# The labour theory of value: a marginal analysis

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# The Labour Theory of Value: A Marginal Analysis

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**Abstract**

*The difficulties of the classical and Marxian labour theory of value are overcome when labour value is understood as cost, analogously to marginal cost as marginal labour value. Marginal labour value is the reciprocal of the marginal productivity of labour. Under "perfect competition" relative prices are equal to the ratio of marginal labour values. Indeed, Pareto-optimality implies the validity of the labour theory of value. But it is shown that, in principle, a capitalist system can never be in a Pareto-optimal state. To assure a maximum productivity of labour, and therefore minimum socially necessary labour values, society has to overcome capitalism and organise the formation and control over capital collectively. This article presents the marginal approach to the labour theory of value.*

**JEL Classification**: B10; B14; B24; B51; D21; D33; D46

**Keywords:** Exploitation; Labour Theory of Value; Marginal Analysis; Marxism; General Economic Equilibrium; Cost Theory

**"Marginal Labor ... is the only kind that can measure value."**

**John Bates Clark (Clark, 1896)**

## I. Introduction

It is a widespread belief that the labour theory of value is inconsistent and in contradiction with modern economic theory. Those economists who still adhere to it are typically regarded as 'heretics'. In the history of economic thought it is referred to the Marginalists, in particular W. S. Jevons, Léon Walras and Carl Menger, who are said to have overcome the impasses of the labour theory of value. Jevons is quoted, accusing Ricardo of having placed political economy on the wrong track. Marginal analysis is presented as being incompatible with the labour theory of value obviously because otherwise questions like how to calculate the minimum socially necessary labour time of producing a commodity would pop up, questions of the highest importance for any real existing socialist system. After all, marginal analysis is a mathematical method to determine optima, e.g. maximum profits, minimum cost, etc., a method extremely important in particular for socialist economists. Those Western Economists, orthodox or Marxist, who are opposed to the marginal analysis of labour values are obviously opposing more than that, they have been and are opposing real existing Socialism all together.

The exchange model of general economic equilibrium without production shows that exchange values exist also in the absence of labour and production processes. However, this argument ignores that without labour there are no quantities of goods to be exchanged[1] and therefore also no people to perform the exchange as they simply could not exist. We still concede that exchange values can exist when there are quantities of goods with use value available to be exchanged. The act of the exchange of goods can increase the use value for the participants, but producing for the market implies that the commodities produced have no direct use value for the producer and therefore it cannot be use values which are exchanged on markets.

1 The free gifts of Nature are ignored as they would not suffice to maintain humanity alive without labour.

It should be noted that the labour theory of value has a meaning only when there is the possibility of humans to produce goods as commodities, that is, not to be consumed by the producer but determined for the market, and in addition, the production processes and the markets must be free from restrictions. It must prevail *perfect competition* as only then we can speak of economic laws equilibrating (marginal) cost and price.

Closer study of the sources, especially the work of Hermann Heinrich Gossen (1854), who is highly praised by Walras and Jevons, reveals that the orthodox interpretation of the history of economic thought is wrong. Marginal analysis has not replaced the labour theory of value but improved on its analysis[2]. Jevons, who most fervently tries to reject the labour theory of value, even writes in one place that commodities exchange according to their labour values: “thus we have proved that commodities will exchange in any market in the ratio of the quantities produced by the same quantity of labour” (Jevons 1871, p. 182)[3].

Of course the discrediting and rejection of the labour theory of value and those who defend it is wanted by the bourgeoisie to avoid the accusation of the exploitation of the labourers. But could orthodox economic theory really be so powerful in guiding the organisation of society without having a labour theoretical foundation? In this paper we present the development of the labour theory of value using marginal analysis as it has been done historically (Clark 1896).

2 Alfred Marshall comments on Jevons "[Jevons'] success was aided even by his faults. For under the honest belief that Ricardo and his followers had rendered their account of the causes that determine value hopelessly wrong by omitting to lay stress on the law of satiable wants, he led many to think he was correcting great errors; whereas he was really only adding very important explanations." (Marshall 1890, pp. 84, 85)

3 Marginal analysis has been introduced long before Jevons, Walras and Menger in the 1870ies. Augustin Cournot (1838) used marginal analysis even before Gossen (1854). It is Cournot's work which has inspired Walras general equilibrium model. Furthermore, also in Germany marginal analysis was well established even if one believes that Gossen's work had been totally ignored.

Properly understood, marginal analysis is not an alternative, opposed to calculating average labour values. On the contrary, it is the appropriate method to resolve the contradictions of the Classical analysis of value. When Marx wrote, commodities exchange according to their labour values provided they have use value, marginal analysis is elaborating on the relationship between use value and labour value and offers a synthesis of use and exchange value by introducing marginal utility and marginal labour value (marginal pain, Gossen 1854).

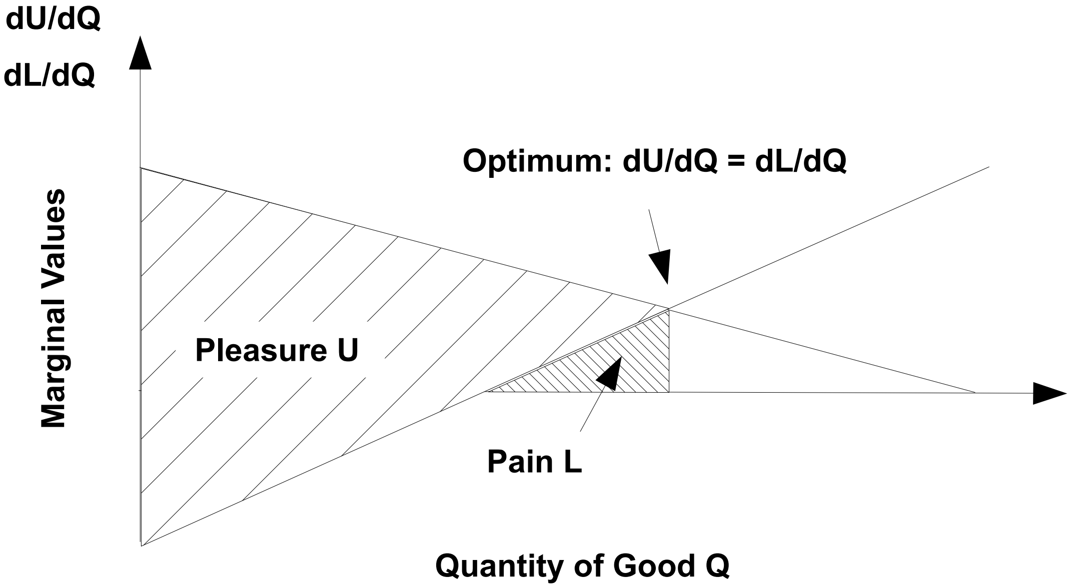



**Figure 1: H. H. Gossen's Marginal Pleasure and Pain Functions**

Equally important is the insight, gained from marginal analysis, under what conditions surplus value is part of the socially necessary labour time and when on the other hand it is simply exploitation of the labourers for the pleasures of the capitalists. The most powerful insight may be that profits gained from the 'returns of capital or land' (natural resources) have to be entirely reinvested in order to assure the optimal use of labour, a requirement in plain contradiction with the institutions of capitalism as the ultimate objective of the capitalist is to make profits in order to consume.

