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Bassetti, Thomas and Favaro, Donata

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Thomas Bassetti[§] Donata Favaro*

Abstract

This paper proposes an endogenous growth model in which gender inequality in employment has an important role in explaining different development dimensions such as socio-political participation, educational attainments, and working hours, in developed countries. Our model's predictions are in line with some stylized facts observed across European countries: more equal societies have higher socio-political participation, devote less time to work, and present higher educational attainments and rates of economic growth than less equal ones. Our model suggests that promoting female employment must be accompanied by pro-family policies in order to sustain economic growth and improve quality of life.

Keywords: Gender inequality, growth, time allocation, socio-political participation.
JEL classification: O1, O4, J16, J22.

[§] Department of Economics M. Fanno, University of Padova, Via del Santo 33, 35123 Padova, Italy. Affiliations: CICSE (Centro Interuniversitario Crescita e Sviluppo Economico) http://cicse.ec.unipi.it/.

* Department of Economics M. Fanno, University of Padova, Via del Santo 33, 35123 Padova, Italy. Affiliations: Child (Centre for Household, Income, Labour and Demographic Economic) http://www.child-centre.it/. Corresponding author: donata.favaro@unipd.it.

1. Introduction

In many advanced countries, female labor market participation is far lower than male. In the EU-27, for example, the activity rate is 77.9% for men and 64.7% for women (cf. Eurostat, Statistics in focus, 8/2011). Though northern European countries have reached female activity rates that are close to the male rates, many EU-15 countries—including the Netherlands, Germany, the United Kingdom, and most southern European countries—show activity gaps that are nearly or more than 10%. Although the employment gap in many countries has narrowed in the last 20 years, much of the female labor market participation is related to part-time jobs, especially in countries with the highest activity rates, i.e., Denmark, Norway, and Sweden (Jaumotte, 2003).

In general, the relationship among gender inequality, per-capita income levels, and/or growth rates has been investigated in the part of the literature that highlighted that the relationship is two-way but also extremely complex. The widening gender gap affects a country's development; on the other hand, development determines the extent of women's participation in a country's socio-economic life. These relationships are particularly significant when countries with widely different development levels are compared.

In this paper, we propose a theoretical framework that highlights the relationship between gender inequality in labor market participation and some development indicators in a country such as the average level of education, the average time devoted to work, and—last but not least—the growth rate of per-capita income. Our model is meant to refer to advanced societies where basic needs are met and time can be devoted to education and other non-working activities such as social participation. Our aim is to propose a model that can capture the relationships emerging in advanced countries where fertility is low compared to developing countries (United Nations data). The Total Fertility Rate (TFR) is never lower than four in central African countries (2008 data) with peaks of five to six children per woman, whereas in OECD countries the rate ranges between 1.2—in eastern European countries—and 2. Furthermore, in advanced countries, the relationship between per-capita GDP and TFR is positive, and the countries with the highest number of children also have the highest level of per-capita income (d'Addio and d'Ercole, 2005). With reference to advanced countries, this runs counter to what is usually maintained in the literature exploring the relationship between fertility and income level in samples of countries that include advanced and developing countries. In this case, the relationship between per-capita GDP and TFR is negative.

We shall focus on European Union countries owing to their peculiarities. To begin with, EU countries are very different from each other regarding female labor market participation, average levels of education, and time devoted to work. More importantly, however, the countries with the highest female employment rates also show the highest population growth rates and fertility rates (TFRs). In Europe, there is a virtuous growth model where higher female labor market participation does not necessarily imply lower fertility and higher fertility can imply higher average levels of education (Del Boca, 2002). Such evidence introduces a new element compared to what is predicted in theoretical models of the relationship between gender inequalities, level of per-capita income, and growth. Considering various kinds of gender differences (differences in employment rates, levels of education, etc.), these models can predict different levels of development or different growth rates between countries with higher or lower gender gaps and fertility rates. For countries with less significant gender differences, many theoretical contributions predict higher growth rates as a consequence of reductions in the fertility rates. Considering what has been outlined above, this does not seem to have been the case in Europe, at least in the last few years: the relationship between gender inequality and fertility does not appear to be negative in European countries. We therefore formulate a model that attempts to propose a different channel through which the extent of the inequality between men and women might affect macroeconomic dimensions and growth.

Our reference framework is Lucas's growth model (1988), which we develop further by hypothesizing that an economy consists of two opposing groups—men and women. As shown in the most recent literature, opposition can originate from different cultural, sociological, and economic factors—which in turn cause gender differences in preferences, kinds of behavior, and attitudes. The recent economic literature applying the experimental approach shows the existence of robust gender differences in risk preferences, social preferences, and competitive preferences (see Croson and Gneezy, 2009, for a recent survey of that literature) that may have economic consequences (Akerlof and Kranton, 2000). Sociological literature provides further evidence of differences between men and women and the impact on society. Masculinity and femininity can identify different social roles. Referring to Hofstede (2001), masculinity is seen to be the trait emphasizing ambition, acquisition of wealth, and differentiated gender roles; femininity is seen to be the trait stressing in general more fluid roles, caring and nurturing behaviors. In a categorization of countries, Hofstede (2001, p. 297) distinguishes between 'feminine' and 'masculine' countries. The former are societies in which social gender roles are clearly distinct: men are supposed to be assertive, and focused on material success; women are supposed to be more modest and concerned with the quality of life. The latter are countries in which social gender roles overlap and men have values similar to those of women. Hofstede reveals that women in feminine countries have the same modest, caring values as the men; in masculine countries, women are somewhat assertive and competitive but not as much as men, so that these countries show a gap between men's values and women's values.

The two groups making up our economy—men and women—interact in society trying to secure for themselves the benefit deriving from control of society itself. The share of the benefit a group appropriates becomes part of the utility function of the representative agent of the group. The benefit is shared based on the relative relevance of the groups as determined by the number of economically active members of each group and the extent of their socio-political participation. The social and economic relevance of each group is the result of two factors: the number of group members with a productive role in the economy—that is, the number of employed members[1]—and the extent to which the members of each group are engaged in socio-political activities with a view to strengthening and asserting their group's values—i.e., the time they devote to such activities. The interplay of the two factors explains why group members need not only to participate productively in the economy by working but also to succeed in asserting their group's values in order to ensure the group's social success. The time devoted to such activities captures the time frame needed to render the group's ideas influential and can be seen as the time devoted not only to socio-political participation but also to securing roles with greater impact on the economic decisions of a society. For simplicity, we shall call this factor time for socio-political participation.

Our model assumes that men and women's employment level is exogenous; so even gender inequality in employment is exogenous. By contrast, the time devoted to socio-political activities is endogenous as is time devoted to work and education. Another parameter in our model is the population growth rate. As has been argued above, our objective is to show how gender inequalities in socio-economic roles can affect the macroeconomic equilibrium in its growth and development components—education and time devoted to work—without influencing the fertility rate.

Our model has been simulated with a sample of European countries that differ from each other first in men and women's employment levels and then in population growth rates. The model captures well the stylized facts observed and the relationships between population growth rates and average time devoted to work, education, and socio-political participation. Countries with higher gender employment ratios have lower growth rates and lower education levels, and devote less effort to socio-political participation and more time to work than countries where the distribution of labor between men and women is more equal. Since in the latter countries population growth rates are the highest in Europe, our numerical simulation has significant policy implications. Equal opportunity policies can stimulate long-term growth through higher rates of human capital accumulation if these policies are accompanied by pro-family policies and more generally by incentives for fertility and population growth. A more equal distribution of labor between men and women—which is what equal opportunity policies strive for—reduces the average time devoted to work and frees up resources—in terms of time—to accumulate more human capital for the younger generations. If the population growth rate increases, a higher growth level can be achieved through pro-family policies.

This paper is organized as follows. In the following section, we briefly discuss the main empirical and theoretical contributions to the analysis of the macroeconomic impact of gender inequalities. The third section is devoted to a discussion of the model and the fourth section to illustrating the simulation results; before that we briefly illustrate European stylized facts that support the model.

2. Gender inequalities and growth: a brief survey of the literature

Several studies have analyzed the relationship between gender inequality in employment and development/growth rates of a country, in particular with a view to showing the importance of greater female participation in paid employment and in raising the education level of women in less developed countries (LDC). This relationship is two-way and is often difficult to identify.

As far as we know, the only study that attempts to analyze the direct impact of gender inequality in the labor force on growth is Klasen and Lamanna's (2009), which analyzes several measures of the gender gap in labor force participation as determinants of growth. The results—based on a sample of countries that does not include members of the European Union—show that regardless of the variable under consideration whether it be the female share of the labor force or the ratio of the female-male labor force participation rates, countries where the female share increased from decade to decade were able to achieve higher rates of subsequent economic growth.

Most empirical research has focused on ascertaining the impact of gender differences in production features—especially the level of education—on growth. The level of education affects a country's macroeconomic performance not only directly but also indirectly through labor force participation and fertility choices. The direct effect of the education gender gap is evidenced by differences in productivity. In the human capital model, a lower level of education for women means lower productivity by part of the people in employment and hence lower income and growth levels compared to what could be achieved if there were no education inequality. In the hypothesis of decreasing returns to scale, a higher education level for women should determine a higher increase in income than a higher education level for men. In addition, the gaps affect per-capita income levels and growth indirectly, too, through participation in the labor force, fertility, and education of future generations. In general, female participation correlates positively with education levels and negatively with fertility rates though these ratios—as we shall see below—depend on the number and types of countries under consideration. A low level of education means low female participation in the labor force, high fertility rates, and—indirectly—lower levels of per-capita income.

