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September 2011

A New-Keynesian Model of the Yield Curve with Learning Dynamics: A Bayesian Evaluation*

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Abstract

We estimate a New-Keynesian macro-finance model of the yield curve incorporating learning by private agents with respect to the long-run expectation of inflation and the equilibrium real interest rate. A preliminary analysis shows that some liquidity premia, expressed as a degree of mispricing relative to no-arbitrage restrictions, and time variation in the prices of risk are important features of the data. These features are, therefore, included in our learning model. The model is estimated on U.S. data using Bayesian techniques. The learning model succeeds in explaining the yield curve movements in terms of macroeconomic shocks. The results also show that the introduction of a learning dynamics is not sufficient to explain the rejection of the extended expectations hypothesis. The learning mechanism, however, reveals some interesting points. We observe an important difference between the estimated inflation target of the central bank and the perceived long-run inflation expectation of private agents, implying the latter were weakly anchored. This is especially the case for the period from mid-1970s to mid-1990s. The learning model also allows a new interpretation of the standard level, slope, and curvature factors based on macroeconomic variables. In line with standard macro-finance models, the slope and curvature factors are mainly driven by exogenous monetary policy shocks. Most of the variation in the level factor, however, is due to shocks to the output-neutral real rate, in contrast to the mentioned literature which attributes most of its variation to long-run inflation expectations.

JEL classifications: E43; E44; E52
Keywords: New-Keynesian model; Affine yield curve model; Learning; Bayesian estimation

*An earlier version of this paper was circulated under the title "Imperfect information, macroeconomic dynamics and the yield curve: an encompassing macro-finance model". We are extremely grateful to Marie Donnay, Refet Gürkaynak, Pablo Rovira Kaltwasser, Raf Wouters, Taner Yigit, and seminar participants at the National Bank of Belgium and Bilkent University for very helpful comments.

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1 Introduction

The modeling of the term structure of interest rates has evolved significanlty since Duffie and Kan (1996) provided a complete characterization of the class of no-arbitrage affine models in which bond yields are a linear function of latent variables. The system proposed by Duffie and Kan was soon extended by a vector autoregressive (VAR) model including both latent factors and observable macroeconomic variables (see Ang and Piazzesi (2003)). This reduced-form framework naturally led to more structural approaches where the macroeconomic dynamics are governed by rational expectations linearized New-Keynesian models, as in Hördahl, Tristani, and Vestin (2006) and Rudebusch and Wu (2008). Although such New-Keynesian models impose a number of restrictions on the macro dynamics, the pricing kernel adopted in these models is still determined exogenously, allowing some flexibility in the specification of the risk premia. Wu (2006) and Bekaert, Cho, and Moreno (2010) come a step closer to the structure implied by dynamic stochastic general equilibrium (DSGE) models by imposing a stochastic discount factor consistent with the utility function of the representative agent of the linearized economy, leading to endogenous and constant prices of risk. The evolution of macro-finance models for the yield curve suggests that a possible benchmark for such models could be described by a New-Keynesian framework characterized by (*i*) rational expectations, (*ii*) the lack of arbitrage opportunities, and (*iii*) consistent and, therefore, constant prices of risk.

Despite the mentioned advances of macro-finance models, their empirical success in fitting the yield curve seems to depend on the inclusion of highly inert latent factors. Kozicki and Tinsley (2001) and Kozicki and Tinsley (2002) suggest that one such factor may be related to the long-run inflation expectation of agents (endpoints). Bekaert, Cho, and Moreno (2010), Dewachter and Lyrio (2006), and Hördahl, Tristani, and Vestin (2006) use a similar approach and introduce a time-varying inflation target of the central bank and show that it is crucial to explain the time variation in long-run yields. Dewachter and Lyrio (2008), on the other hand, propose an alternative model in which the rational expectations assumption is replaced by a learning mechanism which allows private agents to update their long-run expectations about inflation and the equilibrium real interest rate. These expectations seem to be sufficiently volatile to account for most of the variation in long-maturity yields. The inclusion of learning in yield curve models might also help clarify a common rejection of the extended expectations hypothesis. Empirical tests have consistently rejected the joint null hypothesis of rational expectations and the extended expectations hypothesis for the yield curve (see Shiller, Campbell, and Schoenholtz (1983)). In general, these rejections have been interpreted as a rejection of the expectations hypothesis. Kozicki and Tinsley (2005b) point out, however, that the introduction of learning by private agents with respect to the central bank's inflation target might generate sufficiently strong deviations from rational expectations to explain such rejections. This is the case since long-horizon yields depend on long-horizon expectations of the policy rate which incorporates inflation expectations. These expectations, in turn, are anchored by market perceptions regarding the central bank's inflation target.

This paper assesses the empirical success of a New-Keynesian macro-finance model for the yield curve incorporating learning by private agents with respect to the long-run values of macroeconomic variables.

Since our goal is to develop a model which is able to identify the economic sources behind movements in the yield curve with an improved ability to fit the data, we relax the other two restrictions imposed by consistent macro-finance models, i.e. (*i*) the absence of arbitrage opportunities, and (*ii*) the use of endogenous constant prices of risk. In order to assess the empirical implication of each of these restrictions, we first compare the performance of a benchmark model characterized by rational expectations, no-arbitrage, and consistent prices of risk with two extensions to this model. A first extension to the benchmark model allows for time-varying prices of risk and hence does not impose consistency between the pricing kernel and the linearized New-Keynesian macroeconomic framework. In our set-up, the prices of risk are a function of the observable macroeconomic variables. A second extension allows for liquidity premia, i.e. mispricing terms expressed as constant maturity-specific deviations of the actual yield curve from the one implied by no-arbitrage restrictions. These imply arbitrage possibilities which are difficult to justify within a pure macro-finance framework. We interpret these mispricing terms as liquidity or preferred habitat effects. Since both extensions turn out to be important, they are incorporated in our macro-finance model with learning. This final version allows us to evaluate the mentioned conjecutre by Kozicki and Tinsley (2005b) regarding the expectations hypothesis puzzle. The learning dynamics adopted in this paper is an extension to the one proposed by Kozicki and Tinsley (2005b) and Dewachter and Lyrio (2008). It allows private agents to update their perceived long-run expectations of inflation and the equilibrium real rate taking into consideration public and private signals, the latter consisting of exogenous belief shocks and endogenous adaptive learning.

All model versions are estimated on U.S. data using Bayesian techniques.[1] Although computationally intensive, this approach integrates informative priors avoiding unreasonable regions of the parameter space and numerical near singularities. The posterior distribution of the parameters is obtained using standard Markov Chain Monte Carlo (MCMC) methods based on three information sources: macroeconomic variables, the yield curve, and surveys of inflation expectations. The inclusion of survey data in the measurement equation is motivated by the need for the identification of the perceived macroeconomic dynamics.[2] Model versions are compared using the marginal likelihood of the respective models and the Schwarz Bayesian Information Criterion (BIC).

As mentioned before, our results indicate that some liquidity premia (mispricing) and time variation in the prices of risk are important features of the data. These features are, therefore, incorporated in a extended model with learning. Although the estimates for the structural part of this model are in general in line with the literature, the results show that the introduction of a learning dynamics is not sufficient to explain the rejection of the extended expectations hypothesis. The learning mechanism, however, reveals some interesting points. The results show an important disconnection between the inflation target of the central bank and the perceived long-run inflation expectation of private agents, implying that the latter were weakly anchored. This is especially the case from the mid-1970s to the mid-1990s. Also, this disconnection and the variability in the perceived output-neutral real rate seem important to explain

[1] The Bayesian approach is still less common in the macro-finance literature compared with methods which reliy on the Full Information Maximum Likelihood (FIML). Doh (2006, 2007) are exceptions.

[2] Chun (2011) also points out the importance of survey expectations to explain bond yields movements.

a significant part of the variability in long-term yields. Finally, the learning model also allows a new interpretation of the standard level, slope, and curvature factors based on macroeconomic variables. In line with standard macro-finance models, the slope and curvature factors are mainly driven by exogenous monetary policy shocks. Most of the variation in the level factor, however, is due to shocks to the output-neutral real rate, in contrast to the mentioned literature, which attributes most of its variation to long-run inflation expectations.

The remainder of the paper is organized as follows. In Section 2, we present a general macro-finance framework incorporating a New-Keynesian macro model with learning, liquidity premia (mispricing), and flexible prices of risk. Section 3 describes the econometric methodology used in the paper and Section 4 presents the empirical results. The latter includes a comparison among the alternative versions of the macro-finance model, a analysis of the posterior density of our extended learning model, and the implications for the yield curve. Section 5 summarizes the main findings of the paper.

2 The model

The class of macro-finance models for the yield curve is built around (*i*) a macroeconomic framework, described under the historical probability measure, and (*ii*) a financial framework, which models the term structure of interest rates under the risk-neutral measure. This section presents a macro-finance model which extends standard models in both dimensions.

For the macroeconomic dynamics, our benchmark case consists of a standard rational expectations New-Keynesian macro model, including unobserved variables representing the inflation target of the central bank and the output-neutral real interest rate. We extend this framework with the inclusion of learning by private agents with respect to the long-run inflation expectation and the output-neutral real interest rate. This gives rise to the distinction between actual and perceived laws of motion for the macroeconomy.

For the yield curve, our benchmark case assumes no-arbitrage and consistency between the stochastic discount factor and the structural macroeconomic framework, as in Bekaert, Cho, and Moreno (2010), which gives rise to endogenous and constant prices of risk. We extend this case allowing for (*i*) liquidity premia, expressed as constant mispricing terms relative to the no-arbitrage model, and (*ii*) time-varying prices of risk, which implies we do not impose consistency between the pricing kernel and the IS equation.

In the empirical section below, we first assess the separate impact of allowing for mispricing and time-varying prices of risk on the performance of the benchmark model. In this section, we present a model including all three features, i.e. learning, mispricing, and time-varying prices of risk. The benchmark case and its extensions can be easily recovered from this general set-up.

2.1 Macroeconomic dynamics

The macroeconomic dynamics is described by a standard New-Keynesian framework incorporating a Phillips curve (AS equation), an IS equation, and a monetary policy rule. Our structural model also includes two unobserved variables representing the inflation target of the central bank, π_t^*, and the output-neutral real interest rate, ρ_t^*.[3]

Profit maximization by price-setting firms leads to the Phillips curve relating current inflation, π_t, to real marginal costs, s_t. Assuming that real marginal costs are proportional to the output gap, y_t, and a cost-push shock, $v_{\pi,t}$, we obtain the standard aggregate supply curve:

$$\pi_t = c_{\pi,t} + \mu_{1,\pi} E_t \pi_{t+1} + \mu_{2,\pi} \pi_{t-1} + \kappa y_t + v_{\pi,t}, \tag{1}$$

$$c_{\pi,t} = (1 - \mu_{1,\pi} - \mu_{2,\pi}) \pi_t^*, \qquad \mu_{1,\pi} = \frac{\beta}{1 + \beta \delta_\pi}, \qquad \mu_{2,\pi} = \frac{\delta_\pi}{1 + \beta \delta_\pi}, \tag{2}$$

with the cost-push shock following a first-order autoregressive process:

$$v_{\pi,t} = \varphi_\pi v_{\pi,t-1} + \sigma_{v_\pi} \varepsilon_{v_\pi,t}. \tag{3}$$

with $\varepsilon_{v_\pi,t} \sim IID\ \mathcal{N}(0,1)$. This set-up is based on Calvo (1983) sticky price model in which at each period only a fraction of firms reoptimizes prices. Following Galí and Gertler (1999), we assume that nonoptimizing firms use the following indexation scheme:

$$z_{\pi,t} = \pi_t^* + \delta_\pi (\pi_{t-1} - \pi_t^*), \tag{4}$$

where $0 \leq \delta_\pi \leq 1$. We impose a vertical Phillips curve in the long run by restricting the discount factor, β, to 1. In this case, $\mu_{1,\pi} = (1 - \mu_{2,\pi})$.

We adopt a Fuhrer (2000) type of IS equation characterized by endogenous inertia in the output gap dynamics due to the inclusion of habit formation in the consumer's utility function. Maximization of consumer's expected utility leads to the standard IS equation:

$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y) y_{t-1} - \phi (i_t - E_t \pi_{t+1} - \rho_t^*) + v_{y,t}, \tag{5}$$

$$\mu_y = \frac{\sigma}{\sigma + h(\sigma - 1)}, \qquad \phi = \frac{1}{\sigma + h(\sigma - 1)}, \tag{6}$$

where σ and h represent the level of relative risk aversion and habit persistence, respectively, and $v_{y,t}$ follows a first-order autoregressive process:[4]

$$v_{y,t} = \varphi_y v_{y,t-1} + \sigma_{v_y} \varepsilon_{v_y,t}. \tag{7}$$

[3] The introduction of a time-varying equilibrium real rate is motivated by recent estimates for the U.S. by Laubach and Williams (2003), Clark and Kozicki (2004), and Bjørnland, Leitemo, and Maih (2008). Trehan and Wu (2007) also stress the importance of accounting for the time variation in the equilibrium real rate in the analysis of monetary policy. Additional evidence comes from the TIPS market. Gürkaynak, Sack, and Wright (2008) show that long-run real yields display significant and persistent time variation. We model this rate as a purely exogenous process, capturing persistent shocks in productivity, preferences, fiscal policy or financial premia.

[4] Consumer's utility function is given by

$$U(C_s, F_s) = \frac{F_s C_s^{1-\sigma} - 1}{1 - \sigma},$$

where C_s represents consumption and F_s is a combined factor consisting of preference shocks G_s and habit H_s, $F_s = G_s H_s$. Habit is specified as a function of past consumption, $H_s = C_{s-1}^{\eta}$, with $\eta = h(\sigma - 1)$ and $0 \leq h \leq 1$. Furthermore, $v_{y,t} = \phi \ln G_t$.

with $\varepsilon_{v_y,t} \sim IID\ \mathcal{N}(0,1)$. The output-neutral real interest rate is implicitly defined as the long-run equilibrium real interest rate. Ex-ante real rates $(i_t - E_t\pi_{t+1})$ above (below) ρ_t^* lead to a decrease (increase) in output. A relatively strong forward-looking behavior is an implicit characteristic of both the Phillips curve and the IS equation described above. This is due to the fact the parameters $\mu_{1,\pi}$ and μ_y are necessarily betwen 0.5 and 1.

