A critical perspective on heterodox production modeling

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Abstract

Production and distribution needs a proper place in heterodox economics. It has recently been suggested that the construction of production models needs to be empirically grounded. Also it has been stated that empirically grounded production models must be circular production models. This argument then marginalizes the contributions of important economists in heterodox thought. The paper will argue that heterodox production models need not be perfectly circular to make important contributions for heterodox production theory. Furthermore, it will be argued that models which consist of elements of hierarchial structures of production put emphasis on out of equilibrium traverse processes and historical time.

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1 Economics as a Social Provisioning Process

Heterodox economics is a collection of diverse themes. Given the heterogeneity of the field, heterodox economists seem to hold steadfast to these principle tenets: 1) the economic system must be viewed as a non-ergodic historical process, 2) fundamental uncertainty is significant, 3) questions of production and distribution are important, 4) the workings of economic life do not take place in a vacuum, and 5) the economic process is a social process with social classes and political institutions. Heterodox theory stems from a diverse collection of economists with differing and sometimes competing viewpoints, however a survey of the heterodox literature seems to show that these five tenets remain fundamental.[1] One of the principle objectives of heterodox economics is to provide a foundation for a clear understanding of the economic system as a system of social relationships, as well as to describe (and sometimes model) these social relationships and it's processes. This foundation is then applied toward real economic problems.

The methodology of current heterodox theory both stems from, and reflects, the classical tradition of Marx; the contributions of institutionalist theory; the macroeconomics of Keynes, Kalecki, and their Cambridge followers; and the surplus approach of Leontief and later Sraffa.

Heterodox microeconomic theory develops from a grounded theory approach to theory creation (Lee, 2005). Grounded theory is simply the "discovery of the theory from data (Finch, 2002, 215)." Heterodox microeconomic theory is empirically grounded from its construction; and it consists of modeling the economy as a whole and the study of three core areas which include the business enterpriser, the market, and market governance (Lee, 2005). All three areas of focus are of equal importance and interrelated. For present purposes our analysis will be restricted to the business enterpriser, and specifically the modeling of the social relationships of production.

The study of the business enterpriser in heterodox microeconomic theory developed from the institutionalist approach of Veblen (1904). Veblen defines the business enterpriser as "the directing force of the industrial system" and the industrial system is the "material framework of modern civilization (*ibid.*, 1)." The aim of inquiry into the business enterpriser for Veblen was to: "[fully show] in what manner the business methods and business principles, in conjunction with the mechanical industry, influence the modern cultural situation (*ibid.*, 21)." The business method is the combination of mechanical inputs and labor directed towards the acquisition of pecuniary gain for businesses. The business principles of capitalists are pecuniary principles. The wages paid to laborers are simply a means to that end.

> "It might be feasible to set up a theory to the effect that wages are competitively proportioned to the vendibility of the product; but there is no cogent ground for saying that wages in any department or industry, under a business regime, are proportioned to the utility which the output has to any one else than the employer who sells it (Veblen,

[1] See for example: Holt, 2007; , Arestis, 1996; ; Lawson, 1994; and Hamouda and Harcourt, 1990.

1904, 62-63 fn. 1)"

Veblen's theory of the business enterpriser further echoes Marx in this respect that the capitalist process is a class driven, exploitative process, towards the interests of the capitalist class. Clearly Veblen's interest in the business enterpriser is an analysis of both the industrial structure of production, and, more importantly, the motivations of capitalists whose decision guide production. Current heterodox microeconomic theory includes Veblen's theory of the business enterpriser as part of its core content (Lee, 2005). Specifically heterodox microeconomics conceives the business enterpriser as a:

> "... non-static, historically changing going concern, that is, as an entity that has an indefinite life span and which undertakes production, employment, pricing, and investment in this context. It is more than simply a collection of productive resources ... [i]t is also an organization that is structured and contains casual mechanisms that direct these activities. It is within these structures and causal mechanisms (which change over time) that these resources are utilized in various activities in a changing economic environment (Lee, 2005, 7)."

The business enterpriser is seen as a constantly evolving, social process, in historical time that plays a fundamental role in the final distribution of the social surplus. The step forward for heterodox microeconomic theory is to adequately model this process. Input output modeling has become the preferred methodology for heterodox microeconomics modeling (Lee, 2010, 2005, 1998; Eichner, 1991; Arestis, 1996).

Input output models are grounded in reality. From its origination, input output models were used to combine economic facts with economic theory (Leontief, 1987, 1986b). The modeling of the system in an input output framework employs a non-reductionist, circular approach. It perfectly conforms to both the work of Veblen and current heterodox theory by detailing the physical structure of production, and the casual mechanisms which reflect the decisions of capitalists. As such, input output modeling defines the division of labor in the economic system, sets out to determine the exchanges required for reproduction, and sets the requirements for capital accumulation and economic growth. The structural model defines the technical and social framework required for sustainability. The production structure is determined by defined sets of technical and social relationships. These relationships in turn dictate the level of production, price formation, the level of employment, and the level of consumption. The economic system defined by the input output model of production provides an alternative, uniquely heterodox, understanding of the economy and economic processes which are empirically grounded (Bortis, 1990, 64-69).

Through the development of the input output technique, Leontief set out a direct factual study of the structural properties of production and distribution. Input output models are constructed from data that can be directly observed for a particular economic area; normally nations, states, and other smaller, but strictly defined regions (Miller and Blair, 2009; Miernyk, 1969; Leontief, 1987, 1986b).

