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Abstract The present study considers a unionised duopoly with the two most popular labour market institutions, i.e. efficient bargaining (EB) and right to manage (RTM) unions and analyses product market stability under quantity competition. By focusing on the role played by labour market institutions on the market dynamics, we show that when the preference of unions towards wages is fairly low, (*i*) the stability region under RTM is higher than under EB, and (*ii*) a rise in the union power in the Nash bargaining monotonically increases (reduces) the parametric stability region under RTM (EB). In contrast, when the preference of unions towards wages becomes higher, an increase in the union's bargaining power acts: (1) as an economic stabiliser when the union power is still low; (2) as an economic de-stabiliser when the union power is already high. These results shed some light on the effects of how labour market structures affect out-of equilibrium behaviours in a duopoly in addition to the many established results on how they affect equilibrium behaviours in such a context, and thus also constitute policy warnings for the design of the labour market institutions as regards the important issue of economic stability.

Keywords Bifurcation; Cournot; Duopoly; Efficient bargaining; Right to manage

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1. Introduction

The existence of trade unions represents a hard stylised fact in several developed countries, especially in Europe. Indeed, wide evidence exists that relates high rates of unemployment and trade unions in the long run (see Layard et al., 2005), even if such a relationship can actually depend on the way unions operate.

As is known, wage and employment behaviours can be modelled in different ways when workers are organised by a trade union: the "efficient bargaining" (EB) and "right to manage" (RTM) models indeed represent two standard examples. The key feature of the former model is that both the wage and employment are chosen according to a bargaining process between firms and employees' representatives (see McDonald and Solow, 1981), while in the latter one only the wage is subject to negotiation, with firms being free to unilaterally choose employment (see, e.g., Oswald, 1982; Pencavel, 1984, 1985).[1]

The relative importance of wages and employment in the union's preferences may be different in the sense that trade unions can either be wage- or employment-oriented. Furthermore, firm-specific and industry-wide unions can also be distinguished: in the former case, wages are bargained by potentially competitive unions at decentralised or firm-specific level, while an industry-wide union represents all workers of every firms in market, and represents therefore a centralised wage bargaining. The EB and RTM models represent the two most popular alternatives of wage-employment outcomes of collective bargaining. The trade union literature (see Booth, 1995; Layard et al., 2005) has established some clear normative implications which arise from the two models: (1) RTM bargaining brings upon inefficiently low (high) levels of employment (wage), implying that unions may be viewed as socially inefficient institutions so that a weakening of union power would likely enhance social welfare; (2) EB bargaining causes either an efficient employment level or, at least, even in those cases in which employment will either be too high or too low for social efficiency, it causes a social inefficiency lower than that caused by the RTM outcome.

While static outcomes, and their normative implications, in a duopoly with different typologies of unions and bargaining structures has deeply been explored (see, e.g., Dowrick, 1989; Bughin, 1995; Kraft, 1998; Petrakis and Vlassis, 2000; Correa-López and Naylor, 2004; Pal and Saha, 2008; Fanti and Meccheri, 2011), less attention has been paid their stability effects in a dynamic context. This paper therefore aims at filling this gap by taking EB and RTM bargaining between firms and unions into account in the framework of a nonlinear dynamic duopoly (see, e.g., Puu, 1991, 1998; Bischi and Kopel, 2001; Bischi et al., 2010) with quantity competition. The choice of such a framework, which is characterised by the assumption of myopic rather rational expectations by firms, not only, according to several scholars (i.e., Dixit, 1986), may well represent the context of partially "bounded" rationality in which oligopolistic firms operate, but it mainly serves the purposes to allow for rich dynamical outcomes (of course prevented by the assumption of perfect foresight) and thus the novel investigation on the roles played by the labour market institutions in a dynamic context. This analysis is quite relevant because the existing literature on unionised oligopolies as well as on non linear dynamic oligopolies abstracted from the issue whether and how the established labour market institutions affect market stability.

[1] Indeed, a widely studied special case of the RTM model is the so-called "monopoly union" model, where unions hold the whole bargaining power when it takes place.

