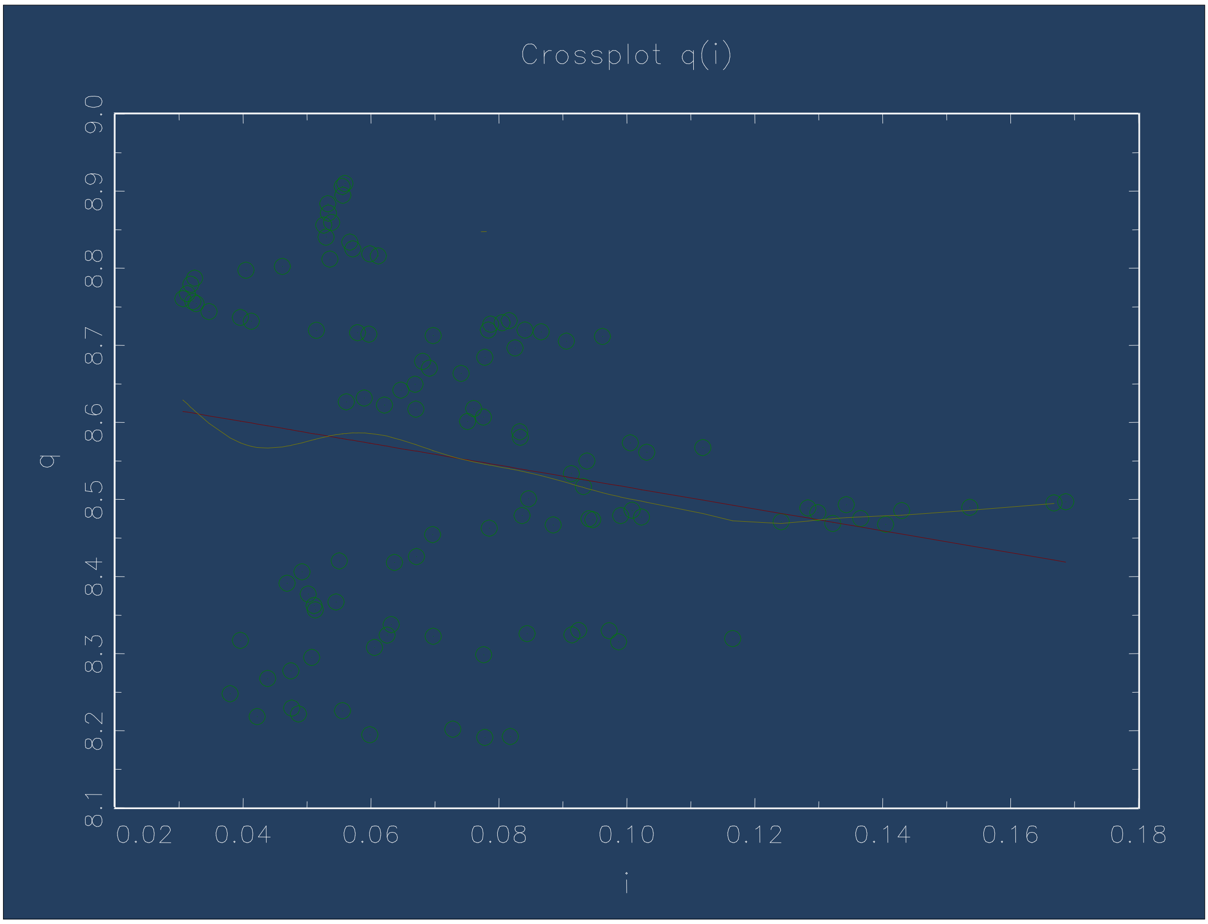
sample range:	[1972 Q4, 1997 Q4], T = 101
	optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion	2
Final Prediction Error	2
Schwarz Criterion	1
Hannan-Quinn Criterion	1

This variable has unit root with a drift term since the coefficient on the trend term is significant, and optimal number of lags are maximum up to 2.