The balance between total output and combined inputs is described by make and use tables and can be generalized as:

$$\mathbf{A} \times \mathbf{Q} + \mathbf{Y} = \mathbf{Q} \tag{1}$$

Where $\mathbf{I}$ is an $(n \times n)$ identity matrix, $\mathbf{A}$ is an $(n \times n)$ interindustry matrix of technical coefficients, $\mathbf{Q}$ is a $(n \times 1)$ unknown quantity column vector, and $\mathbf{Y}$ is $(n \times 1)$ column vector, representing exogenously determined final demand. Solving equation 1 for $\mathbf{Y}$ yields:

$$\mathbf{(I\ \text{-}A)} \times \mathbf{Q} = \mathbf{Y} \tag{2}$$

The $\mathbf{A}$ matrix can be partitioned allowing labor requirements to be isolated to emphasize the level of employment for a given level of aggregate output. Let $\mathbf{L}$ represent aggregate employment, and $\mathbf{l}$ is a $n \times 1$ row vector of labor coefficients. Now aggregate employment can be defined as:

$$\mathbf{l} \times (\mathbf{I} - \mathbf{A})^{-1} \times \mathbf{Y} = L \tag{3}$$

Analogous to this, the pricing model can be demonstrated, where the column vector $\mathbf{V}$ is a $(n \times 1)$ vector of exogenously given values added, which in the present case does not consist of any fixed capital stocks. In this case, the value added in each industry is equal to the amount paid out to wages. Lastly the price vector p_i is unknown. Thus:

$$\mathbf{P} \times \mathbf{A} + \mathbf{V} = \mathbf{P} \tag{4}$$

The solutions for $\mathbf{Q}$ and $\mathbf{P}$[2] are represented by:

$$(\mathbf{I} - \mathbf{A})^{-1} \times \mathbf{Y} = \mathbf{Q} \tag{5}$$

$$\mathbf{V} \times (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{P} \tag{6}$$

The Leontief inverse $(\mathbf{I} - \mathbf{A})^{-1}$ is the matrix multiplier. Final demand is related to output in a very Keynesian fashion. A growth in final demand for consumption and/or investment goods ($\mathbf{Y}$) must be met by a growth in output ($\mathbf{Q}$), proportioned by the inverse of the technology matrix $(\mathbf{I} - \mathbf{A})^{-1}$. I.e.:[3]

$$(\mathbf{I} - \mathbf{A})^{-1} \times \Delta\mathbf{Y} = \Delta\mathbf{Q} \tag{7}$$

[2] The conditions that guarantee positive solutions for quantities $\mathbf{Q}$ and prices $\mathbf{P}$ is that $\mathbf{A}$ is a positive semidefinite, nonsingular matrix with a maximum eigenvalue less than one. (Pasinetti, 1977, 266-267).

[3] See Leontief, 1986, 19-41; Miller et al. 2009, 10-71;

The quantity of total employment[4], L, is given by:

$$L = a_n q_1 + a_n q_2 + \ldots + a_n q_{n-1} + y_n \tag{8}$$

This only reflects the level of employment, it does not suggest that the economy is at full employment. Let L_{max} = total labor force in the economy. The economy is only at full employment if and only if:

$$L_{max} = L \tag{9}$$

From equation 9 it may be noted that if $L_{max} > L$, unemployment exists by the difference $L_{max} - L$. Even if unemployed workers are willing to work at the going wage rate, there is still no desire for firms to hire because all that is being supplied is being sold either as interindustry inputs or as final demand. This relationship is seen in Equation 8 which demonstrates total employment is equal to total output in the economy, which is ultimately driven by the level of final demand of goods (both consumption goods and investment goods) and services (Leontief, 1986b; Pasinetti, 1977; Kurz and Salvadori, 1995).

The effective demand condition in Leontief's model is that the value added component (the sum of wages and profits) must be equivalent to the value of final demand of the economy and it is not necessary for this to correspond to the full employment level.

The input output model effectively outlines the social relationships of production and distribution. "The system reacts to the absence of the information the market cannot provide by creating uncertainty-reducing institutions: wage contracts, debt contracts, supply agreements, administered pricing (Kregel, 1980, 46). The economic system is not portrayed as an ergodic process, but a 'cumulatively unfolding process' (Arestis, 1992, 114). 'Business principles', and 'business method' (Veblen, 1904) are built into the framework and highlight the 'precariousness' of 'conventions' (Keynes, 1936). In this respect, the input output models provide a proper framework for economic analysis in both the institutionalist and Post Keynesian framework. Further, input output models adequately model social relationships of production and distribution and class divisions and class conflict are built into the models, all of which are empirically grounded. Thus it appears on the surface that input output models to be the preferred approach of heterodox microeconomic analysis (Lee, 2010, 2005, 1998; Arestis, 1996; Eichner, 1991).

[4]Labor is assumed to be homogenous

2 Input Output Modeling: A Critique

It seems that input output models are perfectly suitable for modeling production and distribution in the heterodox tradition. However there are some fundamental problems. An initial problem is that one of the tenets of heterodox theory is that the economy is a non-ergodic process that needs to be analyzed in historical time that flows in a unidirectional cumulative process. The theory of the business enterpriser as defined in heterodox microeconomics is “a non-static, historically changing going concern” (Lee, 2005, 7). The problem is that input output models use a static model to study a non-static process. Input output models are a snap shop of an economy at a given time. Historical time is only considered in an input output in this respect.

One way to get around this is to model the economy using a dynamic input output modeling process. However it will be seen that the dynamic input output model has serious problems as well for heterodox economists. First the dynamic input output model will be introduced and then the flaws of this approach will be addressed.

In the dynamic Leontief model all production processes must make use of stocks of input factors. The stock input is not used up in the production process as are the flow inputs represented in the **A** matrix. Although stocks are not ‘used up’ the dynamic input output model may allow for normal deterioration as the stock is being utilized.

2.1 The Dynamic Input Output Model

Leontief (1986b) introduces the capital stock **B** matrix[5] which describes the capital structure of the economy for a given set of technologies. Essentially the **B** matrix lies in the background of the interindustry **A** matrix, which is used to describe the flows of inputs, including labor, *given the current state of technology*, i.e. the **B** matrix. The interindustry **A** matrix must be a functional relationship of the capital stock matrix, **B**. Technological change is represented by one or more changes in the capital structure of the economy, i.e. changes in the **B** matrix. The coefficients of the **B** matrix are not fixed, rather changes to the **B** matrix then cause a related change to the flow matrix **A**. The capital stock matrix is depicted in the matrix in equation 10.

