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The Multiplier Principle, Credit-Money and Time

Sebastian Gechert*†

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Abstract. We analyze the simple fiscal multiplier and extend it in terms of a credit-money framework and in terms of a time dimension, making it applicable to time series data. In order to take care of a credit-money framework, we complement the sources and uses of funds that are available along the multiplier process. In order to tackle the issue of time, we introduce a time component, which captures the time duration of a multiplier round. We argue that both attempts are incomplete on their own, but together they form a new version of the multiplier depending on the time duration of a multiplier period and a leakage that comprises net debt settlement and net accumulation of receivables. While the comparative-static stability condition of the multiplier can be dropped in this framework, our integrated multiplier reveals a dynamic stability condition for the multiplier process. Moreover, the integrated multiplier can be applied to evaluate income effects of transitory stimulus packages for a given time span. Multiplier effects are not calculated via identification of public spending shocks and GDP effects, but via determination of the behavioral parameters.

Keywords. Fiscal multiplier; credit-money; dynamic stability, stock-flow relation

JEL classification. E12, E20, E62

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1 Introduction

The discussion of fiscal multipliers has lasted for decades and still economists struggle on the value of the multiplier. Stimulus packages facing the great recession have brought the matter back on the agenda. The literature on multiplier evaluations is growing fast, while at the same time the range of results is increasing.

One reason for the large range of results is simply the method of measuring the multiplier. Spilimbergo et al. (2009: 2) report several definitions of the multiplier in empirical studies and complain about the lack of comparability. For the single case of VAR models the multiplier effect is measured either as *peak response*, *initial response*, or *cumulative response* to either a permanent or one-shot increase in spending. Moreover, there is no standardized time period, within which the effects are measured.

This heterogeneity stems from the theoretical basis. Multiplier theory does not provide an answer to the question how long the process lasts. As the multiplier is a dimensionless term, the length of a round and the length of the whole adjustment process are undefined *a priori*; they are either set from outside the model or disregarded when the focus is on the equilibrium effects of a permanent increment in autonomous demand. In the current discussion, however, the multiplier principle is applied to evaluating transitory stimulus packages, where the effect on income for a limited period is sought after. It turns out that more emphasize should be given to the time dimension of the multiplier process. If empirical evidence should serve as proof or disproof of the multiplier principle, empirical and theoretical work should run on comparable methodical grounds in this regard. However, this is not yet the case, since the multiplier principle is a theoretical construct in a logical-time comparative static framework, while measurement of multiplier effects draws on historical time series data. This makes the case for transferring the multiplier principle from logical to historical time, i. e. from hypothetical rounds to concrete time periods. Hence, we will introduce a time component to the multiplier formula, which captures the time duration of a multiplier round, or, in other words, displays how many multiplier rounds take place in a given period.

A second issue relates to the working of the multiplier in a credit-money framework, which concerns funding of both the initial spending and the induced spending. We analyse the multiplier process from a monetary perspective, starting with the well-founded assumption that an increase in autonomous spending, be it public or private, is usually financed by credit expansion. Moreover, we complement the sources and uses of funds for induced spending in the multiplier process to derive a fully-fledged budget constraint, drawing on income and changes in the level of assets and liabilities. In contrast to the standard multiplier, where only flow-flow relations matter, the monetary perspective includes stock-flow relations, which again requires the introduction of time.

The aim of this paper is to tackle both the issues of time and credit-money, and to show that they are interrelated. The paper argues that both attempts are incomplete on their own, but taken together they form a more general multiplier formula. This *integrated multiplier* is consistent with a credit-money framework and is applicable to empirical questions.

The paper is organized as follows. The next section reviews the standard multiplier approach. Section three provides a more detailed discussion of the credit-money issue and introduces the monetary perspective on the multiplier. The fourth section introduces the time-component. Section five combines these findings to an integrated multiplier. In section six we deal with the static and dynamic stability conditions of the multiplier in a credit-money economy. The final section concludes.

A short note on assumptions should be given beforehand: (1) Throughout this paper we consider a closed economy with underemployed resources. (2) Commodity and service prices are mainly determined by unit costs outside the model; consumer price inflation is not a result of an increasing money stock. Thus, we do not distinguish between nominal and real terms. (3) There are no central bank reactions. (4) Changes in distribution of income or assets are not considered, see Helmedag (2008) for a discussion of this issue.

2 The standard multiplier process

The multiplier principle has been viewed by most authors as a way to predict the impact of an exogenous expenditure on overall income or employment.[1] It is this major interpretation of the multiplier—the *serial multiplier*—that empirical investigations of stimulus packages draw upon; and it is this interpretation that is dealt with here. In order to account for the nature of stimulus packages, the focus is on one-shot stimuli.

