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Abstract We study the stability issue in a Cournot duopoly with codetermined firms. We show that when both firms codetermine employment together with decentralised employees' representatives, a rise in wages acts as an economic de-stabiliser (stabiliser) when the wage is fairly low (high), while under profit maximisation a rise in wages always stabilises the market equilibrium. Moreover, increasing the union's bargaining power has a de-stabilising effect, except when the wage is low and the firm's power is already high.

Keywords Bifurcation; Codetermination; Cournot; Duopoly; Employment

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1. Introduction

A well known stylised fact concerning labour markets is that in some important countries, such as Germany, codetermination laws, according to which workers in large firms have nearly the same decision rights as capital owners, do exist. On the other hand, codetermination rights mainly concern firm's employment, whereas wages are apart from the field of application of such laws. On the other hand, even if one abstracts from codetermination laws,[1] it is widely observed that: (1) a distinction exists, especially in Europe (e.g., Scandinavian countries and Austria) between centralised (e.g., at a national or economy-wide level) unions that set the wage for an entire national industry, and decentralised (e.g., at a firm or district level) unions that negotiate about employment alone; (2) de-centralised wage setting processes, which however establish wage contracts of long lasting effectiveness (e.g., the three-year contracts often observed in the US), and local bargaining employment of higher periodicity exist. Both make relevant the case of bargaining over employment without considering wage setting.

An interesting study published in this Journal that has tackled this issue out from a point of view of a static bargaining game in a Cournot duopoly is Kraft (1998). The author interestingly shows that: (*i*) bargaining on employment alone is the dominant strategy with respect to the standard profit-maximising firm's behaviour, if the union's power is not too large (which seems to be the case of codetermination laws),[2] and (*ii*) "codetermination is welfare maximizing!" (see Kraft, 1998, p. 200). Therefore, given both the empirical relevance of decentralised bargaining on employment alone and the surprising theoretical features of such a process evidenced by the literature above noted, we observe that so far nobody has considered the effects of codetermination on product market stability in a duopoly with quantity competition. However, this is not an irrelevant issue to be dealt with given the long lasting debate on pros and cons of unions' power in both decentralised and centralised bargaining. This note aims to fill this gap in the economics theoretical literature by extending the duopoly model by Kraft (1998) in a dynamic context. The out-of-equilibrium dynamics is based on the assumption of "bounded rational" firm's behaviour as suggested, for instance, by Dixit (1986) and recently popularised by the literature on dynamic oligopolies (see, e.g., Puu 1998; Bischi and Kopel, 2001; Den Haan, 2001; Bischi et al., 2010; Tramontana 2010).

We find that an exogenous increase in the labour costs under codetermination destabilises the Cournot-Nash equilibrium when the wage is low enough (while playing a stabilising role for further wage increases when the wage is higher, but only whether the unions' power in determining employment is fairly low), while under profit maximisation a rise in wages plays an unambiguous stabilising role. Moreover, raising the relative union's bargaining power tends to destabilise the equilibrium, except when the wage is low and the firm's power is already high.

[1] According to co-determination laws "employment determination are handled by the supervisory board in codetermined firms. On the supervisory board employees have near-parity rights. In the iron and steel industry as well as in mining employees have explicit parity decision rights." (Kraft, 1998, p. 195).

[2] Indeed Kraft (p. 199) notices, at least when co-determination is regulated by law, that a situation in which workers have higher bargaining power than firm owners would rather be unrealistic. Indeed, he argues that "the assumption of $\beta > 0.25$ seems to be acceptable for codetermination in German firms, given the fact of near-parity representation of the employees."

Therefore, to keep the market stable the union's power in determining employment should be as low as possible: it should at most be close to the near-parity when the wage is fairly low. In a similar way, we observe that an increase in wages, which is beneficial for stability when unions are absent or their power in fixing employment is low, tends to destabilise the market equilibrium when the power of unions is high, unless wages are already set at a too high a level. This leads to a counterintuitive remark: under codetermination, when the power of unions in fixing employment is fairly high (namely, higher than that of firms), it is convenient for stability to reduce (increase) wages when they are already low (high).

