A new method to estimate the risk of financial intermediaries

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Abstract

In this paper we reconsider the formal estimation of the risk of financial intermediaries. Risk is modeled as the variability of the profit function of a representative intermediary, here bank, as formally considered in finance theory. In turn, banking theory suggests that risk is determined simultaneously with profits and other bank- and industry-level characteristics that cannot be considered predetermined when profit maximizing decisions of financial institutions are to be made. Thus, risk is endogenous. We estimate the model on a panel of US banks, spanning the period 1985q1-2010q2. The findings suggest that risk was fairly stable up to 2001 and accelerated quickly thereafter and up to 2007. Indices of bank risk commonly used in the literature do not capture this trend and/or the scale of the increase.

Keywords: Risk of financial intermediaries; Endogenous risk; Full information maximum likelihood, Profit function, Duality

JEL codes: G21; C51; C33
1. Introduction

The financial crisis that erupted in 2007 turned the spotlight on financial institutions and their management of the risks they face. A fundamental and timely question is how the total (solventy) risk of a financial intermediary should be measured. In this paper, we propose a new method to estimate this risk, using the profit function and the implications of standard economic and banking theory. The model is quite general and, in fact, applies to all firms. An important element in our framework is that risk is endogenous to internal factors, such as managerial decisions, and external factors, such as the macroeconomic environment. This novelty is essential because, in the literature, measurement of the risk of financial institutions is usually based on accounting ratios that cannot capture this type of simultaneity, nor do they seem to capture the level of risk and its upward trend during the 2000s.

Building on economic theory, we use the implications of the portfolio selection models developed by Markowitz (1952) and Roy (1952), and extended by many others, where the estimates of the simple variance of profits, or the downside variance, can be used to measure risk. In particular, this theory suggests that if an overall measure of risk is sought in the context of expected utility, that measure should be related to the variability of profits or the variability of factors determining the profit function. In this literature, such measures are employed primarily to model asset prices and portfolio value. Here we use the profit function to describe the technology of financial institutions in the context of duality theory.

We augment this framework with the implications of intermediation (banking) theory, which suggests that risk decisions of financial intermediaries are simultaneously made with perceptions on expected profits and, in addition, are affected by certain characteristics of a
bank’s balance sheet and the state of the economic environment. This simultaneity calls for a class of models, where risk is jointly determined along with (i) other decisions of the financial institutions (e.g. concerning the level of capitalization and/or liquidity) and (ii) the macroeconomic environment. To this end, the variability of profits should be endogenous to profits themselves and potentially to other bank-level variables or the structural and macroeconomic conditions. Therefore, an important advantage of the approach presented here is that technology, risk, bank decisions and structural and macroeconomic conditions can be modeled jointly.

The new method is quite general and can, in fact, be applied to any firm. Here, we focus on financial institutions and, in particular banks, because of the clear implications of banking theory concerning the endogeneity discussed above, the important developments in the banking sector before and after the subprime crisis and the key role banks play in the real and the monetary economic spectrum. One important concern for our modeling choice was not to impose more stringent data requirements on the researcher.

The model is applied to a large panel of US banks that covers the period 1985q2-2010q1. The results indicate that bank risk was relatively stable up to 2001 and gradually increased by more than 200%, since then. This pattern is robust, irrespective of the functional form used to estimate the profit function and the variables included to tackle simultaneity. Thus, the new measure captures the buildup of bank risk way before the eruption of the financial turmoil in 2007. In this respect, and besides having a clear theoretical basis, the new measure represents a better alternative to measures widely employed in banking studies to measure risk that, as we

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1 This is recognized by Shrieve and Dahl (1992), Diamond and Rajan (2000), Dangl and Zechner (2004), Berger and Bouwman (2009), Flannery and Rangan (2008), Freixas and Rochet (2008), Degryse et al. (2009), among many others.
show below, do not seem to capture the buildup of risk during the 2000s and/or its substantial increase.

The rest of the paper proceeds along the following lines. Section 2 presents the formal econometric model. Section 3 discusses the application to the US banking sector and presents the empirical findings. Section 4 concludes the paper and offers some ideas for potential extensions.