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1,(n-1)} \\ b_{21} & b_{22} & \dots & b_{2,(n-1)} \\ \vdots & \vdots & \vdots & \vdots \\ b_{(n-1)1} & b_{(n-1)2} & \dots & b_{(n-1),(n-1)} \end{bmatrix} \tag{10}$$

The structure of the **B** matrix of capital coefficients is similar to that of the **A** matrix. It is of rank (n-1). The n^{th} row and column of the technology matrix are all zeros in an open Leontief model. According to Leontief (1951), the capital coefficient b_{ij} is defined as:

> [T]he technologically determined stock of the particular kinds of goods–machine tools, industrial buildings, "working inventories", of primary or intermediate materials–produced

[5]The **B** matrix has the same properties as the **A** matrix. It is an nonsingular, positive semidefinite matrix, with a maximum eigenvalue less than one. This last assumption concerning the **B** matrix is of particular importance. A maximum eigenvalue less than one ensures stability. Furthermore, a maximum eigenvalue less than one also assumes a surplus (Kurz and Salvadori, 1995, 111). Meaning there is not full utilization of capital goods and in terms of productive capacity the system is always elastic. The reason why the economic system does not operate at full capacity is because of the unemployment of labor. With excess capacity, the problem of instability is solved. Arguments against the stability of the dynamic Leontief model has been presented by Jorgenson (1960). Jorgenson proves that the dynamic Leontief model will result in unstable prices and quantities. This is because Jorgenson (1960a, 1960b) begins with the neoclassical assumption that "…(1) all output levels are at capacity and there are no excessive or deficient holdings of stocks; [and] (2) the output of each industry is equal to demands for current consumption and for investment in the expansion of capacity; there are no excess demands or excess supplies for commodities in the economy; (Jorgenson, 1960, 421)." Stability is important in dynamic input-output models because instability leads to price and quantity solutions which tends towards infinity. "If the dynamic input-output system is not macro-economically stable, the economic interpretation of the model cannot be retained (Jorgenson, 1960, 422)." If this was the case, the dynamic Leontief input-output model would not be useful to explain actual economic dynamics. However while Jorgenson's conclusion of the importance of stability is correct, the reasoning which causes the dynamic Leontief model to be unstable is based upon the neoclassical theory of full employment and full utilization. If this was not the case, and excess capacity were allowed to exist (as shown by Leontief (1953)) the dynamic Leontief model is shown to be stable. Steindl (1952) argued that investment decisions were based upon a planned degree of capacity utilization. Planned degree of capacity utilization is less than full utilization to allow for unexpected increases in final output. In *The Economics of Imperfect Competition* (1969) Joan Robinson makes a similar argument as Steindl in her introduction: "Imperfect competition came in to explain the fact, in the world around us, that more or less plants were working part time …firms could work their plants at less than full capacity and still earn a profit (p. vi)." The business enterpriser can increase output without changes to the capital stock because capital operates on varying levels of capacity, thus allowing for system stability.

> by industry i that industry j has to employ per unit of it's output. In other words, each column of matrix **B** describes the physical capital requirements (per unit of it's total output) of a particular industry, *in the same way that the corresponding column of matrix **A** describes it's "current inputs" requirements* (my emphasis) (Leontief, 1986b, 30)

The Leontief system describes the necessary capital stocks and input flows which are required for any given level of aggregate demand. Historical time now can be considered within a dynamic framework. It is unlikely that investment goods produced in the current time period will be available for production within the same time period. To consider the time element, it is assumed that investment made in the current period will not be available for production until the consecutive time period. Following this assumption, from the definitions just given, the input requirements for any level of final demand are:

$$\mathbf{Q}_t - \mathbf{A}_t\mathbf{Q}_t - \mathbf{B}_{(t+1)}(\mathbf{Q}_{(t+1)} - \mathbf{Q}_t) = \mathbf{Y}_t \tag{11}$$

The left hand side of Equation 11[6] represents the current input requirements for all industries in year t, as well as the investment in capital required from year t to year $t+1$ to expand capacity from $\mathbf{Q}_t$ to $\mathbf{Q}_{(t+1)}$.

2.2 Problems of Dynamic Input Output Models

The dynamic input output model now incorporates historical time. However there are some fundamental problems with the dynamic model for heterodox theory. Of greatest concern is that investment is endogenous in dynamic input output models. The models do not incorporate decision making of businesses regarding investment activity. This is a clear drawback of the dynamic Leontief model as it is a clear violation of both institutionalist and Keynesian principles. Investment decisions are not endogenous in heterodox theory. Rather, they are based upon fundamental uncertainty. Stemming from Keynes, investment is guided by the state of confidence of capitalists. Further, Keynes emphasized the many variables, social, political, and economic variables, which influence capitalists decisions when undertaking investment. Because of these violations, the dynamic input output model fails as a methodology to modeling the economy as a social provisioning process with institutions, and institutional change which influence decision making.

2.3 What to do now?

The way around these problems seems to be to revert back to the static input output model to model the dynamic economy. The argument is that historical time can be considered by studying multiple

[6] The **B** matrix does not consist of full capacity utilization. There will always be some planned level of unutilized capacity which allow businesses to respond to changes in demand. This then also resolves issues concerning the stability of the **B** matrix. Now that the **B** matrix consists of investment goods in the dynamic Leontief model, the final demand vector, **Y** now consists of only consumption and government goods.

input output tables (in other words looking at many snap shots) over time. Although with this approach the dynamics of the process are not caught. Another problem with studying many snapshots over time is that the analysis will always be backward looking. But this is insufficient for looking forward.

