MPRA
Munich Personal RePEc Archive

Technology choice and endogenous productivity dispersion over the business cycles

Tian, Can

University of Pennsylvania

31 May 2011

Online at https://mpra.ub.uni-muenchen.de/34480/
MPRA Paper No. 34480, posted 03 Nov 2011 01:03 UTC

Technology Choice and Endogenous Productivity Dispersion over the Business Cycles*

Can Tian†
University of Pennsylvania

November 2, 2011

Abstract

Various empirical works have shown that dispersion of firm-level profitability is significantly countercyclical. I incorporate firms' technology adoption decision into firm dynamics model with business cycle features to explain these empirical findings both qualitatively and quantitatively. The option of endogenous exiting and credit constraint jointly play an important role in motivating firms' risk taking behavior. The model predicts that relatively small sized firms are more likely to take risk, and that the dispersion measured as the variance/standard deviation of firm-level profitability is larger in recessions, which are consistent to the data.

Keywords: Firm Dynamics, Business Cycles, Countercyclical Dispersion.

JEL Classification Codes: D21, D92, E32, L11, L25, L26

*This is a revised version of my third year paper at UPenn. I am deeply indebted to Dirk Krueger for insightful advises and constant encouragement. I am grateful to Ufuk Akcigit, Harold Cole and Jeremy Greenwood for great comments. I also thank Rudiger Bachmann, Jie Cai, Fei Li, Min Ouyang, Pengfei Wang, Yi Wen, Laura Wu, Xiaodong Zhu and participants at 2011 North America Summer Meeting of the Econometric Society, 2011 Asian Meeting of the Econometric Society, and 2011 Tsinghua Workshop in Macroeconomics for Young Economists. All errors are mine.

†333 McNeil Building, 3718 Locust Walk, Philadelphia, PA 19104-6297, USA. (Email: tiancan@sas.upenn.edu)

Contents

1 Introduction

Uncertainty rises in bad times. Recently, this phenomenon attracts growing attention of economists, with numerous new evidences from individual level data sets[1]. However, this significantly negative correlation between uncertainty and aggregate economic condition is often treated as a calibration discipline, while not many works have been done to explain it.

In this paper, I provide a possible mechanism through which the worsened aggregate economic condition leads to an increase in the measured dispersion in individual level productivity. The model at work stands close to the standard industry dynamic model with firm entry and exit built in the seminal work Hopenhayn (1992), with aggregate fluctuations in terms of "technology shocks" as the driving force of model dynamics, which is also a standard approach in real business cycles literature. Meanwhile, it differs from the standard in that in each period, after observing the aggregate "technology realization", a staying firm has the option to adopt a risky technology, in addition to the standard safe technology whose productivity realization is determined by the aggregate state. Given the same capital input, the output and productivity associated to the risky technology is a mean-preserving spread of the safe one's output and productivity. Although firms are risk neutral and the risky technology does not give higher flow payoff, there is a positive fraction of firms that strictly prefer to take the risk. This is because the option of exit provides a lower bound to a firm's continuation value as a function of working capital and creates a local convexity in it. Therefore, firms in this region have the incentive to randomize over their future values by choosing the risky technology, and when the uncertain productivity realizes, dispersion arises. This setup resembles Vereshchagina and Hopenhayn (2009) on occupational choice. In bad times, this region gets larger and the fraction of risky firms then gets larger. Consequently, the average or aggregate riskiness in firms' production increases, so does the realized productivity dispersion. Despite the model is only a standard one with a little twist, it is capable of generating productivity dispersion negatively correlated to aggregate state, with the correlation coefficient in line with data.

This model's mechanism is also strongly motivated by empirical findings. It has features and implications that mirror the following observations: (1) new firms are relatively small and small firms have low survival rate; (2) small and/or young firms tend to bear more risk and/or show larger productivity dispersion; (3) business cycles indicators lead the change in productivity dispersion; and (4) in recessions, more firms become risky and this increases exit rate.

The first two points are closely related, as the exit hazard is a special form of firm level

[1] Examples are Higson, Holly and Kattuman (2002), Higson, Holly, Kattuman and Platis (2004), Bloom (2009), Bloom, Floetotto and Jaimovich (2010), Bachmann and Bayer (2011), Arellano, Bai and Kehoe (2009), Bachmann, Elstner and Sims (2011), Chugh (2010), Kehrig (2011), to name a few.

risk. The relation between firm size and dynamics is well established and can be dated back to, for example, Dunne, Roberts, and Samuelson (1988). This is further discussed in Section 2. The findings on firm size and riskiness mainly come from two directions. Firstly, it is well established in the entrepreneurship literature that entrepreneurs, especially poorer ones, bear substantial amount of risk and tend to hold largely undiversified assets by investing heavily in their own firms, despite no or little premium in doing so. The risk here is interpreted as either the dispersion in small firm owners' personal income, or dispersion in return to private equity. At the same time, privately owned businesses are on average smaller in scale, measured in either capital stock, number of employees, or output[2]. The second stream of empirical findings, more relevant to my work, regards the productivity and firm size differential. Gertler and Gilchrist (1991), using the Quarterly Financial Report for Manufacturing Corporations, find that smaller firms exhibit higher standard deviation in sales growth rates than larger ones do. Dhawan (2001) looks at publicly traded firms in COMPUSTAT and find that small firms have higher failure rate and larger standard deviation in profit rate, while conditional on surviving, small firms show higher average profit rate. The superior profitability in small firms reduces if adjusted according to the failure rates. Here, Dhawan defines the profit rate as operating income per unit of capital, and he defines the firm-level riskiness or volatility as the variance in the random realizations of production. Using his definitions, my model generates the same pattern of profit rate and riskiness differential in size. There is also evidence from outside U.S.. For example, utilizing German data set USTAN, Bachmann and Bayer (2011) find decreasing productivity risk in firm size, where the risk is measured as average cross-sectional standard deviation in log-differences in firm-level Solow residuals.

The latter two points are on the cyclical change. Increase in measured cross-sectional dispersion lags the worsened business cycles indicator, for example, GDP growth rate, as shown in Bachmann, Elstner and Sims (2011) and Kehrig (2011) among others. Similar response is observed on the stock market. The last point relates to the key feature of the model. Although unfortunately I do not have direct observation from the data, there are indirect evidences that imply a larger fraction of risky firms in recessions consisting of mainly small firms. Exit rate raises in bad times. The findings on the relation between firm size and exit rate show that small firms and establishments drive the negative correlation between exit rate and business cycles. This indicates that small firms are more sensitive to the cyclical change, as the model predicts. The increased exit rate in bad times is shown in Section 2. A maybe more direct evidence is from Bachmann and Moscarini (2011), who find higher frequency of price adjustments during recessions, which is interpreted as

[2] Examples for works dedicated in this direction are Barton H. Hamilton (2000), Moskowitz and Vissing-Jorgensen (2002), and Herranz, Krasa and Villamil (2009). See Quadrini (2009) for a detailed review.

more of risky pricing experiments of the firms.

The goal of this paper is to complement existing theories. It is true that, if there is causality between measured uncertainty and aggregate economic condition, the direction can be either. The real option literature that aims at explaining such countercyclicality suggests the opposite direction of causal relationship, from increased uncertainty to decline in aggregate economic activity. An influential paper dedicated in this direction is Bloom (2009), which is later generalized by Bloom et. al. (2009). Bloom shows that increased uncertainty, through the channel of adjustment costs to capital and labor, leads to larger option value of waiting and a pause in investment and employment. A sizable drop in aggregate economic activity occurs because of this "wait-and-see" effect. The time varying uncertainty is twofold in his model: (1) time series standard deviation of productivity can be either high or low, evolving as a Markov process, and (2) the one-step-ahead conditional variance of this Markov process depends on current realization. However, Bachmann and Bayer (2011) and Bachmann, Elstner and Sims (2011) show that there is little evidence of sizeable "wait-and-see" effects in data. In addition, the process of entry and exit is neglected. Arellano, Bai and Kehoe (2009) do consider the entry and exit dynamics that interact with financial constraints, but, again, the causal direction is from time series uncertainty shock to a sizeable response in aggregate variable.

The uncertainty shock is indeed important and the inverted causality may still work, but there is an issue regarding measuring uncertainty, which relates to the lead-lag relationship between uncertainty and cycles. Time series variances of major business condition indicators are often interpreted as uncertainty. In addition, a parallel family of uncertainty measures regards the realized cross-sectional dispersion in individual level performances, which include, among others, cross-sectional variance in measured firm-level total factor productivities, levels or growth rates, and sales growth rates. However, realized cross-sectional dispersion not only lags the aggregate cycles, but is also a controversial measure of uncertainty about future, which in turn casts more doubts on the argument that it is the increased uncertainty that leads to worsened economic activity[3].

The other paper that entertains the same causal direction as mine is Bachmann and Moscarini (2011). They build a model in which firms need to run costly experimentation and hence learn about their own market powers. As a result of lower experimentation costs, the dispersion of productivity measured in sales is larger during recessions due to more experimentations conducted. My model shares a similar feature with theirs, in that the option of exiting promotes the risky performance of firms. At the same time, my model differs from theirs by predicting that smaller firms are the major driving force of countercyclical productivity and entry/exit dynamics. In this

[3] Bloom (2009) and Bloom et. al. (2010) are two representative examples.

paper, I incorporate firms' technology adoption decision into an otherwise standard firm dynamics model with business cycle features. The main feature of this model is that, conditional on staying, an operating firm can choose from two different types of technology: a safe one, and a risky one with no risky premium. The option of exiting naturally forms a lower bound on the value of a firm, and therefore provides a certain degree of incentive on risk taking behavior on the margin resulting from the convexity of the value function.

