A dynamic model of house price

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21 May 2011

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Oct 29th, 2011

Abstract

In this paper, we build the rationale of the financial intermediate's decision of making loans to potential home buyers over an infinite time horizon. In the first period "good" borrowers with stable future income flows receive loans and buy homes. In later periods, the intermediate securitizes the loans to raise new capital and makes loans to some of the "bad" borrowers with uncertain future income flows. Currently, we simplify the securitization as a tool to raise capital without cost over time. This unrealistic simplification should be improved in later work. The financial intermediate calculates the expected payoffs in different scenarios under the realizations of uncertainty to decide whether to make loans to a new borrower and whether to liquidate a house if the owner is short of liquidity in the short run. After clarifying the sequence of moves of different agents within each period, we compute the financial intermediate's decision rule described by a Bellman equation. Then we simulate borrowers' income realization and produce a figure of house price as well as value function over time.

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Contents

1 Introduction

Real estate sector has an important position in economy. Houses are not only a fundamental demand but an investment tool as well. In the past decade, real estate property has increasingly attracted fund managers' attention and become an important part of their portfolios in order to provide diversification and effective risk management. With its increasing importance, more credit derivatives based on real estate sector are found in alternative investments. For example, two of the most popular diversification tools in managers' portfolio are financial products related with REIT (Real Estate Investment Trusts) Index and NCREIF (National Council of Real Estate Investment Fiduciaries) Property Index (NPI). In the meanwhile, kinds of derivatives from securitization significantly increase the market liquidity. Examples are mortgage-backed securities (MBSs), related credit default swaps (CDSs) and collateralized debt obligations (CDOs) based on house mortgage. On the one hand they provide investment opportunities for investors; on the other they reduce the burden of risk-based/regulatory capital that financial intermediates must undertake under the Basel Accord adopted by the G-10 group, therefore making those financial institutions able to provide more liquidity and lead the investors' capital to fields with higher yields.

House prices have been an essential indicator of the economic trend since years ago, one phenomenon is that many economic bubbles start with a lending boom. Not much work related to this finding has been done in literature. Despite of numerous articles explaining bubbles and economic crisis, most of them focus on the stock market sector. They employ models with rational agents or irrational behaviors to give possible explanations.

The recent economic crisis was triggered by sub-prime mortgage crisis beginning from year 2008. In this crisis, the houses have more (harmful) effects and influences than in previous ones because mortgage-based financial products have firmly connected house prices with other financial indicators. This is an interesting phenomenon. Traditional beliefs regard real estate properties as an effective tool of value preservation. In this sense, effects of houses should be counter-cyclical, and this is why fund managers use them to diversify. However, if real estate-based products have been linked with others, the house price fall will trigger price fluctuation of other related financial products. And because of the huge amount of capital in real estate sector, the firmer the link, the greater the influence. This is what Stiglitz (2010) says "contagion of crisis".

This is market participants' analytical false of market and investment diversification. And this time they systematically neglected systematic problems. Historic experience indeed shows that large-scale defaults never happened simultaneously, it only is a small probability that mortgage-based securities would suffer market value reduction by over 10 percent. But those participants did not fully realize that when market risks are linked together, the value of traditional diversification vanishes. When house price falls first, interest rate increases and economy steps into recession, making relative assets more risky and raising the probability of defaults - this further causes price falls in real estate sector. The most recent rise of house price started from around year 2000. Most people believe that reasons include financial liberalization in late 1990s. Part of the Glass- Steagall Act of 1933, which aims to separate businesses of investment banks, commercial banks and insurance companies, was repealed as the Financial Services Modernization Act of 1999, or the Gramm-Leach-Bliley Act, was passed. Financial intermediates therefore are allowed to securitize the loans they possess by issuing mortgage-backed securities, attracting more capital since investors find it less costly (Bolton and Freixas, 2000). The relaxation of financial institutes' supply constraints allow them to lend potential home buyers, triggering a potential rise of property price.

Besides above logic, there are other potential reasons of this crisis. Securitization makes the borrowing-lending chain too long therefore causes serious information asymmetry; inconsistent salary incentive mechanism of managers may encourage shortism behavior; credit rating agencies are employed by the firms they rate, causing wrong incentives; incorrect asset pricing models exist, etc. This paper focuses on the initial house price increase triggered by securitization and tries to endogenize the movement pattern of price increase and crash by analyzing the interaction between potential home buyers with heterogeneous income structures, lending decision of financial institutions and investors' investment on MBS and other house-based financial products.