OLS and Nadaraya-Watson regression

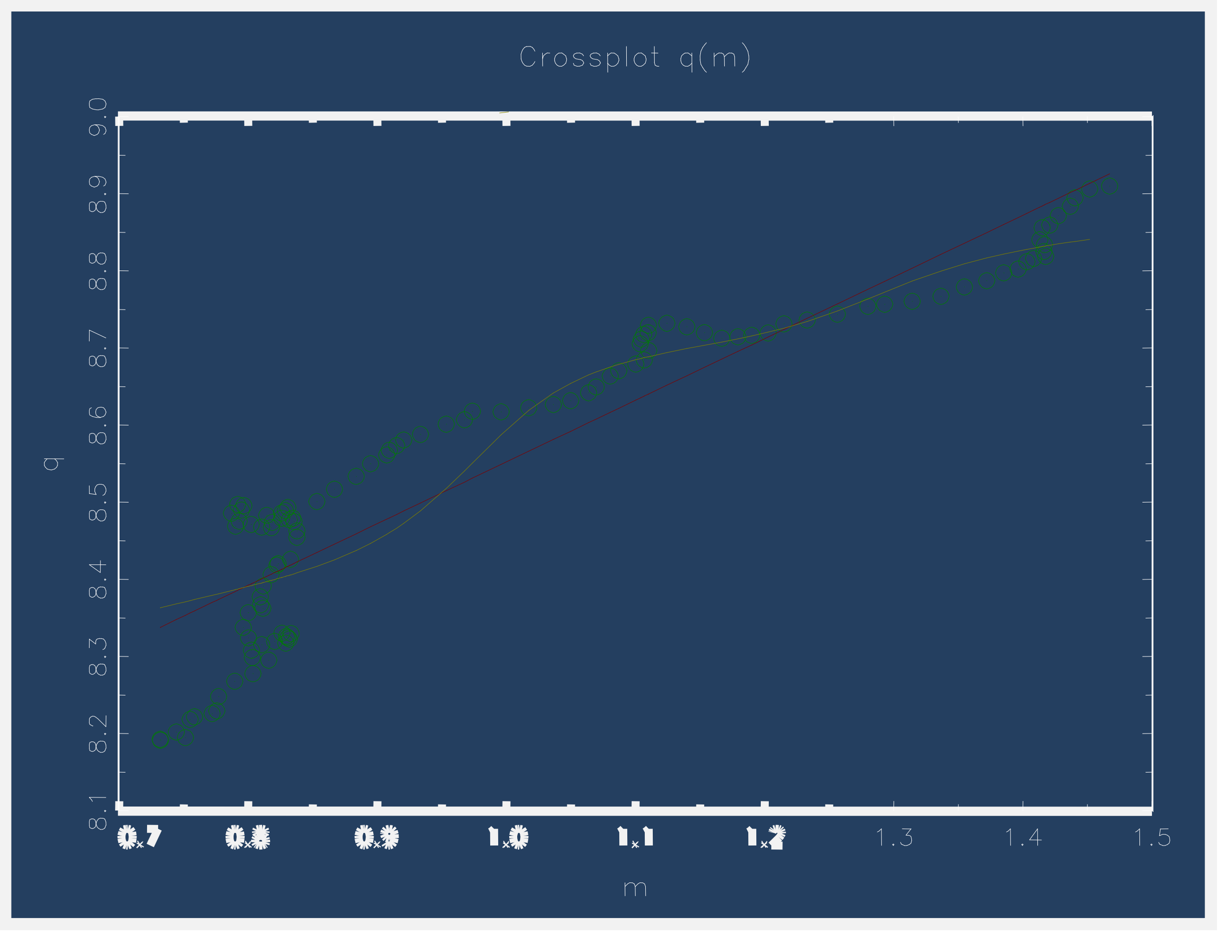
Next we present Nadaraya-Watson plots of OLS regressions

First we regress q on i (log of real US GDP with three months interest rates)



OLS ESTIMATION
sample range: [1970 Q1, 1997 Q4], T = 112
dependent: q
independent: i
q = 8.6576 + -1.4159 *i
t-values = { 174.0741 -2.2817 }
sigma = 0.1962
R-squared = 0.0452

From Nadaraya-Watson OLS regression we can see that the there is negative slope between q and i, trend is also negative. This means that interest rates and GDP are inversely related.



OLS ESTIMATION
sample range: [1970 Q1, 1997 Q4], T = 112
dependent: q
independent: m
q = 7.7525 + 0.7996 *m
t-values = { 242.2745 25.6475 }
sigma = 0.0760
R-squared = 0.8567

q and m are positively related .This means that log of Real GDP and" St. Louis Adjusted Monetary Base"/"GDP Implicit Price Deflator are positively associated.