The first empirical studies that attempted to evaluate the relationship between women's education and growth in a cross-country perspective produced debatable results. Results are heavily dependent on the method used and the structure of the model. Hill and King (1995) show that the level of women's education has a very positive effect on GNP and that more marked gender inequalities in education reduce the GNP. Barro and Lee (1994)[2] found that the ratio between the two is negative while Stokey (1994) showed that this relationship can be accounted for by including Hong Kong and Korea among the countries that are observed. Excluding Hong Kong and Korea from the dataset would make the relationship irrelevant. Moreover, excluding all four Asian Tigers from the dataset would further reduce the asymptotic significance of women's education. Lorgelly and Owen (1999) point out that Barro and Lee's result (1994) is due to the high multiple collinearity between men and women's levels of education and that the negative relationship between growth and women's education and the positive one between growth and men's education is particularly weak and dependent on the number and type of countries taken into consideration. Klasen (2002) partly corrects the multiple collinearity; instead of including variables for male and female educational achievement (which are highly correlated), Klasen includes a variable that measures overall human education as well as one that measures the female-male ratio of education attainment. Results confirm that there are direct and indirect relationships between inequality in education and economic growth; gender inequality in education undermines economic growth.

The results of empirical analyses in the 1990s are not as robust as the sample of countries analyzed is heterogeneous in terms of levels of development and growth rates. An attempt to distinguish between countries at different stages of development can be found in Dollar and Gatti's (1999) study, which evaluates how gender differences in secondary school achievement affect growth. The authors distinguish between countries with very low levels of female education—secondary female attainment covering less than 10.35% of the population—and countries with higher levels of female education. Dollar and Gatti found that while in less educated/developed countries female and male secondary attainment do not affect growth significantly, for the most developed economies in the sample, male education modestly affects growth, in negative terms, and female secondary attainment has a significant, positive impact on growth. In an empirical study estimating the per-capita income level and growth rate, Knowles *et al.* (2002) confirmed results with reference to more developed countries. However, the authors always found significant, positive relationships between the female level of education and per-capita income rates, even as regards the overall sample—consisting of high-income countries and LDC. In this case, though, the incidence on the per-capita income level is lower.

The economic variables and gender differences in socio-economic life can have two-way relationships, and the degree of development of a country can be not only the result but also the cause of a higher or lower degree of women's empowerment or participation in economic life. In the first part of the study, Dollar and Gatti (1999) assess the effect of the degree of development—per-capita GNP level—on female education achievement and on some measures of gender inequality, such as the degree of women's empowerment and economic equality. As to the effect of the stage of development on female education achievement, empirical analysis shows that up to a threshold level of $2000 per capita (in PPP), there is no tendency for female education achievement to catch up with the superior male achievement. After that level of income, on the other hand, there is a strong tendency for female education achievement to catch up.[3] The relationship between women's income level and education achievement is convex for the different specifications chosen. The convex relationship with income is also confirmed when gender inequality is measured by the degree of women's empowerment, proxied by the percentage of seats occupied by women in the lower and upper chambers, or by economic inequality under the law, which assesses whether men and women are entitled to equal pay for equal work.

From a theoretical point of view, part of the recent literature models some aspects of the relationship between gender inequalities and growth (Galor and Weil, 1996; Lagerlöf, 2003; Cavalcanti and Tavares, 2007). In most of these papers, the impact of the differences between men and women on growth depends on fertility choices. Fertility is endogenous, and children have time and goods costs. These models can explain differences in fertility between poor and rich countries and—through different fertility rates—distinctive growth rates (Becker 1960). To account for the fact that countries grow more as they become richer, these models assume that the fertility rate decreases as income increases. This happens because increases in income have a different impact on men and women. Wage increases have an income effect on men's choices to have children. By contrast, wage increases for women have an income and a substitution effect, and the substitution effect prevails over the income one. Wage increases for women increase fertility as an income effect but have a negative effect on fertility as childbearing costs increase if it is assumed that the burden of childbearing falls entirely on women. If countries become richer and men and women's incomes increase, fertility decreases.

A development of this theoretical framework can be found in Galor and Weil (1996) where households' fertility decisions are taken to be a function of men and women's relative wages. Therefore, variations in fertility occur only if there are changes in women's relative wage. As capital intensity in an economy rises, an increase in women's relative wages reduces the fertility rate and hence increases the growth rate of per-capita income. Women's relative wages rise as the economy develops and women's rewards increase in areas where women have a comparative advantage. Within this framework, women have a greater comparative advantage than men in brain-intensive jobs.

A different approach can be found in Lagerlöf's (2003) study, where gender inequalities can arise because of gender roles and gender stereotypes, for example, differences in the value of men's and women's time in non-reproductive activities. As a production function with constant returns to capital is used, sustained growth in per-capita income is feasible. Lagerlöf uses an overlapping-generations framework where men and women, despite having identical abilities, may end up having different levels of human capital. This arises through a coordination process where parents care about the total income of their children's household (therefore, the income of the child and of his or her spouse) and families play a coordination game against one another. Therefore, gender discrimination arises at a Nash equilibrium, though both sexes are completely symmetric: a daughter may need less education than a son, simply because she is expected to marry a man, who may be better educated and earn a higher income. Assuming that there is an equilibrium within couples, Lagerlöf (2003) shows that, as spouses' human capital levels become more equal and women's time becomes more expensive, couples respond by substituting quantity for quality in children. Fertility thus falls, and human capital and per-capita income growth rates rise.

Gender inequality in employment can arise when gender differences in wages turn women away from the labor market. This can lead to a lower per-capita income for two reasons: lower female labor market participation and an increase in fertility. This is the model in Cavalcanti and Tavares's (2007) study, where a growth model is developed with gender wage differentials and endogenous determination of labor market participation and fertility. This model is similar to Galor and Weil's (1996).

Other papers support the idea that gender inequality in employment can adversely affect growth as a consequence of lower international competitiveness in countries where women provide the bulk of labor in the export sector and there are wage differences (Seguino, 2000), lower bargaining power within families that would translate into lower savings, as women and men differ in their savings behavior (Seguino and Floro, 2003), or lower investments in children's health and education (World Bank, 2001).

3. The model

3.1 The optimization problem

We consider a competitive, closed economy populated by identical and infinitely lived agents. Agents are perfectly rational, and the production sector is subject to constant returns to scale. Agents belong to either social group *f* (females) or *m* (males) and at any time *t* group *s=f, m* is made up of $N_s(t)$ individuals working in the only production sector of the economy. Each population $N_s(t)$ is determined exogenously as well as *n*, the growth rate of population, which is equal for the groups.

In each instant, individuals allocate their time among three different activities: education, work, and socio-political participation. The fraction of time allocated to each activity is expressed, respectively, as $e_s(t)$, $u_s(t)$ and $p_s(t)$. Similar to Lucas's (1988), our model does not include the choice of leisure time; that is exogenous and equal between the two groups. This may seem a strong assumption; conversely, data on time use show that the amount of broad leisure time, including caring activities, is generally very similar between men and women; gender differences arise when the distribution of time between caring activities and leisure in strict terms is taken into account.

The representative individual of group *s* has the following preferences over per-capita consumption, $c_s(t)$, and time spent in socio-political participation, $\mathrm{p_s(t)}$, which is the fraction of time the individual spends in non-working activities addressed to socially establish the values of the group he or she belongs to. The intertemporal utility function is as follows:

$$V_s(0) = \int_0^{\infty} e^{-(\rho-n)t}\left[\log c_s(t) + g_s(p_s(t))\right]\mathrm{d}t \qquad (1)$$

where $\rho > 0$ is a common discount rate, with $\rho > n$. The utility function of the representative agent of group *s* depends on the level of consumption and on function $g_s(p_s(t))$, which summarizes how socio-political participation affects the utility of consumption.[4] The objective function of each group's representative agent depends positively on the level of consumption and on the proportion of time allocated to assert the group's own values and principles. In fact, function $g_s(p_s)$ is defined as follows:

$$g_s(p_s) = \frac{N_s p_s}{N_s p_s + N_{-s} p_{-s}} Z \tag{2}$$

where Z represents some total benefit the two groups share. N_m and N_f represent, respectively, the number of men and women in the economy; since leisure time is assumed to be exogenous, population and employment—of whatever sex—overlap. For our purpose, we will refer to N_m and N_f.

Function *g*(.) summarizes the incidence, in terms of utility, of each group's relative socio-economic importance, which depends on the group's relative size in society—measured by the number of individuals belonging to that group relative to the other group's size—and on the time that each group's representative individual allocates to socio-political activities. Given an exogenous number of female and male populations in the economy, men and women can increase their social impact and take a higher proportion of Z for the benefit of the group by increasing the amount of time spent on socio-political activities. Notice that this assumption ultimately implies that the two groups have different sets of values; otherwise, each group would get the same utility even when the society is fully directed by the opposite group. In other words, our formulation is perfectly consistent with the idea that women and men do not share the same values. On the contrary, when a group does not have any control, only consumption enters the utility function.

For simplicity, we assume that both groups can obtain the same maximum level of $g_s(p_s)$ once they fully control the society.

In relative terms, we can define parameter β as describing the representativeness of men with respect to women: $\beta \equiv \frac{N_m}{N_f}$. This allows us to state sex-specific functions $g_f(p_f)$ and $g_m(p_m)$:

$$g_f(p_f) = \frac{p_f}{p_f + \beta p_m} Z \qquad \text{for women} \qquad (3)$$

$$g_m(p_m) = \frac{\beta p_m}{p_f + \beta p_m} Z \qquad \text{for men} \qquad (4)$$

β positively affects men's utility, whereas *it* has a negative effect on women's utility. Notice that when all individuals devote the same amount of time to socio-political participation, β becomes a measure of the different benefits men and women obtain from the social structure.

Parameter β represents the extent of gender inequality in the labor market. The parameter can assume values equal, bigger, or lower than zero. If women are the most represented group in the society, then $\beta < 1$; on the contrary, if men are the biggest group, $\beta > 1$. $\beta = 1$ means that there is numerical equality between men and women in the population and in employment. However, to simplify the exposition and discussion of the propositions' proofs, we will assume $\beta \geq 1$; in doing so, we will confine our analysis to an economy in which the male population can outnumber the female population, but the reverse cannot happen. We are allowed to do that because the situation in which the female population exceeds that of men ($\beta < 1$) exactly mirrors the one we are going to discuss; this can be easily seen if we define $\beta = \frac{N_f}{N_m}$ and assume $\beta \geq 1$. Therefore, the results we obtain can be extended to economies where the majority group is female. However, since in the real world men are the majority group in employment (even in developed economies), our assumption makes it possible to easily compare our model's theoretical predictions to the empirical facts and simulation results we are going to discuss in Section 4.

