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He, Qichun

Central University of Economics and Finance

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The Impact of the Distribution of Property Rights on Inventions on Growth: A Two-Representative-Agent Model with Asymmetric Information*

Qichun He†

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Abstract

We study how entrepreneurs and households share the monopolistic profit from inventions would affect growth. The share to the entrepreneur is called entrepreneur's inventive incentive (EII). First, there are two representative agents (a borrowing entrepreneur and a household who provides the financing capital), both making intertemporal savings decisions. Second, the two agents sign credit contracts to deal with asymmetric information. A larger EII elicits more entrepreneurs' effort, increasing the monopolistic profit from innovations (a "bigger cake" effect); it, however, leaves a smaller share of the cake to households. Initially, the former effect dominates, but beyond a point, the latter effect dominates. As the cake becomes bigger, if the creditor's share gets too small, her return (the product of the size of the cake and her share in the cake) may decrease and she would be less willing to save to finance R&D. Therefore, growth is an inverted-U function of EII.

JEL Classification: O12; O31; O43; E14

Keywords: Two Representative Agents; Credit Market Imperfection; Credit Contract; Entrepreneur's Inventive Incentive; Inverted-U

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†Assistant Professor, China Economics and Management Academy (CEMA), Central University of Finance and Economics (CUFE), No. 39 South College Road, Haidian District, Beijing, 100081, China. Email: qichunhe@gmail.com, heqichun@cufe.edu.cn.

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"The entrepreneurial function is not, in principle, connected with the possession of wealth…He can only become an entrepreneur by previously becoming a debtor…What he first wants is credit…He is the typical debtor in capitalist society." Schumpeter (1961, 101-102)

1 Introduction

One basic issue concerning production is why some countries persistently grow slower than others. For example, the annual growth from 1960 to 2000 was above 4% and below 0% for 3 East Asian countries (China, Japan and South Korea) and 14 sub-Saharan African countries respectively (Heston, Summers and Aten, 2002). There are numerous theories tackling the basic issue. Recently, Aghion and Howitt (2006), for example, use competition in R&D (i.e., firm entry/exit) to explain the substantial cross-country differences in growth rates. Inspired by Schumpeter (1961), we use the difference in the distribution of property rights on inventions to explain the observed long-run growth differentials.

Following Schumpeter, credit-constrained entrepreneurs have to borrow from the households to finance innovations. In the presence of financial imperfections, how the entrepreneurs and the households share the monopolistic profit from innovations (i.e., the property rights on inventions) would affect long-run growth. Similar to the atemporal 'landlord-tenant' problem (Stiglitz, 1974; Laffont and Matoussi, 1995; Cheung, 1968), we define this as an intertemporal 'capital lord-entrepreneur' problem, which is useful for understanding the dynamics of long-run growth and income inequality in capitalist society.[1] In so doing, we apply the insights from the 'landlord-tenant' problem or the linear compensation contracts in the principal-agent literature (particular from authors such as Holmstrom and Milgrom, 1987; Laffont and Tirole, 1986) to the endogenous growth setting.

We contribute to the growth literature by incorporating two important real world features into endogenous growth models. First, there are two representative agents: an entrepreneur and a worker. They both make intertemporal savings and consumption decisions. This differs from the one representative agent assumption in most new growth models (NGMs) (e.g., Romer, 1990; Aghion and Howitt, 1992).[2] Aghion and Howitt (1998, p. 66) criticize that one shortcoming of the NGMs lies in their simple representation of R&D activities. Previous attempts to overcome this shortcoming still keep the one

[1] In terms of issues involved in contracting, the 'landlord-tenant' problem is similar to our problem. The former is about arable land whose supply is fixed, while our problem is concerned with capital that can be accumulated to generate endogenous growth that is dynamic in essence. Malinvaud (1961), for example, analyzes the analogy between atemperoal and intertemporal theories of resource allocation.

[2] For early attempts to endogenize growth, see Acemoglu (2008) and Grossman and Helpman (1991).

representative agent framework.[3] With two representative agents, we are able to study how the distribution of the monopolistic profit from innovations between entrepreneurs and workers-financiers (a richer representation of R&D activity) impacts long-run growth. This differs from Banerjee and Newman (1993) who emphasize the distribution of wealth in affecting economic growth.

Second, we emphasize the role of financial imperfections (i.e., the asymmetric information between entrepreneurs and households).[4] Given financial imperfections, credit contract plays an important role, unlike in many NGMs. Many works (e.g., Townsend, 1979; Williamson, 1987; Bernanke and Gertler, 1989) study credit contracts in the presence of information asymmetry, but few are concerned with endogenous growth.[5] Aghion and Tirole (1994), for example, study the contractual arrangements of R&D when entrepreneur's effort is hidden action, leaving the relationship between entrepreneur's effort and growth unexplored. Many studies that incorporate financial imperfection in a growth setting either investigate issues other than long-run growth (e.g., Aghion and Bolton 1997; Aghion et al., 1999; Aghion et al., 2005)[6] or have "little gain in terms of new economic insights" (Aghion and Howitt, 1998, p. 69) (e.g., King and Levine, 1993).

The new economic insights of our model are as follows. The share of the monopolistic profit from innovations distributed to entrepreneurs is termed as the entrepreneur's inventive incentive (hereafter, EII). Our main prediction is that long-run growth is an inverted-U function of the EII. The mechanism is as follows. On the one hand, a higher EII means that entrepreneurs keep a higher share of the marginal benefit of their effort. Resultantly, they contribute more effort in R&D, yielding a larger monopolistic profit from each innovation.[7] Specifically, with two representative agents and the asymmetric information that exists between them (e.g., entrepreneur's effort is unobservable to others), the entrepreneur chooses effort to maximize *her share* (rather than the whole share) of the profit from innovations. Therefore, the entrepreneur's optimal effort increases with her share, generating a "bigger cake".[8] On the other hand, a higher EII leaves a smaller share to households. This "smaller household share" effect is not captured by the one-representative-agent NGMs. When the EII is low, the former effect dominates, but beyond a point, the latter effect dominates. As in previous NGMs, long-run growth is linear in the return to the capital/savings of the household-creditor. As the cake gets bigger, if the creditor's share in the cake gets too small, her return (the product of the size of the

[3] Aghion and Howitt (1998, p. 66) argue that the relaxation of the one individual assumption is typically made within firms where employee as inventors are subject to assignment contracts with their employers who provide the financing and physical capital. It is still a representative agent model.

[4] Paulson et al. (2006) show the existence of credit market imperfections with Thailand data.

[5] Williamson (1987) and Bernanke and Gertler (1989) focus on explaining business cycles.

[6] Aghion et al. (2005), for example, introduce financial market frictions into NGMs to show that financial development matters for growth only at the early stage of economic development.

[7] Laffont and Matoussi (1995) provide empirical evidence that efficiency is lower when the tenant's share of output is lower given moral hazard and financial constraints.

[8] Even if the entrepreneur saves through financial intermediaries, she, due to the large market effect, will not take into account the effect (externality) of her effort on the whole economy.

cake and her share in the cake) becomes lower. Resultantly, she would be less willing to save to finance R&D, lowering long-run growth. A numerical example confirms the inverted-U result and shows that having the optimal EII could increase annual growth by 0.4 percentage points.[9] We further show that the income ratio between entrepreneurs and households is an increasing function of EII.[10] Moreover, a poverty trap emerges if EII is too low or too high.[11]

Cross-country differences in the distribution of property rights on inventions, therefore, may offer one explanation for the observed substantive country-level growth differentials. Why have some Asian countries been more successful in achieving higher growth? Taking China as an example, we argue that the economic reform of China since 1978 has unleashed entrepreneurs' inventive spirit by allowing them to retain a larger fraction of the profit from innovations (see Li et al., 2009), ending up promoting growth. Meanwhile, the 14 sub-Saharan African countries have negative growth, because the entrepreneurs' inventive spirit may have been constrained at a low level. Therefore, less effort would be forthcoming from the entrepreneurs, which lowers the return to innovations and thereby makes households less willing to finance R&D. A poverty trap emerges.

Our theoretical predictions are general in the sense that they do not depend on the particular information structure of the economy (i.e., the particular form of credit contract). Therefore, the rest of the paper proceeds as follows. In section 2, we develop a model with hidden action (or moral hazard, i.e., entrepreneurs' effort is unobservable to others). To resemble the real world credit market, in section 3, we further show that the inverted-U result exists with both hidden information (two unobserved types of entrepreneurs) and hidden action. Section 4 concludes.

2 A Model with Moral Hazard

To study the intertemporal 'capital lord-entrepreneur' problem, we use the NGM with expanding varieties to include two representative agents, financial imperfections and credit contracts.[12] The economy consists of a final goods sector, an intermediate goods sector, entrepreneurs, financial intermediaries, and workers. Each intermediate good represents an innovation. Each innovation is a project that is conducted by an entrepreneur. The innovation cost (the cost of R&D) of each intermediate good is a fixed amount, η. We assume that the financing capital for R&D (i.e., η) totally comes from financial intermediaries that

[9] Solow (2001) argues: "Adding a couple of tenths of a percentage point to the growth rate is an achievement that eventually dwarfs in welfare significance any of the standard goals of economic policy. Who would not be excited?" Similar arguments are given in Barro and Sala-i-Martion (2004, p. 6).

[10] There is a large literature studying how inequality affects growth (e.g., Aghion et al., 1999). Unlike Persson and Tabellini (1994) who argue that inequality decreases growth, inequality and growth in our model are simultaneously determined by the distribution of property rights on inventions.

[11] There is also a large literature on poverty trap. For one that is concerned with financial intermediaries, please see Berthelemy and Varoudakis (1996).

[12] For the original NGM with expanding varieties, see Barro and Sala-i-Martin (2004, ch. 6).

absorb savings from households (workers as well as entrepreneurs).[13] Entrepreneurs have to borrow from financial intermediaries in the amount of η to finance their innovations. Therefore, following Schumpeter (1961), only external financing is considered.[14]

The first representative agent is an entrepreneur who has double identities: As an entrepreneur, she borrows from financial intermediaries to finance her innovation; as a saver, she makes intertemporal savings decisions and saves through financial intermediaries. The second is a worker who also makes intertemporal savings decisions and saves through financial intermediaries. We refer to savers as households.