In the US housing market, different financial institutions specialize in different services. A mortgage originator, either a mortgage broker or a mortgage banker, works with a borrower to complete a mortgage transaction as the original mortgage lender. Mortgage originators are part of the highly fragmented primary mortgage market. After the transaction, borrowers pay their mortgage loan payments to a mortgage servicer, whose duty includes the acceptance and recording of mortgage payments; calculating variable interest rates on adjustable rate loans; payment of taxes and insurance from borrower escrow accounts; negotiations of workouts and modifications of mortgage upon default and conducting or supervising the foreclosure process when necessary (FDIC Law, Regulations, Related Acts). CDO and MBS issuers either purchase loans from servicers or issue CDSs to originator or servicers as credit protectors, then use the assets as collaterals to sell securities. Market participants also include many other kinds of financial service providers.

In my model, all those financial institutions are simplified into one financial intermediate (this is reasonable if all the service providers merge into a conglomerate, as Gramm-Leach-Bliley Act allows). This intermediate checks the qualifications, like income structures, of potential home buyers; makes loan lending decisions;

collects the mortgage payments; reorganizes the loans and slices the assets into different securities like MBSs to absorb new capital and lends out again. Also this intermediate deals with defaults, renegotiation, foreclosure and liquidation.

The financial intermediate considers two effects when it makes lending decisions. On one hand, without pervasive inflation, when house price increases much faster than income, lenders, usually banks and other financial mortgage service providers, have to consider the borrowers' capacity of repaying the debt. On the other hand, while house price is high and other lenders are lending, lenders may be more willing to lend because once borrowers are forced to default due to short-term liquidity shock, lenders can foreclose and liquidate the houses, and if home price is greater than principal's value of loans plus interest, the lenders profit. The liquidation and negative income shock push down the price while the lending drives the price up.

In this paper, we assume the financial intermediate begins with an initial capital available to lend in the first period. We build the rationale of the financial intermediate's decision of making loans to potential home buyers over an infinite time horizon. In the first period "good" borrowers with stable future income flows receive loans and buy homes. In later periods, the intermediate securitizes the loans to raise new capital and makes loans to some of the "bad" borrowers with uncertain future income flows. Currently, we simplify the securitization as a tool to raise capital without cost over time. This unrealistic simplification should be improved in later work. The financial intermediate calculates the expected payoffs in different scenarios under the realizations of uncertainty to decide whether to make loans to a new borrower and whether to liquidate a house if the owner is short of liquidity in the short run. After clarifying the sequence of moves of different agents within each period, we compute the financial intermediate's decision rule described by a Bellman equation. Then we simulate borrowers' income realization and produce a figure of house price as well as value function over time.

The ultimate aim of this paper is to generate an endogenous price evolution and to produce an endogenous house price curve with first-increase-then-crash shape. Ideal model includes three parts. The first part is the loan making decision of the financial intermediate. The decision depends upon potential home buyers' income flow pattern, current house price and expected house price in the next period and further future. Here the target of the financial intermediate is to maximize (expected) profit. The second part is the house purchasing and loan applying decision of home buyers. The decision also depends upon above factors and the buyer makes decision to maximize utility. Interaction between the financial intermediate and the home buyers determines the equilibrium price. The third part is the investment decision. Investors rely on (lagged) market information to decide whether to continue purchasing the securities that the financial intermediate issues. The investors' decision determines the capital and financial support of house price, therefore affects the rise and fall of the price. Investors' information is lagged for one period, reflecting the fact that information asymmetry and inaccuracy becomes more serious with longer borrowing-lending chain resulting from securitization.

So far the model only considers the first and second parts. Moreover, the home buyers' utility in the second part is extremely simplified. Therefore only the first part has the dynamic analysis. The buyers' dynamic decisions have not been constructed. The behavior of investors is not reflected in the current model.

Furthermore, the model makes some other assumptions like exogenous interest rates (they should be determined in the equilibrium). Those assumptions should be relaxed in further work.