Indeed, we show that labour market institutions can actually have different effects not only on equilibrium outcomes, as established by the existing static literature, but also on market stability. In particular, we stress the existence of four clear-cut results. The first result concerns the unambiguous role played by the relative degree of "wage-aggressiveness" by unions, irrespective of the mode of labour market institution (i.e., RTM and EB): the higher the relative importance of wages in the union's objective, the more likely the Cournot-Nash equilibrium of the duopoly economy is stable. Other two results are claimed by separately considering the two typologies of bargaining: indeed, under RTM we find that the lower the union bargaining power, the more likely the loss of market stability, whereas under EB the union power brings upon either an opposite effect (for fairly low levels of both the union's power in bargaining and preference towards wage in the union's objective) with respect to the case of RTM, or an ambiguous effect on stability (i.e., when the union power in the Nash bargaining is fairly low and/or the union's preference towards wages is low as well, it still remains true that a rise in the union power works for instability, while when the union power is fairly high and the preference towards wages in the union's objective is large enough, a further increase in power of unions in the Nash bargaining acts as a stabilising device. However, it must also be noted that when unions are strongly employment-oriented, higher levels of union power, including the case of monopoly union, always work for market stability. Finally the last, but not least, result concerns the comparison in terms of stability between the two most popular labour market institutions and states that when the union's power is high and/or unions are fairly employment-oriented, the RTM institution is neatly more favourable for market stability. By contrast, when the power of unions in the bargaining is low and unions are fairly wage-oriented, the EB becomes the labour market institution which favours market stability.

Moreover, it must also be noted that in any case when the union power is approximately less than one half the stability region under RTM is wider than under EB, irrespective of the relative size of union's preferences.

The rest of the paper is organised as follows. Section 2 builds on the duopoly model under both RTM and EB. Sections 3 introduces expectations and analyses the local stability properties of the unique positive Cournot-Nash equilibrium. Section 4 compares the stability/instability regions when either RTM or EB takes places. Section 5 concludes.

2. A Cournot duopoly with unions. The cases of efficient bargaining and right to manage

We consider a normalised[2] Cournot duopoly for a single homogenous product with a negatively sloped inverse demand given by $p = 1 - q_1 - q_2$, where $p > 0$ denotes the price and q_1 (q_2) is the output produced by firm 1 (firm 2). The average and marginal costs for each single firm to provide one additional unit of output in the market are equal and given by $0 < w_i < 1$, which represents the wage negotiated by unions at firm-specific level. This hypothesis leads firm i ($i = \{1,2\}$) to produce output q_i through a production function with constant (marginal) returns to labour, that is $q_i = L_i$ (see,

[2] This hypothesis does not imply a loss of generality, while being adopted only for the sake of simplicity.

e.g., Dowrick, 1989; Bughin, 1995; Correa-López and Naylor, 2004), where L_i is labour employed by firm i.

Efficient bargaining. Under EB (see McDonald and Solow, 1981), firms and unions bargain over both employment and wages. The objective of every firm is therefore to maximise profits $\Pi_i(w_i, L_i) = pq_i - w_i L_i$ with respect to employment and wages, while the objective of every firm-specific union is to maximise utility $U_i(w_i, L_i) = (w_i - w^\circ)^\theta L_i$ with respect to employment and wages, where $\theta > 0$ is the relative weight attached by unions to wages[3] and w° is the reservation or competitive wage. Without loss of generality, we set $w^\circ = 0$ henceforth. Since the production function is $q_i = L_i$, the Nash bargaining between firms and unions is summarised by the following functional:

$$V_i = [(1 - q_i - q_j - w_i)q_i]^\beta (w^\theta{}_i \cdot q_i)^{1-\beta}, \tag{1}$$

where the control variables are q_i and w_i, $0 \le \beta \le 1$ is the relative bargaining power of firms. Therefore, the best reply functions of the ith player as regards output and wages are simultaneously determined by maximising Eq. (1) with respect to q_i and w_i, respectively, that is:

$$\frac{\partial V_i}{\partial q_i} = \frac{V_i \cdot [1 - q_i(1+\beta) - q_j - w_i]}{(1 - q_i - q_j - w_i)q_i} = 0 \Leftrightarrow q_i(q_j, w_i) = \frac{1 - q_j - w_i}{1+\beta}, \tag{2.1}$$

$$\frac{\partial V_i}{\partial w_i} = \frac{V_i \cdot \{\theta(1-\beta)(1 - q_i - q_j) - w_i[\beta + \theta(1-\beta)]\}}{(1 - q_i - q_j - w_i)w_i} = 0 \Leftrightarrow w_i(q_i, q_j) = \frac{\theta(1-\beta)}{\beta + \theta(1-\beta)}(1 - q_i - q_j) < 1. \tag{2.2}$$