The present study contributes to two growing strands of literature on: unionised oligopolies (see, e.g., Dowrick, 1989, 1990; Bughin, 1995, Kraft, 1998, Correa-Lopez and Naylor, 2004, Fanti and Meccheri, 2011), and dynamic oligopolies (see, e.g., Bischi et al., 2010), and provides a novel analysis on the dynamic effects of co-determination laws or, more generally, of bargaining on employment alone without any wage negotiations.

The rest of the paper is organised as follows. Section 2 builds on the model Sections 3 introduces expectations and analyses the local stability properties of the unique positive Cournot-Nash equilibrium, showing the local bifurcations and the emergence of regular and complex dynamics with numerical simulations. Section 4 concludes.

2. A Cournot duopoly with codetermined firms

The model is outlined in accordance with Kraft (1998). Without loss of generality, we consider a normalised Cournot duopoly for a single homogenous product with a negatively sloped inverse demand given by $p = 1 - q_1 - q_2$, where p denotes the price and q_1 (q_2) is the output produced by firm 1 (firm 2). The average and marginal costs for each single firm to provide one additional unit of output in the market are equal and constant at $0 < w < 1$, which represents the wage negotiated at the economy-wide level, while employment L_i is determined at the ith firm-specific level, with $i = \{1,2\}$. The hypothesis of constant average and marginal costs implies that firm i produces through a production function with constant (marginal) returns to labour, that is $q_i = L_i$ (see, e.g., Dowrick, 1989, 1990; Bughin, 1995; Correa-López and Naylor, 2004).

The objective of every firm is to maximise profits with respect to employment $\Pi_i(w, L_i) = pq_i - wL_i$, while the objective of unions is to maximise utility $U_i(w, L_i) = (w - w^\circ)^\theta L_i$ with respect to employment, where $\theta > 0$ is the relative weight attached by unions to wages and w° is the reservation or competitive wage. Without loss of generality, we set $\theta = 1$ and $w^\circ = 0$ henceforth. We assume that both firms codetermine employment with firm-specific unions. Since the production function is $q_i = L_i$, the Nash bargaining between firms and unions takes the form:

$$V_i = [(1 - q_i - q_j - w)q_i]^\beta (wq_i)^{1-\beta}, \tag{1}$$

where the control variable is q_i and $0 \le \beta \le 1$ is the relative bargaining power of firms. Therefore, the best reply function of the ith firm is determined by:

$$\frac{\partial V_i}{\partial q_i} = \frac{[(1 - q_i - q_j - w)q_i]^\beta (wq_i)^{1-\beta} [1 - q_i(1+\beta) - q_j - w]}{(1 - q_i - q_j - w)q_i} = 0 \Leftrightarrow q_i = \frac{1 - q_j - w}{1+\beta}. \tag{2}$$

3. Expectations and local stability

Let $q_{i,t}$ be firm i's quantity produced at time $t = 0,1,2,....$ Then, $q_{i,t+1}$ is obtained as:

$$q_{i,t+1} = \arg\max_{q_{i,t}} V_{i,t}\left(q_{i,t}, q^e{}_{j,t+1}\right), \tag{3}$$

where $q^e{}_{j,t+1}$ represents the quantity that the rival, i.e. firm j, today (time t) expects will be produced in the future (time $t+1$) by firm i. Assuming now heterogeneous (i.e., bounded rational[3] and Cournot-naïve[4]) expectations by each firm (see, e.g., Tramontana, 2010) about the quantity to be produced in the future by the rival, the two-dimensional system that characterises the dynamics of the economy is the following:

$$\begin{cases} q_{1,t+1} = q_{1,t} + \alpha q_{1,t} \dfrac{\partial V_{1,t}}{\partial q_{1,t}} \\ q_{2,t+1} = q_{2,t} \end{cases}. \tag{4}$$