Intermediate goods are inputs of the final goods sector. A final goods firm produces a single consumption good using the aggregate production function in equation (1):

$$Y = \sum_{j=1}^{N} X_j^{\alpha} \left(\widetilde{A}_j L\right)^{1-\alpha}, \tag{1}$$

where N is the number of innovations, X_j is the amount of intermediate good j, $\widetilde{A}_j$ is the stochastic productivity of intermediate good j with all $\widetilde{A}_j's$ distributed on the interval $[0, A]$, and L is the labor force. Each j is supplied by a monopolistic entrepreneur. The random variable $\widetilde{A}_j$ has a probability density function (PDF) $f(x, e)$, where $e\,(\geq 0)$ stands for the effort level contributed by the entrepreneurs. The effort level cannot be observed by the financial intermediaries (i.e., hidden action, or a moral hazard problem). We assume that $F_e(x, e) < 0$ (i.e., the cumulative distribution function (CDF) of $\widetilde{A}_j$ with high effort first-order-stochastically dominates that with lower effort. That is, an increase in e raises the expected return of the projects.) and $F_{ee}(x, e) \geq 0$.

When there is only hidden action in situations in which the financial intermediaries are assumed to be risk neutral and the entrepreneurs are assumed to be risk averse, the optimal contract can be a linear sharing contract (i.e., the equity contract here in our model). The linear contracts are shown to be robust in a wild range of situations by, for instance, Holmstrom and Milgrom (1987) and Laffont and Tirole (1986). For any proof on this, please see also, for instance, Laffont and Matoussi (1995), Lacker and Weinberg (1989) and Stiglitz (1974). The equity contract specifies that, the financial intermediaries provide the financing capital for R&D (i.e., η), and share the monopolistic profit from innovations with the entrepreneurs, with the entrepreneurs' share given as $\beta \in [0, 1]$ (the measure of the EII). β is assumed to be exogenously fixed by the structure of the economy (e.g., the legal and policy restrictions on the entry into production and intermediation, and the number of entrepreneurs and financial intermediaries).

[13] The results hold if we only let workers save. Nonetheless, we allow entrepreneurs to save via financial intermediaries. Therefore, households refer to savers.

[14] Self-financing does exist as in Thailand (Paulson et al., 2006), but we follow Schumpeter (1961) (see the quote at the beginning of this paper) to focus on external financing because R&D usually involves large volume of investment that far exceeds the wealth of each entrepreneur and workers can save in the financial system to finance entrepreneurial R&D. In addition, without changing the predictions, we, for simplicity, assume that entrepreneurs do not need collateral in borrowing from financial intermediaries.

In this section, we use the ad hoc assumption that the share of the entrepreneurs (i.e., β) is exogenously given to illustrate the mechanism at play. Even though in the standard sharecropping framework, the share (i.e., β) may be endogenously determined by the bilateral contracting between the entrepreneurs and the financiers, it is also affected by the structure of the economy that shapes the contracting environment and affects the relative bargaining power of the agents. Therefore, β should be viewed as measuring the underlying exogenous primitives of the economy. For this reason, we assume an exogenous β in illustrating the mechanism of our model in a simplest way.

The economy consists of a fixed amount of people. The assigning of occupation is through a simple win-or-lose lottery. Those who lose the lottery become workers who have unit labor endowment and supply it to final goods firms. The total number of workers is L. Those who win the lottery become entrepreneurs. Then people cannot change their occupations, which is different from previous works in which occupational choices are endogenous (e.g., Aghion and Bolton 1997; Banerjee and Newman 1993).[15]

We assume infinite horizon and agents live forever. We assume that the objective functions of the entrepreneurs and the workers have the same form:

$$\underset{e,c}{Max} \int_0^{\infty} \frac{[c - N \cdot h(e)]^{1-\theta} - 1}{1-\theta} \exp(-\rho t)\, dt, \tag{2}$$

where $h(e)$ is the cost of effort, which is additive to consumption c;[16] N is the number of innovations; ρ is the constant rate of time preference; θ is degree of constant relative risk averse (CRRA). We assume that $h(0) = h'(0) = 0$, $h^{'}(e) > 0$, and $h''(e) > 0$. For the entrepreneurs, their objective function consists of two parts: the utility from consumption, and the disutility of effort. Workers maximize the case of $h(0) = 0$.

In the end, the ex post realization of $\widetilde{A}$ is known to everyone. Entrepreneurs pay back their loans in accordance with the contracts. The timing of the model is as follows:

1. The agents make their savings decisions, and if they save, they must save through financial intermediaries. Then the lottery for the patents on the intermediate goods is announced. Those who win the lottery become entrepreneurs.

2. The entrepreneurs borrow credit from financial intermediaries by signing an equity contract. The effort of entrepreneurs cannot be observed by others. The entrepreneurs determine how much effort to put into R&D, which depends on β.

3. $\widetilde{A}$ is realized and the profit from innovations is distributed according to the contracts.

[15] When the entrepreneur's profit increases, more people may choose to be entrepreneurs, which drives down the profit of the entrepreneurs. Since our model does not have occupational choice, this effect does not exist. We leave the consideration of occupational choice to future research.

[16] This assumption is crucial to ensure that the credit contract problem becomes an intratemporal one, so our main intertemporal problem is about consumption and saving. Relaxing this assumption would not yield tractable solutions and is left to future research.

2.1 Final Goods Firms and Intermediate Goods Firms

A final goods firm produces a single final good using the production function in equation (1). It maximizes its profit by taking as given the wage rate, the prices of intermediate goods, and the ex post realization of $\widetilde{A}_j$. The demand for the j-th intermediate is obtained from the FOC (first-order condition) associated with $X_j : X_j = \widetilde{A}_j L \left(\frac{\alpha}{P_j}\right)^{\frac{1}{1-\alpha}}$.

An innovation transforms one unit of final good into one unit of an intermediate good. Normalizing final goods' price as one, the unit cost to intermediate goods firms is also one. After invention is done, an intermediate good firm j maximizes its profit, taking as given the demand from the final good firm. Its price mark-up is $P_j = \frac{1}{\alpha}$, and the monopoly profit is $\pi_j = \widetilde{A}_j L \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} = l \cdot \widetilde{A}_j$, where l is constant and equal to $L\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}}$.

2.2 The Equity Contract

The equity contract specifies that the entrepreneurs and the financial intermediaries share the monopolistic profit from innovations, with the entrepreneurs' share given as $\beta \in [0,1]$, which, therefore, makes the contracting problem become trivial.

The entrepreneurs choose optimal effort by

$$\underset{e}{Max}\ U = \beta \int_0^A l \cdot x f(x,e)\, dx - h(e), \tag{3}$$

where l is constant and equal to $L\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}}$. The first order condition is

$$\frac{dU}{de} = \beta l \left(\int_0^A -F_e(x,e)\, dx\right) - h'(e). \tag{4}$$

The second order condition is $\frac{d^2U}{de^2} = \beta l \left(\int_0^A -F_{ee}(x,e)\, dx\right) - h''(e)$. Given the assumptions, we have $\frac{dU}{de}|_{e=0} > 0$ and $\frac{d^2U}{de^2} < 0$. Therefore, there is a unique $e^* > 0$ that solves equation (3). Differentiating equation (4) with respect to β and e yields

$$\frac{de}{d\beta} = -\frac{l\left(\int_0^A -F_e(x,e)\, dx\right)}{\frac{d^2U}{de^2}} > 0,$$

which states that higher effort would be forthcoming from the entrepreneurs, should they receive a larger share of the profit from innovations (i.e., a higher β).

Given the entrepreneurs' problem, the financial intermediary's profit is

$$R^* = (1-\beta)\int_0^A lxf(x,e)\, dx = (1-\beta)\left[A - \int_0^A F(x,e)\, dx\right]. \tag{5}$$

Taking derivative of R^* with respect to β yields

$$\frac{\partial R^*}{\partial \beta} = -\left(A - \int_0^A F(x,e)\,dx\right) + (1-\beta)\left(-\int_0^A F_e(x,e)\,dx\right)\frac{de}{d\beta}. \quad (6)$$

To move further, we make one additional assumption: $h''(e)$ is sufficiently small (or $F_e(x,e)$ is sufficiently large). Given this assumption, we have $\frac{\partial R^*}{\partial \beta}|_{\beta=0} > 0$. The proof is trivial. Since $\frac{de}{d\beta}|_{\beta=0} = \frac{l\left(\int_0^A -F_e(x,e)dx\right)}{h''(e)}$, when $h''(e)$ is sufficiently small, $\frac{de}{d\beta}|_{\beta=0}$ will be sufficiently large to make sure $\frac{\partial R^*}{\partial \beta}|_{\beta=0} > 0$. Moreover, the assumption ensures that $\frac{\partial^2 R^*}{\partial \beta^2} < 0$. When $\beta = 1$, $\frac{\partial R^*}{\partial \beta}|_{\beta=1} < 0$. Therefore, the return to savings in the financial intermediaries is an inverted-U function of β, which will be used in proving proposition 2.

2.3 The Interest Rate

The financial intermediaries represent those who save. Using the free-entry condition in the intermediary services (i.e., the value of each project to the financial intermediary (V) is equal to the cost of financing each project, η) and the asset equation (i.e., the period capital return is equal to the flow profit of the firms), the expected interest rate is

$$rV = r\eta = R^* \Longrightarrow r = \frac{R^*}{\eta}. \quad (7)$$

2.4 The Behavior of Households and the General Equilibrium

Proposition 1. *The model has a balanced growth path.*

Proof: The model has two representative agents: a worker and an entrepreneur. We first prove that the consumption growth rate of a worker is same as that of an entrepreneur. A typical worker maximizes the present discounted value of her consumption stream

$$\underset{c}{Max} \int_0^\infty \frac{c^{1-\theta}-1}{1-\theta}\exp(-\rho t)\,dt$$
$$s.t.\;\; c + (1-\phi)\eta \dot{N} = wL + (1-\phi)r\eta N,$$

where $(1-\phi) \in (0,1)$ is the share of R&D cost covered by the savings of the workers in the financial intermediaries, which is fixed on a balanced growth path.[17] On a balanced growth path, the workers finance $(1-\phi)$ share of the R&D cost and receive $(1-\phi)$ share of the profits from R&D via the financial intermediary. Solving Hamiltonian: $H = e^{-\rho t}\frac{c^{1-\theta}-1}{1-\theta} + \lambda\left(\frac{wL+(1-\alpha)r\eta N-c}{(1-\alpha)\eta}\right)$ yields the growth rate of workers' consumption as

$$\frac{\dot{c}}{c} = \frac{1}{\theta}(r-\rho) = \frac{(1-\beta)\left[A - \int_0^A F(x,e)\,dx\right]}{\theta\eta} - \frac{\rho}{\theta}, \quad (8)$$