The marginal analysis of labour values allows to determine the minima of socially necessary labour values. By giving a precise meaning to socially necessary labour value, marginal analysis provides a means to understand the underlying economic processes by which the mode of production of capitalism is transformed into the mode of production of socialism, a process of "crowding out capitalism". The labour theory of value is not only at the core of modern economic theory but it is also fundamental for the theory of Historical Materialism. First, we discuss the concept of labour value as it is by no means obvious what has to be understood by labour values.

## II. Labour Values and the Theory of Cost

Few are the writings on the labour theory of value which discuss the meaning of labour values at length[4] and here we have indeed the ultimate cause for the misunderstandings concerning the subject. Commonly, the labour value of a commodity is understood to be the sum of labour time spend upon producing all the components a good consists of as well as the good itself. To compute this value, it is most convenient to use the Leontief input-output model (see **Appendix I**), the solution of which gives the vector of the quantities of labour time per commodity. A more general mathematical method to calculate these values is linear programming[5], but from an economic point of view both methods are essentially the same as can be seen solving a Leontief input-output model via linear programming.

4 An exception is Flaschel (2010). But he provides only an axiomatic definition claimed as being plausible. No discussion of labour values in relation to the theory of cost is found. On the other hand in Soviet economics, in contrast to Western Marxism, this was a core issue.

5 This has been introduced by Morishima (1974). We shall see below that this method can easily be adjusted to calculate labour values as cost by incorporating the "**κ**-rate" which is based on Kantorovich's "norm of effectiveness" (*see* (Kantorovich & Vainshtein, 1976) into the program.

However, it is not at all clear why the values calculated this way should be equal to the socially necessary labour time to produce the commodity. There is a difference between the value of the sum of the values of the inputs of production and total cost per unit of output, regardless of expressing this value in terms of money or labour time. Not only that there can be and usually are externalities but there are costs which occur because of *the use of capital,* i.e. the means of production[6]. It is exactly the precise meaning and understanding of the causes of these costs we have to focus on, in order to provide a useful concept of labour value and to unravel the Gordian knot of the relationship between labour values and prices.

Bourgeois economists offer a wide range of explanations for the causes of the cost of using capital[7], e.g. waiting, abstinence, time preferences etc., which try to justify that the owner of some capital is given the right to claim some return upon it by leaving it to be used by others. We shall not get into the discussion and refusal of all these arguments but simply provide a reason which appears to be self evident when the static analysis of labour values is extended to the dynamic analysis of the process of production.

It is most important to realize that this part of cost or of labour value is surplus labour, in monetary terms, profits[8]. This is already a glimpse that there is no "real" difference between labour values and cost of production, the later simply being the monetary expression of the former. With other words, there is no *transformation problem*[9] once labour values are understood as socially necessary cost,

6 It must be noted here that, contrary to orthodox Marxists but in conformity with reality, we consider wages not as capital.

7 Maurice Allais provides a long list of them (Allais 1947).

8 The reader familiar with Morishima's work will recognize this as a devastating criticism of his approach.

9 This is the Marxian term referring to Marx' vision in *Capital* on how labour values would be transformed into prices of production (= average cost).

expressed in labour time.[10]

**III. The Cost of the Firm in Terms of Labour Values**

In Marx's *Capital* we find a very touching analysis of the "labour day", a penetrating analysis of the exploitation process at the work place (Marx 1867 (1906), p. 255 ff.). We shall attempt here to present the analysis of exploitation in the context of the modern theory of cost but in contrast to the orthodox theory of cost we shall conduct it in terms of labour values. After we have succeeded in presenting the labour theory of value along these lines we shall re-examine and correct the Marxian analysis.

As will become apparent below we have to assume that the economy is in a state of equilibrium of perfect competition, where the firms lack the power to influence the prices of the markets, neither the prices of the factor markets nor of the product markets, i.e. the firms are price takers. We must emphasise that this condition is absolutely crucial for the analysis of labour values as obviously any deviation from perfect competition opens up the chance of extra profits due to some kind of monopoly power and this would create a difference between the cost of production and price and therefore invalidate the labour theory of value because the equality of (marginal) cost – including economic or ordinary profits - and price will be lost. It is the competition within the economic system which enforces the law of value.[11]

10 It needs to be emphasised here that we are discussing not a capitalist economic model but the general economic equilibrium model which is not, in its pure form, a capitalist economic system. In fact, in a capitalist economic system prices are usually not at all proportional to labour values which is one of the outcomes of the inefficient and wasteful use of labour. In order to have a general economic equilibrium model of capitalism important modifications of the basic benchmark model have to be introduced as imperfections, some of them being systemic.

11 This is a criticism of Rosa Luxemburg's position ""In order to find the value of a commodity, we must start by

There is an intrinsic relationship between the process of production and cost; one speaks of cost being the dual to production (McFadden, Fuss 1978). Cost theory traces the costs resulting from the production processes. When the process of production changes so usually does cost. We present here only the most rudimentary elements of cost theory in terms of labour values.

The ultimate source of all costs is labour[12] because labour time is the sacrifice of human life time in order to produce goods with a use value[13]. This is what places the labour theory of value at the centre of political economy. Most generally speaking labour is the act of humans changing matter involving the sacrifice of human life time but gaining conveniences yielding use value supporting human life; it is this the fundamental fact on which is build Political Economy and the theory of Historical Materialism.

Cost can be classified depending on the purpose of analysis. We distinguish cost, depending on what kind of return it offers as *labour cost* (wages), *capital cost* (interest), or *rent*[14]. When we speak

assuming that demand and supply are in a state of equilibrium, that the price of a commodity and its value closely correspond to one another. Thus the scientific problem of value begins at the very point where the effect of demand and supply ceases to operate." (Luxemburg 1913, 1951, p. 36). We recognise supply and demand as the forces which establish the law of value via the establishment of equilibrium prices. One may concede to Rosa Luxemburg that she wanted to exclude the erratic movements of prices from their equilibrium values, but by eliminating supply and demand she throws the baby out with the bath water. Our cost analysis centres on the establishment of supply and demand functions in terms of labour values. The labour value of a commodity is at the intersection of these functions.

12 This is by no means an exclusively Marxian proposition, e.g. (Fisher 1906, p. 173 ff.)

13 Tugan-Baranovsky explains this point at some length (Tugan-Baranowsky 1905), but we do not share his distinction between cost and labour value which is due to his (and Ricardo's) error regarding profits. (see below the explanation of the **κ**-rate).

14 In what follows we ignore rent as our analysis is strictly limited to perfect competition implying the assumption that

of *labour cost* we refer to *direct labour*, the labour directly used in the production process, or we speak of *dead* or *indirect labour*, referring to labour contained[15] in the means of production, i.e. the labour content of the depreciated capital. The sum of these costs, representing total cost, is mathematical expressed in the *cost equation* (*see below*).

In the analysis of exploitation occurring in the production process, labour cost can be classified as *paid labour* or *unpaid labour*. Paid labour is that part of the working day which is used to produce the value equivalent to the wage whereas unpaid labour (surplus labour) is the labour time which is used to produce the value equivalent of profits in all its forms.

Another criterion is the dependence on the quantity of output. If cost is independent of the quantity of output it is *fixed cost* whereas if it changes with output it is *variable cost*. *Total cost* is the sum of all costs occurring through the act of producing a quantity of output. The *classical cost function* expresses total cost as a function of output.

