The propensity function as formal passkey to economic action

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The Propensity Function as Formal Passkey to Economic Action

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Abstract

The purpose of the present paper is to demonstrate how the interaction of the structural axiomatic core and the behavioral propensity function produces plausible outcomes in the product market. The propensity function is a compact formal expression of random, semi-random, and deterministic behavioral assumptions. Its two components are direction and magnitude of the rate of change of an elementary axiomatic variable. A type-C propensity function is the formal container for a familiar conception that Samuelson identified as qualitative prediction. Two type-C functions are sufficient to produce stochastic stability and optimality in the product market.

JEL D01, D03

Keywords New framework of concepts, Structure-centric, Axiom set, Qualitative prediction, Tendency laws, Separability, Determinism–indeterminism, Information function, Action function

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> The undefined and defined terms, the axioms, and all the theorems that can be derived from them constitute a theory. (Stigum, 1991, p. 36)

Standard economics rests on a set of behavioral axioms (Arrow and Hahn, 1991, p. v) which, in an abstract form, rephrase the minimax principle as verbally laid down by J. S. Mill:

> Just in the same manner [as geometry] does Political Economy presuppose an arbitrary definition of man, as a being who invariably does that by which he may obtain the greatest amount of necessaries, conveniences, and luxuries, with the smallest quantity of labour and physical self-denial with which they can be obtained in the existing state of knowledge. (Mill, 2004, p. 110)

Mill, though, regarded this behavioral principle as a working hypothesis with limited applicability:

> In political economy for instance, empirical laws of human nature are tacitly assumed by English thinkers, which are calculated only for Great Britain and the United States. (Mill, 2006, p. 906)

And Mill never defended the minimax principle as a realistic description of human behavior, but on methodological grounds.

> Not that any political economist was ever so absurd as to suppose that mankind are really thus constituted, but because this is the mode in which science must necessarily proceed. (Mill, 2004, p. 106)

Hence any criticism on moral, psychological or sociological grounds remains on the surface and misses Mill's intention. But, by the same token, to invest a rather trivial behavioral hypothesis with the status of an irremovable cornerstone of the whole theoretical edifice is hardly in accordance with Mill's understanding of economics as a scientific endeavor. Value, wealth and the iron laws of distribution were the real issues of the classics and human behavior was a theoretical sideshow. Somewhere in the process, though, analytical priorities changed and homo oeconomicus took over center stage – not to everyone's satisfaction.

> No science has been criticized by its own servants as openly and constantly as economics. The motives of dissatisfaction are many, but the most important pertains to the fiction of *homo oeconomicus.* (Georgescu-Roegen, 1971, p. 1)

It is by now well-understood that the homo oeconomicus approach is not unsatisfactory because of one-sidedness or unrealism, as common sense would have it, but because it cannot work for methodological reasons.

> The predictive weakness of theories couched in intensional vocabulary do not correlate in a manageable way with the vocabulary of other successful scientific theories; they do not divide nature at the joints; .... (Rosenberg, 1994, p. 224)

To properly divide nature at the joints the subject matter is in the following formally separated into the structural axiom set and the propensity function. While the axiomatization of human behavior is in the last instance self-contradictory the axiomatization of the money economy's fundamental structure is feasible. Structural axioms are free of any explicit or implicit behavioral assumptions. This means that a general formal complement is required that captures human behavior, at least insofar as it is relevant for the functioning of the economic system. The purpose of the present paper is to demonstrate how the interaction of *structural* axiomatic core and *behavioral* propensity function produces plausible outcomes in the product market.

Accordingly the formal frame that constitutes the pure consumption economy is set up in section 1. In sections 2 and 3 the propensity function is introduced and related to familiar conceptions. The two components of the propensity function are direction and magnitude of the rate of change of an elementary axiomatic variable. The magnitude is left to chance, the direction is determined by the signum function. The properties of the signum function and the behavioral link between the information function and the action function are established in sections 4 and 5. In section 6 the whole formalism is put to work in a simulation. The directedness that originates from the signum function produces stochastic stability in the product market. In the structural axiomatic context the argument of the information function adopts the role of the first derivative. In section 7 the formal conditions of a general behavioral optimum are defined in structural axiomatic terms. Section 8 summarizes.

1 Axioms

The first three structural axioms relate to income, production, and expenditures in a period of arbitrary length. For the remainder of this inquiry the period length is conveniently assumed to be the calendar year. Simplicity demands that we have at first one world economy, one firm, and one product.

Total income of the household sector Y is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N.

$$Y = WL + DN \quad |t \qquad (1)$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment expenditures, no foreign trade, and no taxes or any other government activity.[1]

The period values of the variables are connected formally by the familiar growth equation, which is added to the structural set as the fourth axiom:

$$Z_t = Z_{t-1}\,(1 + \dddot{Z}_t) \tag{4}$$

The path of the representative variable Z_t, which stands for the seven elementary (*W, P, R, X, L, D, N,*) and the three composed *(Y, O, C)* axiomatic variables is then determined by the initial value Z_0 and the rates of change $\dddot{Z}_t$ for each period:

$$Z_t = Z_0\,(1 + \dddot{Z}_1)\,(1 + \dddot{Z}_2)\ldots(1 + \dddot{Z}_t) = Z_0 \prod_{t=1}^{t} (1 + \dddot{Z}_t) \tag{5}$$

Thus the price in period *t*, for example, is formally derived from the known price in period *t-1* and the actual price *change* in period *t*, which depends in the main on the decision maker's objectives, on the subjective interpretations of the actual situation, on expectations, on opportunities and limits, and on actionability. When the initial values are determined nothing but the rates of change of the elementary variables remain as explananda. Thus structural axiomatization is conductive to a theory of economic change.