[17] Otherwise, in steady state, ϕ approaches either 1 or 0. Both cases are simpler and growth rate is determined by the savings of either entrepreneurs or workers. When ϕ is unity, it becomes a one representative agent model like most NGMs. When ϕ is zero (i.e., the entrepreneurs do not save), the proof of the existence of a balanced growth path will be much simpler and the results of the paper hold.

where the last equality uses equation (7). The objective function of the entrepreneur is

$$\underset{e,c}{Max} \int_0^{\infty} \frac{[c - N \cdot h(e)]^{1-\theta} - 1}{1-\theta} \exp(-\rho t)\, dt$$
$$s.t. \qquad c + \phi\eta\dot{N} = N\pi(e) + r\phi\eta N.$$

Similarly, on a balanced growth path, the entrepreneur will finance ϕ share of the R&D cost and receive ϕ proportion of the profits of innovations via the financial intermediaries. To solve for the entrepreneur's effort, we only need to solve the period-by-period maximization problem of the entrepreneur. To confirm this, we substitute out c in the entrepreneur's objective function with the budget constraint to get an equivalent maximization problem of the entrepreneur as

$$\underset{e}{Max} \int_0^{\infty} \frac{N^{1-\theta}\left[\pi(e) - h(e) + r\phi\eta N - \phi\eta\frac{\dot{N}}{N}\right]^{1-\theta} - 1}{1-\theta} \exp(-\rho t)\, dt.$$

The entrepreneur's choice of effort is independent of her consumption decisions. When the entrepreneur chooses effort, she won't take into account the effect of her effort on the return to the financial intermediaries (or the balanced growth rate, $\frac{\dot{N}}{N}$). In other words, even though the entrepreneur saves through financial intermediaries, she, due to the large market effect, will not take into account the effect (externality) of her effort on the whole economy. Resultantly, the optimal effort of the entrepreneur is governed by maximizing equation (3), which yields e^*. Now defining new variables $\widetilde{c} = c - N \cdot h(e^*)$ and $\widetilde{\pi}(e^*) = N\pi(e^*) - N \cdot h(e^*)$, we rewrite the entrepreneur's problem as

$$\underset{\widetilde{c}}{Max} \int_0^{\infty} \frac{\widetilde{c}^{1-\theta} - 1}{1-\theta} \exp(-\rho t)\, dt$$
$$s.t. \qquad \widetilde{c} + \phi\eta\dot{N} = N\widetilde{\pi}(e^*) + r\phi\eta N.$$

Solving the Hamiltonian yields $\frac{\dot{\widetilde{c}}}{\widetilde{c}} = \frac{1}{\theta}(r - \rho)$, which is the same as that for the workers. Then it is straightforward to show that the model has a balanced growth path on which the consumptions of the workers and the entrepreneurs, the final output, and the number of varieties (innovations) N grow at the same rate. Because of Barro and Sala-i-Martin's (2004, ch. 6) excellent proof of it, we shall omit the proof. Since $\widetilde{c}$ and N grow at the same rate, given that e^* is constant, the consumption growth rate of the entrepreneurs $c_e = \widetilde{c} + N \cdot h(e^*)$ will also equal that of N. Therefore, on a balanced growth path, we have $\frac{\dot{c}}{c} = \frac{\dot{c}_e}{c_e} = \frac{\dot{N}}{N} = \frac{\dot{Y}}{Y}$, where c denotes the consumption of the workers, c_e the consumption of the entrepreneurs, N the number of varieties, and Y the total output. That is, different groups of people share the same growth rate of consumption, although their consumption levels differ. A worker's consumption grows at rate $\frac{\dot{N}}{N}$. An entrepreneur's income from savings grows at rate $\frac{\dot{N}}{N}$, and so does her income from innovations, $\pi^* N$, both of which grow at rate $\frac{\dot{N}}{N}$. Therefore, the consumption of the entrepreneurs grows at rate $\frac{\dot{N}}{N}$.

2.5 The Growth Rate and the Entrepreneur's Inventive Incentive

Proposition 2. *The balanced growth rate is an inverted-U function of β.*

Proof: The balanced growth rate is given in equation (8). Therefore, the balanced growth rate is linear in the return to the savings in the financial intermediaries (R^*). Since R^* is *an inverted-U function of* β (see the proof at the end of subsection 2.2), so is the balanced growth rate. Q.E.D. The detailed mechanism is as follows.

An increase in β (the EII) has two opposing effects on the growth rate. On the one hand, a higher β stimulates the entrepreneurs to contribute more effort in R&D, raising the expected profit from innovations. This "bigger cake effect" pushes up the growth rate. On the other hand, a higher β leaves a lower share to the savers-households (a lower $(1-\beta)$), which makes them less willing to finance innovations. The "smaller households' share effect" decreases the growth rate. In equation (6), the "bigger cake effect" is $(1-\beta)\left(-\int_0^A F_e(x,e)\,dx\right)\frac{de}{d\beta}$, and "smaller households' share effect" is $-\left(A-\int_0^A F(x,e)\,dx\right)$. When $\beta=0$, the former effect dominates, so the balanced growth rate increases as β goes up. This is mainly because a marginal increase in the EII elicits a very large increase in entrepreneur's effort. When β approaches 1, the latter effect dominates because the former effect approaches zero. Although a marginal increase in the EII yields a larger cake, the share to the households is so small that its product with the size of the cake (i.e., the return to the savings of the households) decreases. Resultantly, the growth rate will decrease as β approaches 1. Therefore, an atemporal maximization of the whole cake does not necessarily yield the highest growth in our intertemporal environment.

2.6 The Income Gap and the Possibility of a Poverty Trap

Although different groups of people share the same growth rate of income and consumption on a balanced growth path, their consumption levels are different. A worker's income as well as consumption grow at rate $\frac{\dot{N}}{N}$. An entrepreneur's income from savings grows at rate $\frac{\dot{N}}{N}$, and so does her income from innovations. Therefore, the difference in the income levels between the agents (i.e., the workers and the entrepreneurs) is linear in N, so it will grow at the balanced growth rate. An increase in the EII will make it jump up immediately. After that, it will grow at the rate of the balanced growth rate. The ratios of the income and wealth levels between agents are fixed on a balanced growth path, that is, they are not functions of N. However, following an increase in the EII (β), the income ratio between the workers and the entrepreneurs will also jump up and then stay fixed.

The possible existence of a poverty trap is presented in the following Corollary.

Corollary 1. *When β is too low/high, there may exist a poverty trap with no growth.*

Proof: The growth rate is given in equation (8). Therefore, when the expected return from savings is smaller than the time preference, the agents will not save to finance innovation. There emerges a poverty trap in which there is no endogenous growth. Q.E.D.

In least developing countries, even if the property rights are secure, if the EII is too low, less effort will be forthcoming from the entrepreneurs, lowering the return to innovations and thereby making households unwilling to finance R&D. Similarly, if the EII is too high, more effort from the entrepreneurs yields a larger cake, however, the households share little of the cake, which also makes them unwilling to finance R&D. To get out of the poverty trap, setting an optimal distribution rule between the entrepreneurs who borrow to conduct R&D and the households who save to finance R&D is desirable.

3 The Model with Hidden Information and Hidden Action

The structure of the model here is identical to that in section 2. The differences are as follows. This section deals with two types of asymmetric information: ex-ante hidden information that involves two types of entrepreneurs (the type with good projects called good entrepreneurs) and ex-post hidden action (good entrepreneurs' effort that affects the return of their projects is unobservable to others).[18] This information structure resembles the real world credit market. Therefore, it is meaningful to check whether the inverted-U relationship between long-run growth and the EII holds up. Given the information structure, the optimal contracts between the financial intermediaries and the entrepreneurs are assumed to be debt contracts combined with signals. Here, we prove that the debt contract combined with signals can be supported as an equilibrium, but we leave the proof of whether there may be other types of contracts (or what contracts are optimal) to future research. Debt contract has been proven to be optimal with nonrandom auditing (see Townsend, 1979; Williamson, 1987; Wang and Williamson, 1998). Moreover, in practice, it is an important arrangement between the entrepreneurs and the financial intermediaries argued by Wang and Williamson (1998).

Specifically, the random productivity of each innovation ends up with PDF $f_g\left(\widetilde{A}, e\right)$ with probability λ, and $f_b\left(\widetilde{A}\right)$ with probability $(1-\lambda)$, with $0 < \lambda < 1$, where $e\,(\geq 0)$ stands for the effort level contributed by the entrepreneurs. We call those projects with PDF $f_g\left(\widetilde{A}, e\right)$ "good" projects, whose owners are therefore "good" entrepreneurs. We term those projects with $f_b\left(\widetilde{A}\right)$ "bad" projects, whose owners are "bad" entrepreneurs. For simplicity, we assume that the PDF of bad projects cannot be affected by the effort of bad entrepreneurs. We assume that $F_g\left(x, e\right) < F_b\left(x\right)$, $\forall x \in [0, A]$, that is, the CDF of the random productivity of a good project first-order-stochastically dominates that of a bad one. Therefore, a good project always has a higher mean value of A (a higher expected profit). Moreover, we assume $F_e\left(x, e\right) < 0$ and $F_{ee}\left(x, e\right) \geq 0$.

[18] Ex ante and ex post (i.e., before or after contract is signed) information asymmetry problems have been reviewed in Mas-Colell et al. (1995, chs. 13 and 14).

Before any R&D is conducted, the ex ante types of the projects are unknown to all parties, so the entrepreneurs must sign a basic debt contract with the financial intermediaries to get credit for their R&D. That is, the debt contract is signed as if the projects were bad ones (please see section 3.1 for the detailed terms of a basic debt contract), which turns out to give bad entrepreneurs just their reservation utility. The optimality for the financial intermediaries to do so is given in Lemma 6 and the discussion following Lemma 6. After the entrepreneurs spend their credit in R&D, nature determines the types of their projects. Specifically, some entrepreneurs end up with "good" projects, while the rest end up with "bad" ones. The type is hidden information. Moreover, good entrepreneurs' effort is unobservable to others (i.e., hidden action, or a moral hazard problem).