Inserting now Eq. (2.2) into Eq. (2.1) to eliminate w_i, we definitely obtain firm i's output best-reply function as follows:

$$q_i(q_j) = \frac{1 - q_j}{1 + \beta + \theta(1-\beta)}. \tag{3}$$

Right to manage. Under RTM, the objective of every firm is to maximise profits with respect to employment and wages, while the objective of every firm-specific union is to maximise utility with respect to wages (see Booth, 1995). Therefore, firms and decentralised unions bargain over wages with firms being free to unilaterally choose employment. The Nash product Eq. (1) is therefore maximised by player i with respect to w_i alone to get the optimal wage as determined by Eq. (2.2). As regards employment (i.e., output), firm i's profits maximisation gives the following output best-reply function which depends on the own wage and rival's output, that is:

$$\frac{\partial \Pi_i}{\partial q_i} = 1 - w_i - 2q_i - q_j = 0 \Leftrightarrow q_i(q_j, w_i) = \frac{1 - q_j - w_i}{2}. \tag{4}$$

At this stage of the game, therefore, we can exploit Eq. (2.2) together with Eq. (4) to definitely obtain the following output reaction function of firm i:

$$q_i(q_j) = \frac{\beta(1 - q_j)}{2\beta + \theta(1-\beta)}. \tag{5}$$

By comparing Eqs. (3) and (5) it is clear that output (i.e., employment) under efficient bargaining is higher than under right to manage.

[3] Values of θ smaller (higher) than 1 imply that the union is less (more) concerned about wages and more (less) concerned about jobs (see, e.g., Mezzetti and Dinopoulos, 1991; Fanti and Gori, 2011).

3. Expectations and local stability

Let $q_{i,t}$ be firm i's quantity produced at time $t=0,1,2,...$. Then, $q_{i,t+1}$ is obtained as:

$$q_{i,t+1} = \arg\max_{q_{i,t}} \Theta_{i,t}\left(q_{i,t}, q^{e}{}_{j,t+1}\right), \tag{6}$$

where $q^{e}{}_{j,t+1}$ represents the quantity that the rival, i.e. firm j, today (time t) expects will be produced by firm i in the future (time $t+1$), and $\Theta_{i,t}\left(q_{i,t}, q^{e}{}_{j,t+1}\right)$ is a function which equals the Nash maximand $V_{i,t}\left(q_{i,t}, q^{e}{}_{j,t+1}\right)$ of the ith player under efficient bargaining and profits $\Pi_{i,t}\left(q_{i,t}, q^{e}{}_{j,t+1}\right)$ under right to manage. Assuming now heterogeneous (i.e., bounded rational[4] and Cournot-naïve[5] for firm 1 and 2, respectively) expectations (see, e.g., Agiza and Elsadany, 2003; Zhang et al., 2007; Tramontana, 2010) about the quantity that firm i expects will be produced by the rival in the future, the two-dimensional systems that characterise the dynamics of a duopoly economy under efficient bargaining and right to manage unions are respectively determined by:

$$EB: \begin{cases} q_{1,t+1} = q_{1,t} + \alpha q_{1,t} \dfrac{\partial V_{1,t}}{\partial q_{1,t}}, \\ q_{2,t+1} = q_{2,t} \end{cases} \tag{7.1}$$

and

$$RTM: \begin{cases} q_{1,t+1} = q_{1,t} + \alpha q_{1,t} \dfrac{\partial \Pi_{1,t}}{\partial q_{1,t}}, \\ q_{2,t+1} = q_{2,t} \end{cases} \tag{7.2}$$

where $\alpha > 0$ is a coefficient that captures the speed of adjustment of player 1's quantity with respect to a marginal change either in $V_{1,t}$ under efficient bargaining or $\Pi_{1,t}$ under right to manage, when $q_{1,t}$ varies.

Efficient bargaining. By taking the optimal wage determined by Eq. (2.2) into account, and using Eqs. (2.1), (3) and (7.1) the dynamics of the economy under efficient bargaining is described by:

$$\begin{cases} q_{1,t+1} = q_{1,t} \\ \quad + \dfrac{\alpha \left[\dfrac{\beta(1-q_{1,t}-q_{2,t})q_{1,t}}{\beta+\theta(1-\beta)}\right]^{\beta} \left\{\left[\dfrac{\beta(1-\beta)(1-q_{1,t}-q_{2,t})}{\beta+\theta(1-\beta)}\right]^{\theta} q_{1,t}\right\}^{1-\beta} \left\{1-q_{1,t}\left[1+\beta+\theta(1-\beta)\right]-q_{2,t}\right\}}{1-q_{1,t}-q_{2,t}} \\ q_{2,t+1} = \dfrac{1-q_{1,t}}{1+\beta+\theta(1-\beta)} \end{cases}. \tag{8}$$

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[4] In the standard dynamic Cournot duopoly with profit-maximising firms, each bounded rational player uses information on current profits to increase or decrease the quantity produced at time $t+1$ depending on whether marginal profits are either positive or negative (see Dixit, 1986).