Without loss of generality, we normalize Z to one; moreover, for simplicity of notation, when possible and not confusing, we omit subscript indices $s = f, m$.

Individuals maximize the value function (1) with respect to $c(t)$, $p(t)$, and $u(t)$, subject to the following laws of motion for per-capita capital (Equation 5), and individual human capital (Equation 6):

$$\dot{k}(t) = f\big(k(t), u(t)h(t)\big) - (\delta + n)k(t) - c(t) \qquad (5)$$

$$\dot{h}(t) = B\big(1 - u(t) - p(t)\big)h(t) \qquad (6)$$

Initial levels of physical and human capital, $k(0)$ and $h(0)$, are given.

Following Lucas (1988), human capital accumulation depends on the level of human capital already attained by our representative agent $h(t)$, and time devoted to education $(1 - u(t) - p(t))$ multiplied by parameter B, measuring productivity of the education sector. The production function is assumed to be a Cobb-Douglas with constant returns to scale, $f\big(k(t), u(t)h(t)\big) = k(t)^{\alpha}\big(u(t)h(t)\big)^{1-\alpha}$, where $\alpha \in (0,1)$ is the capital share. Since $u(t)$ represents the fraction of time the representative agent devotes to current production, $u(t)h(t)$ represents the fraction of time devoted to work in terms of efficiency units. In contrast with Lucas (1988), we assume that there are no external effects in the aggregation of human capital.[5]

To solve the model, we maximize the current value Hamiltonian:

$$H = \log c(t) + g\big(p(t)\big) + \lambda_1(t)[f\big(k(t), u(t)h(t)\big) - (\delta + n)k(t) - c(t)] + \lambda_2(t)[B\big(1 - u(t) - p(t)\big)h(t)]$$

where $\lambda_1(t)$ and $\lambda_2(t)$ are the shadow prices of physical and human capital, respectively.[6] The FOCs associated with the current value Hamiltonian are as follows:

$$c(t)^{-1} = \lambda_1(t) \qquad (7)$$

$$g_{p(t)} = \lambda_2(t)Bh(t) \qquad (8)$$

$$\lambda_1(t)f_{L(t)}h(t) = \lambda_2(t)Bh(t) \qquad (9)$$

$$\frac{\dot{\lambda}_1(t)}{\lambda_1(t)} = \rho + \delta - f_{k(t)} \qquad (10)$$

$$\frac{\dot{\lambda}_2(t)}{\lambda_2(t)} = \rho - n - Bu(t) - B(1 - u(t) - p(t)) \qquad (11)$$

The two transversality conditions can be written as follows: $\lim_{t\to\infty} e^{-(\rho-n)t}\lambda_1(t)k(t) = 0$ and $\lim_{t\to\infty} e^{-(\rho-n)t}\lambda_2(t)h(t)$. Together with the transversality conditions, Equations (5)-(11) describe the dynamics of the economy.

Equation (8) represents the novelty with respect to Lucas's model and is crucial for future analysis. Deriving the values of $\lambda_1(t)$ and $\lambda_2(t)$ from (7) and (8), respectively, and putting them into (9), we obtain

$$g_{p(t)} = f_{L(t)} \frac{h(t)}{k(t)} \frac{k(t)}{c(t)} \tag{12}$$

Equation (12) describes the relationship among state and control variables at time t and implies that the marginal utility of an hour devoted to socio-political participation in terms of consumption, $c(t)g_{p(t)}$, must be equal to the hourly wage rate, $h(t)f_{L(t)}$.

To solve our dynamic system, we reduce the number of state variables by considering the following ratios: $x \equiv \frac{k}{h}$ and $q \equiv \frac{c}{k}$.[7] By differentiating x and q with respect to time and considering system (7)-(11), we obtain the following dynamic system:

$$\frac{\dot{x}}{x} = u^{1-\alpha}x^{\alpha-1} - q - (\delta + n) - B(1-u-p) \tag{13}$$

$$\frac{\dot{u}}{u} = \frac{1-\alpha}{\alpha}(n+\delta+B(1-p)) - q + Bu \tag{14}$$

$$\frac{\dot{q}}{q} = (\alpha-1)u^{1-\alpha}x^{\alpha-1} - \rho + q + n \tag{15}$$

Therefore, imposing $\frac{\dot{x}}{x} = \frac{\dot{u}}{u} = \frac{\dot{q}}{q} = 0$, the long-run equilibrium will be:

$$q^* = \frac{n + B(-1+p^*)(-1+\alpha) + \delta + \alpha(-2n-\delta+\rho)}{\alpha}, \quad x^* = \frac{\left(\frac{B+n-Bp^*+\delta}{\alpha}\right)^{\frac{1}{-1+\alpha}}(-n+\rho)}{B}$$

and $u^* = \frac{\rho - n}{B}$.[8]

As in the Lucas model without external effects, Equation (14) implies that the steady-state equilibrium level of time allocated to work depends positively on the rate of intertemporal preference, negatively on the rate of population growth, and inversely on productivity of the education sector.

From (7) and (10), we obtain that along the Balanced Growth Path (BGP) our economy will grow at the following rate:

$$\gamma^* = \alpha\left(x^*\right)^{\alpha-1}\left(u^*\right)^{1-\alpha} - \delta - \rho = B\left(1 - p^*\right) + n - \rho \tag{16}$$

As in Lucas (1988), the population growth rate and the productivity of the education sector enhance growth, whereas the depreciation rate has a negative impact on economic growth. With respect to Lucas, now, we have that time devoted to social participation reduces the effect of B on γ^*. This result is due to the fact that individuals of both groups work, in per-capita terms, the same number of hours. Therefore, as long as the population growth rate remains constant, the only way to increase time for social activities is through reducing the fraction of time devoted to education. At first sight, this may appear empirically unrealistic; however, this is because we do not model the choice between household work and market work. The model determines the allocation of time among education, work, and socio-political participation. In the following section, we discuss each group's socio-political commitment in equilibrium.

3.2 Socio-political participation

We know, from relation (2), that time spent by a group in socio-political activities affects the utility of the other group. Therefore, the optimal allocation of time among different activities ends up being the result of a strategic interaction between the two groups. We assume that each group takes as given the fraction of time devoted to socio-political participation by the other group; that is a typical competition game à la Cournot. By considering the Cobb-Douglas production technology, Equation (12) leads to the following (implicit) reaction functions:

For group *f*:

$$\frac{\beta p_m(t)}{\left[\beta p_m(t) + p_f(t)\right]^2} = (1-\alpha) x_f(t)^{\alpha-1} u_f(t)^{-\alpha} q_f(t)^{-1} \tag{17}$$

For group m: $$\frac{\beta p_f(t)}{\left[\beta p_m(t)+p_f(t)\right]^2}=(1-\alpha)x_m(t)^{\alpha-1}u_m(t)^{-\alpha}q_m(t)^{-1} \tag{18}$$

where x_s and q_s are the values of our reduced state variables, with s=f, m. By substituting the steady-state values of x, u, and q into system (17)-(18), we obtain a Nash-Cournot equilibrium in which the fraction of time allocated to socio-political participation by both groups depends on male relative to the female population (or, in alternative, male to female employment), that is, β, as well as on the other parameters.

Before discussing the main results of our model, we need to introduce the definition of *admissible region*. Given a certain value of u, the admissible region for group s =f, m corresponds to the set of p_s compatible with the time constraint: $\{p_s \geq 0 : 1-u-p_s \geq 0\}$. Directly from this definition and the equilibrium level of u, we know that $p_s \leq \frac{B+n-\rho}{B}$.

Proposition 1: In the steady-state equilibrium, both groups devote the same fraction of time to socio-political participation.

Proof: See Appendix B.

Proposition 1 shows that Nash-Cournot equilibriums are symmetric; that is, men and women allocate the same fraction of time to socio-political activities. This does not imply, however, that men and women obtain the same satisfaction level from participating as long as β is different from 1. In particular, men receive a fraction $\frac{\beta}{1+\beta}$ of the total benefit Z, whereas women get a fraction $\frac{1}{1+\beta}$ of Z.[9]

The fact that a Nash-Cournot equilibrium is symmetric does not necessarily imply that for a certain configuration of parameters the equilibrium is also unique. The next proposition shows that this equilibrium is always unique.

Proposition 2: In the steady-state equilibrium, the Nash-Cournot solution is unique.

Proof: See Appendix C.

Proposition 2 constitutes an important result because equilibrium multiplicity usually leads to selection problems.

The next proposition provides one of the main results of our model, showing how the representativeness of the two groups in the society, captured by β, influences the amount of time devoted by individuals to socio-political participation.[10]

Proposition 3: For any plausible configuration of parameters, as the two groups become equally represented in a society (that is, when parameter β approaches unity), time allocated to socio-political participation increases.

Proof: See Appendix D.

Corollary 1: If parameters n, B, and ρ remain constant, a positive relationship exists between β and time for education.

Proof: See Appendix E.

Proposition 3 and Corollary 1 state that, *ceteris paribus*, increasing equality between men and women in society, time allocated to socio-political participation increases, and time spent on education diminishes. We will be able to show, in Section 4, through numerical simulation, that when greater gender equality is accompanied by a higher rate of population growth, then time allocated to socio-participation and time spent on education increase, while time for work decreases.

The reaction functions described by (17) and (18) are represented in Figures 1 and 2, where parameters' values observed for Italy and discussed in the next section are used. Figures 1 and 2 show the results stated in Propositions 1, 2, and 3: first, equilibriums lie on the 45 degree line; that is, they are symmetric (Proposition 1). Second, there is only one admissible equilibrium for any configuration of parameters (Proposition 2); finally, in

Figure 2 as the proportion of men in the labor market, β, increases (in this case, we suppose that it goes from 1 to 2), the reaction functions flat toward the abscissas and time devoted to social activities by whatever groups decreases (Proposition 3).[11]

[Figure 1]

[Figure 2]

In the next section, we conduct comparative statics analyses to examine the role played by the main parameters of the model—capital share, population growth rate, and productivity of the education sector—in the allocation of time.