One attempt for the static input output model to be forward looking is through the Leontief matrix multiplier. The matrix multiplier describes the intermediate input requirements necessary for any level of final demand. Given the inclusion of intermediate demand in the multiplier process, the multiplier does incorporate capacity conditions required to meet demand requirements. The use of the multiplier is simply a description of necessary input requirements. The problem here is that the multiplier does not assume that the capacity of additional inputs are readily available to increase demand.

Heterodox microeconomic theory takes a leap forward by suggesting that input requirements will always be met following an increase in final demand (through fiscal policy or otherwise). This leap forward is justified through both theoretical and empirical evidence arguing that businesses operating in an oligopolistic competitive environment will always have on hand reserve capacity to accommodate varying levels of final demand. But we must be careful here. Simply suggesting, even if based upon empirical evidence, that businesses operate with reserve capacity is not the same thing as saying that the reserve capacity will always be sufficient to accommodate any given level of final demand (say for example a level of final demand consistent with full employment). However by only focusing on demand constraints, heterodox microeconomics is suggesting that demand creates its own supply, without ever acknowledging potential capacity constraints.

The problem is that perfectly circular models, like the input output model, does not sufficiently model structural bottlenecks on the supply side. Potential capacity constraints mean that the multiplier process will not fully work itself out, and will be lower in reality than theoretically estimated.

2.4 Structural Bottlenecks and Traverse Analysis

Through introducing structural bottlenecks the economy now faces a new set of conditions. There is the effective demand condition, but also the condition of the inappropriateness of the capital stock to adjust to new, higher levels of effective demand (Hagemann, 1990, 144). When the existing fixed capital stock (which is inherited from an earlier time period) is insufficient to respond to a higher rate of growth from an increase in final demand constitutes an important structural barrier. Heterodox theory, to be complete, needs to provide an analysis of both effective demand and structural conditions which identify potential bottlenecks (Forstater, 2002, 2000; Gehrke and Hagemann, 1996; Hagemann, 1990; Lowe, 1976; Pasinetti, 1981). The current development of heterodox microeconomics ignores the problem of structural bottlenecks altogether. Further by utilizing static input output models as a means to study business cycles, input output models fail to capture both capacity and demand constraints in a static system.

> The business cycle problem is no approach *to* but a reproach *against* a static system, since it is an antinomic problem in it. It is soluble only in a system in which the polarity of upswing and crises arises analytically from the conditions of the system, just as the undisturbed adjustment derives from the conditions of the static system. Who wants to solve the business-cycle problem must sacrifice the static system. Who adheres to the static system must abandon the business-cycle problem (Lowe, 1997, 250)

Lowe stated the problem in very clear terms. The consequences of a disturbing factor, such as an exogenous increase in final demand, can not be emphasized in a static system. The problem does not only lie in static input output models, but rather in input output models themselves. As a circular model the system does not incorporate linear stages of production which are necessary for considering capacity constraints. The linear stages of production are founded on earlier Austrian models of Böhm-Bawerk, and his Austrian followers which contends that all finished goods can be traced back to labor and land, and to treat fixed capital as intermediate stages of production (Hicks, 1973). It was Piero Sraffa's 1960 criticism of earlier Austrian models that generally fixed capital can not be traced back to dated quantities of labor (Hagemann, 1990). There is equal criticism of Hicksian vertical integration methodology of neo-Austrian processes put forth and utilized by Post Keynesian economists[7](Trigg and Lee, 2005). The critique is that the neo-Austrian model reducing everything back to labor, and the system is interconnected through final demand. Such an approach to modeling does not represent the system of production and distribution as a set of social relationships.

Given the backlash of linear production models, heterodox theory fails to recognize the importance of the hierarchy of production in modeling a dynamic economy. The hierarchy of production is not recognized in heterodox microeconomics because of the assumption that all firms operate with some normal level of reserve capacity. But once again, it is unwise to assume that a normal level of reserve capacity is sufficient to meet any increase in effective demand. This is especially true with the Post Keynesian emphasis for fiscal policy as a means towards full employment. Is reserve capacity in every sector sufficient to meet the a full employment level of demand brought about by Keynesian policies without any capacity constraints? If the answer is yes, then capacity constraints do not need to be considered. However this is probably unrealistic. Heterodox theory cannot leave out the possibility that it may take real time for additional supply inputs to be available to accommodate an increase in demand brought about by Keynesian policies. The use of the matrix multiplier simply assumes that intermediate inputs can increase without any discussion of how these inputs actually increase.

2.5 Traverse Analysis

Traverse analysis accomplishes just that. It is the study of the movement of the economic system from one growth rate to another. The emphasis here is on the *movement* of the system. This can

[7]The development and utilization of vertically integrated production model stems from Pasinetti 2007; 1993; 1981; 1980.

be caused by changes in level or composition of the labor force, changes in the availability of natural resources, and to changes in the technological structure of production. Traverse analysis is an analysis of both supply and demand conditions for economic growth in historical time. Thus traverse analysis fills the gap that exists in the matrix multiplier used in heterodox theory.

Traverse analysis stems from Hicks (1965) and Lowe (1976) who both set out to construct a detailed analysis of the disequilibrium path through which the economy moves. Hicks' (1965) initial attempt of traverse analysis began as a Marxian-type (or Sraffian) two sector model with one capital good. However later in life, Hicks moved away from the 'classical' traverse, to studying the disequilibrium path in a more neoclassical (neo-Austrian) setting. Hicks' (1973) neo-Austrian model highlights the stages of production to turn original material into final consumer goods. Given the nature of neo-Austrian models, Hicks fails to adequately treat the nature of fixed capital.

The traverse process models both structural bottlenecks and final demand. Given constraints in the supply of intermediate inputs to production, input output analysis comes up short. In order to account for the reproduction and expansion of the economy in the face of structural bottlenecks the Austrian concept of linear production models must be supplemented with the circular flow of the actual economy as represented by input output models (Lowe, 1987, 1976).