Those entrepreneurs who have good projects will find it optimal to spend ς in signaling to renegotiate the terms of their contracts. Receiving the signal, the financial intermediaries will agree to change the terms of the contract to elicit more entrepreneur's effort, ending up with a higher return to both themselves and the entrepreneurs. The new contract is unattractive to the entrepreneurs with bad projects, so a separating signaling equilibrium exists. The good entrepreneurs and the financial intermediaries use costless Nash bargaining to share the monopolistic profit from innovations, and the entrepreneurs' share (i.e., bargaining power) is given as $\beta \in [0,1]$ (the measure of the EII).[19] β, which has a different role to play here, is assumed to be exogenously determined by the state and structure of the economy as discussed in section 2. Each entrepreneur has a reservation utility, $\overline{u}$, for each period.

Although contract renegotiation is often studied in case of a looming bankruptcy (e.g., as a part of a reorganization under Chapter 11 of the US bankruptcy law), many also investigate firm-driven renegotiations in case of success, for example, by switching from single to multiple banking relations. Bannier (2007), for example, argues: "Particularly young and small firms may find it difficult to credibly signal their quality in order to access the capital markets...Increasing information precision along the duration of the relationship [between the firm and the bank] should allow a more efficient renegotiation of credit conditions...First, due to her ability to renegotiate repayment conditions, the relationship bank may ease or tighten the firm's financial constraints by asking for a lower or higher repayment rate...the former leads to a beneficial smoothing of the firm's budget constraints." This supports our type of renegotiation between the successful entrepreneurs (i.e., those with good projects) and the financial intermediaries.

In the end, the ex post realization of $\widetilde{A}$ is known to everyone. Entrepreneurs pay back their loans in accordance with the contracts. The timing of the model is as follows:

1. The agents make their savings decisions, and if they save, they must save through financial intermediaries. Then the lottery for the patents to the intermediate goods

[19] It may seem unclear why need so many elements for our results. However, the assumptions are necessary (and sufficient) (proven in the working paper version of our model).

is announced. Those who win the lottery become entrepreneurs.

2. The entrepreneurs borrow from the financial intermediaries by signing a basic debt contract. After they spend credit on R&D, nature determines the types of the projects. The type and effort of the entrepreneurs cannot be observed by others.

3. If the projects turn out to be good, their owners have incentives to signal themselves to renegotiate with the financial intermediaries. In a separating equilibrium, after choosing effort, the good entrepreneurs determine how much to spend in signaling. Receiving the signal, the financial intermediaries agree to share the difference in the profits between the good projects and bad ones with the good entrepreneurs according to costless Nash bargaining. The share of the good entrepreneurs is β. The owners of the bad projects find it optimal to stick to the original contract.

4. $\widetilde{A}$ is realized and the profit from innovations is distributed according to the contracts.

The problem of final goods firms and that of intermediate goods firms are identical to those in section 2.1. Therefore, in the following we first study the optimal debt contracts.

3.1 Debt Contracts

Initially, a basic debt contract is offered to all entrepreneurs. The basic debt contract is signed according to a project with PDF $f_b\left(\widetilde{A}\right)$, and it states a payback rule as follows. The financial intermediary pays the R&D cost, η. If the ex post $\widetilde{A}$ is on the interval $[D_b, A]$ with $0 < D_b < A$, the entrepreneur pays back $D_b \cdot l$, where l is constant and equal to $L\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}}$; if the realization of $\widetilde{A}$ is on the interval $[0, D_b]$, the entrepreneur announces bankruptcy and the financial intermediary takes over the project without additional cost.

In a separating signaling equilibrium, the good entrepreneurs will find it optimal to signal their revealed type to secure a new debt contract. The new debt contract is unattractive to entrepreneurs whose projects turn out to be bad ones. We use backward induction to solve the separating signaling equilibrium.

3.1.1 Solving the Separating Signaling Equilibrium Taking Effort as Given

Step 1. Solving the Basic Debt Contract

In a separating signaling equilibrium, bad entrepreneurs find it optimal to spend nothing in signaling (i.e., they do not signal themselves) and stick to the basic debt contract:

$$\text{The entrepreneur's profit is} \begin{cases} l \cdot \left(\widetilde{A} - D_b\right) & ; \quad if\ \widetilde{A} \in [D_b, A] \\ 0 & ; \quad if\ \widetilde{A} \in [0, D_b] \end{cases}$$

$$\text{The financial intermediary receives} \begin{cases} l \cdot D_b\ ; & if\ \widetilde{A} \in [D_b, A] \\ l \cdot \widetilde{A} & ; \quad if\ \widetilde{A} \in [0, D_b] \end{cases}$$

where D_b needs to be solved for. The bad entrepreneurs must accept the terms of the basic debt contract, a take-it-or-leave-it offer by the financial intermediary. The reason is that the financial intermediaries have first-mover advantage. By a costless Nash bargaining assumed in our paper, the financial intermediaries will make a take-it-or-leave-it offer that just gives the bad entrepreneurs their reservation utility. The financial intermediary's problem is to maximize its expected profit, subject to the bad entrepreneur's participation constraint: The bad entrepreneur earns at least as much as her reservation utility $\overline{u}$. Solving the financial intermediary's problem in equations (9) and (10) delivers lemma 1.

$$\underset{D_b}{Max}\ R_b = \int_0^{D_b} l \cdot x f_b(x)\, dx + l \cdot D_b \left[1 - F_b(D_b)\right] \tag{9}$$

$$s.t.\ \pi_b = \int_{D_b}^{A} l \cdot (x - D_b) f_b(x)\, dx \geq \overline{u} \tag{10}$$

Lemma 1 *The optimal D_b^* is set to yield that the bad entrepreneur earns her reservation utility, and the financial intermediary gets the total expected profit from a bad project less the entrepreneur's reservation utility.*

Proof: At optimality, the participation constraint in equation (10) binds: $\pi_b^* = \overline{u}$. That is, a bad entrepreneur receives her reservation utility. Integrating (10) by parts yields the optimal D_b^* as in equation (11). Integrating the financial intermediary's objective function by parts and using equation (11), its expected profit from financing a bad project, given in equation (12), is the total expected profit from a bad project, $lE_b\left(\widetilde{A}\right)$, less the bad entrepreneur's reservation utility, $\overline{u}$. Q.E.D.

$$D_b^* - \int_0^{D_b^*} F_b(x)\, dx = E_b\left(\widetilde{A}\right) - \frac{\overline{u}}{l} \tag{11}$$

$$R_b^* = l \cdot E_b\left(\widetilde{A}\right) - \overline{u} \tag{12}$$

Step 2. Solving the Debt Contract for Entrepreneurs with Good Projects

The entrepreneurs with good projects find it optimal to renegotiate with the financial intermediaries for a new contract. To do so, they have to spend ς in signaling and bear all of the cost of signaling. The terms of the new debt contract are

The entrepreneur's profit is $\begin{cases} l \cdot \left(\widetilde{A} - D_g\right) - \varsigma; & if\ \widetilde{A} \in [D_g, A] \\ -\varsigma \quad ; & if\ \widetilde{A} \in [0, D_g] \end{cases}$

The financial intermediary receives $\begin{cases} l \cdot D_g \quad ; & if\ \widetilde{A} \in [D_g, A] \\ l \cdot \widetilde{A} \quad ; & if\ \widetilde{A} \in [0, D_g] \end{cases}$

where D_g and ς need to be determined. The difference between this debt contract and the basic one is that, the good entrepreneurs must signal their type each period to renegotiate D_g with the financial intermediaries. Specifically, the good entrepreneurs and the financial intermediaries use Nash bargaining to divide the difference in profits between a good project and a bad one, with the good entrepreneurs' share being β.

The good entrepreneurs' decision on effort is independent of their consumption decisions, which is proved in proposition 1. The good entrepreneurs' objective is

$$\underset{e,\varsigma}{Max}\ U_g = \int_{D_g}^{A} l \cdot (x - D_g)\, f_g(x,e)\, dx - h(e) - \varsigma \tag{13}$$

subject to

[1] D_g^* is determined by Nash bargaining;

[2] The bad entrepreneurs' incentive constraint, which makes sure the contract of $\{\varsigma^*, D_g^*\}$ is not better than $\{0, D_b^*\}$;

[3] The good entrepreneurs' incentive constraint, which makes sure the contract of $\{\varsigma^*, D_g^*(e^*)\}$ is not worse than $\{0, D_b^*(\widehat{e})\}$;

[4] The good entrepreneurs earn at least as much from contract $\{0, D_b^*(\widehat{e})\}$;

[5] The financial intermediaries' incentive constraint, that is, they earn at least as much from contract $\{\varsigma^*, D_g^*(e^*)\}$ as that from contract $\{0, D_b^*(\widehat{e})\}$.

where $\widehat{e}$ and e^* are the effort levels of the good entrepreneurs corresponding to D_b^* and D_g^* respectively. Now the financial intermediary's profit is

$$R_g = l \int_0^{D_g} x f_g(x,e)\, dx + l \cdot D_g \left[1 - F_g(D_g, e)\right].$$

Now solving the good entrepreneurs' maximization problem proceeds as follows.

Constraints [1], [4] and [5] are considered first.

Constraints [4] and [5] are determined by the case of $\beta = 0$, in which the financial intermediaries stick to offer the same basic debt contract D_b^* to both types of entrepreneurs. The good entrepreneurs will not signal themselves and contribute an optimal amount of effort, denoted as $\widehat{e}$, into their projects. $\widehat{e}$ is solved as

$$\widehat{e} = \arg\max_{e} U_g = \int_{D_b^*}^{A} l \cdot (x - D_b^*)\, f_g(x,e)\, dx - h(e). \tag{14}$$

Given $\widehat{e}$ and D_b^*, the good entrepreneur and the financial intermediaries receive

$$\begin{aligned} \widehat{U}_g &= \int_{D_b^*}^{A} l \cdot (x - D_b^*)\, f_g(x,\widehat{e})\, dx - h(\widehat{e}) & (15)\\ \widehat{R}_g &= l\left(D_b^* - \int_0^{D_b^*} F_g(x,\widehat{e})\, dx\right), & (16) \end{aligned}$$

which are their reservation utilities in Nash Bargaining.

If $\beta > 0$, then the good entrepreneurs have incentives to signal themselves. Since they bear the signaling cost, it is not involved in renegotiation. Receiving the signal, the financial intermediary and the good entrepreneur use a costless Nash bargaining to divide the profit from the good project less the sum of their reservation prices given in equations (15) and (16). The sum of the two parties' reservation utilities is $\widehat{\Pi}(\widehat{e}) = lE_g\left(\widetilde{A}, \widehat{e}\right) - h(\widehat{e})$, which is the total profit from a good project less the cost of good entrepreneur's effort in the case of $\beta = 0$. Given D_b^*, the reservation prices are already determined.

