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Time Preference and Interest Rate in a Dynamic General Equilibrium Model

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Abstract

This paper reexamines the relationship between the time preference rate and the real interest rate in the neoclassical growth model by introducing Keynesian time preference. It is shown that the long-run behavior of the neoclassical growth model persists. When introduucing money by money-in-utility, money is superneutral and the optimal monetary policy is the Friedman rule.

Keywords: Keynesian time preference, Monetary Superneutrality, Optimum Quantity of Money

JEL Classification Numbers: E31, E5, O41

1 Introduction

In Neoclassical economics the rate of time preference is usually taken as an exogenous parameter in an individual's utility function which describes the degree of patience and captures the tradeoff between consumption today and consumption in the future. In the continuous-time model, the larger the time preference rate is, the larger proportion of their income are used in present consumption by the people with less patience. And the interest rate is the

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rate of return of capital and other financial assets, which is equal to the marginal product of capital at any point in time. In the long run steady state, the subjective time preference rate determines and equals the objective real interest rate. It is a mazing result with simplified assumptions. But in reality there seemingly exist some flaws. One of them is that the neoclassical model assumes that the subjective time preference is a given constant. As a kind of individual or social habits, time preference must fluctuate with the evolution of the economy and society. That is to say, the time preference rate must be an (endogenous) variable in the model. The literature on endogenous time preference rate extends the assumptions and results of the standard neoclassical growth model, such as . The other flaw is that the time preference rate determines unilaterally the real rate of interest and correspondingly capital accumulation and growth and the converse is impossible. Does the real rate of interest affects the degree of patience of the people? In his great work, Keynes talks about the "changes in the rate of time-discounting": "it was convenient to suppose that expenditure on consumption (auther: the time discount rate) is *cet. par.* negatively sensitive to changes in the rate of interest, so that any rise in the rate of interest would appreciably diminish consumption. It has long been recognised, however, that the total effect of changes in the rate of interest on the readiness to spend on present consumption is complex and uncertain, being dependent on conflicting tendencies, since some of the subjective motives towards saving will be more easily satisfied if the rate of interest rises, whilst others will be weakened. Over a long period substantial changes in the rate of interest probably tend to modifiy social habits considerably, thus affecting the subjective propersity to spend–though in which direction it would be hard to say, except in the light of actual experience." "Perhaps the most important influence, operating through changes in the rate of interest, on the readiness to spend out of a given income, depends on the effect of these changes on the appreciation or depreciation in the price of securities and other assets." From the cited passages, Keynes roughly points out that the time preference rate depends upon the real interest rate endogenously in the long run, and the higher the real interest rate, the stronger the desire for saving.

Though as a kind of social habits the time preference rate (or the degree of patience) is relatively stable, it is intuitive to see the real interest rate will affect the consumption habit in the long run. In order to consider the effect of the real interest rate on the time preference rate, we impose some assumptions on the time preference rate in the neoclassical growth model based on Keynes' statements. By assuming the time preference rate is a strictly decreasing and strictly convex function of the real interest rate, we find that the long run bahavior of the neoclassical growth model persists. Furthermore, in the neoclassical monetary growth model, money is superneutral and the optimal monetary policy is the Friedman rule.

2 Keynesian Time Preference

Based on the previous analysis, we assume that the time preference rate of the representative agent is a strictly decreasing and stictly convex function of the real interest rate, namely,

$$\rho_t = \rho(r_t), \tag{2.1}$$

which satisfies

$$\rho'(r_t) < 0, \rho''(r_t) > 0, \rho(0) = \rho_f. \tag{2.2}$$

Asumptions (2.1) and (2.2) make the time preference rate endogenous, and they imply that the higher the real interest rate is, the more patient the individual is. But notice that the increase in the patience is at a decreasing rate. Moreover, the discount rate is a positive constant if the inflation rate is zero, just like a "Fisherian" consumer with a constant rate of time preference, i.e., $\rho(0) = \rho_f$. Furthermore, it is also assumed that the time discount factor of the individual at time t depends not only on the current level of inflation, but also on the entire path of past real interest rate $\{r_v\}_{v=0}^{t}$, namely,