The rest of the paper is organized as follows. Section 2 describes the stylized facts on cyclical dispersion of productivity, firm size distribution and dynamics. Section 3 contains a simple three-period model that illustrates the mechanism and shows preliminary results. Section 4 takes the simple model into infinite horizon. Section 6 concludes.

2 Empirical Facts

Cyclical Productivity Dispersion. Eisfeldt and Rampini (2006) use data from COMPUSTAT and find countercyclical movement of dispersion in Tobin's q. At the same time, they show a similar pattern for dispersion of total factor productivity growth rates at four digit SIC level, with correlation being -0.465. Bloom (2009) shows that the US stock market volatility measured as VXO index is positively correlated to the cross-sectional standard deviations of firm profit growth, firm stock return, and industrial TFP growth at four digit SIC level, but its correlation with industrial production is significantly negative. Moreover, Bloom, Floetotto and Jaimovich (2010) take an even closer look at this issue and examine the Census of Manufactures, and find that various measures of uncertainty are significantly countercyclical at all of establishment, firm, industry, and aggregate levels. Bachmann and Bayer (2009a, 2009b) take a long panel of German firm-level micro-data that covers all single digit industries, and show that the correlation between dispersion in growth rates of firm-level TFP, sale, and value added and economic performance is significantly negative. This pattern preserves in subsamples divided by sector and by size. Although a different economy, their USTAN data set has the clear advantage in coverage. Moreover, by looking at different size quantiles, they document that time series averaged productivity dispersion in smaller firms tend to be larger than bigger firms. Chugh (2010) explores the profitability series constructed by Cooper and Haltiwanger (2006) from Longitudinal Research Database and calculates the cyclical correlation between average productivity and the dispersion to be -0.97. However, the sample is of relatively short length as annual data and covers only 1977-1988, a period that exhibits unusually large degree of opposite movement according to my own approximation. Kehrig (2011) focuses more on the dispersion of productivity levels rather than profit rates. He looks at the establishment-level data of the US manufacturing sector that consists of the Annual Survey

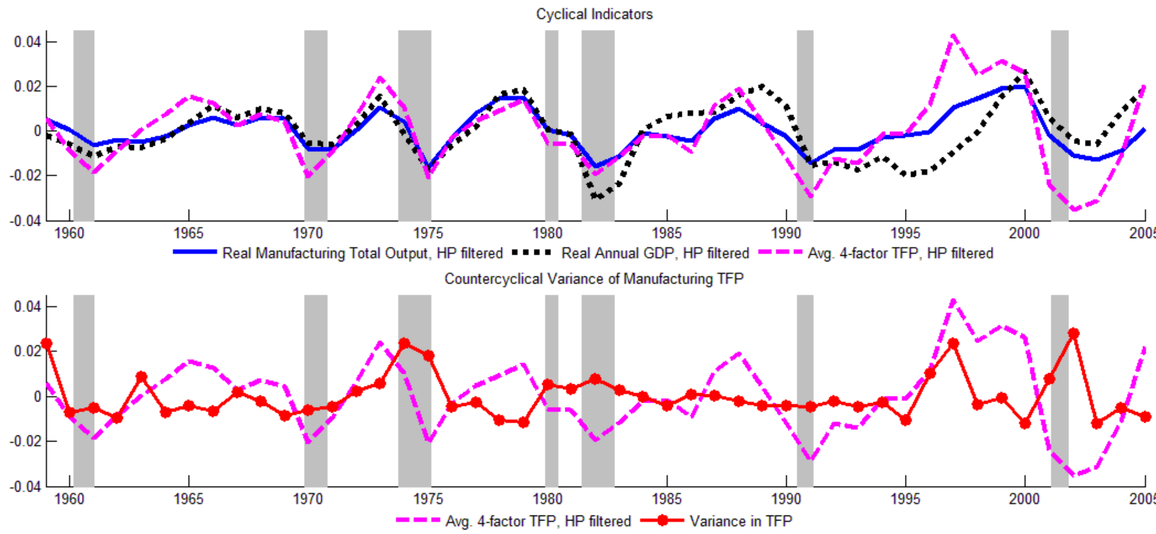

Figure 1: Cyclical Indicators and Variances in TFP. Upper panel plots different cyclical indicators, Real GDP (dotted line), Real total manufacturing output (solid line), Average TFP across industries at SIC 4 Digit level (dashed line). Lower panel shows cyclical behavior of TFP dispersion measured as variance (solid line with dots), together with Average TFP (dashed line). All series are HP-filtered. The shaded bars illustrate official NBER recessions. Real GDP data is from FRED; TFP series are from MIPD, and so is Manufacturing output measured as Real Total Shipment.

of Manufactures, Census of Manufactures, Plant Capacity Utilization Survey, and Longitudinal Business Database. Though the manufacturing sector as a whole shows countercyclical dispersion in establishment-level TFP, the durable industries show stronger cyclicality and it is the firms at bottom quantile of productivity distribution that drive the dispersion dynamics. In the theoretical part, he steps away from uncertainty shocks and uses only preference shock of the representative household as the driving force.

The upper panel of Figure (1) shows the co-movement of different business cycle indicators. In particular, I claim that the average TFP is a valid aggregate state indicator for the manufacturing sector. The correlation coefficient between average TFP (HP filtered) and sectoral output (HP filtered) is 0.86 with p-value of scale 10^{-9}. The average TFP corresponds to the cyclical indicator used through out the model, and the fluctuation in it represents technology or productivity shock, which drives the dynamics of model economy. Following Eisfeldt and Rampini (2006) and Bloom (2009), I use dispersion in cross-sectional TFP distribution at four digit SIC level to approximate that at the individual level, without arguing the validity of the approximation. The result is the lower panel of Figure (1), illustrating countercyclical movement of variance in TFP.[4] The precise correlation coefficients for the US manufacturing sector are documented in detail in both Bloom, Floetotto and Jaimovich (2010) and Kehrig (2011), and are summarized in Table 1 together with my own calculation.

Table 1. Correlations between Dispersion and Cyclical Indicator[5]

	For US Manufacturing Sector	GDP Growth	GDP HP Res.	Avg. ΔTFP
				Kehrig (2011)
(1)	Estab. TFP, Std. Dev. (Durables, HP Residual)	-0.420	-0.528	–
(2)	Estab. TFP, Std. Dev. (Non-durables, HP Residual)	-0.172	–	–
				Bloom et. al. (2010)
(3)	Estab. Output Growth, IQR	-0.364	–	–
(4)	Estab. TFP Growth, Std. Dev.	-0.273	–	–
(5)	Firm Sales Growth, IQR	-0.265	–	–
(6)	Firm Stock Returns, IQR	-0.339	–	–
				Calculated from NBER-CES MIPD
(7)	Ind. TFP Growth, IQR	-0.502 (0.000)	-0.298 (0.021)	-0.184 (0.108)
(8)	Ind. TFP Growth, Std. Dev.	-0.262 (0.038)	-0.241 (0.051)	-0.129 (0.194)
(9)	Ind. TFP Growth, Var.	-0.249 (0.046)	-0.245 (0.048)	-0.123 (0.206)

[4] I obtain data from the same sources as the aforementioned two papers, yet with more recent data up until 2005. I get the same signaficantly negative correlations as in these two papers if I only use the same range of data as they do. However, if I include the newly update data as shown in the figure, I can only a negative correlation that is not significant and is much smaller in absolue scale, which is less than 0.11.

[5] The first column of results show correlation coefficients (p-value) with Real GDP growth rate, the second with residuals of HP-filtered Real GDP, and the last with weighted average TFP growth rate in manufacturing sector. Row (1) and (2) are taken from Table 3 and 4 in Kehrig (2011), in which the microlevel data sources are mainly ASM/CM/LBD continuously covering period of 1972-2005 at annual frequency. Row (3) to (6) are from Table 1 in Bloom, Floetotto and Jaimovich (2010). Establishment-level data are also from ASM/CM/LBD, 1972-2006, while the firm-level infomation is from Compustat at quarterly frequency, 1967:II-2008:III for sales growth and 1969:I-2010:III for stock returns. Row (7) to (9) are TFP dispersions cross industries at four digit SIC level and NBER-CES Manufacturing Industry Productivity Database is the source, covering annually 1959-2005. Except for IQR, all other moments of industrial TFP growth are weighted by real value of total shipment. Numbers in parentheses are one-sided p-values under the null of non-negative correlation.

Due to the limitation of data, I use dispersion measures at TFP growth rate instead of TFP level. The corresponding cyclical indicators are then GDP growth rate, sectoral output growth rate, and average TFP growth rate. To be comparable to other works, I only include GDP growth rate and GDP HP residuals in Table 1.

Firm Dynamics. To illustrate firm dynamics over time, I obtain annual data from 1977 to 2009 in Business Dynamics Statistics (BDS) at CES, a data set that recently became publicly accessible. To be consistent with micro-level evidence on countercyclical dispersion, I only look at the establishments in manufacturing sector. [6]

Table 2 summarizes the property of establishment entry and exit rates by firm size[7]. A firm is classified to be small if it has less than 50 registered employees. A more detailed illustration of entry and exit rates by year and by establishment size can be found in the Appendix.