This paper is organized as follows. The first section introduces the real estate and mortgage market. The second section reviews related literature. The third section sets up the model and clarifies the sequence of moves. The forth gives computation and simulation of the value function and house price. The last section summarizes and points out where to improve in the future work.

2 Literature Review

Related literature at least includes two branches. The first branch is the housing market lending and financing behavior as well as studies of bubbles. Among the great amount of articles on bubbles, economic crisis and asset pricing, few of them study housing market[1]. Most focus on the stock market sector. For example, Scheinkman and Xiong (2003) and Pastor and Veronesi (2003, 2006) studies possible reasons of stock price increases, the former attributes to different beliefs of investors while the latter believes that the uncertainty is the cause.

Other researchers focus on life-cycle models and incomplete markets and housing decisions without aggregate risk. Fernandez-Villaverde and Krueger (2010) study how life-cycle consumption is influenced by consumer durables in an incomplete markets model with production, but limit their focus to equilibria in which prices, wages and interest rates are constant over time. Kiyotaki, Michaelides, and Nikolov (2011) study a life-cycle model with housing and non-housing production, focusing their analysis on the perfect foresight equilibria of an economy without aggregate risk and an exogenous interest rate. Iacoviello and Pavan (2009) combine aggregate risk, production, and incomplete markets. They study the role of housing and debt for the volatility of the aggregate economy in a model with a single production and single saving technology.

[1]For example, Storesletten, Telmer, and Yaron (2007) and Gomes and Michaelides (2008) model the production side of the economy but focus on single-sector economies without real estate sector.

Some researchers study the land market. Kiyotaki and Moore (1995) examine how the amplification and persistence of a negative temporary shock affects the land market. Lustig and Van Nieuwerburgh (2005) employ a two-sector exchange economy model to study the empirical relationships among housing collateral, consumption insurance and risk premia. Bolton and Freixas (2000) build model to compare the costs of different financing methods. Their paper says that with asymmetric information, different financing tools incur different costs, banks choose tools under given conditions. Among the financing channels, internal securitization has a low cost.

One of the most recent and important work is Favilukis, Ludvigson and Van Nieuwerburgh (2010). Their model's main implications are: 1) house prices relative to measures of fundamental value are volatile; 2) a financial market liberalization drives price-rent ratios up because it drives risk premia down; 3) foreign purchases of U.S. bonds play a central role in lower interest rates but a small role in housing booms; 4) financial market liberalization plus foreign capital leads to a shift in the composition of wealth towards housing.

The second branch is dynamic discrete-choice models, which enlightens me to model the financial intermediate's decision choice. One of the classic papers is Rust (1987) which studies how a manager of bus depot makes decision to replace engines. As milage goes up because of long-time use of engines, the maintenance cost is higher if not replacing; replacing incurs a large fixed cost but clear the milage to zero. The manager must compare the tradeoff to make engine replacement decision. Rust constructed a dynamic forward-looking model to examine above what cutoff value the manager will replace.

3 Model

3.1 Notations

In this model, there are infinite periods: $t = 0, 1, 2$. There is an economy consisting of N groups. N is very large so each group can be considered as competing with others competitively. In each group there are three kinds of agents: a financial intermediate F, a "good buyer" G and "bad buyers" B. The financial intermediate lives in every period. The good buyer G was born in the first period at $t = 0$. At the beginning of every period from $t = 1$ to infinity, a single bad buyer B was born. So at $t = T$, there have been in total $T - 1$ bad buyers born because in the first period it was the good buyer born.

In the first period, G was born with an endowment of I_{G0}^{0} which is positive. At $t = 1, 2, 3$, G receives I_{Gt}^{0} in each period. 0 indicates that G was born at $t = 0$, the subscript Gt means that it is G's income at time t. The good buyer G is "good" because her income in each period is certain and positive: $I_{Gt}^{0} = I_G > 0$, $\forall t \geq 0$.

In each period of $t = 1, 2, ...$, B was born with uncertain income. For a B born at $t = t_1$, his endowment is $I_{Bt_1}^{t_1}$, the superscript t_1 indicates that this B was born at time t_1, the subscript says it is the income at $t = t_1$. At $t_2 \geq t_1 + 1$, G receives $I_{Bt_2}^{t_1}$, meaning that it is the income at $t = t_2$ of B who was born at $t = t_1$. B's income is uncertain. In each period, $I_{Bt_2}^{t_1}$ equals $I_B > 0$ with probability p_B and zero with probability $1 - p_B$. The realization of income across time is independent.