where $\alpha > 0$ is a coefficient that captures the speed of adjustment of player 1's quantity with respect to a marginal change in $V_{1,t}$ when $q_{1,t}$ varies. Therefore, through the use of Eqs. (2) and (4) we get:

$$\begin{cases} q_{1,t+1} = q_{1,t} + \dfrac{\alpha\left[\left(1 - q_{1,t} - q_{2,t} - w\right)q_{1,t}\right]^{\beta}\left(w q_{1,t}\right)^{1-\beta}\left[1 - q_{1,t}(1+\beta) - q_{2,t} - w\right]}{1 - q_{1,t} - q_{2,t} - w} \\ q_{2,t+1} = \dfrac{1 - q_{1,t} - w}{1+\beta} \end{cases}. \tag{5}$$

From Eq. (5) it can be seen that a rise in β has a threefold effect on the marginal value of the Nash product of player 1 and, hence, on the quantity it will produce in the future. First, it increases the relative bargaining power of firm 1. Second, it tends to reduce the reaction of player 1 through a direct negative effect. Third, it also reduces the quantity produced by rival (firm 2) at time t and then tends to increase the reaction of player 1 through an indirect positive effect. As regards wages, an exogenous positive shock on w, by increasing production costs, tends to reduce firms' profits while also raising the utility of unions. Moreover, as a direct effect a rise in w plays an ambiguous role on the marginal value of the Nash product and then the reaction of the bounded rational firm is ambiguous through this channel. Indeed, as an indirect effect an increase in wages tends to reduce the output produced by the naïve firm at time t and then it also increase the reaction of the bounded rational firm because the marginal value of the Nash product raises through this channel. Definitely, the effect of a rise either in β or w at time t is potentially uncertain on the quantity produced by the bounded rational firm at time $t+1$.

Equilibrium implies $q_{1,t+1} = q_{1,t} = q_1$ and $q_{2,t+1} = q_{2,t} = q_2$. Then, from Eq. (5) in equilibrium we get:

[3] In the standard dynamic Cournot duopoly with profit-maximising firms, each bounded rational player uses information on current profits to increase or decrease the quantity produced at time $t+1$ depending on whether marginal profits are either positive or negative (see Dixit, 1986).

[4] Cournot (1838) *de facto* was the first to use naïve expectations in an oligopoly model.

$$\begin{cases} \dfrac{\alpha[(1-q_1-q_2-w)q_1]^{\beta}(wq_1)^{1-\beta}[1-q_1(1+\beta)-q_2-w]}{1-q_1-q_2-w}=0 \\ \dfrac{1-q_1-w}{1+\beta}-q_2=0 \end{cases}, \tag{6}$$

and the unique interior fixed point $E(q^*_1, q^*_2)$ of the two dimensional system is therefore characterised by:

$$E=\left(\frac{1-w}{2+\beta},\frac{1-w}{2+\beta}\right). \tag{7}$$

In order to investigate the local stability properties of the Cournot-Nash equilibrium E we build on the Jacobian matrix

$$J=\begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}=\begin{pmatrix} 1-\dfrac{\alpha w(\beta^2+3\beta+2)}{\beta(2+\beta)}\left[\dfrac{\beta(1-w)}{w(2+\beta)}\right]^{\beta} & \dfrac{-\alpha w}{\beta}\left[\dfrac{\beta(1-w)}{w(2+\beta)}\right]^{\beta} \\ \dfrac{-1}{1+\beta} & 0 \end{pmatrix}, \tag{8}$$

where partial derivatives J_{ii} and J_{ij} are evaluated at the equilibrium point defined by Eq. (7). Trace and determinant of J are given by:

$$T:=Tr(J)=J_{11}+J_{22}=1-\frac{\alpha w(\beta^2+3\beta+2)}{\beta(2+\beta)}\left[\frac{\beta(1-w)}{w(2+\beta)}\right]^{\beta}, \tag{9}$$

$$D:=Det(J)=J_{11}J_{22}-J_{12}J_{21}=\frac{-\alpha w}{\beta(1+\beta)}\left[\frac{\beta(1-w)}{w(2+\beta)}\right]^{\beta}<0. \tag{10}$$