> For the production of coal, iron is required; for the production of iron, coal is required; no one can say whether the coal industry or the iron industry is earlier or later in the hierarchy of production (Dorfman et al., 1987, 205)"

The critique of Dorfman, Samuelson and Solow becomes relevant when the assumption that reserve capacity is sufficient to meet any increase in the rate of growth *is relaxed*. The Post-Classical structural model of Adolph Lowe takes into consideration both the circularity of production as in the input-output approach, and the transformation of inputs through successive stages of production. Lowe's model is appropriate for highlighting the bottlenecks in the production process. One of the central concerns is the consideration of historical time. The economy is depicted as always moving in a uni-directional path in historical time.

Lowe's model, unlike Marx's and Sraffa's framework, is a three sector model which combines transferability with specificity (Hagemann, 1992, 236). For the continuity of production there are two conditions which must be fulfilled. Labor and equipment have to operate within a technically defined, predetermined manner and is combined with stocks of original natural resources[8] and natural resources transformed into working capital. The second condition which must be fulfilled is that the stocks of equipment, natural resources, and labor which have been used up in production have to be

[8]Lowe does not make a clear distinction whether natural resources should be taken as given, or whether natural resources are produced means of production. Both interpretations can be inferred, at one point Lowe refers to natural resources being the land which factories and other durable capital equipment rest, other times he refers to natural resources undergoing stages of transformation. In the latter case natural resources must then be replenished and can be thought of as a produced means of production. This latter interpretation is more consistent with the heterodox viewpoint of natural resources.

replenished at the end of the period (Lowe, 1976, 27). This assumption of full capacity on the supply side allows for supply constraints to be considered. Lowe's structural model is given in equations 12 - 14. The complete model contains sets of casual relationships. Stocks of capital (K_i), combined with stocks of labor (L_i) and stocks of natural resources (N_i) to produce (indicated by a right arrow) stocks of output (o_i) in sector i. The capital letters indicate stocks, whereas lower case letters indicate flows. The symbol $\oplus$ indicates that the inputs of capital, labor, and natural resources are combined in fixed proportions (Lowe, 1976, 28).

The model consists of three horizontal stages, and within each sector four successive stages, indicated by subscripts in which a given stock of natural resources is transformed simultaneously by stocks of labor and capital into a final consumption good (Lowe, 1976, 28).

$$K_1 \oplus L_1 \oplus N_1 \rightarrow o_1 \qquad \textbf{Sub-Sector 1a:} \tag{12}$$

Four-Stage Process - Sub-Sector 1a

1. $k_{11} \oplus l_{11} \oplus n_{11} \rightarrow o_{11}$ (Extractive Machinery)
2. $k_{12} \oplus l_{12} \oplus n_{12} \oplus o_{11} \rightarrow o_{12}$ (Blast Furnaces)
3. $k_{13} \oplus l_{13} \oplus n_{13} \oplus o_{12} \rightarrow o_{13}$ (Steel Mills)
4. $k_{14} \oplus l_{14} \oplus n_{14} \oplus o_{13} \rightarrow o_{14}$ (Machine Tools/Steel Mills/Blast Furnaces/Ext. Machinery)

$$K_2 \oplus L_2 \oplus N_2 \rightarrow o_2 \qquad \textbf{Sub-Sector 1b:} \tag{13}$$

Four-Stage Process - Sub-Sector 1b

1. $k_{21} \oplus l_{21} \oplus n_{21} \rightarrow o_{21}$ (Ore)
2. $k_{22} \oplus l_{22} \oplus n_{22} \oplus o_{21} \rightarrow o_{22}$ (*Pig Iron*)
3. $k_{23} \oplus l_{23} \oplus n_{23} \oplus o_{22} \rightarrow o_{23}$ (Steel)
4. $k_{24} \oplus l_{24} \oplus n_{24} \oplus o_{23} \rightarrow o_{24}$ (Gin/Spindles/Looms/Sewing Machines)

$$K_3 \oplus L_3 \oplus N_3 \rightarrow o_3 \qquad \textbf{Sector 2:} \tag{14}$$

Four-Stage Process - Sector 2

1. $k_{31} \oplus l_{31} \oplus n_{31} \rightarrow o_{31}$ (Cotton)
2. $k_{32} \oplus l_{32} \oplus n_{32} \oplus o_{31} \rightarrow o_{32}$ (Yarn)
3. $k_{33} \oplus l_{33} \oplus n_{33} \oplus o_{32} \rightarrow o_{33}$ (Cloth)
4. $k_{34} \oplus l_{34} \oplus n_{34} \oplus o_{33} \rightarrow o_{34}$ (Dress)

The order of production is vertically divided between two sectors, the equipment goods sector (Sector 1) and the consumer goods sector (Sector 2). The consumer goods sector consists of a finished consumer good only, while the equipment goods sector is subdivided into two sectors, 1a and 1b. Sector 1a is circular, it provides working capital to be used back in it's own sector (for the replacement of fixed capital used up) and applied towards sector 1b, whereas sector 1b supplied working capital for sector 2 only. The subdivision of sector 1 implies that the output produced in sector 1a and 1b are qualitatively different. Once production occurs stocks of resources get worn out and need to be replaced. Lowe (1976, 31-36) defines k_{ij}, n_{ij}, and l_{ij}, as flows. The magnitude of which is the amount of fixed capital, labor, and natural resources which is "used up" in the production process.

The example which Lowe gives to bring clarity to his model is the production of a simple dress, which Lowe defines as the final consumer good. The process begins with sector 1a. Within sector 1a consists of four vertical stages producing output in a sequential process. The first stage in sector one combines capital, labor, and natural resources (defined as flows) to produce extractive machinery (o_{11}). Extractive machinery from stage 1 is then used as an input in stage 2, to be combined with capital, labor, and natural resources, to produce blast furnaces (o_{12}). The output from stage 2, blast furnaces, are transferred to the third stage, combined with capital, labor, and natural resources, to produce steel mills (o_{13}). Steel mills are then used in stage 4 to produce machine tools (o_{14}).