Both good buyer and bad buyers want to buy houses which give them utility. Below some price, they are better off from buying and holding the houses than holding an equivalent amount of cash. However, neither one's endowment is enough to afford a house even when house price is low, which is an assumption. Mathematically, $I_{it}^{t} < H_t$, where H_t is the house price at time t. Therefore the buyers need to borrow money from the financial intermediate F.

We assume here that if a B's endowment is zero, $I_{Bt}^t = 0$, F decides not to lend. Only when $I_{it}^t > 0$ $(i = G, B)$ the financial intermediate is willing to make initial loans $L_{i(t+1)}^t = (1 + r_{it})(H_t - I_{it}^t)$ to i. The subscript of L, $i(t + 1)$, means it is the amount i has to repay at $t + 1$. Since G's income is positive and certain, F lends L_{G1}^0 to G for sure at $t = 0$. There is an interest rate $1 + r_{it}$ at time t for person i (G's and B's rates are different, B's is greater).

3.2 Loans, Liquidation and House Price

The process of roll-over of loans is as follows. For buyer who was born at t_1, if $I_{it_1}^{t_1} > 0$, F lends $L_{i(t_1+1)}^{t_1} = (1 + r_{it_1})(H_{t_1} - I_{it_1}^{t_1})$ to i. At the beginning of time $t_1 + 1$, the borrower first repays his debt, wholly or partially, with his realized income at t_1+1, $I_{i(t_1+1)}^{t_1}$. If $I_{i(t_1+1)}^{t_1} \geq L_{i(t_1+1)}^{t_1}$, the borrower will have no obligation from the next period. Otherwise, if $I_{i(t_1+1)}^{t_1} < L_{i(t_1+1)}^{t_1}$, the borrower still has to repay in the next period and the amount is $(1 + r_{i(t_1+1)})(L_{i(t_1+1)}^{t_1} - I_{i(t_1+1)}^{t_1})$. There is an interest rate $r_{i(t_1+1)}$ at time t_1+1 for person i (G's and B's rates are different, B's is greater). Therefore, at time $t_1 + 2$, the loan that i has to repay equals

$$L_{i(t_1+2)}^{t_1} = \max[(1 + r_{i(t_1+1)})(L_{i(t_1+1)}^{t_1} - I_{i(t_1+1)}^{t_1}), 0]$$

Similarly, at time t_2, the borrower i born at t_1 has loan with the amount of

$$L_{i(t_2+1)}^{t_1} = \max[(1 + r_{it_2})(L_{it_2}^{t_1} - I_{it_2}^{t_1}), 0]$$

Now we consider the financial intermediate F's liquidation decision. we assume[2] that if a buyer's realization of income in the current period t is positive,

[2]We will construct micro foundation of these assumptions via utility of agents in the future work.

he must prefer repaying his debt than defaulting. So there is no voluntary default from the buyers' side if $I_{it}^{t'} > 0$. We further assume that if a buyer repays at time t, F has no legal right to liquidate the buyer's house. But when a bad buyer's realization of income is zero, $I_{Bt}^{t'} = 0$, F may consider liquidation. After liquidation, F receives the minimum value of current house price and loans that the buyer needs to repay, $\min(H_t - \delta, L_{it}^{t'})$. F may not want to liquidate if the house price is high enough because if F waits to the next period, the loan of next period will be greater than current loan, $L_{i(t+1)}^{t'}|_{I_{it}^{t'}=0} = (1 + r_{it})L_{it}^{t'}$. Therefore, a bad buyer with zero current income faces a liquidation risk.

In this model, the supply of houses follows an exogenous curve[3]. It may take the form of step function. The quantity of houses depends on how many buyers who have received loans from the financial intermediate F, including those who have repaid all debts and others who are repaying, but not including the people whose houses have been liquidated by F. The supply curve must ensure a non-decreasing relationship between quantity and house price.

3.3 Sequence of Moves

Let me clarify the time line of the moves within every period $t > 0$ now to facilitate understanding of later dynamic analysis.

