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Abstract

In this paper we introduce product demand uncertainty in a mixed oligopoly model and reexamine the nature of sub-game perfect Nash equilibrium (SPNE) when firms decide in the first stage whether to lead or follow in the subsequent quantity-setting game. In the non-stochastic setting, Pal (1998) demonstrated that when the public firm competes with a domestic private firm, multiple equilibria exist but the efficient equilibrium outcome is for the public firm to follow. Matsumura (2003a) proved that when the public firm's rival is a foreign private firm, leadership of the public firm is both efficient as well as SPN equilibrium. Our stochastic model shows that when the leader must commit to output before the resolution of uncertainty, multiple SPNE is possible. Whether the equilibrium outcome is public or private leadership hinges upon the degree of privatization and market volatility. More importantly, Pareto-inefficient simultaneous production is a likely SPNE. Our results are driven by the fact that the resolution of uncertainty enhances the profits of the follower firm in a manner that is well known in real option theory.

JEL classification: C72, D8, L13.

Keywords: Mixed oligopoly; Partial privatization; Demand Uncertainty.

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1 Introduction

In many industries across many countries private and public firms compete in oligopolistic markets. This type of market structure is famously known as mixed oligopoly. Industries characterized by mixed oligopoly range from network (transportation, broadcasting, telecommunication, mail), energy (gas, electricity), to service (insurance, banking, health care, education) sectors. Privatization and liberalization of markets dominated by state enterprises have made mixed oligopoly specially significant in transitional and developing countries. Research on mixed oligopolies have burgeoned since the seminal paper by Merrill and Schneider (1966).[1] The interest in this area has heightened in recent years in view of the liberalization and privatization policies in the so-called transitional economies of Eastern Europe.[2]

In the traditional models of mixed oligopoly, public and private firms are assumed to set output either simultaneously or sequentially giving rise to a Cournot or Stackelberg structure. Crucially, however, whether the public firm led or followed was determined exogenously. The order of moves in a mixed oligopoly model was first endogenized by Pal (1998). He adopted the observable delay game of Hamilton and Slutsky (1990) where firms in the first stage determine whether to lead or follow and then set quantities accordingly in the later stages. The sub-game perfect Nash equilibrium (SPNE) of the extended game then determines whether the basic quantity game is Cournot or Stackelberg. The significance of this approach is that since the order of moves nonmarginally affects the nature of equilibrium in oligopolistic markets, it is more satisfactory to have the order emerge as an outcome of an optimizing process. Matsumura (2003a) adopted a similar procedure to determine the endogenous order of moves in a mixed oligopoly where the private firm is foreign owned. An interesting outcome of this research is that unlike pure oligopolies where the SPNE of the observable delay game results in a Cournot structure, in a mixed oligopoly SPNE generally only admits a Stackelberg model. Thus, Pal (1998) showed that when marginal costs are constant and the public firm is less efficient, equilibria with the public firm as the leader and the follower are both SPNE although social welfare is higher when the public firm follows. By contrast, Matsumura (2003a) demonstrated that when the public firm competes with a foreign private firm, leadership of the public firm is the equilibrium as well as the socially efficient outcome.

A limitation of the considerable literature on mixed oligopoly, including the ones cited above, is that the role of uncertainty is generally ignored. In recent studies, Hirokawa and

[1]See De Fraja and Delbono (1990), Bös (1991), and Nett (1993) for surveys on mixed oligopoly models.
[2]See Megginson and Netter (2001) and the reference therein for recent privatization trend around the world.

Sasaki (2001) and Brown and Chiang (2003) introduced demand uncertainty in the standard, quantity-setting, observable delay game of pure oligopoly. An assumption underlying their model is that uncertainty is resolved with time so that waiting carries positive option value in a manner well known in the finance and investment literature (see Dixit and Pindyck, 1994). This implies, therefore, that when quantity commitment is irreversible leadership involves a trade-off. The usual advantage of leadership and preemption is now potentially compromised by the lost option value of waiting or following. A Cournot structure is thus no longer guaranteed as the SPNE of the extended game.

In the present paper, we introduce demand uncertainty in a mixed oligopoly model. In particular, we revisit the nature of equilibria in the observable delay games analyzed by Pal (1998) and Matsumura (2003a).[3,4] We follow the uncertainty regime in Brown and Chiang (2003) and assume a linear market demand that is subject to an additive disturbance. The market is served by a public firm maximizing a weighted sum of social surplus (the sum of consumer and producer surpluses and its own profit) and a private profit maximizing firm (which could be foreign owned) sharing common technology embodied by quadratic total cost functions. We assume that uncertainty is resolved after the leader's commitment to output but before the follower firm must make its output decision. Leadership, thus, involves a sacrifice of option value.

In a non-stochastic model, it is well known that the leader has no incentive to deviate from the committed output since follower's reaction is incorporated in setting that output. In the stochastic model, however, leader's output may well be sub-optimal, *ex-post*, after the resolution of uncertainty. Since our interest is to highlight the role of option value in determining the order of moves, we must, therefore, rule out ex-post deviation from the committed output by the leader. This scenario is relevant specifically for industries where technology makes quantity adjustment very costly. As pointed out in Hirokawa and Sasaki (2001), quantity stickiness may

[3]Hamilton and Slutsky (1990) suggest two ways of endogenizing the order of moves in duopoly: action commitment and observable delay games. In both timing games the firms have to move in exactly one of two periods. Briefly, in the observable delay game firms announce the period in which they will move before choosing an action. After the announcements, firms then select their actions knowing when the other firm will make its choice. The game admits a unique equilibrium outcome.

[4]Besides the observable delay game of Hamilton and Slutsky (1990), Bagwell (1995) shows that commitment is completely ineffective if firm's observations of commitments in early stages of the game are subject to an arbitrary small noise. van Damme and Hurkens (1997) criticize Bagwell and argue that, under certain regularity conditions, some mixed strategy equilibria preserves the value of commitment, and hence the Stackelberg game is a feasible outcome. Adolph (1996) also shows that commitment retains its value if the communication error is small relative to trembles. Amir and Grilo (1999) ignore communication error in commitments and consider general demand and cost functions. They find that while the Cournot duopoly is a predominant outcome, the sequential Stackelberg game remains a possibility under the restrictive assumptions that the demand function be sufficiently concave. These studies assume that firms are privately owned.

also result from market institution. Thus, many commodities are sold through retail outlets and producing firms may have a contract with retailers to supply a fixed quantity. “Buying shelves” is an example of such quantity commitment. Our stochastic mixed oligopoly model yields results that are significantly different from the corresponding deterministic models. Thus, we demonstrate the existence of multiple SPNE, the nature of which depends on the level of uncertainty and public ownership. Crucially, we show that unlike in Pal (1998) and Matsumura (2003a), Cournot outcome is a part of the SPNE set. Since output and social welfare are usually lower in a Cournot equilibrium than in the Stackelberg leadership case, our result has significant implications for public policy in the context of mixed oligopoly.[5]

The paper is organized as follows. Section 2 presents the basic model in which timing and output games are played between a public firm and a domestic private firm in a stochastic environment. Section 3 characterizes the equilibrium outcomes for a simultaneous-move leadership game. Section 4 takes into account several extensions. Specifically, we consider (i) the competition between a public firm and a foreign private firm; (ii) the sequential-move leadership game with the public firm being the first mover; (iii) a general setting where the public firm is partially nationalized. Finally, the paper ends with concluding remarks.