When we relate *variable* or *fixed cost* to output we speak of *average variable* or *average fixed cost* etc. or if we use ordinary or partial derivatives we speak of *marginal cost*. *Marginal cost* expresses the cost of an incremental unit of output whereas the *cost of production*[16] expresses the *average cost* of a unit of output.

the rent for marginal land (natural resources) is zero.

15 This is loosely speaking as the labour value of the means of production is not the labour time having been spent to produce them but the labour time being socially necessary at the time of evaluation to produce them. So the labour value is not a substance of the commodity but an attribute an economic agent attaches to a commodity.

16 *Cost of production* is the Marxian and Neo-Ricardian term for average cost. Marxian terminology distinguishes between *constant cost* (capital cost), *variable cost* (paid labour cost), and *surplus value* (value of unpaid labour time). Surplus value is either *absolute* or *relative surplus value* depending, if it is obtained via the lengthening of the working hours or the intensification of work.

An important distinction of labour time as cost goes back to Adam Smith. If it is seen from the point of an investor buying labour (in fact, buying the labour force for some time), then one speaks of *labour commanded,* which is calculated simply by dividing the amount of money, spent on buying labour force, by the wage rate. Alternatively, one may look at the commodity from the point of the producer (capitalist) as a product and then the labour time embodied in the product is considered as *labour embodied*. Under perfect competition, when the labour theory of value holds, both are equal[17].

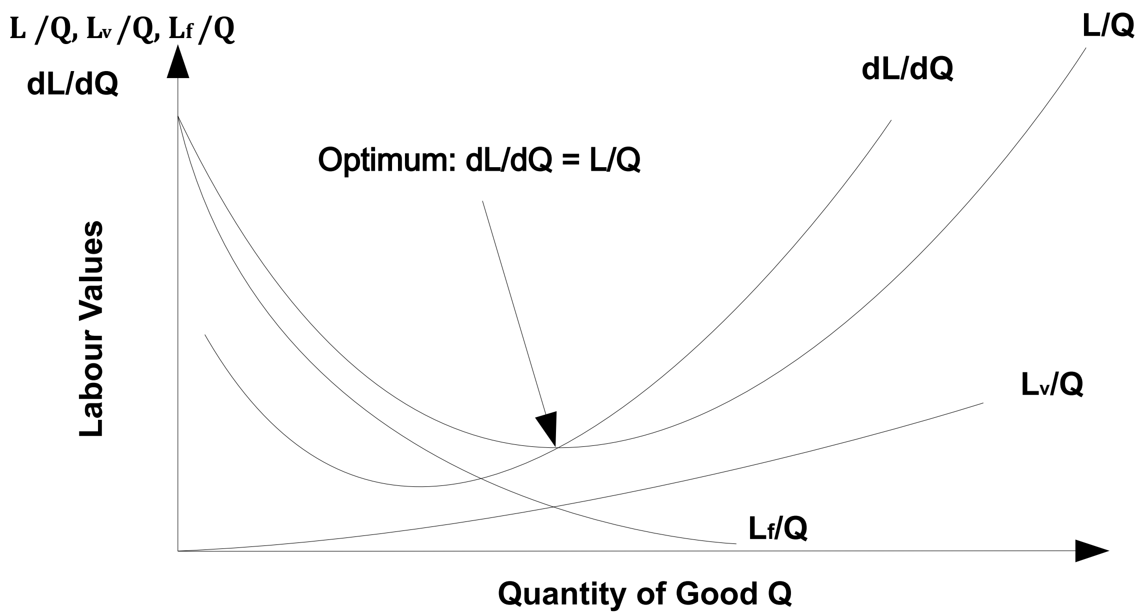
When we have *cost-*, *profit-*, or *revenue functions* we can use marginal analysis to determine the relative extrema of these functions, e.g. minimum average cost, maximum profit or revenue etc. In order to determine the minimum cost per unit of output we need to find the minimum of the *average cost curve*. This is an important exercise and marginal analysis is most useful to find the solution. It shall give us the *minimum socially necessary labour time* of the production of a commodity and anyone claiming this to be anti-Marxist must be regarded as a fool. But this was the typical Western Marxist attitude at the times of the Cold War, and it still is, even now! In fact, it is common practice amongst Western Marxists to discuss models which imply constant average cost[18].

When cost depend on the quantity produced, *fixed costs* are diminishing with the increase of output. With a certain size of the production unit given, it is most likely that above a certain amount of output additional output will be more costly, i.e. *marginal cost* will rise. Both effects, diminishing *average fixed cost* and increasing *marginal cost* lead to an average cost curve of an U-shaped type. This remains true when we consider the costs in terms of labour time. Average labour value, **L/Q**, is

17 Orthodox economists and even Western Marxists usually deny this as they exclude the surplus labour embodied in profits from their definitions of labour value, e.g. Morishima, H. D. Kurz etc.

18 See the previous footnote.

a function of output and this function typically has an U-shape.




## Figure 2: Cost Functions in Terms of Labour Values

**Figure 2** presents the following cost functions in terms of labour values:

1. **$L_f$/Q** – variable fixed cost;
2. **$L_v$/Q** – variable cost;
3. **L/Q** – average total cost (sum of 1. and 2.), cost of production;
4. **dL/dQ** – marginal cost.

The curve of average total cost, **L/Q**, is cut at its minimum by the curve of marginal cost, **dL/dQ**. The point of minimum average cost is the point of minimum socially necessary labour.

We shall prove below that one obtains the functions of average and marginal cost in terms of money, as they are usually presented in orthodox economics, by multiplication of the labour value functions with the wage rate.

With the help of the *cost equation* and the *production function* we establish the optimal quantities of production at minimum average cost, i.e. at *minimum socially necessary labour values*. The cost equation is

$$C = wL + (1+\kappa)K$$

$$\begin{gathered} C - \text{ total cost}, w - \text{ wage rate}, \\ \kappa - \kappa\text{-rate (in orthodox economics the rate of interest)}, \\ K - \text{ value of capital goods} \end{gathered} \tag{1}$$

The production function[19] is

$$\begin{gathered} Q = f(L, K) \\ Q - \text{ output} \end{gathered} \tag{2}$$

We find the quantity of output at minimum cost by differentiating the Lagrangian, **£**, with respect to the factors of production.

The Lagrangian is:

$$£ = wL + (1+\kappa)K + \lambda[Q^* - f(L, K)] \tag{3}$$

The first order conditions are:

$$\begin{gathered} \frac{\partial £}{\partial L} = w - \lambda \frac{\partial Q}{\partial L} = 0 \\ \frac{\partial £}{\partial K} = (1+\kappa) - \lambda \frac{\partial Q}{\partial K} = 0 \\ \frac{\partial £}{\partial \lambda} = Q^* - f(L, K) = 0 \\ \lambda - \text{ Lagrangian multiplier} \end{gathered} \tag{4}$$

And resolved gives:

19 We conduct the analysis using the term K denoting the value of capital goods. This is for convenience only as the analysis can also be conducted by using quantities of heterogeneous capital goods.

$$w = \lambda \frac{\partial Q}{\partial L}$$

$$(1+\kappa) = \lambda \frac{\partial Q}{\partial K} \qquad \textbf{(5)}$$

$$Q^* = f(L, K)$$

We can make use of the inverse of a function rule $\frac{dx}{dy} = \frac{1}{dy/dx}$ and write the term for the factor labour as

$$\lambda = w \frac{\partial L}{\partial Q} \qquad \textbf{(6)}$$

We have on the left side the Lagrange multiplier, ***λ,*** and on the right side the product of the wage rate, **w**, and marginal labour value, $\frac{\partial L}{\partial Q}$ .