(Step 1) Before the realization of income of any buyer born at $t' < t$, F makes a decision rule $D(H_t, L_{it}^{t'})$ to determine whether it should liquidate the borrower's house under the condition that the buyer's realization of income in this period is zero, $I_{it}^{t'} = 0$. $D(H_t, L_{it}^{t'})$ takes the value of either 0 or 1. $D = 1$ means F liquidates. F's decision is based on the current house price H_t, which is determined

[3]We will construct micro foundation for the supply side in the future work, considering housing suppliers' utility.

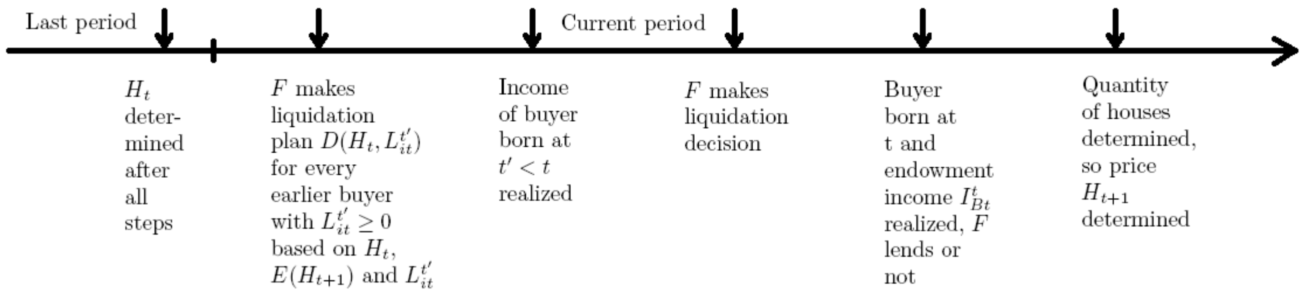

Figure 1: Sequence of Moves

in equilibrium after all the steps done in the last period $t-1$, the expectation of the future house price at $t+1$, $E(H_{t+1})$ and the amount of loans he has at time t, $L_{it}^{t'}$. F does this analysis for every earlier buyer with $L_{it}^{t'} \geq 0$ who hasn't finished repaying all debts. Here we have another assumption that the house price is sticky: after H_t is generated in the last period at time $t-1$, it remains the same for the entire period t.

(Step 2) For any buyer born at $t' < t$, if he hasn't finished the payment of mortgage loans, his income is realized. Different buyers' realization is independent of each other.

(Step 3) For every buyer's different realization of income at step (2), F decides independently whether to liquidate according to its decision rule obtained at step (1).

(Step 4) A new bad buyer B was born at time t and his endowment income I_{Bt}^t is realized, either I_B or zero. If $I_{Bt}^t = 0$, F decides not to lend. If $I_{Bt}^t = I_B$, F considers lending or not based on the current house price and future house price.

(Step 5) F's decisions of lending or not for the earlier buyers who were born at $t' < t$ and for the new born B_t change the quantity of houses, therefore change the house price. The newly generated house price will be taken into account by F in the next period $t+1$.

3.3.1 Step 1

Now we consider at step (1), the decision of F to liquidate the house if agent i's realization of income is zero in current period. Given exogenous interest rates, the value function of agent i born at time t' in the current period t depends on the house price at time t, H_t, the expectation of the future house price at $t+1$, $E(H_{t+1})$ and the amount of loans he has at time t, $L_{it}^{t'}$. We assume no momentum expectation of F and F's $E(H_{t+1})$ is equal to the current house price H_t. Therefore F's value function and decision rule are now determined by two variables.

Since F makes the decision rule at the beginning of each period without the realization of income, the value function takes the form

$$\begin{aligned} V(H_t, L_{it}^{t'}) = {} & p_B\{\min(L_{it}^{t'}, I_{it}^{t'}) + \beta_F E[V(H_{t+1}, L_{i(t+1)}^{t'})|_{I_{it}^{t'}=I_i}]\} \\ & +(1-p_B) \max_{D(H_t, L_{it}^{t'})\in\{0,1\}} \max\{D(H_t, L_{it}^{t'})\min(H_t-\delta, L_{it}^{t'}), \\ & (1-D(H_t, L_{it}^{t'}))\beta_F E[V(H_{t+1}, L_{i(t+1)}^{t'})|_{I_{it}^{t'}=0}]\} \end{aligned}$$

where

$$L_{i(t+1)}^{t'}|_{I_{it}^{t'}=I_i} = (1+r_{it})\max(L_{it}^{t'} - I_{it}^{t'}, 0)$$

and

$$L_{i(t+1)}^{t'}|_{I_{it}^{t'}=0} = (1+r_{it})(L_{it}^{t'} - I_{it}^{t'}) = (1+r_{it})L_{it}^{t'}$$

