
Guido Travaglini

Università degli Studi di Roma "La Sapienza"

6. June 2007

Online at http://mpra.ub.uni-muenchen.de/3419/

Guido Travaglini
Università di Roma ‘La Sapienza’
Istituto di Economia e Finanza
Email: jay_of_may@yahoo.com
First version.
June 2007.

Nulla dies sine linea (Pliny the Elder, XXXV, 84).

Keywords: Generalized Method of Moments, Monetary Policy Rules, Multiple Breaks.

JEL Classification: C12, C61, E58.
Abstract.

This paper combines two major strands of literature: structural breaks and Taylor rules. At first, I propose a nonstandard \( t \)-test statistic for detecting multiple level and trend breaks of I(0) series by supplying theoretical and limit-distribution critical values obtained from Montecarlo experimentation. Thereafter, I introduce a forward-looking Taylor rule expressed as a dynamic model which allows for multiple breaks and reaction-function coefficients of the leads of inflation, of the output gap and of an equity market index. Sequential GMM estimation of the model, applied to the Effective Federal Funds Rate for the period 1984:01-2001:06, produces three main interesting results: the existence of significant structural breaks, the substantial role played by inflation in the FOMC decisions and a marked equity targeting policy approach. Such results reveal departures from rationality, determined by structured and unstructured uncertainty, which the Fed systematically attempts at reducing by administering inflation scares and misinformation about the actual Phillips curve, in order to keep the output and equity markets under control.
1. Introduction.

The literature on Taylor Rules [Taylor, 1993, 1998, 1999, 2001] is by now one of the vastest in the field of Monetary Policy and keeps on gaining momentum in both theoretical and applied analysis. Recently, it has undergone substantial revamping to correct for cointegration and omitted-variable bias, and to include rationality and uncertainty, and also the likelihood of structural breaks.

The literature on the latter topic, in the meantime, has pursued significant progress since Perron’s seminal article [1989] that has modified the traditional approach toward Unit Root (UR) testing [Dickey and Fuller, 1979]. By departing from different null hypotheses that include UR with or without drift, trending series with I(0) or I(1) errors and with or without additive outliers, the alternative hypotheses formulated have accordingly included different combinations that range from one single level and/or trend break to multiple structural breaks of unknown date [Banerjee, Lumsdaine and Stock, 1992; Zivot and Andrews, 1992; Perron and Vogelsang, 1992, 1997; Andrews, 1993; Lumsdaine and Papell, 1997; Bai, 1997, 1999; Vogelsang and Perron, 1998; Bai and Perron, 1998, 2003a, 2003b; Perron and Qu, 2004; Perron, 2005; Deng and Perron, 2005; Kejriwal, 2006; Kejriwal and Perron, 2006].

The present paper, by drawing from this vast experience, and especially from a seminal contribution in the field [Perron and Zhu, 2005], attempts at fitting the Taylor Rule model into the multiple-breaks issue and proposes a novel t-statistic testing procedure for multiple level and trend breaks occurring at unknown dates [Vogelsang, 1997, 1999]. This procedure is easy and fast at identifying break dates, as it exploits the algebraic difference between the critical \( t \) statistic obtained under the null hypothesis of I(0) series with stationary noise and the alternative provided by a I(0) model with a constant, a trend term, the structural breaks and other stationary noise components.

The plan of the paper is the following. Section 2 briefly introduces the scope and goals of the Fed and a chronology of the major events occurred in the recent history of the U.S. monetary policy. Section 3 analyzes the recent literature on Taylor Rules, and aims at reaffirming the absolute relevance of dynamic versions thereof in the presence of cointegration, omitted variables and structural breaks. Section 4 analyzes the null and alternative hypotheses adopted, computes the critical values of the \( t \) statistics of the two breaks and produces their finite-sample Monte Carlo simulations. Section 5 provides the motivations for constructing a dynamic Forward-Looking Taylor Rule that is GMM estimable and embodies rationality, error correction and structural breaks. Section 6 exhibits the main time-series characteristics of the database used and produces some GMM pretesting addressed at the selection of optimal leads, lags and instruments. Along with the deterministic components, the following relevant stationary regressors are included: inflation, output gap and an equity index. Finally, Section 7 provides the results by treating as time series, for the trimmed sample of monthly observations 1984:01-2001:06, the estimated \( t \) statistics of both breaks and the sums of the reaction-
function coefficients of the regressors. Some of their key properties are utilized to evaluate the Fed’s departures from rationality and systematic attempts made to reduce structured uncertainty. Section 8 concludes, while the Appendix supplies the necessary off-text details of Sections 4.

2. The Major Events of the U.S. Monetary Policy.

“The Board of Governors of the Federal Reserve System and the Federal Open Market Committee shall maintain long run growth of the monetary and credit aggregates commensurate with the economy's long-run potential to increase production, so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates”.

So recites the Act of Nov. 16, 1977, added to the Federal Reserve Act of Dec. 23, 1913 at Jekyll Island and amended by the Acts of Oct. 27, 1978, of Aug. 23, 1988 and of Dec. 27, 2000. Within this broad-scoped set of goals, the Board of Governors of the Federal Reserve System (the ‘Fed’) and the Federal Open Market Committee (the FOMC) have been since then officially committed to pursue primarily the goal of stabilizing inflation and real output (henceforth denoted as ‘output’). Stabilization of other key macroeconomic variables, such as employment, exchange rates, financial assets, are let to receive secondary attention as a natural consequence of the evolution of former, or within the context of short-run interventions, or by assigning them to other authorities, chiefly those mandated to pursue fiscal policy [Taylor, 1998].

By abiding to such formal and practical guidelines, thereby adhering to some rule of thumb, the Fed since its foundation has officially elected inflation and output as its two major monetary policy targets [Orphanides, 2003], while making discretionary use of policy price instruments, the discount rate and the official (or ‘intended’) Federal Funds Rate (OFFR), and of quantity instruments, the Open Market Operations and the commercial banks’ reserve requirements1.

The Great Depression episode is the first example of how a price-instrument rule of thumb was applied. The Fed preemptively decided to adopt an "antispeculative" policy tightening since the years 1928-29, by hiking the interest rates to prevent the rise of stock prices and the associated increases in bank loans to brokers. By trying to distinguish between "productive" and "speculative" uses of credit [Bernanke, 2002] and by attempting to avoid inflation and/or widespread economic disruption [Friedman and Schwartz, 1963], the Fed – ironically if not tragically – acted as the Sorcerer’s Apprentice in Wolfgang Goethe's ballad (1797), later popularized in a Walt Disney cartoon (1940).

---

1 Of these, the Board of Governors defines the Federal Funds Rate as the interest rate at which depository institutions lend federal funds to other depository institutions overnight, while the Open Market Operations are the purchases and sales of U.S. Treasury and federal agency securities. The discount rate traditionally receives less weight as an indicator of prime lending, and is defined as the interest rate charged to depository institutions to borrow funds directly from the Federal Reserve.
The Fed faced some degree of unstructured uncertainty as to the likely effects of its moves over the economy, since it was puzzled at the countercyclical behaviour of price inflation with respect to output growth and misinterpreted, by means of structured uncertainty, the technology-driven stock market boom and growing productivity of the early 20’s. Hence, the spate of tightening of credit conditions that lasted until the end of 1930 acted as the infamous broomsticks. However, the apprentice was benevolently pardoned, not fired.

The obvious question, at this juncture, is the following: has the Fed learned how to interpret the major economic events and how to use the appropriate instruments? Wither rationality? How high was the degree of uncertainty? Was uncertainty structured or unstructured or both? By switching rapidly to the quantity policy instrument in conjunction with World War II, the Fed hoped to avoid the mistakes stemming from the previous inaccurate use of the interest rates. However, the Dollar glut and the outbreak of inflation in the late Sixties, the fall of the fixed exchange-rate regime, the rekindled countercyclical behaviour of price inflation with respect to output and productivity during the slumpflation of the years 1968-83 and the boom of the years 1994-2000 have represented serious problems as to the achievement of financial stability, notwithstanding some valuable albeit dogged defence [Greenspan, 2004; Goodfriend and King, 2005].

Given such premises, and in spite of the reswitch to an interest-rate policy rule performed by Volcker in early 1982, there’s no wonder why the stock market crashes of late 1987 (the ‘Black Monday’) and of late 2000 are nothing but a replica of that of 1929, especially in the Fed’s perception that credit would be used by the public to finance speculative and not productive activities [Cecchetti, 2003] and in its inability at measuring, and thus predicting, the stock-market bubbles [Gruen et al., 2005].

However the Fed, apparently, has apparently not yet learned that sitting in the sidelines is no good for its own reputation as a rational utilizer of information and for the stability of the economy, and especially of the financial system. In fact, according to some authors, the Fed still needs lessons to improve its recognition of the key target variables and of their future course in order to comply with the Keynesian policy prescription that ex ante is preferable to ex post action over the target variables [Rudebusch, 1998; Orphanides, 1998, 2000; Orphanides et al., 2000; Orphanides and Williams, 2005b; Swanson, 2004].

In practice, the Fed may have not fully learned the lesson that central bankers should respond to asset prices only when there is enough information of the stock market and of the message it customarily delivers [Bernanke and Gertler, 1999, 2001], not to speak about achieving ‘best estimates’ of the inflation and output gaps, on which more in Sect. 3. This evidence seems to point to frequent ‘departures from

---

2 These authors make a case about two alternative intervention plans of a central bank in the face of an emerging asset-price bubble, denoted as: i) skeptic, ii) activist. The dividing line is represented by the information set about the bubble stochastic process available to the policy makers. If structured uncertainty is high, policy should be attenuated and only inflation targeting should be pursued, otherwise stock puncturing may be desirable if there is sufficient knowledge about the relationship between money and the asset market. There follows, along this line of interpretation, that the Japanese stock market crash of 1989 is an example of activism (i.e. aggressiveness) with full structured uncertainty.
rationality’ [Rudebusch, 1999, 2002a], suggesting that some of Goethe’s magical broomsticks may be still hovering atop the Fed’s headquarters in Washington D.C.

Since Volcker’s main policy instrument change addressed at ensuring financial stability, the interest rate has become the centerpiece of theoretical and applied research as to the behaviour of the Fed and of central banks in general. Hence, a specific ‘rule of thumb’ has been formalized in terms of what is widely known nowadays as the ‘Taylor Rule’ [Taylor, 1993, 1998, 1999, 2001].

The period of interest in the present paper covers the years 1982-2006 by monthly observations, and spans the later Volcker, the entire Greenspan and the first full year of Bernanke’s chairmanships. Although Volcker was appointed in late 1979, the month 1982:04 corresponds to the date when officially the Fed determined to switch to an 'interest rate rule'. Significant enough evidence stands in favor of this date selection [Fair, 2001a, 2001b], considered as a milestone in any analysis regarding modern U.S. monetary policy making.

This period is fraught with relevant episodes in the economic and financial arenas, and is characterized by a gradual shift from ‘indirect targeting’ through Open Market Operations toward direct target announcement of the OFFR since 1995, accompanied by an official assessment issued by the FOMC, initiated in the year 2000, of the macroeconomic risks associated to its decisions. In sum, this period of ‘interest rate policy rule’ testifies of a progressive control over the OFFR and, lately, of some kind of transparency as well.

“The tightening of monetary policy by the Federal Reserve in 1979, then led by Paul Volcker, ultimately (by mid 1986) broke the back of price acceleration in the United States, ushering in a two-decade long decline in inflation that eventually brought us to the current state of price stability” [Greenspan, 2004]. Hence, the Eighties begin with Volcker's targeting of money and non-borrowed reserves, immediately following his appointment, and by a particularly aggressive tightening in monetary policy addressed at stemming a double-digit inflation. However, poor confidence in monetary policy by the public culminates in a near threefold increase of the price of gold by the late 1981, which prompts the chairman to switch to the above mentioned interest rate rule.

After the Plaza Accord on the Dollar devaluation (1985) and a slack in price inflation, the late Eighties feature the appointment of Greenspan in August 1987, soon greeted by a new inflationary spate and by the stock market crash in October 1987. The Chairman by consequence attempts at pursuing an expansionary monetary policy by a massive injection of liquidity and by keeping under control the interest rates to avoid a recession. In fact, the industrial production remains essentially stable. A few months later, however, the OFFR is raised several times and peaks to close 10% in the early Spring 1989. The immediate consequence is a four-year recession which lasts until late 1992 in spite of progressive easing whereby, by September 1992, the EFFR reaches by that period a historical minimum: somewhat less than 3%. The length of the downturn is essentially attributed to “constriction of credit in response to major losses at banks..., coupled with a crisis in the savings and loan industry that had its origins in a serious maturity mismatch as interest rates rose”
[Greenspan, 2004]. In other words, the recession is caused by unprecedented lows (even negative) in the real interest rates, which causes large imbalances between savings and investments.

The subsequent tightening which starts by mid 1993, passing through the Mexican peso crisis (1994), the Russian ruble crisis (1998) and the Asian crisis (1997-1999), is initiated with robust hikes in basis points (bps) on accounts of an inflation scare [Orphanides and Williams, 2005a] probably induced by the upturn of industrial production, and sets the real interest values back to substantial positives. This is attained via an increase of 300 bps of the OFFR that lasts until June 1995. Public confidence on this move that heralds a distinguished, yet unjustified, anti-inflation stance, fosters even more significant economic growth, which trespasses the 8% yearly figure in December 1994, 9% in March 1997 and 10% in January 1998, and definitely marks the beginning of an economic expansion which will culminate in the ‘dotcom’ equity bubble (1999-2000).

Greenspan’s ensuing engagement in a strenuous fight against ‘irrational exuberance’ may be considered, from an outsider’s viewpoint, as nothing more than a crusade against the same puzzle that had baffled his predecessors in the late Twenties, namely, the mix of soaring asset prices, declining price inflation and sustained economic growth and productivity, typical aspects of the ‘perverse’ Phillips curve, [Tetlow and von zur Muehlen, 2001].