2 The Basic Model

Consider a two-stage duopoly game in which players choose to make their output decisions either in stage 1 or stage 2. Firm 1 is a pure public firm with a single objective of maximizing social welfare, while firm 2 is a profit maximizing private firm. Firm's demand function arises from utility maximization of a representative consumer with quasi-linear utility function. Thus, the inverse demand function is given by

$$p = \alpha - (q_1 + q_2) + \theta,$$

where q_i is firm's i's ($i = 1, 2$) output; α is a demand (scale) parameter large enough for the equilibrium quantities to be always positive (i.e., $\alpha > q_1 + q_2 - \theta$); and θ is a random disturbance

[5] Our work relates to a number of recent studies that deal with endogenous timing in mixed oligopoly. Recently, Lu and Poddar (2006) analyze a capacity choice game in mixed duopoly under demand uncertainty. They developed a two-stage framework in which a public firm and a domestic private firm simultaneously choose capacity in stage 1 before uncertainty becomes known. In stage 2, after the resolution of uncertainty both firms simultaneously choose how much output to produce. They obtain clear-cut results including two symmetric and one asymmetric equilibria. In the symmetric case, when the realized demand is high, firms' quantities exceed their capacity, whereas if the realized demand is low, both firms carry idle capacity. By contrast, under mild realized demand, public (private) firm chooses under (excess) capacity. Lu (2006) extends Pal's (1998) model by introducing foreign firms. He finds that in equilibrium public firm always chooses to be the Stackelberg follower.

term distributed according to the density function $f(\theta)$. Note that θ is an idiosyncratic shock, with zero mean and constant variance, i.e., $E(\theta) = 0$ and $Var(\theta) = \sigma^2 > 0$. The value of θ is unknown to all players in stage 1, but it becomes known at the beginning of stage 2.

We assume that firms are risk-neutral, information is perfect, and firms play a pure strategy game. We further assume that before the output game begins, the firm determines simultaneously whether to move early (E) and produce in stage 1 or to follow late (L) and produce output in stage 2. Given that the random variable will not be revealed until the end of the first stage, the early mover would have to make the output decision before the random variable θ becomes known. The late mover, however, makes his output decision after the complete resolution of uncertainty. Given the timing of their moves, there are four possible combination: (i) Both firms choose to move late, denoted by (L, L); (ii) Both firms move early, denoted by (E, E); (iii) Firm 1 moves early and firm 2 moves late, denoted by (E, L); and (iv) Firm 1 moves late and firm 2 moves early, denoted by (L, E). When firm's actions are the same, the Cournot outcome results (i.e., Cases (i) and (ii)). Games with different actions (i.e., Cases (iii) and (iv)) yield the Stackelberg outcome.

All firms have identical technologies, represented by the cost function, $C_i(q_i) = \frac{1}{2}q_i^2, \ \ i = 1, 2.$[6] The profit function of firm i can therefore be written as

$$\Pi_i(q_1, q_2) = (\alpha - q_1 - q_2 + \theta)q_i - \frac{1}{2}q_i^2. \tag{1}$$

Firm 1 maximizes social welfare W which is defined as the sum of consumers' and producers' surplus[7], while firm 2 simply maximizes profit. Both firms are based in the domestic market. Specifically, the W function can be written as

$$\begin{aligned} W(q_1, q_2; \theta) &= \int_0^Q p(x)dx - pQ + \Pi_1(q_1, q_2) + \Pi_2(q_2, q_1) \\ &= \int_0^Q p(x)dx - C_1(q_1) - C_2(q_2) \\ &= \frac{1}{2}Q^2 + (\alpha - Q + \theta)Q - q_1^2/2 - q_2^2/2, \end{aligned} \tag{2}$$

[6] A similar cost function can also be found in Fershtman (1990), Fjell and Pal (1996), Matsumura (2003a), Chang (2004), and Matsumura and Kanda (2005). For simplicity, we ignore the fixed cost. But, including it will not affect our results.

[7] In a stochastic environment, one natural question is whether the standard consumer surplus (CS) or the expected consumer surplus (ECS) is a valid measure of welfare. Stennek (1999) shows that when consumers are risk-neutral and have quasi-linear (zero-income elasticity) preferences, the expected consumer surplus is a good measure of consumer welfare.

where $Q = q_1 + q_2$. Given θ, the payoff functions for firms 1 and 2 are given by

$$U_1 = W(q_1, q_2), \quad (3)$$

$$U_2 = \Pi_2(q_1, q_2), \quad (4)$$

respectively. There are four cases to consider: first two cases involve similar actions by both firms (resulting in the Cournot-Nash equilibria), while the remaining two cases involve different actions by the two firms (yielding the Stackelberg equilibria).

2.1 Case 1: (L, L)

Both firms move late. In this case, firms decide on output level after θ is revealed to all firms at the end of stage 1. Each firm independently maximizes its objective function U_i $(i = 1, 2)$ subject to $q_i \in \Re^+$, given its rival's output q_j, $j = 1, 2$. The first-order conditions associated with (3) and (4) are

$$\begin{aligned} \alpha - 2q_1 - q_2 + \theta &= 0, \\ \alpha - q_1 - 3q_2 + \theta &= 0, \end{aligned}$$

yielding the equilibrium outputs

$$\begin{aligned} q_1^*(\theta) &= \frac{2(\alpha + \theta)}{5}, \\ q_2^*(\theta) &= \frac{(\alpha + \theta)}{5}. \end{aligned}$$

Upon substitution, the expected payoffs are therefore

$$A_{LL} = E(U_1) = \frac{8\sigma^2}{25} + \frac{8\alpha^2}{25}, \quad (5)$$

$$B_{LL} = E(U_2) = \frac{3\sigma^2}{50} + \frac{3\alpha^2}{50}. \quad (6)$$

The first subscript refers to the action taken by firm 1, while the second subscript represents the action chosen by firm 2. Notice that the second term of A_{LL} or B_{LL} is the usual payoffs under certainty. In the presence of uncertainty, taking the output decision after the resolution of the random variable enhances firms' payoffs since firms are now able to make a more well-informed decisions. The benefit of making a well-informed decision is captured by the first term in (5) and (6). We call this the *option value* effect. Its magnitude increases with the degree of uncertainty, σ^2. Clearly, the option value effect ceases to prevail under certainty. In this case, waiting does not carry any information value. One can easily verify this by setting $\sigma^2 = 0$.

2.2 Case 2: (E, E)

Both firms move early. In this case, firms decide on output level before θ becomes known. Firm i chooses q_i to maximize its expected payoffs, $E(U_i)$. This yields

$$\begin{aligned} q_1^* &= \frac{2\alpha}{5}, \\ q_2^* &= \frac{\alpha}{5}. \end{aligned}$$

The expected payoffs when both firms move early are therefore

$$A_{EE} = E(U_1) = \frac{8\alpha^2}{25}, \tag{7}$$

$$B_{EE} = E(U_2) = \frac{3\alpha^2}{50}. \tag{8}$$

Note that if $\sigma^2 = 0$, then $A_{EE} = A_{LL}$ and $B_{EE} = B_{LL}$. Given that A_{LL} and B_{LL} increase with σ^2, we obtain

Corollary 1. $(L, L) \succsim_i (E, E)$ for $i = 1, 2$.

Proof. The proof is straightforward. The gains from waiting for firms 1 and 2 are $A_{LL} - A_{EE} = \frac{8\sigma^2}{25} \geq 0$ and $B_{LL} - B_{EE} = \frac{3\sigma^2}{50} \geq 0$, respectively. Corollary 1 is thus proven. ■

The result is related to the theory of option value in Finance in that information is valuable and net benefits that result from waiting are enhanced when markets become more volatile. That is, waiting is welfare improving. Next, we investigate firms' payoffs when they take different actions.