[5] Cournot (1838) was *de facto* the first author to use naïve expectations in an oligopoly model.

Equilibrium implies $q_{1,t+1} = q_{1,t} = q_1$ and $q_{2,t+1} = q_{2,t} = q_2$. Then, assuming that equilibrium conditions hold, Eq. (8) can be transformed to:

$$\begin{cases} \dfrac{\alpha\left[\dfrac{\beta(1-q_1-q_2)q_1}{\beta+\theta(1-\beta)}\right]^{\beta}\left\{\left[\dfrac{\beta(1-\beta)(1-q_1-q_2)}{\beta+\theta(1-\beta)}\right]^{\theta} q_1\right\}^{1-\beta}\{1-q_1[1+\beta+\theta(1-\beta)]-q_2\}}{1-q_1-q_2} = 0 \\ \dfrac{1-q_1}{1+\beta+\theta(1-\beta)} - q_2 = 0 \end{cases}, \tag{9}$$

and the unique interior fixed point $E_{EB}\left(q^*{}_1, q^*{}_2\right)$ of the two dimensional system (8) is characterised by:

$$E_{EB} = \left(\frac{1}{2+\beta+\theta(1-\beta)}, \frac{1}{2+\beta+\theta(1-\beta)}\right). \tag{10}$$

In order to investigate the local stability properties of the Cournot-Nash equilibrium E we build on the Jacobian matrix

$$J_{EB} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} 1 - \dfrac{\alpha \mathrm{AB}\left[\theta^2(1-\beta)^2 - \theta\left(2\beta^2+\beta-3\right) + (1+\beta)(2+\beta)\right]}{\beta+\theta(1-\beta)} & \dfrac{-\alpha\mathrm{AB}[2+\beta+\theta(1-\beta)]}{\beta+\theta(1-\beta)} \\ \dfrac{-1}{1+\beta+\theta(1-\beta)} & 0 \end{pmatrix}, \tag{11}$$

where partial derivatives J_{ii} and J_{ij} are evaluated at the equilibrium point defined by Eq. (10), $\mathrm{A} := \left\{\dfrac{\beta}{[2+\beta+\theta(1-\beta)]^2}\right\}^{\beta} > 0$ and $\mathrm{B} := \left\{\dfrac{1}{2+\beta+\theta(1-\beta)}\left[\dfrac{\theta(1-\beta)}{2+\beta+\theta(1-\beta)}\right]^{\theta}\right\}^{1-\beta} > 0$.

Trace and determinant of J are given by:

$$T_{EB} := Tr\left(J_{EB}\right) = J_{11} + J_{22} = 1 - \frac{\alpha\mathrm{AB}\left[\theta^2(1-\beta)^2 - \theta\left(2\beta^2+\beta-3\right) + (1+\beta)(2+\beta)\right]}{\beta+\theta(1-\beta)}, \tag{12}$$

$$D_{EB} := Det\left(J_{EB}\right) = J_{11}J_{22} - J_{12}J_{21} = \frac{-\alpha\mathrm{AB}[2+\beta+\theta(1-\beta)]}{[\beta+\theta(1-\beta)]\cdot[1+\beta+\theta(1-\beta)]} < 0. \tag{13}$$

Therefore, the characteristic polynomial of (8) is the following:

$$F_{EB}(\lambda) = \lambda^2 - T_{EB}\lambda + D_{EB}, \tag{14}$$

For the system in two dimensions defined by Eq. (8), the stability conditions that ensure that both eigenvalues λ_a and λ_b of the characteristic polynomial (14) remain within the unit circle are the following:

$$\begin{cases} (i) \quad F_{EB} = \dfrac{2\left\{\theta^2(1-\beta)^2 + \theta\left[1-\beta^2+\beta(1-\beta)\right] + \beta(1+\beta)\right\}}{[\beta+\theta(1-\beta)]\cdot[1+\beta+\theta(1-\beta)]} \\ \quad - \dfrac{\alpha\mathrm{AB}[2+\beta+\theta(1-\beta)]\cdot\left[(1-\beta)^2\theta^2 + 2\left(1-\beta^2\right)\theta + 1 + (1+\beta)^2\right]}{[\beta+\theta(1-\beta)]\cdot[1+\beta+\theta(1-\beta)]} > 0 \\ (ii) \quad TC_{EB} = \dfrac{\alpha\mathrm{AB}[2+\beta+\theta(1-\beta)]^2}{1+\beta+\theta(1-\beta)} > 0 \\ (iii) \quad H_{EB} = \dfrac{\alpha\mathrm{AB}[2+\beta+\theta(1-\beta)] + \theta^2(1-\beta)^2 + \theta\left[1-\beta^2+\beta(1-\beta)\right] + \beta(1+\beta)}{1+\beta+\theta(1-\beta)} > 0 \end{cases}. \tag{15}$$

The violation of any single inequality in (15), with the other two being simultaneously fulfilled leads to: (*i*) a flip bifurcation (a real eigenvalue that passes through -1) when

$F_{EB}=0$; (*ii*) a fold or transcritical bifurcation (a real eigenvalue that passes through $+1$) when $TC_{EB}=0$; (*iii*) a Neimark-Sacker bifurcation (i.e., the modulus of a complex eigenvalue pair that passes through 1) when $H_{EB}=0$, namely $D_{EB}=1$ and $|T_{EB}|<2$. From Eq. (15) it is clear that conditions (*ii*) and (*iii*) are always fulfilled, while condition (*i*) can be violated. We now develop the usual one-parameter bifurcation analysis by studying the stability properties of the unique positive equilibrium E_{EB} of the two-dimensional system Eq. (8). Let $\mathrm{P}_{EB}(\alpha,\beta,\theta)$ represent a boundary at which the Nash equilibrium Eq. (10) of a Cournot duopoly under efficient bargaining looses stability through a flip bifurcation ($F_{EB}=0$, see condition (*i*) in Eq. 15) when:

$$\begin{aligned}\mathrm{P}_{EB}(\alpha,\beta,\theta)&:=2\left\{\theta^2(1-\beta)^2+\theta\left[1-\beta^2+\beta(1-\beta)\right]+\beta(1+\beta)\right\}\\&-\alpha\mathrm{AB}\left[2+\beta+\theta(1-\beta)\right]\cdot\left[(1-\beta)^2\theta^2+2\left(1-\beta^2\right)\theta+1+(1+\beta)^2\right]=0\end{aligned}. \tag{16}$$

Now, define

$$\alpha^F{}_{EB}(\beta,\theta)=\frac{2\left\{\theta^2(1-\beta)^2+\theta\left[1-\beta^2+\beta(1-\beta)\right]+\beta(1+\beta)\right\}}{\alpha\mathrm{AB}\left[2+\beta+\theta(1-\beta)\right]\cdot\left[(1-\beta)^2\theta^2+2\left(1-\beta^2\right)\theta+1+(1+\beta)^2\right]}, \tag{17}$$

as the (unique) flip bifurcation value of α. Then, the following proposition holds.

Proposition 1. *Let* $0<\alpha<\alpha^F{}_{EB}(\beta,\theta)$ *hold. Then, the Cournot-Nash equilibrium* E_{EB} *of the two-dimensional system (8) is locally asymptotically stable. A flip bifurcation emerges if, and only if,* $\alpha=\alpha^F{}_{EB}(\beta,\theta)$. *Let* $\alpha>\alpha^F{}_{EB}(\beta,\theta)$ *hold. Then, the Cournot-Nash equilibrium* E_{EB} *is locally unstable.*

Proof. Since $\mathrm{P}_{EB}(\alpha,\beta,\theta)>0$ for any $0<\alpha<\alpha^F{}_{EB}(\beta,\theta)$, $\mathrm{P}_{EB}(\alpha,\beta,\theta)=0$ if, and only if, $\alpha=\alpha^F{}_{EB}(\beta,\theta)$ and $\mathrm{P}_{EB}(\alpha,\beta,\theta)<0$ for any $\alpha>\alpha^F{}_{EB}(\beta,\theta)$, then Proposition 1 follows. **Q.E.D.**