The risk-free monetary policy interest rate, i_t, is modeled as an extended Taylor (1993) rule:

$$i_t = (1-\gamma_i)i_t^T + \gamma_i i_{t-1} + v_{i,t}, \tag{8}$$

$$v_{i,t} = \varphi_i v_{i,t-1} + \sigma_{v_i}\varepsilon_{v_i,t}, \tag{9}$$

with $\varepsilon_{v_i,t} \sim IID\ \mathcal{N}(0,1)$, and where $v_{i,t}$ is a autoregressive policy shock[5] and the target interest rate, i_t^T, is a function of both the inflation and output gaps:

$$i_t^T = \rho_t^* + E_t\pi_{t+1} + \gamma_\pi(\pi_t - \pi_t^*) + \gamma_y y_t. \tag{10}$$

This specification implies that for $\gamma_\pi > 0$ and $\gamma_y > 0$ central banks follow an active policy. The targeted real interest rate $(i_t^T - E_t\pi_{t+1})$ increases above (decreases below) ρ_t^* in function of positive (negative) inflation or output gaps:

$$i_t^T - E_t\pi_{t+1} - \rho_t^* = \gamma_\pi(\pi_t - \pi_t^*) + \gamma_y y_t. \tag{11}$$

Eq. (10) shows that ρ_t^* can also be interpreted as the real rate target of the central bank. Since in steady state $\pi_t = \pi_t^*$ and $y_t = 0$, the central bank implicitly aims at a real rate equal to ρ_t^*. Finally, the dynamics of π_t^* and ρ_t^* are modeled as random walks:

$$\begin{aligned} \pi_t^* &= \pi_{t-1}^* + \sigma_{\pi^*}\varepsilon_{\pi^*,t}, \\ \rho_t^* &= \rho_{t-1}^* + \sigma_{\rho^*}\varepsilon_{\rho^*,t}. \end{aligned} \tag{12}$$

with $\varepsilon_{\pi^*,t} \sim IID\ \mathcal{N}(0,1)$ and $\varepsilon_{\rho^*,t} \sim IID\ \mathcal{N}(0,1)$.This introduces stochastic endpoints for inflation and the risk-free interest rate.[6] By construction, inflation (π_t) and the real rate $(i_t - \pi_t)$ converge in expectation towards π_t^* and ρ_t^*, respectively ($\lim_{s\to\infty} E_t\pi_{t+s} = \pi_t^*$ and $\lim_{s\to\infty} E_t i_{t+s} = \pi_t^* + \rho_t^*$).

2.1.1 Learning dynamics

We follow Kozicki and Tinsley (2005a) and Doh (2007) and differentiate between the beliefs held by private agents and those held by the central bank.[7] We restrict these differences to the unobserved long-run tendencies of the economy, i.e. the long-run inflation expectation and the output-neutral real interest rate. Such differences can be motivated by the assumption that private agents have imperfect information with respect to the central bank's inflation target and the output-neutral real rate. One

[5] Therefore, next to allowing for endogenous inertia through inflation indexation, habit formation and interest rate smoothing, we also incorporate exogenous inertia induced by the autocorrelation of supply, demand, and policy shocks, respectively.

[6] As shown by Kozicki and Tinsley (2001), stochastic endpoints are crucial in modeling the link between macroeconomic variables and the yield curve. Most macro-finance models, however, only include one stochastic endpoint, i.e. the inflation target of the central bank.

[7] Orphanides and Wei (2010) also stress the importance of allowing for evolving beliefs about the macroeconomy dynamics to explain movements in long-term yields.

could also assume there is imperfect policy credibility of the central bank with respect to its inflation target. Therefore, we distinguish the stochastic trends perceived by private agents, π_t^{*P} and ρ_t^{*P}, from the actual stochastic trends inferred by the central bank, π_t^* and ρ_t^*. Conditional on the perceived stochastic endpoints, agents form their expectations rationally.[8] The perceived stochastic endpoints change over time according to the following rules:

$$\begin{aligned}
\pi_t^{*P} &= \pi_{t-1}^{*P} + \omega_{\pi^*} \underbrace{\sigma_{\pi^*}\varepsilon_{\pi^*,t}}_{\text{public signal}} + (1-\omega_{\pi^*})\underbrace{\left[\sigma_{\pi^{*b}}\varepsilon_{\pi^{*b},t} + g_\pi\left(\pi_t - E_{t-1}^P\pi_t\right)\right]}_{\text{private signal}}, \\
\rho_t^{*P} &= \rho_{t-1}^{*P} + \omega_{\rho^*} \underbrace{\sigma_{\rho^*}\varepsilon_{\rho^*,t}}_{\text{public signal}} + (1-\omega_{\rho^*})\underbrace{\left[\sigma_{\rho^{*b}}\varepsilon_{\rho^{*b},t} + g_\rho\left(i_t - \pi_t - E_{t-1}^P(i_t-\pi_t)\right)\right]}_{\text{private signal}},
\end{aligned} \qquad (13)$$

with initial conditions π_0^{*P} and ρ_0^{*P} and where E^P denotes the subjective expectations operator. The updating rules include two sources of information. (i) *Public signals:* These are the observed shocks ($\varepsilon_{\pi^*,t}$, $\varepsilon_{\rho^*,t}$) to the actual stochastic trends. The parameters ω_{π^*} and ω_{ρ^*} measure the weight attached to these signals and reflect their perceived quality. (ii) *Private signals:* These are subjective inferences regarding the changes in the stochastic trends and are composed of two shocks: (a) exogenous belief shocks ($\varepsilon_{\pi^{*b},t}$, $\varepsilon_{\rho^{*b},t}$); and (b) endogenous adaptive revisions of the perceived endpoints induced by prediction errors for inflation and the ex-post real interest rate with constant gains g_π and g_ρ, respectively.

The learning model in Eq. (13) incorporates a number of standard expectations formation processes. First, the full-information, rational expectations case implies that agents (i) perceive the observed signals $\varepsilon_{\pi^*,t}$ and $\varepsilon_{\rho^*,t}$ as fully informative, such that $\omega_{\pi^*} = \omega_{\rho^*} = 1$, and (ii) have perfect information regarding the initial state, i.e. $\pi_0^{*P} = \pi_0^*$ and $\rho_0^{*P} = \rho_0^*$. In the imperfect information case, where $0 < \omega_{\pi^*} < 1$ and $0 < \omega_{\rho^*} < 1$, agents attach value to both public and private signals. Second, constant gain learning (e.g. Dewachter and Lyrio (2008)) is obtained by setting $\omega_{\pi^*} = \omega_{\rho^*} = 0$, and $\sigma_{\pi^{*b}} = \sigma_{\rho^{*b}} = 0$. In this case, agents attach no value to public signals and only update their perceived stochastic trends as a result of prediction errors.

2.1.2 Actual and Perceived Laws of Motion

The structural model in Eqs. (1), (5), (8), (12) and (13) can be written in state space form. We collect the observable macroeconomic factors in $X_t^m = [\pi_t,\ y_t,\ i_t,\ v_{\pi,t},\ v_{y,t},\ v_{i,t}]'$, the actual stochastic trends in $X_t^* = [\pi_t^*,\ \rho_t^*]'$, and the perceived trends in $X_t^{*P} = [\pi_t^{*P},\ \rho_t^{*P}]'$. Analogously, the structural macroeconomic shocks are grouped in $\varepsilon_t^v = [\varepsilon_{v_\pi,t},\ \varepsilon_{v_y,t},\ \varepsilon_{v_i,t}]'$, the shocks to the stochastic trends in $\varepsilon_t^* = [\varepsilon_{\pi^*,t},\ \varepsilon_{\rho^*,t}]'$, and the belief shocks in $\varepsilon_t^{*b} = [\varepsilon_{\pi^{*b},t},\ \varepsilon_{\rho^{*b},t}]'$.

The Perceived Law of Motion (PLM) expresses the macroeconomic dynamics as perceived by private agents. The PLM is obtained as the rational expectations (RE) solution to the structural equations replacing the actual stochastic endpoints (X_t^*) by their perceived counterparts (X_t^{*P}). The Actual Law of Motion (ALM) is obtained (i) by substituting the expectations $E_t\left[X_{t+1}^m\right]$ by the subjective expectations $E_t^P\left[X_{t+1}^m\right]$ implied by the PLM into the structural dynamics of the model, and (ii) by taking into account

[8] We assume that agents know the structural dynamics and observe the macroeconomic variables π_t, y_t and i_t and the exogenous factors $v_{\pi,t}$, $v_{y,t}$ and $v_{i,t}$.

the dynamics of the perceived stochastic endpoints in Eq. (13). In Appendix A, we show that the ALM can be summarized with respect to the full state vector $X_t = [X_t^{m\prime},\ X_t^{*P\prime},\ X_t^{*\prime}]^\prime$ and shock vector $\varepsilon_t = [\varepsilon_t^{v\prime},\ \varepsilon_t^{*b\prime},\ \varepsilon_t^{*\prime}]^\prime$ as:

$$X_t = C^A + \Phi^A X_{t-1} + \Gamma^A S^A \varepsilon_t. \tag{14}$$

Analogously, the PLM can be described in state space form as:

$$X_t = C^P + \Phi^P X_{t-1} + \Gamma^P S^P \varepsilon_t^P. \tag{15}$$

The learning dynamics described in Eq. (13) has a significant impact on the actual macroeconomic outcome (ALM). To see this, first note that the model described in Section 2.1 implies the existence of an expectations channel. Since subjective expectations with respect to the macroeconomic factors ($E_t^P\left[X_{t+1}^m\right]$) are conditioned on the perceived stochastic endpoints X_t^{*P}, which follow the learning dynamics in Eq. (13), the latter affect the actual macroeconomic dynamics X_t^m. As a consequence, both components of the private signal updating the perceived stochastic endpoints in Eq. (13) affect the ALM. The first component includes exogenous belief shocks ($\varepsilon_{\pi^{*b},t},\ \varepsilon_{\rho^{*b},t}$) and their impact depends on the information content attributed to them (lower values for ω_{π^*} and ω_{ρ^*} imply a higher impact). Such shocks are only neutralized in the full-information RE case, i.e. when $\omega_{\pi^*} = \omega_{\rho^*} = 1$. The second component is endogenous and represents a constant gain learning. It implies a feedback from the actual macroeconomic dynamics, including the actual stochastic endpoints, to the perceived endpoints. If actual endpoints are higher (lower) than perceived endpoints, agents underpredict (overpredict) the inflation rate and the real interest rate, resulting in positive (negative) prediction errors. With adaptive learning, implying positive gains ($g_\pi > 0$ and $g_\rho > 0$), these prediction errors affect the perceived endpoints and generate a trend-wise convergence between actual and perceived stochastic endpoints. This convergence is imposed in the estimation.[9]

2.2 The term structure of interest rates

Bond prices are determined in financial markets by private agents using subjective expectations with respect to the macroeconomic dynamics. The solution for bond prices is, therefore, implied by the PLM in Eq. (15). We follow Ang and Piazzesi (2003) and solve for the affine yield curve representation. We assume a log-normal stochastic discount factor:

$$m_{t+1} = \exp(-i_t - \tfrac{1}{2}\Lambda_t^\prime S^P S^{P\prime}\Lambda_t - \Lambda_t^\prime S^P \varepsilon_{t+1}^P), \tag{16}$$

with prices of risk Λ_t linear in the state variable X_t:

$$\Lambda_t = \Lambda_0 + \Lambda_1 X_t. \tag{17}$$

Imposing the no-arbitrage condition on zero-coupon bond prices with time to maturity τ, $p_t^{(\tau)}$, implies that bond prices satisfy the equilibrium pricing condition:

$$p_t^{(\tau)} = E_t^P\left[m_{t+1}\ p_{t+1}^{(\tau-1)}\right], \tag{18}$$

[9] Denoting the expectations generated under the ALM by E_t^A, we impose that $\lim_{s\to\infty} E_t^A\left[X_{t+s}^{*P}\right] = X_t^*$.

and can be written as an exponentially affine function of the state vector:

$$p_t^{(\tau)} = \exp(a_\tau + b_\tau X_t), \tag{19}$$

where the price loadings can be expressed as:

$$\begin{aligned} a_\tau &= a_{\tau-1} + b_{\tau-1}(C^P - \Gamma^P S^P S^{P\prime}\Lambda_0) + \tfrac{1}{2} b_{\tau-1}\Gamma^P S^P S^{P\prime}\Gamma^{P\prime} b_{\tau-1}' - \delta_0, \\ b_\tau &= b_{\tau-1}(\Phi^P - \Gamma^P S^P S^{P\prime}\Lambda_1) - \delta_1', \end{aligned} \tag{20}$$

with the risk-free interest rate identified as $i_t = \delta_0 + \delta_1' X_t$, and initial conditions $a_0 = 0$ and $b_0 = 0$. Note that the learning dynamics is integrated in the yield curve model through the PLM matrices C^P, Φ^P, Γ^P, and S^P defined in Appendix A.

Since the time t yield on a zero-coupon bond with maturity τ is defined as $y_t^{(\tau)} = -\ln(p_t^{(\tau)})/\tau$, the no-arbitrage yield curve is linear in the state vector:

$$y_t^{(\tau)} = A_\tau + B_\tau X_t, \tag{21}$$

with yield loadings $A_\tau = -a_\tau/\tau$ and $B_\tau = -b_\tau/\tau$. This representation is not necessarily consistent with the macroeconomic part of the model, as noted by Wu (2006) and Bekaert, Cho, and Moreno (2010). Full consistency implies additional restrictions on the prices of risk, aligning the pricing kernel in Eq. (16) with the IS curve in Eq. (5). Within the above linearized and homoskedastic macroeconomic framework, consistency implies constant prices of risk ($\Lambda_0 = \Lambda_0^{IS}$, $\Lambda_1 = 0$), specified along the lines of Bekaert, Cho, and Moreno (2010). Therefore, the expected excess holding return $ehr_t^{(\tau)}$ which is in general affine in the state vector X_t:

$$ehr_t^{(\tau)} = E_t^P \ln(P_{t+1}^{(\tau-1)} - P_t^{(\tau)}) = b_{\tau-1}\Gamma^P S^P S^{P\prime}\Lambda_0 + b_{\tau-1}\Gamma^P S^P S^{P\prime}\Lambda_1 X_t - \frac{1}{2} b_{\tau-1}\Gamma^P S^P S^{P\prime}\Gamma^{P\prime} b_{\tau-1}{}' \tag{22}$$

leads to constant risk premia once consistency is imposed. Finally, the no-arbitrage yield curve is extended with the inclusion of a maturity-specific constant liquidity premium ξ_τ and measurement error $\eta_{y,t}^{(\tau)}$:

$$y_t^{(\tau)} = A_\tau + B_\tau X_t + \xi_\tau + \eta_{y,t}^{(\tau)}, \tag{23}$$

with $\xi_1 = 0$, i.e. no liquidity premium on the one-period bond, and all measurement errors normally distributed (i.i.d.) with mean zero and maturity specific variance $\sigma^2_{\eta_y,\tau}$.

3 Econometric methodology

The empirical analysis is done within a Bayesian framework. In Secton 3.1, we show how the posterior density is computed taking into account the learning dynamics adopted by private agents. Section 3.2 describes the four model versions analyzed in this paper.