At that time, however, few critics would have cast any doubt on the Fed’s ability to muster its own econometric toolkit. By consequence, it is much more likely that the general public had been baffled. In fact, by waiving again the standard of an inflation scare, whereas inflation was at unprecedented historical lows – a mean value of 1.87% during the period 1998:01-1999:12 – the Chairman enacted a spate of OFFR hikes for a total of 175 bps between June 1999 and May 2000 officially to “mitigate the (stock-market) fallout when it occurs and, hopefully, ease the transition to the next expansion” [Greenspan, 2004], but in practice to prick the bubble which would burst few months later.

The new millennium testifies of a rapid reduction of the OFFR, commenced in January 2001 and ended in June 2004, in the meantime accompanied by the ‘911’ episode (2001), by the Iraq war (2003) and by a new recession, initiated in November 2001 by NBER standards. Spurred by the subsequent economic recovery and by growing inflationary pressures, the Fed tights again by means of a sequel of rate hikes that has to date reached a level of 5% with Bernanke in office.

---

3 The data on price inflation (CPI index, all goods) do not support any evidence of overheating, as the mean for the period 1992:01-1996:12 is a bare 2.87% per annum and that for the longer stretch 1992:01-1999:12 is even lower: 2.55%.

4 Greenspan’s approach to the Phillips curve has been apparently characterized by a tradeoff between actual policy practice and official declarations. As to the latter, in fact, the chairman has been poised to affirm several times that the Fed’s goal is to attain sustainable economic growth with zero inflation, thereby adhering to some unconventional (i.e. non-Keynesian) view of the curve in face of the ‘new economy’ [Ball and Tchaidze, 2002; Rasche and Thornton, 2006].

5 Cecchetti [2003] interestingly points to the Fed’s growing concern about the stock market rise since the mid Nineties, by revealing from the minutes and transcripts of the FOMC meetings the number of occurrences of all words related to equities. The keywords search yields a maximum of such occurrences precisely during the years 1998-2000.

The rule of thumb, which has been followed by the Fed since its foundation and in line with the subsequent amendments, is eventually revamped and formalized into what is by now universally known as the ‘Taylor rule’ (TR), so named after his concoctor [Taylor, 1993, 1998, 1999, 2001]. This rule, by remarking the central tenet of stabilizing inflation and output, introduces the benchmark values around which they can be made to deviate within a safe confidence band. The benchmarks are provided by their respective long-run values fixed, but possibly uncommitted, by the Fed [Friedman and Schwartz, 1963].

The TR is therefore simply a standalone linear reaction function of the central bank’s interest rate with respect to inflation and output deviations ('output gap'), which may be used for policy purposes. The author recommends a ‘leaning-against-the-wind’ policy with the EFFR adjusting positively by a coefficient higher than one to inflation deviations and by a coefficient close to one to output deviations [Taylor, 1999]. He thus advocates monetary policy to be ‘activist’ by reacting aggressively to both inflation and output deviations – once these are made observable and reliable to the Central Bank and thus structured uncertainty is absent – just the opposite of what suggested by the Monetarists and by the Lucas critique [Orphanides, 2000].

The TR is utilized by the Fed as a reasonable rule to be followed in order to ensure macroeconomic stability. It is also utilized by econometricians as an interpretive gadget to guess on the Fed’s decisions about future interest rates, given the usually thick veil of secrecy that has surrounded most if not all of its FOMC sessions since the times of Jekyll Island.

In both cases, estimation of the TR is no easy task since it involves selection of the appropriate key variables and computation of their deviations. When expectations about their future course are included, matters may become more complicated and both kinds of uncertainty ensue. The problem may in fact be very serious for central bankers, faced with incomplete knowledge about the evolution of real and financial markets determined by incorrect model specification [Hansen and Sargent, 2003, 2004, 2007].

By consequence, central bankers would be obliged to modify policy targeting or to attempt at fooling the public by means of inflation scares or false news on macroeconomic stability. This behavior corresponds to reducing unstructured uncertainty in their model used for policy making [Brock and Durlauf, 2004].

This is the reason why the TR since its inception has undergone a growing degree of model sophistication, usually associated to the null hypothesis of rationality and perfect database information on the past, present and future economic conditions in the hands of the Board of the Governors. The literature on the TR is by now huge and with many variants, which posit contemporaneous, backward-looking (BLTR) or forward-looking (FLTR) regressors, and oftentimes nest the rule within the framework of theoretical or empirical macroeconomic models, which may also include functions of private sector's behavior, in order to test for stability properties in the presence of learning.
Taylor’s standard contemporaneous model (STR), and the more sophisticated BLTR, are subject to the Lucas critique of parameter instability and subsequent structured uncertainty, as well as to data reliability, that may strongly impinge upon the Fed’s ability to gather information and fix optimal policy guidelines [Rudebusch, 2005]. Some authors maintain that the monetary policy consequences may as well be proportionate to the distorted estimates of inflation and of the output gap [Bullard and Mitra, 2001; Orphanides and Williams, 2005b; Orphanides, 1998, 2000, 2001, Orphanides and van Norden, 1999; Orphanides et al., 2000; Smets, 1998; Österholm, 2005a].

By consequence, to better embed the assumed rational-expectations component in the Fed’s information set utilized in policy decisions, real-time data usage by empirical researchers is highly recommended, as this may be the same method used by the Board of Governors. These data are generally drawn from the Green Book forecasts, from the minutes of the FOMC sessions and from other sources [Gavin and Mandal, 2001; Orphanides, 2001b; Österholm, 2005a] and are found in several cases to outsmart all other private prediction methods because reportedly unbiased and efficient [Romer and Romer, 2000]6.

Other authors, instead, maintain that real-time data suffer from error-in-variables bias and cannot be reputed as ‘best estimates’. Therefore, parameter inconsistency and non certainty equivalence arises [Orphanides, 1998]. In particular, it is shown that if only few reliable regressors in the TR can be used because deriving from ‘best estimates’ – the other regressors being discarded – the policy rule will be based on a constrained and not on a global optimum [Smets, 1998; Orphanides, 1998; Swanson, 2004].

In addition, the BLTR, by including the lagged endogenous variable to ensure 'smoothing' of the monetary policy rule i.e. a gradual adjustment of the interest rate toward its target rate [Clarida et al., 1998, 1999, 2000; Sack, 1998; Sack and Wieland, 1999; Bernanke, 2004], suffers from serious criticism. In fact, some authors maintain that such inclusion is a computational artifact to avoid serial correlation in the disturbance term, rather than an attempt at proving the existence of an inertial interest rate policy [Rudebusch, 2002a, 2002b; Siklos and Wohar, 2004; Bunzel and Enders, 2005; Welz and Österholm, 2005].

Augmentation of the BLTR to avoid serial correlation and the omitted-variable bias has thus prompted several authors to include additional regressors, like consumer sentiment, exchange rates, housing prices and – in many cases – the equity market [Bernanke and Gertler, 1999; Bullard and Schaling, 2002; Cecchetti et al., 2002; Rigobon and Sack, 2001; Fuhrer and Tootell, 2004; Bunzel and Enders, 2005] due to its strict links to monetary policy in the context of the transmission mechanism [Cecchetti, 2003; Bernanke and Kuttner, 2005].

Additional criticism on the STR and BLTR models regards two relatively novel issues: cointegration and structural breaks. Some authors argue in fact that the

---

6 The most used sources (e.g. see Gavin and Mandal, 2001) are the Humphrey Hawkins Forecast periodically released by the Board of Governors of the Federal Reserve System, the Real Potential Gross Domestic Output estimates released by the Congressional Budget Office and the Blue Chip Consensus Forecast.
TR representation in levels may be spurious, since the endogenous variable is likely to incorporate one or more cointegrating relationships [Österholm, 2005b; Siklos and Wohar, 2004], producing asymptotic invalidity of standard inferences [Phillips, 1986]. By consequence, UR and cointegration pretesting of the variables involved is a must in empirical research, since interest and inflation rates are often found to be I(1) [Siklos and Wohar, 2004; Österholm, 2005a], so that absence of an error-correction (EC) term in a first-differenced TR is at least surprising [Söderlind et al., 2005].

In addition, the possibility of breaks in policy regime significantly questions the very nature of a rule, even more so if performed by a rational central bank. In fact, frequent and/or substantial policy regimes in terms of multiple structural breaks are found to exist [Judd and Rudebusch, 1998; Fair, 2001b; Bunzel and Enders, 2005] and may be the rule rather than the exception in long-span time series [Noriega and Soria, 2002].

Given these limitations, the FLTR has gained significant momentum in these last years, both in theoretical and empirical research [Batini and Haldane, 1999; Bullard and Mirra, 2001; Carlstrom and Fuerst, 2000; Christiano and Gust, 2000; Clarida et al., 1998, 1999, 2000; Lansing and Trehan, 2003; Levin et al., 1999; Linde, 2002; Orphanides, 2003; Rotemberg and Woodford, 1999; Rudebusch and Svensson, 1999; Woodford, 2001; Svensson, 2003].

The FLTR embodies rational expectations from the Fed’s standpoint within the context of dynamic optimization, and is therefore considered to be the most appropriate representation of the central bank’s behavior as it assumes rational private agents when the rule is nested into a game-theoretic model (but see Woodford, [2000]). In addition, as is well known, it belongs to the class of forward-filter estimators, whose chief property, from the technical viewpoint, is to orthogonalize residuals with respect to instruments, ensuring strong exogeneity [Hayashi and Sims, 1983].

The FLTR, however, is not problem-free unless duly modified in the light of the problems afflicting the other two rules. It has been in fact demonstrated that a FLTR nested into a macroeconomic model with rational expectations and certainty equivalence may cause instability of the system with indeterminate solutions [Bullard and Mirra, 2001; Carlstrom and Fuerst, 2000]. This occurrence is correctable only if monetary policy were aggressively activist, namely, if the EFFR were made to rise more than proportionately with respect to expected inflation [Bernanke and Woodford, 1997; Taylor, 1999; Clarida et al., 2000; Woodford, 2001; Svensson and Woodford, 2002]. The same kind of aggressiveness is suggested by other authors also if certainty equivalence does not hold and unstructured uncertainty is high [Tetlow and von zur Muehlen, 2001; Swanson, 2004]. In such case, since real-time data do not correspond to their ‘best estimates’, especially when prediction stretches long ahead in the future, sizable departures from rationality and/or irrational noise may emerge [Rudebusch, 1999].

In any case, the FLTR is regarded nowadays in applied research as the most powerful approximation of Taylor rule modeling. Its use of the GMM algorithm is
specifically fit to deal with rational expectations due to the optimizing use of first-order conditions and the simultaneous treatment of endogeneity, autocorrelation and heteroskedasticity [Hansen and Singleton, 1982; Newey and West, 1987; Hansen and West, 2002]. In addition, the GMM effectively disposes of the errors-in-variable problem that plagues expectation models with unreliable data [Orphanides, 1998, 1999; Orphanides et al., 2000], ensures orthogonality and, for a sufficiently long time span of the series, is consistent because characterized by large-sample robustness [Hansen, 1982; Newey and West, 1987; Hansen and West, 2002].

It is for these reasons that the FLTR model, coupled with the GMM, will be adopted in the present paper to estimate the Fed’s interest rate policy during the last quarter century or so. Major modifications to solve the problems outlined above are introduced, such as first differencing [Orphanides and Williams, 2005b; Söderlind et al., 2005], the appropriate choice of leads, the introduction of the EC mechanism [Söderlind et al., 2005] and of a stock market index, and finally the introduction of multiple structural breaks together with a novel technique designed for their detection.


The departing point to test for the existence of structural breaks in a time series function is the null hypothesis given by the I(0) series

\[ \Delta y_t = y_t - y_{t-1} = e_t; \quad y_1 = 0 \]

where \( y_t \) spans the period \( t = 1...T \), and \( e_t \sim N(0,1) \) corresponds to a standard Data Generating Process (DGP) with random draws from a normal distribution whose underlying true process is a driftless random walk.

Let the field of fractional real numbers \( \Lambda = (\lambda_0, 1-\lambda_0) \), where \( 0 < \lambda_0 < 1 \) is a preselect trimming factor, normally required to avoid endpoints spurious estimation in the presence of unknown-date breaks [Andrews, 1993; Andrews and Ploberger, 1994]. Let then the true break fraction be \( \lambda \in \Lambda \) such that \( 0 < \lambda < \lambda < (1-\lambda) \). For given \( T \), define \( \lambda_0 T \leq \lambda T \leq (1-\lambda_0)T \) the field of integers wherein the true break date occurs. Any of the two structural breaks may be formulated as \( TB_t \in \{ \lambda_0 T, (1-\lambda_0)T \} \) [Banerjee, Lumsdaine and Stock, 1992; Carrión-i-Silvestre and Sansò Rossellò, 2005].

Given the null hypothesis of driftless stationarity posited in eq. 1, the proposed alternative in the present paper is provided by a stationary series with a constant, a trend and their respective breaks, plus a noise component. Specifically, the alternative is represented by an augmented AO model, which is usually estimated by Ordinary Least Squares (OLS). For the trimmed time interval \( t = \lambda_0 T, \lambda_0 T + 1,...,(1-\lambda_0)T \), \( \Delta y_t \) is the endogenous variable such that
\[ \Delta y_i(\lambda) = \mu_i + \mu_2 DU_i(\lambda) + \tau_i t + \tau_2 DT_i(\lambda) + \varepsilon_i(\lambda) \]

where the \((\lambda)\) notation attached to the left- and right-hand side variables indicates sequential estimation of the equation for all values of \(\lambda \in \Lambda\), and the disturbance \(\varepsilon_i = I.I.D.(0, \sigma^2)\) is \(I(0)\) with \(E(\varepsilon_i, \varepsilon_j) = 0; \forall t, s, s \neq t\) for the same time interval [Perron and Yabu, 2004; Perron and Zhu, 2005]. The two differently defined unknown-date break dummies \(DU_i\) and \(DT_i\) are:

A) \(DU_i = I(t > TB_i)\) a change in the intercept of \(\Delta y_i\), \((\mu_i - \mu_0)\), namely a break in the mean level of \(\Delta y_i\);

B) \(DT_i = (t - TB_i)I(t > TB_i)\) a change in the trend slope \((\tau_i - \tau_0)\), namely a change in the inclination of \(\Delta y_i\) around the deterministic time trend;

where coefficients \(\mu_0\) and \(\tau_0\) are the respective pre-change values.