The first part on the right side considers the situation under which the income realization is positive. In period t, if the current income $I_{it}^{t'}$ it is greater than the amount of loan the borrower has to pay, F receives full amount of loan, $L_{it}^{t'}$, and from the next period that borrower will have no debt obligation. If $I_{it}^{t'}$ is smaller than $L_{it}^{t'}$, F takes all his income. Hence F's revenue in the current period given positive income realization is the minimum of the two. The second term in the first part takes the standard Bellman equation format. β_F is F's discount factor.

The value of the second part on the right side depends on F's decision choice of whether to liquidate when income realization at t is zero. If F liquidates, the liquidation generates $H_t - \delta$ cash where δ is the liquidation cost. If $H_t - \delta < L_{it}^{t'}$, F will take all the value from the liquidation; otherwise, F only gets $L_{it}^{t'}$ and the rest belongs to the borrower. Thus F's payoff from liquidation is $\min(H_t - \delta, L_{it}^{t'})$. The second term in the second part says if F does not liquidate, F gets a discounted value function in the next period. F compares the values from the two choices and takes decision to obtain the one with greater value.

Under the assumption of the equality between $E(H_{t+1})$ and H_t, the value function can be simplified as:

$$\begin{aligned}V(H_t, L_{it}^{t'}) = p_B\{&\min(L_{it}^{t'}, I_{it}^{t'}) + \beta_F E[V(H_t, (1+r_{it})\max(L_{it}^{t'} - I_{it}^{t'}, 0))|_{I_{it}^{t'}=I_i}]\} \\ &+(1-p_B) \max_{D(H_t, L_{it}^{t'})\in\{0,1\}} \max\{D(H_t, L_{it}^{t'})\min(H_t - \delta, L_{it}^{t'}), \\ &(1 - D(H_t, L_{it}^{t'}))\beta_F E[V(H_t, (1+r_{it})L_{it}^{t'})|_{I_{it}^{t'}=0}]\}\end{aligned}$$

3.3.2 Step 4

Here we also maintain the assumption of the equality between $E(H_{t+1})$ and H_t. When a new born arrives at t with positive endowment $I_{Bt}^{t} = I_B$ or zero, F makes lending decision based on the current house price. If F does not lend, its net payoff is zero; if it lends, the payoff is $\beta_F E[V(H_{t+1}, L_{i(t+1)}^{t})] - (H_t - I_{it}^{t})$, the discounted expected value of the next period's value function net of the amount F lends. F compares the two values. If the net payoff from lending is positive, then F lends;

others F does not lend. Thus the net payoff F gets from a new born buyer $W(H_t)$ is

$$W(H_t) = \max_{d(H_t)\in\{0,1\}} \{d(H_t)[\beta_F E[V(H_{t+1}, L^t_{i(t+1)})] - (H_t - I^t_{it})], 0\}$$

where

$$L^t_{i(t+1)} = (1 + r_{it})(H_t - I^t_{it})$$

$d(H_t) = 1$ means that F lends.

4 Computation and Simulation Results

The purpose of computation is to find 1) F's decision rule under different values of house prices and loans; and 2) house price path with random assigned income realization.

4.1 Value Function

Solving decision rules follows standard Bellman equation numerical computation. We do have some interesting and intuitive findings. One of them is that when loan-price ratio is high enough, the financial intermediate tends not to liquidate. Mathematically, an increase of loans increases $\beta_F E[V(H_t, (1 + r_{it})L^{t'}_{it})|_{I^{t'}_{it}=0}]$ but not $\min(H_t - \delta, L^{t'}_{it})$ since the latter takes a form of minimum. The intuition is that when amount of loans is high enough (possibly because the loans compound period by period and rise fast when a buyer's realized income keeps being zero for several successive periods), liquidation only gives F a given one time payoff but not to liquidate gives F right to receive the borrower's every period's future income, probably more than what it can get from liquidation.