Using Nash bargaining to deal with constraints [4] and [5]:

$$\underset{D_g}{Max}\left(u_g - \widehat{U}_g\right)^{\beta}\left(R_g - \widehat{R}_g\right)^{1-\beta}$$

where u_g and R_g are the utility of the good entrepreneur and the profit of the financial intermediary in the new debt contract respectively. The solution is $u_g - \widehat{U}_g = \frac{\beta}{1-\beta}\left(R_g - \widehat{R}_g\right)$. Given $\left(u_g - \widehat{U}_g\right) + \left(R_g - \widehat{R}_g\right) = \Pi(e) - \widehat{\Pi}(\widehat{e})$, where $\Pi(e) = lE_g\left(\widetilde{A}, e\right) - h(e)$ and $\widehat{\Pi}(\widehat{e}) = \widehat{U}_g + \widehat{R}_g = lE_g\left(\widetilde{A}, \widehat{e}\right) - h(\widehat{e})$, we have

$$u_g - \widehat{U}_g = \beta\left[\Pi(e) - \widehat{\Pi}(\widehat{e})\right] \tag{17}$$

$$R_g - \widehat{R}_g = (1-\beta)\left[\Pi(e) - \widehat{\Pi}(\widehat{e})\right] \tag{18}$$

The Nash bargaining solution in equations (17) and (18) says: besides their reservation prices, the good entrepreneur receives β share of the increase in the total surplus $\Pi(e) - \widehat{\Pi}(\widehat{e})$, while the financial intermediary receives the remaining $(1-\beta)$ share.

If $\Pi(e) > \widehat{\Pi}(\widehat{e})$, receiving the signal ς, the financial intermediary would agree to change the basic debt contract into a new one (i.e., changing D_b^* into D_g^*) for the good entrepreneur. In so doing, the financial intermediary receives a higher return: $R_g > \widehat{R}_g$. Since the good entrepreneur bears all of the signaling cost, her final utility is $U_g = (u_g - \varsigma)$.

Lemma 2 *In a separating equilibrium, constraint [1] is solved, that is, D_g is solved as a function of effort. Further, D_g can be solved for independent of signal ς. Moreover, D_g^* is a decreasing function of β, that is, $\frac{\partial D_g^*}{\partial \beta} < 0$.*

Proof: $R_g = l\left(D_g - \int_0^{D_g} F_g(x,e)\,dx\right) = (1-\beta)\left[\Pi(e) - \widehat{\Pi}(\widehat{e})\right] + \widehat{R}_g \Longrightarrow$

$$l\left(D_g - \int_0^{D_g} F_g(x,e)\,dx\right) = (1-\beta)\left[\Pi(e) - \widehat{\Pi}(\widehat{e})\right] + \widehat{R}_g \tag{19}$$

From equation (19), the optimal D_g is a function of e only, independent of signal ς. Therefore, constraint [1] is solved. The reaction function of D_g as a function of e will be used later on to pin down the optimal effort level e^*. Taking the partial derivative of D_g with respect to β will deliver $\frac{\partial D_g^*}{\partial \beta} < 0$. Q.E.D.

Now we check constraints [2] and [3].

$$\pi_b' = \int_{D_g}^{A} l \cdot (x - D_g) f_b(x) dx - \varsigma \leq \pi_b^* = \overline{u} \qquad [2]$$

$$U_g(\widehat{e}) = \int_{D_b^*}^{A} l(x - D_b^*) f_g(x, \widehat{e}) dx - h(\widehat{e}) \leq U_g(e^*) \qquad [3]$$

where in constraint [3], $e^* = \arg\max_e U_g = \int_{D_g}^{A} l(x - D_g) f_g(x,e) dx - h(e) - \varsigma$; $\widehat{e}$ is determined by equation (6), and $\varsigma(\widehat{e}) = 0$.

Lemma 3 *For a separating equilibrium to exist, we must have $D_g^* < D_b^*$, that is, the interest rate ($\frac{D \cdot l}{\eta}$) is lower for the good entrepreneurs than for the bad entrepreneurs.*

Proof: See the Appendix.

The intuition is that, the profit of the bad entrepreneurs is a decreasing function of D. If $D_g > D_b^*$, then pretending to be good entrepreneurs is unattractive to the bad entrepreneurs, so the good entrepreneurs will not spend anything in signaling (i.e., $\varsigma = 0$). Therefore, no separating equilibrium would exist.

Lemma 4 *The optimal ς is to make constraint [2] bind, and it is positive. In addition, constraint [3] is satisfied if constraint [2] holds with equality.*

Proof: See the Appendix.

The first part of lemma 4 states that the optimal signal ς^* makes the bad entrepreneurs indifferent between the basic debt contract (D_b^*) and the new debt contract $\left(D_g^*\right)$, so they choose to stick to the basic debt contract. Signals lower than ς^* do not separate the two types of the entrepreneurs, and signals higher than ς^* give the good entrepreneurs lower utility. Therefore, the good entrepreneurs will always spend just ς^* in signaling.

The second part of lemma 4 reveals that the good entrepreneurs will always find it optimal to signal themselves to renegotiate with the financial intermediaries. Observing the signal ς^*, the financial intermediaries would agree to offer a new debt contract for the good entrepreneurs. In so doing, both parties are better off.

3.1.2 Solving for the Optimal Effort Levels e_g^* and e_b^*

Taking into account the debt contracts solved, the entrepreneurs choose their optimal effort to maximize their utility, which produces lemma 5.

Lemma 5 *e_g^* and e_b^* can be solved. (1) $e_b^* = 0$. (2) If $F_g(x,e)$ first-order-stochastically dominates $F_b(x)$ and $F_g(x,e)$ is elastic in e, then $e_g^* > \widehat{e}$ for $\forall \beta > 0$, which ensures $\Pi(e^*) > \widehat{\Pi}(\widehat{e})$. Then constraints [1] to [5] are satisfied and a separating signaling equilibrium exists. In addition, e_g^* is an increasing function of β: $\frac{de_g^*(\beta)}{d\beta} > 0$, $\forall \beta \in [0,1]$.*

Proof: See the Appendix.

Lemma 5 ensures the existence of a separating signaling equilibrium. After the effort levels are solved, lemma 2 yields D_g^*, lemma 1 delivers D_b^*, and lemma 4 pins down ς^*. The separating signaling equilibrium with two types of debt contracts is solved.

Lemma 6 *When a separating signaling equilibrium exists, if λ (the share of the good projects) is small enough, there is no pooling equilibrium that dominates the separating equilibrium for the financial intermediaries.*

Proof: The proof is similar to those in a standard textbook. The reason is the same: the benefit of signaling (the information benefit) dominates the cost of signaling. Q.E.D.

Given lemma 6, it is easy to justify why the financial intermediaries offer the basic debt contract instead of a pooling debt contract to all entrepreneurs at the beginning.

To recap, after the entrepreneurs sign a basic debt contract and spend the credit on R&D, they know the types of their projects. If the CDF of the good projects first-order-stochastically dominates that of bad ones, and the CDF of the good projects is affected by entrepreneurs' effort, then the good entrepreneurs find it optimal to signal themselves to renegotiate with the financial intermediaries for a new contract. The financial intermediaries agree to do so, which elicits a higher effort from the good entrepreneurs that yields higher profits to both financial intermediaries and the good entrepreneurs. The cost of signaling is just high enough to make the bad entrepreneurs stick to the basic debt contract. A separating signaling equilibrium emerges (see the illustration in figure 1).

[Figure 1 Here]

3.2 The Interest Rate and the General Equilibrium

With the existence of a separating signaling equilibrium, using the free-entry condition and the capital-asset equation, the expected interest rate is

$$\frac{E(R)}{\overline{r}} = \eta \Longrightarrow \overline{r} = \frac{(1-\lambda) R_b^* + \lambda \left[\widehat{R}_g + (1-\beta)\left[\Pi(e^*) - \widehat{\Pi}(\widehat{e})\right]\right]}{\eta}. \tag{20}$$

The proof of the existence of a balanced growth path is found in section 2.4. On the balanced growth path, the consumptions of the workers and that of the entrepreneurs, the final output, and N all grow at the same rate. That is, $\frac{\dot{c}}{c} = \frac{\dot{c}_g}{c_g} = \frac{\dot{c}_b}{c_b} = \frac{\dot{N}}{N} = \frac{\dot{Y}}{Y}$, where c denotes the consumption of the workers, c_g the consumption of the good entrepreneurs, c_b the consumption of the bad entrepreneurs, N the number of varieties, and Y the total output. Solving the Hamiltonian for the workers: $H = e^{-\rho t}\frac{c^{1-\theta}-1}{1-\theta} + \lambda\left(\frac{wL+(1-\alpha)r\eta N-c}{(1-\alpha)\eta}\right)$ yields the balanced growth rate as

$$g = \frac{1}{\theta}(\overline{r}-\rho) = \frac{(1-\lambda) R_b^* + \lambda \left[\widehat{R}_g + (1-\beta)\left(\Pi(e^*) - \widehat{\Pi}(\widehat{e})\right)\right]}{\theta\eta} - \frac{\rho}{\theta}, \tag{21}$$

where the last equality uses equation (20).