2 The propensity function

Axiomatization provides the bare structural bones. The first question is how the rates of change of the elementary variables are determined. The obvious next step is to invoke both, purposeful agents and pure randomness, to 'animate' (Popper, 1994, p. 164) the formalized economy. To prepare for a general answer the rate of change

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[1] "... we shall be concerned with a particular, but very important, class of economic theories, namely those where the theoretical model consists of a system of (ordinary or functional) equations between certain economic variables. A few remarks may be made as to the common sense of this type of economic theory. Broadly speaking, we may classify such quantitative economic relations in the three groups: I. Definitional identities, II. Technical relations, III. Relations describing economic action. The first group is exemplified by such relations as: Total expenditure = price multiplied by quantity bought, total output = output per worker times the number of workers, and similar types of "bookkeeping identities." To the second group belong, e.g., technical production functions, and other natural or institutional restrictions which are usually taken as data in economic planning. In the third group we find the broad class of relations describing the behavior of individuals or collective units in their economic activity, their decisions to produce and consume." (Haavelmo, 1944, pp. 2-3)

Propensity type	State of knowledge	Set of possible outcomes $\{\dddot{Z}\}$
A: Uncertainty	a) definitive, i.e. proven by an impossibility theorem, or, b) contingent, i.e. may be overcome with better methods or tools	$\{-100\% \leq \dddot{Z} < \infty\}$ P (•) not defined
B: Randomness	The set of possible outcomes is inductively inferred from more or less sophisticated observations $(x_{B1}, x_{B2}, \ldots)$ that display no dominant influence $\{\dddot{Z}\} = f_B(x_{B1}, x_{B2}, \ldots)$	e.g. $\{-3\% \leq \dddot{Z} \leq 3\%\}$ P (•) equally distributed
C: Directed randomness	One conjectures a direction of change $\{-1, 0, 1\}$ and has a hypothesis that explains it in the logical format if [antecedent] then $\{-1; 0; 1\}$; $\{\dddot{Z}\} = f_C(x_{C1}, x_{C2}, \ldots)$	e.g. $\{-1; 0; 1\}\{0\% \leq \dddot{Z} \leq 3\%\}$ P (•) equally distributed
D: Dependency	One has a sufficiently specified hypothesis about the influence of the independent variables $(x_{D1}, x_{D2}, \ldots)$: $\{\dddot{Z}\} = f_D(x_{D1}, x_{D2}, \ldots)$	e.g. $\{7.2\% \pm 0,3\%\}$ P =0.99
E. Determinism	One has a law and knows the boundary conditions: $\{\dddot{Z}\} = f_E(x_{E1}, x_{E2}, \ldots)$	e.g. $\{7.19470\%\}$ P =1

Figure 1: The propensity function covers five logical types to derive the rates of change of the axiomatic variables in period *t*

of the representative variable $\dddot{Z}$ is replaced by a set of possible outcomes $\{\dddot{Z}\}$. Its rationale is outlined in Figure 1.

Figure 1 is consolidated to the propensity function (6) with $\dddot{Z}$ standing for the rates of change of the seven elementary variables.[2] The propensity function yields a definite rate of change for each period:

$$\dddot{Z} = \underbrace{\{-1, 0, 1\}}_{\text{direction}} \underbrace{P(\{\dddot{Z}\})}_{\text{magnitude}} \quad |t \tag{6}$$

The two components of the propensity function are direction and magnitude. In the limiting case of uncertainty, type-A, the propensity function is not defined. In the limiting case of determinism, type-E, everything is defined by the magnitude bracket, the value in the direction bracket is unity by default. At the moment no deterministic behavioral functions are known in economics. Type-A represents the status quo, i.e. that we have *no a priori* knowledge of the set of possible outcomes or any determinants of (6). To get things going we have to take type-B as the minimalist starting point of behavioral analysis:

> The simplest hypothesis is that variation is random until the contrary is shown, the onus of the proof resting on the advocate of the more complicated hypothesis ... . (Kreuzenkamp and McAleer, 1995, p. 12)

[2] The term propensity is here used in the sense of (Popper, 1990).

The magnitude of the rate of change is in the following left to chance. The agents are supposed to determine the direction of change.

3 The qualitative and separable agent

The direction of action is captured in a general form by a signum function that determines the algebraic sign of the random rate of change of the axiomatic variables:

$$\{-1,0,1\} = sgn_{\alpha}\left(\text{if [A]}\right) \quad |\text{t} \tag{7}$$

The direction is conditional on whether one or more antecedents are satisfied or not. The antecedent [A] is a shorthand for conditions of any degree of sophistication expressed by the operators >, <, =, AND, OR etc. and variables with a time index t or prior. Type-C is the formal container for a familiar conception that Samuelson identified as 'qualitative prediction':

> The qualitative predictions that we can test against data are roughly the signs, positive or negative, of the partial differentials of the changes in the values of economic variables we set out to measure. ... In general, qualitative predictions purport to identify the direction in which changes move, without, however, identifying the magnitude of these changes. Of course, as Samuelson notes, we would like to have more than qualitative predictions ... . (Rosenberg, 1992, p. 68)

The notion of directionality is by no means new in economics and can be traced back to J. S. Mill:[3]