3.1 Posterior distribution

Denoting the data set by Z^T and the parameter vector for model version i by θ_i, we can describe the posterior density of θ_i according to Bayes Theorem as:

$$p(\theta_i \mid Z^T) \propto \frac{L(Z^T \mid \theta_i) p(\theta_i)}{p(Z^T)}, \tag{24}$$

with $p(\theta_i)$ the prior for the parameters in model version i, $L(Z^T \mid \theta_i)$ the likelihood function, and $p(Z^T)$ the marginal likelihood of Z^T (given version i of the model). The likelihood function is constructed under the ALM, treating the factors π_t^*, ρ_t^*, π_t^{*P} and ρ_t^{*P} as unobserved variables to the econometrician. The transition equation therefore is given by the ALM dynamics in Eq. (14), which we rewrite making explicit the dependence on the parameter vector θ_i:

$$X_t = C^A(\theta_i) + \Phi^A(\theta_i)X_{t-1} + \Gamma^A(\theta_i)S^A(\theta_i)\varepsilon_t, \qquad \varepsilon_t \sim N(0, I). \tag{25}$$

The likelihood function is based on the prediction errors identified by the measurement equation, which includes three types of information: (i) macroeconomic data from π_t, y_t and i_t; (ii) yield curve data, $y_t^{(\tau_i)}$, $i = 1, \ldots, n_y$; and (iii) survey data on inflation expectations, $S^{(\tau_i)}$, $i = 1, \ldots, n_s$. The measurement equation is affine in the state vector X_t:

$$Z_t = A_Z(\theta_i) + B_Z(\theta_i)X_t + \xi_Z(\theta_i) + \Sigma_Z(\theta_i)\eta_{Z,t}, \qquad \eta_{Z,t} \sim N(0, I), \tag{26}$$

with $Z_t = [\pi_t,\ y_t,\ i_t,\ y_t^{(\tau_1)}, \ldots, y_t^{(\tau_{n_y})},\ S_t^{(\tau_1)}, \ldots, S_t^{(\tau_{n_s})}]'$. The vector ξ_Z contains the mispricing terms ξ_τ. We assume that the variance-covariance matrix of the measurement errors, $\Omega_Z = \Sigma_Z\Sigma_Z'$, is diagonal and the macroeconomic variables are observed without measurement errors, making Ω_Z singular.

The log likelihood function is obtained by integrating out the unobserved latent factors using the Kalman filter:

$$L(Z^T \mid \theta_i) = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left[\ln(\left|V_{Z,t|t-1}\right|) + (Z_t - Z_{t|t-1})'\left(V_{Z,t|t-1}\right)^{-1}(Z_t - Z_{t|t-1})\right], \tag{27}$$

with the prediction and updating equations for the mean given by

$$\begin{aligned} Z_{t|t-1} &= A_Z + B_Z X_{t|t-1} + \mu_Z, \\ X_{t|t-1} &= C^A + \Phi^A X_{t-1|t-1}, \\ X_{t|t} &= X_{t|t-1} + K_{t|t-1}(Z_t - Z_{t|t-1}), \end{aligned} \tag{28}$$

and for the variance V_Z given by

$$\begin{aligned} V_{Z,t|t-1} &= B_Z P_{t|t-1} B_Z' + \Omega_Z, \\ P_{t|t-1} &= \Phi^A P_{t-1|t-1} \Phi^{A\prime} + \Gamma^A S^A S^{A\prime} \Gamma^{A\prime}, \\ P_{t|t} &= (I - K_{t|t-1} B_Z) P_{t|t-1}, \end{aligned} \tag{29}$$

with a Kalman gain

$$K_{t|t-1} = P_{t|t-1} B_Z' (B_Z P_{t|t-1} B_Z' + \Sigma_Z \Sigma_Z')^{-1}. \tag{30}$$

The posterior density of θ_i is in general not known in closed form. We use MCMC methods, in particular the Metropolis-Hastings algorithm, to simulate draws from the posterior. We follow the standard two-step procedure. First, a simulated annealing procedure is used to find the mode of the posterior.[10]

[10] The following additional conditions are imposed in the estimation. We impose determinacy of the solution for RE models. For the learning model, we impose non-explosive behavior of the ALM by excluding explosive roots. We initialize the Kalman filter by estimating the initial values of the unobserved variables. For the initial variance-covariance matrix, we solve for the steady state of the Riccati equation. This steady state exists given the imposed cointegration of the latent factors with the observed macroeconomic factors.

In a second step, the Metropolis-Hastings procedure is used to trace the posterior density of θ_i.[11]

Given the likelihood $L(Z^T \mid \theta_i)$ and the prior $p(\theta_i)$, the marginal likelihood of the data for model version i is obtained by integrating over the parameter vector θ_i. The marginal likelihood and the BIC criterion are used to evaluate the relative performance of the different versions of the model.

3.2 Model versions

We assess four versions of the macro-finance model: (i) the *Benchmark Model*, which assumes rational expectations and full-information by private agents, imposes consistency between the pricing kernel and the IS equation (i.e. implied constant prices of risk), and prices bonds according to no-arbitrage restrictions; (ii) the *Mispricing Model*, an extension of the benchmark model allowing for liquidity premia, expressed as constant mispricing terms relative to the no-arbitrage model; (iii) the *Flexible Price of Risk Model*, an extension of the benchmark model allowing for flexible prices of risk; and (iv) the *Learning Model*, a general model including learning by private agents with respect to the long-run inflation expectation and the equilibrium real rate, and also allowing for liquidity premia and flexible prices of risk. Table 1 summarizes the differences across the models. Below we specify the parameter vector to be estimated for each of the versions of the model.

Insert Table 1

Benchmark Model (B). The parameter vector for the Benchmark Model is:

$$\begin{aligned}\theta_B \quad = \quad & \left[\delta_\pi, \kappa, h, \sigma, \gamma_\pi, \gamma_y, \gamma_i,\ \varphi_\pi, \varphi_y, \varphi_i,\ \sigma_\pi, \sigma_y, \sigma_i, \sigma_{\pi^*}, \sigma_{\rho^*},\ \pi_0^*, \rho_0^*, \right. \\ & \left. \sigma_{\eta_y,1}, \sigma_{\eta_y,2}, \sigma_{\eta_y,4}, \sigma_{\eta_y,12}, \sigma_{\eta_y,20}, \sigma_{\eta_y,40},\ \sigma_{\eta_\pi,4}, \sigma_{\eta_\pi,40}\right]',\end{aligned}$$

where the parameters π_0^* and ρ_0^* are the estimated initial values for the latent variables π_t^* and ρ_t^*.

Mispricing Model (Misp). The first extension to the benchmark model allows for mispricing in terms of constant maturity-specific deviations of the actual yield curve from the one implied by no-arbitrage restrictions. Since the mispricing terms, ξ_τ, are not systematically related to macroeconomic variables, we refer to them as liquidity effects. We include a liquidity effect for yields with maturities of 2, 4, 12, 20 and 40 quarters. The parameter vector in this case is given by:

$$\theta_{Misp} = \left[\theta_B',\ \xi_2, \xi_4, \xi_{12}, \xi_{20}, \xi_{40}\right]'.$$

Flexible Price of Risk Model (FPR). In the benchmark model, the prices of risk are implied by the structural macroeconomic framework and are time invariant. This version of the model relaxes this constraint.[12] Nevertheless, to reduce the number of parameters to be estimated, we impose some structure on the prices of risk. We assume that (i) the prices of risk only load on observable macroeconomic

[11] The Metropolis-Hastings algorithm is based on a total of 200,000 simulations with a training sample of 20,000. An acceptance ratio of 40% is targeted in the algorithm. Parameters are drawn based on the Gaussian random walk model. Finally, we use Geweke (1999)'s test for differences in means and cumulative mean plots to assess convergence.

[12] Hördahl, Tristani, and Vestin (2006) and Rudebusch and Wu (2008) also estimate an alternative version of the structural model where the parameters in Λ_0 and Λ_1 are estimated freely.

variables, π_t, y_t and i_t;[13] and (ii) the prices of risk and the risk premia are stationary under the PLM:

$$\Lambda_0 = \begin{bmatrix} \Lambda_{0,\pi} \\ \Lambda_{0,y} \\ \Lambda_{0,i} \\ \Lambda_{0,\pi^*} \\ \Lambda_{0,\rho*} \\ 0 \\ 0 \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} \Lambda_{1,\pi\pi} & \Lambda_{1,\pi y} & \Lambda_{1,\pi i} & 0 & 0 & 0 & -\Lambda_{1,\pi\pi} - \Lambda_{1,\pi i} & -\Lambda_{1,\pi i} & 0 & 0 \\ \Lambda_{1,y\pi} & \Lambda_{1,yy} & \Lambda_{1,yi} & 0 & 0 & 0 & -\Lambda_{1,y\pi} - \Lambda_{1,yi} & -\Lambda_{1,yi} & 0 & 0 \\ \Lambda_{1,i\pi} & \Lambda_{1,iy} & \Lambda_{1,ii} & 0 & 0 & 0 & -\Lambda_{1,i\pi} - \Lambda_{1,ii} & -\Lambda_{1,ii} & 0 & 0 \\ \Lambda_{1,\rho\pi} & \Lambda_{1,\rho y} & \Lambda_{1,\rho i} & 0 & 0 & 0 & -\Lambda_{1,\rho\pi} - \Lambda_{1,\rho i} & -\Lambda_{1,\rho i} & 0 & 0 \\ \Lambda_{1,\pi^*\pi} & \Lambda_{1,\pi^* y} & \Lambda_{1,\pi^* i} & 0 & 0 & 0 & -\Lambda_{1,\pi^*\pi} - \Lambda_{1,\pi^* i} & -\Lambda_{1,\pi^* i} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Therefore, the parameter vector to be estimated is:

$$\theta_{FPR} = \left[\theta_B',\ \Lambda_{0,\pi}, \Lambda_{0,y}, \Lambda_{0,i}, \Lambda_{0,\pi^*}, \Lambda_{0,\rho^*},\ \Lambda_{1,\pi\pi}, \Lambda_{1,\pi y}, \Lambda_{1,\pi i},\ \Lambda_{1,y\pi}, \Lambda_{1,yy}, \Lambda_{1,yi},\ \Lambda_{1,i\pi}, \Lambda_{1,iy}, \Lambda_{1,ii}\right]'.$$

Learning Model (Learning). The last version of the model includes learning by private agents and allows for the two extensions of the benchmark model discussed above, i.e. mispricing and flexible prices of risk. As a result, the parameter vector is:

$$\theta_{Learning} = [\theta_{FPR}',\ \xi_2, \xi_4, \xi_{12}, \xi_{20}, \xi_{40},\ \omega_{\pi^*}, \omega_{\rho^*}, g_\pi, g_\rho, \sigma_{\pi^{*b}}, \sigma_{\rho^{*b}}, \pi_0^{*b}, \rho_0^{*b}]',$$

where the last eight parameters refer to the learning dynamics explained in Section 2.1.1.

4 Empirical analysis

We describe the data in Section 4.1 and the prior distribution for the parameters in each model version in Section 4.2. In Section 4.3, we compare the performance of the models by means of the marginal likelihood and BIC statistics. Since the *Learning Model* outperforms the other versions of the macro-finance model, we discuss the posterior density and implied factors of this model in Sections 4.4 and 4.5, and the implications for the yield curve in Sections 4.6 and 4.7.

4.1 Data

The data set consists of quarterly observations for the U.S. economy covering the period from 1960:Q2 to 2006:Q4 (187 observations). The data include observations on macroeconomic variables, the term structure of interest rates, and inflation expectations. The macroeconomic variables are the inflation rate, the output gap, and the central bank policy interest rate. Inflation is computed based on the quarterly GDP deflator and is expressed in per annum terms. The output gap is the percentage deviation of GDP from the potential output reported by the Congressional Budget Office (CBO). The policy rate is the effective federal funds rate.[14] The term structure of interest rate data consist of yields of bonds with maturities of 1, 2, 4, 12, 20 and 40 quarters. For 1- and 2-quarter yields, we use data from the secondary market for Treasury bills.[15] For 4-, 12-, 20- and 40-quarter yields, we combine the data sets compiled

[13] This type of restriction is based on the statistical tests rejecting the unit root hypothesis for term and risk premia. In our context, it implies the stationarity of the prices of risk. In the empirical implementation, we restrict Λ_1 further. Statistical analysis also shows that we can set $\Lambda_{1,\pi^*.}$ and $\Lambda_{1,\rho *.}$ to zero.

[14] For inflation and real GDP, the data series GDPDEF and GDPC1 are retrieved from the Federal Reserve Economic Data (FRED) data base, respectively. We use the 2006 vintage of potential output. Data on the effective federal funds rate are obtained from the Federal Reserve Bank of St. Louis FRED database.

[15] We use the FRED series TB3MS and TB6MS for the 1- and 2-quarter yields, respectively.

by Gürkaynak, Sack, and Wright (2007) and McCulloch and Kwon (1993).[16] Inflation expectation data are obtained from the Survey of Professional Forecasters (SPF) provided by the Federal Reserve Bank of Philadelphia. We use the series for the 4- and 40-quarter average inflation expectation.[17]

4.2 Priors

Tables 2 and 3 report the prior distribution, mean, and standard deviation of the parameters of the respective models. Table 2 lists the priors for the parameters which are common to all models, i.e. the structural parameters and parameters related to structural shocks and measurement errors. Table 3 contains the priors for the set of parameters which vary according to each model, i.e. related to mispricing, prices of risk, and learning.

Insert Tables 2 and 3

Phillips curve. We adopt a beta distribution for the inflation indexation parameter (δ_π) with a mean of 0.7 and standard deviation of 0.05. This attributes a significant role to the endogenous backward-looking component in inflation ($\mu_{2,\pi} = 0.41$). We assume a strict prior for the output sensitivity of inflation (κ), represented by a normal distribution with a mean of 0.12 and standard deviation of 0.03.[18]

IS curve. For the level of relative risk aversion (σ), we follow the standard macroeconomic view and adopt a prior with mass concentrated on the lower values of σ. For this, we choose a gamma distribution with a mean of 1.5 and standard deviation of 0.4.[19]

Monetary policy. The priors for the monetary policy rule are obtained from the Taylor rule literature. The inflation gap and output gap parameters (γ_π, γ_y) have normal distributions with a mean of 0.5. The differences in the standard deviations (i.e. $\sigma_{\gamma_\pi} = 0.25$ and $\sigma_{\gamma_y} = 0.4$) reflect the reported uncertainty in the estimates for these parameters. For the interest rate smoothing parameter (γ_i), we use a normal distribution with a mean of 0.8 and standard deviation of 0.2. The high standard deviation reflects the ongoing debate concerning the degree of interest smoothing (see Rudebusch (2002), English, Nelson, and Sack (2003), and Gerlach-Kristen (2004)).

Structural shocks. We use loose priors for the autocorrelation parameters of the three structural shocks, i.e. a normal distribution with mean and standard deviation equal to 0.5. The standard deviation for the permanent shocks (σ_{π^*} and σ_{ρ^*}) are uniformly distributed with support between 0 and 1% per quarter. They prevent permanent shocks from becoming excessively large but are sufficiently wide to include a significant range for these parameters.

Measurement errors. We use a inverted gamma distribution with a mean of 0.005 and standard deviation of 0.003 for the standard deviation of the measurement errors of bond yields and inflation expectations.[20]

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[16] The Gürkaynak, Sack, and Wright (2007) data set starts on the 14th of June 1961 for bonds with maturities of 4, 12 and 20 quarters and on the 16th of August 1971 for the 40-quarter bond. Missing observations are obtained from the McCulloch and Kwon (1993) data set.

[17] Table B1 in the Appendix reports the descriptive statistics of the data.

[18] The value for the mean corresponds to the one found by Bekaert, Cho, and Moreno (2010). This bias towards lower estimates is in line with estimation results using General Method of Moments (GMM) or Maximum Likelihood (ML) techniques (e.g. Cho and Moreno (2006)).

[19] This prior contrasts with estimates from the finance literature which often reports values between 20 and 100.