Testing for cointegration

Π = the equilibrium matrix in the error-correction model.Procedure is as follows: calculate the rank of Π, i.e., number of independent rows or columns there exist 3 possibilities

1. Rank(Π) = 0

–VECM reduces to a VAR in 1st differences

–1st differences are I(0) $\Rightarrow$ no cointegration

2. Rank(Π) = 2 This Occurs only when both variables stationary and what follows no common trend $\Rightarrow$ independent $\Rightarrow$ variables over-differenced and correct model is in levels, not 1st differences

1. Rank(Π) = 1 One independent row $\Rightarrow$ determinant of Π = 0

(Product of Diagonal 1) – (Product of Diagonal 2) = 0

One cointegrating vector (r), Each term in Π is assumed non-zero and long-run or equilibrium coefficient on Y or Z.

- Procedure is as follows : Decompose Π into 2 q×r matrices where α = matrix of short-run "adjustment" coefficients in the EC Model

β' = each row is one of the r

Johansen Trace Test for: m i q

```
unrestricted dummies:      D[1982 Q1] D[1982 Q2]
restricted dummies:        S[1982 Q1]
sample range:              [1970 Q3, 1997 Q4], T = 110
included lags (levels):    2
dimension of the process: 3
intercept included
seasonal dummies included
response surface computed:
 r0   LR        pval      90%       95%       99%
-----------------------------------------------
 0    89.03     0.0000    37.61     39.81     44.17
 1    25.98     0.0242    22.29     24.18     28.00
 2    8.89      0.2126    11.02     12.82     16.66
```

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA	
sample range: [1972 Q4, 1997 Q4], T = 101	
	optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion	6
Final Prediction Error	2
Schwarz Criterion	2
Hannan-Quinn Criterion	2

Since there is unit root between these variables, they are cointegrated of order 1 I(1) as johansen test shows. Optimal number of endogenous lags by info criteria is 2.

ARIMA for i variable

Three months interbank interest rates is being tested for optimal lags by Hannan and Rissanen test. And the optimal number of lags is (1,0)

OPTIMAL LAGS FROM HANNAN-RISSANEN MODEL SELECTION	
(Hannan & Rissanen, 1982, Biometrika 69)	
original variable:	i
order of differencing (d):	0
adjusted sample range:	[1972 Q4, 1997 Q4], T = 101
optimal lags p, q (searched all combinations where max(p,q) <= 3)	
Akaike Info Criterion:	p=1, q=0
Hannan-Quinn Criterion:	p=1, q=0
Schwarz Criterion:	p=1, q=0

For 1st difference of the variable optimal number of lags is zero(0,0).

OPTIMAL LAGS FROM HANNAN-RISSANEN MODEL SELECTION	
(Hannan & Rissanen, 1982, Biometrika 69)	
original variable:	i
order of differencing (d):	1
adjusted sample range:	[1973 Q1, 1997 Q4], T = 100
optimal lags p, q (searched all combinations where max(p,q) <= 3)	
Akaike Info Criterion:	p=0, q=0
Hannan-Quinn Criterion:	p=0, q=0
Schwarz Criterion:	p=0, q=0

ARIMA

Model: ARIMA(0,0,0)				
Final Results:				
Iterations Until Convergence: 1				
Log Likelihood: 237.438744	Number of Residuals: 112			
AIC : -464.877488	Error Variance : 0.000883001			
SBC : -451.284993	Standard Error : 0.029715327			
DF: 107 Adj. SSE: 0.094481072	SSE: 0.094481072			
Dependent Variable: i				
	Coefficients	Std. Errors	T-Ratio	Approx. Prob.
CONST	0.08737120	0.00754583	11.57874	0.00000
S1	-0.00253821	0.00794603	-0.31943	0.75002
S2	-0.00085841	0.00794366	-0.10806	0.91415
S3	0.00009758	0.00794223	0.01229	0.99022
TREND	-0.00021551	0.00008690	-2.48001	0.01470

In the ARIMA models seasonal dummies are not significant, while trend is this variable has unit root with a drift.