3.3 The Growth Rate and the Entrepreneur's Inventive Incentive

Proposition 3. *Given the conditions in lemma 5 are satisfied, then* $\frac{\partial g}{\partial \beta}|_{\beta=0} > 0$ *and* $\frac{\partial g}{\partial \beta}|_{\beta=1} < 0$*: The bigger-cake effect dominates and is dominated by the smaller-household-share effect at* $\beta = 0$ *and* $\beta = 1$ *respectively. If* $\frac{d^2 e}{d\beta^2} < \left[\frac{\frac{\partial \Pi(e^*)}{\partial e} \frac{de}{d\beta}}{(1-\beta)} - \frac{\partial^2 \Pi(e^*)}{\partial e^2} \left(\frac{de}{d\beta} \right)^2 \right] / \frac{\partial \Pi(e^*)}{\partial e}$, *then the balanced growth rate is an inverted-U function of* β *(the EII) and there exists a unique* $\beta^* \in (0,1)$ *that maximizes the balanced growth rate.*

Proof: First, if the conditions in lemma 5 are satisfied, then there exists a separating signaling equilibrium in which the balanced growth rate is given in equation (21). Taking the derivative of the balanced growth rate with respect to β produces

$$\frac{\partial g}{\partial \beta} = \frac{\lambda}{\theta \eta} \left(- \left[\Pi(e^*) - \widehat{\Pi}(\widehat{e}) \right] + (1-\beta) \frac{\partial \Pi(e^*)}{\partial e} \frac{de}{d\beta} \right), \tag{22}$$

where $\Pi(e^*) = l E_g \left(\widetilde{A}, e^* \right) - h(e^*) = l \left(A - \int_0^A F_g(x, e^*)\, dx \right) - h(e^*)$. When $\beta = 0$, then $e^* = \widehat{e}$ and $\Pi(e^*) = \widehat{\Pi}(\widehat{e})$. Given $\frac{\partial \Pi(\widehat{e})}{\partial e}|_{\widehat{e}} > \frac{\partial \widehat{U}_g}{\partial e}|_{\widehat{e}} = 0$, equation (22) delivers

$$\frac{\partial g}{\partial \beta}|_{\beta=\widehat{\beta}, e=\widehat{e}} = \frac{\lambda}{\theta \eta} \left(\frac{\partial \Pi(\widehat{e})}{\partial e}|_{\widehat{e}} \right) \frac{de}{d\beta} > 0 \tag{23}$$

$$\frac{\partial g}{\partial \beta}|_{\beta=1} = -\frac{\lambda}{\theta \eta} \left[\Pi(e^*) - \widehat{\Pi}(\widehat{e}) \right] < 0. \tag{24}$$

Taking derivative of equation (22) with respect to β, we have

$$\frac{\partial^2 g}{\partial \beta^2} = \frac{\lambda}{\theta \eta} \left(- \frac{\partial \Pi(e^*)}{\partial e} \frac{de}{d\beta} + (1-\beta) \left[\frac{\partial^2 \Pi(e^*)}{\partial e^2} \left(\frac{de}{d\beta} \right)^2 + \frac{\partial \Pi(e^*)}{\partial e} \frac{d^2 e}{d\beta^2} \right] \right). \tag{25}$$

Given that $\widehat{U}_g = l \left(A - D - \int_D^A F_g(x, e^*)\, dx \right) - h(e^*)$ is concave in e, then $\Pi(e^*)$ must be concave in e. It is obvious that $\frac{\partial \Pi(\widehat{e})}{\partial e}|_{\widehat{e}} > \frac{\partial \widehat{U}_g}{\partial e}|_{\widehat{e}} = 0$ because $\widehat{e}$ is maximizing $\widehat{U}_g$. Given the concavity of $\Pi(e^*)$, $\frac{de}{d\beta} > 0$ (from lemma 5), and if $\frac{d^2 e}{d\beta^2} < \left[\frac{\frac{\partial \Pi(e^*)}{\partial e} \frac{de}{d\beta}}{(1-\beta)} - \frac{\partial^2 \Pi(e^*)}{\partial e^2} \left(\frac{de}{d\beta} \right)^2 \right] / \frac{\partial \Pi(e^*)}{\partial e}$, we have $\frac{\partial^2 g}{\partial \beta^2} < 0$ (the balanced growth rate is an inverted-U function of β) as long as $\frac{\partial \Pi(e^*)}{\partial e} > 0$. $\frac{\partial \Pi(e^*)}{\partial e} > 0$ is ensured because the good entrepreneurs always equate their marginal cost of effort, $h'(e^*)$, with *their* share (β) of the marginal benefit of effort. Therefore, too little effort will be forthcoming from them, meaning the total (the combined share of the workers and the entrepreneurs) marginal benefit of effort must be larger than the marginal cost of effort, $h'(e^*)$, that is, $\frac{\partial \Pi(e^*)}{\partial e} > 0$. Then, equations (23) and (24) ensure that the balanced growth rate has a maximum at $\beta^* \in (0,1)$.

An increase in β (the EII) has two opposing effects on balanced growth rates. On the one hand, a higher β stimulates the good entrepreneurs to contribute more effort in R&D, raising the expected profit from innovations, $\Pi(e^*)$, because $\frac{\partial \Pi(e^*)}{\partial e} > 0$ and $\frac{de}{d\beta} > 0$. This "bigger cake effect" pushes up the growth rate. On the other hand, a higher β leaves a lower share to households, which makes them less willing to save to finance innovation. The "smaller households' share effect" decreases the growth rate.

In equation (22), the "bigger cake effect" is $(1-\beta)\frac{\partial \Pi(e^*)}{\partial e}\frac{de}{d\beta}$, and the "smaller households' share effect" is $-\left[\Pi(e^*) - \widehat{\Pi}(\widehat{e})\right]$. When $\beta = 0$, the former effect is positive, but the latter effect is zero because $\left[\Pi(e^*|_{\beta=0}) - \widehat{\Pi}(\widehat{e})\right] = \Pi(\widehat{e}) - \widehat{\Pi}(\widehat{e}) = 0$. Therefore, the balanced growth rate is increasing in β. As β increases from 0 to 1, the "bigger cake" effect is strictly decreasing. The "smaller household's share" effect is strictly increasing because, for one additional share given up by the household, the additional share of the cake is bigger beause effort is increasing. At β^*, the "smaller household's share effect" just increases to equal the decreasing "bigger cake" effect, and growth achieves a maximum. When $\beta = 1$, the former effect is zero while the latter effect is negative, so growth is decreasing in β. Therefore, the balanced growth rate is an inverted-U function of β.

The assumptions (two representative agents that make savings decisions, hidden information/type, and hidden action) are necessary for the inverted-U results with debt contract. Without any of the assumptions, we would not get a non-monotone relationship between the balanced growth rate and the EII. The detailed proof is omitted but is available upon request.

3.4 The Income Gap and the Possibility of a Poverty Trap

The dynamics of income inequality between agents and the possible existence of a poverty trap can be analyzed as in subsection 2.6. Although different groups of people share the same growth rate of income and consumption on a balanced growth path, their consumption levels differ. A worker's income grows at rate $\frac{\dot{N}}{N}$. A good entrepreneur's income from savings grows at rate $\frac{\dot{N}}{N}$, and so does her income from innovation, $\pi_g^* \lambda N$. A bad entrepreneur earns dividends from financial intermediaries and a profit from innovations, $(1-\lambda) N\overline{u}$, both of which grow at rate $\frac{\dot{N}}{N}$. Therefore, the difference in the income levels between agents is a function of N, so it will grow at the balanced growth rate. An increase in the EII will make it jump up immediately. After that, it will grow at the balanced growth rate. The ratios of the income and wealth levels between agents are fixed on a balanced growth path, that is, they are not functions of N. However, following an increase in the EII (β), the income ratio between the workers and the entrepreneurs and that between the good and bad entrepreneurs will also jump up and then stay fixed.

The possible existence of a poverty trap is presented in the following proposition.

Proposition 4. *At $\beta = 0$ or $\beta = 1$, if $\left[(1-\lambda) R_b^* + \lambda \widehat{R}_g\right]/\eta < \rho$, then there exists a* *poverty trap in which there is no balanced growth.*

Proof: Given the balanced growth rate in equation (21), we have

$$\text{Growth}|_{\beta=1} = \frac{(1-\lambda) R_b^* + \lambda \widehat{R}_g}{\theta\eta} - \frac{\rho}{\theta} = \text{Growth}|_{\beta=0}.$$

The latter equality holds because $\Pi(e^*)|_{\beta=0} = \widehat{\Pi}(\widehat{e})$. Therefore, when the average expected return from the two types of projects is smaller than the time preference, the agents will not save to finance innovation. There emerges a poverty trap in which there is no balanced growth. Q.E.D.

3.5 A Numerical Example

Here we use Maple to solve a numerical example of the model to evaluate the validity of the theoretical predictions and appreciate the magnitude of the EII's effect on growth. We choose the following distributions:

$$\begin{cases} F_g(x,e) = \begin{cases} 0; \; x \in [0, 10\ln(1+0.15e)] \\ 1-(1+0.15e)\exp(-0.1x) \; ; \; e \geq 0, \; x \in [10\ln(1+0.15e, +\infty) \end{cases} \\ F_b(x) = 1 - \exp(-0.25x) \; ; \; x \in [0, +\infty) \end{cases}$$

where e stands for effort. $F_g(x,e)$ first-order-stochastically dominates $F_b(x)$ (see figure 2). Both assumptions on $F_g(x,e)$: $F_{g,e}(x,e) < 0$ and $F_{g,ee}(x,e) = 0$ are satisfied.

[Figure 2 Here]

The cost function of entrepreneurs' effort is chosen as $h(e) = \frac{1}{2}e^2$, with both assumptions $h'(e) = e > 0$ for $e > 0$ and $h''(e) = 1 > 0$ satisfied. A standard production function assumes $\alpha = \frac{1}{3}$, which produces the constant $l = L\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}} \approx 0.1L$. L is the raw number of workers, which could be 10^8 (hundreds of millions) for large countries such as the US and 10^6 (millions) for smaller countries such as Singapore. For the entrepreneurs' reservation utility, $\overline{u}$, we use per capita GDP as the measure, which is around $10^4 \sim 10^5$ (tens of thousand). Hence, $\frac{\overline{u}}{l} \in [10^{-3}, 1]$. Given $E_b(x) = 4$, we choose $\frac{\overline{u}}{l} = 0.2$, which says that the entrepreneurs' reservation utility is around 5% of the total return of the projects. To calculate the balanced growth rate, θ is chosen to be one, which implies log utility. $\lambda = 0.9$ means the ex post fraction of good projects in whole projects is 90%. Time preference ρ is chosen to be 0.12.[20] $\eta = 50$ is chosen to ensure that balanced growth rates lie round 2% per year. All the functions and parameters are listed in Table 1.

[Table 1 Here]

The numerical model is solved as follows. First, solving the basic debt contract between the financial intermediaries and the bad entrepreneurs using lemma 1 yields $D_b^* = 11.98$

[20] This is consistent with the average time preference factor of 90% in Ventura (2003).

and $R_b^* = 3.80$. Second, we solve the debt contract between the good entrepreneurs and the financial intermediaries. Solving equations (14), (15) and (16) produces $\widehat{\varrho} = 0.45$, $\widehat{U}_g = 3.11$, $\widehat{R}_g = 7.46$, and $\widehat{\Pi} = \widehat{U}_g + \widehat{R}_g = 10.57$. These are the reservation prices in Nash Bargaining. Combining equations $(A11)$ and (19) yields the equilibrium debt contract $\left(D_g^*, \varrho^*\right)$ for each β. Lemma 4 gives ς^*, so the separating signaling equilibrium is solved. The equilibrium values of other endogenous variables are then calculated (see Table 2).

