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Abstract This paper analyses the dynamics of a Cournot duopoly under cross-ownership participation when players have heterogeneous, i.e. bounded rational and naïve, expectations. We find that when the shareholder that owns firm 1 also holds a percentage of firm 2, the parametric stability region of the unique Cournot-Nash equilibrium is larger than when every firm is owned by a unique shareholder, and an increase in the fraction of shares that the shareholder that owns firm 1 has in firm 2 tends to stabilise the market equilibrium. Moreover, when products are (horizontally) differentiated, a rise in the fraction of shares of firm 2 held by the shareholder that owns firm 1 acts as an economic (de)stabiliser when products of variety 1 and 2 are (complements) substitutes between each other. The policy implication is that, despite on the one hand, cross-ownership acts as an anti-competitive device that indeed tends to reduce social welfare with the corresponding anti-trust consequences, on the other hand, it acts as an economic stabiliser (except when products are complements).

Keywords Bifurcation; Cournot; Cross-ownership; Duopoly

JEL Classification C62; D43; L13; L4

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1. Introduction

The effects of cross-ownership participation on equilibrium outcomes in a Cournot duopoly have been investigated by an established literature. For instance, Macho-Stadler and Verdier (1991) study managerial incentives in a Cournot duopoly where owners and managers are separate identities, while Bárcena-Ruiz and Oilazola (2007) and Bárcena-Ruiz and Campo (2011) concentrate, respectively, on the incentives to acquire cost-saving production technologies and the issue of cooperative and non-cooperative environmental taxes with two firms located in different countries under , present cross-ownership. Moreover, Gilo et al. (2006) focus on the incentives by firms to engage in tacit collusion, Pal (2010) studies the level of privatization in a mixed duopoly with differentiated products, while Fanti (2011) analyses the effects on social welfare of cross-ownership. This literature converges around the fact that cross-ownership in a standard Cournot duopoly reduces competition and increases prices.[1]

Despite the increasing interest around the effects of cross-ownership in static duopolies, less attention has been paid to role played by cross-ownership in dynamic nonlinear duopolies (see Bischi et al., 2010). The present paper, therefore, aims to fill this gap and refers to a burgeoning literature where firms are assumed to have heterogeneous expectation formation mechanisms (see, e.g., Leonard and Nishimura, 1999; Agiza et al., 2002; Agiza and Elsadany, 2003, 2004; Zhang et al., 2007; Tramontana, 2010) with myopic rather than rational expectations. According to many scholars (see, e.g., Dixit, 1986), such a dynamical framework not only may well represent the context of partial "bounded" rationality in which oligopolistic firms are involved, but it also allows for novel investigations on the dynamic roles played by cross-shareholdings in an out-of-equilibrium context, which, of course, would be prevented by the assumption of perfect foresight.

The main results of the present paper are the following: an equity participation of one firm in another one (i.e. cross-participation at ownership level) tends to stabilise product market equilibrium. This holds true when products are homogenous as well as products when they are substitutes. However, when products are complements cross-ownership tends to destabilise the equilibrium. While from the static theory it has been established that (passive) cross-ownership may well facilitate tacit collusion and that competition is substantially reduced, our results also provide stability arguments for the judgement of partial cross-ownership practices, thus enlarging the spectrum of useful anti-trust policies.

The rest of the paper is organised as follows. Section 2 sets up the Cournot duopoly under cross-ownership and horizontal product differentiation. Section 3 studies the local stability properties of the unique Cournot-Nash equilibrium. Section 4 concludes.

2. A Cournot duopoly with cross-ownership

We consider a single industry that consists of two firms, namely firm 1 and firm 2 , each of which produces products of different variety, that is there exists horizontal product differentiation (see Hotelling, 1929; Chamberlin, 1933 for the notion of

[1] Many empirical examples of the role played by the cross-ownership may be found especially in automobile, airline, telephone, energy and banking industries (see, for instance, Alley (1997), Airline Business (1998), Parker and Roller (1997), Amundsen and Bergman (2002) and Trivieri (2007), respectively).