The serial multiplier, which Keynes owes to Kahn (1931), is supposed to resemble a real process of spending and receipts through time, by drawing on comparative-static logical-time analysis. It describes a sequence of events: an initial increase in investment in the *first round* generates additional income which is partly spent for consumption and partly saved in the *second round*; what has been spent for consumption, induces additional income that, again, is spent for consumption and saved in a certain proportion in *round number three*, and so on. Given that $0 < s < 1$, this 'converging series of ever diminishing waves of expenditures' (Meade 1975: 84) makes the multiplier an outcome of an equilibrating process in

[1]Besides, there are other interpretations of the multiplier as a logical relation (Gnos 2008) or as a sectoral equilibrium condition (Hartwig 2004, 2008). See Chick (1983: 253-4) for a more detailed discussion of the methodical difference.

logical time:

$$\Delta Y = (1 + c + c^2 + c^3 + ...)\Delta I = \frac{1}{s}\Delta I \quad \text{with} \quad 0 < (c, s) < 1 \tag{1}$$

The process stops as soon as additional planned saving equals additional investment again. It is the increasing income that adjusts saving step by step to investment. In other words, the lower the propensity to save s, the more income is generated before saving is equal to investment again.

At a first glance, the multiplier process looks like an inescapable mechanism. There has, however, been a long discussion why individual voluntary decisions of savers and investors generate that outcome.[2] The prevalent answer states that the generated saving is necessary to finance investment, which can also be found in the General Theory:

> 'An increment of investment in terms of wage-units cannot occur unless the public *are prepared to* increase their savings in terms of wage-units. Ordinarily speaking, the public will not do this unless their aggregate income in terms of wage-units is increasing. Thus their effort to consume a part of their increased incomes will stimulate output until the new level [...] of incomes provides a margin of saving *sufficient to correspond* to the increased investment. The multiplier tells us by how much their employment has to be increased to yield an increase in real income sufficient to induce them *to do the necessary extra saving ...*' (Keynes 1936: 117) (emphases added; S.G.)

Following this rationale, voluntary savings must fund the initial investment which forces them to become equal. According to Shackle (1951: 243), Keynes owes this to Kahn:

> 'For it will be demonstrated [...] that, pari passu with the building of roads, funds are released from various sources at precisely the rate that is *required to pay the cost of the roads.*' Kahn (1931: 174) (emphases added; S.G.)

These explanations are a recurrence to Say's Law: the multiplier is considered to explain how investment is made possible by savings while the core of the General Theory was meant to explain that investment governs savings (Trigg 2003).

Keynes himself overcame this contradiction in his post-General Theory writings (Keynes 1937a: 246-7), (Keynes 1937b: 664-6). These articles laid the foundation for the now widely accepted endogenous money approach which states that finance

[2] See for example Dalziel (1996); Moore (1994); Cottrell (1994); Chick (1983); Warming (1932) and more recently Gechert (2011); Bailly (2008); Rochon (2008)

requires no saving. Financial resources for investment are provided by private banks creating credit *ex nihilo*. When a loan is granted, the borrower holds a debt and a deposit; nobody has saved beforehand. Once the borrower spends the money on newly produced capital goods, the producer receives deposits that can be considered transitory saving. If they are spent later on, someone else earns and transitionally saves them. The overall amount of financial assets is zero at any time because there is still a liability to the bank. Only the investment has created wealth. Thus, finance creates saving via investment and not the other way round. Accepting the endogenous money approach means rejecting the notion that saving finances investment. If investment needs finance but finance does not require saving, there is no market constraint for voluntary saving to be on par with investment. There is only the ever-valid *ex post* identity of actual saving and investment, but that provides no economic explanation for the outcome of the multiplier process (Gechert 2011).

However, there is a more pragmatic answer to the question why saving should adjust to investment. Kahn and Keynes simply modeled the process this way to arrive at a finite multiplier value in a comparative-static framework (Hegeland 1966: 61).[3] From that point of view, savings are a mere residual, a leakage allowing the process to find a position of rest after the initial demand shock. However, to yield that outcome, the serial multiplier process depends on two critical assumptions, namely, (1) Keynes' division of the multiplier and the multiplicand, and (2) his simple consumption function. He considered consumption and investment to be of a completely different nature:

> 'The theory can be summed up by saying that, given the psychology of the public, the level of output and employment as a whole depends on the amount of investment. I put it in this way, not because this is the only factor on which aggregate output depends, but because it is usual in a complex system to regard as the *causa causans* that factor which is most prone to sudden and wide fluctuation.' (Keynes 1937c: 221)

Keynes divides the multiplier ($1/s$) and the multiplicand (ΔI) in order to separate the more stable from the more fluctuating expenditures. As the citation above shows, he also regards this as the right way to separate cause and effect. Investment demand is very volatile and thus the ultimate *cause* of economic fluctuations. In contrast, consumption is a mere *effect*, and is determined by the simple Keynesian consumption function as a stable share of disposable income. Consequently, the

[3]Hegeland also considers a second effect. The notion that a governmental expenditure would eventually create adequate savings may have been convincing to politicians of the non-inflationary effects of expansionary fiscal policy.

leakage in each round of the process is given by savings, that is *current income not consumed*.