[Table 2 here]

The results in Table 2 confirm the validity of the lemmas and the propositions: D_g^* is an decreasing function of β; effort e^* is an increasing function of β, and it increases at a decreasing rate $\left(\frac{d^2e}{d\beta^2} < 0\right)$ that ensures the balanced growth rate g^* is an inverted-U function of β, with $\beta^* \approx 0.298$ maximizing the growth rate. As β increases from zero to β^*, the balance growth rate increases by roughly 0.4 percentage points. If we define $F_g(x, e) = 1-(1+\delta e)\exp(-\lambda_1 x)$ and $h(e) = \phi e^2$, where δ, λ_1 and ϕ are positive constants, then the balanced growth rate increases more by decreasing λ_1 and ϕ or increasing δ.[21] Nonetheless, "who would not be excited" (Solow, 2001) by the 0.4%, considering billions of people are still suffering from hunger and social conflicts (FAO, 2009[22]).

4 Conclusions

In this paper, we incorporate two representative agents (a credit-constrained entrepreneur borrows from a household to finance innovations) and micro-foundations and contractual arrangements on R&D in the presence of hidden information and moral hazard into NGMs. Schumpeter (1961) emphasizes that the possession of wealth is not necessary for the entrepreneurial function because the entrepreneurs can rent capital from the households to achieve creative destruction. The two representative agents assumption allows us to study how agents share the monopolistic profit on inventions affects long-run growth. Moreover, financial imperfection widely exists between the entrepreneurs and the creditors in real world (e.g., Paulson et al., 2006) and has been used to study growth and income inequality (e.g., Aghion et al., 1999, Banerjee and Newman, 1993) and to amplify business fluctuations (Bernanke and Gertler, 1989; Williamson, 1987).

Incorporating contracting into an intertemporal setting yields the "household-entrepreneur" problem, in which income gap and growth are simultaneously determined by primitive parameters. Especially there exists an inverted-U relationship between EII (entrepreneur's share) and the balanced growth rate. The implication is that, the differences in the EII can cause some economies to persistently grow slower than others. The possible existence of a poverty trap has strong implications for least developed countries. In these countries,

[21] For instance, with $\lambda_1 = 0.075$, or $\delta = 0.25$ (let alone the combinations of changes in the parameters), all else equal, as β increases from 0 to β^*, the balance growth rate increases by nearly 1 percentage point.

[22] Please refer to http://www.wfp.org/hunger/stats for details.

if the EII were constrained at a very low/high level, there would be no growth even if there are secure property rights. To get out of the poverty trap, it is desirable to set an optimal distribution rule between the entrepreneurs who borrow to conduct R&D and the households who save to finance R&D.

Our theoretical predictions are general in the sense that many policies and institutions have effects on the distribution of property rights on inventions, which would inevitably affect long-run growth and wealth inequality. One of such issues is taxation, which is left to future research. Nonetheless, parts of the findings of Cagetti and Nardi (2007) on taxation in the US are consistent with our predictions. Nevertheless, our paper has possible extensions. First, which form of contract (debt or equity) is optimal in the growth setting is left unproven. Second, the separation between consumption and leisure/effort in the utility function makes the dynamic contracting become an intratemporal one. Relaxing this assumption would further complicate the model and prevent us from getting analytic solutions. These important extensions are left to future research.

Appendix Proofs.

Lemma 3 *For a separating equilibrium to exist, we must have $D_g^* < D_b^*$, that is, the interest rate ($\frac{D \cdot l}{\eta}$) is lower for good entrepreneurs than for bad ones.*

Proof: The informed good entrepreneurs must spend some of their profits to signal themselves. Given effort, good entrepreneurs' utility is a decreasing function of the signal cost as in equation (13). Good entrepreneurs' utility is higher if the signal is smaller.

$$\underset{\varsigma}{Max}\colon U_g = \int_{D_g}^{A} l\left(x - D_g\right) f_g\left(x, e\right) dx - h\left(e\right) - \varsigma \tag{A1}$$

Then, from the profit function of bad entrepreneurs, we have

$$\pi_b = A - D - l \int_{D}^{A} F_b\left(x\right) dx \Longrightarrow \frac{\partial \pi_b}{\partial D} = -\left[1 - F_b\left(D\right)\right] < 0.$$

Thus the higher D is, the lower will be the profit of bad entrepreneurs. If $D_g > D_b^*$, then $\pi_b' = \int_{D_g}^{A} l\left(x - D_g\right) f_b\left(x\right) dx < \pi_b^*$. Good entrepreneurs will not spend anything in signaling ($\varsigma = 0$) and no separating equilibrium exists. For a separating equilibrium to exist, therefore, we must have $D_g^* < D_b^*$. In this case, good entrepreneurs have to make the signal so large that it will be unprofitable for bad entrepreneurs to pretend that they are good ones. Q.E.D.

Lemma 4 *The optimal ς is to make constraint [2] bind, and it is positive. In addition, constraint [3] is satisfied if constraint [2] holds with equality.*

Proof: From the proof of lemma 3, good entrepreneurs' utility is higher if the signal is smaller. Now we must use constraint [2] (the bad entrepreneurs' IC) to pin down the signal. By observing constraint [2], we get that the higher the signal, the lower the profit for a bad entrepreneur to pretend to be a good entrepreneur. Hence the optimal signal will just make constraint [2] bind. The minimum signal that makes constraint [2] bind is

$$\varsigma^{*}= l\left[\left(D_b^*-D_g^*\right)-\int_{D_g^*}^{D_b^*}F_b\left(x\right)dx\right] \tag{A2}$$

From lemma 3, $D_b^*>D_g^*$, thus $\left[\left(D_b^*-D_g^*\right)-\int_{D_g^*}^{D_b^*}F_b\left(x\right)dx\right]>0$, and hence $\varsigma^*>0$.

Now we can prove that constraint [3] is satisfied if constraint [2] holds with equality. We have $\widetilde{U}_g\left(\widehat{e}\right)=\int_{D_g^*}^{A}l\left(x-D_g^*\right)f_g\left(x,\widehat{e}\right)dx-h\left(\widehat{e}\right)-\varsigma^*\leq U_g\left(e^*\right)$, which is ensured by $e^*=\arg\max_{e}U_g=\int_{D_g^*}^{A}l\left(x-D_g^*\right)f_g\left(x,e\right)dx-h\left(e\right)-\varsigma^*$. Hence constraint [3], that is, $U_g\left(e^*\right)>U_g\left(\widehat{e}\right)$, is satisfied if $\widetilde{U}_g\left(\widehat{e}\right)>U_g\left(\widehat{e}\right)$. And $\widetilde{U}_g\left(\widehat{e}\right)>U_g\left(\widehat{e}\right)$ if and only if

$$\widehat{\pi}_g\left(\widehat{e}\right)=\int_{D_b^*}^{A}l\left(x-D_b^*\right)f_g\left(x,\widehat{e}\right)dx\leq\widetilde{\pi}_g\left(\widehat{e}\right)=\int_{D_g^*}^{A}l\left(x-D_g^*\right)f_g\left(x,\widehat{e}\right)dx-\varsigma^*$$

The above can be simplified as $\varsigma^*\leq l\cdot\left[\left(D_b^*-D_g^*\right)-\int_{D_g^*}^{D_b^*}F_g\left(x,\widehat{e}\right)dx\right]$.

Given that $F_g\left(x,0\right)<F_b\left(x\right)$ for $\forall x\in\left[0,A\right]$, and $F_e\left(x,e\right)<0$, we have $F_g\left(x,\widehat{e}\right)<F_b\left(x\right)$ for $\forall e>0$. This together with (A2) makes sure the above equation is satisfied. Hence constraint [3] is satisfied if constraint [2] holds with equality. Q.E.D.

Lemma 5 *e_g^* and e_b^* can be solved. (1) $e_b^*=0$. (2) If $F_g\left(x,e\right)$ first-order-stochastically dominates $F_b\left(x\right)$ and $F_g\left(x,e\right)$ is elastic in e, then $e_g^*>\widehat{e}$ for $\forall\beta>0$, which ensures $\Pi\left(e^*\right)>\widehat{\Pi}\left(\widehat{e}\right)$. Then constraints [1] to [5] are satisfied and a separating signaling equilibrium exists. In addition, e_g^* is an increasing function of β: $\frac{de_g^*(\beta)}{d\beta}>0$.*

Proof: With $u_g\left(e,\beta\right)$, $D_g\left(e,\beta\right)$ and $\varsigma^*\left(e,\beta\right)$ known to good entrepreneurs, they choose their optimal effort e^* by

$$\underset{e}{Max}\;U\left(e,\beta\right)=u_g\left(e,\beta\right)-\varsigma^*\left(e,\beta\right). \tag{A3}$$

We simplify $\varsigma^*\left(e,\beta\right)$ first. Combining equations (19), (11) and (12), we have

$$l\left(D_b^*-D_g^*\right)=l\left(\int_0^{D_b^*}F_b\left(x\right)dx-\int_0^{D_g^*}F_g\left(x,e\right)dx\right)-\left(1-\beta\right)\left[\Pi\left(e\right)-\widehat{\Pi}\left(\widehat{e}\right)\right]+R_b^*-\widehat{R}_g \tag{A4}$$

Plugging equation (A4) into equation (A2) to simplify $\varsigma^*\left(e,\beta\right)$ as

$$\varsigma^{*}= l\left(\int_0^{D_g^*}\left[F_b\left(x\right)dx-F_g\left(x,e\right)\right]dx\right)-\left(1-\beta\right)\left[\Pi\left(e\right)-\widehat{\Pi}\left(\widehat{e}\right)\right]+R_b^*-\widehat{R}_g \tag{A5}$$

Plugging $u_g\left(e,\beta\right)$ from equation (17) and $\varsigma^*\left(e,\beta\right)$ from equation (A5) into equation (A3), and using $\widehat{\Pi}(\widehat{e}) = \widehat{U}_g + \widehat{R}_g$, we get

$$\underset{e}{Max}: U(e,\beta) = \Pi(e) - l\left(\int_0^{D_g^*} [F_b(x)\,dx - F_g(x,e)]\,dx\right) - R_b^*$$

Using $\Pi(e) = l \cdot E_g\left(\widetilde{A}, e\right) - h(e)$, and equation (12): $R_b^* = l \cdot E_b\left(\widetilde{A}\right) - \overline{u}$, we get

$$\begin{aligned}
\underset{e}{Max}\, U(e,\beta) &= l\left(E_g\left(\widetilde{A}, e\right) - E_b\left(\widetilde{A}\right) - \int_0^{D_g^*} [F_b(x)\,dx - F_g(x,e)]\,dx\right) - h(e) + \overline{u} \\
&= l\left(\int_{D_g^*}^{A} [F_b(x)\,dx - F_g(x,e)]\,dx\right) - h(e) + \overline{u} \qquad \text{(A6)}
\end{aligned}$$