differentiated products). There are two shareholders, A and B. Firm 1 is completely owned by shareholder A, which also owns a share of participation in firm 2. Therefore firm 2 is jointly owned by both shareholders, with shareholder B having the majority of shares and hence the control of such a firm. We let h ($0<h<1/2$) be the fraction of shares that shareholder A has in firm 2. Shareholders are assumed to maximise their total profits. This implies that the objective functions of shareholders A and B are respectively defined by:

$$\Pi_A = \Pi_1 + h\Pi_2, \tag{1.1}$$

$$\Pi_B = (1-h)\Pi_2, \tag{1.2}$$

where profits of the ith firm can be written as:

$$\Pi_i = (p_i - w)q_i, \quad i = \{1,2\}, \tag{2}$$

and $0<w<1$, which can be interpreted as the wage per unit of labour, is assumed to capture all short-run marginal cost in both firms.

The (normalised) market inverse demand functions are linear and given by the following equations (see, e.g., Singh and Vives, 1994; Fanti and Gori, 2011 for details):

$$p_1 = 1 - q_1 - dq_2, \tag{3.1}$$

$$p_2 = 1 - q_2 - dq_1, \tag{3.2}$$

where d captures the relative degree of horizontal product differentiation. Indeed, when $-1<d<0$ ($0<d<1$) products of variety 1 and 2 are complements (substitutes). When $d \to -1$ ($d \to 1$) products tend to become perfect complements (substitutes). The case $d=0$ the case under which each single firm acts as if it were a monopolist in the market.

Shareholders maximise their objective functions. Therefore, the reaction or best reply functions of firms 1 and 2 are computed as the unique solution of the maximisation of Eqs. (1.1) and (1.2) for q_1 and q_2, respectively, and they are given by:

$$\frac{\partial \Pi_A(q_1,q_2)}{\partial q_1} = 1 - w - 2q_1 - d\,q_2(1+h) = 0 \Leftrightarrow q_1(q_2) = \frac{1}{2}[1 - w - d\,q_2(1+h)], \tag{4.1}$$

$$\frac{\partial \Pi_B(q_1,q_2)}{\partial q_2} = -(1-h)q_2 + (1-h)(1 - w - q_2 - d\,q_1) = 0 \Leftrightarrow q_2(q_1) = \frac{1}{2}(1 - w - d\,q_1). \tag{4.2}$$

In order to characterise the dynamics of the economy, we need to set up specific hypotheses about expectations. To this purpose, we strictly follows Zhang et al. (2007) and Tramontana (2010), and assume heterogeneous expectations by players: i.e., firm 1 (2) has bounded rational (naïve) expectations about the quantity to be produced by the rival. Given these expectations formation mechanisms, the two-dimensional system that characterises the dynamics of a differentiated Cournot duopoly under cross-ownership is the following:

$$\begin{cases} q_{1,t+1} = q_{1,t} + \alpha q_{1,t} \dfrac{\partial \Pi_{A,t}}{\partial q_{1,t}} \\ q_{2,t+1} = q_{2,t} \end{cases}, \tag{5.1}$$

where $\alpha > 0$ is a coefficient that captures the speed of adjustment of firm 1's quantity at time $t+1$ with respect to a marginal change in profits of shareholder A when q_1 varies at time t.

Using Eqs. (4.1), (4.2), the two-dimensional system Eq. (5.1) can alternatively be written as follows:

$$\begin{cases} q_{1,t+1} = q_{1,t} + \alpha q_{1,t}\left[1 - w - 2q_{1,t} - d\,q_{2,t}(1+h)\right] \\ q_{2,t+1} = q_{2,t} = \dfrac{1}{2}\left(1 - w - d\,q_{1,t}\right) \end{cases} . \tag{5.2}$$

From Eq. (5.2) we may conclude that the role played by the parameter h, i.e., the fraction of shares that shareholder A has in firm 2, is different depending on whether goods and services are substitutes (complements), that is $0 < d < 1$ ($-1 < d < 0$). Indeed, when products of variety 1 and 2 tend to be substitutable (complements) between each other, for any given value of the speed of adjustment α, an increase in h at time t reduces (increases) the weight of the reaction of (i.e., the quantity of product 1 produced at time $t+1$ by) firm 1, because its marginal profits at time t become lower (higher) since the degree of competition between the two firms weakens (strengthens).