It is important to know that the Lagrange multiplier, **λ**, is marginal cost, **dC/dQ**. This can be shown as follows:

From the cost equation **(1)**, $C = g(L, K)$ we can derive the total differential of cost as

$$dC = \frac{\partial C}{\partial L} dL + \frac{\partial C}{\partial K} dK \qquad \textbf{(7)}$$

and for the production function, $Q = f(L, K)$ , the total differential is

$$dQ = \frac{\partial Q}{\partial L} dL + \frac{\partial Q}{\partial K} dK \qquad \textbf{(8)}$$

Using the cost equation **(1)** we get the derivatives of cost with respect to labour and capital as

$$\frac{\partial C}{\partial L} = w$$

$$\frac{\partial C}{\partial K} = (1+\kappa) \qquad \textbf{(9)}$$

respectively. Substituting these into **(7)** gives

$$dC = w\,dL + (1+\kappa)\,dK \qquad \textbf{(10)}$$

At minimum cost, from the first order conditions **(5)** we can rewrite **(10)** as

$$dC = \lambda \frac{\partial Q}{\partial L} dL + \lambda \frac{\partial Q}{\partial K} dK$$

or

$$dC = \lambda \left[ \frac{\partial Q}{\partial L} dL + \frac{\partial Q}{\partial K} dK \right] \qquad \textbf{(11)}$$

From **(8)** we see that the term in brackets is equal to ***dQ***. Substituted in **(11)** is

$$dC = \lambda\, dQ \qquad \textbf{(12)}$$

Therefore **λ** can be interpreted as marginal cost

$$\lambda = \frac{dC}{dQ} \qquad \textbf{(13)}$$

We conclude that in an optimal economic system marginal cost is nothing else but the monetary expression of marginal labour value

$$\lambda = \frac{dC}{dQ} = w \frac{\partial L}{\partial Q} \qquad \textbf{(14)}$$

From this follows that if prices are equal to marginal costs, which is the condition for profit maximization under perfect competition

$$p_i = \lambda_i ;\, i = 1, \ldots , n \qquad \textbf{(15)}$$

prices are proportional to labour values

$$\lambda_i = \frac{dC}{dQ_i} = p_i = w \frac{\partial L}{\partial Q_i} ;\ for\, i = 1, \ldots n \qquad \textbf{(16)}$$

In orthodox economics one finds equation **(16)** expressed in the following form: the wage rate is equal to the value of the marginal product of labour

$$w = p_i \frac{\partial Q_i}{\partial L} ;\ for\, i = 1, \ldots n \qquad \textbf{(17)}$$

This is equivalent to **(16)** because $\frac{\partial Q_i}{\partial L} = \frac{1}{\partial L / \partial Q_i}$ .

From **(16)** follows relative prices are equal to marginal labour values

$$\frac{p_i}{p_1} = \frac{\partial L / \partial Q_i}{\partial L / \partial Q_1} \qquad \textbf{(18)}$$

which is the labour theory of value.

That profit maximization under conditions of constant prices (e.g. set by planning authorities) requires that marginal cost equals price can be seen using the profit function, expressing the difference of revenue over cost.

$$\Pi = p Q - C$$

$$\Pi - \text{ profits}, p - \text{ price}, Q - \text{ quantity}, C - \text{ cost} \qquad \textbf{(19)}$$

The quantity yielding maximum profits is there where marginal profit is zero:

$$\frac{d\Pi}{dQ} = p - \frac{dC}{dQ} = 0 \qquad \textbf{(20)}$$

and from this follows the first order condition for profit maximization, marginal cost equals price.

$$\frac{dC}{dQ} = p \qquad \textbf{(21)}$$

These results are extremely important also for the calculation of labour values as one simply has to divide the monetary value of some commodity by the wage rate to obtain the corresponding labour value, the labour time embodied as socially necessary labour time to produce the commodity. Furthermore we can also express demand functions in terms of labour value by dividing the ordinary demand function by the wage rate.

From this follows also that the marginal labour function (or in monetary terms, the marginal cost function) is the supply function of the competitive firm in that part of the function which lies above the average labour value function (average cost curve).

Now we prove that the marginal labour value function cuts the average labour value function at its minimum as it has been presented in **Figure 2**.

The cost equation **(1)** can be represented in terms of labour values as

$$L = \frac{C}{w} = L_v + (1+\kappa)\frac{K}{w} \qquad \textbf{(22)}$$

Now we consider that in the short term the means of production are given and so the capital costs are fixed cost as also the **κ**-rate is given. In terms of labour values we denote these embodied labour value as $\mathbf{L_f}$, recalling that it stands for the labour time used to produce the means of production, **K/w**, as well as the labour time equivalent to the cost of using this capital, **κ K/w**. We know already that the latter part represents surplus value.

According to our model the only labour that varies with the quantity produced - the variable cost - is direct labour, $\mathbf{L_v}$. This is the labour time that also turns up in the production function **(2)** but now we have given it a subscript **v** to identify it as variable labour. We can express $\mathbf{L_v}$ as a function of output because the production function is invertible and capital, **K**, is fixed, $\mathbf{\bar{K}}$.

$$L_v = h(Q, \bar{K}) \qquad \textbf{(23)}$$

The right term in equation **(22)** is fixed labour, $\mathbf{L_f}$

$$L_f = (1+\kappa)\frac{\bar{K}}{w} \qquad \textbf{(24)}$$

Substituting the expressions $\mathbf{L_v}$ and $\mathbf{L_f}$ in the equation **(22)** we get the *labour value function*[20]

$$L = L_v + L_f$$

$$\begin{gathered} L \text{ - TOTAL labour (including indirect + surplus labour)}, \\ L_v = h(Q, \bar{K}) \text{ - variable labour}, \\ L_f \text{ - fixed labour (including surplus labour)} \end{gathered} \qquad \textbf{(25)}$$

20 Notice that this is leading to the solution of Adam Smith's paradox of labour values and the adding-up-theorem of labour, profit and rent (we ignore rent) as the **L** comprises them all, including also indirect labour, stored up in the means of production (constant capital).

This function is the *labour value function* and represents total cost in terms of labour time as a function of output.