Therefore, the characteristic polynomial of (8) is the following:

$$F(\lambda)=\lambda^2-T\lambda+D, \tag{11}$$

For the system in two dimensions defined by Eq. (5), the stability conditions that ensure that both eigenvalues λ_a and λ_b of the characteristic polynomial (11) remain within the unit circle are the following:

$$\begin{cases} (i)\quad F=\dfrac{2\beta(1+\beta)(1-w)-\alpha(2+\beta)(\beta^2+2\beta+2)\dfrac{w(1-w)}{2+\beta}\left[\dfrac{\beta(1-w)}{w(2+\beta)}\right]^{\beta}}{\beta(1+\beta)(1-w)}>0 \\ (ii)\quad TC=\alpha w\dfrac{2+\beta}{1+\beta}\left[\dfrac{\beta(1-w)}{w(2+\beta)}\right]^{\beta}>0 \\ (iii)\quad H=1+\dfrac{\alpha w}{\beta(1+\beta)}\left[\dfrac{\beta(1-w)}{w(2+\beta)}\right]^{\beta}>0 \end{cases}. \tag{12}$$

The violation of any single inequality in (12), with the other two being simultaneously fulfilled leads to: (*i*) a flip bifurcation (a real eigenvalue that passes through -1) when $F=0$; (*ii*) a fold or transcritical bifurcation (a real eigenvalue that passes through $+1$) when $TC=0$; (*iii*) a Neimark-Sacker bifurcation (i.e., the modulus of a complex eigenvalue pair that passes through 1) when $H=0$, namely $D=1$ and $|T|<2$. From Eq. (12) it is clear that conditions (*ii*) and (*iii*) are always fulfilled, while condition (*i*) can be violated. The following equation $\mathrm{B}(\alpha,\beta,w)$, i.e. the numerator of F in (12), represents a boundary at which the Nash equilibrium Eq. (7) looses stability through a flip bifurcation ($F = 0$) when:

$$\mathrm{B}(\alpha,\beta,w) := 2\beta(1+\beta) - \alpha w(\beta^2 + 2\beta + 2)\left[\frac{\beta(1-w)}{w(2+\beta)}\right]^{\beta} = 0. \quad (13)$$

Now, define

$$\alpha^F(\beta,w) = \frac{2\beta(1+\beta)}{w(\beta^2 + 2\beta + 2)\left[\frac{\beta(1-w)}{w(2+\beta)}\right]^{\beta}}. \quad (14)$$

as the (unique) flip bifurcation value of α. Then, the following proposition holds.

Proposition 1. *Let $0 < \alpha < \alpha^F(\beta,w)$ hold. Then, the Cournot-Nash equilibrium E of the two-dimensional system (5) is locally asymptotically stable. A flip bifurcation emerges if $\alpha = \alpha^F(\beta,w)$. Let $\alpha > \alpha^F(\beta,w)$ hold. Then, the Cournot-Nash equilibrium E is locally unstable.*

Proof. Since $\mathrm{B}(\alpha,\beta,w) > 0$ for any $0 < \alpha < \alpha^F(\beta,w)$, $\mathrm{B}(\alpha,\beta,w) = 0$ if $\alpha = \alpha^F(\beta,w)$ and $\mathrm{B}(\alpha,\beta,w) < 0$ for any $\alpha > \alpha^F(\beta,w)$, then Proposition 1 follows. **Q.E.D.**

It is now of importance to study how a rise in wages affects stability of the Cournot-Nash equilibrium when both firms bargain on employment together with firm-specific employees' representatives ($0 \le \beta < 1$), in contrast to the standard case of profit maximisation ($\beta = 1$). The results are summarised in the following proposition.