2. The model

A quite important problem faced by the empirical researcher in estimating technology functions of financial intermediaries is that risk should be endogenously determined. The banking theory behind this issue is straightforward. The level of risk is set by bank managers in a way that encompasses information about the level of expected profits, the level of capital and liquidity that banks hold and the state of the regulatory and macroeconomic environment. Therefore, one cannot suggest that risk determines *stricto sensu* current bank profits. In fact, the perceived optimal level of bank risk is simultaneously determined with current profits, also taking into account other endogenous and predetermined variables. This modeling choice, even though fundamental for the robust estimation of the risk of financial intermediaries, is absent in the empirical literature.

Here, we present a model that uses the profit function to estimate endogenous bank profits. We model a representative bank but, this model may, in fact, be applied in its general form to any firm. Bank risk depends on certain endogenous variables and it is itself considered to be endogenous in the profit function. The rest of the endogenous variables are determined in the context of a simultaneous equation model and also depend on profits as well as risk. Therefore, the model is, in fact, very general as it considers all potential types of endogeneity.
We consider a restricted normalized profit function of the form:

\[ y_i = \beta_1' x_{i1} + \beta_2' z_i + \sigma_i v_i, \text{ for } i = 1, \ldots, N, \]  

(1)

where \( y_i \) represents profit of bank \( i \), \( x_{i1} \) is a standard \( k \times 1 \) vector of covariates in the profit function, \( z_i \) is a \( G \times 1 \) vector of endogenous variables, \( v_i \sim N(0,1) \) is the error term, and \( \sigma_i^2 \) is the variance of profits to be estimated. Following the portfolio selection theory, we consider the estimates of profit variability as a formal measure of risk.\(^2\)

Assume the following additional specification for the variance of the profit function:

\[ \sigma_i^2 = f(z_i, \gamma), \]  

(2)

where \( z_i \) is a \( G \times 1 \) vector of variables that determines the risk of banks, \( \gamma \) is a vector of parameters to be estimated and \( f(z_i, \gamma) \) is a functional form differentiable in \( z_i \). For example, \( f \) can take the form \( \sigma_i^2 = z_i' \gamma \) or \( \sigma_i^2 = \exp(z_i' \gamma) \), etc. Note that, despite the fact that we use a “cross-sectional notation”, panel data models of the form \( \sigma_{it}^2 = f(z_t, \sigma_{i,t-1}^2, \ldots, \sigma_{i,t-L}^2; \gamma) \) are fully nested within our general specification in Eq. (2).\(^3\) The dependence on \( y_i \) will be discussed below.

Up to this stage, we formally identify risk with the variability of profits and explain this variability in terms of a vector of variables in \( z_i \). If these variables were predetermined or exogenous, estimation of the profit function in (1) subject to (2) would be straightforward using the method of maximum likelihood. Unfortunately, this is a very strong assumption for financial institutions’ risk-setting behavior, since the \( z_i \)s represent firm (bank) characteristics that are simultaneously determined with the level of risk in the following way:

\[ z_i = f(x_{i2}, y_i, \sigma_i^2). \]

(3)

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\(^2\) Of course, after estimation one could consider only the downside variance of profits as a measure of bank risk.

\(^3\) This also includes the formal possibility of fixed effects in (1) and (2).
For example, bank managers set the optimal level of risk given the levels of capitalization, liquidity, etc, which are naturally included in \( z \). This simultaneity of the \( z \)s with bank risk is a notorious element in the banking literature and should be accounted for in any attempt to estimate risk robustly. Further, risk and other characteristics of bank balance sheets are heavily affected by the regulatory or macroeconomic conditions prevailing at each point in time. Therefore, these elements might also be included in \( z \). Thorough discussions of these issues can be found in the literature cited in footnote 1.

To account for this endogeneity, assume the following general simultaneous equation model:

\[
\Gamma z_i = Bx_{12} + \varphi_1(y_i) \lambda_1 + \varphi_2(\sigma_i^2) \lambda_2 + u_i, \ u_i \sim N(0, \Sigma), \tag{4}
\]

where \( x_{12} \) is a \( k_2 \times 1 \) vector of explanatory variables, which can include \( x_{11} \). Here, \( \varphi_1 \) and \( \varphi_2 \) are known univariate differentiable functions (for example \( \varphi_j(w) = w \) or \( \varphi_j(w) = \log w, \ j = 1, 2 \)), \( \lambda_1, \lambda_2 \) are \( G \times 1 \) vectors of coefficients, and \( \Gamma \) and \( B \) are \( G \times G \) and \( G \times k_2 \), respectively. Of course, restrictions are assumed in place for \( \Gamma \) and \( B \) in view of identification. For example, the diagonal elements of \( \Gamma \) are assumed to be equal to 1 and this matrix must be nonsingular. Moreover, the variance \( \sigma_i^2 \) may depend also on \( x_{12} \).\(^4\)