[6] A noteworthy issue here is how to define an entrant and an exiting establishment. According to the official overview of BDS dataset, "An establishment opening or entrant is an establishment with positive employment in the current year and zero employment in the prior year. An establishment closing or exit is an establishment with zero employment in the current year and positive employment in the prior year. The vast majority of establishment openings are true greenfield entrants. Similarly, the vast majority of establishment closings are true establishment exits (i.e., operations ceased at this physical location). However, there are a small number of establishments that temporarily shutdown (i.e., have a year with zero employment) and these are counted in the establishment openings and closings." Therefore, an inevitable caveat is that, although of relatively small number, an "idling" establishment can show up in the data as exit first, and then as entrant, for potentially many times. However, one clear advantage especially over firm-level data is that merging and acquisition are not reasons for disappearing units. Therefore, I can safely assume that exiting establishments suffer from low realizations of productivity.

[7] The entry and exit rates are indeed calculated utilizing the numbers of new born establishments, closed establishments, and existing establishments. However, the size is classified using the number of employees in a firm, instead of an establishment. One can only argue that large firms tend to own large establishments, and therefore large establishments exhibit similar dynamics to the ones owned by large firms. Otherwise, it is not clear whether this is a valid approximation.

Table 2. Entry and Exit Rates in Manufactures[8]

For US Manufacturing Sector 1977-2009		Total	Large	Small
(1)	Avg. Entry Rate (%)	9.36	5.18	31.18
(2)	Avg. Exit Rate (%)	9.28	6.00	30.06
(3)	Std. Dev. (Entry^{HP}) (%)	0.52	0.64	1.85
(4)	Std. Dev. (Exit^{HP}) (%)	0.67	0.90	1.56
(5)	Corr(Entry^{HP}, (Avg. TFP)^{HP})	0.20 (0.29)	0.19 (0.33)	0.21 (0.29)
(6)	Corr(Exit^{HP}, (Avg. TFP)^{HP})	-0.26 (0.17)	-0.17 (0.37)	-0.23 (0.24)
(5')	Corr(ΔEntry, Avg. ΔTFP)	0.22 (0.26)	0.13 (0.51)	0.31 (0.11)
(6')	Corr(ΔExit, Avg. ΔTFP)	-0.10 (0.62)	0.06 (0.76)	-0.06 (0.73)

Comparing establishment dynamics in small firms to that of large ones, they are of much larger scales, more volatile, and more cyclical. Therefore, in the quantitative model, I only focus on the dynamics in small firms, and treat the entry and exit of large firms mainly as exogenous, and they happen only with small probability.

3 A Simple Model

To highlight the mechanism, I start from a simplified and tractable three period version of the full model. I remove some features of the working model that is not as crucial, and focus only on the incumbents' problem. The main idea is that the option to exit promotes risk taking of small firms by creating a local non-concavity in a firm's continuation value function, which in turn generates a non-degenerate dispersion in productivity. Moreover, as is shown in the comparative statics analysis, such dispersion becomes larger in bad time, due to a larger fraction of risk taking firms. The same mechanism drives the infinite horizon model as well.

3.1 Setup

There are 3 periods, $t = 0, 1, 2$. The length of each period is taken as one year, same as the period length in the full model. There are a continuum of risk neutral firm owners, each of whom

[8] The data source is still BDS. The binary grouping rule in size can be found in caption of Figure (2). In Row (1) and (2), the numbers are simple time series averages. Row (3) and (4) are standard deviations for HP residuals. Row (5) to (6) are correlations for HP residuals, (7) and (8) are for changes. Numbers in parenthesis are p-values. I choose to compute correlation coefficient this way instead of using original entry/exit rates because there is a declining trend in both series. This is an interesting observation on its own sake, but this paper is silent on it.

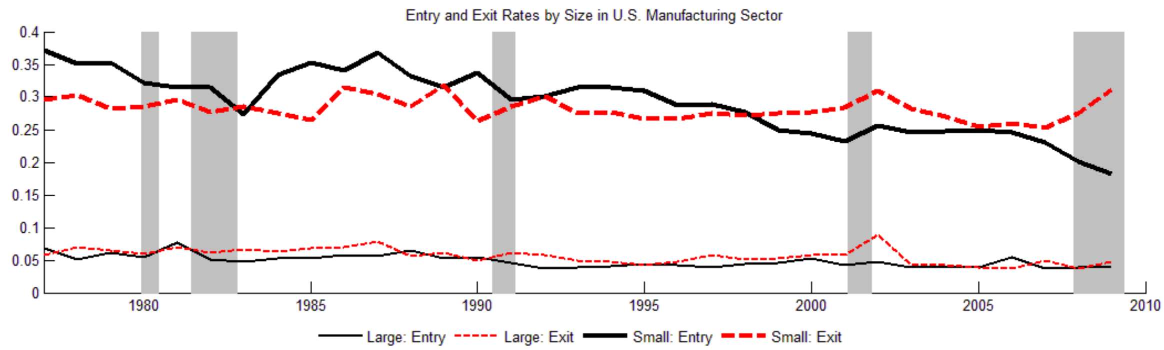

Figure 2: Cyclical behavior of entry and exit in manufacturing sector by size. A small firm is classified as one with less than 50 registered employees, and a large one with at least 50. This figure shows original series of entry (solid lines) and exit (dashed lines) rates by size. The two thinner lines at the bottom are for large firms, and the two thicker ones are for small firms. Data on entry and exit rates are from BDS of CES.

owns a firm with different level of initial resource $w_0 \in [0, \bar{w}]$. Assume that each firm has only one establishment or plant that produces one kind of product. The c.d.f. of owners' initial resource holding is given as $G(w_0)$. At period 0, initial wealth w_0 can be divided into investment k_0 for future wealth and immediate consumption $w_0 - k_0$. If an owner decides to invest k_0, then she will get $w_1 = F(Z, k)$ as period 1 wealth, where

$$F(Z, k) = Zk^{\alpha}, 0 < \alpha < 1,$$

and Z represents the realized productivity of the technology the firm owner chooses after investment decision. A production project is associated with a technology. An owner can choose one and only one out of two available technologies: a safe one and a risky one, differing in the riskiness and realizations of productivity.[9] For the safe technology, $Z = A$ for sure, while for a risky one, with probability $p \in (0, 1)$, $Z = \bar{z} > A$, and with probability $1 - p$, $Z = 0$. Both technologies give the same expected value of Z, that is, $p\bar{z} + (1 - p)\, 0 = A$. The risky technology has a variance in productivity as a function of p and $\bar{z}$, $\sigma^2(p, \bar{z}) = p(1 - p)\bar{z}^2$. As a result of linearity of $F(Z, k)$ in Z, the expected return of risky project is the same as the safe one, i.e., there is no ex ante risk premium. Under this setup, A corresponds to the average establishment-level TFP in data, and plays the role of economic condition indicator (or cyclical indicator in the full model); the riskiness of the risky technology represents the risk at the establishment level, while its aggregated counterpart measures the dispersion in productivity. I assume that production requires full attention

[9] For tractability, I assume only one type of risky technology and binary possible realization of it. In fact, it is possible to allow a continuous range of randomization, and this generalization does not alter the qualitative result.

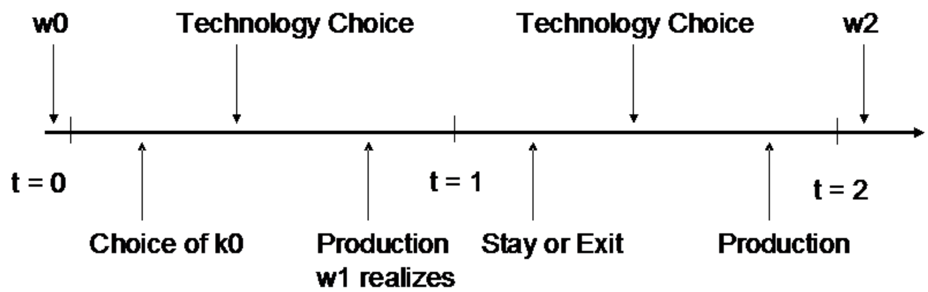

Figure 3: Timing of the Simple Model

of the firm owner, hence a firm cannot undertake multiple production projects simultaneously.

3.2 Analysis

At period 1, after the uncertainty in Z realizes, the agent can decide whether to close her firm, exit the industry and get outside option value V^0, or stay. Conditional on staying, she makes the investment choice k_1 and technology adoption choice again based on period 1 wealth w_1. In the last period, she simply consumes her final wealth w_2. The objective of an agent with initial wealth w_0 is to maximize her discounted consumption, with discount factor β:

$$V_0(w_0) = \max_{0 \leq k_0 \leq w_0} \{(w_0 - k_0) + \beta \max \{V_1(Ak_0^\alpha), (1-p) V_1(0) + pV_1(\bar{z}k_0^\alpha)\}\}$$

where $V_t(w_t)$ is the time t value for an agent with wealth w_t.