Right to manage. In the case of right to manage, by taking the optimal wage bargained by firms and unions into account (see Eq. 2.2), and using Eqs. (4), (5) and (7.1), the dynamics of the economy under right to manage is described by the following two-dimensional system:

$$\begin{cases} q_{1,t+1}=q_{1,t}+\alpha q_{1,t}\dfrac{\beta\left(1-q_{2,t}\right)-\left[2\beta+\theta(1-\beta)\right]q_{1,t}}{\beta+\theta(1-\beta)} \\ q_{2,t+1}=\dfrac{\beta\left(1-q_{1,t}\right)}{2\beta+\theta(1-\beta)} \end{cases}. \tag{18}$$

Assuming now that equilibrium conditions hold, Eq. (18) can be transformed to:

$$\begin{cases} \alpha q_1\dfrac{\beta\left(1-q_2\right)-\left[2\beta+\theta(1-\beta)\right]q_1}{\beta+\theta(1-\beta)}=0 \\ \dfrac{\beta\left(1-q_1\right)}{2\beta+\theta(1-\beta)}-q_2=0 \end{cases}, \tag{19}$$

so that the unique interior fixed point $E_{RTM}\left(q^*{}_1,q^*{}_2\right)$ of the two dimensional system (18) is given by:

$$E_{RTM}=\left(\frac{\beta}{3\beta+\theta(1-\beta)},\frac{\beta}{3\beta+\theta(1-\beta)}\right). \tag{20}$$

Using the same line of reasoning as in the case of efficient bargaining, the stability conditions can now be written as follows:

$$\begin{cases} (i) \quad F_{RTM} = 2 - \dfrac{\alpha\beta\left[(1-\beta)^2\theta^2 + 4\beta(1-\beta)\theta + 5\beta^2\right]}{[\beta+\theta(1-\beta)]\cdot[2\beta+\theta(1-\beta)]\cdot[3\beta+\theta(1-\beta)]} > 0 \\ (ii) \quad TC_{RTM} = \dfrac{\alpha\beta}{2\beta+\theta(1-\beta)} > 0 \\ (iii) \quad H_{RTM} = 1 + \dfrac{\alpha\beta^3}{[\beta+\theta(1-\beta)]\cdot[2\beta+\theta(1-\beta)]\cdot[3\beta+\theta(1-\beta)]} > 0 \end{cases} . \tag{21}$$

From Eq. (21) it is clear that conditions (*ii*) and (*iii*) are always fulfilled, while condition (*i*) can be violated. Therefore, let $\mathrm{P}_{RTM}(\alpha,\beta,\theta)$ represent a boundary at which the Nash equilibrium Eq. (10) of a Cournot duopoly under efficient bargaining looses stability through a flip bifurcation ($F_{RTM}=0$) when:

$$\begin{aligned} \mathrm{P}_{RTM}(\alpha,\beta,\theta) &:= 2[\beta+\theta(1-\beta)]\cdot[2\beta+\theta(1-\beta)]\cdot[3\beta+\theta(1-\beta)] \\ &-\alpha\beta\left[(1-\beta)^2\theta^2+4\beta(1-\beta)\theta+5\beta^2\right]=0 \end{aligned} . \tag{22}$$

Now, define

$$\alpha^F{}_{RTM}(\beta,\theta) = \frac{2[\beta+\theta(1-\beta)]\cdot[2\beta+\theta(1-\beta)]\cdot[3\beta+\theta(1-\beta)]}{\beta\left[(1-\beta)^2\theta^2+4\beta(1-\beta)\theta+5\beta^2\right]}, \tag{23}$$

as the (unique) flip bifurcation value of α. Then, the following proposition holds.

Proposition 2. *Let $0<\alpha<\alpha^F{}_{RTM}(\beta,\theta)$ hold. Then, the Cournot-Nash equilibrium E_{RTM} of the two-dimensional system (18) is locally asymptotically stable. A flip bifurcation emerges if, and only if, $\alpha=\alpha^F{}_{RTM}(\beta,\theta)$. Let $\alpha>\alpha^F{}_{RTM}(\beta,\theta)$ hold. Then, the Cournot-Nash equilibrium E_{RTM} is locally unstable.*

Proof. Since $\mathrm{P}_{RTM}(\alpha,\beta,\theta)>0$ for any $0<\alpha<\alpha^F{}_{RTM}(\beta,\theta)$, $\mathrm{P}_{RTM}(\alpha,\beta,\theta)=0$ if, and only if, $\alpha=\alpha^F{}_{RTM}(\beta,\theta)$ and $\mathrm{P}_{RTM}(\alpha,\beta,\theta)<0$ for any $\alpha>\alpha^F{}_{RTM}(\beta,\theta)$, then Proposition 2 follows. **Q.E.D.**

4. Efficient bargaining versus right to manage

It is now of importance to compare the parametric stability/instability regions under EB and RTM. In this section, therefore, we deal with this subject and, since the comparison between Eqs. (17) and (23) cannot be dealt with in a neat analytical form, we resort to numerical simulations presented in Figures 1-4, which indeed exhaustively describe all the possible cases involved depending on the relative size of the bargaining power of firms, β, and the weight attached by unions to wages in their utility functions, θ. In particular, we let $\alpha^F{}_{EB}(\beta,\theta)$ and $\alpha^F{}_{RTM}(\beta,\theta)$ vary in the (α,β) plane for different values of θ.