For simplicity, we can write \( \Gamma z_i = Bx_{12} + \varphi_1(y_i) \lambda_1 + \varphi_2(\sigma_i^2) \lambda_2 + u_i \equiv B^* x_{12}^* + u_i \). To begin with, we assume \( \lambda_1 = \lambda_2 = 0_{(G \times 1)} \). Then

\[
p(y_i \mid z_i) = \left(2\pi\sigma_i^2\right)^{-1/2} \exp \left[-\frac{(y_i - \beta_1 x_{i1} - \beta_2 z_i)^2}{2\pi\sigma_i^2}\right] \tag{5}
\]

\(^4\) The variance may also depend on \( y_i \). The Jacobian of transformation from \( v_i \) to \( y_i \) can be formally computed. This possibility has been recognized before by Rigobon (2003).
and
\[ p(z_i) = (2\pi)^{-G/2} |\Sigma|^{1/2} \|\Gamma\| \exp \left[ -\frac{1}{2} \left( \Gamma z_i - B^* x_{i2}^* \right)' \Sigma^{-1} \left( \Gamma z_i - B^* x_{i2}^* \right) \right] \] (6)

Therefore, the joint distribution of the observed endogenous variables is
\[ p(y_i, z_i) = (2\pi)^{-(G+1)/2} f(z_i; \gamma)^{-1/2} \exp \left[ -\frac{1}{2} \left( \frac{y_i - \beta_i' x_{i1} - \beta_i' z_i}{2} \right)' \left( \frac{y_i - \beta_i' x_{i1} - \beta_i' z_i}{2} \right) \right] \left| \Sigma \right|^{-1/2}. \] (7)

This likelihood function can be maximized using standard numerical techniques. Formal concentration with respect to parameters \( B^* \) and \( \Sigma \) is also possible so the problem can be simplified in terms of maximizing the log-likelihood function of the sample.\(^5\)

In the general case, where \( \lambda_1, \lambda_2 \neq 0 \), the formulation of \( p(y_i | z_i) \) is straightforward but, the formulation of the inverse distribution \( p(z_i | y_i) \) or \( p(z_i) \) is not trivial. The Jacobian of transformation is given by
\[ D_i = \left\| \frac{\partial (y_i, u_i)}{\partial (y_i, z_i)} \right\| = f(z_i; \gamma)^{-G/2} \left\| \Gamma - \phi \lambda_2 \frac{\partial f(z_i; \gamma)}{\partial z_i'} - \phi \beta_2 \right\|. \] (8)

after accounting for the fact that the variance depends itself on endogenous variables (the \( z, s \)). If \( \sigma_i^2 = z_i' \gamma \), then \( \frac{\partial f(z_i; \gamma)}{\partial z_i'} = \gamma' \). If \( \sigma_i^2 = \exp(z_i' \gamma) \), then \( \frac{\partial f(z_i; \gamma)}{\partial z_i'} = \exp(z_i' \gamma) \gamma' \).

In this case, we have

\(^5\) The details are available on request from the authors.
\[ p(y, z_i) = (2\pi)^{-(G+1)/2} f(z_i; \gamma)^{-G/2} \exp \left[ -\frac{(y_i - \beta'_1 x_i - \beta'_2 z_i)^2}{2\pi f(z_i; \gamma)} \right]. \] (9)

\[ |\Sigma|^{-1/2} \cdot \left[ \Gamma - \varphi'_1 \lambda_2 \frac{\partial f(z_i; \gamma)}{\partial z'_i} - \varphi'_1 \lambda_2 \beta'_2 \right] \cdot \exp \left[ -\frac{1}{2} \left( (\Gamma z_i - B' x'_2)^' \Sigma^{-1} (\Gamma z_i - B' x'_2) \right) \right]. \]