[3] "The laws of economics are to be compared with the law of the tides, rather than with the simple and exact law of gravitation. For the actions of men are so various and uncertain, that the best statement of tendencies, which we can make in a science of human conduct, must needs be inexact and faulty. This might be urged as a reason against making any statements at all on the subject; .... Thus a law of social sciences, or a *Social Law*, is a statement of social tendencies; ..." (Marshall, 2009, pp. 26-27), original emphasis

"Price, and more especially the price of labour, is scarcely ever mentioned without provoking a reference to the 'inexorable', the 'immutable', the 'eternal' laws by which it is governed; to laws which, according to my friend Professor Fawcett, are 'as certain in their operation as those which control physical nature.' It is no small gain to have discovered that no such despotic laws do or can exist; that, inasmuch as the sole function of scientific law is to predict the invariable recurrence of the same effects from the same causes, and as there can be no invariability where – as in the case of price – one of the most efficient causes is that ever-changing chameleon, human character or disposition, price cannot possibly be subjected to law." (Thornton, 1869, p. 65)

"Gustav Rümelin, in a late essay on laws of history ... reflected that two decades of intensive research had yielded nothing that could properly be called a social law. This, he decided, was in retrospect unsurprising, for psychical phenomena are wholly unlike physical, and there is no reason to think the same concept of law applies to each. ... Wilhelm Wundt, similarly, emphasized the relation between intentionally and directionality in arguing for a category of psychical causality that is nondeterministic." (Gingerezer et al., 1997, p. 64)

"According to Schmoller, it was wrong to derive economic laws of nature from human rules of behavior and to speak of a natural economic order." (Klant, 1988, p. 97)

> Doubtless, a man often asserts of an entire class what is only true of a part of it; but his error generally consists not of making too wide an assertion, but in making the wrong *kind* of assertion: he predicated an actual result, when he should only have predicated a *tendency* to the result – a power acting with certain intensity in that direction. (Mill, 2004, p. 123), original emphasis

Type-C does exactly this: it predicates a behavioral tendency or, more specific, a conditional direction of action.

It is important to note that the axiomatic core is not affected when a specific behavioral hypothesis, which is embodied in the propensity function, is falsified. One of the outstanding characteristics of the structural axiomatic approach is the *separability* of the behavioral and non-behavioral fundamentals. This carries with it the option to employ neoclassical or Keynesian or evolutionary behavioral assumptions within one formal framework (cf. Foster, 2005, p. 378). Because it is compatible with any behavioral assumption the structural axiom set is intrinsically general. This generality frees theoretical economics from the necessity to reiterate unpersuasive and in part already empirically refuted behavioral assumptions just because they are irremovable cornerstones of the formal edifice (Boland, 1981).

The magnitude of the rate of change depends on the inductively inferred probability distribution and one has no *a priori* reason to assume either a discrete or a continuous random variable or a specific distribution. Therefore, the uniform discrete distributions in Figure 1 have to be taken as a pragmatic point of departure which in no way prevents the eventual introduction of, for example, a Gauss distribution. This, however, is not only a purely technical matter but touches on some dodgy methodological points that need not concern us here (Mirowski, 2004a, pp. 233-241). The propensity function (6) is a comprehensive blank form open to progressive specification. It demands no ontological commitment to either determinism or indeterminism (cf. Peirce, 1992).

In its general form the propensity function asserts that the directed rates of change $\ddot{Z}$, i.e. the bounded change vectors, are to different degrees dependent on the state of the world at the beginning of period t or earlier. This state is characterized by the difference between the agent's actual condition Z and appropriate objectives or, more general, reference values Z^{θ} of different kinds. The explicit signum function that determines the direction of action has in turn two parts and reads:

$$\{-1,0,1\} = sgn_{\alpha}\left(sgn_{\beta}\left(Z - Z^{\theta}\right)\right) \quad |t \tag{8}$$

The reference value may, for example, be the profit maximum given the actual state of expectations. The underlying intuition of (8) is well-understood since the classics:

"But, it will be said, the existence of trends or tendencies in social change can hardly be questioned: every statistician can calculate such trends. Are these trends not comparable with Newton's law of inertia? The answer is: trends exist, or more precisely the assumptions of trends is often a useful statistical device. *But trends are not laws*." (Popper, 1960, p. 115), original emphasis

> According to Wicksell's hypothesis the course of prices is governed by the difference *i-r*. Wicksell's approach has a highly classical ancestry. Malthus explained the rate of population growth by the difference between the actual wage and the zero growth wage. Ricardo explained capital accumulation by the difference between the actual profit rate and the zero-saving profit rate. Even the application to money was anticipated by Henry Thornton .... (Niehans, 1994, p. 255)

In sum, the propensity function satisfies the demand for a *general* formalism to represent economic man/woman.

> Briefly put, we should expect a conceptual framework that would facilitate the creation of theories or models that contain manipulable variables whose dosages can be raised sufficiently to swamp the environmental effects and sufficiently to obtain accurate directional predictions for a limited period of time. (Leibenstein, 1980, pp. 25-26)

It is worth emphasizing that (8) is purely formal and does not contain more philosophy, sociology or psychology than that human behavior is directed.[4] We have now to determine in more detail how this works.

4 The information and the action function

Economic man/woman in the structural axiomatic context at first chooses an information source and this entails that there is more than one available. It is assumed here that the set of information sources is restricted to the variables of the axiom set. The information function contains two variables: the realized value Z of the chosen variable and the reference value Z^{θ}.