The final working output from sector 1a, machine tools, serve a dual purpose in Lowe's production model. First, after production occurred the fixed capital which was worn out needs to be replaced. Meaning the capital inputs, k_{11}, k_{21}, k_{31}, k_{41} are entirely used up in production. Thus, $k_{11} + k_{21} + k_{31} + k_{41}$ units of machine tools (the output from stage four) is transferred back into Sector 1a among the four successive stages respectively allowing for reproduction in sector 1a.

Output in sector 1a is also used in sector 1b. The working output from sector 1a, stage 1, extractive machinery is used in *both* sector 1a, stage 2, *and* replaces fixed capital used in sector 1b, stage 1. In other words the working capital in sector 1b, stage 1, (k_{21}) is extractive machinery. Likewise the working capital in stage 2 of sector 1b, k_{22} are blast furnaces, (k_{23}) are steel mills, and (k_{24}) are machine tools.

In clearer terms, sector 1b can be read as: Extractive machinery (k_{21}) produced from an earlier stage of production, is combined with labor and other natural resources (i.e. physical land) to produce ore (o_{21}). Blast furnaces ((k_{22}), from an earlier stage of production, combined with labor and other natural resources transform the ore into pig iron (o_{22}). Then steel mills (k_{24}) combined with labor, natural resources, and the pig iron is used to produce steel (o_{23}). Machine tools (k_{24}) combines with labor, natural resources, and the steel to produce intermediate outputs (gin/spindles,looms,sewing machines). Recall machine tools were also used to replace worn out machines in sector 1a. Sector 1a is a circular sector which uses it's own output for inputs and transfers a portion of its output to sector 1b. For the system to remain viable the output of machine tools from sector 1a must be equal to $k_{11} + k_{21} + k_{31} + k_{41} + k_{24}$.

Now turning to the final sector the intermediate output from sector 1b must equal to the fixed capital used up in the production process of sector 2. In other words, $o_{24} = k_{31} + k_{32} + k_{33} + k_{34}$. In sector 2, gin (k_{31}) combines with labor and natural resources to produce cotton (o_{31}). Spindles (k_{32}) combines with labor, natural resources, and cotton to produce yarn (o_{32}). Looms (k_{33}) combines with labor, natural resources and yarn to produce cloth (o_{33}). Sewing machines (k_{34}) combines with labor, natural resources and cloth to produce a dress (o_{34}), the final consumption good.

The stage model depicts a roundabout method of production which can be traced back to the Austrian school of Boehm-Bawerk (Lowe, 1976, 23). However Lowe's model is eclectic. Lowe felt that the Austrian school, made important contributions as to the linear process of production which is not evident in the circular models of Sraffa or Leontief. However the Austrian framework is limited by ignoring circular production. Lowe's model uses both. Putting focus on sector 1b, a sewing machine is one of the final equipment outputs. To produce a sewing machine steel is required as an input, but the converse is not true. To produce steel one needs to produce iron first. However to produce iron ore is required. So there is a specific sequential stage of production. But the concept of roundaboutness is lost because there is also circularity. To produce a sewing machine requires *both* steel and machine tools (from sector 1a). To produce steel requires both steel mills *and* pig iron. But to produce the steel mills in sector 1a required machine tools (produced at a later stage) and blast furnaces. The production of blast furnaces required extractive machinery, but for the production of all output at each of the four stages of sector 1a also requires machine tools as an input. Machine tools are also required as an input in Sector 2, stage 4. Thus machine tools is the crucial circular factor in Lowe's model. Because of the way Lowe designed the model, when the system is operating at full capacity, for the expansion of output first requires the expansion of output of sector 1a, before any other sector can expand output.

2.6 Lowe's Model in Action

Here it will be demonstrated how the Lowe model incorporates both capacity constraints and the effective demand condition of Keynesian economics. Lowe begins as a starting point with the assumption that all non-labor inputs are fully utilized and labor is fully employed (Lowe, 1976, 38). Lowe began this way because his attempt was to show what economic decisions that need to be taken to move from a lower level of production (Lowe's initial state) to an increase of production which accommodates an positive change in final demand (for example because of expansionary fiscal policies).[9] If the economy were not growing at all then the fixed capital worn out in production (denoted by k_{ij}) needs to be replaced at the rate of depreciation of capital equipment. (Thus under this assumption k_{ij} is equal to $K_{ij}d$ where K_{ij} is the stock of fixed capital in sector i, stage j, and

[9] In Lowe's analysis, which we will follow here, Lowe assumes a one-time exogenous increase to the labor force. The problem Lowe investigates is a modeling of economic decisions which need to be taken by both consumers and capitalists, to bring the economy from a previous state of full production and full employment, to an expanded state which will employ the addition to the labor force.