The simplest case is when \( \varphi_1(w) = \varphi_2(w) = w \), and \( f(z_i; \gamma) = z'_i \gamma \). In this case the Jacobian term is simply \( \left\| \Gamma - \lambda_2 \gamma' - \lambda_1 \beta'_2 \right\| \), where \( \lambda_2 \gamma' \) and \( \lambda_1 \beta'_2 \) are rank-one \( G \times G \) matrices. Of course, if \( \lambda_1 \) or \( \lambda_2 \) (or possibly both) are zero, further simplifications arise. The typical case is to have profits, \( y_i \), and the variance, \( \sigma_i^2 \), appearing as determinants of the \( z_i \)s. Part of the reason may be that not all banks have positive profits so that we cannot consider the log of \( y_i \). However, one may have \( \varphi_2(w) = \log w \), with \( \varphi'_1(w) = w^{-1} \). In that case, the Jacobian would be

\[ D_i = \left\| \Gamma - f(z_i; \gamma)^{-1} \lambda_2 \frac{\partial f(z_i; \gamma)}{\partial z'_i} - \lambda_1 \beta'_2 \right\|. \] (10)

In terms of our model, it is instructive to provide a simple example to show that risk can also be a function of profits \( (y_i) \). Indeed, consider for simplicity the following “mean-scale” model \( y_i = \mu + \sigma (y_i) v_i \), where \( v_i \sim N(0,1) \). Apparently, the Jacobian of transformation is

\[ \left\| \frac{\partial y}{\partial y} \right\| = \left| \frac{\sigma(y) - \sigma'(y)(y - \mu)}{\sigma(y)^2} \right| \] (11)

and the density of \( y \) would be

\[ p(y) = (2\pi)^{-1/2} \exp \left[ -\frac{(y - \mu)^2}{2\sigma(y)^2} \right] \left| \frac{\sigma(y) - \sigma'(y)(y - \mu)}{\sigma(y)^2} \right|. \] (12)

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\(^6\) One may think that specifying \( \varphi_2(w) = \log(w) \) is better, since variances are restricted to being positive. This is, of course, correct. However, a large part of the literature on GARCH models simply ignores this constraint and adopts the assumption \( \varphi_2(w) = w \), using parametric restrictions (on \( \gamma \)) to ensure positive variances.
The Jacobian is nonzero, provided $\sigma(y)$ is not a solution of the difference equation

$$\sigma(y) - \sigma'(y)(y - \mu) = 0,$$

that is $\sigma(y)^2$ should not be equal to $C(y - \mu)^2$, where $C$ is a constant. Other specifications for the variance term would be acceptable, for example $\sigma(y)^2 = C_1 + C_2(y - \mu)^2$, $C_1 > 0$. This shows that, in terms of our model, risk can be a function of profits $(y_i)$ themselves, despite the fact that profits are also determined by risk. In that sense, we allow for joint determination of risk and profits.\(^7\)

Suppose, indeed, that $\sigma_i^2 = z'_i \gamma + \alpha y_i$. Then, relative to (8), the only difference is that the Jacobian term is $\Gamma - \varphi'_1 \lambda_1 y' - (\varphi'_2 \lambda_2 + \alpha \varphi'_2 \lambda_2) \beta'_2$. If $\lambda_2 = 0$, the new formulation does not add anything to the Jacobian, otherwise, the contribution depends on $\alpha = \frac{\partial f(z, y_i; y)}{\partial y_i}$.

3. Empirical application to the US banking sector

3.1. Data and empirical setup

Since the recent crisis originated in the USA, focusing on the US banking sector is a natural choice for a contemporary case study of bank risk. The idea here is to develop the new metric without imposing more stringent data requirements compared to the usual empirical research paper. A quick look in the literature will reveal that the most widely used database for studies of the US banking system is the one from the FDIC Call reports.

We build an unbalanced dataset that includes information for commercial banks over the period 1985q1-2010q2. We start from the complete sample of banks in the Call reports, but we

\(^7\) This is different from a GARCH-M type model, where the lagged variance, typically, enters into the mean equation. Here, the current variance can also enter the mean equation, provided that a proper adjustment for the Jacobian term is made. This point seems to be unpublished, at least, to our knowledge.
apply two selection criteria. First, we delete all observations for which data on any of the variables used in our study are missing. Second, we apply an outlier rule to the variables used, corresponding to the 1st and 99th percentiles of the distributions of the respective variables. This deletes extreme values that may drive the results. The final sample consists of 814,253 bank-quarter observations.