It is convenient to work backwards. At time $t = 2$,

$$V_2(w_2) = w_2.$$

At time $t = 1$, an agent with $k_1 > 0$ will be indifferent between operating a safe project and a risky one. Assume that all agents will perform safely in this case, which is consistent with their choice if they were risk averse. For simplicity, I do not allow borrowing in the short model, and the period 1 value for a staying firm will be:

$$V_1^1(w_1) = \max_{0 \leq k_1 \leq w_1} \{(w_1 - k_1) + \beta A k_1^\alpha\}.$$

Let k^* be the optimal capital choice without borrowing constraint. The value of a firm with wealth level w_1 at the beginning of period 1 will be given by

$$V_1(w_1) = \max\left\{V^0, V_1^1(w_1)\right\}.$$

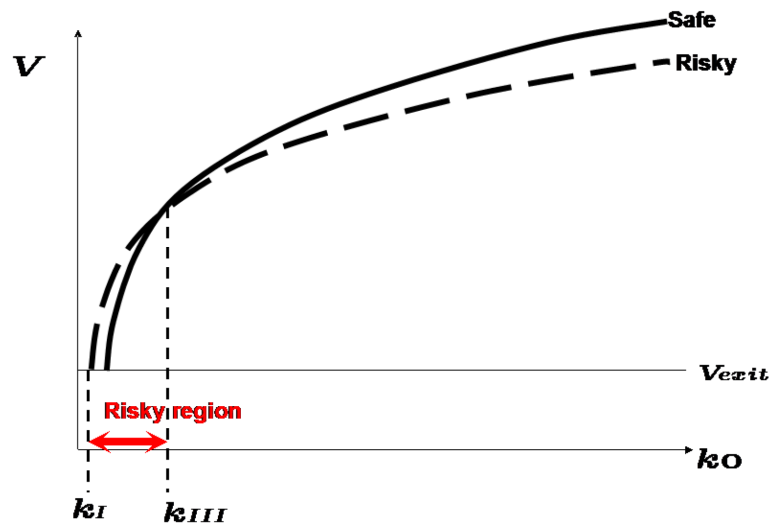

Figure 4: Continuation value as a functions of control variable, k_0. The horizontal axis is k_0, and the vertical axis is the continuation value for each level of k_0. The solid curve is the safe continuation value $V_1\left(Ak_0^{\alpha}\right)$, and the dashed curve is the risky continuation value $(1-p)\,V_1\left(0\right)+pV_1\left(\bar{z}k_0^{\alpha}\right)$. The horizontal line is V^0.

Let w_1^* be such that $V^0 = V_1^1\left(w_1^*\right)$. Note that there is a kink at w_1^* and $V_1\left(w_1\right)$ is convex in a neighborhood of w_1^*. This gives a firm with relatively low wealth level an incentive to take a risky project before it enters period 1. At $t=0$, a firm makes the investment decision and chooses a technology:

$$\begin{aligned} V_0\left(w_0\right) &= \max_{0\leq k_0\leq w_0}\left\{(w_0-k_0)+\beta\max\left\{V_1\left(Ak_0^{\alpha}\right),(1-p)\,V_1\left(0\right)+pV_1\left(\bar{z}k_0^{\alpha}\right)\right\}\right\} \\ &= \max_{0\leq k_0\leq w_0}\left\{(w_0-k_0)+\beta\max\left\{V^0,V_1^1\left(Ak_0^{\alpha}\right),pV_1^1\left(\bar{z}k_0^{\alpha}\right)+(1-p)\,V^0\right\}\right\}. \end{aligned}$$

To explicitly characterize a firm's technology choice, it is useful to introduce the following condition on parameters.

Condition 1. $0<V^0<\alpha^{\frac{2\alpha^2}{1-\alpha^2}}\beta^{\frac{1+\alpha^2}{1-\alpha^2}}\bar{z}^{\frac{1}{1-\alpha}}p^{\frac{2\alpha^2}{1-\alpha^2}}\left(p^{1+\alpha}-p^2\right)/\left(1-p\right).$

The risky and safe continuation values intersect at most once in the region where they are both greater than V^0. This condition ensures the existence of intersection, and makes the analysis tractable as shown in Proposition 1. The intuition is that given $(\bar{z},p)$, the option value V^0 of exiting cannot be too high, otherwise exit becomes very appealing, so does the risky technology. If it is violated, then all staying firms strictly prefer the risky technology. In particular, if V^0 is given, this happens when A is low enough.

Proposition 1. *At $t = 0$, if Condition 1 holds, then $\exists k_0^I$ and k_0^{III} such that $0 < k_0^I < k_0^{III} < k^*$, and the decision rule of a firm owner with initial wealth w_0 will be one of the following:*

1. *If $0 < w_0 \leq k_0^I$, she consumes all w_0 and exits in period 1 for sure;*

2. *If $k_0^I < w_0 < k_0^{III}$, she invests all w_0 in a risky project, then with probability p, $w_1 = \bar{z}k_0^\alpha$, she in turn invests all w_1 in period 1; with probability $1 - p$, $w_1 = 0$, she exits in period 1;*

3. *If $k_0^{III} \leq w_0 \leq k_0^A$, she invests all w_0 in a safe project, then invests all $w_1 = Ak_0^\alpha$ in period 1;*

4. *If $k_0^A < w_0 \leq k^*$, she invests all w_0 in a safe project, then invests k^* and consumes the rest in period 1;*

5. *If $w_0 > k^*$, she invests k^* and consumes the rest in both periods.*

The interesting region, or the "risky region", is the interval $\left[k_0^I, k_0^{III}\right]$. The exiting option forms a lower bound in value function that is higher than in the case without exiting. This new lower bound alters the shape of continuation value function, in particular, the continuation value function has a local convexity if safe technology is chosen. This non-concavity region is roughly the same as the interval $\left[k_0^I, k_0^{III}\right]$, in which firms have limited amount of capital stock. Firms that fall into this region have incentive to smooth out such convexity by entering a lottery and randomizing over possible outcomes, which is exactly the role that risky technology plays in this model. The fraction of risk taking firms will then be determined given the initial distribution $G(w_0)$, and each of these firms bear the same risk in terms of the randomness of productivity[10]. As can be seen below, a change in A drives the changes in the risky region and the the fraction of risk taking firms, and leads to a different productivity dispersion.

Assume a form of "Law of Large Numbers" holds[11], meaning that there are many firms at each w_0, and the fraction of risky project with $\bar{z}$ realized is p. The ex ante aggregate variation in TFP that firms choose to take in period 0, denoted as $\Gamma(p)$, is defined as the average variance in

[10] Once again, the same risk results from the assumption that only one way of randomization is permitted in the model for simplicity. To relax this restriction, one can assume that each firm can choose any distribution on productivity so long as the expection remains A, which results in a risky region larger than $\left[k_0^I, k_0^{III}\right]$. However, while making the model much more complicated, this will not alter the result qualitatively, neither will it provide more insight into the model.

[11] See Judd (1985).

profitability as a function of p, the probability of good realization of risky technology.

$$\begin{aligned}\Gamma(p,\bar{z}) &= \int_W var(Z)\, dG(w_0|k_0>0) \\ &= \int_{k_0^I}^{k_0^{III}} \sigma^2(p,\bar{z})\, dG(w_0|k_0>0) \\ &= \sigma^2(p,\bar{z})\,\Lambda(p,\bar{z}),\end{aligned}$$

where $\Lambda(p,\bar{z}) := \frac{G\left(k_0^{III}\right)-G\left(k_0^I\right)}{1-G\left(k_0^I\right)}$ in which k_0^I and k_0^{III} are functions of p and $\bar{z}$ as well. $\sigma^2(p,\bar{z})$ is simply the variance of the Bernoulli distributed productivity of risky technology, while $\Lambda(p,\bar{z})$ represents the measure of firms in the risky region. At the same time, the aggregate or average output in period 0, $O(p,\bar{z})$, is:

$$\begin{aligned}O(p,\bar{z}) &= \int_W E(F(Z,k_0))\, dG(w_0|k_0>0) \\ &= p\bar{z}\int_{k_0^I}^{k^*} w_0^{\alpha} dG(w_0|k_0>0) + p\bar{z}(k^*)^{\alpha}\frac{1-G(k^*)}{1-G(k_0^I)}.\end{aligned}$$

3.3 Comparative Statics

The nature of the simple model does not permit cyclical features. Therefore, I will instead analyze the comparative statics mimicking different times of business cycles. In particular, I use A, the average productivity, as the economic condition indicator, which corresponds to the average TFP in data. In the model, a change in A can result from either a change in p, or in $\bar{z}$, or in both. Provided that the bad outcome of the risky technology is always zero, $\bar{z}$ then determines the range, the variance of the Bernoulli productivity $\sigma^2(p,\bar{z})$, and the measure of risky region $\Lambda(p,\bar{z})$. At the same time, $\sigma^2(p,\bar{z})$ and $\Lambda(p,\bar{z})$ are also nontrivial functions of p. When A, p, and/or $\bar{z}$ changes, the resulting change in riskiness of a risky technology, that is, $\sigma^2(p,\bar{z})$ or range, is called the "riskiness effect", and the change in the measure of firms in the risky region, $\Lambda(p,\bar{z})$, is the "mean effect". The interesting one is the mean effect, therefore, to show the mechanism, I consider a particular change in A, such that $\bar{z}$ is held unchanged and p is also controlled to fully eliminate the riskiness effect, and examine the resulting mean effect.