3. Equilibrium and local stability

Equilibrium implies $q_{1,t+1} = q_{1,t} = q_1$ and $q_{2,t+1} = q_{2,t} = q_2$. Therefore, the dynamic system defined by Eq. (5.2) can be written in equilibrium as follows:

$$\begin{cases} \alpha q_1\left[1 - w - 2q_1 - d\,q_2(1+h)\right] = 0 \\ \dfrac{1 - w - d\,q_1}{2} - q_2 = 0 \end{cases} , \tag{6}$$

and the unique positive fixed point $E\left(q^*_1, q^*_2\right)$ of the two dimensional system is therefore characterised by:

$$E = \left(\frac{(1-w)\left[2 - d(1+h)\right]}{4 - d^2(1+h)}, \frac{(1-w)(2-d)}{4 - d^2(1+h)} \right), \tag{7}$$

where $q^*_1 < q^*_2$ ($q^*_1 = q^*_2$) for every $0 < h < 1/2$ (if $h = 0$). Under cross-ownership, equilibrium output by firm 1 is lower than equilibrium output by firm 2 as the former firm internalises the fact that both firms compete in quantities and thus the latter one is "more aggressive".

In order to investigate the local stability properties of the fixed point E we build on the Jacobian matrix

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} \dfrac{4 - d^2(1+h) - 2\alpha(1-w)\left[2 - d(1+h)\right]}{4 - d^2(1+h)} & \dfrac{\alpha d(1+h)(1-w)\left[2 - d(1+h)\right]}{4 - d^2(1+h)} \\ \dfrac{-d}{2} & 0 \end{pmatrix}, \tag{8}$$

where partial derivatives J_{ii} and J_{ij} are evaluated at the equilibrium point defined by Eq. (7). Trace and determinant of J are given by:

$$T := Tr(J) = J_{11} + J_{22} = \frac{4 - d^2(1+h) - 2\alpha(1-w)\left[2 - d(1+h)\right]}{4 - d^2(1+h)}, \tag{9}$$

$$D := Det(J) = J_{11}J_{22} - J_{12}J_{21} = \frac{-\alpha d^2(1+h)(1-w)\left[2 - d(1+h)\right]}{2\left[4 - d^2(1+h)\right]} < 0. \tag{10}$$

Therefore, the characteristic polynomial of (8) is the following:

$$F(\lambda) = \lambda^2 - T\lambda + D, \tag{11}$$

For the system in two dimensions defined by Eq. (5), the stability conditions that ensure that both eigenvalues λ_a and λ_b of the characteristic polynomial (11) remain within the unit circle are the following:

$$\begin{cases} (i) \quad F = \dfrac{4\left[4-d^2(1+h)\right]-\alpha(1-w)\left[4+d^2(1+h)\right]\left[2-d(1+h)\right]}{2\left[4-d^2(1+h)\right]} > 0 \\ (ii) \quad TC = \dfrac{1}{2}\alpha(1-w)\left[2-d(1+h)\right] > 0 \\ (iii) \quad H = 1 + \dfrac{\alpha d^2(1-w)(1+h)\left[2-d(1+h)\right]}{2\left[4-d^2(1+h)\right]} > 0 \end{cases} . \tag{12}$$

The violation of any single inequality in (12), with the other two being simultaneously fulfilled leads to: (*i*) a flip bifurcation (a real eigenvalue that passes through -1) when $F=0$; (*ii*) a fold or transcritical bifurcation (a real eigenvalue that passes through $+1$) when $TC=0$; (*iii*) a Neimark-Sacker bifurcation (i.e., the modulus of a complex eigenvalue pair that passes through 1) when $H=0$, namely $D=1$ and $|T|<2$. From Eq. (12) it is clear that conditions (*ii*) and (*iii*) are always fulfilled, while condition (*i*) can be violated.

We now develop the usual one-parameter bifurcation analysis by studying the stability properties of the unique positive equilibrium E of the two-dimensional system Eq. (5.2). Let $\mathrm{B}(\alpha,h,d,w)$ represent a boundary at which the Cournot-Nash equilibrium Eq. (7) looses stability through a flip bifurcation ($F=0$) when:

$$\mathrm{B}(\alpha,h,d,w) := 4\left[4-d^2(1+h)\right]-\alpha(1-w)\left[4+d^2(1+h)\right]\left[2-d(1+h)\right]=0 . \tag{13}$$

We now study the local stability properties of E by comparing: (*i*) the cross-ownership economy ($0<h<1/2$) versus the standard case in which every firm is owned by a single shareholder ($h=0$), when goods and services produced by firms 1 and 2 are *homogeneous* ($d=1$), and (*ii*) the effects on stability of the existence of cross-ownership depending on whether products of varieties 1 and 2 are either substitutes ($0<d<1$) or complements ($-1<d<0$).