In principle, since \(\Delta y_i\) is \(I(0)\), its mean is expected to be zero, and changes in mean can only be a temporary phenomenon. Therefore, case A corresponds to unknown-date structural breaks in terms of temporary change(s) in the level of the endogenous variable (the "crash" model). Similarly, case B corresponds to temporary shifts in the trend slope (the "changing growth" model) [Perron 1997; Banerjee et al., 1992; Vogelsang and Perron, 1989; Zivot and Andrews, 1992]. Eq. 2, by using both cases together, is defined by Perron and Zhu [2004] as a "local disjoint broken trend" model with \(I(0)\) errors (their “Model IIb”).

Eq. 2 is estimated sequentially for all \(\lambda \in \Lambda\), that is, for all possible true break dates occurring in the \(\lambda T\) time interval thereby producing, for some compact coefficient space \(\mathbf{B}\) of real numbers, a time series of length \(1 + (1 - \lambda_0)T\) of the coefficient vector \(\hat{\beta}(\lambda) = [\mu_1, \mu_2, \tau_1, \tau_2]\), such that \(\hat{\beta}(\lambda) \in \mathbf{B}\). Also the \(t\) statistics of \(\hat{\beta}(\lambda)\) for the above trimmed interval may be obtained and defined as \(\hat{t}_\mu(\lambda)\) and \(\hat{t}_\tau(\lambda)\), respectively. They are nonstandard-distributed since the respective breaks are associated to unknown dates and thus appear as a nuisance in eq. 2 [Andrews, 1993; Andrews and Ploberger, 1994; Vogelsang, 1999].


However, different from these methods that identify the break(s) when a supremum (or exponential or weighted average) value is achieved and tested for, all that is required here is to sequentially find as many \(t\) statistics that exceed the appropriately tabulated critical value for a given magnitude of \(\lambda\). In fact, after finding the critical values for different magnitudes of \(\lambda\) by Montecarlo simulation, respectively denoted as \(t_r(\lambda, L)\) and \(t_r(\lambda, T)\), any occurrence for a given confidence level (e.g. 95%) whereby \(|\hat{t}_\mu(\lambda)| > t_r(\lambda, L)\) and \(|\hat{t}_\tau(\lambda)| > t_r(\lambda, T)\) indicates the
existence of a level and a trend break, respectively, just as with standard \( t \)-statistic testing\(^7\). By consequence, rather than identifying a single (local or global) break, the procedure permits detecting their slow overtime evolution, thus providing in general break periods which may be characterized by a more permanent nature than single-dated breaks applied to I(0) series, as discussed above.

In the present paper, for reasons already expressed in the previous Section, eq. 2 must be estimated by GMM. This estimation method is equivalent to OLS in the absence of heteroskedasticity autocorrelation of the error term and of regressor endogeneity. As constructed, eq. 2 meets these requirements so that OLS and GMM estimation at this juncture are equivalent.

By this reasoning, it is sufficient to supply some additional notation. Let the \( K \times 1 \) vector \((K = 4)\) of the determinants of \( \Delta y_t \) in eq. 2 be \( X_t = [1, t, DU_t, DT_t] \) and let the estimated parameter vector be

\[
\hat{\beta}(\lambda) = \frac{\sum_{t=\lambda_0}^{(1-\lambda_0)T} \Delta y_t X_t}{\sum_{t=\lambda_0}^{(1-\lambda_0)T} X_t' X_t}
\]

with variance \( \sigma_e^2 \left[ \sum_{t=\lambda_0}^{T} X_t' X_t \right]^{-1} \). After letting the estimated and the ‘true’ parameter vector respectively be specified as \( \hat{\beta}(\lambda) = [\hat{\mu}_1, \hat{\tau}_1, \hat{\mu}_2, \hat{\tau}_2] \) and \( \beta^* = [\mu_1^*, \tau_1^*, \mu_2^*, \tau_2^*] \), the scaling matrix of the rates of convergence of \( \hat{\beta}(\lambda) \) with respect to \( \beta^* \) is given by \( \Upsilon_t = diag\left[ T^{1/2}, T^{3/2}, T^{1/2}, T^{3/2} \right] \).

Then, by generating \( \Delta y_t \) according to eq. 1 we have, for \( 0 < \lambda < 1 \)

\[
\Upsilon_T \left[ \hat{\beta}(\lambda) - \beta^* \right] \xrightarrow{L} \Theta_T(\lambda)^{-1} \Psi_T(\lambda),
\]

whereby, for \( W(r) \) a standard Brownian motion in the plane \( r \in [0,1] \), the following limit expressions ensue:

\[
\Psi_T(\lambda) = \sigma \left[ W(1), W(1) - \int_0^1 W(r) dr, (1-\lambda)W(1), (1-\lambda) \left( W(1) - \int_0^1 W(r) dr \right) \right]
\]

and

\(^7\) The empirical distribution of the two simulated \( t \) statistics is a standard Normal with positive and negative occurrences entering with equal probability weights. Their variances and variance components are shown in Table 3.
\[
\theta_T(\lambda) = \begin{bmatrix}
1 & 1/2 & 1-\lambda & \frac{(1-\lambda)^2}{2} \\
1/3 & \frac{(1-\lambda^2)}{2} & \frac{(2 - 3\lambda + \lambda^3)}{6} \\
1-\lambda & \frac{(1-\lambda^2)}{2} & \frac{(1-\lambda^3)}{3}
\end{bmatrix}.
\]

From eq. 4 the limit distribution of the coefficient vector is the same as that reported by Perron and Zhu for Model IIb [2005, p. 81] (see the Appendix), while its asymptotic \(t\) statistics are computed as follows:

\[
t_T(\lambda) = \theta_T(\lambda)^{-1}\psi_T(\lambda)/(\Omega_T(\lambda))^{1/2}
\]

where \(\Omega_T(\lambda) = \sigma^2[I(\theta_T(\lambda))]^{-1}\) and I is the 4x4 identity matrix. The \(t\) statistics of the level break \(t_T(\lambda, L)\), and of the trend break \(t_T(\lambda, T)\), are thus

\[
t_T(\lambda, L) = 3\frac{\lambda W(1) - \int_0^1 W(r)dr}{[\lambda(1-\lambda)]^{1/2}}
\]

\[
t_T(\lambda, T) = 3^{1/2}\frac{\lambda(3\lambda - 1)W(1) - 2(2\lambda - 1)\int_0^1 W(r)dr}{[\lambda(1-\lambda)(3\lambda^2 - 3\lambda + 1)]^{1/2}}
\]

Table 1 shows the values taken by eqs. 8.1 and 8.2 for select \(\lambda\). While the coefficients attached to \(W(1)\) markedly rise when passing from low to higher magnitudes of \(\lambda\) in both equations, those of \(\int_0^1 W(r)dr\) decrease and then increase in absolute terms in the first equation and markedly fall by outweighing the effect of \(W(1)\) in the second.

---

\(8\) The values of eqs. 8.1 and 8.2 for the DGP of \(\Delta y\) originated by the null of a trending equation with drift given by the equation \(y = d + \delta t + e\), with noise \(e \sim N(0,1)\) are very similar, by construction, to those reported in Table 1, independent of the magnitudes of the drift and trend components.
The critical values of the $t$ statistics are obtained by Montecarlo simulation$^9$. For select magnitudes of $\lambda$ running from 0.10 to 0.90, and for different sample sizes ($T = 100, 200, 300, 500$), the finite-sample critical values of eqs. 8.1 and 8.2 are reported in Table 2. These are obtained after performing $N = 10,000$ draws ($N = 5,000$ for $T=500$) of the $T$-sized vector of artificial discrete realizations of $\Delta y_i$ of eq. 1. Each of these realizations is in turn given by the cumulative sum of 1,000 values of $\Delta y_i^* \sim NID\left(0,1/\sqrt{1,000}\right)$ with $y_i^* = 0$. Thereafter, the Brownian functionals of eq. 5 are approximated by such sums, which are independently and identically distributed, and eqs. 8.1 and 8.2 subsequently computed. Finally, the critical absolute values are obtained by finding the extremes falling in the 99%, 95% and 90% quantiles together with their 10% upper and lower confidence bands.

The reader will notice from Table 2 that the critical values are independent of the finite-sample size $T$ and that they achieve minimal absolutes at $\lambda=0.50$, exhibiting larger values at both ends of the selected $\lambda$. Finally, except for $\lambda=0.50$, $t_T(\lambda,L)$ is smaller than $t_T(\lambda,T)$ by a factor that reaches 1.2 at both ends.

In addition, the $N$-draws artificially computed $t$ statistics for given values of $\lambda$, considering positives and negatives, are normally distributed with zero mean and variance given by the squares of eqs. 8.1 and 8.2 with the numerators (excluding the integers) replaced by their respective standard error obtained by simulation estimation. These numerators are respectively denoted as $t_T(\lambda,L)_\text{num}$ and $t_T(\lambda,T)_\text{num}$, and are zero-mean Gaussian processes.

Incidentally, the $T$-length $N$-draws series $W(1)$ and $\int_0^1 W(r)dr$ pertaining to both numerators of eqs. 8.1 and 8.2, are both zero-mean I(0) Gaussian processes. However, independent of both $\lambda$ and $T$, the former exhibits unit variance and the latter a variance close to 1/3, being respectively distributed as a standard normal and as a doubly truncated normal distribution (with extremes close to 5% to 95%). These are the only constant variances, since all the others are strictly dependent on the magnitude of $\lambda$.

The results on the numerators and other statistics are reported in Table 3, where for ease of space only the $T=200$ sample case is considered. The variances of the estimated numerators of eqs. 8.1 and 8.2 achieve a minimal value at $\lambda=0.50$, being more than twofold for the first and more than tenfold for the second at both ends. $\lambda W(1)$ most obviously grows since it equals $\lambda^2$, and the same occurs to $\lambda(3\lambda - 1)W(1)$ since it equals $(\lambda(3\lambda - 1))^2$. The variance of the second component of the numerator of eq. 8.2 achieves a minimum of zero at $\lambda=0.50$ and rises at both ends. Similarly for the variances of the simulated $t$ statistics (shown in the last two columns of Table 3), which attain a minimal value in correspondence of $\lambda=0.50$.

---

$^9$ By construction, the squares of the two $t$ statistics, for given $\lambda$, correspond to their respective limit Wald-test statistics. As for the first of the two, see for instance Bai and Perron [2003a] and Perron [2005]. For both see Vogelsang [1999] although the simulation method adopted therein differs from that of the present paper.
where they share an almost equal value, and then increase by eight and ten times at both ends, respectively. Finally, the estimated variance of the first statistic is on average 40% smaller than the second, reflecting the similar albeit smaller gap in their critical values, as reported in Table 2.

Table 4 reports for \( T=200 \) (\( N = 10,000 \)) the values of the one-tailed size and power – evaluated at the 5% percent significance level – for testing the null hypotheses that the \( t \) statistics of eqs. 8.1 and 8.2 are equal to zero and to the standard \( t \)-distribution value of 1.96\(^{10}\). Size is always close to zero and \( \lambda \)-independent for the zero case alternative and well off the nominal value of 5% for the other alternative, especially for values of \( \lambda \) close to 0.50.

In other words, the probability of rejecting the first (second) null when it is true (Type I error) is very low (notably high, even more so for the statistic \( t_T(\lambda, L) \)). Instead, as shown by the power, the probability of rejecting both nulls (Type II error) when they are false is very high and is \( \lambda \)-independent. In essence, both of the \( t \) statistics of eqs. 8.1 and 8.2 are high-powered against both alternatives but share definitely high a size against the alternative of the \( t \) statistics being equal to 1.96\(^{11}\).

**Sect. 5. GMM formulation of a Dynamic Forward-Looking Taylor Rule.**

Let the endogenous variable be the EFFR, denoted as \( r_t \). Let also the regressors be the general price index, output, and a stock market index, respectively denoted as \( p_t \), \( y_t \) and \( s_t \), all in log levels. Finally, let the central bank operate by using information \( j \) steps ahead (\( j=1,\ldots,J \)), for \( J<T \). By virtue of this characteristic, the vector of regressors may be augmented by means of future values of \( r_t \)\(^{12}\), in order to ensure additional information. By consequence, the full vector of regressors may be written as \( Y_{t+j} = (y_{t+j}, p_{t+j}, s_{t+j}, r_{t+j}) \) and, to ensure stationarity, both the endogenous and exogenous variables may be expressed in first differences, respectively denoted as \( \Delta r_t \) and \( \Delta Y_{t+j} \), so that the following generic function ensues

\[
\Delta r_t = f \left[ \sum_{j=1}^{J} (\Delta Y_{t+j}) \right] + \text{noise}.
\]

The FLTR model thus becomes an extension of eqs. 2 and 9 and may be written

---

\(^{10}\) \( T=200 \) was chosen more or less randomly, simply because the sample of the actual data being used in the empirical part is close to that figure. Size and power results for other magnitudes of \( T \) are significantly no different.

\(^{11}\) Size tests of the alternatives that both \( t \)-statistics be equal to 3.0 and 4.0 are unreported for ease of space but exhibit probabilities higher than those of Table 4, and in the range of 0.30-0.50. This evidence confirms the zero-mean value taken by both statistics.