Proposition 2. *Let V^0 and $\bar{z}$ remain unchanged and assume Condition 1 always holds. Let $A \in \{A^H, A^L\} = \{p^H\bar{z}, p^L\bar{z}\}$, p^H and p^L be such that $p^H > p^L > 0$. Suppose the distribution of initial wealth $G(\cdot)$ is Pareto and the lower bound of its support is below k_0^I when risky technology is p^H. Then:*

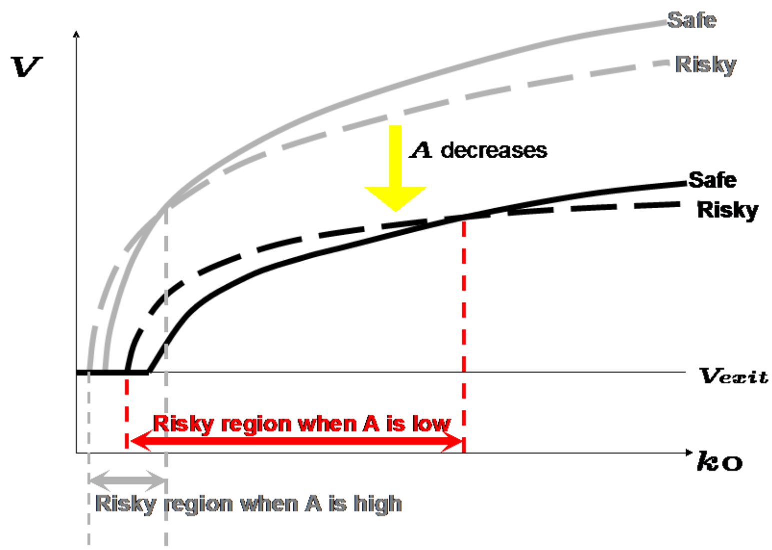

Figure 5: Comparative Statistics.

1. $O\left(p^H, \bar{z}\right) > O\left(p^L, \bar{z}\right)$;

2. $\Lambda\left(p^H, \bar{z}\right) < \Lambda\left(p^L, \bar{z}\right)$.

   To control the riskiness effect, assume $p^H + p^L = 1$, *then:*

3. $\sigma^2\left(p^H, \bar{z}\right) = \sigma^2\left(p^L, \bar{z}\right) = \bar{z}^2 p^H p^L$;

4. $\Gamma\left(p^H, \bar{z}\right) < \Gamma\left(p^L, \bar{z}\right)$.

According to this proposition, given $\bar{z}$ fixed, A (or p) summarizes the aggregate state, higher A then means good times. When the aggregate state is good, the total output is high, and this is always the case whether the riskiness effect is controlled or not. Meanwhile, the risky region is smaller in good times, which in turn leads to smaller fraction of risk taking firms, regardless of the riskiness effect. The assumption of Pareto distribution is to mimic the actually observed size distribution of firms, which is a sufficient but not necessary condition for the desired change in risky region. For example, a uniform distribution will give the same result. When the riskiness effect is controlled, the riskiness of a risky technology remains unchanged, therefore it is the change in fraction of risk taking firms that drives the change in resulting productivity dispersion, or average riskiness that firms choose to take, measured as variance in productivity. In fact, in the calibrated quantitative model, it turns out that the riskiness effect is too small to generate significant difference in simulated results.

Figure (5) illustrates what happens to the model if A decreases, as described in Proposition 2. When A is low, the exiting threshold increases and more firms exit. At the same time, low A also leads to a larger risky region and a greater fraction of risk taking firms, so now there are more firms that strictly prefer to the risky technology. As a result, if the change in A is controlled as specified before, the average risk that firms choose to take is also larger, so is the realized productivity dispersion. To summarize, the key step for the model to generate countercyclical productivity dispersion is the change in the risky region as aggregate state changes. And it is mainly because of an enlarged fraction of risk taking firms that causes a larger productivity dispersion in bad times. This mechanism remains in the quantitative model with infinite horizon. In fact, if the aggregate state follows a Markov process with only two possible outcomes of A^H and A^L controlled in a similar way, then without introducing other features, the negative correlation between aggregate state and productivity dispersion is still almost perfect.

4 Quantitative Model

4.1 Setup

Time is discrete, with infinite horizon. The firms that have survived at least one period are called incumbents. There is a continuum of potential entrant firms every period, each of whom draws their initial capital k_0 from a distribution $G^0(k_0)$. Once entering, an entrant acts as an incumbent thereafter as long as this firm stays. The production function is the same as in the simple model, $F(Z,k) = Zk^{\alpha}$, with $0 < \alpha < 1$ and Z being the realized productivity depending on technology choice.[12] At the beginning of each period, all firms observe average productivity A. An incumbent firm owner makes the choice between staying and exiting. If an incumbent exits, the

[12] In fact, $F(Z,k) = Zk^{\alpha}$ can be interpreted as a firm's profit, that is, the revenue net of the cost for variable factors, for example labor and materials. Specifically, assume a plant faces an inverse demand function $P(y) = By^{-b}$, and therefore its revenue becomes $R(y) = By^{1-b}$. Suppose the actual production function is $y = \tilde{A}k^{\tilde{\alpha}}l^{\tilde{\phi}}$, and the price for other factors is ω, then after optimization of l, the revenue function becomes

$$R = \left(B\tilde{A}^{1-b}\right)^{1/\left(\tilde{\phi}(1-b)\right)} \left[\tilde{\phi}(1-b)/\omega\right]^{\tilde{\phi}(1-b)/\left(\tilde{\phi}(1-b)-1\right)} k^{\tilde{\alpha}(1-b)/\left(\tilde{\phi}(1-b)-1\right)},$$

and profit function

$$\pi = \left(1 - \tilde{\phi}(1-b)\right) R.$$

Redefining variables gives the form of Zk^{α}. Therefore, Z in the model is more appropriately interpreted as measured productivity that includes information from the demand side, instead of actual productivity. For the same reason, parameter A shown later in the model shall also be interpreted as aggregate state of the model economy, rather than production technology.

owner takes away the remaining profit. A staying firm then decides the amount of next period's working capital k' and whether to adopt the safe technology or the risky one. Again, assume full attention of a firm owner as a prerequisite of production. After production, capital depreciates at a random rate $\delta \in \{\delta_1, ..., \delta_{N_\delta}\}$ with probability $\pi(\delta_i)$, $i = 1, ..., N_\delta$, which is assumed to be i.i.d. across firms and over time. Technology choice, investment, and depreciation jointly determine the incumbent's next period disposable resource.

The aggregate state for the model economy A evolves as a Markov chain with $A \in \mathbb{A} = \{A_1, ..., A_{N_A}\}$, and transition probability $\pi_{ij} = \Pr(A^j|A^i)$. In particular, this Markov chain is a discretized AR(1) process, such that $A_t = \rho_A A_{t-1} + \sigma_u u_t$, where $\rho_A \in (0, 1)$ is the serial correlation, and u_t is white noise. Following conventional real business cycles models, I assume time invariant volatility in A, in terms of constant σ_u. This implies that the driving force of this modelled economy is the traditional "technology shocks", that is, the change in "first moment". This is different from Bloom (2009) and Bloom et. al. (2010), who use time varying higher moments as the pure source of aggregate fluctuation. Meanwhile, this also distinct from, for example, Bechmann and Bayer (2009a,b) and Chugh (2010), who allow time varying higher moments in addition to the usual first moment movement to account for the countercyclical dispersion observed in data. I do not allow σ_u to change over time is based on the following considerations that (1) σ_u is time series volatility, which is not the same as observed cross-sectional dispersion, and (2) this model emphasizes a mechanism through which time varying A generates realized productivity dispersion, and it is of no need to introduce additional variation.

Production is costly. In each period, a staying and active firm needs to pay a fixed operating cost, and, if the firm needs increase or decrease its capital stock, it pays a capital adjustment cost as well. Mainly following Cooper and Haltiwanger (2006) and Bloom (2009), I assume the capital adjustment cost consists of two parts: (1) a non-convex cost, and (2) a transaction cost. The non-convex cost represents the opportunity cost when a firm is under capital adjustment. Specifically, this firm foregoes a fraction c_k of its production if there is capital adjustment in a given period. The transaction cost represents the partial irreversibility. When a firm needs to increase capital, the price paid for every unit of new capital is normalized to be one, where the price is interpreted as how many units of output needed to get one unit of capital. However, if a firm wants to reduce capital, the selling price for each unit of capital is $\theta < 1$.

Each time period has several stages, which resembles period 1 in the simple three period model.

- Stage 1: Observation of state variables. Aggregate state A realizes, so does the random capital depreciation for each firm δ. An incumbent firm observes (A, δ), and enters this period with remaining capital, $(1 - \delta) k$, and together with period's production $F(Z_{-1}, k)$, where Z_{-1} is the realization of last period's productivity of this firm. A potential entrant draws k_0 and observes A.

- Stage 2: Entry and exit. An entrant with (k_0, A) enters if there is positive expected profit. An incumbent exits either voluntarily based on continuation values, or exogenously with probability η.

- Stage 3: Investment and technology decision. Both staying incumbents and new born firms decide how much to invest, and then choose between safe and risky technologies. At the same time, the operating cost and capital adjustment cost are paid.