d is the rate of depreciation of the fixed capital.) Now assume that there is growth in the labor supply of the economy. Given the structural relationships described above, for this is to occur first requires an increase in the level of production in Sub-sector 1a. But, because there is full utilization of capital equipment, this can only occur if there is a fall in the aggregate supply of consumer goods. However, this can only occur through a reduction of aggregate demand in period 1. This in turn requires an increase in the level of voluntary or involuntary savings of consumers. In Keynesian terms this requires a decrease in the MPC and an increase in the marginal propensity to save (MPS). Now in period 2, an increase in the MPS (either voluntary or involuntary) results in labor being displaced from production of consumer goods, causing aggregate demand to fall further. A decrease in demand for consumer goods in turn frees up output from Sub-sector 1b. Also demand from Sub-sector 1a by Sub-sector 1b is also reduced in period 2. However these workers are not displaced. Given the twofold transferability of output in Sector 1 expansion is possible and allows the economy to return to full employment. Part of the original stock of extractive machinery, blast furnaces, steel mills, and machine tools which were in period 1 to be used in the production of intermediate capital goods in Sector 1b are now free in period 2 to produce additional extractive machinery, blast furnaces, steel mills, and machine tools. Furthermore, a proportion of working capital in Sub-sector 1b (steel pig iron and ore) that was used in period 1 to produce consumption goods in Sector 2, are now not needed in Sector 2 given an decrease in the demand for consumption goods. These intermediate goods in Sub-sector 1b may now be shifted back into Sector 1a for additional production of extractive machinery, blast furnaces, steel mills, and machine tools. Now, a greater amount of working capital from Sub-sector 1a can be transferred to Sector 1b, which now allows for employment to increase in sector 1b, and then these goods can be transferred to Sector 2, increasing employment in this Sector, and increasing the production of the consumption goods. The traverse to a higher growth rate given an exogenous shock to labor participation is a four-phase process which requires real time to complete. There first must be a change in inputs before there is a change in outputs. The first phase of the adjustment process is that the proportion of output from Sub-sector 1b which delivered to Sub-sector 1a must first be increased relative to that which is deliver to Sector 2. The second phase is that aggregate output in Sub-sector 1a is increased. The third phase is that an increase in output from Sector 1a is delivered to Sector 1b, and output in Sub-sector 1b is increased. Now, the forth phase of the traverse allows for consumption output to adjust to the higher rate of growth and employ the unemployed labor to produce those additional consumption goods (Lowe, 1976; Hagemann, 1992). The dual-nature of working capital between Sub-sector 1a and Sub-sector 1b allows an increase in the speed at which adjustment to a higher rate of growth is possible.

2.7 Rectifying Lowe's Model with Today's Framework

Lowe's three sector/four stage model is not presented in a manner that is consistent with the previous frameworks considered. However Lowe's three-sector model can follow a framework similar to that of Leontief and Sraffa. Following the framework of the previous models considered, let a_{ij} be machine input coefficients (i, j, = 1, 2, 3). Furthermore, let l_i be the labor per unit of output coefficients, w be the wage rate, d be the fixed rate of depreciation, and r being the profit rate, g the growth rate, p_i the price of output of sector i, Q_i be output of sector i, and L equal the labor force employed. Focusing on just the three-sectors, Lowe's three-sector model can be put into a contemporary framework. The pricing equations become:

$$\begin{aligned} a_{11}(d+r)p_1 + l_1 w &= p_1 \\ a_{21}(d+r)p_1 + l_2 w &= p_2 \\ a_{32}(1+r)p_2 + l_3 w &= p_3 \end{aligned} \tag{15}$$

Or in matrix form:

$$\mathbf{A}(d+r) + \mathbf{l}w = \mathbf{p} \tag{16}$$

The quantity equations can be given in a similar manner. The growth rate g of the labor force must also be the rate of capital accumulation. The quantity equations become:

$$\begin{gathered} a_{11}(d+g)Q_1 + a_{21}(d+g)Q_2 = Q_1 \\ a_{32}Q_3 = Q_2 \\ l_1(d+g)Q_1 + l_1(d+g)Q_2 + l_1(d+g)Q_3 = L \end{gathered} \tag{17}$$

The inclusion of the pricing equations given in equations 15 and 16 become imperative with a discussion of the traverse. The technical-structural conditions required for the economic system to adjust to higher rate growth has been considered above. Recall for the system to adjust to a higher rate of growth requires intermediate goods from Sub-sector 1a to be transferred back to Sub-sector 1a. But why is this transaction done at all? Investment is not a reaction to current conditions given in period 1. In fact, there was a reduction of aggregate demand due to an increase in the rate of savings by consumers. However, there must be an anticipation of future profits based upon an increase in final demand in the upcoming period.

Capitalists may or may not have these expectations. If not, capitalists will not engage in the necessary investments which are required to bring the system to full employment given a higher labor force participation rate. Private businesses are motivated by their own pecuniary desires. They are not motivated by macroeconomic policies which enhance social welfare. Because of this result, Lowe suggests that the achievement and maintenance of full employment requires "public controls". Public controls are similar to Keynes' proposition for the socialization of investment. Public controls are direct government intervention which essentially forces capitalists within the sectors to engage in the necessary investments which ensure the macro-economy will maintain a level of full employment.

It has just been addressed that the Lowe model, while unique, considers both important structural bottlenecks and effective demand. Furthermore the model puts emphasis on the social relationships of production between capitalists within the sectors and stages. Furthermore Lowe considers that capitalists may not independently engage in behaviors which allow the economic system to maintain production at the full employment level.The Lowe model incorporates fundamental principles of both the Veblenian and Keynesian traditions.

3 Conclusion – incomplete

By marginalizing non-linear models in heterodox microeconomics, the heterodox microeconomic approach to production modeling is only capable of modeling an economic system that is not subject to capacity constraints and structural bottlenecks. It has been shown that in order for structural bottlenecks to be considered the circular production model needs to be supplemented with the vertical stage model. The approach of Lowe has been shown to be both consistent with heterodox microeconomics and also offer important contributions missing in the current literature.

References

Arestis, P. (1992). *The Post Keynesian Approach to Economics*. Cheltenham, UK: Edward Elgar Publishing Ltd.

Arestis, P. (1996). Post keynesian economics: towards coherence. *Cambridge Journal of Economics 20*, 111–135.

Bortis, H. (1990). Structure and change within the circular theory of production. In M. Baranzini and R. Scazzieri (Eds.), *The Economic Theory of Structure and Change*, pp. 64–92. Cambridge, UK: Cambridge University Press.

Dorfman, R., P. Samuelson, and R. Solow (1987). *Linear Programming and Economic Analysis*. New York: Dover Publications.

Eichner, A. (1991). *The Macrodynamics of Advanced Market Economies*. M.E. Sharpe.