We estimate the system of Eqs. (1), (2) and (3), using both a log-linear and a translog specification. Following the paradigm of Humphrey and Pulley (1997) and Koetter et al. (2011), we use an alternative profit function that models profits as a function of outputs and input prices. We provide formal definitions for the variables used to estimate the profit function in Table 1 and summary statistics in Table 2. To define outputs and inputs we follow the intermediation approach.\(^8\) As profits contain both positive and negative values, taking logs of profits becomes an issue. We, primarily, use the approach of Bos and Koetter (2011), who left-censor \(y\) and construct a negative profit indicator variable, say \(y_1\), as an additional right-hand side variable.\(^9\)

As discussed above, we assume that the variance of profits (risk) is endogenous to profits themselves and other bank or industry characteristics. Bank characteristics used as \(z\) are the basic equity capital ratio (total equity capital to total assets) and/or a liquidity ratio (liquid assets to total assets). Therefore, we assume that banks make risk decisions simultaneously with the levels

\(^8\) We impose linear homogeneity by dividing profits and input prices by \(w_3\). One could include securities and non-interest income or off-balance sheet items as outputs. This would reduce the time frame of the analysis from 1997 onwards. Changes in average values of estimated risk are not larger than 5%, thus, we choose to use the full sample period.

\(^9\) For banks that exhibit positive profits, \(y_1=1\) and for banks that exhibit negative profits, \(y_1=\)absolute value of these negative profits. The left hand side variable, \(y\) equals the true value of \(y\) for positive profits, and \(y=1\) for negative profits. Following relevant literature, we also carry out sensitivity analysis by (i) using only positive profits and (ii) adding up the maximum negative profits observed in our sample to all banks plus 1 (to make an index of only positive profits). We report the results from the method of Bos and Koetter (2011) but, the rest are available on request.
of capitalization and/or liquidity in their balance sheets.\textsuperscript{10} Besides $z$, in all models, risk is endogenously determined by inst1, the first lags of pre1 and pre2 and profits, themselves. Further, banks might decide upon their level of risk, given changes in the macroeconomic environment. Therefore, we also use the three month T-bill rate and the industrial production index as determinants of bank risk. Both these macroeconomic variables enter Eq. (2), lagged once to allow information to reach the market and, therefore, we consider them to be predetermined variables.\textsuperscript{11}

To identify $z$, we assume that capital and liquidity ratios also depend on inst1 and the fourth lag of inst2. This is a reasonable assumption in the literature of the determinants of bank capital and liquidity (e.g., Flannery and Rangan, 2008; Berger and Bouwman, 2009). Including other determinants of $z$ is plausible and in specification (4) we also use the macroeconomic variables; however, since identification is achieved, including more variables as determinants of $z$ is beyond the scope of the present paper.

3.2. Empirical results

Table 3 reports estimation results for the main determinants of the risk equation, which show very good fit. The results on the rest of the coefficients are available on request. We report the results for four specifications. The first two are log linear specifications and the last two are translog specifications.

\textsuperscript{10} To motivate this, consider two banks with similar initial risk levels but, different levels of capitalization or liquidity. Now if e.g. an exogenous shock hits the banking sector, the more liquid or capitalized bank will be able to buffer risk more easily, while the less liquid or less-capitalized bank will have to re-determine its risky position to a greater extent. Many other similar arguments can be found in Freixas and Rochet (2008) and Degryse et al. (2009).

\textsuperscript{11} One can, very easily, experiment with many other predetermined variables to be included in Eq. (2) and examine the sensitivity of the results. We experimented with some regulatory dummies, characterizing major regulatory events, with institutional variables, etc. The main results are unaffected. Thus, because our main effort here is to measure risk and not analyze an exhaustive list of its determinants, we decided to keep the empirical framework as simple as possible.
The results of interest are those on the variance of the profit function, which in our model represents individual bank risk. In Figure 1, we plot the quarterly average of the bank-quarter values of risk (log of variance) obtained from the four specifications. All models capture the increase in bank risk that took place in the 2000s. Irrespective of the functional form used, or whether we specify \( z_1 \) or \( z_2 \) as endogenous, bank risk was fairly stable until 2001 and increased more than 200%, thereafter. This pattern is robust to the inclusion of equity capital or a time trend also inside Eq. (1), and alternative determinants of \( z_1 \) or \( z_2 \) in Eq. (3).