- Stage 4: Production. Production takes place in the form $F(Z, k')$, where k' is the new working capital, and Z is the productivity. If a firm chooses safe technology, then the productivity is deterministic, $Z = A$. Otherwise, with probability $p(A)$, the risky technology turns out to be a success, $Z = \bar{z}$, and with probability $1 - p(A)$, it fails, and $Z = 0$.

4.2 Individual Decision

An Incumbent's Problem. At the beginning of each period, an incumbent firm is characterized by (Z_{-1}, k, δ, A), where $Z_{-1} \in \{A_{-1}, 0, \bar{z}\}$ is the realized productivity of last period for a specific firm, which can be either of the safe productivity A_{-1}, the bad realization 0, or the good realization $\bar{z}$, k is the total amount of capital that was used in last period, δ is the realized random depreciation rate, and A represents the economic condition of current period.

The first choice an incumbent firm owner makes is between keeping operating and closing the firm and leaving.

$$V(Z_{-1}, k, \delta, A) = \max (1 - \chi) V^1 (Z_{-1}, k, \delta, A) + \chi V^0 (Z_{-1}, k, \delta, A),$$

where $\chi \in \{\eta, 1\}$ is the exiting choice, and η is the exogenous exiting hazard. If a firm with (Z_{-1}, k, δ, A) chooses to exit, the value is:

$$V^0 (Z_{-1}, k, \delta, A) = \theta(A) (Z_{-1} k^{\alpha} + (1 - \delta) k);$$

where $\theta(A) < 1$ is the fraction of resource a firm owner can take away when exiting, which is actually a resale price and is potentially a function of A. If this firm chooses to stay, the owner must then decide on investment, i, and technology choice, safe or risky. The capital stock evolves as follows

$$\gamma k' = (1 - \delta) k + i,$$

where, following Khan and Thomas (2008), $\gamma > 1$ determines the growth rates on the balanced growth path[13]. The operating cost $C(i; Z_{-1}, k, \delta, A)$ of a firm consists of a fixed cost c_f and a capital adjustment cost:

$$C(i; Z_{-1}, k, \delta, A) = c_f + c_k F(Z_{-1}, k) 1_{\{i \neq 0\}} + (1 - \theta(A))(-i) 1_{\{i<0\}}.$$

Actively adjusting capital stock and choosing $i \neq 0$, costs a firm c_k fraction of its revenue from last period's production. In addition, if a firm reduces its scale, it can only sell its current capital possession at price $\theta(A) < 1$. Combining these pieces gives the flow profit of this firm $D(k'; Z_{-1}, k, \delta, A)$, and

$$P(i; Z_{-1}, k, \delta, A) = F(Z_{-1}, k) - i - C(i; Z_{-1}, k, \delta, A) \geq 0.$$

I enforce non-negative profit as a constraint. The firm also has to choose between safe and risky technology. A safe technology produces $F(A, k')$ for sure; while a risky technology results in productivity at $\bar{z}$ with probability $p(A)$ and 0 with $1 - p(A)$. If the safe one is chosen, the firm gets:

$$V^1_{safe}(i; k, \delta, A) = \mathbb{E}_{A',\delta'}[V(A, k', \delta', A') | A],$$

and likewise,

$$V^1_{risky}(i; k, \delta, A) = p(A)\mathbb{E}_{A',\delta'}[V(\bar{z}, k', \delta', A') | A] + (1 - p(A))\mathbb{E}_{A',\delta'}[V(0, k', \delta', A') | A].$$

Therefore, conditional on staying, an incumbent firm solves the following maximization problem:

$$V^1(Z_{-1}, k, \delta, A) = \max_i \left\{P(i; Z_{-1}, k, \delta, A) + \beta \max\left\{V^1_{safe}(k'; Z_{-1}, k, \delta, A), V^1_{risky}(k'; Z_{-1}, k, \delta, A)\right\}\right\}.$$

Denote the state variables of an incumbent as $\psi = (Z_{-1}, k, \delta, A) \in \Psi$, with Ψ being the set of all possible states. Solution to an incumbent's question with state ψ is a list of policy functions $\{\chi(\psi), \tau(\psi), \iota(\psi)\}$ such that (1) $\chi(\psi)$ is the exiting choice, $\chi : \Psi \to \{\eta, 1\}$; and conditional on surviving, (2) $\tau(\psi)$ is the technology choice, $\tau : \{\psi \in \Psi : \chi(\psi) = \eta\} \to \{0, 1\}$, where 0 represents the safe technology and 1 the risky one, and (3) $\iota(\psi)$ is the investment level, $\iota : \{\psi \in \Psi : \chi(\psi) = \eta\} \to \mathbb{R}$.

[13] This assumption is not crucial for generating countercyclical variance in productivity. The quantitative results on countercyclicality does not alter if $\gamma = 1$ as in standard business cycles models. The only reason of introducing this parameter is to make the simulated model moments comparable to data moments. The average annual growth rate of per capital output, assuming balanced growth path, is $\gamma = 1.016$, which is not removed from the data moments, especially those of investment dynamics.

A Potential Entrant's Problem. A potential entrant draws initial capital holding k_0 from a invariant Pareto distribution $G^0(k_0)$ with parameter ξ. The value of staying outside the market is

$$V_0^0(k_0, A) = \theta(A) k_0.$$

To start up a business, one must pay a setup cost c_e from initial capital, and thereafter acts as an incumbent with state (Z_{-1}, k, δ, A) being $\psi_0 = (0, (k_0 - c_e)/(1-\delta), \delta, A)$, where, without loss of generality, $\delta = \delta_0 := \mathbb{E}(\delta)$. Hence, the payoff of opening a firm will be:

$$V_0^1(k_0, A) = V^1(0, (k_0 - c_e)/(1-\delta), \delta, A).$$

A new firm will be born if

$$V_0^1(k_0, A) > V_0^0(k_0, A).$$

Solution to this problem is a binomial entry choice $\varepsilon : \Psi_0 \subset \Psi \to \{0, 1\}$, where Ψ_0 contains all possible ψ_0, and $\varepsilon(\psi_0) = 1$ if an entrant enters and 0 otherwise.

4.3 Aggregate Dynamics

Given the solutions to the individual problems described before, $\{\chi(\cdot), \tau(\cdot), \iota(\cdot); \varepsilon(\cdot)\}$, it is straightforward to write down the transition dynamics for the distribution over $\psi = (Z_{-1}, k, \delta, A)$.

For an arbitrary $\psi \in \Psi$, it is either $\psi \in \Psi_0$ or ψ can only be the state of an incumbent. I denote $\phi(\psi)$ as the measure or density of point $\psi = (Z_{-1}, k, \delta, A)$ at Stage 1 of a typical period with aggregate state A, before entry and exit takes place. If $\chi(\psi) = 1$, then a firm with this state exits for sure, and no other transition can happen. If $\chi(\psi) = \eta$, then with probability η this firm exogenously exits, and with a complementary probability, it stays. Conditional on staying, if the firm chooses the safe technology, $\tau(\psi) = 0$, then with probability $\pi(\delta')$ its individual state becomes $(A, (k + \iota(\psi))/\gamma, \delta')$. On the other hand, if the firm chooses the risky technology, $\tau(\psi) = 1$, then with probability $p(A)\pi(\delta')$ its individual state becomes $(\bar{z}, (k + \iota(\psi))/\gamma, \delta')$, and with probability $(1 - p(A))\pi(\delta')$ it becomes $(0, (k + \iota(\psi))/\gamma, \delta')$. Now turn to the new borns. Denote $g^0(\psi_0)$ the entrant's measure or density at point ψ_0 determined by $G^0(\cdot)$. A new born with ψ_0 enters if $\varepsilon(\psi_0) = 1$. After entering, this firm acts exactly the same as a surviving incumbent with $\psi = \psi_0$. To summarize, starting from $\phi(\psi)$, a fraction $\chi(\psi)\phi(\psi)$ exits, $(1 - \chi(\psi))(1 - \tau(\psi))\pi(\delta')\phi(\psi)$ goes to individual state $(A, (k + \iota(\psi))/\gamma, \delta')$, $(1 - \chi(\psi))\tau(\psi)p(A)\pi(\delta')\phi(\psi)$ goes to $(\bar{z}, (k + \iota(\psi))/\gamma, \delta')$, and the rest to $(0, (k + \iota(\psi))/\gamma, \delta')$; starting from $g^0(\psi_0)$, $(1 - \tau(\psi))\pi(\delta')g^0(\psi_0)$ goes to $(A, (k + \iota(\psi_0))/\gamma, \delta')$, $\tau(\psi)p(A)\pi(\delta')g^0(\psi_0)$ goes to $(\bar{z}, (k + \iota(\psi_0))/\gamma, \delta')$, and the rest to $(0, (k + \iota(\psi_0))/\gamma, \delta')$. Finally, the aggregate states becomes A' with probability $\Pr(A'|A)$, $A' \in \mathbb{A}$. Formally, suppose the aggregate state at Stage 1 of a period is $A' = A_j$, and that of last period is $A = A_i$, meaning the realized productivity Z is one of $\{A_i, \bar{z}, 0\}$. Every state not on the realization path has zero measure, or

$$\phi'(A, k', \delta', A') = 0 \text{ if } A \neq A_i \text{ or } A' \neq A_j,$$

where primed variables are ones realized at the same period as A'. The rest of the states can then be divided into three groups by realization of Z, all of which come from both incumbents and new borns. For $Z = A_i$,