[20] To prevent singularity problems in the estimation, we impose a lower bound of 5 basis points on all measurement errors.

Mispricing. For the liquidity premia (ξ_τ), we use a normal distribution with a mean of 0 and standard deviation of 0.005. This reflects a belief on relatively small average mispricing errors.

Prices of risk. We choose relatively uninformative priors for the prices of risk. The priors are set such that at the mean the model implies (i) a positive constant risk premium ($E(\Lambda_0) < 0$), and (ii) a risk premium increasing with the inflation and the interest rate gaps, ($\pi_t - \pi_t^*$) and ($i_t - \pi_t^* - \rho_t$), while decreasing with the output gap, y_t.

Learning. We impose relatively strict priors for the parameters w_{π^*} and w_{ρ^*}. We adopt beta distributions with support on the interval [0, 1] with a mean of 0.85 and standard deviation of 0.10. The prior is thus biased towards the full-information RE model.[21] For the constant gains (g_π, g_ρ), we apply a uniform distribution on the interval [0, 0.25]. This support is sufficiently large to contain most of the estimates reported in the literature (e.g. Kozicki and Tinsley (2005a) and Milani (2007)).

4.3 Relative performance of the models

The marginal likelihood of the data and the BIC statistics for each model are reported in Table 1. The BIC serves as a goodness-of-fit measure up to a penalty for model dimensionality. We assess the empirical relevance of allowing for mispricing in the macro-finance model by comparing the performance of the *Benchmark Model* with that of the *Mispricing Model*. We observe that the marginal likelihood of the *Mispricing Model* (7628) is significantly higher than the one implied by the *Benchmark Model* (7381). This shows the importance of allowing for mispricing terms in the modeling of the yield curve. Section (4.6) below analyses the estimated mispricing parameters for the *Learning Model* together with the analysis of the implied fit of the yield curve.

We now evaluate the effect of time-varying prices of risk on the performance of the macro-finance model. Since the marginal likelihood of the *Flexible Price of Risk model* (7638) is significantly higher than that of the *Benchmark Model* (7381), we can reject the implied constant prices of risk which guarantee consistency between the macroeconomic framework and the pricing kernel ($\Lambda_0 = \Lambda_0^{IS}$, $\Lambda_1 = 0$). Nevertheless, in order to assess whether our results also imply a rejection of the extended expectations hypothesis, which simply postulates time-invariant prices of risk ($\Lambda_0 \neq 0$, $\Lambda_1 = 0$), we need to determine if the parameters in Λ_1 are statistically different from zero. This is indeed the case as can be seen in Table C4 in the Appendix. The results then point to the need to allow for time-varying prices of risk and, therefore, risk premia in the modeling of bond yields.

The above results indicate that one should incorporate both extensions (mispricing and time-varying prices of risk) in a macro-finance model. This is done in the *Learning Model*. The results show that this version outperforms all other versions with a marginal likelihood (7741) substantially higher than that for the alternative models. Therefore, the three extensions combined significantly improve the overall fit of the model. Assuming a uniform prior over the alternative model versions, the posterior odds ratio of the learning version equals its Bayes factor of (approximately) 1, suggesting the superiority of this version

[21] This is especially justified for the output-neutral real rate since the data set does not contain much information identifying the difference between the private sector and the central bank assessment about this rate. We, therefore, effectively penalize real rate processes that deviate too much from the one implied by the restrictions of a RE equilibrium.

relative to all other versions of the model. Results for the BIC statistics in Table 1 lead to a similar conclusion. Despite the fact that the *Learning Model* is the largest model, it is clearly preferred in terms of the BIC statistic.

Table 1 also decomposes the performance of the models in terms of the macroeconomic, yield curve and inflation expectations dimensions. We use the likelihood of the prediction errors of the respective data subsets as performance measure. This decomposition also shows that the *Learning Model* outperforms all other model versions in each dimension. In the sections below, therefore, we only assess the posterior distribution of the parameters in the *Learning Model* and its implications for the yield curve.

4.4 Posterior distribution of the *Learning Model*

Tables 4 and 5 report the mean, standard deviation, mode, and 90% confidence interval for the posterior distribution of the parameters in the *Learning Model*.[22] We focus on four sets of parameters related to: (*i*) the New-Keynesian macro model; (*ii*) the structural and belief shocks; (*iii*) the prices of risk; and (*iv*) the learning dynamics. The mispricing parameters are discussed in Section (4.6) where we examine the implied fit of the yield curve. When not stated differently, the estimates refer to the mode of the posterior distribution.

Insert Tables 4 and 5

Structural model. The estimates for the structural model in Eqs. (1), (5), and (8) shown in Table 4 are in line with findings in the macro literature. The results reject the purely forward-looking New-Keynesian model in favor of a hybrid version containing also a backward-looking component. The following remarks can be made with respect to each structural equation.

Phillips curve. The inflation indexation parameter (δ_π) with a mode of 0.53 implies a relatively high weight on the forward-looking inflation component, i.e. $\mu_{1,\pi} = 0.66$. Such value is typically not recovered in the macro-finance literature. For instance, Bekaert, Cho, and Moreno (2010) report implied values in the range of [0.53, 0.63]. Nevertheless, our estimate is aligned with results reported in the macro literature (e.g. Galí and Gertler (1999) and Galí, Gertler, and David Lopez-Salido (2005)). The estimate for the mode of the inflation sensitivity to the output gap (κ) is relatively small (0.012), despite the strict prior around a mean value of 0.12. This indicates a weak link between detrended output and inflation and reflects the mismatch in the persistence of the two variables. Although lower than theoretically expected, this estimate is high compared to other General Method of Moments (GMM) or Maximum Likelihood (ML) based studies (e.g. Cho and Moreno (2006) report a value of 0.001).

IS curve. The habit persistence parameter (h) is estimated at a mode of 0.76 and with a relatively high precision. The estimate for the risk aversion parameter (σ) also seems reasonable from a macroeconomic perspective. Its mode is equal to 2.55 with 90% of the support contained in the interval [1.9, 3.2].[23]

[22] Appendix C presents the posterior distribution of the parameters for the other versions of the model.

[23] This range of values is quite different from the ones found in the macro-finance literature. Dewachter and Lyrio (2008), for example, estimate a number of models and obtain risk aversion parameters between 22 and 62.

Combined, these values result in a relatively strong forward-looking component with a weight on the expected future output gap (μ_y) equal to 0.69.

Monetary policy. Our estimates imply an active monetary policy rule both in the inflation and output gaps. Both γ_π and γ_y (with modes at 0.44 and 0.63, respectively) are positive, statistically significant, and close to the values implied by the standard Taylor rule. We also find a relatively low value for the interest rate smoothing parameter (γ_i) with a mode of 0.69 in the confidence interval [0.64, 0.78]. This implies that it would take the FED less than two quarters to halve the gap between the actual and the target interest rate. We believe this estimate is more realistic than the ones commonly reported in the literature (around 0.9 on a quarterly frequency) which suggest a halving time of more than six quarters. Our results are in line with the macro literature, e.g. Trehan and Wu (2007), and underscore the importance of omitted variable bias in Taylor rule estimations.[24]

Structural and belief shocks. We estimate the standard deviation of seven shocks in the *Learning Model*: three temporary structural macroeconomic shocks, i.e. supply, demand and policy rate shocks (σ_{v_π}, σ_{v_y}, σ_{v_i}); two permanent shocks associated with the inflation target and the output-neutral real rate (σ_{π^*}, σ_{ρ^*}); and two belief shocks related to the inflation target and the output-neutral real rate ($\sigma_{\pi^{*b}}$, $\sigma_{\rho^{*b}}$).

The estimates in Table 4 indicate that the supply and policy rate shocks are relatively large ($\sigma_{v_\pi} = 0.012$ and $\sigma_{v_i} = 0.012$) and negatively autocorrelated ($\varphi_\pi = -0.38$ and $\varphi_i = -0.15$). Although the negative autocorrelation might be surprising, note that the model incorporates two additional channels modeling persistence: (*i*) the endogenous persistence due to inflation indexation (δ_π) and interest rate smoothing (γ_i); and (*ii*) the dependence of inflation and the interest rate on the processes modeling the perceived stochastic endpoints for inflation and the output-neutral real rate. Finally, a low first-order correlation for supply and policy rate shocks has also been reported by Ireland (2007). The demand shock, on the other side, is relatively small ($\sigma_{v_y} = 0.003$) and with a autocorrelation (φ_y) of 0.65.

An important feature of the learning dynamics described in Eq. (13) is the introduction of exogenous belief shocks. We find that, for inflation, belief shocks (see Table 5) are relatively large in comparison with actual inflation target shocks ($\sigma_{\pi^{*b}} = 0.58\%$ compared to $\sigma_{\pi^*} = 0.04\%$). This implies a relatively smooth inflation target dynamics while still allowing for substantial variation in the perceived long-run inflation expectation. On the contrary, shocks to the output-neutral real rate are larger than belief shocks to this rate ($\sigma_{\rho^*} = 0.73\%$ compared to $\sigma_{\rho^{*b}} = 0.50\%$). Although this highlights the importance of output-neutral real rate shocks for the the yield curve dynamics, especially for long-term yields, it could also point to some form of misspecification of our model. This source of variation is typically ignored in standard macro-finance models by assuming a constant equilibrium real rate. Both findings are important departures from the results of standard macro-finance models.

[24] The omitted variable bias argument in the interest rate smoothing parameter has been put forward by Rudebusch (2002). Subsequent studies using latent factors in the Taylor rule found that a substantial part of the interest rate inertia could be attributed to omitted variables. For instance, Gerlach-Kristen (2004) estimates this parameter around 0.6 with a standard error of 0.2, while English, Nelson, and Sack (2003) report values around 0.6 with a standard error of 0.15.

Prices of risk. As mentioned before, it has been argued by Kozicki and Tinsley (2005b) that models containing asymmetric information and learning dynamics can explain the rejection of the expectations hypothesis. These authors take into consideration the fact that private agents' perception about the central bank's inflation target might deviate from the central bank's true target. This is relevant since long-horizon yields are related to long-horizon expectations of the policy rate which includes inflation expectations and the latter are anchored by market perceptions of the central bank's inflation target. The authors conclude that the common rejection of the expectations hypothesis might reflect incorrect assumptions about expectations formation process and not an incorrect link between long and short rates. Our estimation results of the *Learning Model* suggest this is not the case. This can be seen by the fact that even allowing for learning some of the time-varying prices of risk parameters in Λ_1 remain significant (see Table 5). In order to reproduce the data dynamics, therefore, it seems crucial to allow for time-varying risk premia in the dynamics of bond yields.

Learning dynamics. The size and significance of the learning parameters in Table 5 indicate substantial deviations from the full-information RE model. Noting that the latter model is embedded in the *Learning Model*, i.e. for $\omega_{\pi^*} = \omega_{\rho^*} = 1$, it is clearly rejected in favor of the alternative of learning, i.e $0 < \omega_{\pi^*} < 1$, $0 < \omega_{\rho^*} < 1$. The deviation from the full-information case is especially pronounced for the perceived long-run inflation expectation suggesting that this expectation is weakly anchored. The significant learning effect is due to (*i*) a relatively large weight attached to private signals in the updating rule for the perceived stochastic endpoint for inflation ($1 - \omega_{\pi^*} = 0.35$), (*ii*) a relatively large size of belief shock ($\sigma_{\pi^{*b}} = 0.58\%$), and (*iii*) a significant constant gain ($g_\pi = 0.22$). As can be seen in Eq. (13), multiplying the constant gain (g_π) by ($1 - \omega_{\pi^*}$) yields the total impact of the subjective forecast error on the perceived long-run inflation rate, which in our case is equal to 0.07 and in line with estimates reported in the literature. For instance, Milani (2007) finds values in the range $0.02 - 0.03$, while Kozicki and Tinsley (2005a) find higher values (around 0.10) using a similar learning model. For the output-neutral real rate, the estimates imply only marginal effects of learning on its dynamics due to the low weight given to private signals in the updating rule for the perceived equilibrium real rate ($1 - \omega_{\rho^*} = 0.03$).

4.5 Macro factors implied by the *Learning Model*

Figure 1 displays the filtered time series for the ten macroeconomic factors implied by the mode of the posterior distribution of the *Learning Model*. They include three observable factors (inflation, the output gap and the federal funds rate), three exogenous shocks (supply, demand and policy rate shocks), and four stochastic endpoints (actual and perceived) for inflation and the output-neutral real rate. Figure 2 depicts the actual and perceived stochastic endpoints and the respective 90% confidence intervals together with the respective observed macroeconomic variables.

Insert Figure 1 and 2

From Figure 1, we observe the mentioned disconnection between the inflation target of the central bank and the the perceived long-run inflation expectation of private agents. This can be seen by the significant and persistent differences between the two series. The series for the perceived long-run inflation expectation displays substantial time variation, while the time path of the inflation target is mostly contained within the confidence interval between $1\% - 3.8\%$ (see top-right panel of Figure 2). A similar type of disconnection between subjective inflation expectations and the inflation target is found in Kozicki and Tinsley (2005a) and Dewachter and Lyrio (2008). The results suggest that subjective inflation expectations were not well anchored, especially over the first part of the sample.

For the output-neutral real rate, we notice a strong similarity between the actual and perceived rates. As mentioned before, this is implied by the estimate for w_{ρ^*} (0.97) in Eq. (13) which assigns a marginal role to the learning dynamics. Figure 2 shows that the filtered output-neutral real rate is typically contained in the interval between $0\% - 5\%$ p.a. (with a historical average close to 2.5% p.a.) and displays significant persistence with relatively low rates in the 1970s and substantially higher rates in the 1980s. The perceived output-neutral real rate (bottom-left panel of Figure 2) also displays significant volatility and persistence, features also reported by e.g. Laubach and Williams (2003), Clark and Kozicki (2004) and Bjørnland, Leitemo, and Maih (2008). Figure 2 also illustrates the substantial differences in the uncertainty surrounding the estimated time paths of the two series. The perceived real rate (bottom-left panel) is identified with significant more precision than the actual rate (bottom-right panel) due primarily to the yield curve dynamics. The dynamics of the actual real rate is only weakly identified by the ALM, as can be observed from its large confidence interval. In fact, the 90% confidence interval reported in the bottom-right panel of Figure 2 does not exclude a constant or very smooth actual output-neutral real rate.

The variability in the perceived output-neutral real rate and the disconnection between the inflation target of the central bank and the subjective inflation expectations of private agents help explain a significant part of the variation in long-term yields. This is done without having to assign an excessively large standard deviation for the inflation target of the central bank.[25] The estimated mode for the inflation target is around two percent while the filtered inflation expectations are in line with survey data (see Figure 2). For the output-neutral real rate, however, its estimated standard deviation might be excessively large (see Figure 1).

4.6 The fit of the yield curve

The yield curve model implied by the *Learning Model* includes eight factors: three observable macroeconomic factors, three exogenous shocks, and two latent factors tracking the perceived stochastic endpoints for inflation and the output-neutral real interest rate.[26] Figure 3 shows the yield curve loadings, i.e. the sensitivity of the yield curve with respect to each macroeconomic factor. As can be seen, long-term yields are affected almost one-to-one by both stochastic endpoints. The factor loadings on the policy rate reveal a slope factor response, while other macroeconomic variables, i.e. inflation and demand shocks, affect

[25] For instance, Doh (2006) reports standard deviations between 30 and 35 basis points per quarter for U.S. data for the period 1960-2005. Dewachter and Lyrio (2006) and Bekaert, Cho, and Moreno (2010) find values ranging from 30 to more than 73 basis points per quarter.