The latter equality using $E_g\left(\widetilde{A}, e\right) - E_b\left(\widetilde{A}\right) = \int_0^A [F_b(x)\,dx - F_g(x,e)]\,dx$. Now, the objective of good entrepreneur will be maximizing $U(e,\beta)$ by choosing e and taking into account the reaction of D_g^* as a function of e as in equation (19):

$$\begin{aligned}
&\underset{e}{Max}\, U(e,\beta) = l\left(\int_{D_g^*}^{A} [F_b(x)\,dx - F_g(x,e)]\,dx\right) - h(e) + \overline{u} \\
&s.t.\; l\left(D_g - \int_0^{D_g} F_g(x,e)\,dx\right) = (1-\beta)\left[\Pi(e) - \widehat{\Pi}(\widehat{e})\right] + \widehat{R}_g \qquad \text{(11)}
\end{aligned}$$

FOC:

$$\frac{dU}{de} = l\left(\int_{D_g^*}^{A} -F_{g,e}(x,e)\,dx - [F_b(D_g) - F_g(D_g,e)]\frac{\partial D_g}{\partial e}\right) - h'(e) \qquad \text{(A7)}$$

From equation (19), we have

$$\frac{\partial D_g}{\partial e} = \frac{(1-\beta)\left(\int_0^A -F_{g,e}(x,e)\,dx - h'(e)\right) - \int_0^{D_g} -F_{g,e}(x,e)\,dx}{1 - F_g(D_g,e)} \qquad \text{(A8)}$$

Given $\beta = 0$, then the solution corresponds to $e = \widehat{e}$, and $D_g^* = D_b^*$ with

$$\frac{d\widehat{U}_g}{de}|_{e=\widehat{e},\beta=0} = l\left(\int_{D_b^*}^{A} -F_{g,e}(x,\widehat{e})\,dx\right) - h'(\widehat{e}) = 0 \qquad \text{(A9)}$$

To see how the equilibrium values of e^* and D_g^* change with β, we can proceed as follows. First, evaluating $\frac{dU}{de}$ in equation (A7) using equation (A8) at $\beta = 0$ and $\widehat{e}$ produces

$$\frac{dU}{de}|_{e=\widehat{e},\beta=0} = \frac{F_b(D_g) - F_g(D_g,\widehat{e})}{1 - F_g(D_g,\widehat{e})}\left[l\left(\int_{D_g}^{A} -F_{g,e}(x,\widehat{e})\,dx\right) - h'(\widehat{e})\right] \qquad \text{(A10)}$$

Given an infinitely small increase in β, good entrepreneurs and financial intermediaries negotiate over D_g given that good entrepreneurs will choose e by maximizing their utility subject to equation (19). From lemma 2, D_g is a decreasing function of β, so the increase in β causes D_g to drop from D_b^*. Given equation (A9), the decrease of D_g will yield $\frac{dU}{de}|_{e=\widehat{e},\beta=0} > 0$ since $\int_{D_g}^{A} -F_{g,e}(x,\widehat{e})\,dx > \int_{D_b^*}^{A} -F_{g,e}(x,\widehat{e})\,dx$ given $D_g < D_b^*$ and $F_{g,e}(x,\widehat{e}) < 0$. Now, good entrepreneurs will adjust their effort. Given $U(e,\beta)$ is concave in effort e (see section 4.3), optimization given $\frac{dU}{de}|_{e=\widehat{e},\beta=0} > 0$ requires good entrepreneurs to increase their effort from $\widehat{e}$ to e_g^*. Therefore, optimal effort e_g^* is an increasing function of β: $\frac{de_g^*(\beta)}{d\beta} > 0$. Alternatively, total differentiating equation (A10) with respect to e and β, rearranging, we can show that $\frac{de_g^*(\beta)}{d\beta} > 0$ if $F_g(x,e)$ first-order-stochastically dominates $F_b(x)$ and $F_g(x,e)$ is elastic in e.

Since $U(e,\beta)$ is concave in e, $\Pi(e)$ is also concave in e. Given $\frac{dU}{de}|_{e=\widehat{e},\beta=0} > 0$, then

$$\frac{d\Pi}{de}|_{e=\widehat{e},\beta=0} = \left[l \left(\int_{D_g}^{A} -F_{g,e}(x,\widehat{e})\,dx \right) - h'(\widehat{e}) \right] > \frac{dU}{de}|_{e=\widehat{e},\beta=0} > 0$$

which produces $\Pi(e^*) > \widehat{\Pi}(\widehat{e})$. According to the Nash bargaining solution in (17) and (18), if $\Pi(e^*) > \widehat{\Pi}(\widehat{e})$, then $R_g^* > \widehat{R}_g$ and $U_g^* > \widehat{U}_g$. Therefore, constraints [4] and [5] are satisfied. Since the constraints [1] to [5] are satisfied, a separating signaling equilibrium exists.

Substituting equation (A8) into equation (A7), and simplifying, we get

$$\begin{aligned} \frac{dU}{de} &= \frac{[F_b(D_g) - F_g(D_g,e)]\,\beta \left[\int_0^A -F_{g,e}(x,e)\,dx\right] + (1 - F_b(D_g)) \left[\int_{D_g}^A -F_{g,e}(x,e)\,dx\right]}{1 - F_g(D_g,e)} \\ &\quad - \left(1 - \frac{(1-\beta)[F_b(D_g) - F_g(D_g,e)]}{1 - F_g(D_g,e)}\right) h'(e) \end{aligned} \tag{A11}$$

Now combining equation (A11) and equation (19) produces a two equations-two unknowns system, which delivers the equilibrium values of $\left(D_g^*, e^*\right)$.

With $\pi_b^* = \overline{u}$ and D_b^* are known to bad entrepreneurs, they choose their optimal effort level according to $\underset{e}{Max} U(e) = \overline{u} - h(e)$, which generates $e_b^* = 0$. Q.E.D.

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Table 1. Parameter Values and Functions

for Calculating Equilibrium Debt Contracts		for Calculating Growth Rates
$\alpha = \frac{1}{3}$	$h(e) = \frac{1}{2}e^2$	$\theta = 1$
$L = 10^7$	$F_g(x, e) = 1 - (1 + 0.15e)\exp(-0.1x)$	$\lambda = 0.9$
$l = 10^6$	$F_b(x) = 1 - \exp(-0.25x)$	$\eta = 50$
$\overline{u} = 2 \times 10^5$	$x = \widetilde{A} \in [0, +\infty)$	$\rho = 0.12$

Table 2. Equilibrium Values of Debt Contracts and Balanced Growth Rates

Given $\beta =$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$D_g^* =$	$11.98(D_b^*)$	11.50	11.29	11.09	10.90	10.73	10.57	10.42	10.27	10.14	10.01
$e^* =$	$0.45(\widehat{e})$	0.73	0.86	0.95	1.01	1.06	1.10	1.13	1.15	1.18	1.19
$g^* =$	2.18	2.41	2.52	2.54	2.52	2.49	2.44	2.38	2.32	2.25	2.18
$\frac{de^*}{d\beta} =$	4.29	1.73	1.04	0.72	0.54	0.43	0.35	0.29	0.25	0.21	0.19
$\frac{dg^*}{d\beta} =$	4.60	1.06	0.30	-0.00	-0.16	-0.24	-0.30	-0.33	-0.36	-0.38	-0.39
$U_g^* =$	$3.12(\widehat{U}_g)$	3.22	3.24	3.27	3.30	3.33	3.36	3.39	3.43	3.46	3.49
$R_g^* =$	$7.46(\widehat{R}_g)$	7.58	7.64	7.66	7.65	7.63	7.60	7.57	7.53	7.49	7.46
$R_b^* =$	3.80										
$D_g^* \leq D_b^*$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: g^* stands for the balanced growth rate; e^* for good entrepreneurs' effort.

$D_g^* \leq D_b^*$ means signaling separating equilibrium exists.

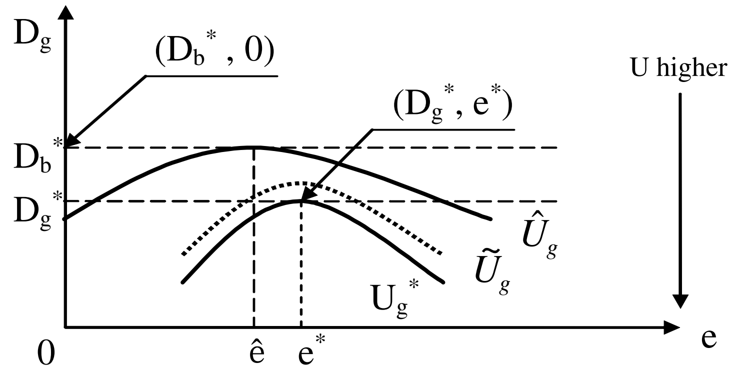

Figure 1. Entrepreneurs' Indifference Curves and Equilibrium Debt Contracts

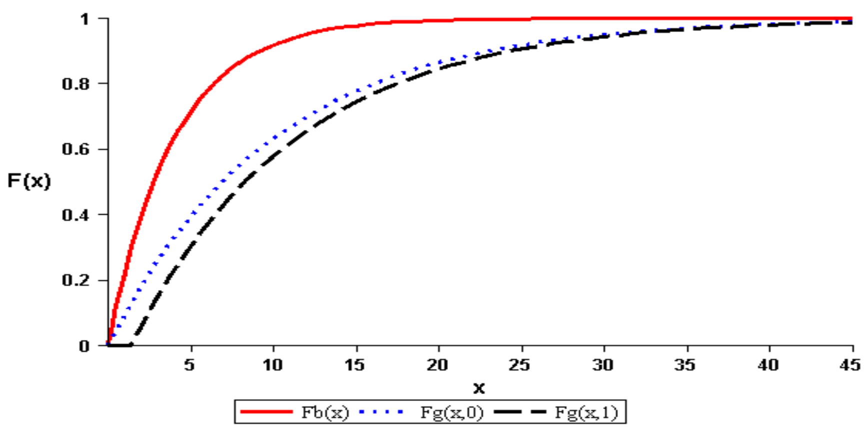

Figure 2. First-order-stochastic dominance of $F_g(x, e)$ over $F_b(x)$.