2.3 Case 3: (E, L)

Firm 1 moves early and firm 2 moves late. In this case, the public firm acts as a Stackelberg leader, while the private firm is a follower. As usual, we start with follower's maximization problem. Given θ and q_1, firm 2 maximizes

$$\Pi_2(q_2, q_1) = (\alpha - q_1 - q_2 + \theta)q_2 - \frac{1}{2}q_2^2,$$

yielding the first order condition, $q_2(q_1, \theta) = (\alpha - q_1 + \theta)/3$. Given this, firm 1 (i.e., the leader) then chooses q_1 to maximize its expected payoff function given by

$$E[W(q_1, q_2(q_1, \theta))].$$

The associated first-order condition is

$$\partial E(W)/\partial q_1 - [\partial E(W)/\partial q_2]/3 = 0.$$

Solving yields

$$q_1^* = \frac{5\alpha}{14}.$$

By substitution, the follower's (i.e., firm 2's) optimal output is therefore

$$q_2^* = \frac{3\alpha}{14} + \frac{\theta}{3}.$$

Given these, it is straightforward to obtain the expected payoffs for firms 1 and 2:

$$A_{EL} = E(U_1) = \frac{2\sigma^2}{9} + \frac{9\alpha^2}{28}, \tag{9}$$

$$B_{EL} = E(U_2) = \frac{\sigma^2}{6} + \frac{27\alpha^2}{392}. \tag{10}$$

2.4 Case 4: (L, E)

In this case, firm 1 acts as a follower and firm 2 is the Stackelberg leader. By following the same procedure, one can obtain the expected payoffs for firms 1 and 2 as follows:

$$A_{LE} = E(U_1) = \frac{\sigma^2}{4} + \frac{21\alpha^2}{64}, \tag{11}$$

$$B_{LE} = E(U_2) = \frac{\alpha^2}{16}. \tag{12}$$

3 Equilibria in a Simultaneous-Move Leadership Game

We are now ready to characterize the equilibria for a game with players setting their leadership strategy simultaneously. Here, we are interested in knowing which combinations of firm's strategies will emerge as an equilibrium outcome. To this end, we consider a 2-player pure-strategy game $\Gamma = \langle S_1, S_2, U_1, U_2 \rangle$, where S_i $(i = 1, 2)$ is player i's finite set of pure strategy. Table 1 summarizes the expected payoff functions of firms associated with each strategy combination. where A_{ij} and B_{ij} $(i,\ j = E,\ L)$ are given in (5)-(12).

The following definition presents the solution concept used to characterize the equilibria.

Definition 1. *A (pure-strategy) Nash equilibrium consists of a strategy $s_i^* \in S_i$ for each player i (i=1,2) such that*

$$U_i(s_i^*, s_j^*) \succeq_i U_i(s_i, s_j^*) \ \textit{for all} \ s_i \in S_i.$$

Table 1: Public vs Domestic Private Firm

		Firm 2	
	S_i	Early	Late
Firm 1	Early	(A_{EE}, B_{EE})	(A_{EL}, B_{EL})
	Late	(A_{LE}, B_{LE})	(A_{LL}, B_{LL})

Thus, for s_i^* to be a Nash equilibrium it must be that each player's strategy yields an outcome that is at least as high a payoff as any other strategy of the player, given that every other player j chooses his equilibrium strategy s_j^*. In other words, no player has an incentive to deviate, given the actions of the other player. In what follows, we calculate the Nash equilibria of the game presented in Table 1.

In this game, each player has two strategies available: early (E) and late (L). The payoffs associated with a particular pair of strategies are given in the appropriate cell of the bi-matrix. By convention, the first payoff belongs to the row player (here, firm 1), followed by the payoff of the column player (here, firm 2). Thus, if both firms 1 and 2 choose E, then firm 1 receives A_{EE} and firm 2 receives B_{EE}. Similarly, if firm 1 chooses L and firm 2 chooses E, then firm 1 receives A_{LE} and firm 2 receives B_{LE}. When both firms choose the same action, the Cournot game results. In the case of different actions, the Stackelberg game results.

Whether the equilibrium outcome is Nash or Stackelberg can be shown to depend on the degree of uncertainty, characterized by σ^2. Let $\widehat{z}_3$ ($\widehat{z}_4$) solves $A_{EL} - A_{LL} = 0$ ($B_{LE} - B_{LL} = 0$), where $\widehat{z}_3 = \frac{9\alpha^2}{616}$ ($\widehat{z}_4 = \frac{\alpha^2}{24}$). The following proposition summarizes our main results:

Proposition 1. *Consider a mixed duopoly in which the demand is linear and the cost functions are quadratic; (i)* (E, L) *is a Nash equilibrium if* $0 \leq \sigma^2 \leq \widehat{z}_3$*; (ii)* (L, E) *is a Nash equilibrium if* $0 \leq \sigma^2 \leq \widehat{z}_4$*; and (iii)* (L, L) *is a Nash equilibrium if* $\sigma^2 \geq \widehat{z}_4$.

Proof. We first show that (E, E) cannot be an equilibrium. To see this, calculate $\triangle_1 = A_{EE} - A_{LE} = -\frac{\sigma^2}{4} - \frac{13\alpha^2}{1600} < 0$ and $\triangle_2 = B_{EE} - B_{EL} = -\frac{\sigma^2}{6} - \frac{87\alpha^2}{9800} < 0$. For ($E$, E) to be an equilibrium, it requires that $\triangle_1 > 0$ and $\triangle_2 > 0$. Clearly, the requirement for (E, E) to be an equilibrium is violated.

For (E, L) to be a Nash equilibrium, it must be that $\triangle_3 = A_{EL} - A_{LL} = -\frac{22\sigma^2}{225} + \frac{\alpha^2}{700} \geq 0$ and $-\triangle_2 = B_{EL} - B_{EE} = \frac{\sigma^2}{6} + \frac{87\alpha^2}{9800} \geq 0$. The former implies that firm 1 prefers E to L, while the latter implies that firm 2 chooses L over E. Note that $-\triangle_2 > 0$. Thus, whether (E, L) is as an equilibrium depends on the sign of $\triangle_3$. Let $\widehat{z}_3$ solves $\triangle_3 = 0$, where $\widehat{z}_3 = \frac{9\alpha^2}{616} > 0$. Given that $\partial\triangle_3/\partial\sigma^2 < 0$, we have $\triangle_3 \lesseqgtr 0$ if $\sigma^2 \gtreqless \widehat{z}_3$. It is evident that (E, L) is a Nash equilibrium if $0 \leq \sigma^2 \leq \widehat{z}_3$. Both firms will have no incentive to deviate from (E, L). This proves Proposition 1 (i).

For (L, E) to emerge as an equilibrium, we need $-\triangle_1 = A_{LE} - A_{EE} \geq 0$ and $\triangle_4 = B_{LE} - B_{LL} \geq 0$. Firm 1 has no intention to deviate from L if $-\triangle_1 > 0$. Likewise, firm 2 would prefer to stick to E if $\triangle_4 > 0$. As shown above, $\triangle_1 < 0$ or $-\triangle_1 > 0$. Thus, whether (L, E) is as an equilibrium depends on the sign of $\triangle_4$. Recall that $\widehat{z}_4$ solves $\triangle_4 = 0$, where $\widehat{z}_4 = \frac{\alpha^2}{24} > 0$. It is easy to verify that B_{LE} is independent of σ^2 and B_{LL} is an increasing function of σ^2; that is, $\partial\triangle_4/\partial\sigma^2 < 0$. For $\sigma^2 \lesseqgtr \widehat{z}_4$, we have $\triangle_4 \gtreqless 0$. Hence, (L, E) is a Nash equilibrium if $0 \leq \sigma^2 \leq \widehat{z}_4$. This proves Proposition 1 (ii).