$$\phi'(A_i, k', \delta', A_j) = \pi(\delta') \begin{bmatrix} \int (1 - \chi(\psi))(1 - \tau(\psi)) \mathbf{1}_{\{\psi : \gamma k' = (1-\delta)k + \iota(\psi)\}} \phi(\psi) \, d\psi \\ + \int \varepsilon(\psi_0)(1 - \tau(\psi_0)) \mathbf{1}_{\{\psi_0 : \gamma k' = (1-\delta_0)k_0 + \iota(\psi_0)\}} g^0(\psi_0) \, d\psi_0 \end{bmatrix},$$

where variables with no prime are ones observed one period back, with $\psi = (Z_{-1}, k, \delta, A_i)$ and $\psi_0 = (0, (k_0 - c_e) / (1 - \delta_0), \delta_0, A_i)$. For $Z = \bar{z}$ or 0,

$$\phi'(\{\bar{z}, 0\}, k', \delta', A_j) = \pi(\delta') \begin{bmatrix} \int (1 - \chi(\psi)) \tau(\psi) \mathbf{1}_{\{\psi : \gamma k' = (1-\delta)k + \iota(\psi)\}} \phi(\psi) \, d\psi \\ + \int \varepsilon(\psi_0) \tau(\psi_0) \mathbf{1}_{\{\psi_0 : \gamma k' = (1-\delta_0)k_0 + \iota(\psi_0)\}} g^0(\psi_0) \, d\psi_0 \end{bmatrix}.$$

By independence, a fraction $p(A_i)$ has $Z = \bar{z}$, and the rest gets $Z = 0$, that is,

$$\begin{aligned} \phi'(\bar{z}, k', \delta', A_j) &= p(A_i) \phi'(\{\bar{z}, 0\}, k', \delta', A_j), \\ \phi'(0, k', \delta', A_j) &= (1 - p(A_i)) \phi'(\{\bar{z}, 0\}, k', \delta', A_j). \end{aligned}$$

The difficulty in obtaining a closed form transition function is due to the aggregate fluctuation in A, therefore I turn to numerical solutions. In the following subsection, I first pick the parameter values that generate reasonable moments when the model is simulated at a stationarity. The stationarity here means the following. The aggregate state sequence, $\{A_t\}$, is set to be constant at its mean, but the firms still expect the future states to be changing according to a transition probability of A, π_{ij}. At the same time, the time-moving average inflow and outflow of the pool of incumbents roughly equal. Then, leaving all parameters unchanged, I simulate a long sequence of $\{A_t\}$ that actually evolves following π_{ij}, and throw away the first burnt-in periods. It is now that I can look at the generated correlations and other business cycles features of the model.

4.4 Quantitative Results

Table 3. Parameter Values and Rationale

Parameters	Explanation
Aggregate Fluctuation	
$\bar{z} = 1$	Normalization
A	Tauchen (1986). $N_A = 5, \mathbb{E}(A) = 0.5, \rho_A = 0.9, \sigma_\varepsilon = 0.1$
Production	
$\alpha = 0.5$	Capital share
$\beta = 0.9$	Discount factor
δ	Random depreciation rate, with $\mathbb{E}(\delta) = 0.1$.
$\theta = 0.6$	Resale price, temporarily assumed to be constant.
$\eta = 0.05$	Exogenous exiting probability
$c_f = 0.2$	Fixed cost
$c_k = 0.05$	Capital adjustment cost as fraction of revenue
$\gamma = 1.016$	Growth rate on the balanced growth path
Entrants	
$c_e = 0.01$	Entry cost
$\xi = 0.3$	Pareto distribution parameter for G^0

For the purpose of an illustration, I set the parameters as are described in Table 3. In order to set up a baseline for the model, I shut down the fluctuation and simulate a long history of the model under the selected parameter values. Specifically, the realization of A is controlled to be its mean level through the whole history, while all firms actually expect aggregate fluctuation in A_t with transition probabilities π_{ij}. To make sure the scale of the economy is not exploding or shrinking, I let the number of new born firms and that of exiting firms be balanced such that the time-moving average of entry and exit rates are roughly the same. The simulated sequence is then truncated to remove the burnt-in stage. The generated moments and data counterparts are listed in Table 4. The moments on investment dynamics are from Cooper and Haltiwanger (2006), average entry and exit rates are taken from Table 2.

Table 4. Moments from Model

Moments Generated from Model		Data Moments
Investment		
Mean of investment rate	0.166	0.122
Fraction of inaction	0.090	0.081
Fraction with positive investment	0.816	0.815
Fraction with positive investment burst	0.106	0.18
Fraction with negative investment burst	0.088	0.018
Entry and Exit		
Mean entry rate	0.086	9.36
Mean exit rate	0.086	9.28
Productivity		
Mean of productivity, A	0.5	–
Productivity dispersion, Std. Dev. of Z	0.17	–
Firm Size Reshuffle		
Fraction of large-to-small	tbd	tbd
Fraction of small-to-large	tbd	tbd

The main goal of this numerical exercise is to generate countercyclical firm-level productivity dispersion as a result of firm's risk taking behavior, without introducing any time varying volatility in the driving force, A_t. I add the aggregate fluctuation by simulating a sequence of realizations of productivity level A, and let the model evolves accordingly without changing other parameter values. The fluctuation in productivity A follows the Markov process specified in the Table 3, and not surprisingly, it is positively correlated with the total output with correlation coefficient 0.78 (p-value = 1.5e-41). Therefore, the cross-sectionally averaged productivity can serve as a valid cyclical indicator. The measures for productivity dispersion are chosen to be (1) standard deviation of cross-sectional distribution of realized Z, productivity, (2) fraction of firms that prefer risky technology, and (3) the 95% to 5% interpercentile range of realized Z, which is the value of Z at 95% percentile minus the value of Z at 5% percentile.

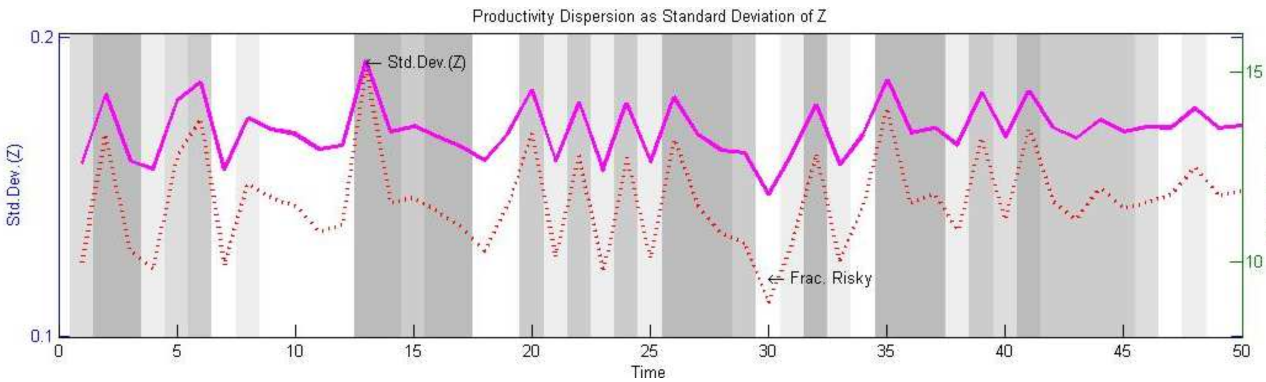

Figure 6: Simulated sequences of (1) cross sectional productivity dispersion measured as standard deviation in realized productivity Z (solid line, left axis), and (2) fraction of firms that choose risky technology (dotted line, right axis, in %). The grey bars indicate the economic condition as value of A. In particular, darker bars represent lower values of A.

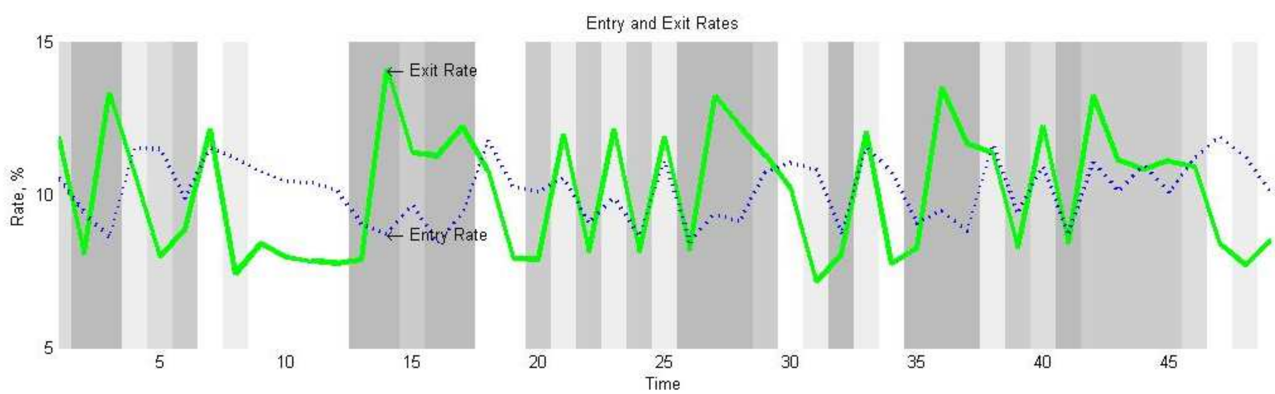

Figure 7: Simulated sequences of entry and exit rates. The solid line represents the exit rates, and the dashed line records entry rates. Grey bars indicate the value of A as in previous figure.