ARIMA for m- log("St. Louis Adjusted Monetary Base"/"GDP Implicit Price Deflator")

This variable is first difference variable. And the optimal number of lags is (1,1)

OPTIMAL LAGS FROM HANNAN-RISSANEN MODEL SELECTION	
(Hannan & Rissanen, 1982, Biometrika 69)	
original variable:	m
order of differencing (d):	1
adjusted sample range:	[1973 Q1, 1997 Q4], T = 100
optimal lags p, q (searched all combinations where max(p,q) <= 3)	
Akaike Info Criterion:	p=1, q=1
Hannan-Quinn Criterion:	p=1, q=1
Schwarz Criterion:	p=1, q=1

Model: ARIMA(0,1,0)				
Final Results:				
Iterations Until Convergence: 1				
Log Likelihood: 378.975787	Number of Residuals: 111			
AIC : -747.951574	Error Variance : 0.000066376			
SBC : -734.403923	Standard Error : 0.008147132			
DF: 106 Adj. SSE: 0.007035831	SSE: 0.007035831			
Dependent Variable: m				
	Coefficients	Std. Errors	T-Ratio	Approx. Prob.
CONST	0.00086107	0.00208108	0.41376	0.67989
S1	-0.00148637	0.00219761	-0.67636	0.50029
S2	0.00107037	0.00217795	0.49146	0.62412
S3	0.00111285	0.00217755	0.51106	0.61037
TREND	0.00009783	0.00002414	4.05246	0.00010

This above table presents ARIMA (01,0) model for st.louis monetary base adjusted for CPI deflator. Trend is only variable that is significant while others including seasonal dummies and constant are not significant. This is unit root with a drift variable.

ARIMA for q variable (log of real US GDP)

This variable is 1st difference variable optimal lags are (1,0)

OPTIMAL LAGS FROM HANNAN-RISSANEN MODEL SELECTION	
(Hannan & Rissanen, 1982, Biometrika 69)	
original variable:	q
order of differencing (d):	1
adjusted sample range:	[1973 Q1, 1997 Q4], T = 100
optimal lags p, q (searched all combinations where max(p,q) <= 3)	
Akaike Info Criterion:	p=1, q=0
Hannan-Quinn Criterion:	p=1, q=0
Schwarz Criterion:	p=1, q=0

Model: ARIMA(0,1,0)

Final Results:

Iterations Until Convergence: 1

Log Likelihood: 374.802067 Number of Residuals: 111

AIC : -739.604135 Error Variance : 0.000071560

SBC : -726.056484 Standard Error : 0.008459306

DF: 106 Adj. SSE: 0.007585344 SSE: 0.007585344

Dependent Variable: q

	Coefficients	Std. Errors	T-Ratio	Approx. Prob.
CONST	0.00624187	0.00216082	2.88866	0.00469
S1	0.00104755	0.00228182	0.45909	0.64711
S2	0.00040380	0.00226140	0.17856	0.85862
S3	-0.00030865	0.00226098	-0.13651	0.89168
TREND	-0.00000097	0.00002506	-0.03875	0.96916

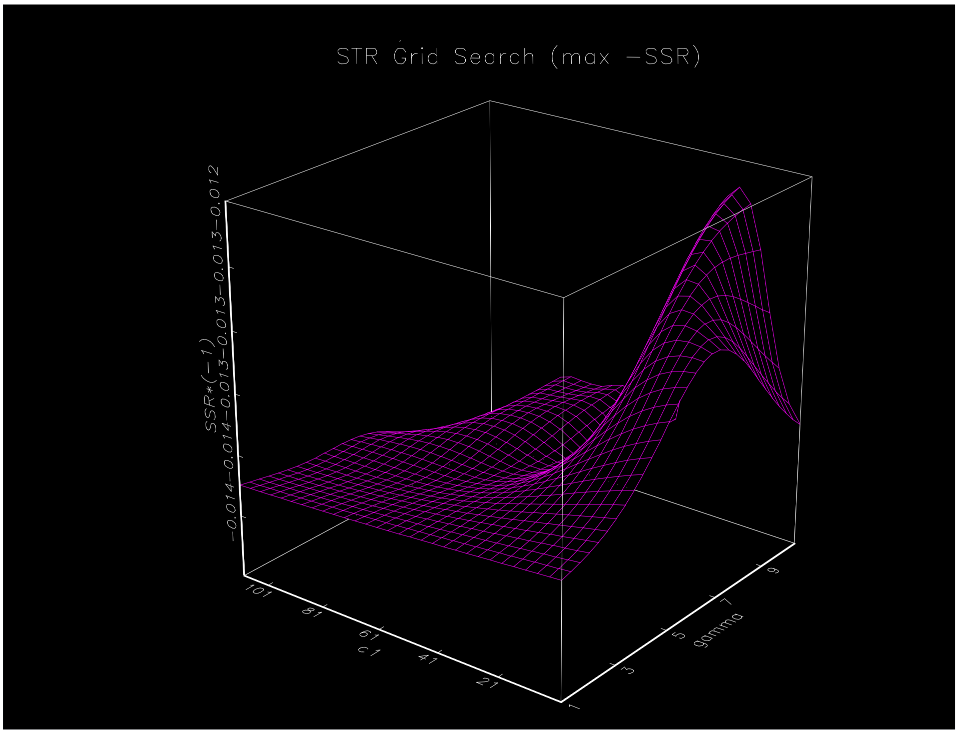
In the arima model for log of real US GDP only constant term is significant.