\(^{12}\) While the OFFR is definitely a control variable, the EFFR is an observable. Yet the latter, which historically differs from the former by a zero-mean \( IID \) disturbance, may be interpreted as the market interest-rate reaction to FOMC decisions and thus comfortably included as a regressor, especially and most interestingly if expressed in expectation terms.
out, for the time interval \( t \in [\lambda_0 T, (1 - \lambda_0)T] \), \( \forall \lambda \in \Lambda \), in the following way:

\[
\Delta r_i(\lambda) = \mu_1 + \mu_2 DU_i(\lambda) + \tau t + \tau_2 DT_i(\lambda) + \sum_{j=1}^{J} \Xi_j r Y_{t+j}(\lambda) + \phi \eta_{t-1}(\lambda) + e_i(\lambda)
\]

where \( \eta_{t-1} \) is the EC term, with \( E(\phi) < 0 \) if the null of cointegration holds true\(^{13}\), and the structural breaks together with the error term belong to the set \( f(.) \) of eq. 9. Finally, \( e_i(\lambda) = I.I.D.(0, \sigma^2_e) \).

Eq. 10, along the lines established in Sect. 4, enables constructing a time series of length \( 1 + (1 - \lambda_0)T \) of the coefficient vector \( \hat{\beta}(\lambda) = [\mu_1, \mu_2, \tau_1, \tau_2, \sum_{j=1}^{J} \Xi_j, \phi] \) and of the two \( t \) statistics \( \hat{t}_\mu(\lambda) \) and \( \hat{t}_\tau(\lambda) \).\(^{14}\) In particular, also the time series of the coefficient sums, up to lead \( J \) of \( \Xi_j \) for each regressor included in the vector \( \Delta Y_{t+j} \), can be obtained.

To obtain the GMM parameter vector \( \hat{\beta}_{GMM}(\lambda) \) in the given setting, some additional notation must be introduced. For the usual trimmed interval \( t = \lambda_0 T, \lambda_0 T + 1, \ldots, (1 - \lambda_0) T \), let now the vector of regressors included in eq. 10 be \( X_t = [1, t, DU_t, DT_t, \Delta Y_{t+j}, \eta_{t-1}] \) with size \( K \times 1 \). Finally, for \( h = 1, \ldots, H \) lags where \( J + 4 \leq H < T \), let there be a vector of size \( L \times 1 \) \((L \geq K)\) denoted as \( Z_t = [1, \Delta Y_{t-h}] \), which includes a constant and the selected stationary instruments, here expressed as the lags of the regressors.

By dropping for an instant the \( (\lambda) \) notation, the \( L \)-sized vector of sample moments evaluated at \( \hat{\beta} \) for the above-given time interval is

\[
g_t(\hat{\beta}) = \sum_{t=\lambda_0 T}^{(1-\lambda_0)T} Z_t \hat{e}_t
\]

where \( \hat{e}_t \) are the first-stage residuals of a (possibly) consistent Instrumental-Variable estimation of eq.10. The sample means of the above are

\[
\overline{g}(\hat{\beta}) = \left[(1 - \lambda_0)^{-1}T\right] g_t(\hat{\beta})
\]

\(^{13}\) The EC coefficient provides an estimate of the magnitude and sign of the speed of mean reversion of the endogenous variable. In fact, if the EC term is stationary, then the first differences of the cointegrated variables are all stationary and mean reverting. This is a typical aspect of efficient markets, e.g. exchange-rate and asset markets [Engle and Patton, 2000]. Inclusion of the EC coefficient, at least intuitively, also disposes of the drawback represented by smoothing.

\(^{14}\) This feature allows eq. 10 to belong to the class of partial structural change models as envisaged, for instance, by Bai and Perron [1998, 2003].
with the orthogonality property that $E\left[\mathbf{g}(\hat{\beta})\right] = 0$.

Let also the ensuing $L \times L$ weight matrix be

$$W(\hat{\beta}) = [(1 - \lambda_0)^{-1}] \sum_{t = \lambda_0}^{(1 - \lambda_0)T} g_t(\hat{\beta})g_t(\hat{\beta})'$$

then:

$$\hat{\beta}_{GMM} = \arg \min_{\beta \in \mathbf{B}} \left( \mathbf{g}(\hat{\beta})W(\hat{\beta})^{-1}\mathbf{g}(\hat{\beta}) \right)$$

Computation of the partial first derivatives of the sample moments, defined as $\partial g_t(\hat{\beta})/\partial \beta$, yields the $L \times K$ Jacobian matrix

$$G_t = [(1 - \lambda_0)^{-1}] \sum_{t = \lambda_0}^{(1 - \lambda_0)T} z_tx_t'$$

where $z_t, x_t$ respectively are the $L$.th and the $K$.th element, for any given $t$, of vectors $Z_t$ and $X_t$. Finally, the efficient GMM estimator, by letting $Z'y = \sum_{t = \lambda_0}^{(1 - \lambda_0)T} z_t\Delta y_t$ and after reintroducing the sequential ($\lambda$) notation, is

$$\hat{\beta}_{GMM}(\lambda) = \left(G_t'(\lambda)W(\hat{\beta})^{-1}G_t(\lambda)\right)^{-1}G_t'(\lambda)W(\hat{\beta})^{-1}Z'y(\lambda)$$

whose asymptotic variance is given by the ‘sandwich matrix’

$$\left[G_{(1 - \lambda_0)'}(\lambda)W(\hat{\beta})^{-1}G_{(1 - \lambda_0)}(\lambda)\right]^{-1}$$

If $Z_t$ is stationary, the mean value of $G_t(\lambda)$ is zero and the following holds: $W(\hat{\beta})^{-1} \overset{p}{\rightarrow} Q$, where $Q$ is a positive definite matrix whose maximum eigenvalue $E(Q)_{\text{max}}$ is such that $\lim_{T \rightarrow \infty} E(Q)_{\text{max}} = 0$ for a sufficiently large number of instruments.$^{15}$

The dynamic format taken by the FLTR in eq. 10 estimated by GMM is preferable to its traditional ‘level rule’ counterpart [Orphanides and Williams, 2005], used, e.g. by Taylor [1999] and Clarida et al. [1998, 2000], for the following reasons: i) the model perfectly suits the I(0) model of eq. 2, so that the estimated relevant $t$ statistics are easily comparable to their simulated critical values of Table 2;

$^{15}$ By means of some applied experimenting conducted with 10,000 Montecarlo draws and $T$ running from 100 to 400, after fixing the number of instruments $L=5,25,50$, the convergence rates of $E(Q)_{\text{max}}$ are found to be $T^{-1/3}, T^{-1/4}, T^{-1/2}$, respectively. This finding implies that $\lim_{L \rightarrow \infty, T \rightarrow \infty} E(Q)_{\text{max}} = 0$ and that, for $L/T \rightarrow 1$, the convergence rate of $E(Q)_{\text{max}}$ is $T^{-1/2}$. 

18
ii) the reaction-function coefficients are scale-free relative to the equation in levels, as the regressors in origin are differently indexed and may produce spurious coefficient results;

iii) the model automatically disposes, by including $\eta_{i-1}$, of $t$-statistic spuriousness deriving from cointegrated variables (e.g. Österholm, 2005b);

iv) GMM estimation automatically corrects for autocorrelation and heteroskedasticity of the error term by using the Heteroskedasticity and Autocorrelation Consistent (HAC) method [Newey and West, 1987; Andrews, 1991];

v) By accordingly selecting the optimal instrument vector, GMM disposes of parameter inconsistency deriving from error-in-variables estimation.

While the first two aspects are self explanatory, the third point implies the well known fact that nonstationary series, unless cointegrated [Choi, 1994; Choi et al., 2004], produce spurious coefficient $t$ statistics, error autocorrelation and a bloated $R^2$ [Granger and Newbold, 1974; Phillips, 1986]. Spuriousness is also found between series generated as independent stationary series with or without linear trends and with seasonality or structural breaks [Granger et al., 2001; Hassler, 2003; Kim et al., 2004; Noriega and Ventosa-Santaulària, 2005, 2006]. These occurrences are found within the context of Ordinary Least Squares (OLS) regressions where the $t$ statistics – in particular those of the deterministic components – diverge as the number of observations gets large.\(^{16}\)

In the context of IV regressions with a stationary endogenous variable, however, spuriousness of the coefficient $t$ statistics arises when many instruments and/or a large kernel bandwidth are used. This is the reason why the appropriate bandwidth and number of instruments must be chosen [Koenker and Machado, 1999; Hansen and West, 2002; Kiefer and Vogelsang, 2002]\(^{17}\).

6. The data base used and some pretesting of the GMM equation.

The key variables used in this paper are four: the EFFR, the rate of inflation of the overall consumer-price index, the total industrial index of capacity utilization and the Standard&Poor 500 equity index. The data sources are described below and the full sample used is the period of monthly observations spanning the period 1982:04-2006:12, for a total of 297 observations.

\(^{16}\) By means of the same applied experimenting, it is shown that in a standard OLS ($T=200$) model with a I(0) endogenous variable and $T \geq K \geq 1$ regressors, the $t$ statistics of the coefficients of the deterministic components, by departing from values below unity at $K=1$, diverge toward a value of 2.00 at a rate of $K^{1/6}$. With an I(1) endogenous variable, the same $t$ statistics depart at $K=1$ from values over 8.0 and 15.0 for the constant and the trend, respectively, and remain virtually unchanged with increasing $K$. Finally, the $t$ statistics of the coefficient sum of the regressors, with the endogenous variable either I(0) or I(1), diverge at the same rate as that of the deterministic components of the first model.

\(^{17}\) In a setting characterized by an I(0) endogenous variable, selection of the appropriate HAC bandwidth ($HB$) and number of instruments ($L$) is crucial, since large values of both give rise to spurious $t$ statistics of the regressor coefficients and of their higher-fractile values. In fact, by means of the same kind of applied experimenting as that of fns. 15 and 16, it is found that for $T=200$, three regressors (constant, trend and a stationary variable) and select $HB = 0,1,5,30$, the $t$ statistics grow respectively at rates $L^{1/5}$, $L^{1/4}$, $L^{1/3}$ and $L^{1/2}$ and in general $L^{1/2}$ for HB, $L \rightarrow T$.\(^{17}\)
Figures 1, 2 and 3 respectively illustrate the levels, the yearly and the monthly percent changes of the key variables, while their basic descriptive statistics are reported in Table 5. When measured in levels, the EFFR is shown to exhibit the highest volatility after the equity index, while capacity utilization is by far the most stable variable. However, when measured in terms of yearly percent changes, not to speak of monthly percent changes, the EFFR is by far the most volatile, a robust indicator of the Fed’s frenzy addressed at controlling the economy by means of the OFFR\textsuperscript{18}. Worth of notice, finally, is the volatility of the capacity utilization index, second in rank within the same percent changes considered.

Table 6 exhibits the ADF test statistic for UR of the key variables taken separately, the Gregory-Hansen (GH) test statistics for the null hypothesis of no cointegration of the EFFR with the other key variables under different regime shifts [Gregory and Hansen, 1996a; 1996b], and the Bai-Perron minimum Bayesian Information Criterion (BIC) detection method of multiple breaks for I(0) series [Bai and Perron, 2003]. The period covered, in consideration of the lag procedure adopted, is 1984:01-2006:12 and, obviously, the break dates reported are included within this time span.

The reader is warned of the low power exhibited by standard ADF testing in the presence of a level or trend shift, which forms the basic contention advanced by Perron [1989], and by GH testing in the presence of multiple breaks [Kejriwal, 2006]. This happens because the cointegrating relationship may exhibit several breaks overtime, a fact downplayed also by more recent research on the topic [Arai and Kurozumi, 2005]. Hence, the ADF and GH statistics reported in Table 6 are simply illustrative. As to the latter, for instance, cointegration is found for both levels and yearly first differences, and the break dates reported of 1991:09 and 1993:08 for the endogenous variable are of difficult interpretation from the historical and statistical viewpoints. Similar conclusions appear to be valid also for the break dates obtained with the BH testing procedure.

All that is necessary, however, is simply a confirmation of the need to first-difference the data to obtain I(0) series in order to avoid mixtures of I(0) and I(1) series, which customarily provide distorted coefficients [Banerjee et al., 1993]. Also, confirmation is needed about the existence of cointegration with at least one break and of possibly two breaks at least, even without cointegration. First differencing is thus essential from the purely statistical viewpoint as well as from the economic viewpoint. It is in fact proven that this procedure avoids the Fed’s misperceptions of the long-run natural rates of the policy variables emerging from use of ‘level rules’, and that it produces a rule that is robust to both kinds of uncertainty [Orphanides and Williams, 2005].

Given the previous information about the time-series properties of the key variables used, the original series are transformed to enable as much as possible achieving I(0) series. The endogenous variable $\Delta r_t$, and its expectation counterpart,

\textsuperscript{18} The tally of changes of the OFFR performed by the FOMC during the period 1990:01-2006:06 is 68 while, for the period 1983:01-1990:12 of indirect targeting the tally is 44. In total, 112 changes along a stretch of nearly a quarter century, that is, a mean of 5 yearly changes out of an average of 8 yearly FOMC meetings.
are expressed as $\log(1+r_t/100) - \log(1+r_{t-1}/100)$ on a monthly basis, while the exogenous variables are all log differences on a monthly basis of their original levels. To eliminate any remaining persistence in the series, which causes error autocorrelation and heteroskedasticity [Bai and Perron, 2000], the appropriate HAC correction is required.

The candidate maximum number of the monthly leads ($J$) of the regressors $\Delta Y_{t+j}$ and of the monthly lags ($H$) of the instruments set $\Delta Y_{t-h}$ is selected on the basis of the reputed informational horizon utilized by the Fed [Batini and Nelson, 2000]. The following maximum leads/lags ($J/H$) combinations appear to be the most likely: 6/24, 6/36, 6/48; 12/24, 12/36, 12/48; 18/24, 18/36, 18/48. Such choice is dictated by the principle that, while preserving the general criterion of GMM overidentifying restrictions, the candidate combinations appear reasonable in the face of alternatives that may reduce or exaggerate the Fed’s actual future and past information database.