In fact, we identify only two different patterns of risk through time. The first, which is quantitatively less important, comes from the specification with liquidity as \( z \) (line 2 on Figure 1) instead of equity capital (line 1). The specification with liquidity shows that risk reached its maximum as soon as early 2005 and remained at very high levels until the end of our sample period. In contrast, line 1 shows that risk was increasing up until 2009. If we add both \( z_1 \) and \( z_2 \) into the same model, the results are very close to those reflected by line 2. Also, the specification with \( z_2 \) shows a higher value of risk. This pattern is explained by the presence of capital requirements in the US banking sector as early as 1989. The capital requirement does not allow bank capital to fluctuate as much as liquidity, which is subject to only limited regulation. Therefore, bank liquidity is, probably, a more important factor in determining bank risk and is the one used in the rest of the specifications reported in Table 3.

The second difference comes from using a translog specification, as opposed to a log-linear one. The flexibility of the translog profit function captures a decline in the variability of profits after the eruption of the crisis in 2007 (see lines 3 and 4). This looks sensible, as banks

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12 One could in fact estimate bootstrap confidence intervals for the values reported in Figure 1. These are available on request.
13 An alternative would be to use the distance of equity capital from the minimum requirement. When doing so, the results are, indeed, closer to those with the use of the liquidity ratio.
started lowering their exposure to very risky assets, as soon as they could, after the eruption of
the crisis, while prudential regulation became tighter with an increased number of inspection
audits and sanctions. However, we should note that risk remains quite high, compared to the
period before 2001. Given the above evidence, we favor the translog specification.

The value of the new method proves quite significant if one compares the results to
indices of bank risk, widely used in the relevant empirical literature. In Figure 2 we report four
such indices, namely (i) the ratio of risky assets to total assets (ra), (ii) the z-index defined as
\((\text{inst2}+z1)/\sigma(\text{inst2})\) and using a 12 quarter window to calculate the variance, (iii) the ratio of loan
loss provisions to total loans (llp) and (iv) the ratio of non-performing loans to total loans (npl).
We graph both the industry and bank average by quarter (see the figure’s legend for a definition).
Evidently, the first two measures cannot capture the extent of the increase in bank risk, while the
requirement for the z-index to include information from the past might bias the true level of
current risk. In addition, to calculate the variance for the z-index one needs to trim the sample,
which might be important in e.g. studies using annual data. The last two indices seem to
completely fail to capture the upward trend of risk prior to 2007 and, in fact, the bank-level
averages (which are the ones employed in bank panel data studies and are denoted by the dashed
lines) completely fail to even capture a significant increase in bank risk. Further, in Table 4 we
report simple correlation coefficients between the values of the four newly constructed indices
(r1 to r4) and the four existing indices (ra, z-index, llp, npl). Evidently, correlation coefficients
between the newly constructed indices and the existing ones are very low. We attribute the
limitations of existing indices to the fact that they do not follow standard economic theory (with
the exception of z-index), to the fact that they reflect a static picture of accounting data and to
their inability to account for the endogeneity/simultaneity issue.
4. Conclusions

This study proposes a new method for the estimation of the risk of financial institutions, which is very general and can be applied to any firm. Two important features of the model are that it is based on standard economic theory and that risk is endogenously determined with certain characteristics of the intermediary and with the macroeconomic environment. Unlike measures of risk, widely used in the empirical literature, the new method captures the perceived increase of bank risk after 2001 and shows that this increase was gradual and higher than 200%. This comparison renders the results of previous literature on bank risk, and its determinants, questionable.

The results of the model could be very easily used to calculate downside variance or look at the standard deviation of expected profits in a fashion similar to the Sharpe ratio. Another natural extension to this paper is to use the model for the estimation of risk of other types of firms or financial intermediaries. On the econometric front, the determination of stochastic risk through a CARCH-type process for the variance seems, also, a reasonable extension. We leave these for future research.