3.3 How the main parameters affect time allocation: comparative statistics

(a) *The effect of the capital share*

The effect of a change in the capital share is ambiguous. Indeed, a higher capital share should lead to a lower marginal productivity of labor that may cause an increase in time for socio-political activities; however, at the same time, a higher α implies a lower ratio h^*/c^* and consequently less time for socio-political participation. The net effect will depend on the magnitudes of these two components that are influenced by the levels of the other parameters. The next claim, however, shows that the relationship between the capital share and the equilibrium level of p is always positive in the admissible region.

Claim 1: In the admissible region, the greater the capital share, the greater the fraction of time allocated to socio-political activities.

Proof: See Appendix F.

By using the same set of parameters used in Figures 1 and 2, we propose a numerical example (Figure 3) that shows how α and p are positively related in the space (p^*, u^*). A lower capital share causes a reduction in time for socio-political activities in favor of time allocated to human capital accumulation. This result partially confirms what we observe in advanced economies (see the following sections), typically characterized by higher capital shares, higher stocks of human capital, and higher socio-political participation (see the comparison between Finland and Italy in the following sections).

[Figure 3]

(b) The role played by the population growth rate

According to our model, when n increases, time devoted to socio-political participation decreases. In fact, a higher population growth rate reduces the ratio c^*/k^*, and causes an increase in the RHS of Equation (12); in order to meet Equation (12), the marginal utility of socio-political participation increases through a reduction in time for socio-political activities. Moreover, given the higher population growth rate, time devoted to work decreases, and p^* further is reduced through this channel. This intuition is better formalized in the next claim.

Claim 2: For $\alpha < 1$, the higher the population growth rate, the lower the socio-political participation.

Proof: See Appendix G.

Figure 4 provides a numerical example for Claim 2. Economies with higher population growth rates should exhibit, *ceteris paribus*, lower consumption per unit of physical capital and socio-political activity compared to economies with lower population growth rates.

[Figure 4]

Notice that Claim 2, together with Proposition 3, implies the existence of a certain substitutability between gender equality, β, and the population growth rate, n, in terms of economic growth. Indeed, when β tends to 1, socio-political participation p increases, reducing the long-run growth rate, γ^* [see Equation (16)]; then, the rate of economic growth can be kept constant if n increases.[12]

(c) Education sector productivity

When the education sector becomes more productive, socio-political participation decreases. This happens because the higher the productivity of the education sector, the higher the opportunity cost of leaving education. Thus, individuals invest a larger fraction of their time in acquiring human capital. To prove this result is sufficient to observe that, according to the equilibrium equation for u^*, the higher B, the lower u^*; hence, we can conclude that, as long as there is a positive relationship between u^* and p^*, B and p^* will be negatively related. In Figure 5, we show the effect on p of an increase in B from 0.0769 to 0.1.

[Figure 5]

In this section, we proposed the first numerical assessment of the relationship between the main parameters of the model and allocation of time in the long-run equilibrium. The results show that the allocation of time between work and other activities is differently affected by changes in the capital share, the rate of population growth, and efficiency in the education sector; while time for socio-politics increases—and education decreases—as the capital share increases, an increase in the rate of population growth negatively affects socio-political participation and stimulates education. Moreover, education is enhanced with better efficiency in the education sector. It is also clear that there is some substitutability between gender equality and population growth in terms of economic growth. This result has important consequences in terms of gender policies: not to adversely influence economic growth, strategies to increase women's work must be managed in such a way that fertility is not negatively affected. Policies devoted to promoting equal opportunities in employment and increasing female employment must be accompanied by pro-family policies.

In the next section, we calibrate the model on different parameters' configurations, defined for selected European countries. This further simulation will allow the strength of the model in explaining the performance of European countries to be tested; in addition, we will show that countries where gender quotas have been accompanied by very significant support of motherhood—such as mandatory parental leave for all dads—

experience higher rates of economic growth, less working time, and higher education levels; this is the case for Finland.

Before presenting the numerical simulations, we will spend the first part of the section discussing some stylized facts of interest.

4. Gender inequalities in employment, education, socio-political participation, and growth: stylized facts and numerical simulations on European countries

4.1 Stylized facts

The model we discussed in the previous section explains the relationship between the relative presence of women in employment and some factors strictly related to countries' socio-economic performance. At the theoretical level, our framework predicts that a society characterized by an equal proportion of men and women in employment achieves a long-run equilibrium in which average time for work is lower than in countries where women are underrepresented in the labor market.[13] Essentially, as the economy becomes more gender balanced, it goes through a redistribution of hours of work from men to women; it follows that individuals reallocate their time, on average, in favor of other activities, such as education and socio-political participation, with positive effects on quality of life and development indicators. In a model of endogenous growth, which is ours, these effects have implications for growth; however, the outcome will depend not only on gender inequality in employment but also on other parameters, such as the population growth rate and the intensity of use of physical capital in production (the capital share), whose values will be crucial for the long-run equilibrium.

We already discussed, in the previous section, numerical examples that illustrate the effects of changing parameters, individually, on the variables determined in the model: in particular, allocation of time among work, socio-political activities, and education. This analysis, although useful for understanding the theoretical framework, does not verify the model's ability to represent the complexity of facts we observe across European countries. To meet this goal, we propose a simulation example, parameterized on representative countries, in the second part of this section. Before going through the simulation results, we will discuss some stylized facts on our variables of interest across Europe. We will show that clear cross-country relationships exist among gender employment ratios, average educational levels, and time for different activities; these results may suggest that female employment and its relative weight in the economy can influence the socio-economic model of reference, confirming our theoretical results.

First, we will propose a focus on the relationship observed across European countries between gender inequality ratios in employment and proxies of variables determined in the model: time allocated to work, socio-political participation, and education (here proxied by average years of education). Then, we will discuss the cross-country relationship between gender inequality in employment and the population growth rate, which is assumed exogenous in the model; this focus will draw attention to the counterintuitive correlation between the two variables and will motivate some considerations on the validity of the model, to be discussed in the second part of the section.

[Figure 6]

In Figure 6, we plot data on the countries' average years of school[14] versus male-to-female employment ratios.[15] The relationship between countries' educational levels and gender employment ratios appears to be clearly negative, as noticeably shown by the estimated regression line: educational levels increase as the male-to-female employment ratios decrease. Not surprisingly, the highest male-to-female ratio in employment, together with the lowest educational levels, is observed in the southern countries (Greece, Italy, and Spain); in contrast, the most equal distribution of work between the two sexes can be observed in the Scandinavian region, some eastern countries, and, to some extent, in France and the United Kingdom (UK). These countries rank among the best in educational levels. Surprisingly, the Netherlands does not rank among the first according to either gender equality (its male-to-female employment ratio is 1.23) or education.

One of the predictions of our theoretical model concerns the effect of gender inequality on average hours of work, predicting a decrease in working time with higher female employment. Stylized facts seem to validate these predictions; in fact, as shown in Figure 7,[16] countries with a more equal gender allocation of work show, in general, lower average weekly hours of work. Countries with the highest levels of working time (more than 40 hours per week) are the eastern and Baltic new entrants and Greece, the country ranking first in gender inequality in employment and average hours of work. Scandinavian countries plus the UK perform the lowest levels of working time and have some of the lowest levels in gender inequality. Notice that the relationship would be positive even if only the sample of EU-12 countries were considered. Indeed, in that case the positive correlation between gender inequality and time for work would be higher; adding Baltic and eastern countries weakens the relationship due to their high engagement in work activities accompanied by relatively low gender inequality ratios.

[Figure 7]

A further result of the theoretical framework explains that individuals engage more in socio-political activities as gender equality in employment increases. Therefore, as female employment increases, the proportion of time allocated to socio-political participation rises. In Figure 8, we show the relationship between the two variables for some European countries, using data on time for socio-political activities collected by Eurostat, through a special survey (Eurostat, European Social Survey [ESS]).[17] From the survey, we extract information on time allocated to "volunteer activities" and "other participatory activities," for which the dataset contains an explicit entry. Figure 8[18] clearly shows the existence of a negative relationship, at least for the European countries included in the ESS, between the proportion of male-to-female employment and time spent in socio-political activities; the relationship is validated by a significant regression line with a negative slope.

[Figure 8]

The empirical evidence discussed so far shows that in the European region a clear relationship exists between the share of women in employment, time allocated to work, and average educational levels. Facts show that the lower the gender inequality in employment, the higher the average education level in the population and the lower average time for work. In addition, in those countries with higher female employment and lower time allocated to work, the commitment to socio-political activities increases. These figures seem to suggest that countries where women and men are equally represented in economic activities are characterized by a higher commitment (in average terms) to activities to assert the individuals' own ideas and values—or, alternatively, time to affect socio-economic decisions—and, at the same time, a lower work commitment than countries with a higher gender inequality in employment. Moreover, countries with less unequal gender ratios in employment are those with the highest levels of schooling.

At this point, the natural question arising is whether the higher female participation in employment, accompanied by better levels of schooling and more time for socio-political participation, is obtained at the expense of a lower fertility rate, with negative effects on the population growth rate. The deepening of this issue is rather important to validate our model. In fact, the population growth rate is a parameter of our model affecting the growth rate of the economy and interacts with variables that depend on gender inequality in employment, to determine the long-run equilibrium. According to Figure 9,[19] the relationship between the total fertility rate and gender inequality in employment, in the European area, is negative: total fertility rates increase as female employment rises. Therefore, there is no trade-off, in the European region, between fertility and female employment.

[Figure 9]

Figure 9 together with Figure 8 determine the negative relationship between n and p stated in Claim 2.

To conclude, we may say that in the European Union three different social models exist: a *Nordic model* where women and men are nearly balanced in employment, levels of schooling are high, working time is contained, and fertility rates are high; a *Mediterranean model* where, on the contrary, women's employment rates are much lower than men's, working time is quite high, average educational levels rank the lowest, and, moreover, fertility rates are low. The third model, the *Continental* one, is in between the Nordic and the Mediterranean.