Differentiating the labour value function with respect to output, the fixed term, $\mathbf{L_f}$, is eliminated and it remains

$$\frac{dL}{dQ}=\frac{d\,L_v}{d\,Q} \qquad \textbf{(26)}$$

We see that the derivative of the labour value function, is equal to the reciprocal of the *marginal productivity of labour* as calculated via the production function **(2)**, because

$$\frac{d\,L_v}{d\,Q}=\frac{1}{\frac{\partial Q}{\partial L_v}} \qquad \textbf{(27)}$$

$\frac{\partial Q}{\partial L_v}$ - partial derivative of the production function with respect to labour

Now we are ready to find the minimum of the average cost function in terms of labour, i.e. the minimum of the *average labour value function*. The *average labour value function* is equation **(25)** divided by output, **Q**.

$$\frac{L}{Q}=\frac{L_v}{Q}+\frac{L_f}{Q} \qquad \textbf{(28)}$$

The derivative of this function with respect to **Q** is

$$\frac{d(L/Q)}{dQ}=\frac{\left[\frac{\partial L_v}{\partial Q}Q-L_v-L_f\right]}{Q^2} \qquad \textbf{(29)}$$

And as $L_v+L_f=L$

$$\frac{d(L/Q)}{dQ}=\frac{\left[\frac{\partial L_v}{\partial Q}Q-L\right]}{Q^2} \qquad \textbf{(30)}$$

The first order condition for the *average labour value function* to be at a minimum is that equation **(30)** is equal to zero. As the denominator, $\mathbf{Q^2}$, must be positive, the nominator must be equal to zero

$$\frac{\partial L_v}{\partial Q} Q - L = 0 \qquad \textbf{(31)}$$

From this follows that at the quantity which is produced with a minimum of average labour values these average labour values are equal to marginal labour value.

$$\frac{L}{Q*} = \frac{\partial L_v}{\partial Q} \qquad \textbf{(32)}$$

$Q*$ - optimal quantity of output

This is an important result as it means that a firm under perfect competition or in a planned economy produces that amount of output at which marginal labour value equals average labour value[21]. This value multiplied with the wage rate is equal to the price. Equilibrium prices in perfect competition are just monetary expressions of marginal labour values.

$$p = w \frac{L}{Q*} = w \frac{\partial L_v}{\partial Q} \qquad \textbf{(33)}$$

$L$ - average TOTAL (including indirect + surplus labour) labour ,
$Q*$ - optimal quantity of output

Notice, $p = w \frac{\partial L_v}{\partial Q}$ is equation **(17)**, combining the first order condition of minimizing cost for labour (**λ** = marginal cost equation **(6))** with profit maximization (equation **(21)**). When we use equation **(32)** and divide both sides by average labour value we obtain:

$$1 = \frac{\partial L / \partial Q}{L / Q} \qquad \textbf{(34)}$$

the inverse of which is the output elasticity of total labour, **L**

21 The Reader must be careful here. Commonly the term *average productivity of labour* refers to the productivity of *direct labour* only, the indirect labour in the capital goods (constant capital) is ignored and therefore the reciprocal of the so defined productivity is *not average labour value*. *Average labour value* is the reciprocal of the average productivity of the total labour (direct, indirect and surplus labour).

$$\varepsilon_{Q,L} = \frac{\partial Q}{\partial L}\frac{L}{Q} = 1 \quad \textbf{(35)}$$

This must be distinguished from the output elasticity of *direct labour*, $\mathbf{L_v}$, which is generally less than **1** at minimum cost

$$\varepsilon_{Q,L_v} = \frac{\partial Q}{\partial L_v}\frac{L_v}{Q} \leq 1 \quad \textbf{(36)}$$

When the output elasticity of total labour is equal to **1** we have constant returns to scale. A percentage increase of labour leads to an equal percentage increase in output. From this follows also that the sum of the output elasticities of the inputs of the production function is also equal to **1** (Euler's Theorem).

$$1 = \frac{\partial Q}{\partial L_v}\frac{L_v}{Q} + \frac{\partial Q}{\partial K}\frac{K}{Q} \quad \textbf{(37)}$$

When a factor is paid the value of its marginal product (equation **(17)**), the output elasticity of that factor is equal to the share of the factor in the value of output.

If $w = p\partial Q/\partial L$ than multiplying the nominator of **(36)** with **w** and the denominator with

$p\dfrac{\partial Q}{\partial L_v}$ we get

$$\varepsilon_{Q,L_v} = \frac{\partial Q}{\partial L_v}\frac{L_v}{Q} = \frac{w L_v}{pQ} \quad \textbf{(38)}$$

and if $(1+k) = p\dfrac{\partial Q}{\partial K}$ (from equation **(5)** setting $\boldsymbol{\lambda} = \mathbf{p}$) we get

$$\varepsilon_{Q,K} = \frac{\partial Q}{\partial K}\frac{K}{Q} = \frac{(1+k)K}{pQ} \quad \textbf{(39)}$$

The relative shares of equations **(38)** and **(39)** substituted into **(37)** gives

$$1 = \frac{w L_v}{pQ} + \frac{(1+k)K}{pQ} \quad \textbf{(40)}$$

Dividing **(33)** by **p** we get

$$1 = \frac{w L}{pQ^*} \quad \textbf{(41)}$$

or

$$p\,Q^{*}=w\,L \qquad \textbf{(42)}$$

The monetary value of output is equal to the total labour time used to produce it multiplied with the wage rate.

Multiplying **(40)** with ***pQ***, and observing that ***pQ*** = ***wL*** we get

$$p\,Q=w\,L=w\,L_{v}+(1+k)\,K \qquad \textbf{(43)}$$

This is the remarkable result that the value of output is equal to the sum of wages and (gross) profits. This is the solution of the paradox in Adam Smith's *Wealth of Nations* of the labour theory of value and the adding-up-theorem of wages, profits and rent[22].

That the value of total output is equal to wages plus profits can also be expressed in Marxian terms as being equal to the sum of variable capital, constant capital and surplus value.

$$pQ=v+c+s$$

$$v \text{ - variable capital}\,,\; c \text{ - constant capital}\,,\; s \text{ - surplus value} \qquad \textbf{(44)}$$

where $pQ=wL\,,\, v=wL_{v}\,,\, c=K\,,\, s=k\,K$ and therefore

$$pQ=v+c+s=wL+(1+\kappa)K \qquad \textbf{(45)}$$

But the rate of surplus value

$$\frac{s}{v}=\frac{k\,K}{w\,L_{v}}=k\,\frac{K/w}{L_{v}} \qquad \textbf{(46)}$$

is not unique as orthodox Marxists want it, but dependent on the wage rate, **w**, the capital-labour ratio, $\mathbf{K/L_v}$, and the **κ**-rate.

We take the labour used in production as being determined by cost minimization and profit maximization. From the inverse of the production function we can always determine the amount of

22 We have omitted rent but there is no problem of incorporating it, attributing some value to land.

direct labour to be used when the optimal quantity to be produced is known and this is determined via the first order conditions **(6).** The stock of capital has been treated as being exogenously given (short run). The wage rate is being determined on the labour market, influenced by industrial relations and collective bargaining and in our model is also treated as a fixed parameter, exogenously given. But the cost of using capital is left to be explained. Only when we are able to explain the **κ**-rate and with it surplus labour our theory of labour values is complete.

## IV. The Dynamic Characteristic of Labour Value

A crucial distinction between our labour theoretical marginal analysis and vulgar economic analysis consists in the treatment of the *cost of the use of capital*. From our point of view capital represents indirect labour, socially necessary labour, stored up in the means of production. The problem to explain properly why there is a cost involved using this sort of labour, not just replacing it, is a very difficult one indeed as neither Smith and Ricardo nor Marx, Rosa Luxemburg or Tugan-Baranovsky and his critics have managed to find the proper answer.

The problem is that the provision of the right amount of capital concerns the future and therefore it is necessary to perform a dynamic analysis, involving time, to resolve it. But this was done only rather late. Rosa Luxemburg criticised vulgar economists in not having provided a proper analysis of the accumulation of capital (Luxemburg, 1951). Although they were quite aware fo the problem they had stuck to the static analysis of markets but failed to take properly account of the dynamics of the economic system, an analysis which Marx had begun in volume II of *Capital* with his schemes of simple reproduction and the enlarged scheme of the accumulation of capital. However, Rosa Luxemburg's analysis has to be considered as “prescience” as she has not provided a consistent labour theory of value neither. Modern economic theory treats the accumulation of capital in the theory of economic growth.