3 Credit-money and the multiplier process

Keynes' separation of the multiplier and the multiplicand may not suffice to separate cause and effect. Explaining causality with volatility has been questioned by Villard (1941: 229-33), Lutz (1955: 40-2), and Machlup (1965: 10), among others. Additionally, the Keynesian consumption function has come under criticism from various strands (Modigliani and Brumberg 1954), (Godley and Lavoie 2007: 70), (D'Orlando and Sanfilippo 2010: 1044). Taken together, it is questionable whether the causality intended by the multiplier principle resembles the actual series of spending and receipts and whether it accounts for the sources and uses of funds in a credit-money framework. We thus make the following extensions to the standard multiplier:

(1) It is illogical that consumption triggers further expenditures in any round of the multiplier process but the first round. According to the standard multiplier formula, consumption can only continue the process, yet not initiate it. We therefore implement initial consumption spending (and of course also public spending) by generally referring to additional autonomous expenditures (ΔA) starting the process; ΔA contains initial investment, consumption and governmental expenditures.

(2) We add induced investment to the expenditures during the multiplier process. Clearly, excluding induced investment controls for a finite multiplier value in a comparative static framework, but this is a mere method-based reason. As regards content, Keynes may have left induced investment out because he argued in a situation far from full capacity utilization where there is no incentive for the private sector to increase capacity; the marginal propensity to invest may be close to zero under these conditions. However, given that initial spending is for consumption purposes—be it public or private—it increases capacity utilization, and therefore may induce investment. Moreover, the majority of the empirical literature applies the multiplier principle regardless of the phase of the business cycle. Thus, their estimations comprise phases of high capacity utilization, too. As many empirical studies find crowding-in of private investment (Guajardo et al. 2011; Beetsma and Giuliodori 2011; Burriel et al. 2010; Tenhofen et al. 2010; Blanchard and Perotti 2002), the theoretical model should not exclude these effects *a priori*. Of course, there are multiplier-accelerator models that employ an investment function as well (Hicks 1959). However, the present paper combines the marginal propensity to consume and the marginal propensity to invest which yields the *marginal propen-*

sity to spend (ε). So far, the modified multiplier formula reads

$$\Delta Y = \frac{1}{1-\varepsilon}\Delta A. \tag{2}$$

The usual assumption that induced expenditures solely stem from income generated in the previous round, controls for $\varepsilon \leq 1$, whereby the series is most likely to converge.

(3) However, we argue that in a credit-money economy induced spending is not limited by current income. We generally use a monetary approach, looking at the flow of funds accompanying the multiplier process. The factual budget constraint of households at any time in the multiplier process is based on cash flows (Brown 2008: 3), and thus includes wealth (Godley and Lavoie 2007: 66), (Zezza 2008: 376), and credit (van Treeck 2009: 475-6), (Zezza 2008: 379), (Bhaduri 2011: 10) as additional sources to spend from. Thus, we allow for leak-ins during the multiplier process, and consequently the stability condition of the multiplier (now: $0 < \varepsilon < 1$) may not hold.

(4) As connected to point (3), we also reformulate the leak-outs based on the cash-flow approach. In the simple multiplier model consumption leads to further income, while saving is the leakage out of the circuit that does not re-enter (Palley 1998: 96). With point (2) we take into account induced investment, whereby we distinguish consumption, investment and the residual that is not spent. From a demand perspective, the residual is simply considered a flow out of active circulation; from a monetary perspective, the flow consists of two parts, namely, accumulation of receivables (deposits, bonds, ...) and reduction of liabilities.[4] These outflows are the counterparts to the inflows introduced in point (3), namely, additional credit and reduction of receivables during the multiplier process.

(5) We apply the cash-flow based thinking to the initial spending as well. As for initial spending, we assume that any expansion of autonomous expenditure comes with new credit creation. This is based on the financing process of investment as described by Davidson (1986: 102) and Chick (1983: 176, 262-3). Initial net-investment is usually financed by new loans or drawing on overdraft facilities provided by the banking system. Internal finance (retained earnings, depreciation) plays a subordinate role for financing net-investment. The argument has been extended to autonomous spending in general by (Wray 2011: 8), Seccareccia (2011: 12-3) and Polak (2001: 7). It certainly applies to public deficit spending; it also applies to consumption, as credit-financed consumption has become increasingly important over the last decades (Fontana 2009: 100), (Dutt 2006: 341-3), (Brown 2008: 20), (Cynamon and Fazzari 2008: 8), (Akerlof 2008: 1). In this respect, the monetary perspective makes another case for including consumption in the multiplicand as described in point (1).

[4] We substantiate this distinction in Section 6.