Before discussing the simulations' results, we propose a last focus, on the relationship between gender inequality in employment and growth rate in per-capita GDP.[20] Figure 10 shows how the relationship is negative; countries with more gender inequality pay a price in terms of growth in per-capita income.

[Figure 10]

4.2 Model's numerical simulations

The model was simulated for three European countries: Finland, Germany, and Italy. The choice was determined by the need to compare countries with different levels of gender inequalities in employment, rates of population growth, and capital shares. Finland is the European country, out of the ex EU-15, with the highest ratio of women-to-men employment; Finland is second, after Estonia, if we consider the recently constituted EU-27. Unsurprisingly, for economists studying the relationship between female labor market participation and fertility, Finland also has the highest total fertility rate and population growth rate across the European Union; this, despite the high proportion of women employed. We can say that Finland is very representative of the *Nordic model*. On the other hand, Italy belongs to the *Mediterranean model* and performs among the last countries in terms of female relative presence in economic activities, with a male-to-female employment ratio second only to Malta—a dead heat with Greece—and one of the lowest population growth rates (and total fertility rates) among European countries. Finally, Germany is in between and is the most representative country of what we called the *Continental model*.

The parameters we calibrate on the three countries are gender inequality in employment β, the growth rate of population n, and the capital share α. The remaining parameters were set at levels normally used in growth model simulations.[21]

In Table 1, we show the assumed values of the three parameters for the countries considered.

[Table 1]

In Table 2, we show the simulation results. Notice that differences across the three countries in the level of simulated values are, in some cases, very small; this is not important for interpreting the results. Indeed, to validate the model, the simulation results should simply respect the cross-country rank observed for the variables' true values.

We can see that our theoretical framework predicts, by simulation, that Finland, the country with the lowest gender inequality in employment and the highest population growth rate, converges to a steady-state where time allocated to education is higher and working time is lower than in all other countries. Moreover, time for socio-political activities is higher than in Germany and Italy. In contrast, Italy meets in the long-run the higher average time for work, and the lowest commitment in education and in socio-political activities compared to the other countries, due to the higher ratio of gender inequality and the lower fertility rate of the group. Germany ranks between Finland and Italy with respect to all variables. In addition, the three countries behave differently in terms of long-run growth rates: Finland exhibits the best income growth rate, followed by Germany and then Italy. Therefore, the model predicts that countries with lower gender inequality in employment grow faster in the long-run; this happens when higher equality in employment is accompanied by a population growth rate, as happening in the most egalitarian European countries (Finland). The model predicts that, when female employment is higher, average working time declines, and individuals free up resources—in terms of time—for other activities different from work. If, at higher female employment, we also observe a higher rate of population growth, then this free time converts into more time for education of new generations, with positive effects on the income growth rate[22].

[Table 2]

From the standpoint of a pure evaluation of the theoretical model, we can state that the model well reproduces the ordinal relationships observed across the European countries, and discussed in the previous section.[23] The growth rate predicted for Italy corresponds to the real value observed in the 2005, while for Finland and Germany our results underestimate the evidence (in 2005, the GDP growth rate was 1.7% for Germany and 3% for Finland). Thus, we can conclude that our results confirm the qualitative relations that emerged in the stylized facts.

5. Conclusions

In this article, we proposed a theoretical model to explain the effect of gender inequality in employment on the allocation of time among work, education, and socio-political activities, and on growth. Our model—of endogenous growth with human capital accumulation à la Lucas—assumes that women and men have different sets of values and, therefore, belong to different groups. The two groups share a total benefit according to their relative weight in society that depends on two factors: gender composition of employment (assuming full employment, population, and employment coincide) and time that each group's representative individual allocates to socio-political activities to affect society values. Socio-political engagement is determined endogenously and depends on the level of gender inequality in the economy.

The model explains how gender imbalance in employment can discourage socio-political participation, and can result in a greater average effort for work in the economy as a whole. This happens at the expense of quality of life.

The numerical simulations, calibrated on some European countries, have allowed country heterogeneity in some factors assumed exogenous at the theoretical level to be taken into account: in particular, the population growth rate. The simulation results show how the model well reproduces the empirical facts observed across the European countries. Countries with lower gender inequality in employment better perform in terms of economic growth; at the same time, compared to other countries with more inequality, countries with lower gender inequality experience models of reallocation of time toward education and activities of social participation, and enhanced economic growth. This result is partially due to the presence, in those countries, of high female employment and high population growth rates. Our results suggest clear policy implications: not to adversely influence economic growth, policies devoted to promote equal opportunities in employment and increase female employment must be accompanied by pro-family policies.

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Figures and Tables

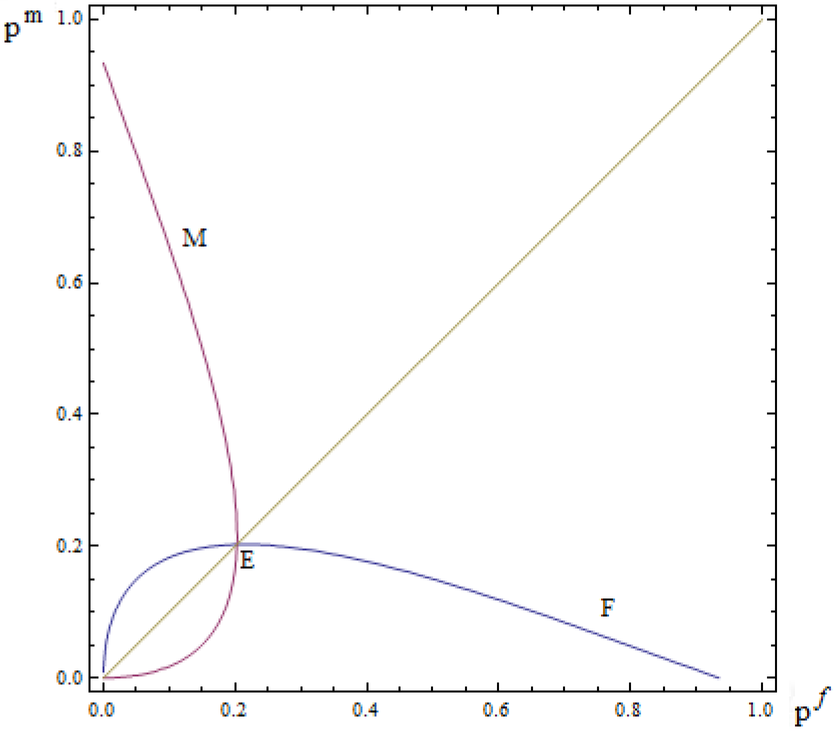


Figure 1. Nash-Cournot equilibriums with β=1

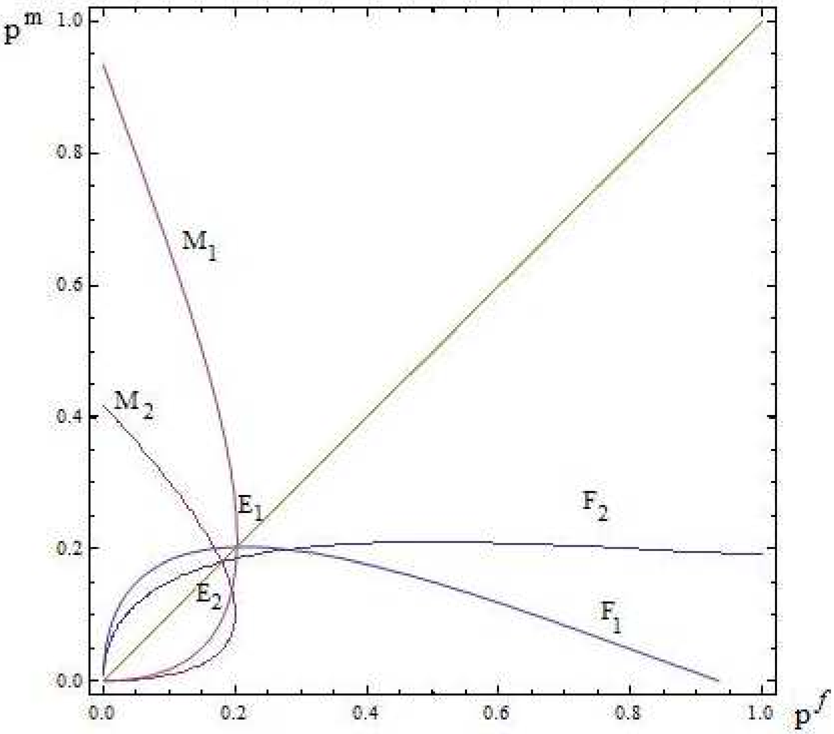


Figure 2. Nash-Cournot equilibriums with β=1 (equilibrium E_1) and β=2 (equilibrium E_2)

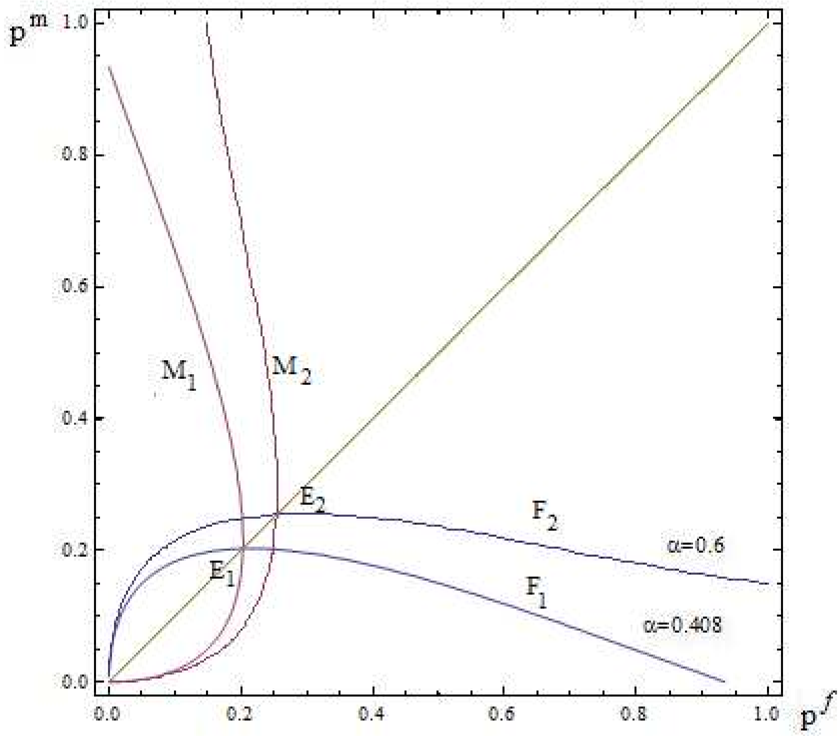


Figure 3: The effect of an increase in the capital share (parameter α) on time for socio-political activities