References


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Table 1
Definitions of variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank profits</td>
<td>y</td>
<td>Total profits before tax (US$)</td>
</tr>
<tr>
<td>Output 1</td>
<td>o1</td>
<td>Commercial and industrial loans (US$)</td>
</tr>
<tr>
<td>Output 2</td>
<td>o2</td>
<td>Loans to individuals (US$)</td>
</tr>
<tr>
<td>Output 3</td>
<td>o3</td>
<td>Loans secured by real estate (US$)</td>
</tr>
<tr>
<td>Output 4</td>
<td>o4</td>
<td>Other loans (US$)</td>
</tr>
<tr>
<td>Input price 1</td>
<td>w1</td>
<td>Salary expenses/ total assets</td>
</tr>
<tr>
<td>Input price 2</td>
<td>w2</td>
<td>Interest expenses/ total deposits</td>
</tr>
<tr>
<td>Input price 3</td>
<td>w3</td>
<td>Expenses on fixed assets/total fixed assets</td>
</tr>
<tr>
<td>Endogenous variable 1</td>
<td>z1</td>
<td>Equity capital/ total assets</td>
</tr>
<tr>
<td>Endogenous variable 2</td>
<td>z2</td>
<td>Liquid assets/ total assets</td>
</tr>
<tr>
<td>Predetermined variable 1</td>
<td>pre1</td>
<td>3-month T-bill rate (in %)</td>
</tr>
<tr>
<td>Predetermined variable 2</td>
<td>pre2</td>
<td>US industrial production index</td>
</tr>
<tr>
<td>Instrument 1</td>
<td>inst1</td>
<td>Bank size calculated as the natural log of total assets</td>
</tr>
<tr>
<td>Instrument 2</td>
<td>inst2</td>
<td>Bank profitability calculated as profits before tax/ total assets</td>
</tr>
</tbody>
</table>

Notes: Variables y, o1, o2, o3, o4 and inst1 are in real terms.

Table 2
Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min.</th>
<th>Max.</th>
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</thead>
<tbody>
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<td>y</td>
<td>5,395.9</td>
<td>121,227.6</td>
<td>-1.81e+07</td>
<td>2.30e+07</td>
</tr>
<tr>
<td>o1</td>
<td>70,340.8</td>
<td>1,143,058</td>
<td>1</td>
<td>1.42e+08</td>
</tr>
<tr>
<td>o2</td>
<td>42,828.3</td>
<td>761,145.9</td>
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<td>9.43e+07</td>
</tr>
<tr>
<td>o3</td>
<td>178,448.6</td>
<td>3,168,042</td>
<td>1</td>
<td>4.75e+08</td>
</tr>
<tr>
<td>o4</td>
<td>33,906.3</td>
<td>798,096.5</td>
<td>1</td>
<td>8.88e+07</td>
</tr>
<tr>
<td>w1</td>
<td>0.0099</td>
<td>0.0053</td>
<td>0.0017</td>
<td>0.0325</td>
</tr>
<tr>
<td>w2</td>
<td>0.0248</td>
<td>0.0147</td>
<td>0.0027668</td>
<td>0.0733</td>
</tr>
<tr>
<td>w3</td>
<td>0.0027</td>
<td>0.0018</td>
<td>0.000198</td>
<td>0.0119</td>
</tr>
<tr>
<td>z1</td>
<td>0.0960</td>
<td>0.0298</td>
<td>0.032091</td>
<td>0.4600</td>
</tr>
<tr>
<td>z2</td>
<td>0.9410</td>
<td>0.0446</td>
<td>0.5944798</td>
<td>0.9978</td>
</tr>
<tr>
<td>pre1</td>
<td>4.543</td>
<td>2.058</td>
<td>0.070</td>
<td>8.533</td>
</tr>
<tr>
<td>pre2</td>
<td>75.525</td>
<td>14.622</td>
<td>54.706</td>
<td>100.44</td>
</tr>
<tr>
<td>inst1</td>
<td>11.289</td>
<td>1.298</td>
<td>8.501</td>
<td>21.293</td>
</tr>
<tr>
<td>inst2</td>
<td>0.0084</td>
<td>0.0071</td>
<td>-0.0356</td>
<td>0.0312</td>
</tr>
</tbody>
</table>

Notes: Variables are defined in Table 1. y, o1, o2, o3 and o4 are in US$. Number of observations equals 814,253 for all variables.
## Table 3
### Estimation results on the main determinants of risk