The reference value can be taken as the concrete result of an optimization procedure, such that the value of Z^{θ} in period t is the subjectively most preferred among all other possible values given the subjective knowledge of the situation in period t and given the actual expectations. The reference value, however, has a wider meaning as we shall see in the following. We refrain here from speculating about what goes on in the head of an agent and treat the reference value as open interface to any promising behavioral approach that is capable of providing the value. Correspondingly we have at the moment no indication of how alterations of the reference value are brought about. It is assumed in the interim that it follows a type-B random path. The information function that yields one of three possible values is specified as:

$$\{-1,0,1\} = sgn_{\beta}\left(Z - Z^{\theta}\right) \quad |\mathrm{t} \tag{9}$$

[4] "Now the rationality principle, which in the social sciences plays a role somewhat analogous to the universal laws of the natural sciences, is false, and if in addition the situational models are also false, then both the constituent elements of social theory are false." (Popper, 1994, p. 173)

It is certainly not the case that real world agents expect that the actual value of a variable is *exactly* equal to the corresponding reference value and that they immediately react to the smallest deviation. On the contrary, infinitesimal precision, which is an implicit property of standard equilibrium models, has to be rejected as an inadmissible idealization.[5] Every action entails set-up costs and therefore inertia.[6] To explicitly account for the inexactitude of the information function the notion of inertia is introduced with the parameter ι:

$$\{-1,0,1\} = sgn_\beta\left(Z - \left(Z^\theta \pm \iota\right)\right) \quad |t \tag{10}$$

Since only the sign of the difference of the realized value Z and the reference value Z^θ is of interest there is no urgent need for economic man/woman to measure these values with high precision. All that is needed is that economic man/woman can ascertain one of the relations: greater, smaller, or roughly equal (cf. March and Simon, 1958, pp. 137-142). So measurement errors or a rule of thumb attitude does not impair the functioning of (10) which fits Bateson's well-known definition of information as 'a difference that makes a difference'.

In a random environment it makes good economic sense *not* to react to small changes and deviations because it is to be expected that they, more often than not, cancel out in the following periods. The magnitude of the inertia parameter in (10) therefore determines the sensitivity of the whole system. In order to keep the following signum functions legible the inertia parameter is omitted with the understanding that it normally has a value greater than zero.

Next, economic man/woman has to choose an action variable. It is assumed that the set of action variables consists of the variables of the axiom set. The subsets, however, are different for the business and the household sector. The quantity bought X belongs to the households's subset but not the firms's. The action variable is dependent upon the output of the information function. The direction for the random rate of change of the action variable X is then given by:

$$\{-1,0,1\} = sgn_X\left(sgn_\beta\left(Z - Z^\theta\right)\right) \quad |t \tag{11}$$

The sign of the rate of change of the action variable depends on the sign that is provided by the information function. We thus have nine combinatorial variants of the relations between information and action as exhibited in Figure 2.

From the nine possible combinations four are singled out as momentarily not feasible. The agent is not allowed *not* to act (a_{nf}, c_{nf}) and he is only allowed to

[5] "Because of its superficial facility of measurement, economics has allowed its modes of thought to become in some degree divorced from its subject-matter. It has tried for a precision, certainty and reach of prediction whose basis is not there." (Shackle, 1972, pp. 361-362)

[6] "The inert area concept is related to human inertia. A very broad concept of utility cost is envisioned in which the elements that contribute to inertia are an indifference range between effort utilities, an insensitivity range, the uncertainty of the utility of a new point over the existing one, as well as over its "effortfulness," exaggerated assessments of the temporary cost of moving, the fear of newness, and fear of disappointment, and so on." (Leibenstein, 1980, p. 112)

$$\underbrace{\{-1,0,1\} = sgn_{\alpha}}_{\text{action}}\underbrace{\left(sgn_{\beta}\left(Z - Z^{\theta}\right)\right)}_{information}$$

		output input	information action	function function
		-1	0	1
output	-1	**a_1**	b_{nf}	**c_1**
action	0	a_{nf}	**b_2**	c_{nf}
function	1	**a_3**	b_{nf}	**c_3**

Figure 2: The general signum function as product of the information and action function

act with a good reason, that is, he is not allowed to act spontaneously (b_{nf}). For a plus or minus input from the information function economic man/woman has to choose between two possibilities (a_1, a_3 respectively c_1, c_3). For a zero input, i.e. the realized value is equal to the reference value, no action results (b_2).

If the information function refers, for example, to the stock of final products $\bar{O}$ and if the price P has been chosen as action variable (instead of, say, employment) and the output of the information function is a plus, i.e. $\bar{O} > \bar{O}^{\theta}$, then the firm has to choose either a price increase (a_3) or a price reduction (a_1). This is an ordinal choice and it is understood that the agent believes that his action will bring him closer to the reference value, that is, from worse off to better off (cf. von Mises, 2006, pp. 39-40). Of course we do not *know* which of the alternatives the firm will choose. Because of this, the missing behavioral link is amended by the standard assumption that the firm will lower the price in order to sell off. The magnitude of the price cut is left to chance.

Even if the signum function is – from the perspective of an outside observer – in principle correctly specified there is, because of other influences, no guarantee that the price cut will bring the stock of products closer to the reference value in period *t*. The signum function is not an unfailing formula for success but one element in a trial and error process with practically unpredictable effects on the agent's operating experience and expectations. We arrive here at an open interface to a theory of learning (Dosi et al., 2005), (Boland, 2003, pp. 271-274).