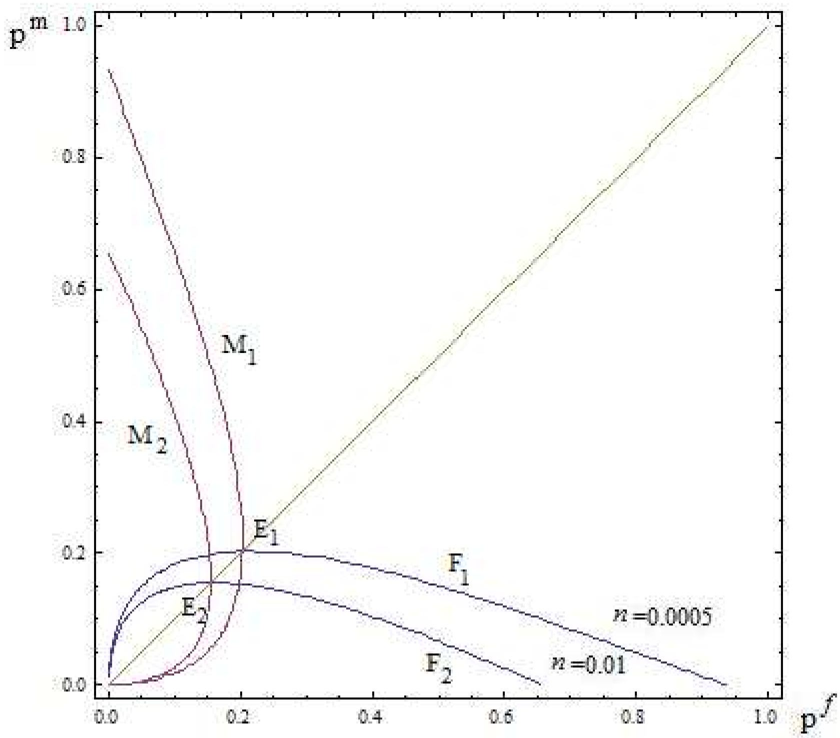


Figure 4. The effect of an increase in the population growth rate (parameter n) on socio-political participation

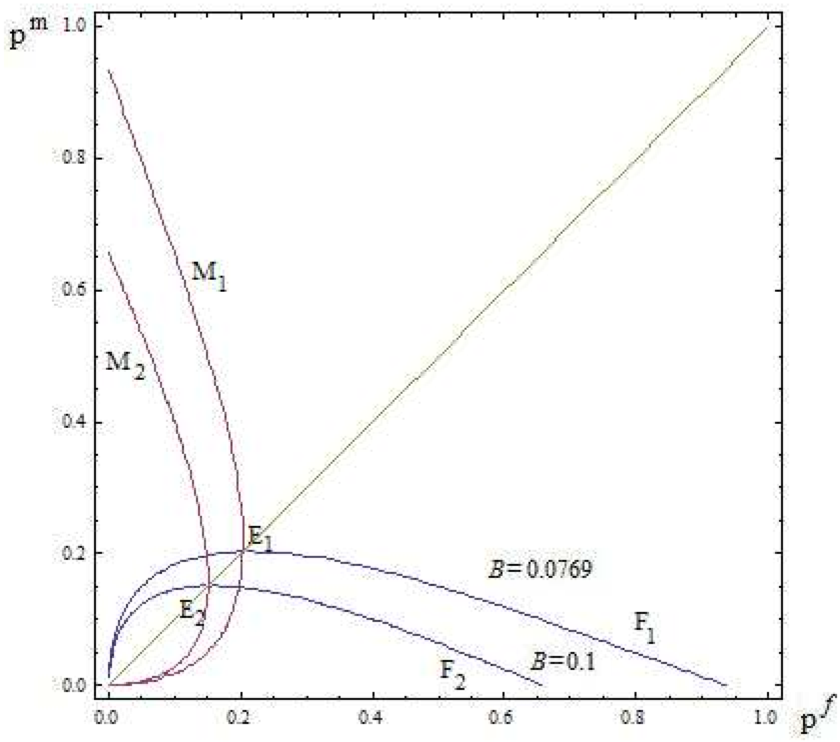


Figure 5. The effect of an increase in the productivity of the education sector (parameter B) on socio-political participation

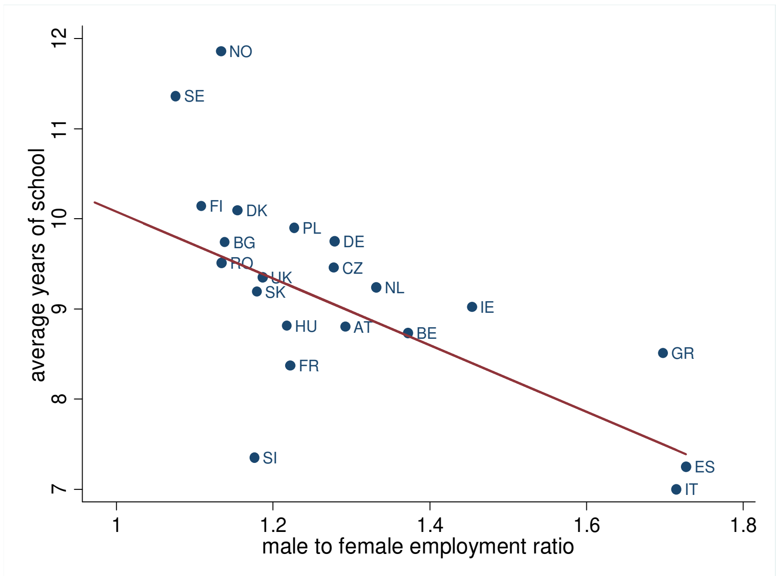


Figure 6. European cross-country relationship between male-to-female employment ratios and average years of school

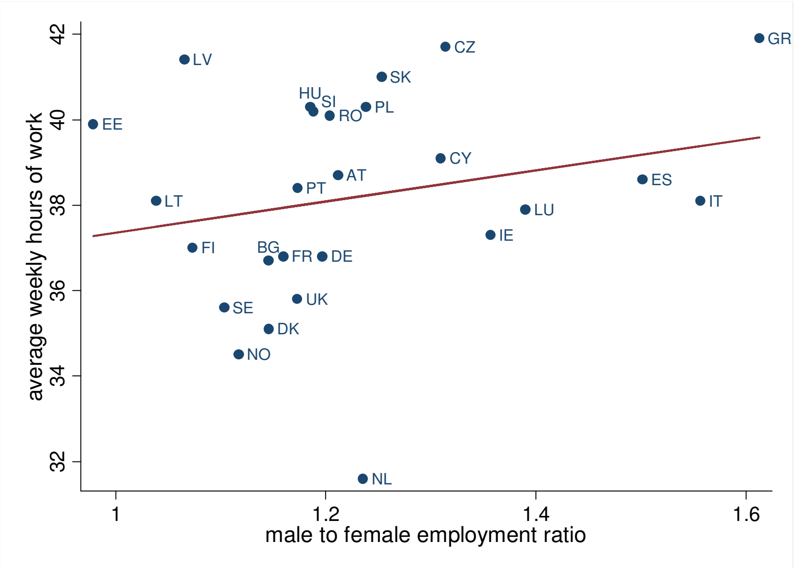


Figure 7. European cross-country relationship between male-to-female employment ratios and average weekly hours of work

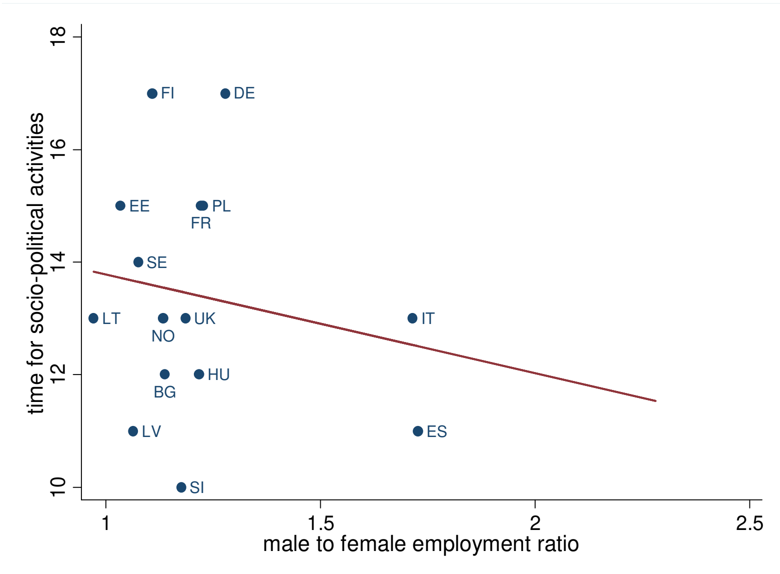


Figure 8. European cross-country relationship between male-to-female employment ratios and time allocated to socio-political activities