Now, define

$$\alpha^F(h,d,w) = \frac{4\left[4-d^2(1+h)\right]}{(1-w)\left[4+d^2(1+h)\right]\left[2-d(1+h)\right]}, \tag{14.1}$$

and

$$\alpha^F(h,1,w) = \frac{4(3-h)}{(1-w)(1-h)(5+h)} . \tag{14.2}$$

as the (unique) flip bifurcation value of α when, under cross-ownership, products 1 and 2 are differentiated and homogeneous, respectively. Then, the following propositions hold.

Proposition 1. *Let $-1<d\le 1$. Then, (1) if $0<\alpha<\alpha^F(h,d,w)$ the Cournot-Nash equilibrium E of the two-dimensional system (5.2) is locally asymptotically stable; (2) a flip bifurcation emerges at $\alpha=\alpha^F(h,d,w)$; (3) if $\alpha>\alpha^F(h,d,w)$ the Cournot-Nash equilibrium E is locally unstable.*

Proof. Since $\mathrm{B}(\alpha,h,d,w)>0$ for any $0<\alpha<\alpha^F(h,d,w)$, $\mathrm{B}(\alpha,h,d,w)=0$ if $\alpha=\alpha^F(h,d,w)$ and $\mathrm{B}(\alpha,h,d,w)<0$ for any $\alpha>\alpha^F(h,d,w)$, then Proposition 1 follows. **Q.E.D.**

Proposition 2. *[Homogeneous products, $d=1$]. The parametric stability region under cross-ownership is wider than when every firm is owned by a single shareholder, and* *an increase in the fraction of shares that shareholder* A *has in firm* 2 *tends to stabilise the market equilibrium.*

Proof. Since $\alpha^F(h,1,w) > \alpha^F(0,1,w)$ and $\frac{\partial \alpha^F(h,1,w)}{\partial h} = \frac{4(7-h)(1+h)}{(1-w)(1-h)^2(5+h)^2} > 0$, then Proposition 2 follows. **Q.E.D.**

Proposition 3. *[Differentiated products,* $-1 < d < 1$*]. An increase in the fraction of shares that shareholder* A *has in firm* 2 *acts as an economic (de)stabiliser when products are (complements) substitutes.*

Proof. Since $\frac{\partial \alpha^F(h,d,w)}{\partial h} = \frac{4d\{16(1-d) + d^2(1+h)[8 - d^2(1+h)]\}}{(1-w)[4 + d^2(1+h)]^2[2 - d(1+h)]^2} < 0$ (>0) for every $-1 < d < 0$ ($0 < d < 1$), then Proposition 3 follows. **Q.E.D.**

When products are either substitutes or homogeneous, a rise in the fraction of shares of firm 2 held by shareholder A reduces the relative importance of profits of the second firm, because the degree of competition between the two firms becomes lower, and then it acts as an economic stabiliser. In contrast, when products are complements, the opposite result is obtained because a rise in h acts as if the degree of competition between firms had increased.

4 Conclusions

This study analysed the dynamics of a Cournot duopoly under cross-ownership participation when players have heterogeneous, i.e. bounded rational and naïve, expectations, and focused on the role played by cross-shareholding on market stability. While it is known that in equilibrium cross-ownership reduces competition, favours tacit collusion and harms social welfare, we found in a dynamic context that out-of-equilibrium cross-ownership acts as an economic stabiliser. However, our results also revealed that when products of different varieties are complements, a rise in the fraction of shares of firm 2 held by the shareholder that owns firm 1 acts as an economic de-stabiliser. These results also provide additional (dynamic) policy insights as regards the anti-trust analysis of markets: indeed, while, on the one hand, cross-ownership acts as an anti-competitive practice being subject to anti-trust concerns, on the other hand, it favours market stability (except when products are complements).

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