To clarify the formal relationships is one thing, to find the right behavioral link between the information and the action function is quite another. For the outcome of the action function in each period we have three values {-1, 0, 1} with zero as the most probable outcome in the case of inertia. Assumed that the probabilities for inaction or for a plus or a minus change are all *equal* then a random guess will deliver the right prediction with a 33 percent probability. This, then, would be the rule of thumb benchmark for one period which a qualitative behavioral prediction must outperform (cf. Rosenberg, 1989). The signum function is a blank form for behavioral assumptions. Clearly, the new economic man/woman is more versatile than the single-minded homo economicus.

5 Properties of the behavioral link

The signum function establishes a link between hitherto independent variables and thus intensifies the systemic interdependencies that are given with the structural axiom set. Its formal constituents are the realized and the reference value of a chosen variable, the information function, the action function, and the resulting direction of change.

The signum function is also the logical interface between the structural axiomatic approach and the 'leading principle of folk psychology' that more often than not provides the explanatory link between desires, beliefs, and action in the social sciences. This link can be stated in one sentence [L]:

> Something like the following oversimplified general statement seems to lie behind our ordinary explanation of human action, our predictions about how people will behave in the future, and explanations in social science that trade on folk psychology:
> [L] If any agent, *x*, wants *d*, and *x* believes that *a* is a means to attain *d* under the circumstances, then *x* does *a*. ...
> For it is [L] that connects the environment to our actions via the desires and beliefs the environment fosters in us. ... Economic theory and parts of political science employ a formal theory of rational choice. But, as we shall see, rational choice theory is folk psychology formalized. (Rosenberg, 2008, pp. 34-35, 80)

It needs only a slight reformulation of [L], i.e. the substitution of d by $Z - Z^{\theta} = 0$, to see that the leading principle of folk psychology is formally embodied as a limiting case in the signum function.

An outside observer can only observe the realized value of the chosen variable and the direction of change. The rest of the signum function is a black box and the gap between the two observables has to be interpolated by assumptions. An alternative to this theoretical approach is to shortcut the signum function and to poll the agents at the beginning of the period about the direction and magnitude of the variable in question (Koopmans, 1957, p. 206) and to feed the results into a simulation. For practical purposes the *first* thing to find out is *how* the agents will act and not to understand *why*.

Since the axioms refer to a finite period length the action that follows from the signum function has to take place at the beginning of period t. From the methodological point of view an inquiry that starts with a finite period length has a broader scope because it entails continuous time as limiting case (for details see 2011c, pp. 11-12).

The signum function reflects the lumpiness of the real world. The smooth variations of quantities in dependence of smooth variations of prices, which is an defining property of standard equilibrium models, has to be rejected as an inadmissible idealization. Being eliminative, instead of merely being neglective (Klant, 1988, p. 90), (Boland, 2003, pp. 79-80), it idealizes away discreteness and fuzziness which are the *very* elements of economic reality the agents have to cope with. As Popper put it:

> ... the following maxim holds for all sciences: Never aim at more precision than is required by the problem at hand. (Popper, 1983, quoted in Redman, 1993, p. 105)

6 Stochastic stability

Being equipped with the propensity function economic man/woman is now left on his/her own in the drifting economy (for details about the pure random economy see 2011b). The household- and the business sector have different sets of variables to act upon. From their respective subsets of elementary variables the household sector is supposed to choose the quantity bought X as action variable and the business sector price P.

By stipulating that the sign of the random price change is negative if the stock of products $\bar{O}$ is above the firm's reference value $\bar{O}^{\theta}$, and vice versa, we provide the missing link in the signum function. The signum function is assumed to be independent and identical in each period. This assumption is not a mere technicality. Implicitly it precludes memory and learning and makes that the stipulation invariably holds for *all* periods, that is, the assumed behavior transmutes into a routine (Nelson and Winter, 1982, pp. 14-19). Thus the firm adapts the price according to a simple rule and is not a price-taker. There is no convincing reason to adopt this central behavioral tenet of standard economics (Eichner, 1983, p. 213). Prices are changed by real people and not by occult market forces. As Arrow pointed out many years ago 'perfect competition and prices changing by the decision of actual agents cannot be reconciled' (Hahn, 1984, p. 186). Hence the standard and the structural approach are irreconcilable exactly at this point. The signum function for the price change in period t is given by:

$$-1_t = sgn_P\left(sgn_\beta\left(\bar{O}_{t-1} - \bar{O}^{\theta}_{t-1}\right)\right) \tag{12}$$

Of course, there is virtually no upper limit for more sophisticated stipulations. Somewhat more differentiated the household sector acts upon X to price and income changes and to the difference of the actual stock of money and the reference value. The rate of change of the quantity bought is negative if the price P increases, positive if income Y increases, positive if the household sector's stock of money $\bar{M}$ is above the reference value. Hence we have three signum functions with a combined effect on the quantity bought X. Note that the reference value $\bar{M}^{\theta}_{t-1}$ is absolute, i.e. an objective, and the reference values P_{t-1} and Y_{t-1} are relative:

$$
\begin{aligned}
-1_t &= sgn_X\left(sgn_\beta\left(P_t - P_{t-1}\right)\right) \\
1_t &= sgn_X\left(sgn_\beta\left(Y_t - Y_{t-1}\right)\right) \qquad (13)\\
1_t &= sgn_X\left(sgn_\beta\left(\bar{M}_{t-1} - \bar{M}^\theta_{t-1}\right)\right)
\end{aligned}
$$