Finally, for (L, L) to be an equilibrium outcome, it requires $\triangle_4 = B_{LE} - B_{LL} \leq 0$ and $\triangle_3 = A_{EL} - A_{LL} \leq 0$. Note that $\triangle_4 \leq 0$ and $\triangle_3 \leq 0$ when $\sigma^2 \geq \widehat{z}_4$ and $\sigma^2 \geq \widehat{z}_3$. It is easily verified that

$$\widehat{z}_4 - \widehat{z}_3 = \frac{25\alpha^2}{924} > 0.$$

Hence, (L, L) results if $\sigma^2 \geq \widehat{z}_4$. This proves Proposition 1 (iii). ■

Proposition 1 characterizes the Nash equilibria for various degree of uncertainty. The equilibrium outcomes include (E, L), (L, E), and (L, L). Interestingly, two types of equilibria can coexist. Specifically, (E, L) and (L, E) coexist when $0 \leq \sigma^2 \leq \widehat{z}_3$. In mixed oligopoly under certainty, Pal (1998) demonstrates that simultaneous production by both firms can never be a SPNE. He also shows that both Stackelberg outcomes (public leadership and private leadership) are equilibrium outcomes. Under uncertainty, these results are shown to be robust as long as the degree of uncertainty is moderate (Proposition 1(i)).

As the degree of uncertainty increases, (L, E) becomes a unique equilibrium. It occurs when $\widehat{z}_3 < \sigma^2 < \widehat{z}_4$ (see Proposition 1(i) and 1(ii)).[8] Pal (1998) shows that when the number of private firm is more than one in oligopoly, the public leadership never appears in equilibrium. Along the same line, Matsumura (2003b) uses a two-production period model formulated by Saloner (1987) and shows that only private leadership is robust. Our result is consistent with these findings but through a different mechanism.

As σ^2 increases beyond $\widehat{z}_4$, (L, L) will emerge as the equilibrium outcome (see Proposition 1(iii)). For sufficiently higher degree of uncertainty, information value is enhanced and conse-

[8]Note that $\widehat{z}_4 > \widehat{z}_3 > 0$.

quently, the option value effect begins to dominate the early moving advantage. This provides a temptation for these two firms to choose L at the same time. Hence, (L, L) (i.e., the Cournot competition) results.

The intuition underlying our results is the following. It may be recalled that in Pal's (1998) model of mixed oligopoly, with 100% government ownership of the public firm, the unique equilibrium is Stackelberg where private firm leads and public firm follows. The underlying reason is that by moving first, the private firm is able to expand production and preempt the market to its advantage. Since higher output raises social welfare it is also a preferred outcome for the public firm. Put differently, the private firm wants to produce more to preempt, whereas public firm wants to produce more to raise social welfare. Thus, if private firm were to move in late, it would lower production to raise price and thereby lower welfare. With demand uncertainty, there is an additional benefit to waiting for uncertainty to be resolved, stemming from the well-known concept of option value in finance and investment. Depending on the degree of uncertainty, therefore, multiple equilibria can emerge. In particular, when uncertainty is high, option value effect dominates and both firms prefer to move in late. When uncertainty is absent Pal's (1998) result obtains. For moderate levels of uncertainty firms trade off benefits of moving first against the option value of waiting.

The following two corollaries are the immediate consequences of Proposition 1.

Corollary 2. In a mixed duopoly model with a pure public firm competing against a private domestic firm, (E, E) ceases to exist.

Proof. Recall from Proposition 1 that $\triangle_1 = A_{EE} - A_{LE} = -\frac{\sigma^2}{4} - \frac{13\alpha^2}{1600} < 0$ and $\triangle_2 = B_{EE} - B_{EL} = -\frac{\sigma^2}{6} - \frac{87\alpha^2}{9800} < 0$. This violates the requirements for $(E,\, E)$ to be an equilibrium, thus proving Corollary 2. ■

Corollary 3. For a given σ^2 such that (L, E) and (E, L) coexist, the social welfare is higher under (L, E) than under (E, L).

Proof. Calculate $A_{LE} - A_{EL} = \frac{\sigma^2}{36} + \frac{3\alpha^2}{448} > 0$. Corollary 3 is thus proven. ■

In order to get a feel for the quantitative impact on firms' payoff, Table 2 presents some numerical examples to highlight our findings (assuming $\alpha = 10$). In the absence of any uncertainty (i.e., $\sigma^2 = 0$), both Stackelberg outcomes (private leadership and public leadership) emerge as the Cournot equilibria (indicated by asterisk *). This finding is consistent with the

Table 2: Equilibrium Payoffs: Public vs. Domestic Private Firm

σ^2	(A_{EE}, B_{EE})	(A_{LL}, B_{LL})	(A_{EL}, B_{EL})	(A_{LE}, B_{LE})
0	(32.00, 6.00)	(32.00, 6.00)	$(32.14^*, 6.88^*)$	$(\mathbf{32.81}^*, 6.25^*)$
4	(32.00, 6.00)	(33.28, 6.24)	(33.03, 7.55)	$(\mathbf{33.81}^*, 6.25^*)$
6	(32.00, 6.00)	$(33.92^*, 6.36^*)$	(33.47, 7.88)	(**34.31**, 6.25)

Notes: * denotes equilibrium outcome. Boldface indicates highest welfare.

results obtained by Pal (1998), Jacques (2004), and Lu (2007). The numerical examples also indicate that the social welfare is higher (lower) with the public firm as a Stackelberg follower (leader), a result that is well-known in the literature (see Corollary 3).

In the presence of uncertainty, the equilibrium pattern begins to change. For a moderate level of uncertainty (e.g., $\sigma^2 = 4$), the model predicts a unique Nash equilibrium, (L, E). That is, the "private leadership" is optimal. This equilibrium outcome is efficient since the social welfare is maximized.

However, further increase in the degree of uncertainty (e.g., $\sigma^2 = 6$) leads both firms to move late, resulting in a Cournot equilibrium (L, L). This is in contrast to Pal's (1998) result that the simultaneous-move outcome does not constitute an equilibrium in mixed duopoly. Table 2 also indicates that the (L, L) equilibrium is associated with a lower social welfare. That is, the private leadership is more efficient, but it fails to be an equilibrium.

4 Extensions

In this section, three extensions are considered. First, we modify our basic model by allowing a foreign private firm to compete against the public firm. This extension is not trivial since it produces non-marginal impacts on the equilibrium outcomes. Second, we extend our basic model to a sequential setting where the public and private firms are making their choices in sequence. It turns out that the equilibrium outcome is either (L, E) or (L, L), depending on the size of uncertainty. Unlike our results in the previous section, the sequential equilibrium is unique. Finally, we consider a more general setting in which the public firm is partially privatized. As shown below, the degree of privatization (g) and market volatility (σ^2) jointly determine the equilibrium outcomes. All four different equilibrium outcomes, (E, E), (E, L),

(L,E), and (L,L) are possible.