Smooth transition regressions

Firs we will run this regression for interbank interest rate here transition variable is trend and two lags in AR part. Results are below followed by the graphical presentation.

STR GRID SEARCH

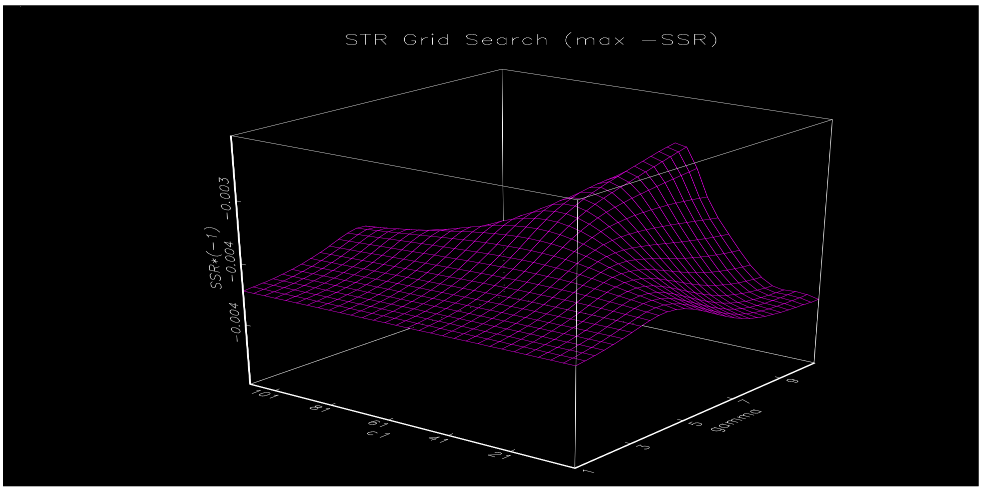
```
variables in AR part:      CONST i(t-1) i(t-2)
restriction theta=0:
transition variable:       TREND
sample range:              [1970 Q3, 1997 Q4], T = 110
transition function:       LSTR1
grid c                     { 1.00, 110.00, 30}
grid gamma                 { 0.50, 10.00, 30}
SSR           gamma         c1
0.0124        10.0000       23.5517
```



On the Table below is presented ST Regression for interbank interest rate.

Smooth transition regression for monetary base variable (m) is given in a table belowe

STR GRID SEARCH		
variables in AR part:		CONST m(t-1) m(t-2)
restriction theta=0:		
transition variable:		m(t-1)
sample range:		[1970 Q3, 1997 Q4], T = 110
transition function:		LSTR1
grid c		{ 0.73, 1.45, 30}
grid gamma		{ 0.50, 10.00, 30}
SSR	gamma	c1
0.0037	10.0000	0.8560



Smooth transition regression for log of real GDP

This regression is given below as well as graphical presentation

STR GRID SEARCH

variables in AR part: CONST q(t-1) q(t-2)

restriction theta=0:

transition variable: TREND

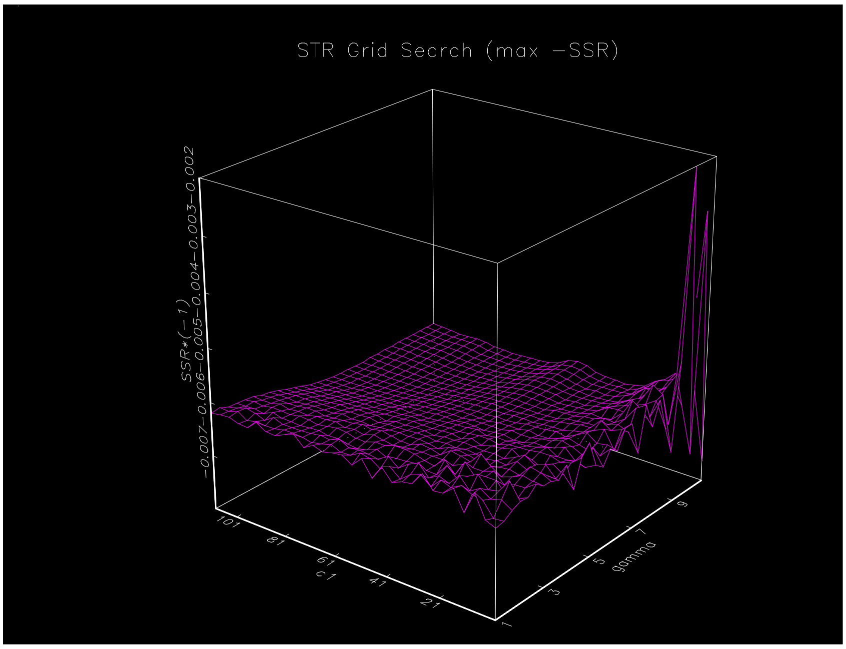
sample range: [1970 Q3, 1997 Q4], T = 110

transition function: LSTR1

grid c { 1.00, 110.00, 30}

grid gamma { 0.50, 10.00, 30}

SSR	gamma	c1
0.0020	7.3352	1.0000



VAR model

VAR is a Relationship between 2 or more variables modelled as a VAR.Vector Auto-Regression where each variable regressed on lags of itself and the other variables, X = vector of q variables of interest, both endogenous and exogenous variables, distinction determined by the analysis

- Π = matrix of coefficients
- k = maximum lag
- ε = an error term ("white noise")

$$X_t = \Pi_{t-1} X_{t-1} + \Pi_{t-2} X_{t-2} + \ldots + \Pi_{t-k} X_{t-k} + \varepsilon_t$$

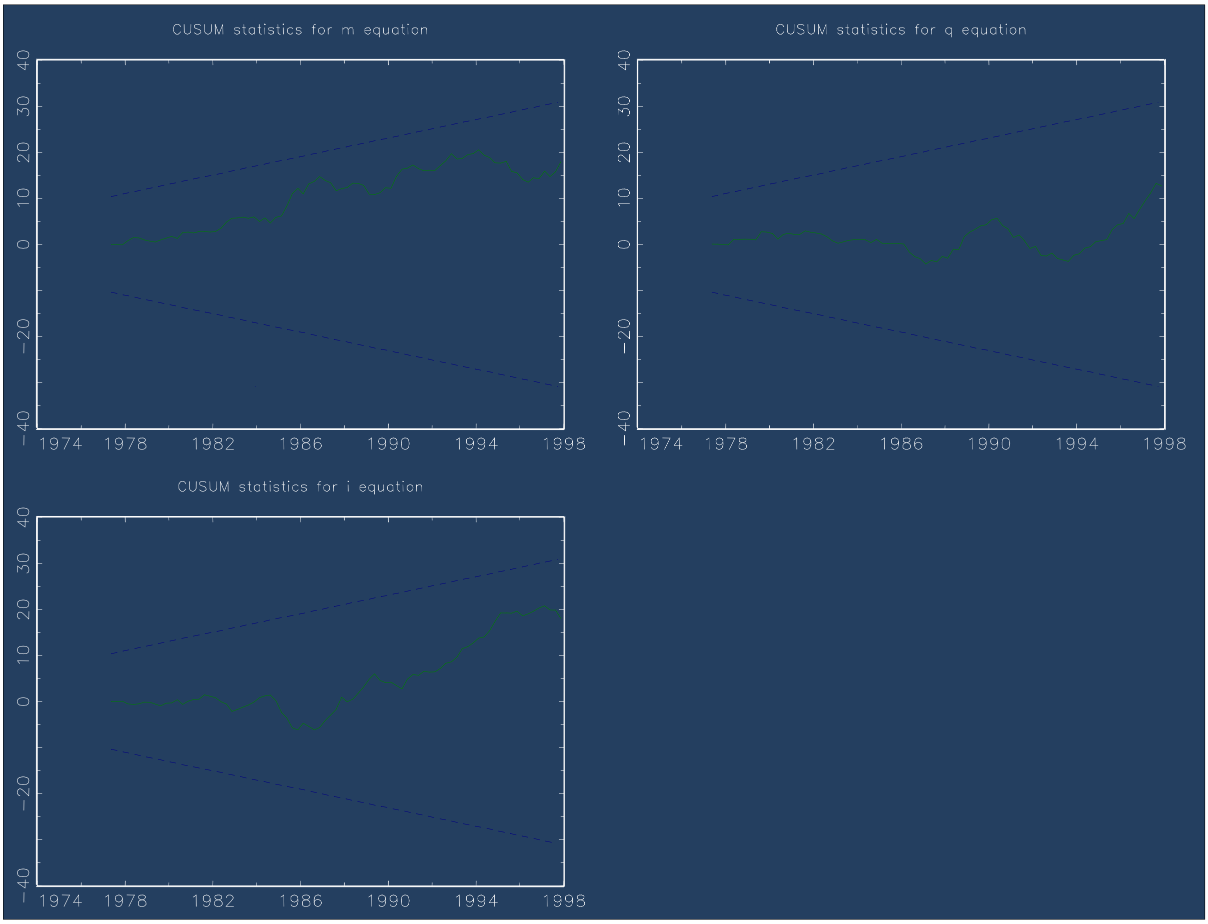
$$\begin{bmatrix} m(t) \\ q(t) \\ i(t) \end{bmatrix} = \begin{bmatrix} 1.213 & -0.235 & -0.187 \\ -0.202 & 1.142 & -0.094 \\ 0.577 & 0.804 & 1.017 \end{bmatrix} \begin{bmatrix} m(t-1) \\ q(t-1) \\ i(t-1) \end{bmatrix} + \begin{bmatrix} -0.174 & 0.391 & 0.064 \\ -0.165 & 1.198 & -0.271 \\ -0.890 & -0.620 & -0.220 \end{bmatrix} \begin{bmatrix} m(t-2) \\ q(t-2) \\ i(t-2) \end{bmatrix}$$