We shall make use of economic growth theory in order to determine the cost of using capital. A simple starting point is that all economic processes can be represented by production functions, i.e. all outputs are the results of some use of inputs combined in production processes which use time, labour time. We further assume that production is cost-minimizing. Above we have specified the first order conditions of this optimal production. We were able to determine the optimal quantity of output to be produced as well as the quantity of the variable input labour, needed to produce it under the condition that the means of production were given, the short-run assumptions. This production is performed at a specific capital-labour ratio. In fact it is the optimal capital-labour ratio for a given wage rate.[23]

A simple approach to economic growth and capital accumulation is to assume that the labour force is given and its growth rate too, that is we treat the growth of the labour force as exogenously given. This is not a very realistic assumption but a handy one as it is obviously above our means to establish a realistic demographic theory. Furthermore we ignore all issues of labour force participation. In addition we do away with technological process or we assume it also to be of a labour augmenting kind - in the literature this is referred to as Harrod neutral technical progress - incorporated in the growth rate of the labour force. Then labour units represent efficiency units.

23 We have not explicitly worked it out in the previous section, but from the first order conditions **(6)** one can eliminate $\lambda$ and then one receives an expression where the factor price ratio, $\mathbf{w/\kappa}$, is equal to the ratio of marginal productivities of the inputs, labour and capital. The ratio of marginal productivities depends on the capital labour ratio, so there is a one-to-one relationship between relative factor prices and the optimal capital labour ratio. As capital is fixed in the short run, one can determine the optimal amount of direct labour input, the demand for labour.

When we assume the labour force growing with the proportional rate **n**, our problem becomes rather simple. We just have to assure that the optimal capital-labour ratio remains constant over time and this occurs only when capital is accumulating at the same rate **n**. We can conclude that the cost of using capital is to levy the funds in order to provide sufficient capital to assure an optimal productivity of capital in the future. So the **κ**-rate has to be equal to the rate of growth of the labour force, **n**.

At this point it is very important to emphasise that under such conditions of production with optimal capital-labour ratios, the ratios of marginal labour values of commodities being equal to relative prices, this production is Pareto-optimal as it involves the optimal allocation of economic resources and therefore the value of production cannot be augmented. Bourgeois economists just do not express Pareto-optimality in terms of labour values (Mas-Colell, Whinston & Green 1995), but once the proper meaning is understood the equivalence becomes obvious.

Bourgeois economists argue that the rate of interest, **r**, has to be taken as the cost of using capital and consequently they find that if the rate of interest is equal to the rate of growth of the labour force, than the economy is growing along the *golden rule path*. One can prove that along the golden rule path of economic growth *consumption per capita* is optimal. A less stringent condition is that the capital-labour ratio remains constant (and consequently also labour values and prices) and than the economy is growing along a *balanced growth path*, but the rate of interest, **r**, is not necessarily equal to the rate of growth of the labour force, the rest of the returns to capital are being consumed by the capitalists when the rate of interest is greater than **n**.

It is precisely here where vulgar economists commit the *faux pas.* The dynamic economic equilibrium model is a theoretical construct in order to figure out the essential relations of economic growth, i.e. the savings rate, interest rate, investment and consumption rates, accumulation of capital, technological progress and the growth of the labour force. But such a model in it's pure form cannot be taken as a realistic representation of a capitalist economic system. And this is precisely because, if and only if all returns on capital are reinvested in order to maximize the productivity of labour and all rents on natural resources are reinvested in order to maintain these resources, there can be economic growth along the golden rule path. Under such conditions there is *no exploitation* of the labourers because all revenue is used in a socially optimal way. But this is surely not the case in a capitalistic system as in this system it is the aim of the capitalists and rentiers to live on profits and rent, i.e. to exploit the labourers[24].

It is in one of the last chapters of Marx Capital (1867 (1906), chap. 24, p. 648 ff.) where Marx criticises harshly the spending habits of the new class of the bourgeoisie. With other words, capitalistic accumulation requires that the rate of return on capital is higher than the rate of growth of the labour force in order to assure balanced full-employment growth,

$$\mathbf{r} > \mathbf{n}. \qquad \mathbf{(47)}$$

Furthermore, the rate of interest is determined on the money and credit markets and there is no economic mechanism yet to assure that capital is growing at a rate guaranteeing full-employment at minimum cost. Any investment rate lower than the rate **n** increases cost and/or creates unemployment. This is known as stagflation[25].

24 This point is very important in the theory of economic growth. Maurice Allais, who has first introduced the golden rule takes the general equilibrium model as a model of capitalism (Allais 1962). Allais is not analysing a capitalist economy but an optimal one. And there he analyses the conditions for the Pareto optimality over time.

25 The Reader may observe that the period of stagflation of the 1970ies appeared after the break-down of *Bretton-Woods,* the rise of international capital movements and the failure of the labour movements to assure an appropriate rate of capital accumulation via public or wage-earner investment funds. It was the capitulation and surrender of social democracy to neo-liberalism, both determined to put an end to real-existing socialism. War, economic

In order to clarify the issues involved in the dynamic development of the economic system we introduced the **κ**-rate as a special variable. And we name it **κ**-rate in honour of Leonid V. Kantorovich who has put forward this kind of analysis in the framework of a socialist economic system where he introduced the "norm of effectiveness" as a norm for the cost of using capital (Kantorovich & Vainshtein 1976). The **κ**-rate is the rate of capital accumulation, necessary to maintain an optimal productivity of labour and full-employment over time, given a growing labour force and technological progress. The **κ**-rate is not a variable established by market forces but has to be set by economic policy authorities. The difficulty is that it deals with the future and this is unknown also to the economic policy makers. It is most likely that deviations of the market costs of using capital, **r**, from this **κ**-rate are amongst the primary factors causing the typical cyclical fluctuations of the capitalist economic systems.

## V. Critique of Marx' Conception of Labour Value

With the theoretical explanation of the **κ**-rate we have concluded our analysis of labour values in the context of an idealized general economic equilibrium model where labour values correspond to the socially necessary labour time as ultimate cost to produce a commodity. In this model labour values are proportional to prices and there is no *transformation of values into prices*; prices are simply the monetary expressions of labour values, labour values multiplied with the wage rate, the price of a unit of labour. In this context *all* surplus labour is labour time devoted to the accumulation of capital to assure the optimal productivity of labour in the context of a growing economy. All surplus labour is part of the socially necessary labour time.

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warfare, the arms race, terrorism (RAF, Action direct), and assasinations (John Lennon, Olof Palme, …) have played an important part in it.

We have contrasted this optimal economic system with the real existing capitalist system and elaborated on the fact that capitalists strive for profits (surplus labour) in order to consume them being in fundamental contradiction to the requirements of an optimal economic system. But this aspect of the capitalist mode of production is not the only feature which distinguishes the capitalistic system from an optimal economic system. Another central issue arising from the private strive for profits is the divergence of prices from actual or optimal (socially necessary) cost. In our model we have assumed that prices are parameters and the firms being unable to exercise any control over them. In practice this is rarely the case and prices are subject to deliberate manipulations. When prices depend and change with the quantity produced, the rule for profit maximization is no longer price equals marginal cost but rather marginal revenue equals marginal cost. This leads to a mark-up of prices above marginal cost and the break-down of the labour theory of value and with it the optimal use of labour and other economic resources.