Table 7 reports the mean values of a battery of statistical indicators that can be used for optimal $J/H$ selection within the given set of candidates. These indicators are obtained via different trial runs of eq. 10 for the select $J/H$ combinations for all values of $\lambda_0 \leq \lambda \leq (1-\lambda_0)$ with bandwidth equal to 119 and a trimming factor $\lambda_0 = .15$.

The indicator list of Table 7 is the following:
1) the standard $t$ statistic of parameter $\varphi$ of the EC term $\eta_{t-1}$, to evaluate the null hypothesis of no cointegration between the key variables expressed in levels;
2) the joint and separate F tests of the null hypothesis of both breaks being equal to zero;
3) the $p$-value of Hansen’s J significance test for overidentifying restrictions;
4) the value of the BIC to assess for the correctness of the leads selected;
5) the minimum eigenvalue of the inverse weight matrix $W(\hat{\beta})^{-1}$ to assess – in the manner suggested by Stock et al. [2002] – its magnitude and to test for the sample moments mean magnitude;
6) the minimum eigenvalue of the matrix $[G_t'(\lambda)W(\hat{\beta})^{-1}G_t(\lambda)]/L$, which is the cross product of the moments and their partial first derivatives, to similarly assess its magnitude and to test for the weakness of the instrument set;
7) the sum of the squared correlation coefficients between the regressor and the instrument set to further test for such weakness;
8) the minimum eigenvalue of the sandwich matrix $[G_{(1-\lambda_0)T}'(\lambda)W(\hat{\beta})^{-1}G_{(1-\lambda_0)T}(\lambda)]^{-1}$, to assess its magnitude and to test for the variance of the estimated parameter vector.

Simple eyeballing of Table 7 reveals that the optimal $J/H$ choice is 6/48, namely 6 leads and 48 lags, found to be the best representation of the informational horizon used by the Fed under the assumption of rationality. Technically, the selected

---

19 The HAC bandwidth selected represents a compromise between a value of zero, which could not correct for heteroskedasticity, and larger values which produces spuriousness of the $t$ statistics of the regressor coefficients, as shown in fn. 17.
combination outsmarts the other combinations by exhibiting very large $t$ and $F$ statistics that reject the respective nulls and the largest $p$-value of Hansen’s $J$ significance test for overidentifying restrictions, as well as the largest absolute value of the BIC statistic.

In addition, even more interesting in terms of selection criterion, are the results regarding the other tests. The 6/48 combination exhibits in fact by far the largest minimum eigenvalue of indicator 6, which significantly rejects the null hypothesis of weak instruments according to the appropriately tabulated critical values [Stock and Yogo, 2005] and the largest value of indicator 7, which also points to a comparatively stronger instruments. Finally, within the $J/H$ combinations that bear a lag of 48, the 6/48 combination stands out also by exhibiting the smallest mean moments (indicator 5) and the smallest variance (indicator 8).

7. Empirical Results of the Selected GMM Model.

The empirical results of eq. 10, estimated for the trimmed period 1984:01-2001:06 ($\lambda_0 = 0.15$) and with the selected combination $J/H = 6/48$, are exhibited in Figure 4 and in Table 8. Panels $a$ and $b$ show the time series, of length $1 + (1 - \lambda_0)T$, of the estimated $t$ statistics vis-à-vis the monthly changes of the EFFR, graphed as the discontinuous line. The other two panels show the time series of the coefficient sums of inflation, capacity utilization and the S&P 500 Index, as well as the corresponding Principal-Component Analysis (PCA) shares.

The coefficient sums represent the ‘policy bias’ of the components, namely, their respective impact upon changes of the EFFR, while the PCA shares represent the relative weights assigned to each component. Both are viewed as indicators of policy targeting by the Fed.

The following three subsections are devoted to the treatment in sequence of these results, together with some interesting additions concerning inflation scares, mismeasurements of the short-run Phillips curve and the “antispeculative” policy approach followed by the Fed.

7.1. The structural breaks.

The level and trend break time series, $\hat{t}_\mu(\lambda)$, and $\hat{t}_\tau(\lambda)$, respectively, share a cyclical if not jagged pattern with several troughs and peaks, evidenced by the shaded areas which represent the break periods where they exceed their respective absolute critical values shown in Table 2. The same is applicable, although with lower frequency, to the coefficient sums.

Table 8, where the top four ranking local extremes of the structural breaks are

---

[20] The pattern is mostly jagged, especially during some periods, as expected by the transient nature of both breaks in the presence of I(0) series. Yet, their slow overtime evolution pointing to the existence of break periods, and not simple episodes, is unquestionable (see Sect. 4).
reported, shows that, while $\hat{t}_\mu(\lambda)$ exhibits four maxima and two minima, $\hat{t}_\tau(\lambda)$ exhibits three maxima and three minima, all found to be significant by the standards supplied in Table 2. While the first two top positive level breaks occur during the Volcker chairmanship as a result of his anti-inflation stance, the other two occur in occasion of Greenspan’s OFFR hikes in response to increases of the output gap that had peaked in 1992 and in occasion of the ‘dotcom bubble’. These findings are almost in line with those of Bunzel and Enders [2005] and the second definitely marks the end of Greenspan’s purportedly ‘unconventional’ approach to the Phillips curve [Ball and Tchaidze, 2002; Rasche and Thornton, 2006].

The most significant negative level break occurs in 2000:12, which ignites the long spate of reductions of the OFFR that would last until mid-2004. The other break dates 1984:09, and characterizes Volcker’s first short-lived interest rate easing of the Eighties.

The first two top positive trend breaks occur in conjunction with their level counterparts occurred under Volcker, while the other trend breaks follow already established spates of rising or falling EFFR. Hence, trend breaks occur either synchronously or with lags with respect to level breaks and, apparently, do not affect but are affected by changes in the EFFR ($\Delta r$). To prove this, a test for the null hypothesis of no strict exogeneity is conducted with $\Delta r$ featuring as the endogenous variable in a distributed lead/lag (of length $P$) regression à la Sims [1972], with the two $t$ statistics as arguments, as follows

\[
11) \quad \Delta r_t = a + \sum_{p=-P}^{P} \theta_p \hat{t}_\mu(\lambda)_{t-p} + \sum_{p=-P}^{P} \vartheta_p \hat{t}_\tau(\lambda)_{t-p} + \varepsilon_t
\]

where $a$ is the constant term, and $\varepsilon_t$ is an IID disturbance. The parameters $\theta_p$ and $\vartheta_p$ refer to the arguments for $p \in [-P,P]$. After setting a broad range of $P$ from 1 to 36, the BIC optimally selects a lead of 2, whereby the null is rejected for $\hat{t}_\mu(\lambda)$, but not for $\hat{t}_\tau(\lambda)$, 21. In other words, the time series of the $t$ statistic for level breaks is strictly exogenous with respect to monthly changes in the EFFR.

Given this finding, a $\chi^2$-distributed Ljung-Box test of the null of no weak exogeneity is conducted between the two $t$ statistics themselves, to assess whether either of the two determines the other’s past course. The lead/lag range is set to length $P = 36$, and an interesting result emerges: $\hat{t}_\tau(\lambda)$ significantly lags $\hat{t}_\mu(\lambda)$, that is, the null is significantly rejected for the $t$ statistic of trend breaks, peaking between 10 and 20 lags 22.

---

21 The computed F(2,199) statistics for 2 leads respectively are 10.75 and 0.75 with significance levels equal to 0.00 and to 0.47. Use of the AIC, which selects a lead of 3, produces the computed F(3,195) statistics respectively of magnitude 7.77 and .89, with significance levels very close to the above.

22 The Ljung-Box statistic, distributed as $\chi^2(P)$ under the null of no exogeneity running from the $t$ statistic of trend breaks to the other $t$ statistic and viceversa,, with $P=36$ and significance levels in brackets is 672.75 (.00) and 23.69.
Both findings demonstrate that structural trend breaks affect past level breaks and that these, in turn, significantly anticipate with a short lead the FOMC decisions about the OFFR\textsuperscript{23}. This implies the following important policy conclusion: the Fed decides the course of the EFFR by means of the future expected level breaks after having experienced substantial trend break effects, which represent the carryover effect of past mistakes in prediction and estimation of the macroeconomic variables\textsuperscript{24}. It comes to no surprise, in this setting, the reason why – for the Greenspan’s period – the most significant trend breaks tend to offset contemporaneous level breaks, since the Fed is concerned to keep on track interest rates with respect to their long-run values.

7.2. The coefficient sums.

Panel $c$ of Figure 4 show the time series of the coefficient sums of inflation, capacity utilization\textsuperscript{25} and of the S&P 500 Index,

The top four ranking demeaned extremes of the coefficient sums are exhibited in Table 9 and definitively produce interesting results as to the conduct of the Fed’s policy during the period under scrutiny. The largest positively-signed anti-inflation bias dates 1987:04 under Volcker’s last months of chairmanship, an indication that “the back of price acceleration in the United States” had not yet been broken and that the usher to “a two-decade long decline in inflation that eventually brought us to the current state of price stability” [Greenspan, 2004] had not yet been wide open. The second largest positively-signed anti-inflation bias dates 2000:09, while the other two pertain to the first half of the Nineties. As to negative biases of this kind, worth of notice is the second largest, which is dated 2001:3 and marks the end of the ‘dotcom’ bubble struggle.

It is easy to compute at this point the amount of inflation scares [Orphanides and Williams, 2005a] and of the misinformation produced by the Fed on the actual behavior of the Phillips curve, both usually heralded to the general public via standard media like conferences, speeches, meetings, etc., all with the official imprint of the Board of Governors.

The corresponding series are obtained as a byproduct of the coefficient sums, which are known to the policymaker together with the breaks deriving from unstructured uncertainty, but are supposedly unknown to the general public. In other words, both series are tools of informational superiority in the hands of the Fed that can be exploited to fool private agents by igniting fears of an overheating economy,

\textsuperscript{(94)}, respectively. Another more brute, yet equally revealing indicator is provided by the sum of the squares of the correlation coefficients over the $P$ range. The sum respectively amounts to 161.14 and 10.35.

\textsuperscript{23} In support of this finding, PCA conducted on eq. 11 over the shares of both breaks reveals a 60% to 40% weight in favor of the level break.

\textsuperscript{24} Orphanides [1998] makes a case for this evidence by demonstrating, in particular, that prediction errors of the real-time data of the output gap made in the late Seventies have become evident to the Fed only several years after.

\textsuperscript{25} Surprising as it may seem, the sign of the capacity-utilization regressor is negative, contrary to that obtained in other empirical work and, obviously, to Taylor’s own policy prescriptions [Taylor ,1999]. Maybe this result is due to differencing, certainly it is consistent with viewing the TR as an IS reduced-form equation, a veritable Keynesian piece of theory indeed!
namely, of inflationary pressures that may derive from too low interest rates and excess production.

The inflation scare series consists of the squared differences between actual inflation and its coefficient sums, and is plotted in Figure 5, panel a. All local maxima are marked graphically as vertical lines. The two highest are centered under Volcker in 1987:03 and under Greenspan in 1997:11, but other sizable scares were administered by Greenspan during the years 1993 and 1999-2000, as already advanced in Sect. 2.

The misinformation series requires knowledge of the timely evolution of the coefficients of the Phillips curve. These are obtained by sequential structural-break estimation of a standard OLS inflation-output gap regression, and are exhibited in Figure 5, panel b. The shaded areas indicate existence of a ‘perverse’ Phillips curve, namely, a negative relationship tying its two arguments. The Fed’s misinformation of the Phillips curve consists of the series of the squared differences between the inflation coefficient sums and the Phillips curve coefficients, and is plotted in Figure 5, panel c.

Misinformation peaks in 1988:05 and indicates unjustified anti-inflationary bias manifested by the post-crash tightening imposed by Greenspan, in the presence of a weak economy, that would in fact tank only a few months later. In the following period, virtually until the late Nineties, the Phillips curve is ‘correct’ and so is the anti-inflation bias. Later, in coincidence with the last business cycle that straddles the millennium and which associates low inflation to high economic growth, the Fed’s misinformation sizably appears again in conjunction with the last two inflation scares.

As to the other coefficient sums, most of the demeaned values of the output gap (the ‘output-gap bias’) occur in association with sustained annual output growth rates. The largest positive-sign bias definitely belongs to the just mentioned struggle period and indicates, as is clear from the same sum regarding the S&P 500 Index, that production and stock prices were both targeted at the same time during that occasion. In other words, simultaneous excess (i.e. above mean) capacity utilization and stock prices had prompted interest rate tightening by the Fed which culminated in 2000:04 with the 0.75 bps hike that brought the OFFR to 6.50.

Only the second largest positive output-gap bias is associated with modest output growth, since it occurs in 1992:12, just at the end of a four-year recession and of interest rate easing commenced one year before. The other two (1996:11 and 1987:07), while characterized by sustained output growth, occur during stalling interest rates. Finally, the two largest negatively signed biases (1984:12 and 1988:03) occur at the end of growth cycles and in conjunction with falling interest rates.

The equity bias is provided by the demeaned coefficient sum of the S&P 500

26 A large variety of short-run Phillips curve specifications is adoptable. Quite reasonable – yet highly imperfect and naïve – appears to be the Distributed-Lags format, which endogeneizes inflation (excluding food and energy) with respect to preselect lags ($P$) of the output gap. Different specifications tried have led to similar results, and in any case to the same pattern, similar to that obtained by Atkeson and Ohanian [2001]. The select model includes $P=8$, where the lowest are the $p$-values of the standard null hypothesis. The associated mean ADF test statistic with no drift and trend is -3.48, indicating no significant residual autocorrelation.
Index. The most relevant positive occurs in 1984:12, toward the end of a pronounced stock-market crisis caused by double-digit EFFR values ignited by Volcker’s tightening strategy. This event – i.e. higher rates in face of rising stock prices – is replicated, albeit to a minor extent, by Greenspan in the aftermath of the 1987 Black Monday when the equity market had just began to redress (1988:11) and during the ‘dotcom’ bubble struggle mentioned above (2000:04).