$$+ \begin{bmatrix} 0.044 & -0.180 & -0.085 \\ 0.410 & 0.060 & 0.263 \\ 0.563 & -0.013 & 0.400 \end{bmatrix} \begin{bmatrix} m(t-3) \\ q(t-3) \\ i(t-3) \end{bmatrix} + \begin{bmatrix} -0.035 & 0.015 & 0.099 \\ -0.133 & -0.074 & -0.255 \\ 0.146 & -0.158 & 0.069 \end{bmatrix} \begin{bmatrix} m(t-4) \\ q(t-4) \\ i(t-4) \end{bmatrix} +$$

$$\begin{bmatrix} -0.086 & 0.044 & -0.120 \\ 0.173 & 0.006 & 0.179 \\ -0.691 & 0.011 & 0.155 \end{bmatrix} \begin{bmatrix} m(t-5) \\ q(t-5) \\ i(t-5) \end{bmatrix} + \begin{bmatrix} 0.015 & -0.050 & 0.163 \\ -0.132 & -0.063 & -0.067 \\ 0.312 & 0.147 & -0.394 \end{bmatrix} \begin{bmatrix} m(t-6) \\ q(t-6) \\ i(t-6) \end{bmatrix} +$$

$$\begin{bmatrix} 0.147 & -0.001 & 0.001 & 0.001 & 0.000 \\ 1.00 & -0.002 & -0.001 & -0.003 & 0.001 \\ -1.408 & -0.003 & 0.000 & -0.004 & -0.001 \end{bmatrix} \begin{bmatrix} CONST \\ S1(t) \\ S2(t) \\ S3(t) \\ TREND(t) \end{bmatrix} + \begin{bmatrix} u1(t) \\ u2(t) \\ u3(t) \end{bmatrix}$$

This VAR model contains data form 1971 Q3 to 1997Q4. CUSUM test below shows that m,q,and i equation do not leave the margins of normal distribution.



CHOW test for VAR

Chow test for VAR shows structural stability of the model and if the model is not stable we should continue testing.

CHOW TEST FOR STRUCTURAL BREAK

```
On the reliability of Chow-type tests..., B. Candelon, H. Lütkepohl, Economic Letters 73
(2001), 155-160
sample range:                [1971 Q3, 1997 Q4], T = 106
tested break date:           1978 Q1 (26 observations before break)
break point Chow test:       555.1126
 bootstrapped p-value:       0.0000
 asymptotic chi^2 p-value:   0.0000
 degrees of freedom:         75
sample split Chow test:      213.1091
 bootstrapped p-value:       0.0000
 asymptotic chi^2 p-value:   0.0000
 degrees of freedom:         69
Chow forecast test:          25.6641
 bootstrapped p-value:       0.0000
 asymptotic F p-value:       0.0103
 degrees of freedom:         240, 3
```

From the above table for Chow test , break point chow test showed that the model is not stable, also sample split test showed that, while chow forecast test is only significant at 10%, this means we have to continue with VECM model.