$$\Delta_t = \int_{v=0}^{t} \rho(r_v) dv. \tag{2.3}$$

Then the modelling strategy has generated a new state variable, the real time discount factor Δ_t. Differentiating Δ_t with respect to t in equation (2.3), we obtain the dynamic accumulation equation of the time discount factor, namely,

$$\dot{\Delta}_t = \rho(r_t). \tag{2.4}$$

With these new elements introduced, this paper will reexamine the neoclassical growth model.[1]

[1] For simplicity, we just consider the case without population growth.

3 The Neoclassical Growth Model

The representative individual's optimization problem is to maximize

$$\int_{t=0}^{\infty} u(c_t)e^{-\Delta_t}dt \tag{3.1}$$

subject to the budget constraint

$$\dot{k}_t = f(k_t) - c_t, \tag{3.2}$$

the no-Ponzi-game condition

$$\lim_{t\to\infty} k_t \exp(-\int_{v=0}^{t} r_v dv) = 0, \tag{3.3}$$

and the initial value of the physical capital k_0, where c_t and k_t are consumption and physical capital stock respectively; r_t and Δ_t are the real interest rate and the time discount factor; $u(c_t)$ and $f(k_t)$ are the standard utility function and neoclassical production function. And the no-Ponzi-game condition rules out unlimited borrowings.

To proceed, the optimization problem of the representative consumer is to maximize (3.1), subject to (3.2), (2.4) and (3.3). The Hamiltonian associated with this problem is

$$H = u(c_t)e^{-\Delta_t} + \lambda_t[f(k_t) - c_t] + \mu_t\rho(r_t),$$

where λ_t and μ_t are the multiplier associated with the constraints (6) and (4), representing the shadow values of capital and time discount factor respectively. The FOCs are as follows

$$\begin{aligned} u'(c_t)e^{-\Delta_t} &= \lambda_t, & (3.4)\\ \lambda_t f'(k_t) &= -\dot{\lambda}_t, & (3.5)\\ e^{-\Delta_t}u(c_t) &= \dot{\mu}_t. & (3.6) \end{aligned}$$

Substituting equation (3.4) and $r_t = f'(k_t)$ into (3.5) leads to

$$\dot{c}_t = -\frac{u'(c_t)}{u''(c_t)}[f'(k_t) - \rho(f'(k_t))]. \tag{3.7}$$

Similiar to the standard neoclassical growth model, equations (3.2) and (3.7) form the dynamic system that we concern together with the initial condition k_0 and the transversality condition. Define the steady state (c^*, k^*) by $\dot{c}_t = \dot{k}_t = 0$. We get a form of algebraic equations

$$\begin{aligned} f'(k^*) &= \rho(f'(k^*)), & (3.8) \\ f(k^*) &= c^*. & (3.9) \end{aligned}$$

From these assumptions on the time preference rate and the neoclassical assumptions of the production function, there exits a unique k^* satisfying equation (3.8) and correspondingly a unique c^* satisfying equation (3.9). Linearizing equtions (3.2) and (3.7) results in the linearized system

$$\begin{pmatrix} \dot{c} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{u'(c^*)}{u''(c^*)} f''(k^*)(1 - \rho'(f'(k^*)) \\ -1 & f'(k^*) \end{pmatrix} \begin{pmatrix} c - c^* \\ k - k^* \end{pmatrix}. \tag{3.10}$$

It is easy to find the determinant of the Jacobian of the coefficient matrix is negative, i.e., $J = -\frac{u'(c^*)}{u''(c^*)} f''(k^*)[1 - \rho'(f'(k^*)) < 0$, which tells that there is a nagative eigenvalue and a positive eigenvalue. Hence, the steady state of the linearized system is a saddle. As is hyperbolic, the linearized system is conjugate to the original nonlinear system in a neighborhood of the steady state similiar to the standard model. Therefore we have the following propostion.