[26] The *Learning Model* features a total of ten factors. However, only the factors entering the PLM are relevant for the yield curve, given that the yields are formed under the subjective expectations operator.

primarily the intermediate maturities.

Insert Figure 3

The performance of the model in fitting the yield curve can be assessed by the standard deviation of the measurement errors ($\sigma_{\eta_y,\tau}$) in Table 4. For yields with maturity above 2 quarters, this value is below 40 basis points.[27] These values are small relative to the total variation of the yields, which exceed 240 basis points (see Table B1 in the Appendix). These values are also in line with estimates reported in the macro-finance literature. For instance, Bekaert, Cho, and Moreno (2010) report measurement error standard deviations of 45 and 54 basis points for the 4- and 40-quarter yields. Estimates based on comparable models presented in Dewachter and Lyrio (2008) are around 50 basis points. Interestingly, De Graeve, Emiris, and Wouters (2009) report significantly lower measurement errors (of the order of 10 to 20 basis points). Comparing these statistics, we conclude that the *Learning Model* is relatively successful in explaining the yield curve variation in terms of macroeconomic shocks. More than 95% of the unconditional variance in yields with maturities beyond 4 quarters is explained by the model. Figure 4 illustrates the yield curve fit implied by the model and Figure 5 decomposes the fit of the 40-quarter yield into a expected real rate, expected inflation and risk premium component. The top panel in this figure shows the significant contribution of the expected inflation in the composition of the 40-quarter yield. The bottom panel shows the fit of the model-implied inflation expectation with respect to the one provided by the Survey of Professional Forecasters.

Insert Figures 4 and 5

Despite the large number of factors included in the model, the average mispricing (ξ_τ) seems economically important and increasing with maturity up to 20 quarters (see Table 5), i.e. model-implied yields are too high at the short end (negative liquidity premium) and too low at the long end of the yield curve (positive liquidity premium).[28] Nevertheless, only the negative mispricing term at the short end of the yield curve (ξ_2) is statistically significant.

Finally, Figure 6 displays the expected excess holding return (per annum for a quarterly holding period) expressed in Eq. (22) together with the NBER recession dates.[29] As can be observed, risk premia have an important time-varying component ranging from -2% p.a. in 1965 to more than 6% p.a. in 1984 for the 40-quarter maturity bond. Similar time patterns and orders of magnitude have been reported by Duffee (2002) and Campbell, Sunderam, and Viceira (2009). In line with intuition, the observed risk premia are countercyclical, generating large and positive risk premia during recessions and

[27] A remarkable aspect of the data is the bad fit of the short end of the yield curve with fitting errors around one percent. This finding is due to the choice of policy rate. With the federal funds rate representing the policy rate, there is an obvious tension with short-term Treasury rates, given that on average these have been below the federal funds rate. This persistent gap is picked up in the measurement error.

[28] The negative liquidity premium at the short end of the yield curve should not come as a surprise. The positive spread between the federal funds rate and the short-term treasuries is well documented and is typically attributed to a risk premium in the federal funds rate reflecting private banks' uncertainty over reserve management.

[29] The average excess holding return and standard deviation (in brackets) implied by the data are: 1.1% (0.2%), 1.5% (0.7%), 1.7% (1%) and 2% (1.7%) for the 4-, 12-, 20- and 40-quarter bonds, respectively. The average risk premium implied by the model (at the mode) are, respectively, 0.4%, 1.2%,1.6% and 2.1%.

smaller and even negative risk premia during expansions.[30]

Insert Figure 6

4.7 What factors drive the yield curve?

We turn to the identification of the macroeconomic factors driving monetary policy and the yield curve. Monetary policy is identified by the federal funds rate and the yield curve is decomposed into its level, slope and curvature factors. We follow the literature (e.g. Bekaert, Cho, and Moreno (2010)) by identifying (*i*) the level factor as the average yield across maturity, (*ii*) the slope factor as the 40-quarter maturity yield spread (relative to the 1-quarter yield), and (*iii*) the curvature factor as the sum of the 40-quarter and 1-quarter yields minus two times the 4-quarter yield. Table 6 presents the variance decomposition for the federal funds rate and the level, slope and curvature factors for horizons of 1, 4, 20 and 40 quarters.

The results show that the high frequency variation in the monetary policy is largely due to independent monetary policy shocks. Such shocks account for over 80% of the 1-quarter variation in the federal funds rate, with macroeconomic shocks having only a marginal contribution.[31] Supply and demand shocks become more important for intermediate horizons, which is explained by the presence of interest rate smoothing. At the 4-quarter horizon, supply and demand shocks account for 25% of the total variation in the policy rate. For longer horizons, monetary policy is dominated by long-term equilibrium forces. For a 40-quarter horizon, movements in the federal funds rate are mostly due to movements in the output-neutral real rate (75%), with belief shocks to long-run inflation expectations accounting for another 10%.

Insert Table 6

The variance decomposition of the level factor contradicts the results of standard macro-finance models, which attribute most of its variation to long-run inflation expectations, e.g. Doh (2006) and De Graeve, Emiris, and Wouters (2009). Our results indicate that, for all horizons, shocks to the output-neutral real rate are responsible for most of the variation in the level factor, explaining 85% of the variation for the 40-quarter horizon. Policy rate shocks are also significant for short-term horizons (up to 4 quarters) with a smaller contribution of supply shocks. Finally, belief shocks to long-run inflation expectations become relevant for intermediate and long horizons.

The variance decomposition for the slope and curvature factors are more in line with standard macro-finance models (e.g. Bekaert, Cho, and Moreno (2010)). The variation in both factors and for all horizons is dominated by exogenous monetary policy shocks. For the slope factor, demand shocks have a significant impact for horizons of 4 quarters and above, while for the curvature factor this impact is present for all horizons.

[30] Rudebusch, Sack, and Swanson (2007) show that a decline in the term premium has been associated with a stimulus to the economy.

[31] This is in line with e.g. Bekaert, Cho, and Moreno (2010).

5 Conclusion

We estimate a New-Keynesian macro-finance model of the yield curve incorporating learning by private agents with respect to the long-run expectation of inflation and the equilibrium real interest rate. Private agent's perception about these two variables are updated taking into consideration their own belief shocks and a constant gain learning process. A preliminary analysis shows that some liquidity premia, expressed as some degree of mispricing relative to no-arbitrage restrictions, and time variation in the prices of risk are important features of the data. These features are, therefore, included in our *Learning Model*.

The *Learning Model* succeeds to some extent in explaining the yield curve movements in terms of macroeconomic shocks. Interestingly, the variability in the perceived stochastic endpoints for inflation and the equilibrium real rate turn out to be important in explaining the variability of long-term yields. The results for this model also show an important difference between the estimated inflation target of the central bank and the perceived long-run inflation expectation of private agents. This is especially the case for the period from the mid-1970s to the mid-1990s and show that private agents' perceptions about long-term inflation were weakly anchored.

The structural decomposition of the yield curve into its macroeconomic components also provides new insights concerning the interpretation of the level, slope and curvature factors. For the slope and curvature factors, the decomposition generated by the *Learning Model* is in line with standard macro-finance models. These factors are primarily affected by exogenous monetary policy shocks, with demand shocks contributing substantially. For the level factor, standard models attribute most of its variation to long-run inflation expectations. We find, however, that shocks to the output-neutral real rate are responsible for most of the variation in this factor. We should emphasize that the pronounced variability of the output-neutral real rate could be a sign of model misspecification.

Several extensions of the model could be undertaken. First, our results document the significance of mispricing terms within a structural macro-finance model. This mispricing can be quite substantial, especially at the short end of the yield curve, suggesting the need for further analysis of these results. In recent research, Dewachter and Iania (2010) show the importance of including financial factors related to the overall liquidity and counterparty risk in the money market in the modeling of the yield curve. The inclusion of such factors in our framework could eliminate the significance of such mispricing terms. Second, in this paper, we use a short-cut to identify the output-neutral real rate. Given the importance of this factor for long-term yields, an important task is to verify further the interpretation of this factor within a learning model. To this end, our model could be extended with the introduction of a complete micro-founded supply side. Such an extension would facilitate the identification of the long-run real interest rate and would refine the set of observable macroeconomic shocks, as in De Graeve, Emiris, and Wouters (2009).

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Tables and Graphs

Table 1: PROPERTIES AND PERFORMANCE OF ALTERNATIVE MODEL VERSIONS

	Model			
	Benchmark	Mispricing	FPR	Learning
Properties				
Mispricing	No	Yes	No	Yes
Prices of risk	Consistent: Λ_0^{IS}	Consistent: Λ_0^{IS}	Free: Λ_0, Λ_1	Free: Λ_0, Λ_1
Expectations	Full-info RE	Full-info RE	Full-info RE	Learning
Overall performance				
Marginal likelihood	7381	7628	7638	7741
BIC	-14815	-15333	-15384	-15442
BIC decomposition				
Macro (-2lnlik)	-3760	-3711	-3745	-3772
Yields (-2lnlik)	-9037	-9392	-9504	-9538
Infl. exp.(-2lnlik)	-2227	-2197	-2182	-2256
Penalty	131	157	194	272

Note: The marginal likelihood is computed using the modified harmonic mean procedure of Geweke. The findings are robust to alternative cut-off levels. The BIC refers to the Schwarz Bayesian Information Criterion and is computed at the mode of the posterior distribution. The decomposition of the likelihood and the BIC are based on the likelihood of the prediction errors of the respective data series. *FRP*: Flexible Price of Risk Model.

Table 2: Prior distribution of parameters for alternative model versions (part I)

Parameter	Prior distribution Type	Mean	Stdev	Model Benchmark	Mispricing	FPR	Learning
			Structural parameters				
δ_π	$\mathcal{B}$	0.700	0.050	yes	yes	yes	yes
κ	$\mathcal{N}$	0.120	0.030	yes	yes	yes	yes
h	$\mathcal{B}$	0.700	0.050	yes	yes	yes	yes
σ	$\mathcal{G}$	1.500	0.335	yes	yes	yes	yes
γ_π	$\mathcal{N}$	0.500	0.250	yes	yes	yes	yes
γ_y	$\mathcal{N}$	0.500	0.400	yes	yes	yes	yes
γ_i	$\mathcal{N}$	0.800	0.200	yes	yes	yes	yes
			Autocorrelation structural shocks				
φ_π	$\mathcal{N}$	0.500	0.500	yes	yes	yes	yes
φ_y	$\mathcal{N}$	0.500	0.500	yes	yes	yes	yes
φ_i	$\mathcal{N}$	0.500	0.500	yes	yes	yes	yes
			Standard deviation structural shocks				
σ_{v_π}	$\mathcal{IG}$	0.010	0.003	yes	yes	yes	yes
σ_{v_y}	$\mathcal{IG}$	0.010	0.003	yes	yes	yes	yes
σ_{v_i}	$\mathcal{IG}$	0.010	0.003	yes	yes	yes	yes
σ_{π^*}	$\mathcal{U}$	0.005	0.003	yes	yes	yes	yes
σ_{ρ^*}	$\mathcal{U}$	0.005	0.003	yes	yes	yes	yes
			Initial values stochastic endpoints				
π_0^*	$\mathcal{N}$	0.020	0.010	yes	yes	yes	yes
ρ_0^*	$\mathcal{N}$	0.020	0.010	yes	yes	yes	yes
			Standard deviation measurement errors yields				
$\sigma_{\eta_y,1}$	$\mathcal{IG}$	0.005	0.003	yes	yes	yes	yes
$\sigma_{\eta_y,2}$	$\mathcal{IG}$	0.005	0.003	yes	yes	yes	yes
$\sigma_{\eta_y,4}$	$\mathcal{IG}$	0.005	0.003	yes	yes	yes	yes
$\sigma_{\eta_y,12}$	$\mathcal{IG}$	0.005	0.003	yes	yes	yes	yes
$\sigma_{\eta_y,20}$	$\mathcal{IG}$	0.005	0.003	yes	yes	yes	yes
$\sigma_{\eta_y,40}$	$\mathcal{IG}$	0.005	0.003	yes	yes	yes	yes
			Standard deviation measurement errors inflation expectations				
$\sigma_{\eta_\pi,4}$	$\mathcal{IG}$	0.005	0.003	yes	yes	yes	yes
$\sigma_{\eta_\pi,40}$	$\mathcal{IG}$	0.005	0.003	yes	yes	yes	yes

Note: This table reports the prior distributions used in the estimation. Column 1 presents the parameters. The second column specifies the type of distribution function: $\mathcal{B}$ (beta), $\mathcal{G}$ (gamma), $\mathcal{IG}$ (inverted gamma), $\mathcal{N}$ (normal), and $\mathcal{U}$ (uniform). Columns 3 and 4 report the mean and standard deviation as implied by the respective prior distributions. Columns 5 to 8 indicate whether or not the parameter is estimated in the respective model. The entry 'yes' indicates the parameter is estimated and 'no' it is not.

Table 3: Prior distribution of parameters for alternative model versions (part II)

	Prior distribution			Model			
Parameter	Type	Mean	Stdev	Benchmark	Mispricing	FPR	Learning
				Average mispricing yields			
ξ_2	$\mathcal{N}$	0.000	0.005	no	yes	no	yes
ξ_4	$\mathcal{N}$	0.000	0.005	no	yes	no	yes
ξ_{12}	$\mathcal{N}$	0.000	0.005	no	yes	no	yes
ξ_{20}	$\mathcal{N}$	0.000	0.005	no	yes	no	yes
ξ_{40}	$\mathcal{N}$	0.000	0.005	no	yes	no	yes
				Prices of risk: $\Lambda_0(\times 10^{-2})$			
$\Lambda_{0,\pi}$	$\mathcal{N}$	-0.050	0.150	impl	impl	yes	yes
$\Lambda_{0,y}$	$\mathcal{N}$	-0.050	0.150	impl	impl	yes	yes
$\Lambda_{0,i}$	$\mathcal{N}$	-0.050	0.150	impl	impl	yes	yes
Λ_{0,π^*}	$\mathcal{N}$	-0.050	0.150	impl	impl	yes	yes
Λ_{0,ρ^*}	$\mathcal{N}$	-0.050	0.150	impl	impl	yes	yes
				Prices of risk: $\Lambda_1(\times 10^{-2})$			
$\Lambda_{1,\pi\pi}$	$\mathcal{N}$	-0.050	0.150	no	no	yes	yes
$\Lambda_{1,\pi y}$	$\mathcal{N}$	0.000	0.500	no	no	yes	yes
$\Lambda_{1,\pi i}$	$\mathcal{N}$	0.000	0.500	no	no	yes	yes
$\Lambda_{1,y\pi}$	$\mathcal{N}$	0.000	0.500	no	no	yes	yes
$\Lambda_{1,yy}$	$\mathcal{N}$	0.050	0.150	no	no	yes	yes
$\Lambda_{1,yi}$	$\mathcal{N}$	0.000	0.500	no	no	yes	yes
$\Lambda_{1,i\pi}$	$\mathcal{N}$	0.000	0.500	no	no	yes	yes
$\Lambda_{1,iy}$	$\mathcal{N}$	0.000	0.500	no	no	yes	yes
$\Lambda_{1,ii}$	$\mathcal{N}$	-0.050	0.150	no	no	yes	yes
				Learning parameters			
ω_{π^*}	B	0.850	0.100	no	no	no	yes
ω_{ρ^*}	B	0.850	0.100	no	no	no	yes
g_π	$\mathcal{U}$	0.125	0.075	no	no	no	yes
g_ρ	$\mathcal{U}$	0.125	0.075	no	no	no	yes
$\sigma_{\pi^{*b}}$	$\mathcal{U}$	0.010	0.006	no	no	no	yes
$\sigma_{\rho^{*b}}$	$\mathcal{U}$	0.010	0.006	no	no	no	yes
π_0^{*P}	$\mathcal{U}$	0.02	0.012	no	no	no	yes
ρ_0^{*P}	$\mathcal{U}$	0.02	0.012	no	no	no	yes

Note: This table reports the prior distributions used in the estimation. Column 1 presents the parameters. Column 2 specifies the type of distribution function: $\mathcal{B}$ (beta), $\mathcal{G}$ (gamma), $\mathcal{IG}$ (inverted gamma), $\mathcal{N}$ (normal), and $\mathcal{U}$ (uniform). Columns 3 and 4 report the mean and standard deviation as implied by the respective prior distributions. Columns 5 to 8 indicate whether or not the parameter is estimated in the respective model. The entry 'yes' indicates the parameter is estimated and 'no' it is not. Finally 'impl' implies the parameter is implied by other structural parameters.