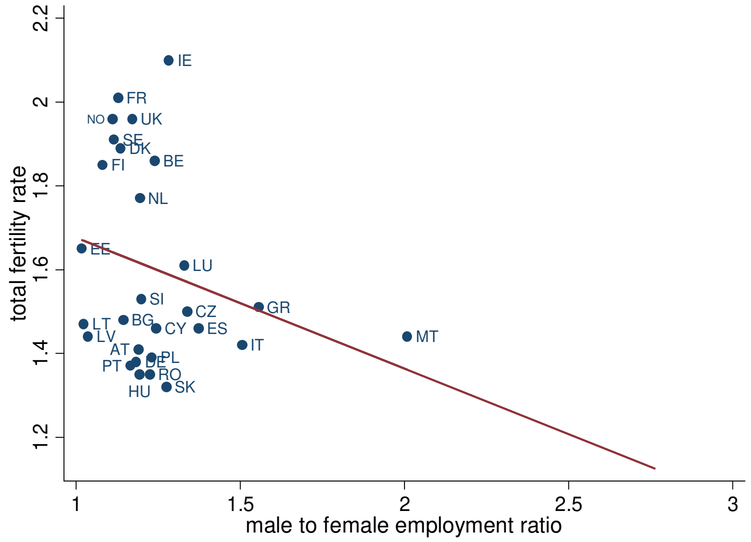


Figure 9. Relationship between male-to-female employment ratio and total fertility rate

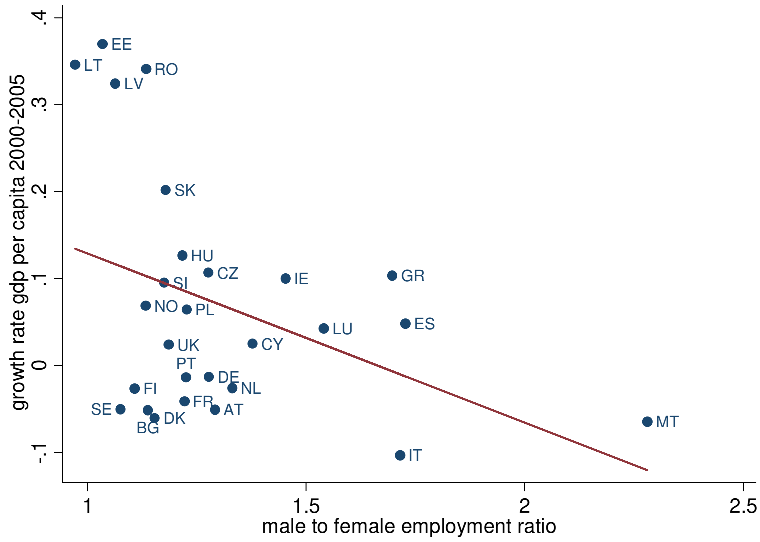


Figure 10. Relationship between male-to-female employment ratio and growth in per-capita income

Table 1. Parameter calibration

Country	Gender inequality in employment (β)	Rate of population growth (n)	Capital share (α)
Finland	1.10826	0.0021	0.418
Germany	1.1969	0.0006	0.367
Italy	1.7153	0.0005	0.408

Source of data: see Section 3. Data on capital shares are from Sturgill (2009).

Table 2. Simulation results

	Finland	Germany	Italy
k/h	4.3993	3.167	4.2794
c/k	0.2345	0.2784	0.2424
$u*$	0.6229	0.6371	0.6437
$p*$	0.1952	0.1937	0.1882
$e*$	0.1819	0.169	0.1681
γ^*	0.014	0.013	0.0129

Appendix

A Transitional dynamics

Here, we briefly examine the transitional dynamics of the economy. In particular, we show that the dynamic system exhibits saddle-path stability. According to (17) and (18) and given Proposition 1, we can write $p(t) = \dfrac{\beta q(t) u(t)^{\alpha} x(t)^{1-\alpha}}{(1-\alpha)(\beta+1)^2}$. That is, we assume that individuals always play the stable Nash-Cournot equilibrium; that is, the adjustment process of social participation is instantaneous. By linearizing the dynamic system (13)-(15) in a neighborhood of the steady-state, we obtain

$$\begin{bmatrix} \frac{\dot{x}}{x} \\ \frac{\dot{u}}{u} \\ \frac{\dot{q}}{q} \end{bmatrix} = \begin{bmatrix} \frac{(\alpha-1)u^{1-\alpha}}{x^{2-\alpha}} + \frac{B\beta q u^{\alpha}}{(\beta+1)^2 x^{\alpha}} & B + \frac{(1-\alpha)}{u^{\alpha} x^{1-\alpha}} - \frac{B\alpha\beta q u^{\alpha-1} x^{1-\alpha}}{(\alpha-1)(\beta+1)^2} & -\frac{B\beta q u^{\alpha} x^{1-\alpha}}{(\alpha-1)(\beta+1)^2} - 1 \\ \frac{B(\alpha-1)\beta q u^{\alpha} x^{-\alpha}}{\alpha(\beta+1)^2} & B - \frac{B\beta q u^{\alpha-1} x^{1-\alpha}}{(\beta+1)^2} & -\frac{B\beta u^{\alpha} x^{1-\alpha}}{\alpha(\beta+1)^2} - 1 \\ u^{1-\alpha} x^{\alpha-2} (\alpha-1)^2 & -u^{-\alpha} x^{\alpha-1} (\alpha-1)^2 & 1 \end{bmatrix} \begin{bmatrix} x - x^* \\ u - u^* \\ q - q^* \end{bmatrix}$$

The characteristic equation can be written as follows

$$\begin{aligned} &\alpha\lambda^3 + [(\alpha-1)d - \alpha(1+a+b+B)]\lambda^2 \\ &+ [(1-\alpha)(1+a)d + \alpha B + b(c + B + \alpha - \alpha c) + \alpha a(B + e + \alpha - \alpha e) + (\alpha-1)c(e + \alpha - \alpha e)]\lambda \\ &- B[b + a(e + \alpha - \alpha e)] = 0 \end{aligned}$$

With $a \equiv (\alpha-1)u^{1-\alpha} x^{\alpha-2}$, $b \equiv \dfrac{B\beta q u^{\alpha} x^{-\alpha}}{(\beta+1)^2}$, $c \equiv (1-\alpha)u^{-\alpha} x^{\alpha-1}$, $d \equiv \dfrac{B\alpha\beta q u^{\alpha-1} x^{1-\alpha}}{(\alpha-1)(\beta+1)^2}$ and $e \equiv \dfrac{B\beta u^{\alpha} x^{1-\alpha}}{(1-\alpha)(\beta+1)^2}$.

To study the stability of the above dynamic system, we evaluate the eigenvalues of the Jacobian matrix at the point (x^*, u^*, q^*). The easiest way to study the roots of the characteristic equation is by using a graphical analysis. We divide the characteristic equation into two parts: a negatively sloped cubic form, $C(\lambda) = -\alpha\lambda^3$, and the remaining quadratic form, $Q(\lambda)$. Notice that the quadratic function has a positive intercept on the vertical axis. Therefore, we must look at the coefficient of the quadratic form in order to understand the concavity of this function. In terms of our calibration, at the steady-state point we represent the situation as follows.

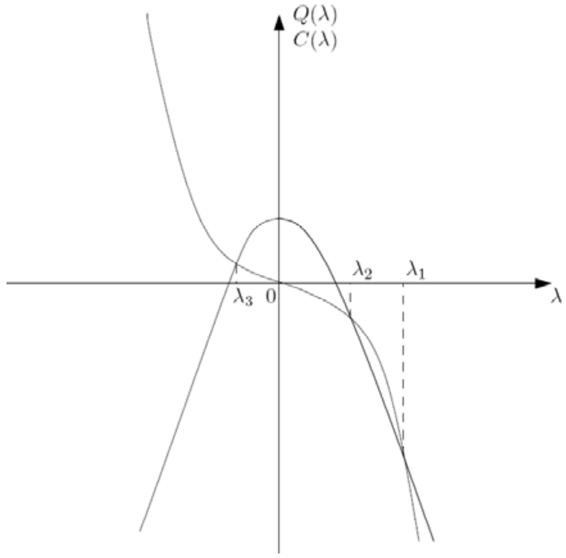
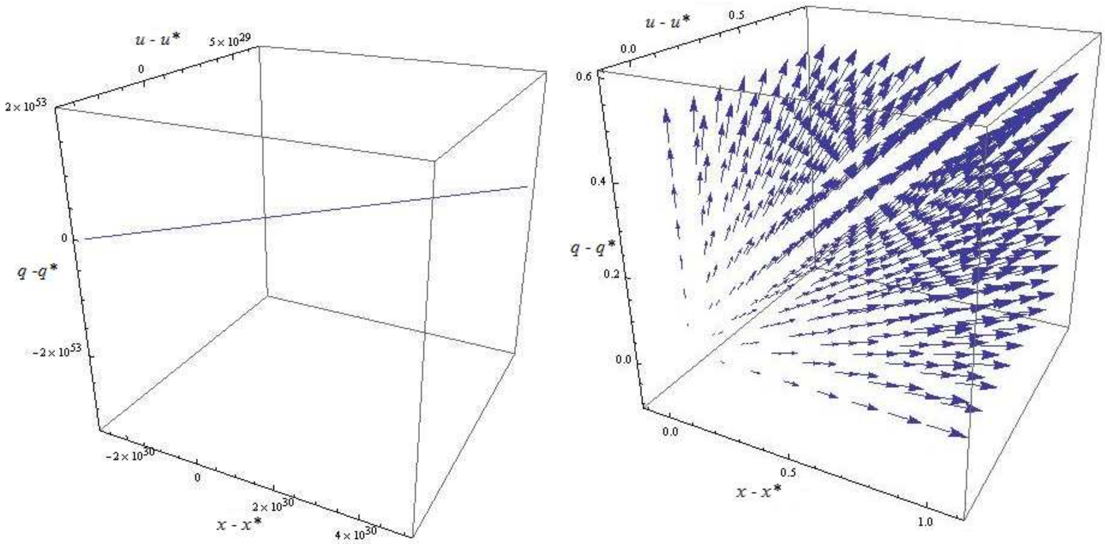


Figure A.1. Eigenvalues of the Jacobian matrix in the steady-state

Therefore, for a given initial value of $k(0)$ and $h(0)$, the levels of $c(0)$ and $u(0)$ must be on the saddle path in order to move toward the steady-state equilibrium.

Figure A.2 shows a graphical representation of the BGP and the phase diagram for Italy. On the axes are reported the differences between the value of a variable at time t and its stationary level. The same figures for Finland and Germany as well as the BGP of each variable are available upon request.