4.1 Public Firm vs Foreign Private Firm

In this subsection, we examine the case where the public firm is competing against a foreign rival.[9] Since foreign firms profit is excluded from social surplus, the social welfare function can be rewritten as

$$\begin{aligned}\widetilde{W}(q_1;\theta) &= \int_0^{q_1} p(x)dx - pq_1 + \Pi_1(q_1,\widetilde{q}_2),\\ &= \int_0^{q_1} p(x)dx - C_1(q_1),\end{aligned}$$

where $\widetilde{q}_2$ is the output produced by the foreign rival firm and $\widetilde{W}$ is the associated welfare. The objective functions of the public and foreign firms are

$$\begin{aligned}U_1^f &= \widetilde{W}(q_1,\widetilde{q}_2),\\ U_2^f &= \Pi_2(q_1,\widetilde{q}_2),\end{aligned}$$

respectively. The payoff matrix can be obtained by replacing A_{ij} and B_{ij} in Table 1 by $\widetilde{A}_{ij}$ and $\widetilde{B}_{ij}$ $(i,j=E,L)$, where

$$\begin{aligned}\widetilde{A}_{EE} &= \frac{19\alpha^2}{72},\ \widetilde{B}_{EE}=\frac{\alpha^2}{24},\\ \widetilde{A}_{EL} &= \frac{\sigma^2}{18}+\frac{9\alpha^2}{34},\ \widetilde{B}_{EL}=\frac{\sigma^2}{6}+\frac{27\alpha^2}{578},\\ \widetilde{A}_{LE} &= \frac{\sigma^2}{4}+\frac{19\alpha^2}{72},\ \widetilde{B}_{LE}=\frac{\alpha^2}{24},\\ \widetilde{A}_{LL} &= \frac{19\sigma^2}{72}+\frac{19\alpha^2}{72},\ \widetilde{B}_{LL}=\frac{\sigma^2}{24}+\frac{\alpha^2}{24}.\end{aligned}$$

As before, the equilibrium outcome of this game depends on the degree of uncertainty, σ^2. Let $\widetilde{z}_3$ solves $\widetilde{\triangle}_3=\widetilde{A}_{EL}-\widetilde{A}_{LL}=0$, where $\widetilde{z}_3=\frac{\alpha^2}{255}>0$. The results are summarized in

Proposition 2. *In a simultaneous-move game in which a public firm is competing against a foreign private firm, the SPNE entails (i)* (L,E) *if* $\sigma^2=0$, *(ii)* (E,L) *if* $0\le\sigma^2\le\widetilde{z}_3$, *and (iii)* (L,L) *if* $\sigma^2\ge\widetilde{z}_3$.

Proof. First, we show that (E,E) cannot be an equilibrium outcome. To see this, calculate $\widetilde{\triangle}_1=\widetilde{A}_{EE}-\widetilde{A}_{LE}=-\frac{\sigma^2}{4}\le 0$ and $\widetilde{\triangle}_2=\widetilde{B}_{EE}-\widetilde{B}_{EL}=-\frac{\sigma^2}{6}-\frac{35\alpha^2}{6936}<0$. The conditions for

[9] For simplicity, we assume that the foreign firm does not pay tariff so that the entry to the domestic market is free. The analysis of tariff in the context of mixed oligopoly can be found in Pal and White (1998) and Chang (2004).

(E,E) to be an equilibrium outcome, $\widetilde{\triangle}_1 \geq 0$ and $\widetilde{\triangle}_2 \geq 0$, are clearly violated. Thus, (E,E) is not sustainable.

Next, we show that (L,E) is an equilibrium only when $\sigma^2 = 0$. To see this, recall that $-\widetilde{\triangle}_1 = \widetilde{A}_{LE} - \widetilde{A}_{EE} = \frac{\sigma^2}{4} \geq 0$ and calculate $\widetilde{\triangle}_4 = \widetilde{B}_{LE} - \widetilde{B}_{LL} = -\frac{\sigma^2}{24} \leq 0$. For (L,E) to be an equilibrium outcome, it requires that $-\widetilde{\triangle}_1 \geq 0$ and $\widetilde{\triangle}_4 \geq 0$. Clearly, these two conditions hold with equality when $\sigma^2 = 0$. Hence, (L,E) can (weakly) emerge as an equilibrium when $\sigma^2 = 0$. This proves Proposition 2 (i).

For (E,L) to be a Nash equilibrium, it must be such that $\widetilde{\triangle}_3 = \widetilde{A}_{EL} - \widetilde{A}_{LL} = \frac{-5\sigma^2}{24} + \frac{\alpha^2}{1224} \geq 0$ and $-\widetilde{\triangle}_2 = \widetilde{B}_{EL} - \widetilde{B}_{EE} = \frac{\sigma^2}{6} + \frac{35\alpha^2}{6936} \geq 0$ (which holds with inequality). The former implies that firm 1 weakly prefers E over L, while the latter says that firm 2 would choose L over E. A deviation from (E,L) can only reduce the payoffs of the players. Let $\widetilde{z}_3$ solves $\widetilde{\triangle}_3 = 0$, where $\widetilde{z}_3 = \frac{\alpha^2}{255} > 0$. Note that $\partial\widetilde{\triangle}_3/\partial\sigma^2 = -5/24 < 0$. Therefore, $\widetilde{\triangle}_3 \lesseqgtr 0$ if $\sigma^2 \gtreqless \widetilde{z}_3$. Given that $-\widetilde{\triangle}_2 > 0$, it is evident that (E,L) is a Nash equilibrium if $0 \leq \sigma^2 \leq \widetilde{z}_3$. The condition ensures that both firms have no incentive to deviate from (E,L), thus proving Proposition 2 (ii).

Finally, for (L,L) to be an equilibrium outcome, it requires $\widetilde{\triangle}_4 = \widetilde{B}_{LE} - \widetilde{B}_{LL} = -\frac{\sigma^2}{24} \leq 0$ (which always holds) and $\widetilde{\triangle}_3 = \widetilde{A}_{EL} - \widetilde{A}_{LL} = \frac{-5\sigma^2}{24} + \frac{\alpha^2}{1224} \leq 0$. Recall that $\widetilde{\triangle}_3 \leq 0$ when $\sigma^2 \geq \widetilde{z}_3 > 0$. Hence, (L,L) results if $\sigma^2 \geq \widetilde{z}_3$. This proves Proposition 2 (iii). ■

Proposition 2 implies that there are three different equilibrium outcomes, (L,E), (E,L) and (L,L), depending on the degree of uncertainty. Under certainty (i.e., $\sigma^2 = 0$), both (E,L) and (L,E) can coexist as the equilibrium outcomes (see Proposition 2(i) and 2(ii)). When uncertainty is moderate, the public firm behaves as a Stackelberg leader, while the foreign firm acts as a Stackelberg follower (see Proposition 2(ii)). Matsumura (2003a) shows that the public leadership outcome is optimal. Here, we demonstrate that his result hold even when a mild demand uncertainty is introduced. Intuitively, allowing the foreign firm to act as a Stackelberg leader is equivalent to giving the foreign firm a right to preempt. Since the profit earned by the foreign firm is excluded from the welfare calculation, higher profit of the foreign firm does not translate into higher social welfare. The outcome is thus sub-optimal. To prevent this, the public firm would prefer to lead so that higher social welfare is guaranteed. The foreign firm is willing to settle for the Stackelberg follower because of the gain in option value.

When uncertainty begins to evolve, both firms end up producing outputs after the resolution of uncertainty. This is again driven by higher option values, as we explain earlier.

Table 3 presents some numerical examples to highlight our findings (again, letting $\alpha = 10$).

Table 3: Equilibrium Payoffs: Public vs. Foreign Private Firm

σ^2	$(\widetilde{A}_{EE}, \widetilde{B}_{EE})$	$(\widetilde{A}_{LL}, \widetilde{B}_{LL})$	$(\widetilde{A}_{EL}, \widetilde{B}_{EL})$	$(\widetilde{A}_{LE}, \widetilde{B}_{LE})$
0	(26.38, 4.16)	(26.38, 4.16)	$(\mathbf{26.47}^*, 4.67^*)$	$(26.38^*, 4.16^*)$
0.3	(26.38, 4.16)	(26.46, 4.17)	$(\mathbf{26.48}^*, 4.72^*)$	(26.46, 4.16)
2	(26.38, 4.16)	$(\mathbf{26.91}^*, 4.25^*)$	(26.58, 5.00)	(26.88, 4.16)

Notes: * denotes equilibrium outcome. Boldface indicates highest welfare.

In the absence of any uncertainty (i.e., $\sigma^2 = 0$), the equilibrium outcome can be either (E, L) or (L, E). As uncertainty beings to increase (i.e., $\sigma^2 = 0.3$), the public leadership (E, L) becomes the only equilibrium outcome. This equilibrium is socially efficient. Further increase in σ^2 to 2 will lead both firms to move late, resulting in (L, L). These results are consistent with Proposition 2.