These points (1)-(5) are interconnected. They augment the possible sources and uses of funds at the beginning and during the multiplier process. The standard multiplier allows for credit-financed expenditures in the initial round only, whereas in any further round current income is the only available source. Consumption and unspecified saving are the only uses. In contrast, in our extended model the following budget constraint holds in any round of the multiplier process:

$$\Delta Y_{r-1} + \Delta D_r + \Delta N_r = \Delta C_r + \Delta I_r + \Delta H_r + \Delta R_r \tag{3}$$

ΔY_{r-1} is the additional disposable income (generated in the previous round); in the initial round ΔY_{r-1} equals ΔA. ΔD_r is additional funding out of wealth (reduction of receivables) and ΔN_r is additional credit (accumulation of liabilities) in round r. ΔC_r is additional consumption and ΔI_r is additional investment; they sum up to the generated income ΔY_r. ΔH_r is additional accumulation of wealth (accumulation of receivables), and ΔR_r is additional debt settlement (reduction of liabilities).[5] The marginal propensity to spend ε is defined as

$$\varepsilon = \frac{\Delta C_r + \Delta I_r}{\Delta Y_{r-1}} = \frac{\Delta Y_r}{\Delta Y_{r-1}} \tag{4}$$

From (3) and (4) follows

$$1 - \varepsilon = \frac{(\Delta H_r - \Delta D_r) + (\Delta R_r - \Delta N_r)}{\Delta Y_{r-1}} \tag{5}$$

where $1-\varepsilon$ depicts the net outflow from the circuit by net debt settlement ($\Delta R_r - \Delta N_r$) and net hoarding ($\Delta H_r - \Delta D_r$).[6] We can define the propensity to net debt settlement (λ) and the propensity to net hoarding (μ):

$$\lambda = \frac{\Delta R_r - \Delta N_r}{\Delta Y_{r-1}} \qquad \mu = \frac{\Delta H_r - \Delta D_r}{\Delta Y_{r-1}} \tag{6}$$

The propensities are then related as follows:

$$\varepsilon = 1 - (\lambda + \mu) \tag{7}$$

It is now obvious that $\varepsilon > 1$ can occur whenever $\lambda + \mu < 0$, i. e. the sum of the propensities to net debt settlement and net hoarding becomes negative. From a demand perspective that means an accelerating growth effect. From a monetary perspective, more funds are floated into the circuit than withdrawn from the circuit, which corresponds to a negative propensity to save for the standard multiplier. However, this infringes the stability condition of the comparative-static

[5] Note: All Δ depict differences to the preceding round.

[6] "Hoarding" includes deposits, bonds, cash, derivatives, ...

model ($0 < \varepsilon < 1$); the multiplier wouldn't converge to a finite value. The usual solution is to restrict the analysis to stable cases.

As we are concerned with making the multiplier principle empirically applicable, it is not viable to look at stable cases only. In the present paper, we develop a method to determine the multiplier without such a restriction. In order to do so, point (5) above, which is not included in the formula yet, becomes relevant. In conjunction with that, we introduce a time-component to the multiplier formula. By this, we also tackle our second issue, namely, transferring the multiplier from logical to historical time.

4 Introducing time to the multiplier principle

In the former section we emphasized the need for a time-dependent multiplier when it comes to evaluating income effects of autonomous demand shocks for a given period, especially when these shocks are transitory. The standard multiplier principle does not provide an answer to the question how long the process lasts, therefore it can not evaluate effects on income for a given period. As the multiplier is a dimensionless term, the length of a round and the length of the whole adjustment process are undefined *a priori*; they are either set from outside the model or disregarded when the focus is on the equilibrium effects of a permanent increment in autonomous demand. In the discussion on deficit spending, however, the multiplier principle is applied to evaluating transitory stimulus packages, where the effect on income for a limited period is sought after. The prevalent ways to transfer the multiplier principle from hypothetical rounds to concrete time periods are either setting one round as one period (Dalziel 1996), or by assigning the whole adjustment process to one period (Godley and Lavoie 2007; Pusch and Rannenberg 2011). In order to refrain from an arbitrary choice, our multiplier model is augmented by a time-component, which captures the time duration of a multiplier round, or, in other words, displays how many multiplier rounds take place in a concrete period.

The first step is to apply the variables to a concrete period. The effect on income due to a multiplier process is an additional flow within a given period in comparison to a baseline scenario. Thus, in the first period t we have ΔY_t, where Δ depicts the deviation from the baseline value Y_t; in any following period we have ΔY_{t+n}, which is the deviation from the baseline value Y_{t+n}. Given the logic of the multiplier, the increase in income stems from an increase in the flow of autonomous expenditures ΔA_t. Following the monetary perspective on the multiplier we suppose the initial spending is financed by newly created bank loans or by drawing on overdraft facilities (point (5) of the former section). ΔA_t thus comes with ΔL_t, an increase in the overall amount of debt at the beginning of period t. ΔL_t is an increment in a stock, created by someone who is willing to borrow and by a bank that is willing to lend. When the additional amount of credit-money is spend for the first time, the multiplier process sets in and induces a succession of expenditures and receipts. Thus the initial spending, amounting to ΔL_t, sets off the multiplier process. The effect on income in period t does not only depend on the leakages, but also on the number of multiplier rounds in that period. With a given leakage, the multiplier effect is higher, the more multiplier rounds take place in a period. The number of multiplier rounds per period shall be given by φ. When we assume for a while that leakages are zero ($\lambda + \mu = 0$), the effect on income in the first period is