Table 5. Generated Cyclicality

Cyclicality: Correlations (p-value) with Cyclical Indicators			
		Cyclical Indicators	
Variables of Interests		Avg. Productivity, A	Total Output, O
Productivity Dispersion	$std.dev.(Z)$	-0.385 (1.6e-8)	-0.374 (4.7e-8)
Frac. of Risky Firms	Λ	-0.389 (1.2e-8)	-0.372 (5.3e-8)
Interpercentile Range 95%-5%	IPR_5^{95}	-0.273 (8.6e-5)	-0.26 (1.7e-4)
Entry Rate	r^{EN}	0.495 (8.1e-14)	-0.096 (0.174)
Exit Rate	r^{EX}	-0.386 (1.5e-8)	-0.562 (0)

The correlation coefficients of productivity dispersion is significantly negative, and the absolute values are in line with the data counterparts. Moreover, the cyclicality of productivity dispersion measured is in comparable scale to that of the fraction of firms that choose risky technology, and the movements are in very similar patterns as can be read off from Figure (6). This implies that it is the change in fraction of risk taking firms that drives the cyclicality of productivity dispersion. In bad times, more firms are willing to take the risk and randomize their future values. Consequently, the resulting dispersion measured as standard deviation of cross-sectional productivity distribution is larger, so is the interpercentile range.[14] The assumed binomial outcome of a risky technology has the potential to impact the behavior of the dispersion, however, such impact is of a much smaller scale and does not alter the main pattern.

5 Discussions and Extensions

The Case with Heterogeneity. I have assumed a common risk-free productivity level for all firms in each period. In order for this paper to be comparable to models of heterogeneous firms with idiosyncratic productivity change, one option is to simulate it many times using different

[14] Due to the model assumption, cross-sectional IPR in productivity can only be either $\bar{z}$, $\bar{z} - A_t$, or A_t, and does not have very interesting dynamics, although it is still countercyclical. This can be overcome by allowing a richer set of productivity lotteries and keeping the expected productivity to be A. For example, in addition to $(p(A), \bar{z})$, firms can also choose any $(p, \bar{z}_A)$ pair with binary outcomes such that $p\bar{z}_A = A$. Intuitively, the IPR measure in this case will again be negatively correlated to A_t because smaller firms have incentive to take even more risk in bad times than in the original case. Therefore, the range of realized productivities is wider, and potentially the IPR is larger and has more possible values.

but correlated processes of A, and then combined the results. This practice will not result in an essentially different relationship between productivity dispersion and cyclical indicators.

The Case with Collateralized Borrowing Constraints. Firms are not allowed to raise fund externally in this model. However, the qualitative result will be the same if firms can borrow and are subject to collateralized borrowing constraints against capital stocks. It is still the small firms that will be constraint. Consequently, small firms choose the risky technology, and they need to bear both the riskiness in the random productivity realizations and the default risk. The cyclical pattern therefore remains. Moreover, tighter borrowing constraint in bad times acts as an amplification device in the resulting productivity dispersion. In fact, the tightness of the constraint moves in the same direction as productivity dispersion, with or without aggregate state controlled.

The Case in General Equilibrium. To mimic the case of general equilibrium, especially the change in prices, I set the capital resale price θ as an increasing function of A. When the economy is in better condition, a firm can sell its capital stock at a higher price. This modification can be supported by empirical finding by Balasubramania and Sivadasan (2009), who find positive correlation between capital resalability and mean of productivity distribution, and negative correlation between capital resalability and productivity dispersion across industries at four digit SIC level. Such modification impacts the model economy by directly affecting the entry and exit decisions through the option values of not entering or exiting, which in turn may change the risk taking behavior of the marginal firms. Intuitively, whether partial equilibrium results can survive, especially the cyclicality, depends on two forces in opposite directions of this modification. The first force in favor of the cyclicality result is due to the local non-concavity in firm's value function. Higher θ at higher A results in higher curvature in the value function and a smaller region for risk taking. Lower θ has the opposite effect. The other force is twofold. For incumbents, θ moving in the same direction as A increases the option value of exiting in good times and decreases it otherwise, therefore it promotes risk taking in good times and depresses it in bad times. For entrants, it also leads to a higher option value of not entering the market and reduces the incentive of opening new firms. The simulation suggests that the first force dominates the other one, and the countercyclicality actually becomes more pronounced in this case.

Table 6. Generated Cyclicality when $\theta = \theta(A)$

Cyclicality with Fluctuations in Capital Resale Price			
		Cyclical Indicators: $Corr(A, O) = 0.71$	
Variables of Interests		Avg. Productivity, A	Total Output, O
Productivity Dispersion	$std.dev.(Z)$	-0.441	-0.601
Frac. of Risky Firms	Λ	-0.441	-0.598
Entry Rate	r^{EN}	0.708	0.038
Exit Rate	r^{EX}	-0.359	-0.576

6 Conclusion

Empirical works have shown that dispersion in firms' profitability measured as the second moment of firm level TFP evolves countercyclically over time. I explore a mechanism in which the time-varying and countercyclical second moment is a natural result of the standard first moment change. I incorporate firms' technology adoption decision into firm dynamics model with business cycle features to explain these empirical findings both qualitatively and quantitatively. The option of endogenous exiting and credit constraint jointly play an important role in motivating firms' risk taking behavior. The model predicts that relatively small sized firms are more likely to take risk, and that the dispersion measured as the variance/standard deviation of firm-level profitability is larger in recessions, which are consistent to the data.

References

[1] Arellano, C., Y. Bai, and P. Kehoe (2009): "Financial Markets and Fluctuations in Uncertainty," *Working Paper*, Federal Reserve Bank of Minneapolis.

[2] Bachmann, R., and C. Bayer (2011): "Uncertain Business Cycles - Really?" *NBER Working Paper* No. 16862.

[3] Bachmann, R., and S. Elstner and E. Sims (2011): "Uncertainty and Economic Activity: Evidence from Business Survey Data," *Working Paper*, University of Michigan.

[4] Bachmann, R., and G. Moscarini (2011): "Business Cycles and Endogenous Uncertainty," *Working Paper*, University of Michigan.

[5] Balasubramanian, N. and J. Sivadasan (2009): "Capital Resalability, Productivity Dispersion, and Market Structure," *The Review of Economics and Statistics*, August 2009, 91(3): 547–557.

[6] Bloom, N. (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77(3), 623-685.

[7] Bloom, N., M. Floetotto, and N. Jaimovich (2010): "Really Uncertain Business Cycles," *Working Paper*, Stanford University.

[8] Chugh, S. (2010): "Firm Risk and Leverage-Based Business Cycles," *Working Paper*, Univeristy of Maryland.

[9] Clementi, G.L., and D. Palazzo (2010): "Entry, Exit, Firm Dynamics, and Aggregate Fluctuations," *Working Paper*, New York University.

[10] Cooper, R., and J. Haltiwanger (2006): "On the Nature of Capital Adjustment Costs," *Review of Economic Studies*, 73(3), 611-633.

[11] Dhawan, R. (2001): "Firm size and productivity differential: theory and evidence from a panel of US firms," *Journal of Economic Behavior & Organization*, Vol. 44, Issue 3, 269-293.

[12] Dunne, T., M. Roberts, and L. Samuelson (1988): "Patterns of Firm Entry and Exit in U.S. Manufacturing Industries," *The RAND Journal of Economics*, Vol. 19, No. 4 (Winter, 1988), 495-515.

[13] Eisfeldt, A.L., and A.A. Rampini (2006): "Capital Reallocation and Riquidity," *Journal of Monetary Economics*, 53 (2006), 369-399.

[14] Gertler, M., and S. Gilchrist (1994): "Monetary Policy, Business Cycles, and the Behavior of Small Manufacturing Firms," *The Quarterly Journal of Economics*, MIT Press, vol. 109(2), pages 309-40.

[15] Hamilton, B.H. (2000): "Does entrepreneurship pay? An empirical analysis of the returns to self-employment." *Journal of Political Econnomy*, 108(3), 604–631.

[16] Higson, C., S. Holly and P. Kattuman (2002): "The Cross-sectional Dynamics of the US Business Cycle: 1950–1999," *Journal of Economic Dynamics & Control*, 26 (2002), 1539-1555.

[17] Hopenhayn, H. (1992): "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 60(5), 1127-1150.

[18] Kehrig, M. (2011): "The Cyclicality of Productivity Dispersion," *Working Paper*, Northwestern University.

[19] Moskiwitz, T.J., and A. Vissing-Jorgensen (2002): "The returns to entrepreneurial investment: a private equity premium puzzle?" *American Economic Review* 92(4), 745–778.

[20] Ouyang, M. (2009): "The scarring effect of recessions," *Journal of Monetary Economics*, Vol. 56, Issue 2, March 2009, 184-199.

[21] Quadrini, V. (2009): "Entrepreneurship in macroeconomics," *Annals of Finance*, Volume 5, Numbers 3-4, 295-311.

[22] Vereshchagina, G., and H. Hopenhayn (2009): "Risk Taking by Entrepreneurs," *American Economic Review*, 99(5), 1808-1830.