Remark: Under certainty (i.e., $\sigma^2 = 0$), L is a weakly dominated strategy for firm 1, while E is a weakly dominated strategy for firm 2. Following Luce and Raiffa (1957), one may legitimately argue that players would never choose the weakly dominated strategy, (L, E). With this additional argument, (L, E) is ruled out and Proposition 2(i) becomes redundant. However, Proposition 2(i) is derived by following the definition of Nash equilibrium given in the text.

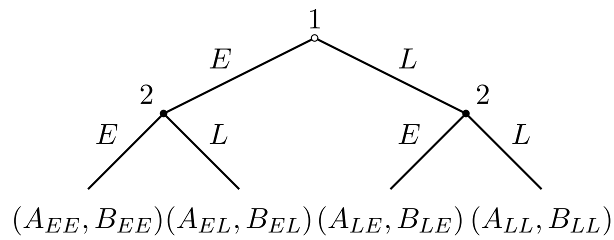
4.2 Equilibrium in a Sequential-Move Leadership Game

The game we consider so far assumes that firms choose their leadership strategies simultaneously and this yields multiple equilibria. In this subsection, we consider the sequential-move leadership game in which the decision to lead or follow is taken sequentially. We consider the game between the public and the domestic private firm.[10] It turns out that the Nash equilibrium becomes unique. To highlight our assertion, assume that the public firm is the first mover who chooses between E and L. Firm 2 (private firm) observes the strategy taken by firm 1 and then responds to firm 1's action accordingly. The game tree is given in Figure 1.

The game is solved, as usual, by backward induction. If firm 1 chooses E, firm 2 will respond by choosing either E or L, which yields (E, E) or (E, L). In this case, firm 2 would choose E

[10] A similar exercise can be done with a foreign private firm competing against the public firm. It is omitted to avoid repetition.

Figure 1: The sequential game



(L) if $\triangle_2 = B_{EE} - B_{EL} > (<) 0$. As shown earlier, $\triangle_2 < 0$, implying that firm 2 would choose L in this case. Hence, (E, L) results. Alternatively, if firm 1 chooses L, two possible outcomes are (L, E) or (L, L). In this case, firm 2 will choose E (L) if $\triangle_4 = B_{LE} - B_{LL} > (<) 0$. Note that $\triangle_4 \gtreqless 0$ if $\sigma^2 \lesseqgtr \widehat{z}_4$, where $\widehat{z}_4 = \alpha^2/24 > 0$. Evidently, (L, E) $((L, L))$ results if $\sigma^2 < \widehat{z}_4$ $(\sigma^2 > \widehat{z}_4)$. In short, σ^2 in relation to $\widehat{z}_4$ determines firm 1's optimal choice.

In anticipation of what firm 2 may do later in the game, firm 1 as the first mover will determine its strategy that gives him the highest payoff. Two cases are considered:

Case 1: $0 \leq \sigma^2 < \widehat{z}_4 = \alpha^2/24$. As discussed above, firm 2 prefers L to E if firm 1 chooses E. Given that $0 \leq \sigma^2 < \widehat{z}_4$, firm 2 prefers E to L if firm 1 chooses L. Knowing this, firm 1 would choose E (L) if $A_{EL} - A_{LE} > (<) 0$. It is easy to verify that $A_{EL} - A_{LE} = -\sigma^2/36 - 3\alpha^2/448 < 0$, suggesting that firm 1 will unambiguously chooses L over E. This yields (L, E).

Case 2: $\sigma^2 > \widehat{z}_4 = \alpha^2/24$. In this case, firm 2 would choose L over E if firm 1 choose L. Given this, firm 1's would therefore choose E (L) if $A_{EL} - A_{LL} > 0$ $(A_{EL} - A_{LL} < 0)$. Note that $A_{EL} - A_{LL} = -22\sigma^2/225 + \alpha^2/700 \lesseqgtr 0$ if $\sigma^2 \gtreqless 9\alpha^2/616$. Given that $\sigma^2 > \alpha^2/24 > 9\alpha^2/616$, we can conclude that $A_{EL} - A_{LL} < 0$ must hold. Therefore, (L, L) emerges as an equilibrium outcome.

This can be summarized in[11]

Proposition 3. *For a sequential-move leadership strategy game in which the public firm is the first mover, the unique equilibrium outcome is* (L, E) *if* $0 \leq \sigma^2 \leq \alpha^2/24$ *and* (L, L) *if* $\sigma^2 > \alpha^2/24$.

Proposition 3 states that when the decision to lead or follow is made sequentially, the public firm would choose L and the private firm would respond by choosing E when σ^2 is sufficiently low. In this case, the private leadership appears to be equilibrium. This result is robust

[11]If firm 2 is the first mover, a similar analysis can be conducted.

whenever $\sigma^2 \in [0, \alpha^2/24]$. By moving late, the public firm can control its output to ensure that the welfare is maximized. But when the market is sufficiently volatile such that $\sigma^2 > \alpha^2/24$ holds, strong option values will persuade the private firm to act as the Stackelberg follower as well. Hence, (L, L) will eventually emerge as the equilibrium outcome.

4.3 Partially-Privatized Public Firm vs Domestic Private Firm

In this subsection, we consider a game between a partly nationalized firm and a domestic private firm.[12] Let firms 1 and 2 be the partly nationalized firm and the privately owned domestic firm, respectively. Let g be the proportion of government's control of firm 1, where $g \in [0, 1]$. Thus, when $g = 0$, firm 1 behaves like a private firm and its objective is to maximize profits. However, when $g = 1$, firm 1 is fully nationalized and its behavior is therefore dictated by social welfare maximization as analyzed in the basic model (see Section 3). The payoff functions for firms 1 and 2, respectively, are given by

$$\begin{aligned}
\overline{U}_1 &= gW + (1-g)\Pi_1(q_1, q_2), \\
\overline{U}_2 &= \Pi_2(q_1, q_2),
\end{aligned}$$

where Π_i $(i = 1, 2)$ and W are given by (1) and (2). That is, firm 1 maximizes a weighted sum of social welfare and its own profit, while firm 2 maximizes its own profit. As before, there are four cases to consider: (L, L), (E, E), (E, L), and (L, E). A straightforward calculation yields the following expected payoff functions:[13]

$$\begin{aligned}
\overline{A}_{LL} &= \frac{2\sigma^2(g-3)(g^2-2g-1)}{(3g-8)^2} + \frac{2\alpha^2(g-3)(g^2-2g-1)}{(3g-8)^2}, \\
\overline{B}_{LL} &= \frac{3\sigma^2(g-2)^2}{2(3g-8)^2} + \frac{3\alpha^2(g-2)^2}{2(3g-8)^2}, \\
\overline{A}_{EE} &= \frac{2\alpha^2(g-3)(g^2-2g-1)}{(3g-8)^2}, \\
\overline{B}_{EE} &= \frac{3\alpha^2(g-2)^2}{2(3g-8)^2},
\end{aligned}$$

[12] Again, the game between a rival foreign firm and the public firm is omitted to avoid repetition. Recently, Chao and Yu (2006) analyzed a mixed oligopoly game with one public and one or more foreign firms. They obtain that foreign competition lowers the optimal tariff rate while partial privatization raises it. Matsumura and Kanda (2005) show that privatization of the public firm is not optimal in a free entry market.