$$\Delta Y_t = \varphi \Delta L_t. \tag{8}$$

φ has a time-dimension which relates the change in the flow (ΔY_t) to the change in the stock (ΔL_t). Without any leak-ins or leak-outs, the income creation would be repeated in every period $t + n$, thus the sum would be infinite for an infinite period. However, there is a definite value for a definite number of periods:

$$\sum_{i=0}^{n} \Delta Y_{t+n} = n\varphi \Delta L_t. \tag{9}$$

The serial multiplier is the antipode: it yields a finite value for an infinite succession of rounds while it does not tell how long one round actually takes. The time-component embeds the multiplier process in historical time and allows for evaluating one-shot impulses to aggregate demand. The introduction of a time component yields a second advantage: even without the usual stability condition of the serial multiplier ($0 < \lambda + \mu < 1$), a concrete multiplier value is calculable for a given period.

We made two critical assumptions until now. (1) The assumption of a zero leakage will be relaxed in the next section, where we put together the analysis of sections 3 and 4.

(2) The assumption of a credit-financed autonomous expenditure is crucial to our extension of the multiplier model. It should be emphasized that the volume of additional credit-money itself is not causal to the multiplier process, but only comes with effective demand. Taking the credit-impulse as a proxy for the initial spending could entail three failures that we try to rebut in the following.

First, spending could be financed by retained earnings, but there are empirical foundations (Friedman 1986; Polak 2001; Biggs et al. 2009; Keen 2010, 2011), as well as static theoretical (Wray 2011: 8), (Seccareccia 2011: 12-3), (Davidson 1986: 102), (Chick 1983: 262-3) and dynamic theoretical (Fisher 1933; Palley 1994; Biggs et al. 2009; Keen 2010; Raberto et al. 2011) foundations of growth processes linked to a growth in overall debt; moreover, deficit spending certainly fits to stimulus packages.

Second, the additional credit could be simply held idle from the start. However, the so-called *reflux principle*, which basically says that an initial borrower is supposed to spend the loan, rules out such cases. Debtors do not hold idle money because debit interest rates usually exceed credit interest rates. If there was no usage for excess credit-money, it would be repaid (Rochon 2008; Lavoie 1999; Kaldor and Trevithick 1981).

Third, the credit impulse could be spent for non-GDP transactions (Arestis and Howells 1999: 118), but this should not apply to stimulus packages. With respect to induced spending, non-GDP transactions (existing financial and non-financial assets, durables) are among the leakages that are reintroduced in the next section.

5 An integrated multiplier

The budget constraint for a credit-money economy is also valid in historical time. Now we consider the case where at the beginning of a period t there is an additional credit-financed demand impulse, generating an amount of credit money (ΔL_t) that circulates for a number of rounds (φ) in the period. At the end of the period (or at the beginning of the next period) agents can draw on further funds from additional credit (ΔN_t) and dishoarding (ΔD_t) that will be available in the next period. Funds are used for debt settlement (ΔR_t), hoarding (ΔH_t) and additional aggregate demand, summing up to ($\Delta C_t + \Delta I_t = \Delta Y_t$) by the end of the period:

$$\varphi \Delta L_t + \Delta D_t + \Delta N_t = \Delta Y_t + \Delta H_t + \Delta R_t \tag{10}$$

Net debt settlement equals ($\Delta R_t - \Delta N_t$) and net hoarding equals ($\Delta H_t - \Delta D_t$).[7] Equation (10) now reads

$$\varphi \Delta L_t - (\Delta R_t - \Delta N_t) - (\Delta H_t - \Delta D_t) = \Delta Y_t \tag{11}$$

With net debt settlement as a constant ratio λ (propensity to settle debt), and net hoarding as a constant ratio μ (propensity to hoard) of the credit flow $\varphi \Delta L_t$, this yields

$$\Delta Y_t = \varphi(1 - \lambda - \mu)\Delta L_t \tag{12}$$

The share of the credit impulse that is not used for net debt settlement and net hoarding induces additional income (ΔY_t) in that period. The sum of income until

[7] Hoarding and dishoarding not only concern idle money, but also all transactions of existing assets (financial and non-financial). In a closed economy as a whole, these transactions net out to zero, since they are mere changes in ownership. Only funds that flow from idle balances to aggregate demand or debt settlement are dishoarded on the macro level. Only funds that do not flow into aggregate demand or debt settlement are hoarded on the macro level.

period $t+n$ induced by a one-shot credit impulse makes the integrated multiplier formula:

$$\sum_{i=0}^{n} \Delta Y_{t+i} = \varphi \Delta L_t \sum_{i=0}^{n} (1-\lambda-\mu)^i \tag{13}$$

For $n \to \infty$, the integrated multiplier formula has a similar structure to the standard multiplier, while taking into account the length of the multiplier period via φ and the different kinds of saving λ, μ:

$$\sum_{i=0}^{\infty} \Delta Y_{t+i} = \frac{\varphi}{\lambda+\mu} \Delta L_t \qquad \text{with } 0 < (\lambda+\mu) < 1 \tag{14}$$

However, for an empirical application it is more useful to determine the multiplier effect for a finite period, which is possible with (13). Additionally, equation (13) is also applicable in situations, where $0 < (\lambda+\mu) < 1$ does not hold.