Table 4: Posterior density estimates I - Learning Model

Parameter	Posterior Mean	Stdev	Mode	5%	95%
		Structural parameters			
δ_π	0.5337	0.0327	0.5288	0.4829	0.5891
κ	0.0137	0.0042	0.0117	0.0078	0.0212
h	0.7512	0.0445	0.7566	0.6741	0.8179
σ	2.6779	0.4004	2.5551	1.9344	3.2328
γ_π	0.3707	0.1081	0.4389	0.2296	0.5824
γ_y	0.6673	0.1638	0.6341	0.4931	1.0214
γ_i	0.6827	0.0406	0.6896	0.6462	0.7849
		Autocorrelation structural shocks			
φ_π	-0.3657	0.0839	-0.3781	-0.5056	-0.2332
φ_y	0.6285	0.0472	0.6489	0.5675	0.7230
φ_i	-0.1609	0.0603	-0.1531	-0.2175	-0.0228
		Standard deviation structural shocks			
σ_{v_π}	0.0118	0.0010	0.0120	0.0101	0.0136
σ_{v_y}	0.0032	0.0003	0.0031	0.0027	0.0038
σ_{v_i}	0.0117	0.0007	0.0119	0.0110	0.0131
σ_{π^*}	0.0015	0.0010	0.0004	0.0001	0.0034
σ_{ρ^*}	0.0074	0.0014	0.0073	0.0039	0.0083
		Initial values stochastic endpoints			
π_0^*	0.0182	0.0085	0.0184	0.0051	0.0332
ρ_0^*	0.0204	0.0072	0.0197	0.0061	0.0308
		Standard deviation measurement error yield curve			
$\sigma_{\eta_y,1}$	0.0103	0.0005	0.0101	0.0094	0.0111
$\sigma_{\eta_y,2}$	0.0044	0.0003	0.0044	0.0040	0.0049
$\sigma_{\eta_y,4}$	0.0040	0.0002	0.0040	0.0037	0.0043
$\sigma_{\eta_y,12}$	0.0020	0.0001	0.0019	0.0018	0.0022
$\sigma_{\eta_y,20}$	0.0008	0.0001	0.0008	0.0006	0.0010
$\sigma_{\eta_y,40}$	0.0035	0.0002	0.0034	0.0032	0.0039
		Standard deviation measurement error inflation expectation			
$\sigma_{\eta_\pi,4}$	0.0052	0.0004	0.0051	0.0046	0.0058
$\sigma_{\eta_\pi,40}$	0.0010	0.0001	0.0010	0.0008	0.0012

Note: This table reports the posterior density estimates for the parameters of the *Learning Model*. Column 1 presents the parameters, Columns 2, 3, and 4 report the mean, standard deviation, and mode of the posterior distribution, respectively. Columns 5 and 6 report the 5-th and 95-th percentile of the posterior distribution, respectively. All results are obtained using the Metropolis-Hastings algorithm.

Table 5: Posterior density estimates II - Learning Model

	Posterior				
Parameter	Mean	Stdev	Mode	5%	95%
	Average mispricing yields				
ξ_2	-0.0034	0.0016	-0.0032	-0.0054	-0.0001
ξ_4	-0.0001	0.0019	0.0003	-0.0023	0.0038
ξ_{12}	0.0010	0.0021	0.0017	-0.0010	0.0058
ξ_{20}	0.0011	0.0022	0.0018	-0.0008	0.0063
ξ_{40}	0.0010	0.0038	0.0013	-0.0023	0.0095
	Prices of risk: $\Lambda_0(\times 10^{-2})$				
$\Lambda_{0,\pi}$	-0.0700	0.1379	-0.1257	-0.3218	0.1365
$\Lambda_{0,y}$	-0.0675	0.1359	0.0432	-0.2558	0.1843
$\Lambda_{0,i}$	-0.0844	0.1456	-0.0193	-0.3322	0.1576
Λ_{0,π^*}	-0.0576	0.1680	-0.0559	-0.2970	0.2323
Λ_{0,ρ^*}	-0.1026	0.0796	-0.1144	-0.2128	0.0385
	Prices of risk: $\Lambda_1(\times 10^{-4})$				
$\Lambda_{1,\pi\pi}$	0.0728	0.0779	-0.0030	-0.0456	0.1973
$\Lambda_{1,\pi y}$	0.3139	0.0944	0.2782	0.1737	0.4842
$\Lambda_{1,\pi i}$	-1.1067	0.2303	-0.9401	-1.5123	-0.7743
$\Lambda_{1,y\pi}$	-0.1302	0.3327	-0.0016	-0.8500	0.2734
$\Lambda_{1,yy}$	0.1051	0.1363	0.0575	-0.1463	0.3112
$\Lambda_{1,yi}$	-0.4329	0.4250	-0.5493	-1.0148	0.3596
$\Lambda_{1,i\pi}$	-0.0445	0.0469	-0.0382	-0.1268	0.0337
$\Lambda_{1,iy}$	-0.0274	0.0363	-0.0198	-0.0887	0.0326
$\Lambda_{1,ii}$	0.5592	0.0718	0.5353	0.4471	0.6808
	Learning parameters				
w_{π^*}	0.6083	0.0645	0.6550	0.4897	0.6907
w_{ρ^*}	0.7789	0.1441	0.9746	0.5858	0.9872
g_π	0.2191	0.0255	0.2190	0.1668	0.2479
g_ρ	0.1200	0.0746	0.0443	0.0117	0.2392
$\sigma_{\pi^{*b}}$	0.0045	0.0014	0.0058	0.0015	0.0067
$\sigma_{\rho^{*b}}$	0.0121	0.0058	0.0050	0.0017	0.0194
π_0^{*b}	0.0057	0.0039	0.0037	0.0006	0.0125
ρ_0^{*b}	0.0240	0.0087	0.0231	0.0078	0.0366

Note: This table reports the posterior density estimates for the parameters of the *Learning Model*. Column 1 presents the parameters, Columns 2, 3, and 4 report the mean, standard deviation, and mode of the posterior distribution, respectively. Columns 5 and 6 report the 5-th and 95-th percentile of the posterior distribution, respectively. All results are obtained using the Metropolis-Hastings algorithm.

Table 6: Variance decomposition of the yield curve - Learning Model

Type of shock	Fed funds rate	Level	Slope	Curvature
	Horizon: 1 quarter			
Supply (ε_{v_π})	0.04	0.08	0.01	0.13
Demand (ε_{v_y})	0.04	0.04	0.03	0.22
Policy rate (ε_{v_i})	0.81	0.33	0.88	0.63
Inflation target (ε_{π^*})	0.00	0.00	0.00	0.00
Eql. real rate (ε_{ρ^*})	0.11	0.52	0.06	0.02
Belief inflation ($\varepsilon_{\pi^{*b}}$)	0.00	0.03	0.01	0.00
Belief real rate ($\varepsilon_{\rho^{*b}}$)	0.00	0.00	0.00	0.00
	Horizon: 4 quarters			
Supply (ε_{v_π})	0.09	0.08	0.05	0.13
Demand (ε_{v_y})	0.16	0.07	0.24	0.23
Policy rate (ε_{v_i})	0.43	0.12	0.65	0.62
Inflation target (ε_{π^*})	0.00	0.00	0.00	0.00
Eql. real rate (ε_{ρ^*})	0.30	0.68	0.05	0.02
Belief inflation ($\varepsilon_{\pi^{*b}}$)	0.02	0.05	0.01	0.00
Belief real rate ($\varepsilon_{\rho^{*b}}$)	0.00	0.00	0.00	0.00
	Horizon: 20 quarters			
Supply (ε_{v_π})	0.06	0.04	0.06	0.13
Demand (ε_{v_y})	0.07	0.02	0.32	0.24
Policy rate (ε_{v_i})	0.12	0.03	0.55	0.60
Inflation target (ε_{π^*})	0.00	0.00	0.00	0.00
Eql. real rate (ε_{ρ^*})	0.66	0.82	0.04	0.02
Belief inflation ($\varepsilon_{\pi^{*b}}$)	0.09	0.09	0.03	0.01
Belief real rate ($\varepsilon_{\rho^{*b}}$)	0.00	0.00	0.00	0.00
	Horizon: 40 quarters			
Supply (ε_{v_π})	0.04	0.03	0.06	0.13
Demand (ε_{v_y})	0.04	0.01	0.32	0.23
Policy rate (ε_{v_i})	0.07	0.02	0.53	0.59
Inflation target (ε_{π^*})	0.00	0.00	0.00	0.00
Eql. real rate (ε_{ρ^*})	0.75	0.85	0.04	0.02
Belief inflation ($\varepsilon_{\pi^{*b}}$)	0.10	0.09	0.05	0.03
Belief real rate ($\varepsilon_{\rho^{*b}}$)	0.00	0.00	0.00	0.00