The largest negative bias (1988:03) occurs soon after the Black Monday as the result of a policy move to avoid the kind of policy tightening already experienced after the Wall Street crash in 1929. A replication of this sound, yet belated, approach occurs also after the ‘dotcom’ bubble burst (2000:12), and the reverse thereof (1994:07) in a period of flat stock prices and stalling rates.

7.3. Principal Component Analysis.

PCA reveals from Panel d of Figure 4 that the Fed has been primarily targeting the equity market all the way through the period considered, by assigning to stock prices a mean relative weight of over 30% vis-à-vis means of 15% for inflation and for the EC component, and 13% for the output gap. Needless to say, the Fed has constantly kept an eye over the equity market by significantly sticking to the “antispeculative” policy approach, an old remembrance that dates back to late Twenties, on which more in Sect. 2.

Specifically, the equity market relative weight peaks close to 40% in occasion of the ‘dotcom’ bubble [Bordo and Wheelock, 2007; Rigobon and Sack, 2003], much in line with Cecchetti’s finding [2003] of significant concern of the Fed over the stock market behavior since after 1992 – while actually of much older date – as emerging from thorough reading and interpretation of the FOMC minutes and transcripts.

The rationale for the “antispeculative” policy approach by the Fed would be justified, under the assumption of the Fed’s rationality, by the volatility which oftentimes characterizes asset prices, whereby a preemptive strike on expected stock bubbling is institutionally mandated to the policy maker. However, inspection of Table 5 demonstrates that the equity index, as compared to the other key variables included in eq. 10, is the most volatile only in level terms while being outsmarted by monthly changes in both the EFFR and in the output gap.

By consequence, the “antispeculative” policy approach bears no justifying foundations whenever higher frequency transformation of the variables is considered. The series may be constructed by similar means used to produce the inflation scares and the misinformation series, and is given by the squared differences of the equity

---

27 PCA is conducted by standard eigenvalue method on the entire vector of regressors of eq.10, including all deterministic components but the constant term. The cross-moment matrix uses correlation coefficients to avoid undesired parameter bloating due to differently measured variables. The shares reported do not tally 1.0, in consideration of the fact that the mean shares of trend and of forward rate changes are both close to 9%, and the other shares (including breaks) are on average 5%. Interestingly enough, the relatively high mean share of the EC implies the existence of a sizable effect of mean reversion on the endogenous variable, maybe more than what customarily expected and definitely more appropriate than interest rate smoothing techniques.
PCAs and monthly volatility. The series is exhibited in Figure 5, panel d, which shows two sizable maxima in 1987:10 and 1998:08, not far from those found for the inflation scare series. A test of the null of no weak exogeneity between these two time series is provided by the $\chi^2$-distributed Ljung-Box test with the lead/lag range set to $P = 36$. There emerges that inflation scares significantly anticipate heavier relative weights placed on the equity market targeting, which inevitably produce stock market lows or even crashes.28

8. Conclusions.

In an effort to interpret the Fed’s reaction function parameters of the Taylor Rule, several steps have been treaded and tests performed to test for rationality, structural breaks and other features related to its empirical estimation.

At first, a nonstandard $t$-test statistic is introduced to detect multiple level and trend breaks of stationary series, by supplying theoretical and limit-distribution critical values obtained from Montecarlo experimentation. Secondly, the Taylor Rule is expressed as an estimable GMM model of an augmented dynamic forward-looking representation. The model is then applied to the Effective Federal Funds Rate with trimming, multiple breaks and reaction-function coefficients of the leads of inflation, the output gap and changes in the equity market, and estimated on a monthly basis for the period 1983-2001.

The aggressiveness of the Fed’s interest-rate policy, as measured by the sizable and frequent changes of the official Federal Funds Rate by the FOMC, should presuppose rationality and thus limited or absent structured uncertainty.

Unfortunately this is not the case. In fact, there exist several sources of violation of rationality. The first is provided by the frequent structural breaks, especially those in level, found to prevail during the late Volcker, and the early and late Greenspan chairmanships. The second source consists of the systematic attempts at fooling the public by means of inflation scares and misinformation about the actual behavior of the Phillips curve. Other sources – unjustified on the basis of current volatility standards – are represented by the anti-inflation bias and by excess equity targeting.

All in all, the U.S. monetary policy for the period under scrutiny, dominated by Greenspan’s chairmanship, has been characterized by substantial aggressiveness addressed at keeping in check output growth and stock market prices, and by exploiting informational superiority over the general public. Not necessarily this behavior has produced benefits to both the Fed’s credibility and to the overall economy. Improvement in techniques addressed at reducing structured uncertainty in the sense proposed by Hansen and Sargent [2003, 2004, 2007] is required.

28 The Ljung-Box statistic, distributed as $\chi^2(P)$ under the null of no exogeneity running from inflation scares and the antispeculative policy approach and viceversa, with $P=36$ and significance levels in brackets is 245.14 (.00) and 36.64 (.44), respectively.
Appendix.

Limit Distributions of the $t$ Statistics of a Break in Level and of a Break in Trend.

The elements of eq. 5, for $\varepsilon_i$ and $\sigma$ from $\Delta y_i^*$ given in the text (Sect. 4), are obtained as follows

$$T^{-1/2} \sum_{t=1}^{T} \varepsilon_i \to \sigma W(1), \quad T^{-1/2} \sum_{t=\lambda_0 T}^{(1-\lambda_0)T} \varepsilon_i \to \sigma(1-\lambda)W(1)$$

$$T^{-3/2} \sum_{t=1}^{T} t \varepsilon_i \to \sigma W(1) - \sigma \int_0^1 W(r) dr, \quad T^{-3/2} \sum_{t=\lambda_0 T}^{(1-\lambda_0)T} t \varepsilon_i \to \sigma(1-\lambda) \left( W(1) - \int_0^1 W(r) dr \right)$$

while $\sigma^{-1} T^{-1/2} \sum_{t=1}^{T} \varepsilon_i \to W(1)$ and $\sigma^{-1} T^{-3/2} \sum_{t=1}^{T} \Delta y_{t}^* \to \int_0^1 W(r) dr$ in eqs. 8.1 and 8.2.

These two Brownian functionals, for the draws run and for all values of $\lambda \in \Lambda$, are $I.I.D.(0,\nu)$, with $\nu$ finite variance. Hence, for $E(.)$ the expectation operator, if

$$E(W(1)) = E \left( \int_0^1 W(r) dr \right) = 0,$$

then

$$\lim_{T \to \infty} \left( \sigma^{-1} T^{-1/2} \sum_{t=1}^{T} \varepsilon_i \right) = 0 \quad \text{and} \quad \lim_{T \to \infty} \left( \sigma^{-1} T^{-3/2} \sum_{t=1}^{T} \Delta y_{t}^* \right) = 0.$$  

which implies that, independent of $\lambda$, the two functionals tend to be zero-mean as $T$ grows with different rates of convergence. In other words, the Central Limit Theorem applies independent of $\lambda$.

For the null model represented by eq. 1, suppose the alternative I(0) model with constant and trend and no breaks were given by

A.1) $\Delta y_i = \mu_t + \tau_t t + \varepsilon_i$.

so that, for $\varepsilon_i \sim I.I.D.(0,\sigma^2)$ the coefficients’ limit distributions are

$$T^{1/2}(\hat{\mu}_i - \mu_i^*) \sim N(0, 4\sigma^2) \quad \text{and} \quad T^{3/2}(\hat{\tau}_i - \tau_i^*) \sim N(0, 12\sigma^2),$$

while for the same null and for the alternative given by eq. 2, here replicated as follows:

A.2) $\Delta y_i(\lambda) = \mu_1 + \mu_2 DU_i(\lambda) + \tau_{1i} t + \tau_{2i} DT_i(\lambda) + \varepsilon_i(\lambda); \quad \forall \lambda \in \Lambda$
so that, for $\epsilon_i(\lambda) \sim I.I.D.(0, \sigma^2)$, the coefficients’ limit distributions [Perron and Zhu, 2004] are

$$T^{1/2}(\hat{\mu}_1 - \mu^*_1) \sim N(0, 4\sigma^2 / \lambda), \quad T^{3/2}(\hat{\tau}_1 - \tau^*_1) \sim N(0, 12\sigma^2 / \lambda^3),$$

$$T^{1/2}(\hat{\mu}_2 - \mu^*_2) \sim N\left(0, 4\sigma^2 / \lambda(1 - \lambda)\right) \text{ and } T^{3/2}(\hat{\tau}_2 - \tau^*_2) \sim N(0, 12\sigma^2 \Phi),$$

where $\Phi = (3\lambda^2 - 3\lambda + 1)/(1 - \lambda)^3\lambda^3$.

The variances of the coefficients’ limit distributions of eq. A.1 are lower than their break counterparts of eq. A.2 (those of $\hat{\mu}_2$ and $\hat{\tau}_2$) and by consequence their standard $t$ statistics must be lower than the nonstandard $t$ statistics derived from A.2. In addition, by construction, the latter symmetrically fall then rise for increasing values of $\lambda \in \Lambda$ and achieve their minimum at $\lambda = 0.50$, with expected values of eq. 8.1 slightly smaller than those of eq. 8.2.

The $t$ statistics of eq. A.1, respectively denoted as $t^*_T(L)$ and $t^*_T(T)$ are thus

A.1.1) \hspace{1cm} t^*_T(L) = -\left(\frac{\lambda W(1) - 3}{\lambda^{1/2}}\int_{0}^{1} W(r) dr\right)

A.1.2) \hspace{1cm} t^*_T(T) = 3^{1/2}\left(\frac{\lambda W(1) - 2}{\lambda^{1/2}}\int_{0}^{1} W(r) dr\right).

For both statistics to be asymptotically equal to the standard value of 1.96, the 95% fractile-values of $W(1)$ and $\int_{0}^{1} W(r) dr$ must respectively equal 7.31 and 3.09.

Of interest it is worth noticing that the $t$ statistics of the constant ($\mu_1$) and of the trend ($\tau_1$) terms of eq. A.2 (eq.2 in the text), respectively denoted as $t^*_T(\lambda, L)$ and $t^*_T(\lambda, T)$, are

A.2.1) \hspace{1cm} t^*_T(\lambda, L) = -\frac{\lambda W(1) - 3}{\lambda^{1/2}}\int_{0}^{1} W(r) dr

A.2.2) \hspace{1cm} t^*_T(\lambda, T) = 3^{1/2}\frac{\lambda W(1) - 2}{\lambda^{1/2}}\int_{0}^{1} W(r) dr
which correspond to those of eqs. A.1.1 and A.1.2, respectively, if \( \lambda = 1 \).

With a similar reasoning, if the alternative I(0) model were made of only the two breaks, i.e.

\[
A.3) \quad \Delta y_t(\lambda) = \mu_2 DU_t(\lambda) + \tau_2 DT_t(\lambda) + \varepsilon_t(\lambda)
\]

the resulting \( t \) statistics, respectively denoted as \( t^*_T(\lambda, L) \) and \( t^*_T(\lambda, T) \), \( t^*_T(\lambda, T) \), would be

\[
A.3.1) \quad t^*_T(\lambda, L) = \frac{(1 + 2\lambda)W(1) - 3\int_0^1 W(r)dr}{(1 - \lambda)^{1/2}}
\]

\[
A.3.2) \quad t^*_T(\lambda, T) = 3^{1/2} \frac{(1 + \lambda)W(1) - 2\int_0^1 W(r)dr}{(1 - \lambda)^{1/2}}
\]

which correspond to those of eqs. A.1.1 and A.1.2, respectively, if \( \lambda = 0 \).

If the disturbance \( \varepsilon_t \) in Eq. 2 is I(1) as in Perron and Zhu [2004], then eq. 6 is

\[
\Theta_T(\lambda) = \begin{bmatrix}
2\lambda/15 & -1/10 & -\lambda/30 & 1/10 \\
6/5\lambda & -1/10 & -6/5\lambda & \\
2/15 & 0 & \\
5\lambda(1 - \lambda)
\end{bmatrix}
\]

whereby the \( t \)-statistics of the breaks, the counterparts of eqs. 8.1 and 8.2, are given by the following

\[
A.4.1) \quad t_T(\lambda, L) = 30^{1/2} \frac{\lambda W(1) - \int_0^1 W(r)dr}{\lambda(\lambda - 1)}
\]

\[
A.4.2) \quad t_T(\lambda, T) = 30 \frac{\lambda(3\lambda - 1)W(1) - 2(2\lambda - 1)\int_0^1 W(r)dr}{\lambda(\lambda - 1)[30\lambda(1 - \lambda)]^{1/2}}
\]

which are, for same values of \( \lambda \), distinctively larger than their I(0) counterparts, reflecting the spuriousness of the equation they are derived from.
Data Sources.

4. **SP500**: "Standard & Poor 500 Index, Monthly Averages of Daily Closes, Bloomberg*.

References.


Kejriwal M., 2006, “Cointegration with Structural Breaks: An Application to the Feldstein-Horioka Puzzle”, Boston University, manuscript.


Perron P., 2005, "Dealing with Structural Breaks", Department of Economics, Boston University, manuscript.