For the scenario of a permanent credit impulse ΔL_t in each period, the effects cumulate. The income-flow in the $t+n$-th period would be equivalent to the sum of income-flows until period $t+n$ for a one-shot impulse:

$$\Delta Y_{t+n} = \varphi \Delta L_t \sum_{i=0}^{n} (1-\lambda-\mu)^i \tag{15}$$

The cumulative effect on income until period $t+n$ now reads

$$\sum_{i=0}^{n} \Delta Y_{t+i} = \varphi \Delta L_t \Big(n \sum_{i=0}^{n} (1-\lambda-\mu)^i - \sum_{i=0}^{n} i(1-\lambda-\mu)^i\Big). \tag{16}$$

The integrated multiplier is time dependent via φ and allows for net inflows and outflows via λ and μ. The more multiplier rounds per period, the higher is the multiplier. The more intense the leakage through net debt settlement and net hoarding, the lower is the multiplier. Suppose the creation of an additional amount of credit-money at the beginning of period t. After the money is spent, it will induce a succession of receipts and expenditures. The economy's average frequency φ determines, how often the additional money circulates for newly produced goods and assets during a given period. The leakage λ determines on average, how much of the credit-money flow is used for net debt settlement and μ determines on average how much of it is used for net accumulation of receivables in each period. Together, they determine, how much additional income is generated out of the initial loan within a given period.

It can be shown that formula (13) is an integrated or general version of the serial multiplier, as it allows for measurement of the multiplier in historical time and is not constrained to $0 < (\lambda+\mu) < 1$, i. e. it allows for additional net inflows to the circuit via induced credit expansion ($\lambda < 0$) and net dishoarding ($\mu < 0$). Since the integrated multiplier refers to a concrete time span, it is still calculable. In other words: the serial multiplier can be derived from the integrated multiplier formula by making the following constraints. The basis of measurement now is not a concrete time period, but one multiplier round. Thus, every subscript t becomes an r and necessarily $\varphi = 1$, since there is always one round per round. Moreover, the serial multiplier adds up the leakages to the marginal propensity to save ($\lambda + \mu = s$), and sets additional credit-financed demand to investment of the initial round ($\Delta L_t = \Delta I_0$). For $r \to \infty$ the formula becomes

$$\sum_{i=0}^{r} \Delta Y_r = \Delta I_0 \sum_{i=0}^{r} (1-s)^i = \frac{1}{s} \Delta I_0 \tag{17}$$

which resembles equation (1). The transformation reveals that the usual multiplier formula is only applicable to concrete time periods when the duration of a multiplier round is set to one per period, a point which was already made by Tsiang (1956: 555-6). Thus the standard multiplier makes *ad hoc* assumptions concerning parameter values that should rather be determined empirically in order to calculate the multiplier effect for a given time span properly. The integrated multiplier makes that possible because it is not just a theoretical construct, but it can be directly applied to empirical questions. Multiplier effects are not calculated via identification of public spending shocks and GDP effects, but via direct determination of the behavioral parameters φ, λ and μ for a given scenario.

An empirical application, however, goes beyond the scope of this paper. Nevertheless, it should be possible to determine the parameters, and it should be worthwhile against the background of the dissent in the empirical debate on the multiplier. In any event, the integrated multiplier lays open the problem of the length of the multiplier period.

6 Dynamic stability of the multiplier process

So far, we did not discuss the impact of the distinction between net debt settlement ($\Delta R_t - \Delta N_t$) and net hoarding ($\Delta H_t - \Delta D_t$). Clearly, both ways of net saving are *non-demand* and thus they have the same short-run effects on income. However, they differ concerning their impact on the stock of credit-money. Net debt settlement is a definite leakage because the economy's gross debt level and the amount of credit-money shrinks; net hoarding, on the contrary, is not a leakage in the strict sense. It maintains the stock of credit-money (and the liabilities to the banking system), but the hoarded receivables are not used for aggregate demand anymore, i. e. they are not in active circulation.[8]