After applying the propensity function to each element of (13) the resulting random rate of change of the quantity bought $\ddot{X}$ is given by:

$$\ddot{X} = \pm\ddot{X}_1 \pm \ddot{X}_2 \pm \ddot{X}_3 \qquad (14)$$

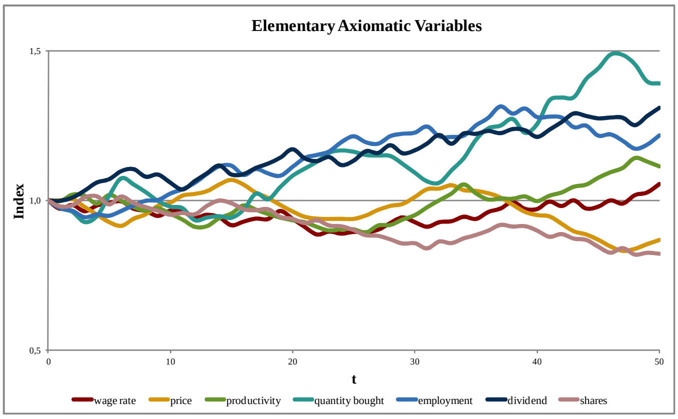
The signum functions incorporate factual behavioral stipulations without reference to a preference order (but see J. S. Mill for the 'composition of causes' 2006, p. 371). The behavioral stipulations reiterate in this first round the conventional commonplace assumptions. The arguments of equation (13) are familiar from the standard demand schedule. It is important to note, however, that with equation (14) the quantity bought in period *t* is determined and nothing else. No attempt is made to speculate about which quantity the households would buy if the price were higher or lower. The conceptual and logical problems of notional demand functions are well known (Mirowski, 2004b, pp. 329-334), (Vickers, 1995, pp. 110-116).

A simulation makes every single action following from (12) and (13) visible (see Appendix). The example in Figure 3 shows that the two type-C propensity functions are sufficient to bring both stocks in a random environment repeatedly back to their reference values, which at first remain unchanged. The system is stable – at least over the observed time span – irrespective of persistent random perturbations of all other variables.

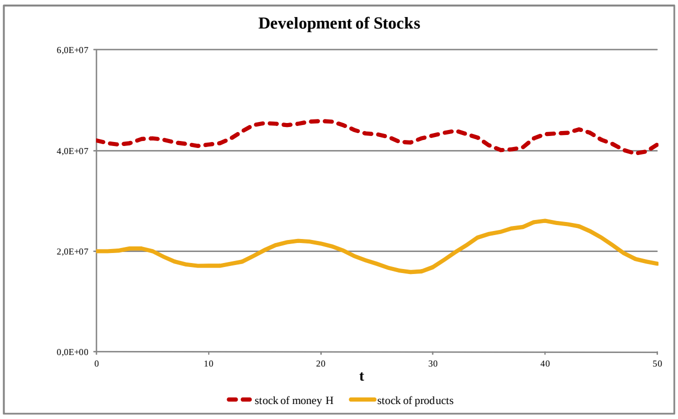
The two reference values $\bar{O}^\theta$ and $\bar{M}^\theta$ are reached in different periods. It can be said, then, that the product market is cleared momentarily in period *t*, i.e. all previous changes of the stock of products cancel out, and that the household sector's budget is momentarily balanced in period $t \pm x$, i.e. all previous changes of the stock of money cancel out. But there is no such thing as a simultaneous behavioral equilibrium.[7]

The whole process, the random changes of productivity, employment, wage rate etc. in combination with the household- and business sector's actions, which follow from their respective signum functions, produce the familiar negatively sloped price-quantity relation in the product market as shown in Figure 3c. This relation looks like the law of demand but it is obvious that each dot represents one *realized* price–quantity configuration while the law of demand states a *hypothetical* ceteris paribus relation for the household sector's demand in *one* period.

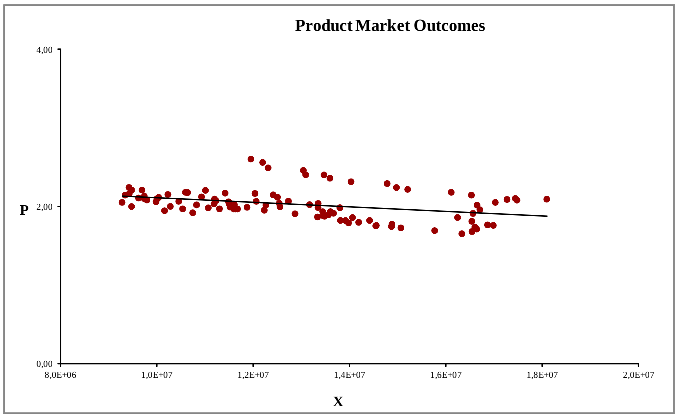
[7] "Just as classical General Equilibrium Theory has never been able to provide a definitive account of how equilibrium prices come to be established, so Rational Expectation Theory has not shown how, starting from relative ignorance, everything that can be learned comes to be learned." (Hahn, 1980, p. 133)



(a) Paths of the elementary axiomatic variables: X and P are of type-C directed randomness, the rest is of plain type-B randomness



(b) Type-C behavior of the household- and business sector produces order out of randomness and keeps the monetary and real stocks close to their desired values



(c) Price-quantity configurations over fifty periods as outcome of type-C behavior with regard to the stock of output and the stock of money

Figure 3

Simultaneity is not a necessary condition for the proper functioning of the economy. All that is needed is market clearing and budget balancing within a time span the agents can cope with. Being no feature of the real world simultaneity has to be regarded as the inoperative artifact of an awkward modeling strategy (Backhouse, 2007, p. 156).