Proposition 2. *(Codetermination). Let $0 \le \beta < 1$. Then an increase in wages acts as an economic de-stabiliser (stabiliser) if, and only if, $w < 1-\beta$ ($w > 1-\beta$). (Profit-maximisation). Let $\beta = 1$. Then, an increase in wages always acts as an economic stabiliser.*

Proof. Since

$$\frac{\partial \alpha^F(\beta,w)}{\partial w} = \frac{2\beta(1+\beta)(w-1+\beta)}{w^2(1-w)(\beta^2 + 2\beta + 2)\left[\frac{\beta(1-w)}{w(2+\beta)}\right]^{\beta}}, \quad (15)$$

then, for any $0 \le \beta < 1$, $\frac{\partial \alpha^F(\beta,w)}{\partial w} < 0$ (> 0) if, and only, if $w < 1-\beta$ ($w > 1-\beta$). In the particular case $\beta = 1$, $\frac{\partial \alpha^F(1,w)}{\partial w} = \frac{12}{5(1-w)^2} > 0$. **Q.E.D.**

Proposition 2 shows the importance of codetermination in causing an ambiguous role of an exogenous shock in wages on stability of the Cournot-Nash equilibrium. Indeed, while a rise in wages (production costs) in the case of profit maximisation monotonically tends to stabilise the market equilibrium, because the direct negative of a rise in wages on the reaction of the bounded rational firm is stronger than the positive one due to the reduction in output produced by the naïve firm, in the case of codetermined firms, a positive exogenous shock in wages (due for instance to an increase in the bargaining power of unions at the economy-wide level), acts as an economic de-stabiliser (stabiliser) when wages are fairly low (already high). Moreover, the higher the bargaining power of firms in determining employment, the lower the threshold value of the wage beyond which an exogenous positive shock in wages stabilises the market equilibrium, and the wider the range of values of w within which stability is guaranteed.

We now study how a reduction in the bargaining power of firms in determining employment with firm-specific unions affects market stability. Since $\partial\alpha^F(\beta,w)/\partial\beta$ cannot be dealt with in a neat analytical form, we present numerical simulations which are summarised in Table 1. In particular, the table shows how the flip bifurcation value $\alpha^F(\beta,w)$ reacts to a reduction in β for different values of w.

Table 1. Flip bifurcation values $\alpha^F(\beta,w)$ when β varies.

$\alpha^F(\beta,w)$						
	$\beta=1$	0.8	0.6	0.4	0.2	0.1
$w=0.1$	2.6666	3.1907	3.4785	3.2172	2.0477	1.0834
0.2	3	3.0521	2.8292	2.225	1.2041	0.5875
0.3	3.4285	3.1316	2.6063	1.8402	0.8941	0.4133
0.4	4	3.3446	2.5481	1.6469	0.7325	0.324
0.5	4.8	3.7009	2.6	1.5495	0.6355	0.2699
0.6	6	4.2658	2.7634	1.5187	0.5743	0.2342
0.7	8	5.2067	3.0876	1.5533	0.5378	0.2098
0.8	12	7.0119	3.7332	1.6862	0.5241	0.1938
0.9	24	11.9242	5.3981	2.0732	0.5479	0.1868

Table 1 clearly shows that a reduction in β has two different effects on stability depending on the relative size of wages. If w is fairly low, a reduction in β (when β is fairly high) first acts as an economic stabiliser by increasing the flip bifurcation values $\alpha^F(\beta,w)$. Then, when β further decreases (i.e., the power of unions in the Nash objective becomes higher), it acts as an economic de-stabiliser. If w is fairly high, a reduction in β monotonically reduces the flip bifurcation value $\alpha^F(\beta,w)$ and then it unambiguously acts as an economic de-stabiliser.[5]

To sum up, as regards the problem of market stability, we may observe that: (*i*) when the exogenously determined wage is low or, alternatively the price-cost margin is high (e.g., the case of industries with low-skilled manpower, or, alternatively, with an exogenously given high profitability), stability of the Cournot-Nash equilibrium is more likely when the firm's bargaining power is fixed at not too high a level (namely, near-parity representation between parties, as neatly shown by the first line in Table 1, when $w=0.1$); (*ii*) when the exogenously determined wage is not too low (and a fortiori either when the wage is high or the price-cost margin is low), stability requires that the unions' power in co-determining employment with firms should be low enough. Alternatively interpreted, these results constitute a warning (e.g., for