Suppose a one-shot credit-financed demand impulse (ΔL_t) at the beginning of period t. Suppose further that there are φ multiplier rounds within period t and that there is a positive propensity to settle debt (λ>0) and a positive propensity to hoard (μ>0) for each period t. Let us first look at the development of the stock of additional credit money. At the beginning of period $t+1$ the remainder of the credit impulse is[9]

$$\Delta L_{t+1} = \Delta L_t - (\Delta R_r - \Delta N_r) = (1-\lambda)\Delta L_t, \tag{18}$$

as only net debt settlement can alter the stock of money. In the first period the whole credit impulse is in active circulation. For the next period, however, a part of the money has been hoarded, while the remainder is still in active circulation. Only the money that is still in active circulation can be used for debt repayment in this period. With positive net hoarding ($0 < \mu < 1$), an ever decreasing share of the current money stock is in active circulation, and thus available for debt repayment. After $t+n$ periods, the stock of additional credit-money is

$$\Delta L_{t+n} = (1-\lambda\sum_{i=0}^{n}(1-\lambda-\mu)^i)\Delta L_t \tag{19}$$

while the income generated in period n is

$$\Delta Y_{t+n} = \varphi(1-\lambda-\mu)^n \Delta L_t. \tag{20}$$

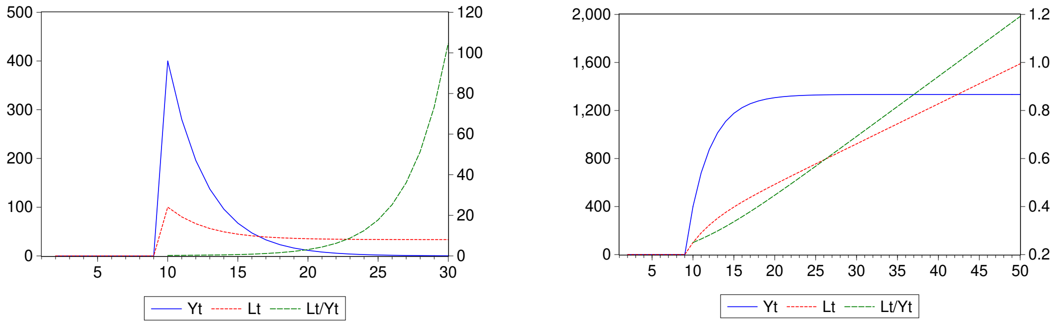
In a dynamic setting, sustained net hoarding gives rise to instability. For $0 < (\lambda,\mu) < 1$, receivables (and liabilities) ΔL_{t+n} converge to a positive limit, i.e. they never return to the baseline, because net hoarding prevents complete debt settlement; ΔY_{t+n} converges to zero, i. e. the additional income effect runs out and income returns to its baseline value. The ratio of $\Delta L_{t+n}/\Delta Y_{t+n}$ explodes, which marks a stock-flow incoherence. See Figure 1(a) for a simulation of a one-shot credit impulse. For the effects of a permanent credit impulse, see 1(b). Income converges to a higher level, while receivables (and liabilities) accumulate on and on. Again, the ratio of $\Delta L_t/\Delta Y_t$ grows infinitely, making the process stock-flow incoherent.

Thus, even if the static stability condition holds ($0 < (\lambda+\mu) < 1$), this is not sufficient for dynamic stability. Only if saving is done in terms of net debt settlement, a dynamic equilibrium can occur. Given a one-shot impulse, both the

[8] This does not mean that the money is held idle though. It may well circulate with a high frequency for financial and non-financial assets, but it is not active for current production.

[9] Again, ΔL_{t+1} depicts the additional amount of credit money compared to the baseline scenario. It is *not* the difference between $L_{t+1} - L_t$.

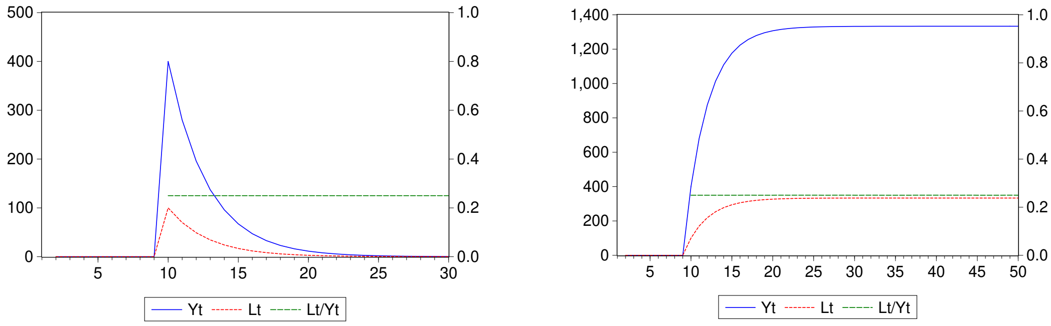
Figure 1: Effects of credit impulse on income and stock of money, unstable case ($\Delta L_t = 100, \lambda = 0.2, \mu = 0.1$)



(a) One-shot credit impulse

(b) Permanent credit impulse

Figure 2: Effects of credit impulse on income and stock of money, stable case ($\Delta L_t = 100, \lambda = 0.3, \mu = 0$)



(a) One-shot credit impulse

(b) Permanent credit impulse

income effect and the amount of circulating credit will pass off (Figure 2(a)); given a permanent impulse, both the income effect and the amount of credit-money will converge to a finite value (Figure 2(b)). This gives rise to a new understanding of the multiplier process from a stock-flow perspective: the multiplier *does not* show the income generating process until an initial investment is financed or paid by savings. What it *does* show, is the income generating process until an additional amount of credit-money is repaid.