Table 1.
Values of eqs. 8.1 and 8.2 for select magnitudes of $\lambda$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$t_T(\lambda, L)$</th>
<th>$t_T(\lambda, T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 0.20$</td>
<td>$1.50W(1) - 7.50 \int_0^1 W(r) dr$</td>
<td>$-0.48W(1) + 7.20 \int_0^1 W(r) dr$</td>
</tr>
<tr>
<td>$\lambda = 0.30$</td>
<td>$1.97W(1) - 6.55 \int_0^1 W(r) dr$</td>
<td>$-0.19W(1) + 4.95 \int_0^1 W(r) dr$</td>
</tr>
<tr>
<td>$\lambda = 0.40$</td>
<td>$2.45W(1) - 6.12 \int_0^1 W(r) dr$</td>
<td>$0.53W(1) + 2.67 \int_0^1 W(r) dr$</td>
</tr>
<tr>
<td>$\lambda = 0.50$</td>
<td>$3.00W(1) - 6.00 \int_0^1 W(r) dr$</td>
<td>$1.73W(1)$</td>
</tr>
<tr>
<td>$\lambda = 0.60$</td>
<td>$3.67W(1) - 6.12 \int_0^1 W(r) dr$</td>
<td>$3.20W(1) - 2.67 \int_0^1 W(r) dr$</td>
</tr>
<tr>
<td>$\lambda = 0.70$</td>
<td>$4.59W(1) - 6.55 \int_0^1 W(r) dr$</td>
<td>$4.77W(1) - 4.95 \int_0^1 W(r) dr$</td>
</tr>
<tr>
<td>$\lambda = 0.80$</td>
<td>$6.00W(1) - 7.50 \int_0^1 W(r) dr$</td>
<td>$6.71W(1) - 7.20 \int_0^1 W(r) dr$</td>
</tr>
</tbody>
</table>

$t_T(\lambda, L)$ is the $t$ statistics of a break in level and $t_T(\lambda, T)$ the $t$ statistics of a break in trend.
### Table 2.
Critical values (in boldface) and 10% confidence intervals of the $t$ statistic of a break in level $t_{p}(\lambda, L)$ and of the $t$ statistic of a break in trend $t_{p}(\lambda, T)$. 10,000 Montecarlo draws of eq.1 for each select sample size $T$ and break fractions $0.10 \leq \lambda \leq 0.90$.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda = 0.10$</th>
<th>$\lambda = 0.20$</th>
<th>$\lambda = 0.30$</th>
<th>$\lambda = 0.40$</th>
<th>$\lambda = 0.50$</th>
<th>$\lambda = 0.60$</th>
<th>$\lambda = 0.70$</th>
<th>$\lambda = 0.80$</th>
<th>$\lambda = 0.90$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{p}(\lambda, L)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_{p}(\lambda, T)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\lambda = 0.10$</th>
<th>$\lambda = 0.20$</th>
<th>$\lambda = 0.30$</th>
<th>$\lambda = 0.40$</th>
<th>$\lambda = 0.50$</th>
<th>$\lambda = 0.60$</th>
<th>$\lambda = 0.70$</th>
<th>$\lambda = 0.80$</th>
<th>$\lambda = 0.90$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{p}(\lambda, L)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_{p}(\lambda, T)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>4.283</td>
<td>4.045</td>
<td>3.807</td>
<td>3.149</td>
<td>2.911</td>
<td>2.673</td>
<td>2.549</td>
<td>2.312</td>
<td>2.074</td>
</tr>
<tr>
<td>200</td>
<td>4.280</td>
<td>4.053</td>
<td>3.826</td>
<td>3.132</td>
<td>2.905</td>
<td>2.678</td>
<td>2.482</td>
<td>2.255</td>
<td>2.029</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\lambda = 0.10$</th>
<th>$\lambda = 0.20$</th>
<th>$\lambda = 0.30$</th>
<th>$\lambda = 0.40$</th>
<th>$\lambda = 0.50$</th>
<th>$\lambda = 0.60$</th>
<th>$\lambda = 0.70$</th>
<th>$\lambda = 0.80$</th>
<th>$\lambda = 0.90$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{p}(\lambda, L)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>4.769</td>
<td>4.513</td>
<td>4.258</td>
<td>3.432</td>
<td>3.176</td>
<td>2.92</td>
<td>2.764</td>
<td>2.509</td>
<td>2.253</td>
</tr>
<tr>
<td>$t_{p}(\lambda, T)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>5.696</td>
<td>5.394</td>
<td>5.092</td>
<td>4.089</td>
<td>3.787</td>
<td>3.484</td>
<td>3.288</td>
<td>2.986</td>
<td>2.684</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\lambda = 0.10$</th>
<th>$\lambda = 0.20$</th>
<th>$\lambda = 0.30$</th>
<th>$\lambda = 0.40$</th>
<th>$\lambda = 0.50$</th>
<th>$\lambda = 0.60$</th>
<th>$\lambda = 0.70$</th>
<th>$\lambda = 0.80$</th>
<th>$\lambda = 0.90$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{p}(\lambda, L)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>8.028</td>
<td>7.610</td>
<td>7.192</td>
<td>5.664</td>
<td>5.245</td>
<td>4.827</td>
<td>4.551</td>
<td>4.133</td>
<td>3.715</td>
</tr>
<tr>
<td>$t_{p}(\lambda, T)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda )</td>
<td>( T_{tL} )</td>
<td>( T_{tT} )</td>
<td>( \lambda )</td>
<td>( T_{tL} )</td>
<td>( T_{tT} )</td>
<td>( \lambda )</td>
<td>( T_{tL} )</td>
<td>( T_{tT} )</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0.30</td>
<td>5.866</td>
<td>5.556</td>
<td>5.246</td>
<td>4.181</td>
<td>3.871</td>
<td>3.561</td>
<td>3.299</td>
<td>2.988</td>
<td>2.678</td>
</tr>
<tr>
<td>0.80</td>
<td>5.812</td>
<td>5.505</td>
<td>5.199</td>
<td>4.137</td>
<td>3.830</td>
<td>3.524</td>
<td>3.260</td>
<td>2.954</td>
<td>2.647</td>
</tr>
</tbody>
</table>
Table 3.

Variances of the \( t \) statistic of a break in level \( t_T (\lambda, L) \) and of the \( t \) statistic of a break in trend \( t_T (\lambda, T) \) and of their components. 10,000 Montecarlo draws of eq.1 for sample size \( T=200 \) and break fractions \( 0.10 \leq \lambda \leq 0.90 \).

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( t_T (\lambda, L)_{\text{num}} )</th>
<th>( t_T (\lambda, T)_{\text{num}} )</th>
<th>( \lambda W(1) )</th>
<th>( \lambda (3\lambda - 1)W(1) )</th>
<th>( 2(2\lambda - 1)\int_0^1 W(r)dr )</th>
<th>( t_T (\lambda, L) )</th>
<th>( t_T (\lambda, T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.25</td>
<td>0.76</td>
<td>0.01</td>
<td>0.00</td>
<td>0.86</td>
<td>24.85</td>
<td>34.76</td>
</tr>
<tr>
<td>0.20</td>
<td>0.18</td>
<td>0.40</td>
<td>0.04</td>
<td>0.01</td>
<td>0.48</td>
<td>10.08</td>
<td>14.45</td>
</tr>
<tr>
<td>0.30</td>
<td>0.13</td>
<td>0.19</td>
<td>0.09</td>
<td>0.00</td>
<td>0.21</td>
<td>5.32</td>
<td>7.29</td>
</tr>
<tr>
<td>0.40</td>
<td>0.10</td>
<td>0.09</td>
<td>0.16</td>
<td>0.00</td>
<td>0.05</td>
<td>3.65</td>
<td>4.15</td>
</tr>
<tr>
<td>0.50</td>
<td>0.09</td>
<td>0.06</td>
<td>0.25</td>
<td>0.01</td>
<td>0.00</td>
<td>3.23</td>
<td>3.15</td>
</tr>
<tr>
<td>0.60</td>
<td>0.10</td>
<td>0.09</td>
<td>0.36</td>
<td>0.23</td>
<td>0.05</td>
<td>3.52</td>
<td>4.05</td>
</tr>
<tr>
<td>0.70</td>
<td>0.13</td>
<td>0.19</td>
<td>0.49</td>
<td>0.60</td>
<td>0.21</td>
<td>5.33</td>
<td>7.21</td>
</tr>
<tr>
<td>0.80</td>
<td>0.18</td>
<td>0.40</td>
<td>0.64</td>
<td>1.26</td>
<td>0.48</td>
<td>9.89</td>
<td>14.12</td>
</tr>
<tr>
<td>0.90</td>
<td>0.25</td>
<td>0.76</td>
<td>0.81</td>
<td>2.35</td>
<td>0.86</td>
<td>24.94</td>
<td>34.79</td>
</tr>
</tbody>
</table>

\( t_T (\lambda, L)_{\text{num}} \) and \( t_T (\lambda, T)_{\text{num}} \) are the simulation estimated numerator of eq. 8.1 and 8.2, respectively. \( W(1) \) and \( \int_0^1 W(r)dr \) are defined in the text (Sect. 4) and bear overall constant variances equal to unity and to roughly 1/3, respectively. \( \lambda W(1) \) is the first term of the numerator of eq. 8.1, while the other two elements, \( \lambda (3\lambda - 1)W(1) \) and \( 2(2\lambda - 1)\int_0^1 W(r)dr \), are the components of the numerator of eq. 8.2.
Table 4.
Size and power of the $t$ statistic of a break in level $t_{\lambda}(\lambda, L)$ and of the $t$ statistic of a break in trend $t_{\lambda}(\lambda, T)$ to test for two null hypotheses.
10,000 Montecarlo draws of eq.1 for sample size $T=200$, nominal size=5% and break fractions $0.10 \leq \lambda \leq 0.90$.

<table>
<thead>
<tr>
<th>Lambda=0.10</th>
<th>Lambda=0.20</th>
<th>Lambda=0.30</th>
<th>Lambda=0.40</th>
<th>Lambda=0.50</th>
<th>Lambda=0.60</th>
<th>Lambda=0.70</th>
<th>Lambda=0.80</th>
<th>Lambda=0.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{\lambda}(\lambda, L)$</td>
<td>0.004</td>
<td>0.949</td>
<td>0.004</td>
<td>0.949</td>
<td>0.001</td>
<td>0.950</td>
<td>0.003</td>
<td>0.949</td>
</tr>
<tr>
<td>$t_{\lambda}(\lambda, T)$</td>
<td>0.003</td>
<td>0.951</td>
<td>0.004</td>
<td>0.951</td>
<td>0.005</td>
<td>0.949</td>
<td>0.005</td>
<td>0.948</td>
</tr>
</tbody>
</table>

Null hypothesis of $t_{\lambda}(\lambda, L) = 0.0$ and $t_{\lambda}(\lambda, T) = 0.0$.

| $t_{\lambda}(\lambda, L)$ | 0.152 | 0.979 | 0.232 | 0.988 | 0.306 | 0.994 | 0.337 | 0.996 | 0.372 | 0.997 | 0.352 | 0.996 | 0.305 | 0.994 | 0.238 | 0.989 | 0.156 | 0.980 |
| $t_{\lambda}(\lambda, T)$ | 0.138 | 0.977 | 0.203 | 0.986 | 0.265 | 0.991 | 0.335 | 0.996 | 0.372 | 0.997 | 0.334 | 0.996 | 0.269 | 0.991 | 0.203 | 0.985 | 0.132 | 0.976 |

Null hypothesis of $t_{\lambda}(\lambda, L) = 2.0$ and $t_{\lambda}(\lambda, T) = 2.0$. 45
Table 5*.


<table>
<thead>
<tr>
<th>Key variable</th>
<th>Mean</th>
<th>S.E.</th>
<th>Vol.</th>
<th>Maxdate</th>
<th>Max.</th>
<th>Mindate</th>
<th>Min.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Levels</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective FFR</td>
<td>5.44</td>
<td>2.48</td>
<td>45.55</td>
<td>1984:08</td>
<td>11.64</td>
<td>2003:12</td>
<td>0.98</td>
</tr>
<tr>
<td>Consumer Price Index</td>
<td>149.66</td>
<td>29.86</td>
<td>19.96</td>
<td>2006:08</td>
<td>203.70</td>
<td>1983:05</td>
<td>99.20</td>
</tr>
<tr>
<td>Capacity Utilization</td>
<td>80.70</td>
<td>2.81</td>
<td>3.48</td>
<td>1995:01</td>
<td>85.07</td>
<td>1983:05</td>
<td>73.48</td>
</tr>
<tr>
<td>Standard &amp; Poor 500</td>
<td>691.35</td>
<td>426.07</td>
<td>61.63</td>
<td>2000:08</td>
<td>1517.68</td>
<td>1984:05</td>
<td>150.55</td>
</tr>
<tr>
<td><strong>Yearly percent changes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective FFR</td>
<td>0.05</td>
<td>0.44</td>
<td>930.88</td>
<td>2005:05</td>
<td>2.00</td>
<td>2001:12</td>
<td>-0.72</td>
</tr>
<tr>
<td>Consumer Price Index</td>
<td>3.11</td>
<td>1.08</td>
<td>34.75</td>
<td>1990:10</td>
<td>6.38</td>
<td>2002:06</td>
<td>1.07</td>
</tr>
<tr>
<td>Capacity Utilization</td>
<td>0.56</td>
<td>3.23</td>
<td>573.96</td>
<td>1984:02</td>
<td>11.57</td>
<td>2001:11</td>
<td>-8.41</td>
</tr>
<tr>
<td>Standard &amp; Poor 500</td>
<td>11.39</td>
<td>15.76</td>
<td>138.30</td>
<td>1983:06</td>
<td>52.94</td>
<td>2001:09</td>
<td>-27.54</td>
</tr>
<tr>
<td><strong>Monthly percent changes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective FFR</td>
<td>0.00</td>
<td>0.05</td>
<td>NA</td>
<td>2004:07</td>
<td>0.22</td>
<td>2002:11</td>
<td>-0.23</td>
</tr>
<tr>
<td>Consumer Price Index</td>
<td>0.25</td>
<td>0.22</td>
<td>85.72</td>
<td>2005:09</td>
<td>1.22</td>
<td>2005:11</td>
<td>-0.65</td>
</tr>
<tr>
<td>Capacity Utilization</td>
<td>0.04</td>
<td>0.54</td>
<td>1292.99</td>
<td>1984:01</td>
<td>1.92</td>
<td>2005:09</td>
<td>-1.77</td>
</tr>
<tr>
<td>Standard &amp; Poor 500</td>
<td>0.85</td>
<td>4.22</td>
<td>495.63</td>
<td>1987:01</td>
<td>13.18</td>
<td>1987:10</td>
<td>-21.76</td>
</tr>
</tbody>
</table>

* FFR is the Federal Funds Rate. Mean is the arithmetic mean, S.E. the standard error, Vol. the volatility index, i.e. the normalized standard error (S.E./Mean). Maxdate and Mindate respectively are the monthly dates of the maximum occurrence (Max.) and of the minimum occurrence (Min.). NA is an indefinite number.
Augmented Dickey-Fuller (ADF)*, Gregory-Hansen test statistic for cointegration with break (GH)**, and Bai-Perron multiple breaks (BP) of the key variables in levels (1), in yearly differences (2), and in monthly differences (3). Period 1983:05-2006:12. In brackets the lags selected by the AIC method for ADF and BIC method for GH.