Proposition 1 *There exists a unique steady state in the neoclassical growth model with Keynesian time preference, which is a saddle, similiar to the standard model with a constant time preference rate.*

From equation (3.8) and $r^* = f'(k^*)$, we have $r^* = \rho(r^*)$. That is, in the long run, the equilibrium interest rate equals the equilibrium time preference rate and accurally the unique fixed point of the time preference function. The formular does embody not only the original idea that the time preference rate determines the real interest rate, but also the idea of how the real interest rate affects the time preference.

4 The Neoclassical Monetary Growth Model

In order to show the usefulness and comprehensiveness of the idea, we introduce money into the model and reexamine the original model. Following Sidrauski (1967), we use the assumption of money-in-utility in a dynamic general equilibrium framework. The objective of the consumer is maximizing

$$W = \int_{t=0}^{\infty} e^{-\Delta_t} u(c_t, m_t) dt, \tag{4.1}$$

subject to the budeget constraint

$$\dot{a}_t = r_t k_t + w_t + x_t - c_t - \pi_t m_t, \tag{4.2}$$

wealth constraint

$$a_t = k_t + m_t, \tag{4.3}$$

and the no-Ponzi-game condition

$$\lim_{t \to \infty} a_t \exp(-\int_{v=0}^{t} r_v dv) = 0, \tag{4.4}$$

where c_t , m_t, k_t, and a_t are consumption, real money balances, physical capital stock, and total wealth, respectively; r_t and w_t are the real interest rate and real wages; Δ_t and r_t are the time discount factor and the real interest rate; and x_t denotes lump-sum real money transfer payments. The stock constraint requires that the total wealth a_t be allocated between capital k_t and real balances m_t. And the no-Ponzi-game condition rules out unlimited borrowings. The instantaneous utility function $U_t = u(c_t, m_t)$ is assumed to be well-behaved, satisfying $u_c > 0$, $u_m > 0$, $u_{cc} < 0$, $u_{mm} < 0$, $u_{cc}u_{mm} - u_{cm}^2 > 0$ and the Inada conditions. Following Sidrauski (1967), Fischer (1979), Shi (2001), and Miao (2004), we assume that both commodities are not inferior[2]. Furthermore, to reach a definitive conclusion, following Calvo (1979), Wang and Zou (2011), we assume that consumption and real money balances are Edgeworth-complementary, i.e., $u_{cm} > 0$.[3]Intuitively, an increase

[2]It is not hard to prove that the normality of the two goods is equivalent to the following two conditions, respectively, $u_{mm} - \frac{u_{cm}u_m}{u_c} < 0$, $\frac{u_{cc}u_m}{u_c} - u_{cm} < 0$.

[3]Wang & Chong (1992)■Aiyagariv & McGrattan (2003), and Barro (2003) called the assumption pareto complementarity between consumption and money.

in real balances raises the marginal valuation of consumption and increases consumption; and a lower level of money holdings decreases the marginal valuation of consumption and lowers consumption. Hence, in the steady state, consumption and real money balances move in the same direction.

To proceed, the optimization problem of the representative consumer is to maximize (4.1), subject to (4.2), (2.4), (4.3) and (4.4). The Hamiltonian associated with this problem is

$$H = u(c_t, m_t)e^{-\Delta_t} + \lambda_t[r_t k_t + w_t - c_t - \pi_t m_t + \tau_t] + \mu_t \rho(r_t) + q_t(k_t + m_t - a_t), \tag{4.5}$$

where λ_t and μ_t are the multiplier associated with the constraints (4.2) and (2.4), representing the shadow values of wealth and time discount factor, respectively; q_t is the Lagrangian multiplier attached to the stock constraint (4.3), representing the marginal value of total wealth.[4]