A stock-flow incoherent setting may entail repercussions from stocks on flows that influence the parameters of the multiplier in upcoming periods. More precisely, an increasing debt-to-income ratio in the economy may enhance the propensity to settle debt or to hoard and thus reduce the multiplier effect in the future. To capture these effects, the integrated multiplier can be extended to a dynamic model, where φ_t, λ_t and μ_t are endogenously determined. This is linked to the growing literature on credit cycles and their influence on the business cycle (Fisher 1933; Palley 1994; Biggs et al. 2009; Keen 2010; Raberto et al. 2011).

However, for multiplier effects to be measurable the steady state does not need to come into being because empirical measurement usually refers to a concrete short-term period. So, even if the debt settlement is incomplete, the income effects can still be measured for this time span via the integrated multiplier. Moreover, the leakage could even be $(\lambda + \mu) < 0$, and still the integrated multiplier would be calculable for a finite period. There may be times of optimism, when an autonomous demand triggers accelerating credit-financed demand expansion, which comes with $(\lambda + \mu) < 0$, yielding a relatively large multiplier value. In contrast, there may be times of debt deflation and strong hoarding with an even negative integrated multiplier value due to an intense leakage $(\lambda + \mu) > 1$.

7 Conclusion

The present paper discussed the shortcomings of the simple Keynesian multiplier model with respect to the characteristics of a credit-money economy and its applicability to empirical questions. Indeed, the well-known serial multiplier is a comprehensible way to model the process of expenditures and receipts stemming from an initial demand for capital goods, but the formula merely looks as though it entails an ever-valid mechanism. We try to capture the multiplier process from a monetary perspective by taking into account that an expansion of aggregate demand usually comes with an expansion of liabilities (credit impulse). Reconsidering the sources and uses of funds in a credit-money economy reveals some degrees of freedom that should not be set *ad hoc*: when induced investment and credit/wealth-financed consumption are taken into account, the comparative-static stability condition of a positive leakage may not hold. Additionally, the serial multiplier formula provides no information regarding the length of the process, causing arbitrariness when it comes to empirically determining the multiplier from time series data.

Thus, we develop an alternative approach where the stability condition is not needed to calculate a finite multiplier value. We do this by introducing a time component that captures the number of multiplier rounds proceeding in a concrete time period. The time component is established by the credit impulse, which sets up a stock-flow relation between additional credit and additional income that requires a time dimension. We combine the extensions concerning credit-money and time to our *integrated multiplier*, which has two channels of influence—the number of multiplier rounds (φ) per period and the magnitude of the net leakage per period, the latter comprising net debt settlement and net accumulation of receivables ($\lambda + \mu$).

This new multiplier has several advantages in comparison to the standard multiplier. First, it takes into account a time dimension, whereby it allows for a rule-based conversion from hypothetical rounds to concrete periods. Second, with the time component the multiplier value is calculable for a given period, even when the comparative-static stability condition ($0 < \lambda + \mu < 1$) does not hold, i. e. in times of accelerating growth, when demand induces further credit-financed demand ($\lambda < 0$) and net dishoarding ($\mu < 0$), and in times of debt deflation and hoarding, when the initial impulse is more than offset by net outflows ($\lambda + \mu > 1$). Third, the effects of one-shot increases in aggregate demand can be measured, which is better suited for the evaluation of transitory stimulus packages. Fourth, the budget constraint that comes with our multiplier version fits to a credit-money economy. Fifth, it allows for induced investment.

With these advantages the integrated multiplier is not just a theoretical construct, but it can be directly applied to empirical questions. Using this method, multiplier effects are not calculated via identification of public spending shocks and GDP effects, but via determination of the behavioral parameters φ, λ and μ for a given scenario. An empirical application of this method is an issue for future research, which would also need an extension to open economy considerations.

As we look at the multiplier from a monetary perspective, the conditions of a dynamic equilibrium are revealed, namely, a stock-flow coherent liabilities-to-income ratio (and, as a mirror image, a stock-flow coherent receivables-to-income ratio). The dynamic stability condition replaces the comparative-static one. This makes a new understanding of the multiplier process from a stock-flow perspective: the multiplier *does not* show the income creating process until an initial investment is financed or paid by savings. What it *does* show, is the income creating process until an additional amount of credit-money is repaid.

This relates the multiplier analysis to processes of leveraging and deleveraging that may entail repercussions on the parameters of the multiplier, which would then depend on the overall debt-to-income ratio in the economy. In a next step, a dynamic model could tackle this issue by endogenising the parameters φ_t, λ_t and μ_t. Of course, that will make it necessary to identify their determinants.

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