Pure or directed stochastic processes are by their very nature unpredictable. Observations over longer time spans may therefore stumble across a configuration of random changes that carry the economy beyond a boundary. We could try to devise more sophisticated signum functions to account for all kinds of contingencies; this, indeed, would amount to endowing the agents with faculties they might not possess. Or we can decide not to be too much irritated by eventual casualties. After all, bankruptcies and market crashes happen and economic theory has to account for it.[8]

Arrow and Hahn have posed the fundamental question that theoretical economics has to answer: "What will an economy motivated by individual greed and controlled by a very large number of different agents look like?" (1991, p. vii). The tentative answer is: somehow like Figure 3,[9] with the annotation that greed is not the same thing as Adam Smith's notion of self-love (Smith, 2008, p. 22) or self-interest. This semantic shift, to the contrary, indicates that the genealogical claim 'that Smith was a creator of general equilibrium theory' (Arrow and Hahn, 1991, p. 3) is vacuous.[10] It would be more in accordance with historical fact to bestow Adam Smith in retrospect with the honor of having been the creator of behavioral economics (Ashraf et al., 2005). General equilibrium theory is as far away from Adam Smith as it is from reality.

Randomness relativizes the distinction between rational and irrational actions because the outcome depends not only on the complementary behavior of other agents and may result in a *folie à deux* but also on coincidental random changes in the economic surround. Adam Smith was well aware of this.[11] It is important to remember that Figure 3 depicts a simulation with arbitrarily fixed absolute reference values. This, of course, is a first round simplification. As a matter of fact these reference values, too, alternate from period to period thus amplifying or damping the agents's actions. In comparison to standard economics it is important to point out that the agents's behavior depends on cumulated residuals (Clower, 1966, p. 114), that is, on the actual history of the system (for details see 2011a).

[8] "However, case studies of entrepreneurial failures are infrequent, even though it seems likely that many, perhaps even most entrepreneurial ventures are unsuccessful." (Rosen, 1997, p. 149)

[9] "Hahn ... has defended the Arrow-Debreu theorems on the grounds that, though they describe no feasible real-world economy, they show 'what the world would have to look like' if the invisible hand were to operate properly. This claim is nonsense ...." (Backhouse, 1998, p. 1853)

[10] "Smith never proclaimed in favor of 'selfishness', ... an attribute of Bernard Mandeville's philosophy ..., which Smith regarded as 'licentious' ...." (Kennedy, 2009, p. 251)

[11] "Smith turns from these self-deceptions to the role that the striving in pursuit of such mirages means for society. For society's sake, he assures us, it is well that these 'deceptions' are widespread because '*this deception rouses and keeps in continual motion the industry of mankind*'" (Kennedy, 2009, p. 247), original emphasis

7 General stochastic and deterministic optimality

There is at first sight nothing in Figure 3 that resembles the notion of a marginalistic behavioral equilibrium. This could not be otherwise. The vision of a system heading towards equilibrium is *not* an *a priori* in the structural axiomatic context. This does not preclude that it is an *a posteriori* with some best-of-all-worlds properties. As Bertalanffy noted with regard to General Systems Theory:

> Already Maupertuis considered his minimum principle as proof that the world, where among many virtual movements the one leading to maximum effect with minimum effort is realized, is the "best of all worlds" and work of a purposeful creator. Euler made a similar remark: "Since the construction of the whole world is the most eminent and since it originated from the wisest creator, nothing is found in the world which would not show a maximum or minimum characteristic." ... The conceptual error of an anthropomorphic interpretation is easily seen. The principle of minimum action and related principles simply result from the fact that, if the system reaches a state of equilibrium, the derivatives become zero; this implies that certain variables reach an extremum, minimum or maximum; . . . (von Bertalanffy, 1969, pp. 75-76)

General equilibrium theory is the worldly echo of Maupertuis and Euler (Nelson, 2006). Having learned from Laplace that he/she was outdated as a hypothesis, the wisest creator returned as invisible hand of the market ratifying Euler's claim that there is nothing in the world, which would not show a maximum or minimum characteristic. However, physics went on:

> Although the law of least action has thus attained a rank among the highest theorems of physics, yet its pretensions to a cosmological necessity, on the grounds of the economy in the universe, are now generally rejected. (Hamilton 1833, quoted in Kline, 1982, p. 73)

Since then the former cosmic economy gained a new lease of life on the microeconomic scale as constrained minimum or maximum.

> There is nothing obvious about the definition of human rationality as the maximization of an objective function over a conserved entity. . . . In its evolution from Maupertuis to Euler to Hamilton, the principle of least (or varying) action shed its theological skin, but the notion of absolute efficiency persisted, and it was this connotation that was recruited to tame the multiform and unruly concept of rationality. (Mirowski, 1988, p. 152)

The salient point of maximization as a behavioral assumption is not the lack of realism but, more serious, that welfare theorems which rely on the minimax principle and general equilibrium cannot be taken as a property of the competitive system but as 'an anthropomorphic interpretation' of the mathematical triviality that derivatives are zero in equilibrium.

Being just a formal implication of equilibrium, optimality is by logical necessity an implicit property of the type-C process of directed randomness. In a stochastic world we are entitled to describe the stable meandering around the reference value over a given time span as stochastic behavioral optimum. This granted we can infer from the results of the simulation as reported in Figure 3 that both sectors realize at the best under the given conditions their planned stocks in the type-C process of interdependent adaption that is governed by their respective signum functions.