Starting from the case $\theta=0$ and $\beta=1$, which perfectly replicates the case of profit-maximising firms with competitive labour markets, Figure 1-4 show that the behaviour of the flip bifurcation boundaries in the (α,β) plane under EB and RTM unions is different and θ plays a dramatic role on stability.[6] In particular, under RTM, the relationship between the flip bifurcation boundary of α and β is monotonically negative, and the curve $\alpha^F{}_{RTM}(\beta,\theta)$ shifts upwards and to the right as long as θ increases, while under EB, the relationship between the flip bifurcation boundary of α and β is monotonically positive when θ is close to zero, and the curve $\alpha^F{}_{EB}(\beta,\theta)$ becomes hump-shaped as long as θ increases.

Therefore, the following results can be established.

Result 1. *[RTM and EB]. An increase in the relative importance of wages in the union's objective, θ, acts as an economic stabiliser in both RTM and EB cases. Indeed, an increase in θ under RTM (EB) shifts the flip bifurcation boundary $\alpha^F{}_{RTM}(\beta,\theta)$ ($\alpha^F{}_{EB}(\beta,\theta)$) upwards in the (α,β) plane [see Figures 1-4].*

Result 2. *[RTM]. Under RTM, an increase in the union's bargaining power, i.e. β moves from 1 to 0, unambiguously acts as an economic stabiliser for any $\theta > 0$ [see Figures 1-4].*

Result 3. *[EB]. Under EB, the relationship between α and β is ambiguous and depends on the relative size of θ. When θ is fairly low [see Figure 1], an increase in the union's bargaining power unambiguously acts an economic de-stabiliser. When θ becomes higher [see Figures 2-4], an increase in the union's bargaining power acts: (1) as an economic stabiliser when the union power is still low (i.e., high values of β); (2) as an economic de-stabiliser when the union power is already high (i.e., low values of β).*

Result 4. *[Comparison between RTM and EB]. When θ is fairly low, the stability region in the (α,β) plane in a Cournot duopoly under RTM is larger than under EB irrespective of the relative importance of the union power in the Nash bargaining [see Figure 1]. When θ becomes higher, the stability region under RTM is lower (higher) than under EB when the union power is fairly low (high), i.e. for fairly high (low) values of β.*

[6] Note that the stability (instability) regions in the (α,β) plane are those below (above) the flip bifurcation boundaries $\alpha^F{}_{EB}(\beta,\theta)$ and $\alpha^F{}_{RTM}(\beta,\theta)$.

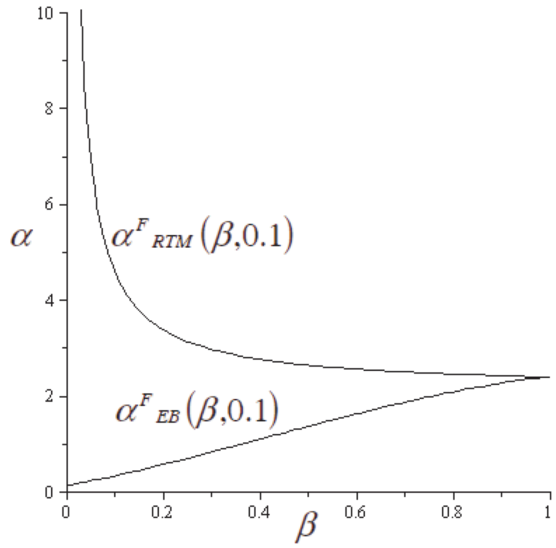

Figure 1. Flip bifurcation boundaries in the (α,β) plane ($\theta = 0.1$).