[5] It is important to note that we have chosen to present the model with heterogeneous (i.e., bounded rational and naïve expectations) for analytical tractability. Indeed, the results of the present study holds even when both codetermined firms have bounded rational expectations as well as when one firm is profit-maximising and the rival is codetermined and both are bounded rational players.

centralised wage setters) about the peril for market stability of raising wages (unless they are fairly low) when the power of unions within the supervisory board that co-determine employment is high. The lesson drawn by these results is that in contrast with the case in which employment is not co-determined and unions only care about wages – indeed, in the latter case an increase in wages always stabilises the equilibrium (i.e. wage-interested unions are beneficial for stability) –, when codetermination laws exist or under separation between a centralised wage setting and de-centralised employment bargaining, a high firm-specific unions' power (often included even in the case of near-parity) to determine employment is harming for stability (except when the exogenously determined wage is low).

3.1. A numerical example of dynamical outcomes under profit maximisation and codetermination[6]

As a simple numerical illustration we now show the different dynamic events that can be observed depending on whether firms are profit-maximising or, alternatively, they are subject to codetermination laws. To this purpose, Figures 1 and 2 depict the bifurcation diagrams for α and portrait the limit point of q^*_1 when $w = 0.5$ and the initial conditions are $q_{1,0} = 0.03$ and $q_{2,0} = 0.01$. Figure1 clearly shows that under profit maximisation ($\beta = 1$), the Cournot-Nash equilibrium is locally asymptotically stable for any $0 < \alpha < 4.8$. Then, a flip bifurcation occurs at $\alpha^F(1, 0.5) = 4.8$. Beyond such a threshold, a two-period cycle emerges, followed by four-period cycles broken off when $\alpha \cong 7$. Then, eight-period cycles followed by high periodicity and a cascade of flip bifurcations that ultimately lead to chaotic motions emerge. Figure 2 instead depicts the case of near-parity codetermination ($\beta = 0.5$), showing that the equilibrium is locally asymptotically stable when $0 < \alpha < 2.064$. A flip bifurcation occurs at $\alpha^F(0.5, 0.5) = 2.064$. Then, a two-period cycle broken off at $\alpha \cong 3.25$ is observed. Then, the Cournot-Nash equilibrium under profit maximisation is more likely to be stable than under codetermination as in the former case the flip bifurcation occurs at a higher value of the speed of adjustment α, and the range of values of α for which the system does not "explode" is higher than in the latter case.

[6] It is worth noting that we do not enter into details of the analysis of complex dynamics as this is not the focus of the present study. It could however be interesting to advise the reader of the possible complicated dynamic events that can occur.