Yet we can even go one step further. In the structural axiomatic context the argument of the information function adopts the role of the first derivative (cf. Frisch, 1936, 101). The formal criterion of a *behavioral optimum* is that the value of the respective information function is *zero*. In this case no further adaptive action results. The formal properties of the first and second derivative are well-known and have to be applied analogously. A *general* behavioral optimum is attained if *all* signum functions realize in the *same* period the configuration b_2 in Figure 2. In this case the system becomes stationary. This, though, is somewhat improbable.

Within the structural axiomatic framework a general behavioral optimum can be defined with absolute formal precision, which, to be sure, does not entail that this interesting *analytical limiting case* will ever be observed in the real world.

It is worth emphasizing that the formal criterion of a behavioral optimum is generally applicable, that is, it does not presuppose decreasing marginal productivity, convex indifference sets or the perfect independence of the individual decision maker. These are formal requirements of the standard optimization calculus that are illegitimately imposed upon reality in order to secure the applicability of the chosen tool.[12] The applicability of the propensity function does not depend on these idiosyncratic restrictions. The propensity function works with all roughly quantifiable objectives in all possible economic worlds.

8 Summary

Behavioral assumptions, rational or otherwise, are not solid enough to be eligible as first principles of theoretical economics. Hence all endeavors to lay the formal foundation on a new site and at a deeper level actually need no further vindication.

[12] "There are two basic interpretations of the maximization postulate, a "factual" interpretation and a "tautological" one. ... The essential argument of the tautological interpreter is that an individual *always* maximizes, although all the objectives may not be immediately apparent. No criticism of this view of the maximization postulate is *possible*. By the same token a nonmaximization postulate cannot make sense. We should note that this approach not only deflects criticism from maximization but also from more complicated arguments that include the postulate. It can always be argued that one has not accurately specified the objective function. This approach makes almost all deductions based on conventional microeconomics into elusive targets." (Leibenstein, 1980, pp. v-vi), original emphasis

The present paper submits four non-behavioral axioms as groundwork for the theoretical reconstruction of the evolving money economy. To formally capture human behavior, that is, to determine the *direction* of action, the axiom set is complemented by the propensity function.

The propensity function is a compact formal expression of random, semi-random, and deterministic behavioral assumptions (type-A to type-E). Thus the *structural* axiomatic core and the *behavioral* propensity function constitute the formal building blocks of any structural axiomatic model.

The two components of the propensity function are direction and magnitude of the rate of change of an elementary axiomatic variable. The magnitude is left to chance, the direction is determined by the signum function.

Economic man/woman is formally portrayed as a bundle of signum functions. The signum function is in turn the product of the information and the action function. The information function refers to the difference between the agent's actual and desired state. The agent's decision constitutes the behavioral link between the output of the information function and the output of the action function, that is, the course of action.

As a constituent of the propensity function the signum function determines the direction of change. The information function as a constituent of the signum function takes the role of the first derivative; a value of zero indicates in the axiomatic context a behavioral optimum.

The directedness that originates from the signum function produces stochastic stability and optimality out of randomness and keeps the economy within viable boundaries for a reasonable time span. Two type-C propensity functions – one for the business- and one for the household sector – are sufficient to clear the product market and to balance the household sector's budget momentarily at different points in time. This structural axiomatic account of market coordination precludes the fictitious notion of a simultaneous marginalistic equilibrium. The propensity function works with all roughly quantifiable objectives in all possible economic worlds.

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Appendix: Simulation

Download-Link for the Excel File

https://docs.google.com/leaf?id=0B57dPq2RYclpZTEzYzNkMzEtYzllOC00MzU5LThhNWItZTc3NTJhYjljMDUw&hl=en_US

Worksheet Sections

(A) Data input (in green cells),

(B)(C) Structural axioms,

(D) Residuals and stocks,

(E) Reference values,

(F) Key ratios and checks,

(G) Data for diagrams.

The prepared graphics display more details of the system's interdependencies in addition to the diagrams presented in the text.

Technical Annotations

- The initial values of the variables are chosen for modeling purposes and do not represent any concrete economy. These numbers can be changed at will.
- The range of the random rates of change is set in column D and E (= set of possible outcomes).
- After changing the parameters press F9 for recalculation.
- If a graphic looks implausible check input data first.
- Changes may be set to zero after an arbitrary period in column F (= until period) for each variable separately. The default value is 1.000 periods.

- By making both values of the set of possible outcomes equal, random variations are excluded and the system becomes deterministic.
- The default propensity in column G is type-B, i.e. all changes are random. When changing from type-B to type-C in column G the respective column-D value has to be set to 0 (and vice versa at reset to type-B). Since the algebraic sign of the respective variable is set according to a specified condition only the positive values of the rate of change are needed.
- Unforeseeable parameter constellations or combinations of random changes may lead to surprising results. These are the interesting cases if the effect persists after multiple recalculations.
- Negative values of the stock of products are not excluded here. They are regarded as virtual and can always be avoided by introducing an additional constraint or by an appropriate initial stock.
- Row 23 (t=1) contains all formulas, that is, the logic of the system. This row is simply copied downward for an arbitrary number of periods.
- For full transparency and to avoid lengthy formulas each step of a calculation is carried out in a new column.
- B-rate of change means that negative values of the rate of change are transformed in order to get an unbiased process.

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