Bearing in mind these qualifications of our analysis we can turn to a comparison with Marx' original theory of surplus value. Marx, like Ricardo understood his analysis as being conducted in the framework of an abstract economic system in which certain assumptions are introduced to reveal the essentials of the capitalist mode of production. His abstract model, unlike ours, was meant to serve directly to explain the capitalist system. In this sense our system is more revolutionary as it allows to distinguish between the "optimal" and the "real capitalist" economy. For Marx there was no **κ**-rate perceivable, but well an average rate of profit.

In fact, it is probably quite useful to make use of the actual average rate of profit to calculate average labour values as these labour values could be perceived as the ones towards which the actual particular values gravitate. Depending on the purpose at hand the concept of averages may be abandoned altogether taking the individual rates of profit to calculate what would be *actually socially labour time* to produce a commodity as opposed to *socially necessary labour time*.

It is very much this concept, the *actually socially labour time,* which Marx was analysing. But here he tracked himself into something difficult: he did not perceive that there is a social cost of using capital to be taken account of. He accepted that the indirect labour represented in the means of production had to be included, but that is all. When one does not consider the cost involved of using a greater or smaller value of means of production, which obviously must be calculated for in the price exceeding direct labour cost (variable capital), then the whole difference of price minus direct plus indirect labour must be profit or surplus labour and this independent of the amount of indirect labour.

In Marxian terminology, the ratio of indirect labour to direct labour is the organic composition of capital. For Marx, as for the Classics, wages were considered part of capital. Apparently this is due to the fact that labourers were considered very much as cattle, as stock belonging to the farmer. With the full establishment of wage labour and industry this was abandoned and wages were excluded from the balance sheet and taken account of in the income statement of the company. What matters is that Marx did not recognize a relationship between value of constant capital (indirect labour) and profits. But as we have shown this does exist and is determining the rate of surplus value.

To elaborate on this it is important to understand the considerations leading to the substitution of direct labour by means of production in order to augment the productivity of the remaining direct labour. Obviously cost-minimizing requires that as long as additional capital goods introduced are less expensive than the direct labour saved this substitution is continued. If no further cost of using this capital is involved the substitution is continued up to the point where the value of constant capital is just equal to the value of wages saved by reducing direct labour time.

But if cost of the use of that capital has to be also taken into account – the interest which has to be paid for it – then the substitution of capital for labour goes on only until the value plus interest to be paid for are equal to the saved wage costs. Here the interest as part of profits (or surplus labour) is proportional to constant capital. The higher the value of that constant capital the higher the surplus labour. Therefore the ratio of profits (surplus labour) to wages (variable capital), which is the rate of surplus labour in Marxian terminology, is a function of the capital-labour ratio (equation **(46)** above).

Orthodox Marxists, ignoring this argument, insist that the rate of surplus labour must be equal for all labourers because of competition amongst them. One should notice that here we have one of those 'abstractions' leading to some model which is supposed to represent the essential of capitalism. However, this modelling is less than convincing as obviously the labourer does not bother at all how the working day is divided into paid and unpaid labour time. He considers only the wage in relation to the whole labour time.

In Vol. III of *Capital,* posthumously edited and published by Engels (Marx 1894 (1909), one finds this thesis of a unique rate of surplus labour and the so called *transformation problem.* If surplus labour (surplus value) is strictly proportional to direct labour time, the working day is partitioned for all labourers in the same proportions of paid labour time and unpaid labour time, exploited by the capitalists, and then the surplus labour value (profits) cannot generally be proportional to the value of constant capital (capital goods) as is required by an average rate of profit but only in the special case of equal capital-labour ratios. Under this view there is no proportionality between labour value and cost of production (= price). It is unbelievable, but the discussion continues up to the present. Of course the discussants never refer or quote Kantorovich or Novozhilov. One must have the impression that these pseudo-theoretical debates are maintained only in order to isolate and disable critical intelligence.

Considering Marx first volume of *Capital* and the discussion of the working day and exploitation his reasoning is a very appropriate analysis deeply reflecting the daily antagonistic conflict in capitalist reality. Marx describes in detail the motives and methods of the capitalists to reduce cost by extending the working day, increasing absolute surplus value or intensifying the production processes, by this increasing relative surplus value. The capitalist does not know about the **κ**-rate or the golden rule path of economic development and he cares only about the difference between revenue and cost, selling at the highest price possible and paying the lowest wages for a working day as long as and as efficient as possible. The essential point is that that part of value which is created in the production process but not appropriated by the labourers - the producers of that value - is exploited labour.

## VI. Conclusion

It is surely a sobering insight that there is a labour theoretical foundation of orthodox economics. Marginal analysis has overcome various difficulties of classical economics, but not, as so many economists of all colours claim, by abandoning the labour theory of value. On the contrary, modern general equilibrium analysis has perfected it and insofar there is some considerable continuity in the development of economic theory. However, we should always be aware that the model of general economic equilibrium of “perfect competition” is not and cannot be in principle a representation of the real capitalist world. To reach the state of dynamic Pareto-optimality Capitalism has to be abandoned. Only where all profits and all rents are productively used and reinvested society can claim to use its economic resources in an efficient manner. But this condition is identical with the abolishment of exploitation (in production) and the overcoming of capitalism.

Let us finish with the words of Simonde de Sismondi “Je prie qu'on y fasse attention; ce n'est point contre les machines, ce n'est point contre les découvertes, ce n'est point contre la civilisation que portent mes objections, c'est contre l'organisation moderne de la société, organisation qui, en dépouillant l'homme qui travaille *de toute autre propriété que celle de ces bras* [emphasis by the editor], ne lui donne aucune garantie contre une concurrence, contre une folle-enchère dirigée à son préjudice, et dont il doit nécessairement être victime.” (Sismondi 1827, vol. 2, p 433).

It is precisely the prevention of the impoverishment of the individuals by their social emancipation, it is the collective formation of capital and the social and democratic control over capital which overcomes the antagonistic contradictions of capitalism and breaks the *ultima ratio* of the capitalists, the supply of and control over capital. There is no alternative for humanity to “Crowding out Capitalism”.

Paris, November 12, 2011

Klaus Hagendorf

# Appendix I

## The Vector of Labour Values in an Open Leontief Input-Output Model

In his book "Lectures on the Theory of Production" Luigi Pasinetti (1977) presents amongst others an analysis of the Leontief input-output models, the 'closed', the 'open' as well as the dynamic Leontief models. We concentrate only on the open Leontief model as this is most suitable to compute labour values.

The input-output model is a simplified general economic equilibrium model assuming linear constant returns to scale production functions.

The basic equation of the open Leontief model, considering quantities is

$$\boldsymbol{A}\boldsymbol{x}+\boldsymbol{y}=\boldsymbol{x}$$

$\boldsymbol{A}$ - Technology matrix (n x n square matrix of capital-output coefficients), **(AI.1)**
$\boldsymbol{x}$ - vector of quantities of n products produced in n sectors,
$\boldsymbol{y}$ - vector of quantities of n products for final use

The first term of the equation, **Ax**, represents the intermediate consumption of **n** commodities used up in the **n** sectors of the system in order to produce **y** quantities of the products for final use. **x** is the vector of the total quantities which have to be produced overall in order to have **y** quantities available for final use. Assumed is that all products may serve as inputs in the production of all output. Solving for the quantities of final use we get

$$\boldsymbol{y}=\boldsymbol{x}-\boldsymbol{A}\boldsymbol{x}=[\boldsymbol{I}-\boldsymbol{A}]\boldsymbol{x} \qquad \textbf{(AI.2)}$$

And this resolved for total output, **x**, is

$$\boldsymbol{x} = [\boldsymbol{I} - \boldsymbol{A}]^{-1}\,\boldsymbol{y} \qquad \textbf{(AI.3)}$$

This shows the total quantities of output which have to be produced in order to have the quantities **y** available for final use.