Figure 1: Macroeconomic factors - Learning Model

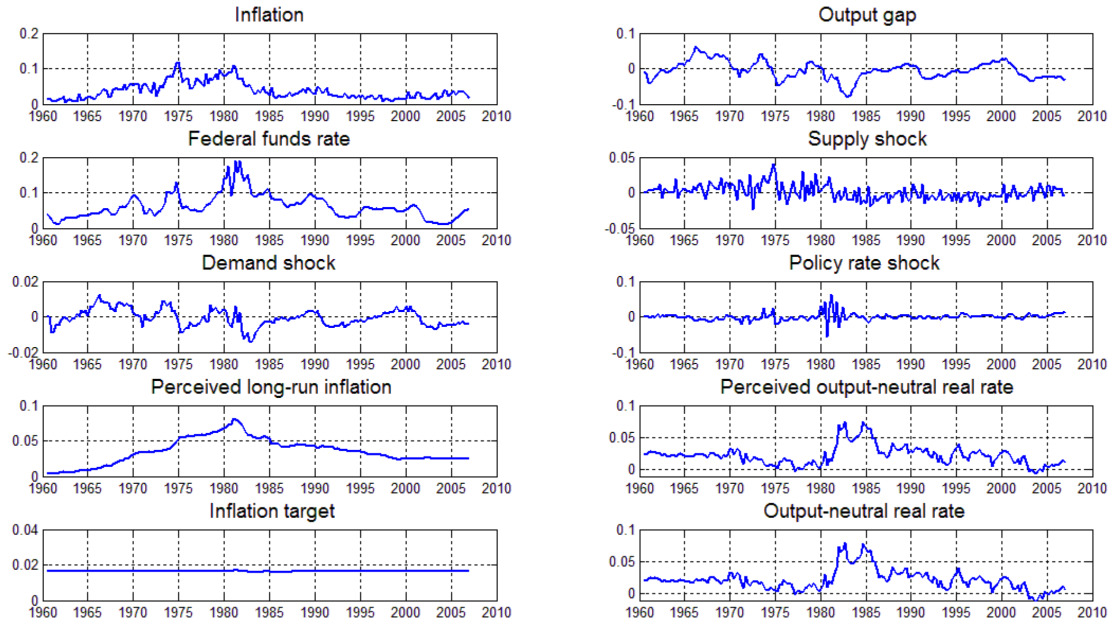


Figure 2: Perceived and actual stochastic endpoints - Learning Model

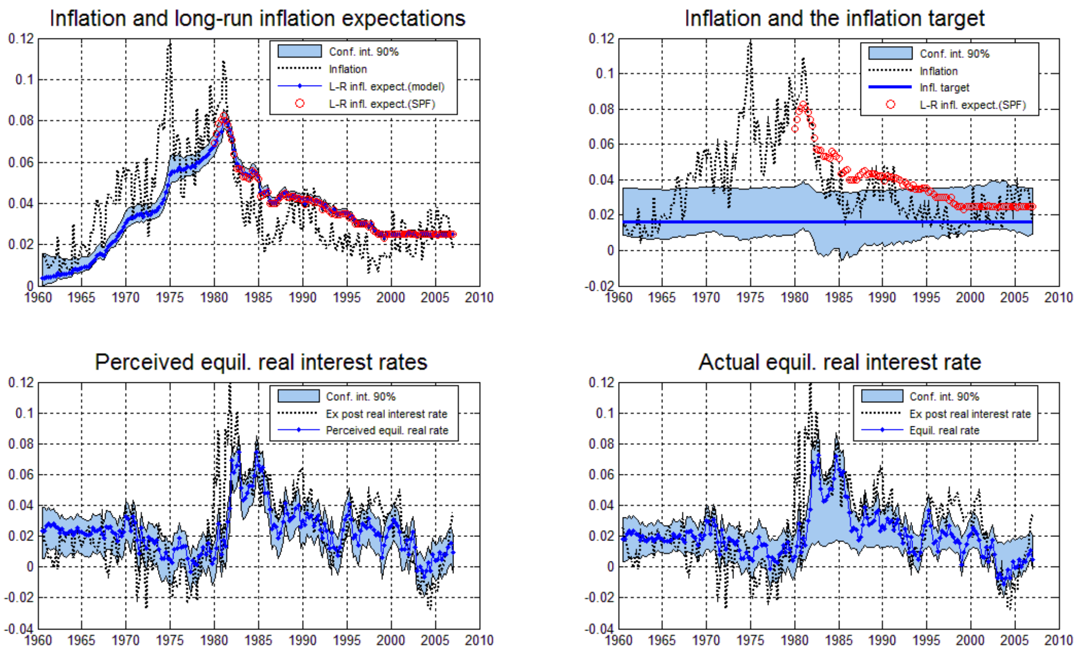


Figure 3: Factor loadings of the yield curve - Learning Model

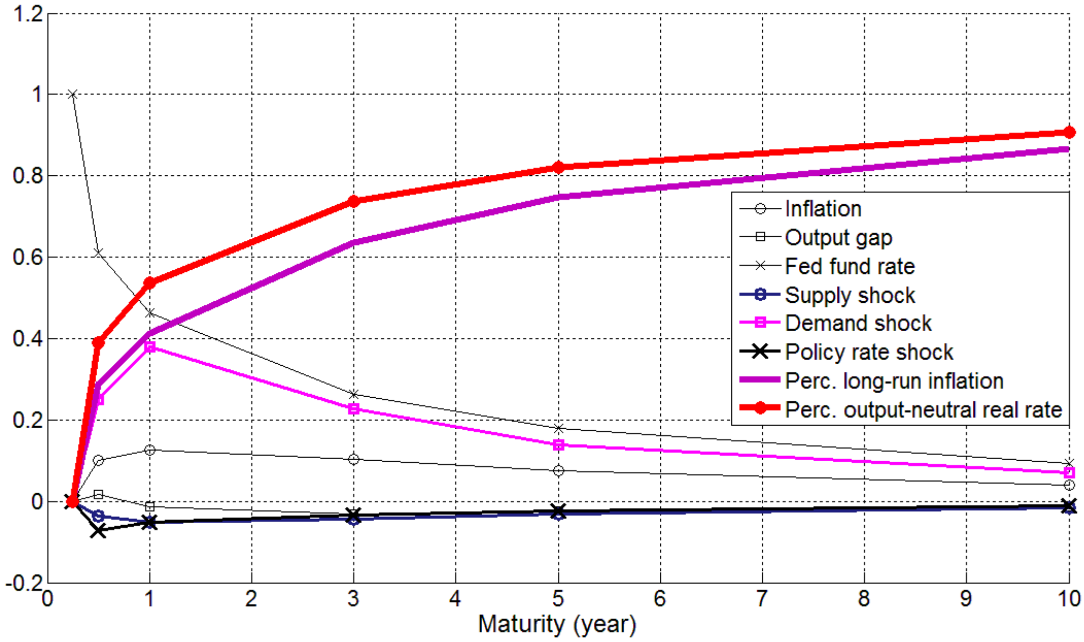


Figure 4: Observed and fitted yield curve - Learning Model

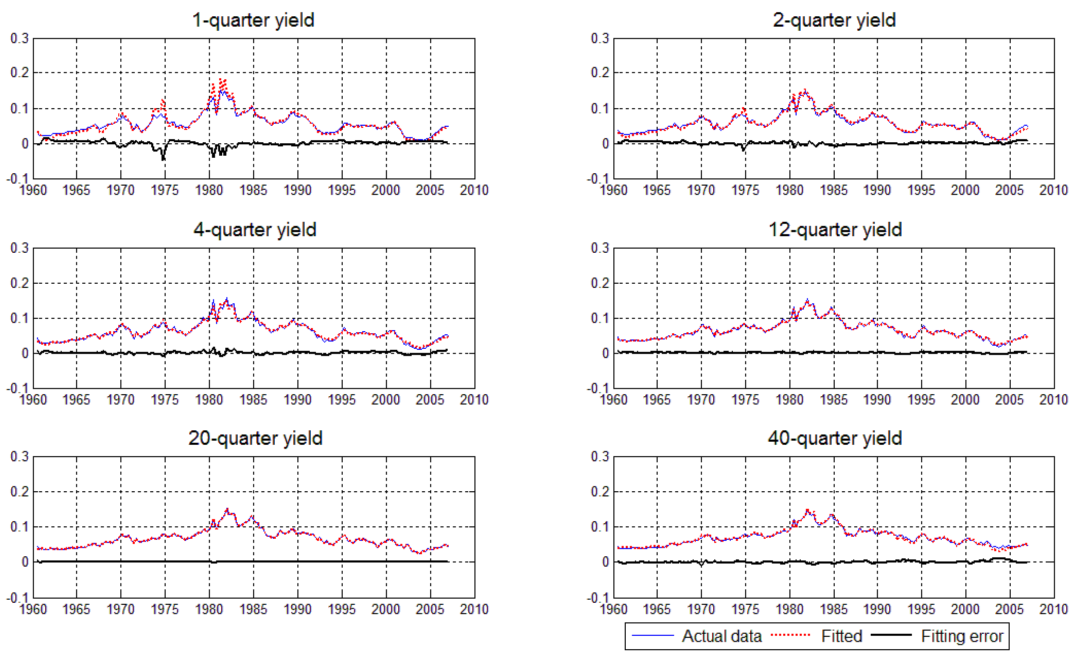


Figure 5: Decomposition of the 40-quarter yield and inflation expectation - Learning Model

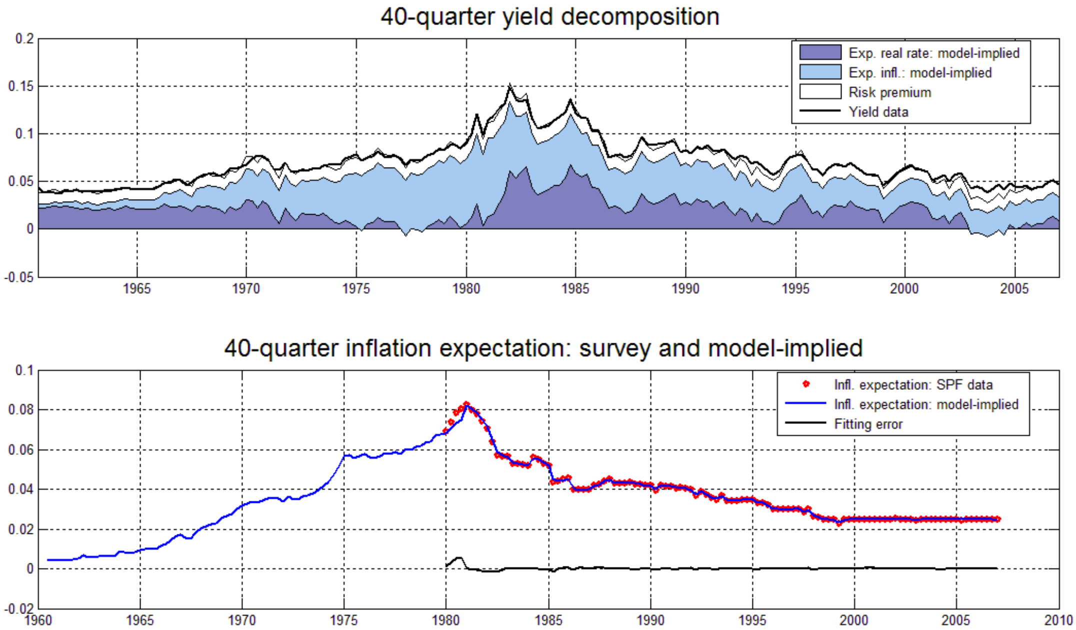
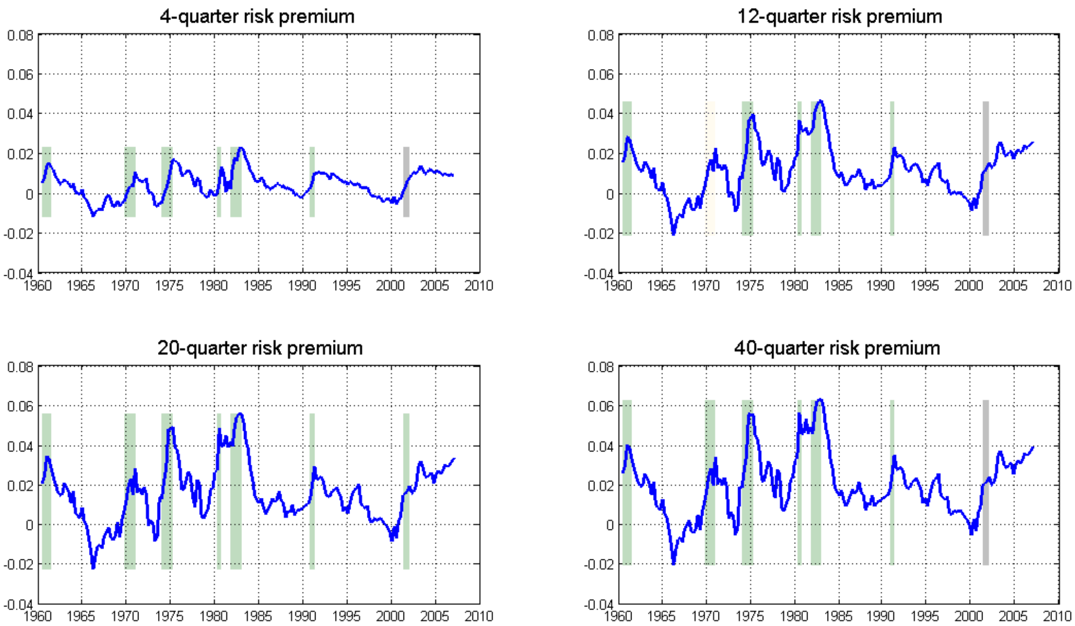


Figure 6: Expected excess holding return - Learning Model



Appendix A: Derivation of the Actual and Perceived Laws of Motion

As mentioned before, the structural model in Eqs. (1), (5), (8), (12) and (13) can be written in state space form. Collecting the observable macroeconomic factors in $X_t^m = [\pi_t,\ y_t,\ i_t,\ v_{\pi,t},\ v_{y,t},\ v_{i,t}]'$, the actual stochastic trends in $X_t^* = [\pi_t^*,\ \rho_t^*]'$, the perceived trends in $X_t^{*P} = [\pi_t^{*P},\ \rho_t^{*P}]'$, and the shocks in $\varepsilon_t^v = [\varepsilon_{v_\pi,t},\ \varepsilon_{v_y,t},\ \varepsilon_{v_i,t}]'$, $\varepsilon_t^* = [\varepsilon_{\pi^*,t},\ \varepsilon_{\rho^*,t}]'$, and $\varepsilon_t^{*b} = [\varepsilon_{\pi^{*b},t},\ \varepsilon_{\rho^{*b},t}]'$, the structural dynamics can be rewritten using matrices A, B, C, D, F, Σ^v and Σ^* as:

$$\begin{aligned} AX_t^m &= C + BE_t\left[X_{t+1}^m\right] + DX_{t-1}^m + FX_t^* + \Sigma^v\varepsilon_t^v, \\ X_t^* &= X_{t-1}^* + \Sigma^*\varepsilon_t^*, \end{aligned} \tag{31}$$

with F defined as $(A-B-D)H$ and H being a matrix containing the cointegrating relations, i.e. the dependence of X_t^m on X_t^*.[32] The full-information rational expectations (RE) solution (conditional on X_t^*) can be written as a reduced-form VAR:

$$\begin{bmatrix} X_t^m \\ X_t^* \end{bmatrix} = \begin{bmatrix} C^{RE} \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi^{RE} & (I-\Phi^{RE})H \\ 0 & I \end{bmatrix}\begin{bmatrix} X_{t-1}^m \\ X_{t-1}^* \end{bmatrix} + \begin{bmatrix} \Sigma_{1,1}^{RE} & \Sigma_{1,2}^{RE} \\ 0 & \Sigma^* \end{bmatrix}\begin{bmatrix} \varepsilon_t^v \\ \varepsilon_t^* \end{bmatrix}. \tag{32}$$

The PLM is obtained as the RE solution to the structural equations replacing the actual stochastic endpoints (X_t^*) by their perceived counterparts (X_t^{*P}):[33]

$$\begin{bmatrix} X_t^m \\ X_t^{*P} \end{bmatrix} = \begin{bmatrix} C^{RE} \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi^{RE} & (I-\Phi^{RE})H \\ 0 & I \end{bmatrix}\begin{bmatrix} X_{t-1}^m \\ X_{t-1}^{*P} \end{bmatrix} + \begin{bmatrix} \Sigma_{1,1}^{RE} & \Sigma_{1,2}^{RE} \\ 0 & \Sigma^{*P} \end{bmatrix}\begin{bmatrix} \varepsilon_t^v \\ \varepsilon_t^{*P} \end{bmatrix}. \tag{33}$$

The ALM is obtained (i) by substituting the expectations $E_t\left[X_{t+1}^m\right]$ by the subjective expectations $E_t^P\left[X_{t+1}^m\right]$ implied by the PLM in Eq. (33) into the structural Eq. (31):

$$\begin{aligned} (A - B\Phi^{RE})X_t^m &= C + B(C^{RE}) + B(I-\Phi^{RE})HX_t^{*P} + DX_{t-1}^m + FX_t^* + \Sigma^v\varepsilon_t^v, \\ X_t^* &= X_{t-1}^* + \Sigma^*\varepsilon_t^*, \end{aligned} \tag{34}$$

and (ii) by taking into account the dynamics of the perceived stochastic endpoints in Eq. (13) expressed in state-space form as:[34]

$$X_t^{*P} = X_{t-1}^{*P} + W\Sigma^*\varepsilon_t^* + (I-W)\left\{\Sigma^{*b}\varepsilon_t^{*b} + GV\left(X_t^m - E_{t-1}^P\left[X_t^m\right]\right)\right\}. \tag{35}$$

The ALM in Eqs. (34) and (35) can subsequently be summarized with respect to the full state vector $X_t = [X_t^{m\prime},\ X_t^{*P\prime},\ X_t^{*\prime}]'$and shock vector $\varepsilon_t = [\varepsilon_t^{v\prime},\ \varepsilon_t^{*b\prime},\ \varepsilon_t^{*\prime}]'$as:

$$X_t = C^A + \Phi^A X_{t-1} + \Gamma^A S^A \varepsilon_t, \tag{36}$$

with

$$C^A = \begin{bmatrix} (A - B\Phi^{RE}) & -B(I-\Phi^{RE})H & -F \\ -(I-W)GV & I & 0 \\ 0 & 0 & I \end{bmatrix}^{-1}\begin{bmatrix} (C + BC^{RE}) \\ -(I-W)GVC^{RE} \\ 0 \end{bmatrix}, \tag{37}$$

[32] The matrix H is given by $H' = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$ such that $\lim_{s\to\infty} E_t\left[X_{t+s}^m\right] = HX_t^*$.

[33] In line with the literature, we assume that agents ignore the dynamics of the updating rule. In solving the model, agents do not consider the implications of current forecast errors on subsequent inferences of the stochastic endpoints. Instead, agents regard the perceived stochastic endpoints as purely exogenous martingale processes, with impact matrix Σ^{*P}.