Finch, J. H. (2002). The role of grounded theory in developing economic theory. *Journal of Economic Methodology 9*(2), 213–234.

Forstater, M. (2000). Full employment and economic flexibility. In E. Carlson and W. Mitchell (Eds.), *The Path to Full Employment*, pp. 67–87. Sydney, AU: The Economic and Labour Relations Review.

Forstater, M. (2002). Full employment policies must consider effective demand and structural and technological change: A prime point of Pasinetti's political economy. In P. Davidson (Ed.), *A Post Keynesian Perspective on 21st Century Economic Problems*. Cheltenham, UK: Edward Elgar Publishing Ltd.

Gehrke, C. and H. Hagemann (1996). Efficient traverses and bottlenecks: a structural approach. In M. Landesmann and R. Scazzieri (Eds.), *Production and Economic Dynamics*, pp. 140–166. Cambridge, UK: Cambridge University Press.

Hagemann, H. (1990). The structural theory of economic growth. In M. Baranzini and R. Scazzieri (Eds.), *Beyond the Steady State: A Revival of Growth Theory*, pp. 144–171. Cambridge, UK: Cambridge University Press.

Hagemann, H. (1992). Traverse analysis in a post-classical model. In J. Halevi, D. Laibman, and E. Nell (Eds.), *Beyond the Steady-State: A Revival of Growth Theory*, pp. 235–263. London, UK: Macmillan Ltd.

Hamouda, O. and G. Harcourt (1990). Post-keynesianism: From criticism to coherence? In J. Pheby (Ed.), *New directions in post-Keynesian economics*, pp. 1–37. Cheltenham, UK: Edward Elgar Publishing Ltd.

Hicks, J. (1965). *Capital and Growth*. Oxford, UK: Oxford University Press.

Hicks, J. (1973). *Capital and TIme*. Oxford, UK: Clarendon Press.

Holt, R. (2007). What is post keynesian economics. In M. Forstater, G. Mongiovi, and S. Pressman (Eds.), *Post Keynesian Macroeconomics: Essays in Honour of Ingrid Rima*, pp. 89–107. New York, NY: Routledge.

J., S. (1952). *Maturity and Stagnation in AMerican Capitalism*. Oxford, UK: Basil Blackwell Ltd.

Jorgenson, D. (1960). Growth and fluctuations: A causal interpretation. *The Quarterly Journal of Economics 74*, 416–436.

Keynes, J. (1964 [1936]). *The General Theory of Employment Interest and Money*. New York, NY: Harcourt Brace and Co.

Kregel, J. (1980). Markets and institutions as features of a capitalist production system. *Journal of Post Keynesian Economics 3*(1), 32–48.

Kurz, H. and N. Salvadori (1995). *A Theory of Production: A Long-Period Analysis*. Cambridge, UK: Cambridge University Press.

Lawson, T. (1994). The nature of Post Keynesian and its links to other traditions: a realist perspective. *Journal of Post Keynesian Economics 16*(4), 503–538.

Lee, F. (1998). *Post Keynesian Price Theory*. Cambridge, MA: Cambridge University Press.

Lee, F. S. (2005). Teaching heterodox microeconomics. *Post-autistic economics review 31*(16).

Lee, F. S. (2010). Alfred eichner's missing "complete model". In M. Lavoie, L.-P. Rochon, and M. Seccareccia (Eds.), *Money and Macrodynamics: Alfred Eichner and Post Keynesian Economics*, pp. 23–42. Armonk, NY: M.E. Sharpe.

Leontief, W. (1953). Dynamic analysis. In W. Leontief (Ed.), *Studies in the Structure of the American Economy*, pp. 53–92. New York, NY: 0xford University Press.

Leontief, W. ([1970] 1986a). The dynamic inverse. In *Input-Output Economics* (Second ed.)., pp. 294–320. New York, NY: Oxford University Press.

Leontief, W. (1986b). *Input-Output Economics* (Second ed.). New York, NY: Oxford University Press.

Leontief, W. (1987). Input-output analysis. In J. Eatwell, M. Milgate, and P. Newman (Eds.), *The New Palgrave: A Dictionary of Economics*, Volume 2, pp. 860–864. London, UK: The Macmillan Press Ltd.

Lowe, A. (1976). *The Path of Economic Growth*. Cambridge, UK: Cambridge University Press.

Lowe, A. (1987). A structural model of production. In A. Oakley (Ed.), *Essays in Political Economics: Public Control in a Democratic Soicety*, pp. 27–59. New York: New York University Press.

Lowe, A. (1997, June). How is business cycle theory possible at all? *Structural Change and Economic Dynamics 2*, 245–270.

Miernyk, W. (1969). *The Elements of Input Output Analysis*. New York, NY: Random House.

Miller, R. and P. Blair (2009). *Input-Output Analysis: Foundations and Extensions* (2nd ed.). Cambridge, UK: Cambridge University Press.

Pasinetti, L. ([1973] 1980). The notion of vertical integration in economic analysis. In L. Pasinetti (Ed.), *Essays on the Theory of Joint Production*, pp. 16–43. New York, NY: Columbia University Press.

Pasinetti, L. (1977). *Lectures on the Theory of Production*. London, UK: The Macmillian Press Ltd.

Pasinetti, L. (1981). *Structural Change and Economic Growth*. Cambridge, UK: Cambridge University Press.

Pasinetti, L. (1993). *Structural Economic Dynamics*. Cambridge, UK: Cambridge University Press.

Pasinetti, L. (2007). *Keynes and the Cambridge Keynesians*. Cambridge, UK: Cambridge University Press.

Sraffa, P. (1960). *Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory*. Cambridge, UK: Cambridge University Press.

Trigg, A. and F. Lee (2005). Pasinetti, keynes and the multiplier. *Review of Political Economy 17*(1), 29–43.

Veblen, T. (1978 [1904]). *The Theory of Business Enterprise*. New Brunswick, NJ: Transaction Publishers.