The first-order conditions for a maximum are given by equations (10)-(13) together with the transversality conditions:

$$u_c(c, m)e^{-\Delta} = \lambda, \tag{4.6}$$

$$u_m(c, m)e^{-\Delta} = (r + \pi)\lambda, \tag{4.7}$$

$$\dot{\lambda} + r\lambda = 0, \tag{4.8}$$

$$u(c, m)e^{-\Delta} = \dot{\mu}, \tag{4.9}$$

$$\lim_{t\to\infty} e^{-\Delta}\lambda k = 0, \lim_{t\to\infty} e^{-\Delta}\mu\Delta = 0. \tag{4.10}$$

The behavior of the firm is simple. Competitive pricing gives

$$r = f'(k), w = f(k) - kf'(k). \tag{4.11}$$

In order to examine macroeconomic equilibrium, we introduce the government's behavior. It is assumed that the government maintains a constant rate of monetary growth

[4]For notional simplicity, we will omit the time subscript in the following mathematical presentations.

$$\frac{\dot{M}}{M} = \theta, \tag{4.12}$$

and keeps its budget balanced

$$x = \frac{\dot{M}}{P}, \tag{4.13}$$

where θ and g are two constants denoting the monetary growth rate and government expenditure, respectively. By the definition of real money balances, $m = \frac{M}{P}$. Substituting equation (4.12) into equation (4.13) results in $x = \theta m$. We impose the assumption of perfect foresight which says that the expected rate of inflation is equal to the real rate of inflation, namely,

$$\frac{\dot{P}}{P} = \pi. \tag{4.14}$$

Taking the derivative of $m = \frac{M}{P}$ with respect to t, rearranging, and substituting equtions (4.12) and (4.14) into it, we have

$$\dot{m} = (\theta - \pi)m. \tag{4.15}$$

Equations (4.6), (4.7) and (4.11) imply that:

$$\frac{u_m(c,m)}{u_c(c,m)} = (f'(k) + \pi). \tag{4.16}$$

From equation (4.16), we solve π as a function of c, m,and k, i.e., $\pi_t = \pi(c, k, m)$. And it is easy to show that

$$\pi_c = \frac{u_{mc}u_c - u_{cc}u_m}{u_c^2} > 0, \pi_m = \frac{u_{mm}u_c - u_{cm}u_m}{u_c^2} < 0, \pi_k = -f''(k) > 0. \tag{4.17}$$

Putting $\pi_t = \pi(c, k, m)$ into equation (4.15) gives the dynamics of real money balances

$$\dot{m} = (\theta - \pi(c, k, m))m. \tag{4.18}$$

Equation (4.6), (4.8), (4.11) and (4.18) give rise to

$$\dot{c} = -\frac{u_c(c,m)}{u_{cc}(c,m)}[f'(k) - \rho(f'(k))] - \frac{u_{cm}(c,m)}{u_{cc}(c,m)}(\theta - \pi(c,k,m))m. \tag{4.19}$$

From equation (4.2), (4.3) and (4.15), we have

$$\dot{k} = f(k) - c. \tag{4.20}$$

Therefore, equations (4.18), (4.19) and (4.20) describe the whole dynamics of the model.

In order to examine the three-dimension dynamic system, we would find the steady state (c^*, k^*, m^*) by defining $\dot{c} = \dot{k} = \dot{m} = 0$. The resulting form of algebraic equations is

$$f'(k^*) = \rho(f'(k^*)), \tag{4.21}$$

$$f(k^*) = c^*, \tag{4.22}$$

$$\theta = \pi(c^*, k^*, m^*). \tag{4.23}$$

Equation (4.21) gives the familiar modified golden-rule level of capital accumulation, which shows that, in the steady state, the marginal product of physical capital equals the subjective time preference rate; equation (34) tells that the steady-state production equals the steady-state level of consumption; and equation (4.23) shows that the steady-state level of inflation is equal to the exogenous level of monetary growth.