[13] The derivation of these results can be obtained from the authors upon request.

$$
\begin{aligned}
\overline{A}_{EL} &= \frac{2g\sigma^2}{9} + \frac{\alpha^2(3g^2-8g-4)}{14(g-3)}, \\
\overline{B}_{EL} &= \frac{\sigma^2}{6} + \frac{3\alpha^2(2g-5)^2}{98(g-3)^2}, \\
\overline{A}_{LE} &= -\frac{\sigma^2}{2(g-3)} + \frac{(2g-5)\alpha^2(2g^3-10g^2+10g+5)}{2(3g-7)^2(g-3)}, \\
\overline{B}_{LE} &= \frac{\alpha^2(g-2)^2}{2(3g-7)(g-3)},
\end{aligned}
$$

where $\overline{A}_{ij}$ and $\overline{B}_{ij}$ denote the expected payoffs for firm 1 and 2, respectively. Whether the equilibrium outcome is Cournot or Stackelberg can be shown to depend on the government's share of the public firm, characterized by $g \in (0,1)$ and the degree of uncertainty, characterized by σ^2. Let $\overline{z}_1$ ($\overline{z}_3$) solves $A_{EE} - A_{LE} = 0$ ($A_{EL} - A_{LL} = 0$), where $\overline{z}_1 = -\frac{\alpha^2(g-2)(6g^4-54g^3+172g^2-219g+82)}{(3g-8)^2(3g-7)^2}$ ($\overline{z}_3 = -\frac{9\alpha^2(g^2-4g+2)^2}{28(3g^2-19g+27)(g-3)}$). Likewise, let $\overline{z}_2$ ($\overline{z}_4$) solves $B_{EE} - B_{EL} = 0$ ($B_{LE} - B_{LL} = 0$) , where $\overline{z}_2 = \frac{9\alpha^2(13g^2-66g+82)(g^2-4g+2)}{49(3g-8)^2(g-3)^2}$ ($\overline{z}_4 = \frac{\alpha^2}{3(g-3)(3g-7)}$). The following proposition summarizes the Nash equilibria:

Proposition 4. *Consider a mixed duopoly with a partially privatized public firm and a domestic private firm facing linear demand and quadratic cost functions. The Nash equilibrium is (i) (E, E) if $\sigma^2 \leq \min[\overline{z}_1,\ \overline{z}_2]$;*[14] *(ii) (E, L) if $\overline{z}_2 \leq \sigma^2 \leq \overline{z}_3$; (iii) (L, E) if $\overline{z}_1 \leq \sigma \leq \overline{z}_4$; and (iv) (L, L) if $\sigma^2 \geq \overline{z}_4$.*

Proof. Let $\overline{\triangle}_1 = A_{EE} - A_{LE}$ and $\overline{\triangle}_2 = B_{EE} - B_{EL}$. By definition, (E, E) results if $\overline{\triangle}_1 \geq 0$ and $\overline{\triangle}_2 \geq 0$. Let $\overline{z}_1$ ($\overline{z}_2$) solves $\overline{\triangle}_1 = 0$ ($\overline{\triangle}_2 = 0$), where

$$
\overline{z}_1 = -\frac{\alpha^2(g-2)(6g^4-54g^3+172g^2-219g+82)}{(3g-8)^2(3g-7)^2} \gtreqless 0 \text{ if } g \lesseqgtr 0.6257; \tag{13}
$$

$$
\overline{z}_2 = \frac{9\alpha^2(13g^2-66g+82)(g^2-4g+2)}{49(3g-8)^2(g-3)^2} \gtreqless 0 \text{ if } g \lesseqgtr 2-\sqrt{2}. \tag{14}
$$

Since $\partial A_{EE}/\partial\sigma^2 = 0$ and $\partial A_{LE}/\partial\sigma^2 > 0$, we have $\partial\overline{\triangle}_1/\partial\sigma^2 < 0$. Thus, if $\sigma^2 \lesseqgtr \overline{z}_1$, $\overline{\triangle}_1 \gtreqless 0$. Similarly, $\partial B_{EE}/\partial\sigma^2 = 0$ and $\partial B_{EL}/\partial\sigma^2 > 0$, we have $\partial\overline{\triangle}_2/\partial\sigma^2 < 0$. Thus, if $\sigma^2 \lesseqgtr \overline{z}_2$, $\overline{\triangle}_2 \gtreqless 0$. We can therefore conclude that (E, E) results if $\sigma^2 \leq \min[\overline{z}_1, \overline{z}_2]$.[15] This proves Proposition 4 (i).

[14] The relative value between $\overline{z}_1$ and $\overline{z}_2$ depends on g. Both functions are decreasing in $g \in [0,1]$. Note that $\overline{z}_1 = \overline{z}_2$ when $g = 0$ and $g \cong 0.38433$. Further note that $\overline{z}_1 >$ $(<)$ $\overline{z}_2$ if $g >$ $(<)$ 0.38433. Thus, $\min[\overline{z}_1,\ \overline{z}_2] = \overline{z}_2$ if $g > 0.38433$ and $\min[\overline{z}_1,\ \overline{z}_2] = \overline{z}_1$ if $0 < g < 0.38433$.

[15] Note that $\overline{z}_1$ and $\overline{z}_2$ can be of either sign. The condition for (E, E) to be an equilibrium may not hold for all $g \in [0,1]$. For example, when $g = 1$, we have $\overline{z}_1 < 0$ and $\overline{z}_2 < 0$. We cannot find any $\sigma^2 \geq 0$ such that $\sigma^2 \leq \min[\overline{z}_1, \overline{z}_2]$ holds. In this case, Proposition 4(i) becomes redundent and (E, E) can never appear as an equilibrium. Conversely, when $g = 0$, then $\overline{z}_1 > 0$ and $\overline{z}_2 > 0$. In this case, (E, E) can be an equilibium since there exists a $\sigma^2 \geq 0$ such that $\sigma^2 \leq \min[\overline{z}_1, \overline{z}_2]$ holds. It is clear from (13) and (14) that for $\sigma^2 \leq \min[\overline{z}_1, \overline{z}_2]$ to hold, $g < \min[0.6257,\ 2-\sqrt{2}] = 2-\sqrt{2}$.

For (E, L) to be a Nash equilibrium, it must be that $\overline{\triangle}_3 = A_{EL} - A_{LL} \geq 0$ and $-\overline{\triangle}_2 = B_{EL} - B_{EE} \geq 0$. The former implies that firm 1 prefers E over L, while the latter implies that firm 2 chooses L over E. That is, any deviation from (E, L) will make the players worse off (or at least no better off). Substituting A_{ij} and B_{ij} given in the text, we obtain

$$\begin{aligned} \overline{\triangle}_3 &= -\frac{2\sigma^2(3g^2 - 19g + 27)}{9(3g-8)^2} - \frac{\alpha^2(g^2 - 4g + 2)^2}{14(g-3)(3g-8)^2}, \\ -\overline{\triangle}_2 &= \frac{\sigma^2}{6} + \frac{3\alpha^2(13g^2 - 66g + 82)(g^2 - 4g + 2)}{98(g-3)^2(3g-8)^2}. \end{aligned}$$

Recall that $\overline{z}_2$ (given above) solves $\overline{\triangle}_2 = 0$. Let $\overline{z}_3$ solves $\overline{\triangle}_3 = 0$, where

$$\overline{z}_3 = -\frac{9\alpha^2(g^2 - 4g + 2)^2}{28(3g^2 - 19g + 27)(3 - g)} > 0.$$

Note that $\partial\overline{\triangle}_3/\partial\sigma^2 < 0$ and $\partial(-\overline{\triangle}_2)/\partial\sigma^2 > 0$. Therefore, for $\sigma^2 \gtreqless \overline{z}_3$, $\overline{\triangle}_3 \lesseqgtr 0$; for $\sigma^2 \gtreqless \overline{z}_2$, $-\overline{\triangle}_2 \gtreqless 0$. It is evident that (E, L) is a Nash equilibrium if $\max(\overline{z}_2, 0) \leq \sigma^2 \leq \overline{z}_3$; that is, when this condition holds, both firms will have no incentive to deviate from (E, L). This proves Proposition 4 (ii).