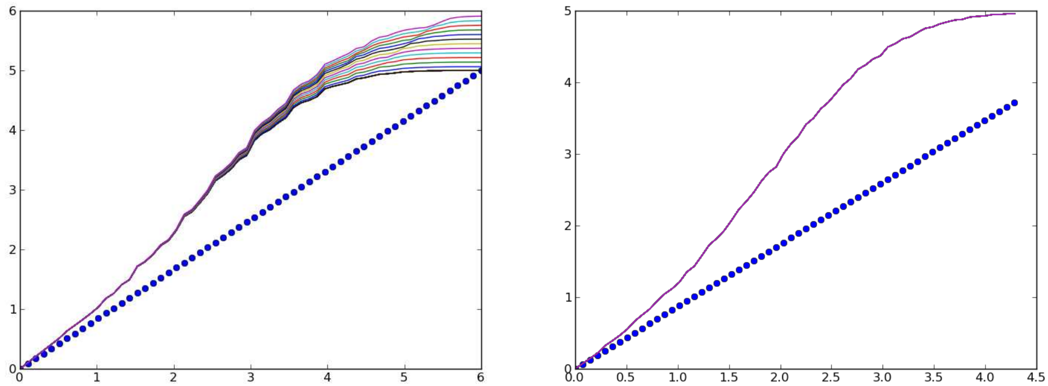

Figure 2: Value Function (vertical axis), Loans (horizontal axis) and House Prices (color)

Figure 2 shows how different values of loans and house price produce $V(H_t, L_{it}^{t'})$. Value of loans is represented in the horizontal axis and value function is in the vertical axis. There are a couple of curves with different color representing different house prices. Upper curves represent higher house price. We draw this way to avoid the complexity of programming 3-dimension figure. The left figure takes parameters $p_B = .5$, $\beta_F = .9$, $r_{it} = r = .2$ and $\delta = .5$. It shows that when loans increase to a high level, the value function stops increasing as the curve becomes flat. Intuitively, for a common lender, owing one billion dollars and owing one trillion dollars to a bank give the bank no different decisions because the amount he owes is too large. The right figure takes parameters $p_B = .5$, $\beta_F = .99$, $r_{it} = r = .28$ and $\delta = .5$. In this figure the curves with different house prices collapses into a single curve since the interest rate is "too" high.

A 3-D figure is shown in Figure 3. It presents that the value function increases with house price and amount of loans, more vividly characterizing the points in Figure 2.

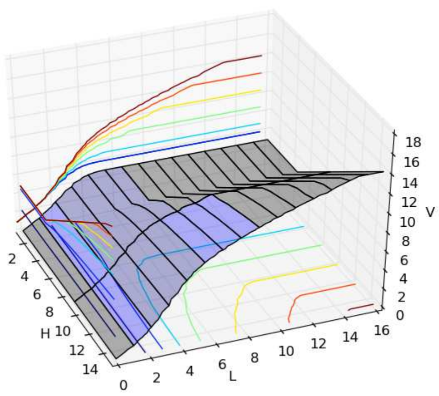

Figure 3: Value Function (V), Loans (L) and House Prices (H)

4.2 House Price

Figure 4 and 5 show the simulated path of house price (vertical axis) over time (horizontal axis) under different parameter settings. Price going down means the number of houses liquidated is greater than additional new lending in the period (the first ten periods in Figure 4 Right and the second ten periods in Figure 5 Right). It is possible (though rare in the current parameter setting) that prices keep staying at a low level (red line in Figure 4 Left). But more cases show the increasing pattern of prices, fast or slow, with fluctuations in some periods. Some paths increase linearly and some exponentially.