Similiar to equation (3.8), equation (4.21) determines uniquely the steady state level of capital and real interest rate r^*. Then equation (4.22) gives the steady state level of consumption and equation (4.23) determines the steady state level of real money balances implicitly. To examine the stibility of the steady state, we linearize equations (4.18)-(4.20) around the steady state (c^*, k^*, m^*)

$$\begin{bmatrix} \dot{c} \\ \dot{k} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \frac{u^*_{cm}}{u^*_{cc}}\pi^*_c m^* & -\frac{u^*_c}{u^*_{cc}} f''(1-\rho') + \frac{u^*_{cm}}{u^*_{cc}}\pi^*_k m^* & \frac{u^*_{cm}}{u^*_{cc}}\pi^*_m m^* \\ -1 & f'(k^*) & 0 \\ -\pi^*_c m^* & -\pi^*_k m^* & -\pi^*_m m^* \end{bmatrix} \begin{bmatrix} c - c^* \\ k - k^* \\ m - m^* \end{bmatrix}. \tag{4.24}$$

Let us define the Jacobian matrix of the linearized system as J. It is not hard to find that

$$\prod_{i=1}^{3} \lambda_i = \det(J) = \frac{u^*_c}{u^*_{cc}} f''(k^*)\pi^*_m m^*(1 - \rho'(f'(k^*))) < 0, \tag{4.25}$$

and

$$\sum_{i=1}^{3}\lambda_i = tr(J) = f'(k^*) + \frac{(u^*_{cc}u^*_{mm} - u^{*2}_{mc})}{-u^*_c u^*_{cc}} > 0. \tag{4.26}$$

Equation (4.25) implies that there exists one negative real eigenvalue or three eigenvalues with negative real parts, and equation (4.26) tells that there exists one eigenvalue with a positive real part at least. Hence there exists an eigenvalue with a negative real part exactly and the steady state is a saddle.

Proposition 2 *In the Sidrauski model with Keynesian time preference, the steady state exists uniquelly and is locally saddle-point stable.*

Totally differentiating equations (33)-(35) gives us a three-dimensional linear system as follows:

$$\begin{bmatrix} 0 & (1-\rho'(f'(k^*)))f''(k^*) & 0 \\ 1 & -f'(k^*) & 0 \\ \pi^*_c & \pi^*_k & \pi^*_m \end{bmatrix} \begin{bmatrix} \frac{dc^*}{d\theta} \\ \frac{dk^*}{d\theta} \\ \frac{dm^*}{d\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{4.27}$$

Applying Cramer's rule results in

$$\frac{dc^*}{d\theta} = 0, \tag{4.28}$$

$$\frac{dk^*}{d\theta} = 0, \tag{4.29}$$

$$\frac{dm^*}{d\theta} = \frac{1}{\pi^*_m} < 0. \tag{4.30}$$

Equations (4.28) and (4.29) shows that monetary superneutrality guarantees in the Sidrauski model with Keynesian time preference, i.e., a permanent change in the monetary growth rate has no effect on the steady state level of consumption and capital.

To examine the optimality of Friedman's rule for optimum quantity of money, let us write down the steady-state utility:

$$W^* = \int_{t=0}^{\infty} e^{-\rho(r^*)t}u(c^*, m^*)dt = \frac{u(c^*, m^*)}{\rho(r^*)}. \tag{4.31}$$

Taking the derivative of W^* with respect to θ in equation (44) yields

$$\frac{dW^*}{d\theta} = \frac{(u_c^* \frac{dc^*}{d\theta} + u_m^* \frac{dm^*}{d\theta})r^* + u^* f''(k^*) \frac{dk^*}{d\theta}}{r^{*2}} = \frac{u_m^*}{\pi_m^* r^*}. \tag{4.32}$$

Setting $\frac{dW^*}{d\theta} = \frac{u_m^*}{\pi_m^* r^*} = 0$ and reminding equations (4.16) and (4.21) give rise to

$$\theta = -\rho(r^*) = -r^*. \tag{4.33}$$

Equation (4.33) gives the standard result of Friedman's rule for optimum quantity of money, i.e., the optimal monetary growth rate is equal to the negative of the time preference rate. Different from the literature, the time preference rate in our model is endogenously determined by the real interest rate. For the time preference function, the long run real interest rate is a fixed point by equation (4.33). Hence, we have the following proposition.