BGP　　　　　　　　　　Phase Diagram

Figure A.2. Balanced Growth Path (BGP) and phase diagram for Italy

B　Proof of Proposition 1

We must prove that all acceptable equilibriums lie on the locus $p_f = p_m$. Let us write (17) and (18) as follows:

$$\frac{u_f^{\alpha}}{(\beta p_m + p_f)^2} = \frac{1}{\beta p_m}(1-\alpha)x_f^{\alpha-1}q_f^{-1}$$

$$\frac{u_m^{\alpha}}{(\beta p_m + p_f)^2} = \frac{1}{\beta p_f}(1-\alpha)x_m^{\alpha-1}q_m^{-1}$$

Since u^* does not depend on *p*, in the steady-state equilibrium all pairs (p_f, p_m) must satisfy the following condition:

$$\frac{1}{p_m}x_f^{\alpha-1}q_f^{-1} = \frac{1}{p_f}x_m^{\alpha-1}q_m^{-1}$$

By substituting the steady-state values of x_s and q_s into the last equality, we obtain $p_m = p_f$.

C Proof of Proposition 2

Given Proposition 1, we can restrict our reasoning on the locus $p^f = p^m = p$. By replacing x^* and q^* into (17), we obtain:

$$u = \frac{p(\alpha-1)(1+\beta)^2(Bp-B-n-\delta)}{\beta[n+B(1-p)(1-\alpha)+\delta+\alpha(\rho-2n-\delta)]}$$

This function has two critical points such that $\frac{du(p)}{dp} = 0$:

$$p_{1,2} = 1 + \Gamma \pm \Lambda$$

with $\Gamma \equiv \frac{n(2\alpha-1)+(\alpha-1)\delta-\alpha\rho}{B(\alpha-1)} > 0$ and

$\Lambda \equiv \frac{\sqrt{\alpha(n-\rho)[(B+\delta)(\alpha-1)+n(2\alpha-1)-\alpha\rho]}}{B(\alpha-1)} < 0$, assuming $\rho > n$.

Since $|\Gamma| > |\Lambda|$, both solutions p_1 and p_2 are greater than 1. Therefore, for $p \leq 1$, the relationship between u and p is monotone. Given the uniqueness of u^*, we will have only one equilibrium level of p^*.

D Proof of Proposition 3

The goal of the proof is to show that $\frac{dp}{d\beta} < 0$ as β goes to 1. Given Proposition 1, we can restrict our reasoning on the locus $p^f = p^m = p$. By putting the expression of x^* into (17), we have

$$\frac{\beta}{(\beta+1)^2} = p(1-\alpha)\frac{B(1-p)+n+\delta}{\alpha}u^{-1}q^{-1}$$

that is

$$\phi(\beta) = \psi(p).$$

By totally differentiating both sides, we get

$$\frac{dp}{d\beta} = \frac{\phi'(\beta)}{\psi'(p)}$$

Where

$$\phi'(\beta) = \frac{1-\beta}{(1+\beta)^3}$$

And, substituting u^* with its expression, we can write:

$$\psi'(p) = B\left\{\frac{1}{\rho - n} + \frac{\alpha\left[B(\alpha-1) + n(2\alpha-1) + (\alpha-1)\delta\right] - \alpha\rho}{\left[n + B(1-p)(1-\alpha) + \delta + \alpha(\rho - \delta - 2n)\right]^2}\right\}$$

We know that as $\beta > 1$ goes to 1, $\phi'(\beta)$ goes to zero from negative values; therefore, the proof is complete if we show that $\psi'(p)$ assumes positive values.

Notice that $\psi'(p) > 0$ for $\alpha < 1$ and

$$\alpha > \frac{(B + n - Bp + \delta)^2}{B^2(p-1)^2 + (n+\delta)(2n + \delta - \rho) - B[n(4p-3) - 2(1-p)\delta + \rho - 2p\rho]}$$ together

with

$$(B + n - Bp + \delta)^2 > B^2(p-1)^2 + (n+\delta)(2n + \delta - \rho) - B[n(4p-3) - 2(1-p)\delta + \rho - 2p\rho]$$

if and only if

$$\bar{p} = \frac{B + n + \delta}{2B}$$

However, the maximum amount of time devoted to socio-political participation lies, at the same time, on the 45-degree line and on the time constraint:

$$p^* = \frac{B + n - \rho}{2B} < \frac{B + n + \delta}{2B} \quad for \quad \rho, \delta > 0$$

Therefore, for $\alpha \in (0,1)$, we know that $\psi'(p) > 0$, and consequently, in a right neighborhood of $\beta = 1$, we can conclude that $\frac{dp}{d\beta} < 0$.

E Proof of Corollary 1

If *n*, *B*, and ρ do not change, u^* will remain constant, and from Proposition 3, we will have a negative relationship between β and p^*, and then a positive relationship between β and time for education $(1 - u^* - p^*)$.

F Proof of Claim 1

By using the same notation we used to prove Proposition 3, we have

$$\frac{dp}{d\alpha} = -\frac{\psi'(\alpha)}{\psi'(p)}$$

Where $\psi'(\alpha) = \frac{Bp(Bp - B - n)}{[n + B(1-p)(1-\alpha) - 2n\alpha + \alpha\rho]^2}$ for $p \in \left(0, \frac{B + n + \delta}{B}\right)$. Therefore, given the sign of $\psi'(p)$ previously discussed, we can conclude that for admissible values of *p*, $\frac{dp}{d\alpha} > 0$.

G Proof of Claim 2

By using the same notation used in previous proofs, we know that

$$\frac{dp}{dn} = -\frac{\psi'(n)}{\psi'(p)}$$

With $\psi'(n) = Bp\left\{\frac{1}{(n-\rho)^2} + \frac{\alpha - 2\alpha^2}{[n + B(1-p)(1-\alpha) + \delta + \alpha(\rho - 2n - \delta)]^2}\right\} < 0$ if and only if $\alpha \in \left(1, \frac{(B + n - B\rho + \delta)^2}{2n^2 + B^2(1-p)^2 + 4n\delta + \delta^2 + 2B(1-p)(2n + \delta - \rho) - 2\delta\rho - \rho^2}\right)$.

Therefore, given the sign of $\psi'(p)$ previously discussed, we can conclude that for admissible values of *p*, $\frac{dp}{dn} < 0$.

[1] As we shall explain in the next section, our model does not envisage unemployment. Thus, the number of employed members corresponds to the group population.

[2] Barro and Lee analyzed a sample of 116 countries between 1965 and 1985.

[3] The authors use a range of models and definitions of the dependent variable. They carry out OLS and 2SLS estimates with adjustments for endogenous income.

[4] We suppose that, at any time *t*, the representative individual of group *s* has an instantaneous utility function $U_s(t) = c_s(t)\exp g_s(p_s(t))$. That is, consuming the final good in a society in which the individual is more represented increases his or her satisfaction. Taking the logarithmic transformation, we obtain the expression that enters in the overall utility shown in Equation (1).

[5] In the Lucas model, the existence of human capital externalities is relevant only for the comparison between a centralized and a decentralized economy. Nonetheless, including external effects would not change our qualitative results in terms of time allocation.

[6] Since the current value Hamiltonian function is concave in $k(t)$ and $h(t)$, Arrow's theorem implies that the FOCs, jointed to the transversality conditions, are necessary and sufficient to characterize a maximum solution.

[7] The reduced dynamic system involves four variables: $p(t)$, $u(t)$, $x(t)$, and $q(t)$. However, as we will see, the growth rate of p(t) can be expressed as a linear combination of system (13)-(15). Therefore, to solve the system, we first determine our steady-state solutions as functions of $p(t)$, and then we use these solutions to find the steady-state value of p by using Equation (12).

[8] See Appendix A for the stability analysis of the system.

[9] A possible extension of this model can be a von Stackelberg game in which one group behaves as a leader whereas the other behaves as a follower. This framework would allow for asymmetric equilibriums in the allocation of time between the two groups.

[10] In next section, we will show that this result is in line with the empirical evidence.

[11] For realistic values of β, the Nash-Cournot equilibrium is stable.

[12] We can supply iso-growth curves of β and n to show that a trade-off exists, in terms of growth, between gender equality and the population growth rate.

[13] Of course, the same is true compared to countries in which men are under-numbered.

[14] Data from Barro and Lee (2001).

[15] Data from Labour Force Statistics, Eurostat (year 2000). The percentage is calculated on total population aged 25-64.

[16] Data from Labour Force Statistics, Eurostat (year 2005). Working time is referred to the main job.

[17] Data available on the web at the address: http://ess.nsd.uib.no. Data were collected across the period 1998-2002 in Belgium, Germany, Estonia, France, Hungary, Slovenia, Finland, Sweden, the United Kingdom, and Norway (for a summary of the results, see Eurostat, 2005). For Spain, Italy, Latvia, Lithuania, and Poland, data were collected across the period 2002-2004 (for a summary, see Eurostat, 2006).

[18] Since the survey was conducted around 2000, but at different times depending on the country, we compare ESS data with gender inequality in employment evaluated in 2000 (Eurostat, Labour Force Statistics).

[19] Data from Labour Force Statistics, Eurostat (year 2008).

[20] The gender employment ratio was evaluated in 2000; the yearly growth rate of the GDP per capita is made for the time interval 2000-2005.

[21] Following Ladròn-de-Guevera *et al.* (1997); time preference (parameter ρ) has been assumed equal to 0.05; productivity in the human capital sector (parameter B) has been assumed equal to 0.0769; the depreciation rate of physical capital (parameter δ) has been valued 0.07.

[22] In Appendix H, we show the convergence paths along the simulation for the three countries. The behavior of the three countries along the path to the long-run equilibrium can be interpreted as a robust check of our model, given the fact that each country has different parameter configurations.

[23] The German physical-to-human capital ratio simulated is very low. This result comes from the levels of capital shares we applied. However, capital shares are lower in advanced economies as well as the ratio k/h, given the relative abundance of human capital (see, e.g., Gundlach, 1997).