<table>
<thead>
<tr>
<th>Key variable</th>
<th>(1)</th>
<th>Key variable</th>
<th>(2)</th>
<th>Key variable</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ADF</td>
<td></td>
<td>ADF</td>
<td></td>
<td>ADF</td>
</tr>
<tr>
<td>Effective FFR</td>
<td>-1.12 (6)</td>
<td>Effective FFR</td>
<td>-3.06 (17)</td>
<td>Effective FFR</td>
<td>-9.96 (5)</td>
</tr>
<tr>
<td>Consumer Price Index</td>
<td>9.46 (14)</td>
<td>Consumer Price Index</td>
<td>-1.08 (15)</td>
<td>Consumer Price Index</td>
<td>-7.93 (14)</td>
</tr>
<tr>
<td>Capacity Utilization</td>
<td>0.39 (3)</td>
<td>Capacity Utilization</td>
<td>-3.13 (15)</td>
<td>Capacity Utilization</td>
<td>-7.54 (11)</td>
</tr>
<tr>
<td>Standard &amp; Poor 500</td>
<td>2.85 (0)</td>
<td>Standard &amp; Poor 500</td>
<td>-1.94 (12)</td>
<td>Standard &amp; Poor 500</td>
<td>-7.446 (11)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GH</th>
<th>-3.86 (2)</th>
<th>GH</th>
<th>-5.33 (0)</th>
<th>GH</th>
<th>-7.88 (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cointegration</td>
<td>YES</td>
<td>Cointegration</td>
<td>YES</td>
<td>Cointegration</td>
<td>NO</td>
</tr>
<tr>
<td>Break date 1</td>
<td>1991:09</td>
<td>Break date 1</td>
<td>1993:08</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Break date 2</td>
<td>1988:12</td>
<td>Break date 2</td>
<td>1990:02</td>
<td>Break date 1</td>
<td>1984:09</td>
</tr>
<tr>
<td>Break date 2</td>
<td>1994:07</td>
<td>Break date 2</td>
<td>1993:06</td>
<td>Break date 2</td>
<td>1984:12</td>
</tr>
</tbody>
</table>

* ADF test with no drift nor trend components. ** GH test with two regime changes.

For ease of space, the expression “levels” refers to log levels for all key variables except for the Effective FFR. The same rationale applies to the term “differences”.

Critical values of ADF: 1% = -2.58 5% = -1.95. Critical values of GH: 1% = -6.89 5% = -6.32.
Table 7.

Select regression statistical indicators for different leads/lags (J/H) combinations. Regression runs of eq.10 with 0.15 trimming. Period 1984:01-2001:06.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$ statistic of EC term</td>
<td>-4.334</td>
<td>-1.873</td>
<td>-0.682</td>
<td>-4.334</td>
<td>-2.792</td>
<td>-1.726</td>
<td>-8.512</td>
<td>-2.851</td>
<td>-2.626</td>
</tr>
<tr>
<td>F test (1,226) of null level break</td>
<td>10.225</td>
<td>3.500</td>
<td>0.973</td>
<td>10.225</td>
<td>6.657</td>
<td>1.672</td>
<td>34.395</td>
<td>9.996</td>
<td>4.760</td>
</tr>
<tr>
<td>F test (1,226) of null trend break</td>
<td>6.700</td>
<td>2.025</td>
<td>0.587</td>
<td>6.700</td>
<td>5.452</td>
<td>1.468</td>
<td>32.711</td>
<td>8.851</td>
<td>9.833</td>
</tr>
<tr>
<td>$p$-value of Hansen’s J statistic</td>
<td>0.965</td>
<td>0.780</td>
<td>0.882</td>
<td>0.965</td>
<td>0.935</td>
<td>0.825</td>
<td>1.000</td>
<td>0.988</td>
<td>0.846</td>
</tr>
<tr>
<td>Minimum eigenvalue of $W(\hat{\beta})^{-1}$</td>
<td>24.427</td>
<td>38.182</td>
<td>90.399</td>
<td>24.427</td>
<td>31.427</td>
<td>54.483</td>
<td>29.646</td>
<td>30.514</td>
<td>51.318</td>
</tr>
<tr>
<td>Minimum eigenvalue of $G_t(\lambda)W(\hat{\beta})^{-1}G_t(\lambda)$</td>
<td>1.984</td>
<td>0.348</td>
<td>0.044</td>
<td>1.984</td>
<td>1.303</td>
<td>0.236</td>
<td>25.171</td>
<td>5.077</td>
<td>1.505</td>
</tr>
<tr>
<td>Sum of correlated XZ</td>
<td>165.576</td>
<td>135.140</td>
<td>122.104</td>
<td>165.576</td>
<td>141.311</td>
<td>122.760</td>
<td>193.157</td>
<td>155.178</td>
<td>155.623</td>
</tr>
<tr>
<td>Minimum eigenvalue of $\left[ G_{t-\lambda}\lambda W(\hat{\beta})^{-1}G_{t-\lambda}\lambda \right]^{-1}$</td>
<td>166.058</td>
<td>177.281</td>
<td>78.741</td>
<td>166.058</td>
<td>109.850</td>
<td>121.064</td>
<td>95.648</td>
<td>110.853</td>
<td>94.912</td>
</tr>
</tbody>
</table>

48
Table 8 #.

The four major maxima and minima of the $t$ statistics of structural breaks by rank of magnitude. Regression run of eq.10 with 6 leads of the regressors and 48 lags of the instruments ($J/H = 6/48$) and 0.15 trimming. Period 1984:01-2001:06.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Variable</th>
<th>Max.</th>
<th>Coeff. sum</th>
<th>$\lambda$</th>
<th>Min.</th>
<th>Coeff. sum</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\hat{t}_n(\lambda)$</td>
<td>1985:01</td>
<td>15.87 ♦</td>
<td>0.06</td>
<td>2000:12</td>
<td>-11.77 ♦</td>
<td>0.93</td>
</tr>
<tr>
<td>2.</td>
<td>$\hat{t}_n(\lambda)$</td>
<td>1987:04</td>
<td>11.04 ♦</td>
<td>0.18</td>
<td>1984:09</td>
<td>-9.05 ♦</td>
<td>0.04</td>
</tr>
<tr>
<td>3.</td>
<td>$\hat{t}_n(\lambda)$</td>
<td>1993:04</td>
<td>10.41 ♦</td>
<td>0.51</td>
<td>1989:06</td>
<td>-1.59</td>
<td>0.30</td>
</tr>
<tr>
<td>4.</td>
<td>$\hat{t}_n(\lambda)$</td>
<td>1999:06</td>
<td>8.26 ♦</td>
<td>0.85</td>
<td>1991:10</td>
<td>-1.34</td>
<td>0.43</td>
</tr>
<tr>
<td>1.</td>
<td>$\hat{i}_t(\lambda)$</td>
<td>1985:01</td>
<td>19.80 ♦</td>
<td>0.06</td>
<td>1999:06</td>
<td>-11.18 ♦</td>
<td>0.85</td>
</tr>
<tr>
<td>2.</td>
<td>$\hat{i}_t(\lambda)$</td>
<td>1987:04</td>
<td>9.23 ♦</td>
<td>0.18</td>
<td>1995:09</td>
<td>-5.80 †</td>
<td>0.64</td>
</tr>
<tr>
<td>3.</td>
<td>$\hat{i}_t(\lambda)$</td>
<td>2000:12</td>
<td>6.68 ♦</td>
<td>0.93</td>
<td>1993:08</td>
<td>-3.38 †</td>
<td>0.53</td>
</tr>
<tr>
<td>4.</td>
<td>$\hat{i}_t(\lambda)$</td>
<td>1991:02</td>
<td>1.07</td>
<td>0.39</td>
<td>1994:08</td>
<td>-0.69</td>
<td>0.03</td>
</tr>
</tbody>
</table>

# $\hat{t}_n(\lambda)$ and $\hat{i}_t(\lambda)$ are the time series of the breaks in level and in trend, respectively. Max. and Min. are the dates at which the extremes occur. $\lambda$ is the fraction of the period considered at which the break occurs. † Significant at 95%, ♦ Significant at 99%.

Table 9 ‡

The four major maxima and minima by rank of magnitude of the demeaned coefficient sums of inflation, output gap and changes of the Standard & Poor 500 Index. Regression run of eq.10 with 6 leads of the regressors and 48 lags of the instruments ($J/H = 6/48$) and 0.15 trimming. Period 1984:01-2001:06.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Variable</th>
<th>Max.</th>
<th>Coeff. sum</th>
<th>$\lambda$</th>
<th>Min.</th>
<th>Coeff. sum</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Inflation</td>
<td>1987:04</td>
<td>0.084</td>
<td>0.18</td>
<td>1984:02</td>
<td>-0.077</td>
<td>0.00</td>
</tr>
<tr>
<td>2.</td>
<td>Inflation</td>
<td>2000:09</td>
<td>0.025</td>
<td>0.92</td>
<td>2001:03</td>
<td>-0.021</td>
<td>0.94</td>
</tr>
<tr>
<td>3.</td>
<td>Inflation</td>
<td>1991:02</td>
<td>0.022</td>
<td>0.39</td>
<td>1994:05</td>
<td>-0.020</td>
<td>0.57</td>
</tr>
<tr>
<td>4.</td>
<td>Inflation</td>
<td>1995:09</td>
<td>0.020</td>
<td>0.64</td>
<td>1990:06</td>
<td>-0.006</td>
<td>0.35</td>
</tr>
<tr>
<td>1.</td>
<td>Output gap</td>
<td>2000:04</td>
<td>0.029</td>
<td>0.89</td>
<td>1984:12</td>
<td>-0.082</td>
<td>0.05</td>
</tr>
<tr>
<td>2.</td>
<td>Output gap</td>
<td>1992:12</td>
<td>0.015</td>
<td>0.49</td>
<td>1988:03</td>
<td>-0.008</td>
<td>0.23</td>
</tr>
<tr>
<td>3.</td>
<td>Output gap</td>
<td>1996:11</td>
<td>0.008</td>
<td>0.71</td>
<td>1991:10</td>
<td>-0.004</td>
<td>0.43</td>
</tr>
<tr>
<td>4.</td>
<td>Output gap</td>
<td>1987:07</td>
<td>0.008</td>
<td>0.19</td>
<td>1995:09</td>
<td>0.001</td>
<td>0.64</td>
</tr>
<tr>
<td>1.</td>
<td>S&amp;P 500</td>
<td>1984:12</td>
<td>0.007</td>
<td>0.05</td>
<td>1988:03</td>
<td>-0.003</td>
<td>0.23</td>
</tr>
<tr>
<td>2.</td>
<td>S&amp;P 500</td>
<td>2000:04</td>
<td>0.002</td>
<td>0.89</td>
<td>1994:07</td>
<td>-0.003</td>
<td>0.58</td>
</tr>
<tr>
<td>3.</td>
<td>S&amp;P 500</td>
<td>1988:11</td>
<td>0.001</td>
<td>0.27</td>
<td>2000:12</td>
<td>-0.002</td>
<td>0.93</td>
</tr>
<tr>
<td>4.</td>
<td>S&amp;P 500</td>
<td>1992:01</td>
<td>-0.000</td>
<td>0.44</td>
<td>1997:10</td>
<td>-0.001</td>
<td>0.76</td>
</tr>
</tbody>
</table>

‡ Coeff. sum is the coefficient sum, and S&P 500 stands for monthly changes of the Standard & Poor 500 Index. Max. and Min. are the dates and $\lambda$ is the fraction of the period considered at which the extremes occur.
FIGURE 1.
Levels of key variables: 1982:04–2006:12

- Effective Federal Funds Rate
- Consumer Price Index
- Industrial capacity utilization
- Standard & Poor 500
Figure 2:

- Effective Federal Funds Rate
- Consumer Price Index
- Industrial capacity utilization
- Standard & Poor 500
FIGURE 4.

Panel a) t statistic of break in level and monthly changes of the Federal Funds Rate. (Shaded areas represent critical t statistic)

-1.5 -1.0 -0.5 0.0 0.5 1.0

Panel b) t statistic of break in trend and changes of the Federal Funds Rate. (Shaded areas represent critical t statistic)

-15 -10 -5 0 5 10 15 20

Panel c) Coefficient sums of inflation, output gap and Standard & Poor 500.

-0.24 -0.16 -0.08 0.00 0.08 0.16 0.24 0.32

Panel d) Shares of inflation, output gap and Standard & Poor 500.

0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40

FIGURE 4.
FIGURE 5. Inflation scares, Phillips curve and antispeculative policy approach. 1984:01-2001:06.
Panel a) Inflation scares.
Panel b) Phillips curve coefficients.
Panel c) Mismeasurement of the Phillips curve.
Panel d) Antispeculative policy approach.