[34] The matrices W, G and Σ^{*b} are diagonal matrices containing the weights ω_{π^*} and ω_{ρ^*}, the gains g_π and g_ρ, and the standard deviations $\sigma_{\pi^{*b}}$ and $\sigma_{\rho^{*b}}$, respectively. Finally, V is a transformation matrix selecting from $(X_t^m - E_{t-1}^P(X_t^m))$ the inflation and real interest rate forecast errors.

$$\Phi^A = \begin{bmatrix} (A - B\Phi^{RE}) & -B(I-\Phi^{RE})H & -F \\ -(I-W)GV & I & 0 \\ 0 & 0 & I \end{bmatrix}^{-1} \begin{bmatrix} D & 0 & 0 \\ -(I-W)GV\Phi^{RE} & I-(I-W)GV(I-\Phi^{RE})H & 0 \\ 0 & 0 & I \end{bmatrix}, \tag{38}$$

$$\Gamma^A S^A = \begin{bmatrix} (A - B\Phi^{RE}) & -B(I-\Phi^{RE})H & -F \\ -(I-W)GV & I & 0 \\ 0 & 0 & I \end{bmatrix}^{-1} \begin{bmatrix} \Sigma^v & 0 & 0 \\ 0 & (I-W)\Sigma^{*b} & W\Sigma^* \\ 0 & 0 & \Sigma^* \end{bmatrix}. \tag{39}$$

Analogously, the PLM in Eq. (33) can be stated in state space form as:

$$X_t = C^P + \Phi^P X_{t-1} + \Gamma^P S^P \varepsilon_t^P, \tag{40}$$

with

$$C^P = \begin{bmatrix} C^{RE} \\ 0 \\ 0 \end{bmatrix}, \quad \Phi^P = \begin{bmatrix} \Phi^{RE} & (I-\Phi^{RE})H & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma^P S^P = \begin{bmatrix} \Sigma_{1,1}^{RE} & \Sigma_{1,2}^{RE} & 0 \\ 0 & \Sigma^{*P} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{41}$$

In both cases, the structural model implies zero vectors of constants, i.e. $C^A = C^P = 0$. [35]

[35] The cointegration between actual and perceived endpoints requires adaptive learning ($G > 0$). In case of no endogenous updating ($G = 0$), cointegration does not hold since the perceived endpoints also depend on the independent belief shocks.

Appendix B: Descriptive statistics of the data

Table B1: Descriptive statitistic of the data

	Macro variables			Yields						Infl. exp.	
	Infl.	O. gap	FF rate	1q	2q	4q	12q	20q	40q	4q	40q
Data	187	187	187	187	187	187	187	187	187	147	109
Mean	0.037	-0.006	0.061	0.055	0.057	0.061	0.065	0.067	0.070	0.040	0.039
Std. Dev.	0.024	0.025	0.034	0.028	0.027	0.028	0.027	0.026	0.024	0.020	0.015
Skewness	1.167	-0.041	1.233	1.031	0.951	0.899	0.891	0.921	0.945	0.909	1.247
Kurtosis	3.881	3.329	5.122	4.460	4.189	4.031	3.756	3.628	3.563	2.958	3.999
Correlation matrix											
Inflation	1										
O. gap	-0.311	1									
FF rate	0.737	-0.152	1								
Yield (1)	0.717	-0.156	0.992	1							
Yield (2)	0.708	-0.169	0.987	0.998	1						
Yield (4)	0.666	-0.192	0.969	0.982	0.989	1					
Yield (12)	0.619	-0.248	0.928	0.950	0.962	0.984	1				
Yield (20)	0.601	-0.283	0.900	0.924	0.938	0.963	0.995	1			
Yield (40)	0.595	-0.327	0.864	0.887	0.901	0.930	0.976	0.992	1		
Infl.exp.(4)	0.873	-0.335	0.896	0.890	0.887	0.863	0.847	0.839	0.836	1	
Infl.exp.(40)	0.835	-0.367	0.876	0.872	0.872	0.857	0.862	0.867	0.875	0.983	1

Note: The data set consists of quarterly observations for the U.S. economy covering the period from 1960:Q2 to 2006:Q4 (187 observations). Inflation is computed based on the quarterly GDP deflator and is expressed in per annum terms. The output gap is the percentage deviation of GDP from the potential output reported by the Congressional Budget Office (CBO). The policy rate is the effective federal funds rate. The term structure of interest rate data consist of yields of bonds with maturities of 1, 2, 4, 12, 20 and 40 quarters. For 1- and 2-quarter yields, we use data from the secondary market for Treasury bills. For 4-, 12-, 20- and 40-quarter yields, we combine the data sets compiled by Gürkaynak, Sack, and Wright (2007) and McCulloch and Kwon (1993). The 4- and 40-quarter average inflation expectation data are obtained from the Survey of Professional Forecasters (SPF). O. gap denotes output gap, FF rate the federal funds rate, Mean the sample arithmetic average, expressed as p.a. percentage, and Std. Dev. the standard deviation.

Appendix C: Posterior distribution of alternative models

This appendix presents the posterior distribution of the parameters of the Benchmark Model, the Mispricing Model, the Flexible Price of Risk Model.

Table C1: Posterior density estimates - Benchmark Model

	Posterior				
Parameter	Mean	Stdev	Mode	5%	95%
Structural parameters					
δ_π	0.5343	0.0703	0.5063	0.4405	0.6723
κ	0.0085	0.0037	0.0072	0.0035	0.0153
h	0.7022	0.0483	0.7057	0.6185	0.7764
σ	2.1574	0.354	2.0651	1.6220	2.7805
γ_π	0.4105	0.1098	0.4056	0.2295	0.596
γ_y	0.3765	0.0575	0.3697	0.2855	0.4729
γ_i	0.6205	0.0429	0.6296	0.5450	0.6817
Autocorrelation structural shocks					
φ_π	-0.2476	0.0858	-0.2326	-0.395	-0.1189
φ_y	0.666	0.0384	0.6702	0.6009	0.7282
φ_i	-0.2417	0.0653	-0.2409	-0.3473	-0.1357
Standard deviation structural shocks					
σ_{v_π}	0.0097	0.0009	0.0096	0.0085	0.0113
σ_{v_y}	0.0031	0.0003	0.0031	0.0027	0.0037
σ_{v_i}	0.0111	0.0006	0.0111	0.0102	0.0121
σ_{π^*}	0.0029	0.0001	0.0029	0.0027	0.0032
σ_{ρ^*}	0.0049	0.0002	0.0049	0.0045	0.0052
Initial values stochastic endpoints					
π_0^*	0.0291	0.0042	0.0291	0.0222	0.0359
ρ_0^*	0.0158	0.0050	0.0161	0.0074	0.0241
Standard deviation measurement error yield curve					
$\sigma_{\eta_y,1}$	0.0102	0.0005	0.0102	0.0094	0.0112
$\sigma_{\eta_y,2}$	0.0080	0.0004	0.008	0.0073	0.0088
$\sigma_{\eta_y,4}$	0.0054	0.0003	0.0054	0.005	0.0059
$\sigma_{\eta_y,12}$	0.0027	0.0002	0.0026	0.0024	0.0029
$\sigma_{\eta_y,20}$	0.0010	0.0002	0.001	0.0008	0.0013
$\sigma_{\eta_y,40}$	0.0059	0.0003	0.0058	0.0053	0.0065
Standard deviation measurement error inflation expectation					
$\sigma_{\eta_\pi,4}$	0.0060	0.0010	0.0055	0.0047	0.0081
$\sigma_{\eta_\pi,40}$	0.0011	0.0002	0.0010	0.0008	0.0014

Note: This table reports the posterior density estimates for the parameters of the *Learning Model*. Column 1 presents the parameters, Columns 2, 3, and 4 report the mean, standard deviation, and mode of the posterior distribution, respectively. Columns 5 and 6 report the 5-th and 95-th percentile of the posterior distribution, respectively. All results are obtained using the Metropolis-Hastings algorithm.

Table C2: Posterior density estimates - Mispricing Model

	Posterior				
Parameter	Mean	Stdev	Mode	5%	95%
		Structural parameters			
δ_π	0.4453	0.0292	0.4468	0.4006	0.4956
κ	0.0098	0.0039	0.0089	0.0042	0.0167
h	0.7559	0.046	0.7589	0.6772	0.8300
σ	2.7394	0.411	2.7017	2.1159	3.4655
γ_π	0.1963	0.0801	0.1692	0.0762	0.3359
γ_y	0.2019	0.0384	0.2015	0.1358	0.2637
γ_i	0.5232	0.0404	0.5326	0.4496	0.5822
		Autocorrelation structural shocks			
φ_π	-0.1676	0.0634	-0.1618	-0.2736	-0.0648
φ_y	0.5894	0.0494	0.5923	0.5022	0.6624
φ_i	-0.1800	0.0762	-0.1887	-0.2984	-0.0521
		Standard deviation structural shocks			
σ_{v_π}	0.0104	0.0009	0.0102	0.0089	0.0119
σ_{v_y}	0.0031	0.0003	0.0031	0.0026	0.0037
σ_{v_i}	0.0117	0.0006	0.0117	0.0107	0.0127
σ_{π^*}	0.0033	0.0002	0.0033	0.0030	0.0037
σ_{ρ^*}	0.0065	0.0004	0.0065	0.0059	0.0073
		Initial values stochastic endpoints			
π_0^*	0.0202	0.0043	0.0205	0.013	0.0273
ρ_0^*	0.0191	0.0059	0.0188	0.0097	0.0293
		Standard deviation measurement error yield curve			
$\sigma_{\eta_y,1}$	0.0102	0.0005	0.0101	0.0094	0.0111
$\sigma_{\eta_y,2}$	0.0057	0.0003	0.0057	0.0052	0.0063
$\sigma_{\eta_y,4}$	0.0041	0.0002	0.0041	0.0038	0.0045
$\sigma_{\eta_y,12}$	0.0021	0.0001	0.0021	0.0019	0.0023
$\sigma_{\eta_y,20}$	0.0008	0.0001	0.0008	0.0006	0.0010
$\sigma_{\eta_y,40}$	0.0036	0.0002	0.0036	0.0033	0.0040
		Standard deviation measurement error inflation expectation			
$\sigma_{\eta_\pi,4}$	0.0047	0.0003	0.0047	0.0042	0.0052
$\sigma_{\eta_\pi,40}$	0.0010	0.0002	0.0010	0.0008	0.0013
		Average mispricing yields			
ξ_2	-0.0048	0.0006	-0.0047	-0.0057	-0.0038
ξ_4	-0.0016	0.0007	-0.0016	-0.0028	-0.0004
ξ_{12}	0.0014	0.0011	0.0014	-0.0003	0.0034
ξ_{20}	0.0041	0.0011	0.0041	0.0022	0.0062
ξ_{40}	0.0106	0.0013	0.0106	0.0085	0.0130

Note: This table reports the posterior density estimates for the parameters of the *Learning Model*. Column 1 presents the parameters, Columns 2, 3, and 4 report the mean, standard deviation, and mode of the posterior distribution, respectively. Columns 5 and 6 report the 5-th and 95-th percentile of the posterior distribution, respectively. All results are obtained using the Metropolis-Hastings algorithm.

Table C3: Posterior density estimates I - Flexible Price of Risk Model

	Posterior				
Parameter	Mean	Stdev	Mode	5%	95%
			Structural parameters		
δ_π	0.5010	0.0298	0.4726	0.4440	0.5373
κ	0.0117	0.0038	0.0095	0.0057	0.0184
h	0.7574	0.0444	0.7136	0.6770	0.8285
σ	2.9300	0.3636	3.4849	2.4057	3.6332
γ_π	0.3493	0.1138	0.3273	0.2161	0.5339
γ_y	0.7060	0.1433	0.7402	0.4649	0.9662
γ_i	0.7529	0.0262	0.7798	0.7047	0.7976
			Autocorrelation structural shocks		
φ_π	-0.2552	0.0701	-0.2370	-0.3501	-0.1186
φ_y	0.6202	0.0501	0.5858	0.5300	0.6845
φ_i	-0.1275	0.0423	-0.0864	-0.1891	-0.0494
			Standard deviation structural shocks		
σ_{v_π}	0.0115	0.0010	0.0118	0.0098	0.0132
σ_{v_y}	0.0032	0.0004	0.0033	0.0026	0.0038
σ_{v_i}	0.0128	0.0006	0.0138	0.0121	0.0141
σ_{π^*}	0.0035	0.0002	0.0034	0.0031	0.0039
σ_{ρ^*}	0.0069	0.0003	0.0069	0.0064	0.0075
			Initial values stochastic endpoints		
π_0^*	0.0194	0.0048	0.0241	0.0123	0.0268
ρ_0^*	0.0133	0.0033	0.0179	0.0085	0.0193
			Standard deviation measurement error yield curve		
$\sigma_{\eta_y,1}$	0.0103	0.0005	0.0099	0.0095	0.0113
$\sigma_{\eta_y,2}$	0.0049	0.0003	0.0050	0.0045	0.0053
$\sigma_{\eta_y,4}$	0.0042	0.0002	0.0041	0.0037	0.0045
$\sigma_{\eta_y,12}$	0.0020	0.0001	0.0019	0.0018	0.0022
$\sigma_{\eta_y,20}$	0.0008	0.0001	0.0007	0.0006	0.0010
$\sigma_{\eta_y,40}$	0.0035	0.0002	0.0034	0.0032	0.0038
			Standard deviation measurement error inflation expectation		
$\sigma_{\eta_\pi,4}$	0.0046	0.0003	0.0043	0.0042	0.0051
$\sigma_{\eta_\pi,40}$	0.0011	0.0002	0.0010	0.0008	0.0014

Note: This table reports the posterior density estimates for the parameters of the *Learning Model.* Column 1 presents the parameters, Columns 2, 3, and 4 report the mean, standard deviation, and mode of the posterior distribution, respectively. Columns 5 and 6 report the 5-th and 95-th percentile of the posterior distribution, respectively. All results are obtained using the Metropolis-Hastings algorithm.

Table C4: Posterior density estimates II - Flexible Price of Risk Model

	Posterior				
Parameter	Mean	Stdev	Mode	5%	95%
		Prices of risk: $\Lambda_0(\times 10^{-2})$			
$\Lambda_{0,\pi}$	-0.2077	0.0906	-0.0889	-0.2905	-0.0447
$\Lambda_{0,y}$	-0.1883	0.1236	-0.1213	-0.3643	0.0026
$\Lambda_{0,i}$	-0.1716	0.0609	-0.2017	-0.2918	-0.1126
Λ_{0,π^*}	-0.1159	0.0930	-0.1315	-0.2956	0.0751
Λ_{0,ρ^*}	-0.0759	0.0266	-0.0620	-0.1304	-0.0325
		Prices of risk: $\Lambda_1(\times 10^{-4})$			
$\Lambda_{1,\pi\pi}$	0.1580	0.0661	0.1628	0.0673	0.2798
$\Lambda_{1,\pi y}$	0.4902	0.0904	0.4948	0.3262	0.6406
$\Lambda_{1,\pi i}$	-1.3656	0.2117	-1.5239	-1.7286	-1.0038
$\Lambda_{1,y\pi}$	-0.4380	0.3497	-0.1740	-0.9482	0.2550
$\Lambda_{1,yy}$	-0.0363	0.0820	0.0768	-0.1751	0.0592
$\Lambda_{1,yi}$	-0.3625	0.5253	-0.5938	-1.6301	0.1121
$\Lambda_{1,i\pi}$	-0.0557	0.0339	-0.0749	-0.1232	-0.0194
$\Lambda_{1,iy}$	-0.0797	0.0314	-0.0802	-0.1317	-0.0319
$\Lambda_{1,ii}$	0.5578	0.0539	0.5094	0.4805	0.6577

Note: This table reports the posterior density estimates for the parameters of the *Learning Model*. Column 1 presents the parameters, Columns 2, 3, and 4 report the mean, standard deviation, and mode of the posterior distribution, respectively. Columns 5 and 6 report the 5-th and 95-th percentile of the posterior distribution, respectively. All results are obtained using the Metropolis-Hastings algorithm.