VECM model

VECM model can be introduced in matrix connotation also

$$\begin{aligned}
\Delta Y_t &= -(1-\Pi_{11})\Delta Y_{t-1} + \Pi_{12}\Delta Z_{t-1} && \text{Differences} \\
&\quad -(1-\Pi_{11}-\Pi_{13})Y_{t-2} + (\Pi_{12}+\Pi_{14})Z_{t-2} && \text{Levels} \\
&\quad +\mu_1 + \varepsilon_{1t} && (1') \\
\Delta Z_t &= \Pi_{21}\Delta Y_{t-1} - (1-\Pi_{22})\Delta Z_{t-1} && \text{Differences} \\
&\quad +(\Pi_{21}+\Pi_{23})Y_{t-2} - (1-\Pi_{22}-\Pi_{24})Z_{t-2} && \text{Levels} \\
&\quad +\mu_2 + \varepsilon_{2t} && (2')
\end{aligned}$$

In matrices

$$\begin{bmatrix}\Delta Y_t \\ \Delta Z_t\end{bmatrix} = \underbrace{\begin{bmatrix}-(1-\Pi_{11}) & \Pi_{12} \\ \Pi_{21} & -(1-\Pi_{22})\end{bmatrix}}_{\equiv \Gamma}\begin{bmatrix}\Delta Y_{t-1} \\ \Delta Z_{t-1}\end{bmatrix} \qquad \text{Differences}$$

$$+\underbrace{\begin{bmatrix}-(1-\Pi_{11}-\Pi_{13} & (\Pi_{12}+\Pi_{14}) \\ (\Pi_{21}+\Pi_{23}) & -(1-\Pi_{22}-\Pi_{24})\end{bmatrix}}_{\equiv \Pi}\begin{bmatrix}Y_{t-2} \\ Z_{t-2}\end{bmatrix} \qquad \text{Levels}$$

$$+\begin{bmatrix}\mu_1 \\ \mu_2\end{bmatrix} + \begin{bmatrix}\varepsilon_{1t} \\ \varepsilon_{2t}\end{bmatrix}$$

$$\begin{bmatrix} dm(t) \\ dq(t) \\ di(t) \end{bmatrix} = \begin{bmatrix} -0.055 \\ -0.046 \\ -0.004 \end{bmatrix} \begin{bmatrix} 1.000 & 0.856 & 4.292 \end{bmatrix} \begin{bmatrix} m(t-1) \\ q(t-1) \\ i(t-1) \end{bmatrix}$$

$$+\begin{bmatrix} -8.063 & -0.027 & -0.029 & -0.020 & -0.012 \end{bmatrix} \begin{bmatrix} CONST \\ S1(t) \\ S2(t) \\ S3(t) \\ TREND(t) \end{bmatrix} + \begin{bmatrix} u1(t) \\ u2(t) \\ u3(t) \end{bmatrix}$$

From the above VECM model, i.e from its VECM mechanism we can see that if the system is in disequilibrium alteration in the change of interbank interchange interest rate, log of real US gdp , and monetary base will be downward 5,5%,4,6% and 0,4% respectively.

Chow test for VECM

These results below show that CHOW test implies stability here which means that VECM models is stable.

CHOW TEST FOR STRUCTURAL BREAK

```
On the reliability of Chow-type tests..., B. Candelon, H.
Lütkepohl, Economic Letters 73 (2001), 155-160
sample range:                [1970 Q2, 1997 Q4], T = 111
tested break date:             1973 Q2 (12 observations before
break)
break point Chow test:       19.7045
 bootstrapped p-value:       0.1200
 asymptotic chi^2 p-value:   0.0198
 degrees of freedom:         9
sample split Chow test:      6.6088
 bootstrapped p-value:       0.1000
 asymptotic chi^2 p-value:   0.0855
 degrees of freedom:         3
Chow forecast test:          0.2300
 bootstrapped p-value:       0.4900
 asymptotic F p-value:       0.9997
 degrees of freedom:         297, 6
```

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