Analogously we present the value system as

$$\boldsymbol{p} = \boldsymbol{p}\,\boldsymbol{A} + \kappa\,\boldsymbol{p}\,\boldsymbol{A} + w\,\boldsymbol{a}_n = (1+\kappa)\,\boldsymbol{p}\,\boldsymbol{A} + w\,\boldsymbol{a}_n$$

$\boldsymbol{p}$ - vector of prices of n commodities ,
$\kappa - \kappa$-rate (rate of capital accumulation) ,
$w$ wage rate , $\boldsymbol{a}_n$ - labour coefficients

**(AI.4)**

This price equation represents the cost of production. The **κ**-rate represents the cost of using capital. The cost of production consist of the cost of the capital goods used up in production, **pA**, the cost of using capital in production, **κpA**, and wages, $\mathbf{wa_n}$. In Marxian terminology **pA** is constant capital, **κpA** profits, and $\mathbf{wa_n}$ variable capital.

The solution of this value system is

$$\boldsymbol{p} = w\,\boldsymbol{a}_n[\boldsymbol{I} - (1+\kappa)\,\boldsymbol{A}]^{-1} \qquad \textbf{(AI.5)}$$

This model can be used to determine the total amount of labour time needed as inputs to produce a unit of output, the socially necessary labour time, $\boldsymbol{v}^{i.}$

The total labour time used as inputs of production is equal to the labour time needed to produce the commodities which serve as inputs plus the direct labour time used up to produce a unit of output

$$\boldsymbol{v^i} = \boldsymbol{v^i}\,\boldsymbol{A} + \boldsymbol{a_n} \qquad \textbf{(AI.6)}$$

$\boldsymbol{v^i}$ - vector of labour time per unit

The expression $\mathbf{v^iA}$ stands for the indirect labour stored up in the means of production whereas $\mathbf{a_n}$ is the direct labour per unit of output. Solved for $\mathbf{v^i}$ we get

$$\boldsymbol{v^i} = \boldsymbol{a_n}[\boldsymbol{I} - \boldsymbol{A}]^{-1} \qquad \textbf{(AI.7)}$$

Equation **(AI.7)** is commonly interpreted by Western Marxists as *average labour values* because it represents all labour time used as inputs in the production of outputs. However, this is a very serious error of Western Marxism as it is clear that this equation does not include surplus labour, it just includes the paid labour, as $\mathbf{wa_n}$ is wages (per unit of output), and it includes constant capital in terms of labour values $\mathbf{v^iA}$, but surplus labour is not included. Indeed, surplus labour is that part of labour which is embodied in the *cost of using capital*, $\mathbf{\kappa v^iA}$, which corresponds to the money value $\mathbf{\kappa pA}$ in the cost of production equation **(AI.4)** above. In fact $\mathbf{p = w\, v^i}$.

Pasinetti treats the vector $\mathbf{v^i}$ in equation **(AI.7)** as the vector of *vertically integrated labour coefficients*, and that's what it is, all the labour directly used up as inputs in the production of output, ignoring the cost of the use of capital, or in other words, providing for growth.

We defend the thesis that labour values must include the cost of the use of capital, they must represent all cost in terms of labour time and the use of capital does cost labour time, i.e. $\mathbf{kv^iA}$. The proper definition of *average labour values* using the Leontief input-output model is therefore

$$v = a_n[I-(1+k)A]^{-1}$$

$v$ - vector of vertically integrated labour coefficients,
$a_n$ - vector of direct labour coefficients,
$k$ - k-rate (in orthodox theory the rate of interest, r)
$A$ - technology matrix (capital-output coefficients), **(AI.8)**

Equation **(AI.7)** defines labour values only for a stationary, static economic system where there is no economic growth.

To put the point more clearly, let's suppose that there is an economy with continuous production functions (neoclassical production functions) in all of its ***n*** sectors. In equilibrium the allocation of its resources is optimal. One can still analyse this economic system in terms of linear algebra and describe it with the technology matrix ***A*** and the vector of labour inputs $\boldsymbol{a_n}$. We then arrive at equation **(V.3.1a)** p. 73 in the "Lectures", our cost of production equation **(AI.4)**

$$(1+\kappa)pA + a_n w = p$$

in Pasinetti r is used instead of $\kappa$ **(AI.9)**

and this can be written as

$$p = a_n[I-(1+\kappa)A]^{-1}w \qquad \textbf{(AI.10)}$$

This corresponds to equation **(V.5.18)** p. 80 in the 'Lectures'.

It is important to realize that under the assumptions above, the row vector

$$v = a_n[\boldsymbol{I} - (1+\kappa)\boldsymbol{A}]^{-1}$$

represents *average labour values* and is equal to the vector of *marginal labour values*

$$v = a_n[I - (1+\kappa)A]^{-1} = [\partial L/\partial x_1, \ldots, \partial L/\partial x_n] \qquad \textbf{(AI.11)}$$

where $\partial L/\partial x_i$ is the *marginal labour value* of sector **i**.

That this must be so can easily be shown. If labour is optimally allocated, the uniform wage rate is equal to the value of the marginal product of each sector.

$$w = p_i \partial x_i/\partial L\,,\ for\, i = 1,2,\ldots,n \qquad \textbf{(AI.12)}$$

We can write equation **(AI.10)** as

$$p = a_n[I - (1+\kappa)A]^{-1} w\, I \qquad \textbf{(AI.13)}$$

***w I*** is a diagonal matrix with the wage rate on its major diagonal. We replace the wage rate for each sector by its value of the marginal product $p_i \partial x_i/\partial L$ and call that matrix ***W*** so that our equation **(AI.13)** becomes

$$p = a_n[I - (1+\kappa)A]^{-1} W \qquad \textbf{(AI.14)}$$

**W** being

$$W = \begin{bmatrix} p_1 \dfrac{\partial x_1}{\partial L} & 0 & \ldots & 0 \\ 0 & p_2 \dfrac{\partial x_2}{\partial L} & \ldots & 0 \\ 0 & 0 & \ldots & \ldots \\ 0 & 0 & \ldots & p_n \dfrac{\partial x_n}{\partial L} \end{bmatrix} \qquad \textbf{(AI.15)}$$

Now it is evident that the elements of $a_n[I-(1+\kappa)A]^{-1}$ must be the marginal labour values as in **(AI.11)** to cancel out with the marginal productivities of $\boldsymbol{W}$ to yield the price vector $\boldsymbol{p}$.

The vector of marginal labour values can also be represented as a power series. Equation **(AI.11)** can be written as

$$v=[\partial L/\partial x_1,\ldots,\partial L/\partial x_n]=a_n[I-(1+\kappa)A]^{-1}=a_n+(1+\kappa)a_nA+(1+\kappa)^2a_nA^2+\ldots \qquad \textbf{(AI.16)}$$

in which the elements $\kappa a_n A ; \kappa^2 a_n A^2, \ldots$ represent surplus labour.

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