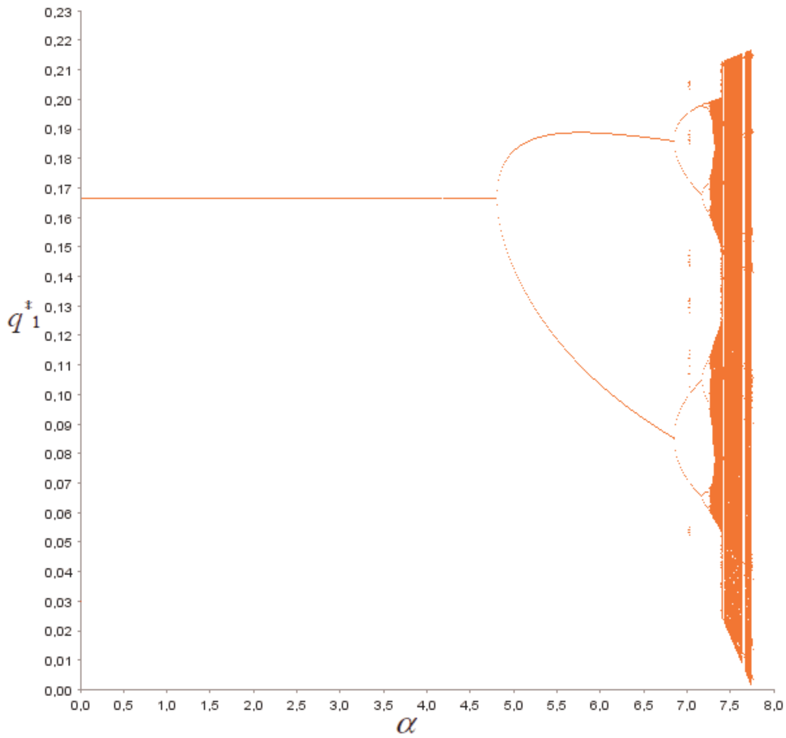

Figure 1. Profit-maximising firms ($\beta = 1$). Bifurcation diagram for α.

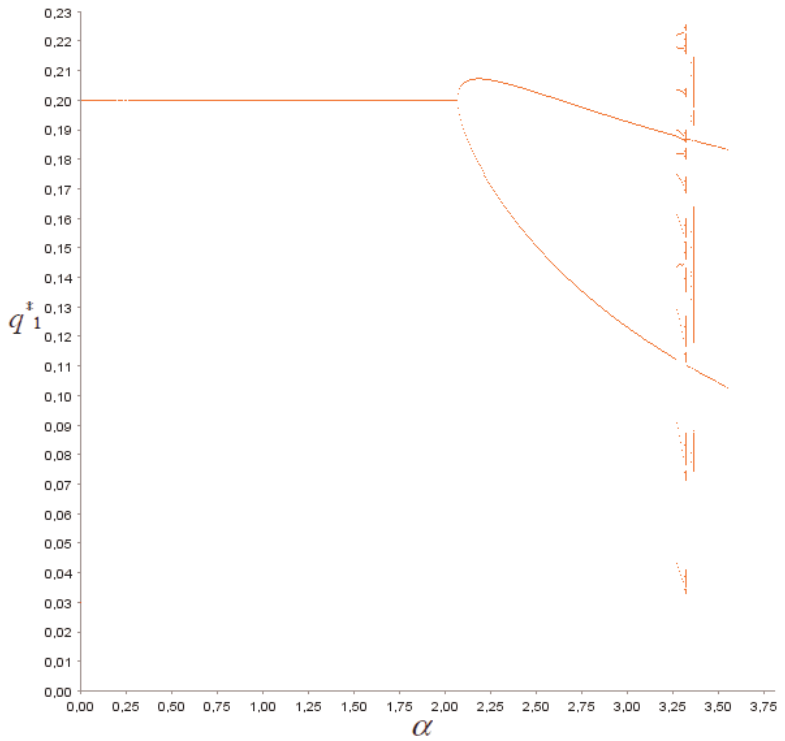

Figure 2. Codetermined firms ($\beta = 0.5$). Bifurcation diagram for α.

4. Conclusions

While traditional economic theories assume that the single aim of competing firms is profit maximisation, in some important countries, such as Germany, workers in large firms have nearly the same decision rights as capital owners, due to the existence of codetermination laws as regards employment setting at the firm level. More in general, a bargaining over employment without considering wages is widely observed, especially in Europe.

The present study analysed the dynamics of a Cournot duopoly where both firms (one of which is “bounded rational” and the other has the standard Cournot-naïve expectations) codetermine employment together with firm-specific workers’ representatives and compared the results with the standard case of profit-maximising firms. We found that a rise in wages under codetermination acts as an economic destabiliser (stabiliser) when the extent of the exogenously determined wage is still fairly low (already high), while under profit maximisation an increase in the labour cost always stabilises the product market equilibrium.

Therefore, on the one hand the existing literature (Kraft, 1998) established that, in equilibrium, co-determination may be preferred by the society as a whole with respect to profit maximisation, on the other hand we found that, out-of-equilibrium, the bargaining power of firms should be fairly high in order to ensure market stability, except when the wage is low: in such a case in fact near-parity decision rights would be better for stability.

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