To obtain the conditions under which (L, E) results, recall $-\overline{\triangle}_1 = A_{LE} - A_{EE}$ and define $\overline{\triangle}_4 = B_{LE} - B_{LL}$. Firm 1 has no intention to deviate from L if $-\overline{\triangle}_1 \geq 0$. Likewise, firm 2 would prefer to stick to E if $\overline{\triangle}_4 \geq 0$. As shown above, $\overline{z}_1$ solves $\overline{\triangle}_1 = 0$. The cutoff value of variance for firm 2 to act as a leader can be obtained by solving $\overline{\triangle}_4 = 0$ for σ^2, which yields

$$\overline{z}_4 = \frac{\alpha^2}{3(g-3)(3g-7)} > 0.$$

Note that A_{EE} is independent of σ^2, while A_{LE} is an increasing function of σ^2. Hence, $\partial(-\overline{\triangle}_1)/\partial\sigma^2 > 0$. For $\sigma^2 \geq \overline{z}_1$, we have $-\overline{\triangle}_1 \geq 0$, meaning that firm 1 will continue to choose L. Moreover, B_{LE} is independent of σ^2 and B_{LL} is an increasing function of σ^2. Thus, $\partial\overline{\triangle}_4/\partial\sigma^2 < 0$. For $\sigma^2 \leq \overline{z}_4$, we have $\overline{\triangle}_4 \geq 0$, implying that firm 2 will stick with E. In sum, (L, E) is a Nash equilibrium if $\max(\overline{z}_1, 0) \leq \sigma^2 \leq \overline{z}_4$. This proves Proposition 4 (iii).

Finally, for (L, L) to be an equilibrium outcome, it requires $\overline{\triangle}_4 = B_{LE} - B_{LL} \leq 0$ and $\overline{\triangle}_3 = A_{EL} - A_{LL} \leq 0$. Note that $\overline{\triangle}_4 \leq 0$ and $\overline{\triangle}_3 \leq 0$ when $\sigma^2 \geq \overline{z}_4$ and $\sigma^2 \geq \overline{z}_3$. It is easily verified that

$$\overline{z}_4 - \overline{z}_3 = \frac{\alpha^2 g(3g-4)(3g-11)(3g-8)^2}{84(3g-7)(g-3)(3g^2 - 19g + 27)} \geq 0. \tag{15}$$

Hence, (L, L) results if $\sigma^2 \geq \max(\overline{z}_3, \overline{z}_4) = \overline{z}_4$. This proves Proposition 4 (iv). ■

Proposition 4 shows that the equilibrium pattern varies with σ^2 and g. In a standard duopoly game with private firms facing no uncertainty (i.e., $g = 0$ and $\sigma^2 = 0$), it is known that the early mover (or the leader) has an advantage over the late mover (or the follower). Therefore, every firm would want to be a leader, hoping that the rival firm becomes a follower. Therefore, it is inevitable that both are caught by a prison's dilemma, meaning that firms are engaged in a "Stackelberg warfare," each trying not to become a Stackelberg follower. This yields (E, E). In our mixed duopoly model under uncertainty, we show that apart from (E, E), the Stackelberg leadership and the Cournot competition can also emerge as the equilibrium outcomes. Intuitively, while the advantage of being a early mover remains in effect, there are benefits to the late mover as well. We call this the *option value* effect, which increases with the degree of uncertainty and runs counter to the usual first mover advantage. The increased option value effect may reach a point such that one of the two firms may prefer L to E. This results in either (E, L) or (L, E).[16] As the degree of uncertainty continues to increase, information value is enhanced and consequently, the option value effect begins to dominate the early mover advantage. This can lead both firms to choose L, resulting in (L, L). In short, the equilibrium outcomes range from the Cournot equilibrium ((E, E) or (L, L)) to the Stackelberg leadership equilibrium ((E, L) or (L, E)). It is worth noting that for all four these equilibrium patterns to emerge, it requires that $g < \min[0.6257,\ 2 - \sqrt{2}] = 2 - \sqrt{2}$. But if $g > 2 - \sqrt{2}$, (E, E) will never emerge as an equilibrium outcome since $\sigma^2 \leq \min[\overline{z}_1,\ \overline{z}_2]$ does not hold for $\sigma^2 \geq 0$ (see Proposition 1(i)). In this case, the equilibrium patterns outlined in Propositions 1 and 4 are therefore identical. However, Proposition 4 is more general than Proposition 1 (the basic case with $g = 1$) since the results hold for $1 \geq g > 2 - \sqrt{2}$.

5 Conclusions

In this paper we have introduced demand uncertainty in a mixed oligopoly model and revisited the nature of endogenous equilibria in such a model. In the context of an observable delay game framework, the standard non-stochastic models generally suggest that the SPNE has a Stackelberg structure. In particular, Pal (1998) and Matsumura (2003a) demonstrated respectively that a pure public firm would follow and lead when in competition with a domestic and a foreign private firm, respectively. These outcomes are also the socially efficient ones. By contrast, we show that with demand uncertainty, equilibrium mixed oligopoly structure is not unique and includes simultaneous production or Cournot structure which is not socially efficient.

[16]Note that, (E, L) and (L, E) coexist if $\max[\overline{z}_1,\ \overline{z}_2] < \sigma^2 < \min[\overline{z}_3,\ \overline{z}_4] = \overline{z}_3$. However, (L, E) becomes the only equilibrium outcome if $\overline{z}_3 < \sigma^2 < \overline{z}_4$.

The assumption that drives this result is that the leader must irreversibly commit to output before the resolution of uncertainty. This implies, therefore, that moving late carries positive option value. When uncertainty is high, option value effect can induce firms to wait for the uncertainty to be revealed and move in late. We also examine the sensitivity of the equilibrium mixed oligopoly structure to levels of uncertainty and state ownership of the public firm. The standard non-stochastic results are then shown to be special cases of our general model where uncertainty parameter is zero and the public firm is fully nationalized.

This analysis can be extended in many important ways. In this paper, for tractability we confine our analysis to linear demand and quadratic demand functions. It would be interesting to check the robustness of our model under more general demand and cost condition. Furthermore, under uncertainty, one might assume that private firms are risk-averse, such risk-aversion can play crucial role when market demand is uncertain. Intuitively, the more risk-averse the firms are, the less likely it is for the firms to move early. This interaction between the uncertainty and risk parameter can generate some new and interesting market structures. Finally, the present paper considers one production period. Another possible extension could be to introduce two production periods model (e.g., Saloner (1987)) in the context of mixed oligopoly. Matsumura (2003b) analyzed a two production periods in mixed duopoly under no uncertainty. He found many equilibria including the Cournot equilibrium and the Stackelberg equilibrium with the public firm acting as the follower. It is worthwhile to check the robustness of his results under uncertainty. Of course, this implies that we no longer can utilize the observable delay game.

Finally, some comments on the empirical relevance of our model is in order. Clear example of mixed oligopoly, where equilibrium leadership structure is shaped by the stochastic environment in which the firms operate, is difficult to identify. Indeed, the literature on the equilibrium structure of mixed oligopolies rarely associates particular equilibria with distinct examples. This is partly due to the fact that while timing of entry is observable, strategic leadership structure is often not. Firm level survey coupled with equilibrium output and price information may reveal the underlying pattern of leadership in mixed oligopolies. Empirical literature has begun to address this issue. The importance of our paper is that if this research reveals an underlying Cournot pattern in output setting behavior in a mixed oligopoly then, contrary to the traditional non-stochastic models, this would be consistent in the context of our model if uncertainty is high. Sectors like transportation and energy, where mixed oligopoly structure is common, are the potential examples of market with significant demand uncertainty.

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