<table>
<thead>
<tr>
<th>Equation:</th>
<th>(1) Risk endogenous to z1</th>
<th>(2) Risk endogenous to z2</th>
<th>(3) Risk endogenous to z2</th>
<th>(4) Risk endogenous to z2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk equation</td>
<td>Log-linear</td>
<td>Log-linear</td>
<td>Translog</td>
<td>Translog</td>
</tr>
<tr>
<td>inst1</td>
<td>-0.036***</td>
<td>-0.036***</td>
<td>-0.189***</td>
<td>-0.151***</td>
</tr>
<tr>
<td>(15.98)</td>
<td>(-16.15)</td>
<td>(-74.95)</td>
<td>(-55.27)</td>
<td></td>
</tr>
<tr>
<td>lag of pre1</td>
<td>-0.028***</td>
<td>-0.029***</td>
<td>-0.130***</td>
<td>-0.121***</td>
</tr>
<tr>
<td>(13.09)</td>
<td>(-13.81)</td>
<td>(-46.11)</td>
<td>(-46.30)</td>
<td></td>
</tr>
<tr>
<td>lag of pre2</td>
<td>-0.042***</td>
<td>-0.043***</td>
<td>-0.187***</td>
<td>-0.172***</td>
</tr>
<tr>
<td>(19.40)</td>
<td>(-20.50)</td>
<td>(-46.37)</td>
<td>(-43.70)</td>
<td></td>
</tr>
<tr>
<td>z1</td>
<td>0.273***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(22.16)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z2</td>
<td></td>
<td>0.379***</td>
<td>0.278***</td>
<td>0.255***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(61.15)</td>
<td>(59.33)</td>
<td>(48.89)</td>
</tr>
</tbody>
</table>

Notes: The table reports estimation results (coefficients and t-statistics) for Equation (2) obtained from the joint estimation of equations (1), (2) and the equation on z, using maximum likelihood. We use 814,253 bank-quarter observations, covering the period 1985q1-2010q2. Variables are defined in Table 1. In all regressions, risk is endogenous to profits and to z1 or z2 as specified on the top of the table. In specifications (1) to (3) the endogenous variables z1 and z2 are identified using inst1 and lagged inst2. In specification (4) lagged pre1 and lagged pre2 also identify z2.

## Table 4
### Correlation matrix between indices of bank risk

<table>
<thead>
<tr>
<th></th>
<th>r1</th>
<th>r2</th>
<th>r3</th>
<th>r4</th>
<th>ra</th>
<th>z-index</th>
<th>llp</th>
<th>npl</th>
</tr>
</thead>
<tbody>
<tr>
<td>r1</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r2</td>
<td>0.900</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r3</td>
<td>0.559</td>
<td>0.671</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r4</td>
<td>0.594</td>
<td>0.715</td>
<td>0.993</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ra</td>
<td>0.031</td>
<td>0.025</td>
<td>0.015</td>
<td>0.017</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z-index</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.002</td>
<td>-0.001</td>
<td>0.146</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>llp</td>
<td>-0.027</td>
<td>-0.021</td>
<td>-0.012</td>
<td>-0.013</td>
<td>-0.070</td>
<td>0.043</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>npl</td>
<td>-0.054</td>
<td>-0.041</td>
<td>-0.026</td>
<td>-0.028</td>
<td>-0.095</td>
<td>-0.061</td>
<td>0.098</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Notes: The table presents simple correlation coefficients between the four indices of bank risk constructed using the equivalent specifications of Table 3 (denoted as r1 to r4) and the bank level values of the indices of bank risk shown in figure 2 (ra is risky assets to total assets, llp is loan loss provisions to total loans, npl is non-performing loans to total loans).
Figure 1
Evolution of bank risk (log of variance) over the period 1985q1-2010q2

Notes: The figure presents the quarterly average of the bank-quarter values of risk as obtained from the specifications (1)-(4) presented in Table 3.
Figure 2
Evolution of bank risk indices commonly found in the banking literature over the period 1985q1-2010q2

(b) Risky assets (risky assets/total assets)  
(c) Credit risk (loan loss provisions/total loans)

(d) Credit risk (problem loans/total loans)

(e) $z$-index = (ROA+EA)/σ(ROA)

Notes: For figures (a) industry average is (total industry risky assets at quarter t)/(total industry assets at quarter t). Average by bank is calculated as the ratio of average risky assets to assets for all banks at time t. The same definition of industry vs. bank average applies to all other measures.