The readers need to realize that there is an implied upper bound of house price. Ignoring the interest rate, the maximum amount of repayment F receives equals discounted value of I_B over infinite time horizon, which is $I_B/(1-\beta_F)$. When house price exceeds this level, F will never lend any more, exerting a downward pressure to the price. In addition, too low interest rates lower the upper bound. The intuition is that too low rates make lending unprofitable to F. This is proved

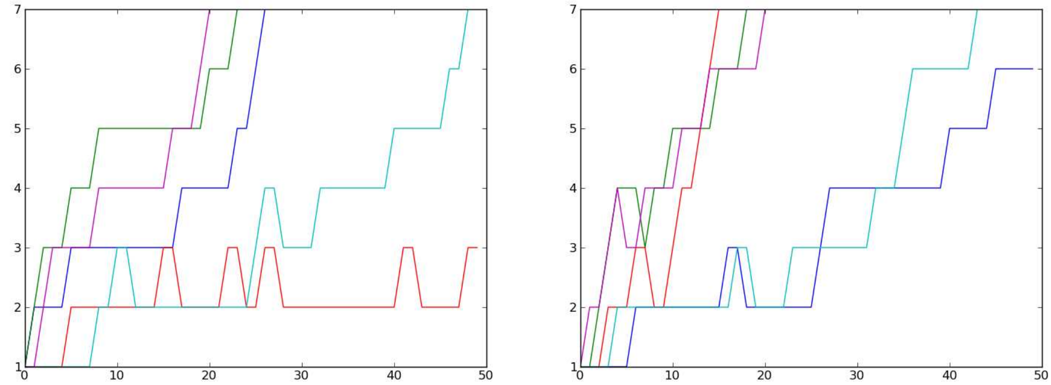

Figure 4: Both: $p_B = .3$, $\beta_F = .9$, $r = .2$, $\delta = .5$, $I_G = I_B = 1$

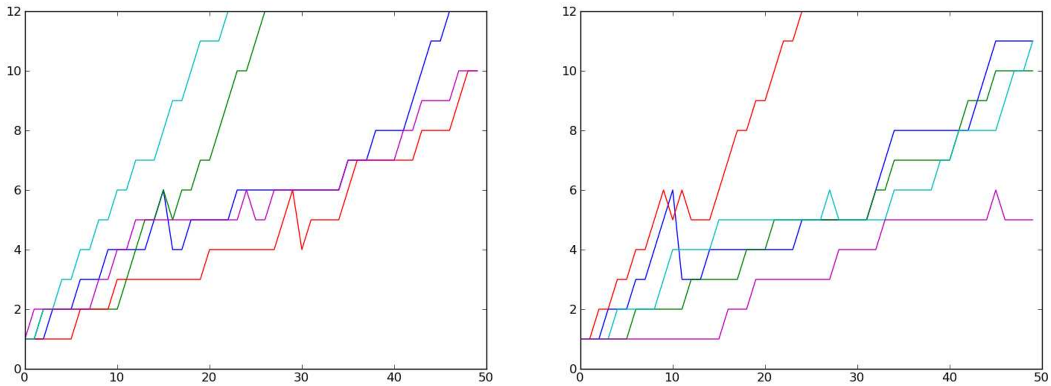

Figure 5: Both: $p_B = .3$, $\beta_F = .94$, $r = .8$, $\delta = .5$, $I_G = I_B = 1$

in the simulated price path. There are other factors affecting the highest possible house price.

5 Concluding Remarks

In this paper, we construct a dynamic decision process of financial intermediate based on the information of house price, expectation of income realization and debt history. The decision rules include whether to renew loans to "earlier" borrowers and whether to make loans to new arriving borrowers. The dynamic process is characterized by a Bellman equation.

We compute the Bellman equation to obtain the decision rules. When loans are small, the value function increases with the amount of loans and house prices. When loans increase to a high level, the value function stops increasing as the curve becomes flat. The simulated house price paths over time generally show an increasing pattern, some exponentially, some linearly and some even stays at a certain low level. In the end, we explain the existence of an upper bound of house price under my current model construction.

There are at least four aspects worthy of further improvement. Firstly, this paper makes several assumptions (though we think they are reasonable and meet reality) to characterize the buyers' borrowing decision; also the supply curve of housing market is exogenous. Dropping these exogenous assumptions by building micro foundation to describe and derive the assumed behaviors with utility functions should make the model more convincing. Secondly, exogenous interest rate may be endogenized via competition of financial intermediates. This change must complicate the value function and the iteration process, therefore substantially increase the complexity of analysis and computation. Thirdly, there is only one kind of sub-prime borrowers (the "bad" buyers) in this model as their uncertain income follows an identical distribution. If borrowers are with different income uncertainty, the model may reflect the reality more. Of course, it is also possible that the changes might not vary the results significantly, then the changes would be unnecessary. Last but not least, investors' decisions should be modeled.

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