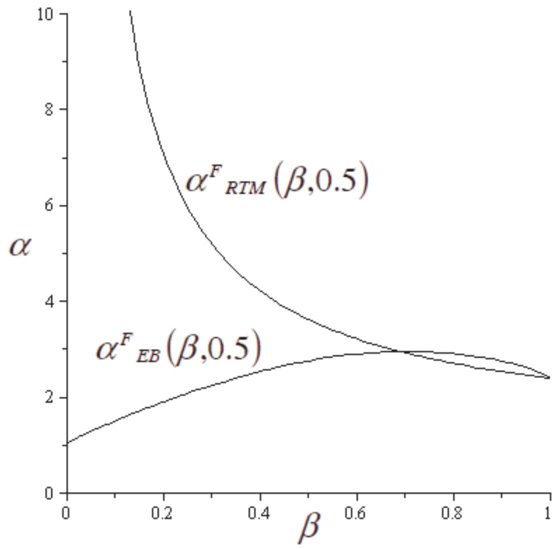

Figure 2. Flip bifurcation boundaries in the (α,β) plane ($\theta = 0.5$).

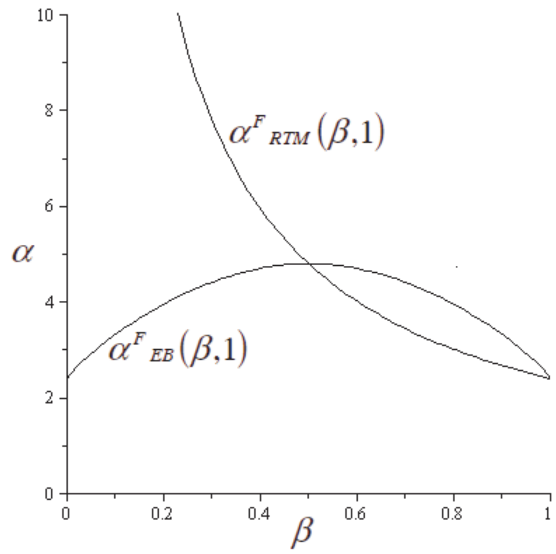

Figure 3. Flip bifurcation boundaries in the (α,β) plane ($\theta = 1$).

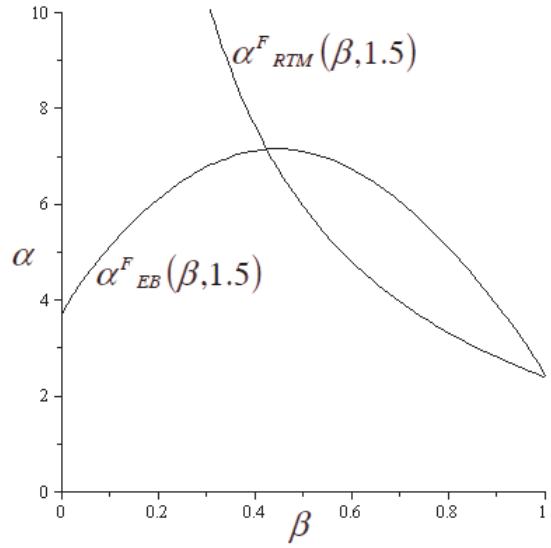

Figure 4. Flip bifurcation boundaries in the (α,β) plane ($\theta=1.5$).

5. Conclusions

The present study analysed the stability issue in a unionised duopoly with the two most popular labour market institutions, i.e. efficient bargaining (EB) and right to manage (RTM) unions, under quantity competition. By focusing on the role played by labour market institutions on market dynamics, we show that such institutions have different effects on market stability, which actually depend on the relative preference of unions either towards wages or employment. In particular, when the preference of unions towards wages is fairly low, (*i*) the stability region under RTM is higher than under EB, and (*ii*) a rise in the union power in the Nash bargaining monotonically increases (reduces) the parametric stability regions under RTM (EB). In contrast, when the preference of unions towards wages becomes higher, an increase in the union's bargaining power acts: (1) as an economic stabiliser when the union power is still low; (2) as an economic de-stabiliser when the union power is already high.

Therefore, as regards economic stability, provided that it is shown that the unions' "wage-aggressiveness" works for stability under both institutions, RTM is the one that should actually be preferred when union power is large and/or unions are fairly wage-oriented and in any case, under RTM, it is always better that the union power is fixed at the largest possible level. By contrast, under EB the region of stability is the largest when firm and union power are near-parity than when one significantly prevails on the other.

These results shed some light on the effects of how labour market structures affect out-of-equilibrium market behaviours, and also constitute interesting warnings for the policy design of labour market institutions as regards the important issue of economic stability.

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