Proposition 3 *In the Sidrauski model with Keynesian time preference, money is superneutral and optimal monetary policy is the Friedman rule.*

5 Summary

In this short paper, by introducing the inflation rate into the representative agent's time preference rate, we have reexamined the effects of monetary and fiscal policies in the money-in-utility model. The comparative static analysis has demonstrated: neither monetary superneutrality nor Friedman's rule for optimum quantity of money holds. Specifically, with an increase of the money growth rate, the steady-state consumption, physical capital stock, real money balance holdings, and welfare all decrease. In addition, with a rise in government expenditure, the steady-state consumption, real money balances, and welfare will be reduced, whereas the steady-state capital stock remains unchanged.

References

[1] Aiyagari, Rao, and Ellen, McGrattan, 2003. The Optimum Quantity of Debt: Technical Appendix. Annals of Economics and Finance 4, 193-217.

[2] Barro, Robert, 2003. Optimal Management of Indexed and Nominal Debt. Annals of Economics and Finance 4,1-15.

[3] Bohm-Bawerk, E., 1891. The Positive Theory of Capital. (W. Smart, Trans.), Macmillan, London.

[4] Epstein, L., 1983. Stationary Cardinal Utility and Optimal Growth under Uncertainty. Journal of Economic Theory 31, 133-152.

[5] Epstein, L., 1987. The Global Stability of Efficient Intertemporal Allocations. Econometrica 55, 329-355.

[6] Epstein, L., Hynes, J., 1983. The Rate of Time Preference and Dynamic Economic Analysis. Journal of Political Economy 91, no. 4.

[7] Fisher, S., 1979. Capital Accumulation on the Transition Path in a Monetary Optimizing Model. Econometrica 47, 1433-1439

[8] Friedman, M., 1969. The Optimum Quantity of Money and Other Essays. Chicago: Aldine.

[9] Gahvari, F., 2007. The Friedman Rule: Old and New. Journal of Monetary Economics 54, 581-589.

[10] Gootzeit, M., Schneider, J., Smith, W., 2002. Marshallian Recursive Preferences and Growth. Journal of Economic Behavior and Organization 49, 381-404.

[11] Ireland, P., 1996. The Role of Countercyclical Monetary Policy. Journal of Political Economy 104, 704-723.

[12] Miao, Jianjun., 2004. A Note on Consumption and Savings under Knightian Uncertainty. Annals of Economics and Finance 5, 299-311.

[13] Katona, G., 1975. Psychological Economics. New York: Elsevier.

[14] Keynes, M. J., 1936. The General Theory of Employment, Interest, and Money. A Harvest/HBJ Book, Harcourt Brace Jovanovich, Publishers.

[15] Mises Ludwig Von., 1963. HUMAN ACTION: A Treatise on Economics. Fourth Revised Edition. Fox & Wilkes, San Francisco

[16] Shi Shouyong., 2001. 2001. Liquidity, Bargaining, and Multiple Equilibria in a Search Monetary Model. Annals of Economics and Finance 2, 325-351.

[17] Sidrauski, M., 1967. Rational Choice and Patterns of Growth in a Monetary Economy. American Economic Review 57, 534-544.

[18] Tobin, J., 1965. Money and Economic Growth. Econometrica 33, 671-684.

[19] Uzawa, H., 1968. Time Preference, the Consumption Function, and Optimal Asset Holdings. In: Wolfe, J.N.(Ed), Value, Capital and Growth; Papers In Honor of Sir John Hicks. University of Edinburgh Press.

[20] Wang, Gaowang, and Heng-fu Zou, 2011, Inflation Aversion and the Optimal Inflation Tax. Annals of Economics and Finance 12-1, 13-30.